Assessing the validity of the Born-Oppenheimer approximation in potential models for doubly heavy hadrons

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Outlines

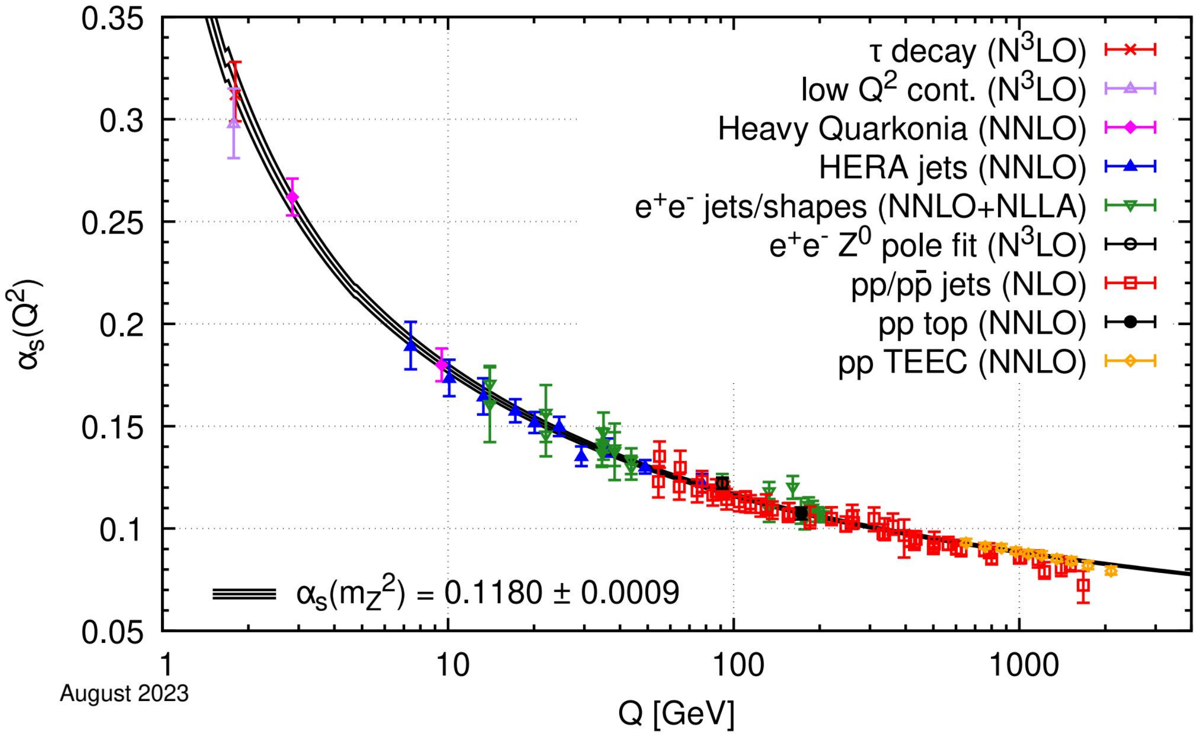
1. Background
2. Formalism
3. Results for doubly heavy systems
4. Summary

1

PART

Background

Hadron properties are difficult to derive directly from QCD



High energy: asymptotic freedom

Low energy: non-perturbative become important

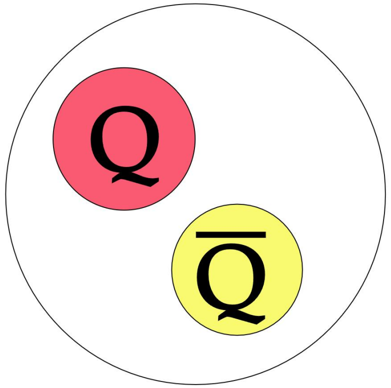
Hadron properties difficult to derive from QCD

Methods: Lattice QCD, Quark Model, Effective field theory et al.

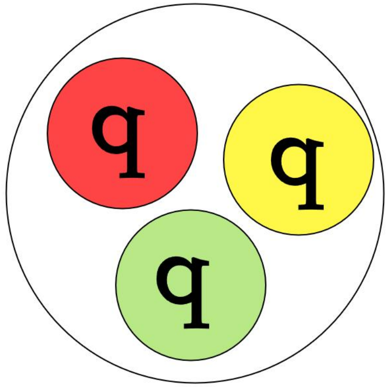
Particle Data Group, Phys. Rev. D 110, 030001 (2024)

Hadronic structure

meson

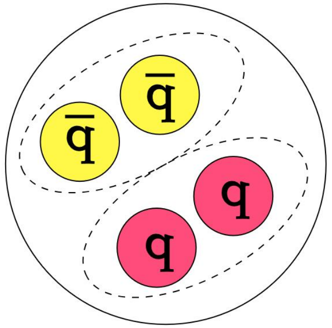


baryon

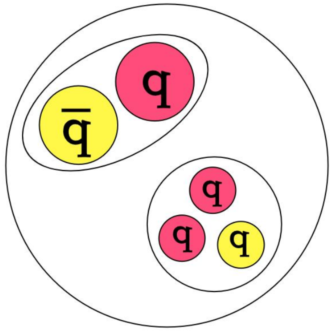


Traditional hadronic states

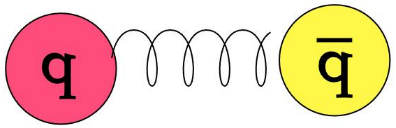
compact picture



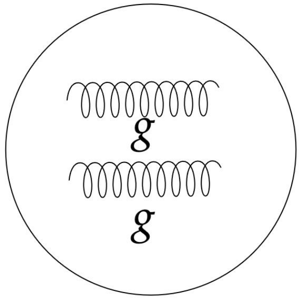
molecule picture



hybrid state



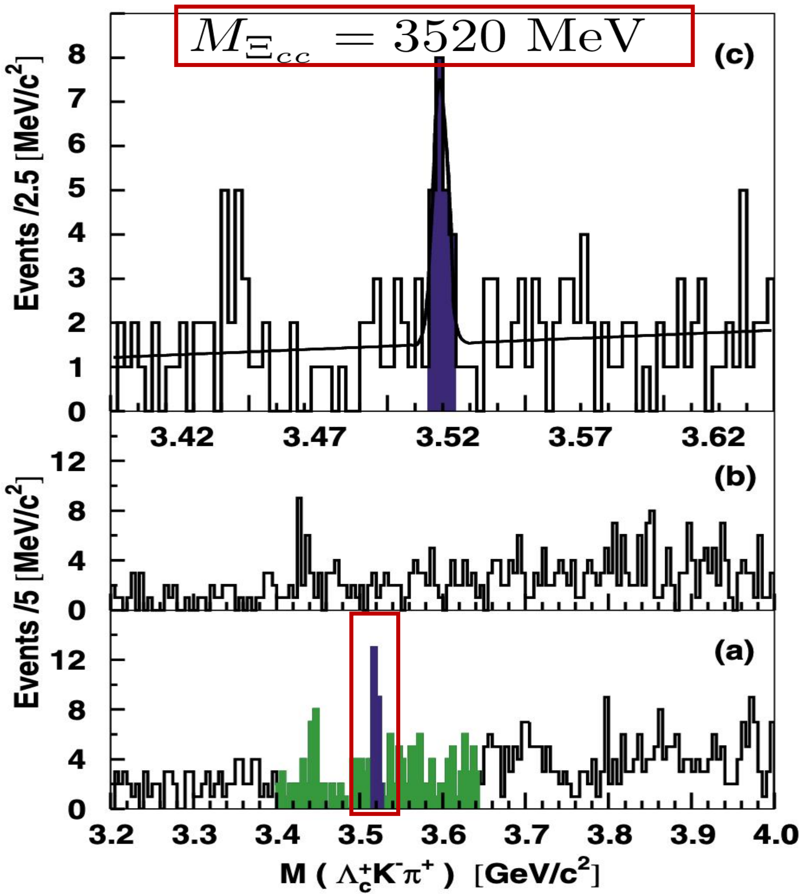
glueball



Exotic hadronic states

Short lifetime of Ξ_{cc}

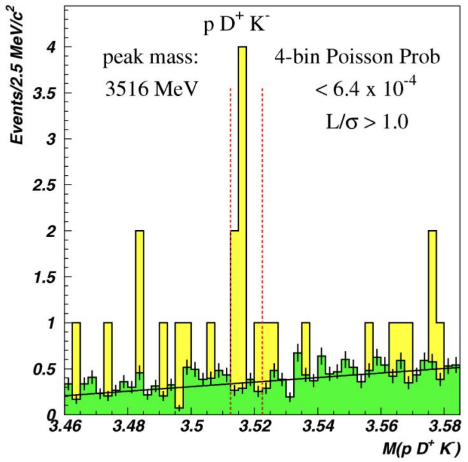
$\Lambda_c^+ K^- \pi^+$



$\tau(\Xi_{cc}^+) < 33$ fs

SELEX, Phys. Rev. Lett. 89,112001 (2002)

pD^+K^-



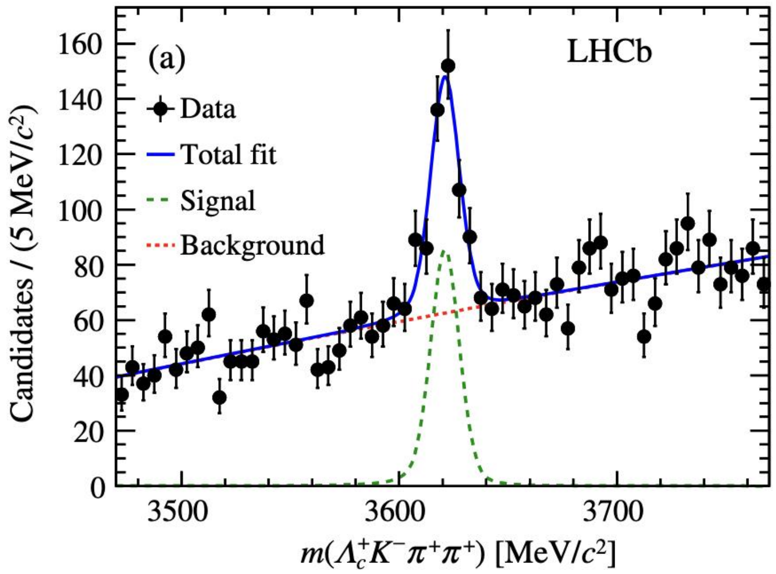
SELEX, Phys. Lett. B 628 (2005)

FOCUS,
Nucl. Phys. B Proc. Suppl. 115, 33 (2003)
BaBar, Phys. Rev. D 74, 011103 (2006)
Belle, Phys. Rev. Lett. 97, 162001 (2006)

not confirm

Longer lifetime of Ξ_{cc}

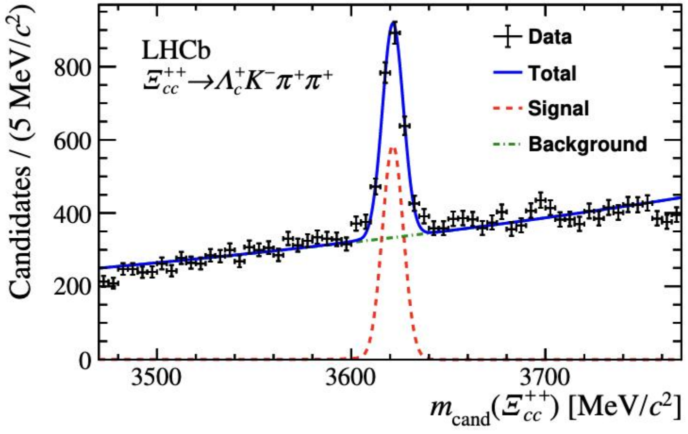
$\Lambda_c^+ K^- \pi^+ \pi^+$



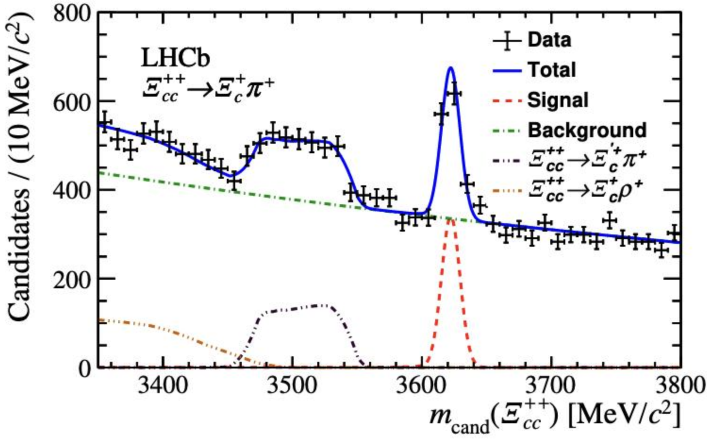
$$3621.40 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.}) \pm 0.14(\Lambda_c^+)\ \text{MeV}/c^2$$

$$\tau(\Xi_{cc}^{++}) = 0.256^{+0.024}_{-0.022}(\text{stat}) \pm 0.014(\text{syst})\ \text{ps}$$

$\Lambda_c^+ K^- \pi^+ \pi^+$



$\Xi_c^+ \pi^+$

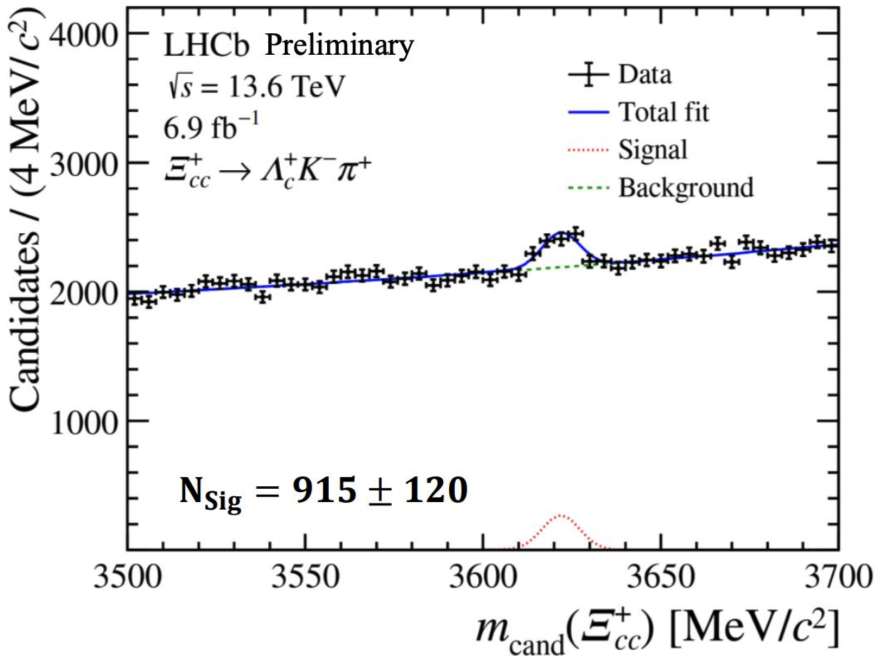


$$3621.55 \pm 0.23\,(\text{stat}) \pm 0.30\,(\text{syst})\ \ \text{MeV}/c^2.$$

LHCb, Phys. Rev. Lett. 119, 112001 (2017)
LHCb, Phys. Rev. Lett. 121, 052002 (2018)
LHCb, JHEP 02 (2020) 049

Isospin partner of Ξ_{cc}^{+}

$\Lambda_c^+ K^- \pi^+$



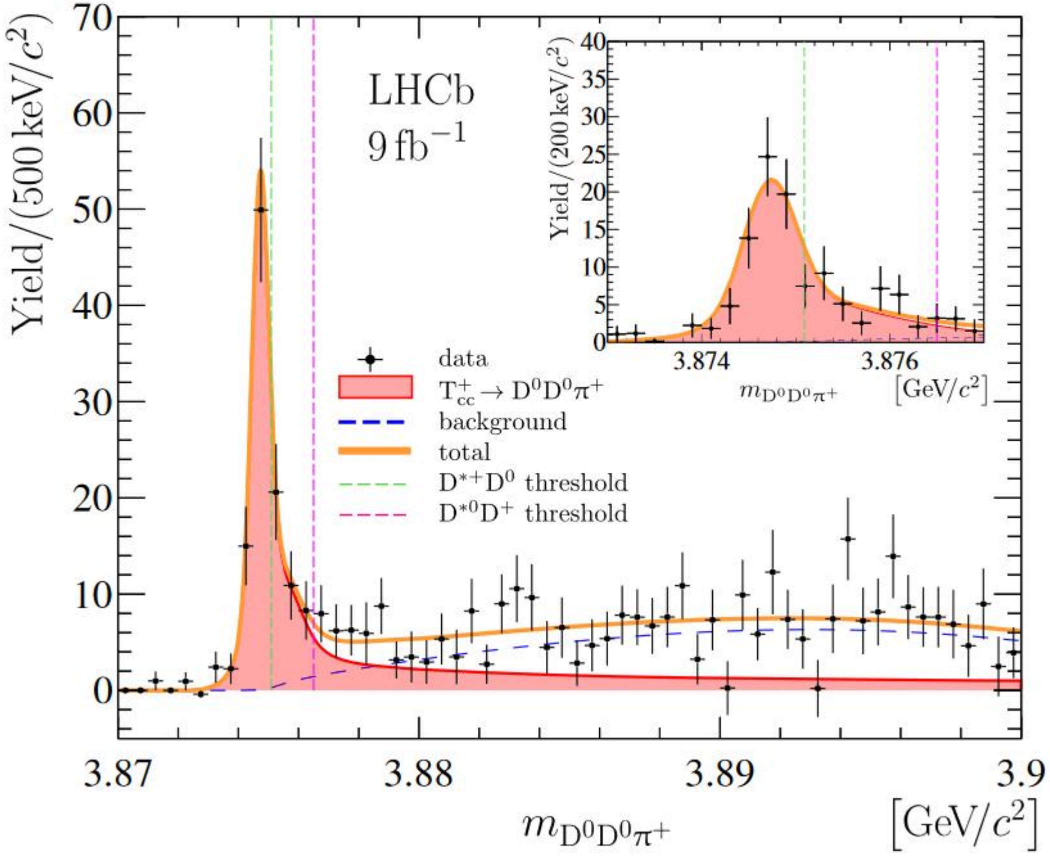
参考袁熙昊老师报告

$3619.97 \pm 0.83\,(\text{stat}) \pm 0.26\,(\text{syst})^{+1.90}_{-1.30}\,(\text{lifetime})\ \text{MeV}/c^2.$

LHCb, S. Y. Han, Moriond EM (2026)

T_{cc}^{+}

$$T_{cc}^{+} \to D^0 D^0 \pi^+$$



$J^P = 1^+ \quad cc\bar{u}\bar{d}$

Breit-Wigner parameterization

$$\delta m_{BW} = m_{T_{cc}} - m_{D^{*+}D^0}$$
$$= -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV}$$
$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}$$

LHCb, Nature Phys18, 751-754 (2022)

A unitarised three body Breit-Wigner function

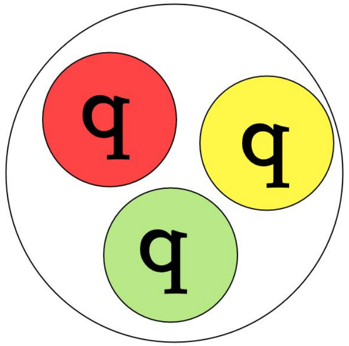
$$\delta m_{U} = m_{T_{cc}} - m_{D^{*+}D^0}$$
$$= -360 \pm 40^{+4}_{-0} \text{ keV}$$
$$\Gamma_{U} = 48 \pm 2^{+0}_{-14} \text{ keV}$$

LHCb, Nature Commun. 13, 3351 (2022)

Theoretical explanations

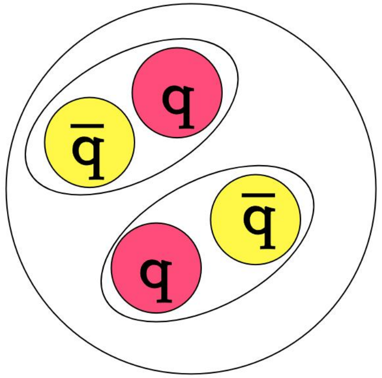
Ξ_{cc}

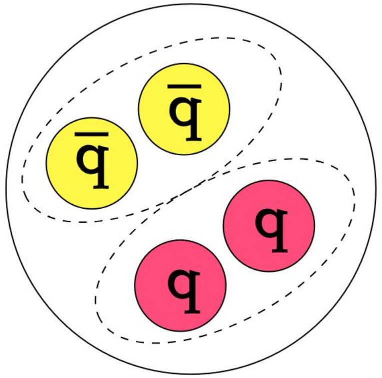
$$3_q \otimes 3_q \otimes 3_q \rightarrow 1_{1c}$$



T_{cc}^{+}

$$(3_q \otimes 3^*_{\bar{q}})_{1c} \otimes (3_q \otimes 3^*_{\bar{q}})_{1c} \rightarrow 1_{1c} \quad (6_{qq} \otimes 6^*_{\bar{q}\bar{q}})_{1c} \rightarrow 1_{1c}, (3_{\bar{q}\bar{q}} \otimes 3^*_{qq})_{1c} \rightarrow 1_{1c}$$





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Application of the B-O Approximation in Doubly Heavy Hadrons

QCD-based approaches: the calculate potential using lattice QCD and BOEFT

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E. Braaten, C. Langmack, and D. H. Smith, Phys. Rev. D 90, 014044 (2014)
P. Bicudo, J. Scheunert, and M. Wagner, Phys. Rev. D 95, 034502 (2017)
M. Berwein, N. Brambilla, A. Mohapatra, and A. Vairo, Phys. Rev. D 110, 094040 (2024)
E. Braaten and R. Bruschini, Phys. Lett. B 863, 139386 (2025).

Potential models:

L. Maiani, A. D. Polosa, and V. Riquer, Phys. Rev. D 100, 014002 (2019)
B. Grinstein, L. Maiani, and A. D. Polosa, Phys. Rev. D 109, 074009 (2024)
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Motivation for Applying the B-O Approximation to Hadrons

1927 № 20

ANNALEN DER PHYSIK

VIERTE FOLGE. BAND 84

1. ***Zur Quantentheorie der Molekeln;***
von M. Born und R. Oppenheimer

Es wird gezeigt, daß die bekannten Anteile der Terme einer Molekel, die der Energie der Elektronenbewegung, der Kernschwingungen und der Rotationen entsprechen, systematisch als die Glieder einer Potenzentwicklung nach der vierten Wurzel des Verhältnisses Elektronenmasse zu (mittlerer) Kernmasse gewonnen werden können. Das Verfahren liefert u. a. eine Gleichung für die Rotationen, die eine Verallgemeinerung des Ansatzes von Kramers und Pauli (Kreisel mit eingebautem Schwungrad) darstellt. Ferner ergibt sich eine Rechtfertigung der von Franck und Condon angestellten Betrachtungen über die Intensität von Bandenlinien. Die Verhältnisse werden am Beispiel der zweiatomigen Molekeln erläutert.

Einleitung

Die Terme der Molekelspektren setzen sich bekanntlich aus Anteilen verschiedener Größenordnung zusammen; der größte Beitrag rührt von der Elektronenbewegung um die Kerne her, dann folgt ein Beitrag der Kernschwingungen, endlich die von den Kernrotationen erzeugten Anteile. Der Grund für die Möglichkeit einer solchen Ordnung liegt offensichtlich in der Größe der Masse der Kerne, verglichen mit der der Elektronen. Vom Standpunkte der älteren Quantentheorie, die die stationären Zustände mit Hilfe der klassischen Mechanik berechnet, ist dieser Gedanke von Born und Heisenberg[1]) durchgeführt worden; es wurde gezeigt, daß die aufgezählten Energieanteile als die Glieder wachsender Ordnung hinsichtlich des Verhältnisses $\sqrt{\frac{m}{M}}$ erscheinen, wo m die Elektronenmasse, M eine mittlere Kernmasse ist. Dabei traten aber Kernschwingungen und Rotationen in der gleichen (zweiten) Ordnung auf, was dem empirischen Befund (bei kleinen Rotationsquantenzahlen) widerspricht.

1) M. Born u. W. Heisenberg, Ann. d. Phys. 74. S. 1. 1924.

Annalen der Physik. IV. Folge. 84. 30

$$m_Q \gg m_q$$

- How reliable is the BOA in potential models for doubly heavy hadrons?

$$m_p/m_e \sim 1836 \quad m_c/m_q \sim 5$$

- Which basis functions are more suitable for describing hadronic systems:Slater-Type Functions (STFs) or Gaussian-Type Functions (GTFs)?

STFs: e^{-Ar} GTFs:e^{-Ar^2}

- Does the confinement term and the attractive interaction between two heavy quarks modify the applicability of the BOA in hadrons?

$$V(r_{AB}) = -\frac{2\alpha_s}{3r_{AB}} + \frac{1}{2} b r_{AB}$$

2

PART

Formalism

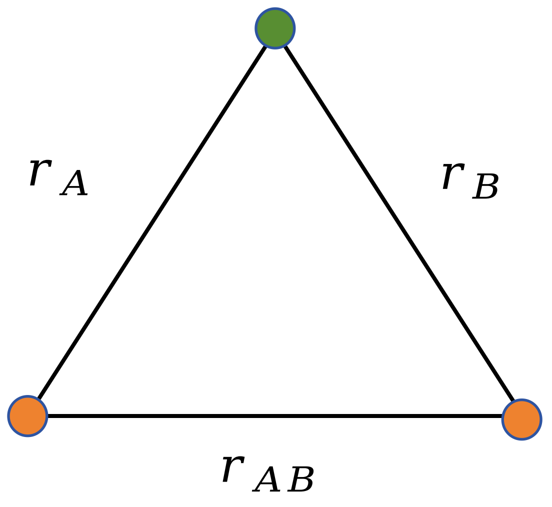
The BOA Hamiltonian

General Hamiltonian

$$H = \sum_i \frac{p_i^2}{2m_i} + \sum_{i<j} (V_{ij}^{\rm conf} + V_{ij}^{\rm hyp}),$$

$$V_{ij}^{\rm conf} = -\left[-\frac{\alpha_s}{r_{ij}} + \frac{3}{4} b r_{ij} + \frac{3}{4} C\right] \boldsymbol{F}_i \cdot \boldsymbol{F}_j, \qquad V_{ij}^{\rm hyp} = \left[\frac{8\pi\alpha_s}{3m_i m_j} \tilde{\delta}(r_{ij}) \boldsymbol{S}_i \cdot \boldsymbol{S}_j\right] \boldsymbol{F}_i \cdot \boldsymbol{F}_j,$$

BOA Hamiltonian for baryon



$$H_{\rm BOA} = H_{heavy} + H_{light}.$$

$$H_{heavy} = T_{heavy} + V(r_{AB}). \qquad H_{light} = T_{light} + V_l.$$

$$V(r_{AB}) = -\frac{2\alpha_s}{3r_{AB}} + \frac{1}{2} b r_{AB} + \frac{1}{2} C. \qquad V_l = -\frac{2\alpha_s}{3r_A} - \frac{2\alpha_s}{3r_B} + \frac{1}{2} b r_A + \frac{1}{2} b r_B + C.$$

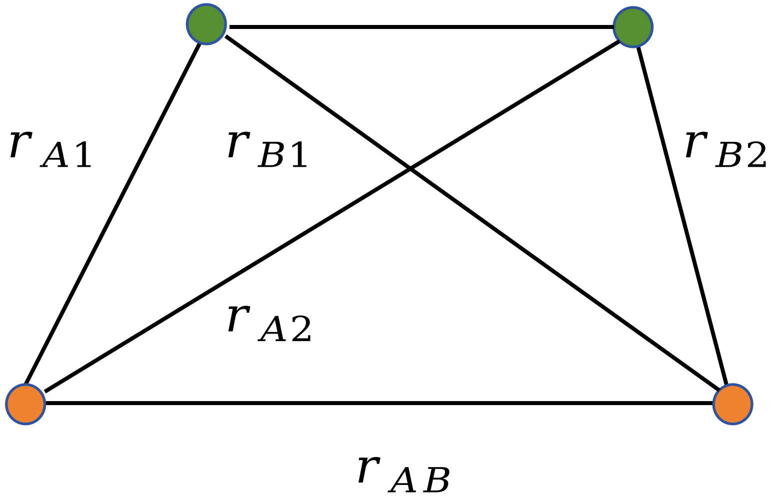
$$\left(T_{light} + V_l\right)\Phi = \varepsilon(r_{AB})\Phi.$$

$$H_{BOA} = -\frac{\nabla^2}{2m_Q} - \frac{2\alpha_s}{3r_{AB}} + \frac{1}{2} b r_{AB} + \frac{1}{2} C + \varepsilon(r_{AB})$$

$$H_{BOA} f(r_{AB}) = E_{BOA} f(r_{AB}),$$

Double heavy systems BOA

BOA Hamiltonian for tetraquark



$$V(r_{AB}) = -\frac{2\alpha_s}{3r_{AB}} + \frac{1}{2}br_{AB} + \frac{1}{2}C$$

$$V_l = -\frac{1\alpha_s}{3r_{A1}} - \frac{1\alpha_s}{3r_{B2}} - \frac{1\alpha_s}{3r_{A2}} - \frac{1\alpha_s}{3r_{B1}} - \frac{2\alpha_s}{3r_l} + \frac{1}{4}br_{A1} + \frac{1}{4}br_{B2} + \frac{1}{4}br_{A2} + \frac{1}{4}br_{B1} + C.$$

$$H_{BOA}f(r_{AB}) = E_{BOA}f(r_{AB}),$$

$$M = \sum m_i + E_{BOA} + \langle H_{CMI} \rangle.$$

$$H_{CMI} = -\sum_{i<j} C_{ij}\lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j$$

The choice of basis functions

STFs: $$R(r) = \frac{A^{3/2}}{\sqrt{\pi}} e^{-Ar}.$$

GTFs: $$\phi_n^G(\mathbf{r}) = \frac{1}{4\pi} N_n e^{-\nu_n r^2}, \quad N_n = \left(\frac{4(2\nu_n)^{3/2}}{\sqrt{\pi}} \right)^{1/2}$$

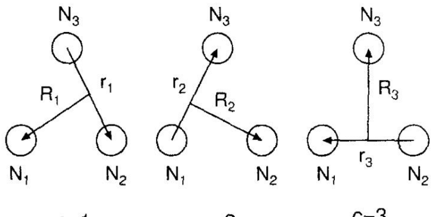
Gaussian Expansion Method as a Benchmark for the BOA

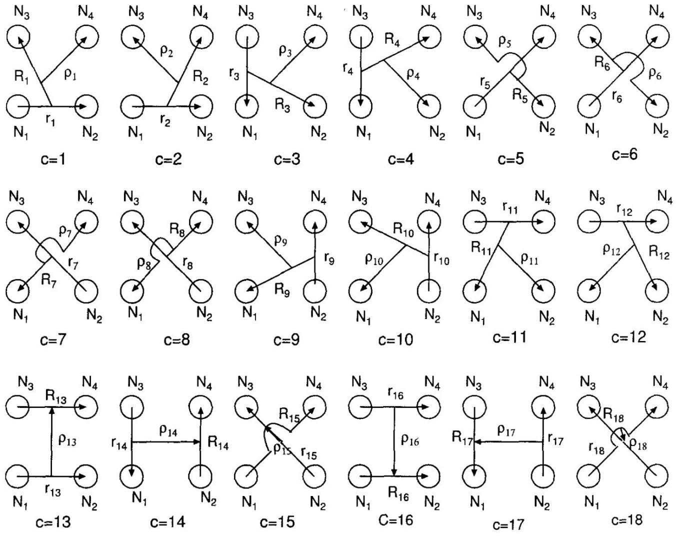
GEM provides highly reliable solutions to the full multi–body Schrödinger equation without assuming a separation of scales

S.-Q. Luo and X. Liu, Phys. Rev. D 108, 034002 (2023)
H. Zhou, S.-Q. Luo, and X. Liu, Phys. Rev. D 112, 074007 (2025)
Z.-L. Zhang and S.-Q. Luo, Phys. Rev. D 112, 074020 (2025)
S.-Q. Luo and X. Liu, Phys. Rev. D 112, 014047 (2025)
H.-T. An, S.-Q. Luo, and X. Liu, Phys. Rev. D 112, 054041 (2025)
Y.-X. Peng, S.-Q. Luo, and X. Liu, Phys. Rev. D 110, 074034 (2024)

States	$M^{\text{The.}}$ (MeV)	$M^{\text{Exp.}}$ (MeV)	$M^{\text{Err.}}$ (MeV)
$D^{\pm}$	1864.4	1869.5	0.4
$D^{*\pm}$	2010.2	2010.26	0.05
$D_s^{\pm}$	1968.5	1968.35	0.07
$D_s^{*\pm}$	2101.2	2106.6	3.4
$B^{\pm}$	5282.5	5279.41	0.07
B^{*}	5300.5	5324.75	0.20
B_s^0	5366.8	5366.91	0.11
B_s^*	5385.8	5415.4	1.4
	$\chi^2/n = 2155.7$		

Jacobian coordinates





		GEM	BOA
Quark masses	m_n(GeV)	0.3971	same
	m_s(MeV)	576.1	same
	m_c(MeV)	1598.8	same
	m_b(MeV)	4893.1	same
	m_t(MeV)	172560	same
Confinement	α_s		same
	b (GeV^2)	0.15	same
	C (MeV)	-606.9	same
Hyperfine	σ (MeV)	3772.2	—
	C_{nn}(MeV)	—	18.3
	C_{cc}(MeV)	—	3.5
	C_{cn}(MeV)	—	4.0
	C_{cs}(MeV)	—	4.3
	C_{bb}(MeV)	—	1.9
	C_{bn}(MeV)	—	1.3
	C_{bs}(MeV)	—	1.3
	$C_{c\bar{n}}$(MeV)	—	6.6
	$C_{b\bar{n}}$(MeV)	—	2.1
	C_{ss} (MeV)	—	6.5
	$C_{c\bar{s}}$(MeV)	—	6.7
	$C_{b\bar{s}}$(MeV)	—	2.3

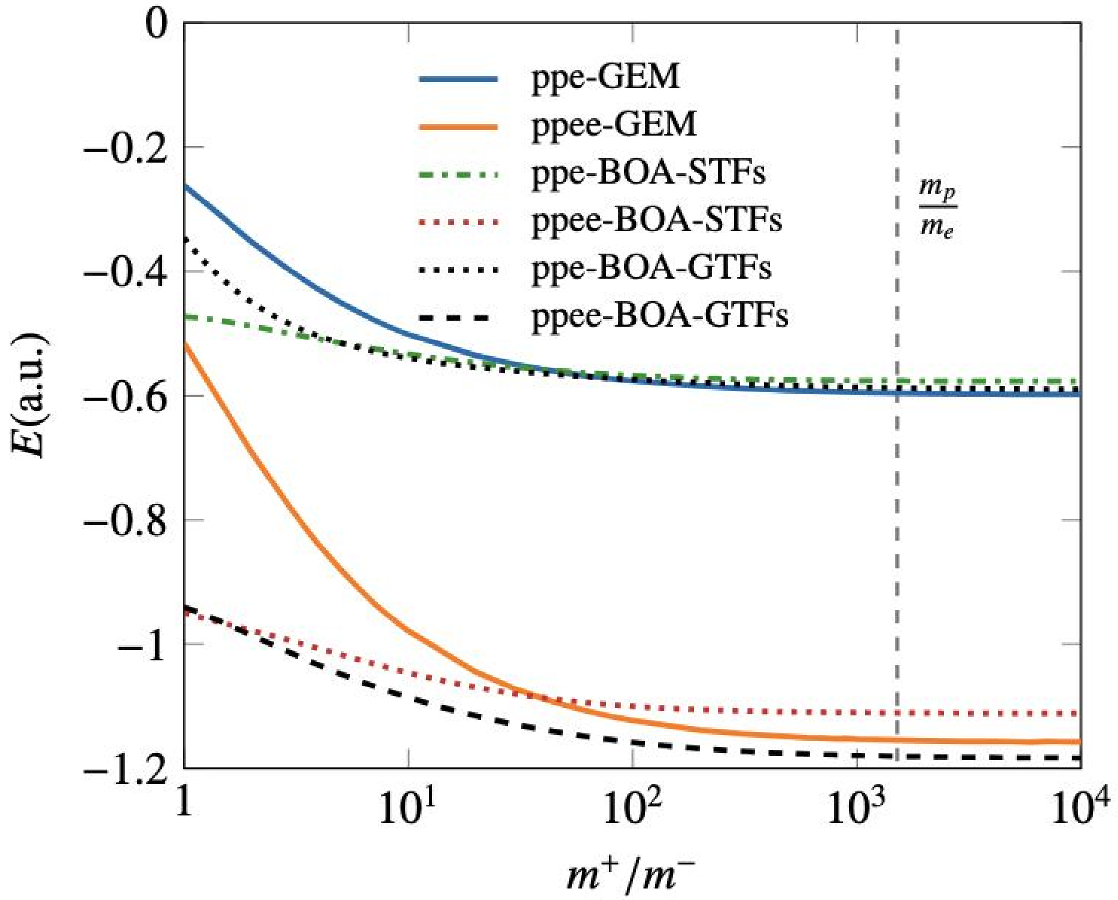
E. Hiyama, Y. Kino, and M. Kamimura, Prog. Part. Nucl. Phys. 51, 223 (2003).

3

PART

Results doubly heavy systems

Hydrogen molecular ion and hydrogen molecule

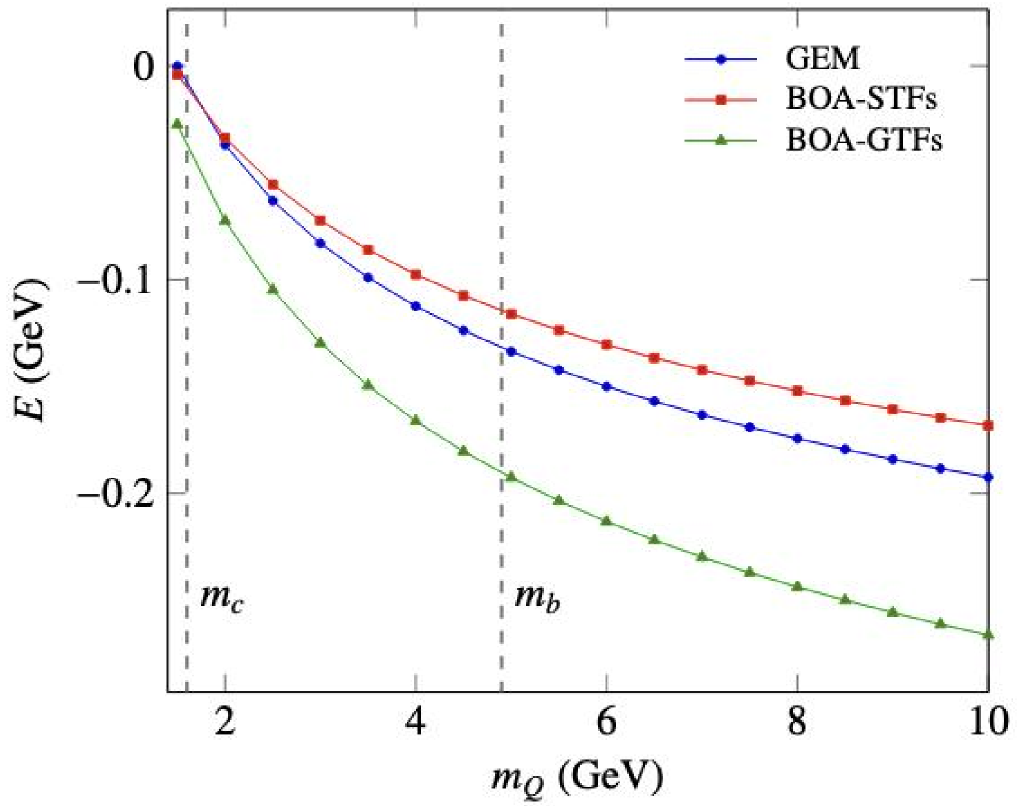


$$H = \sum_i \frac{p_i^2}{2m_i/m_e} + \sum_{i<j} \frac{1}{r_{ij}}$$

Hydrogen molecular ion

Hydrogen molecule

The reliability of the B–O approximation is governed by the mass ratio between the heavy and light constituent.



- Small heavy-quark masses：

  GEM～BOA-STFs～BOA-GTFs

- Large heavy-quark masses:

  BOA-STFs>GEM>BOA-GTFs

GEM>BOA

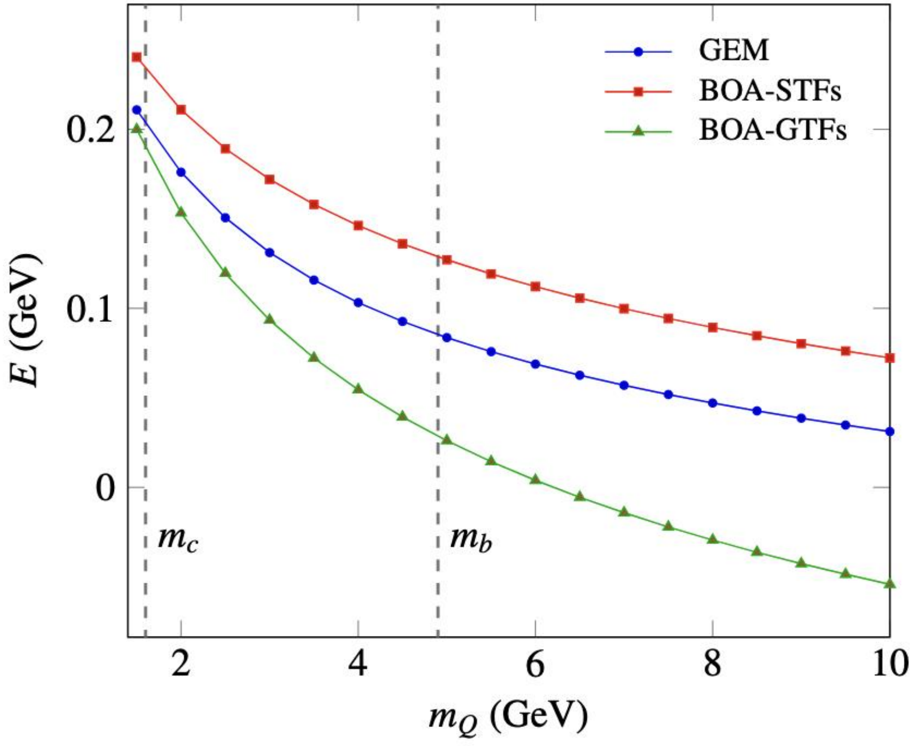
The underestimation in the BOA–GTFs results primarily originates from the neglect of non–adiabatic contributions.

T. K. Das, H. T. Coelho, and V. P. Brito, Phys. Rev. C 48, 2201 (1993)

Doubly heavy baryon

States	BOA-STFs		BOA-GTFs		GEM	
	E_{BOA}	Mass	E_{BOA}	Mass	E	Mass
Ξ_{cc}	-10.9	3550.5	-37.8	3525.5	-7.9	3566.4
Ξ_{cc}^*	-10.9	3614.5	-37.8	3587.4	-7.9	3600.3
Ξ_{bb}	-114.4	10060.1	-190.2	9984.4	-131.4	10046.5
Ξ_{bb}^*	-114.4	10080.9	-190.2	10005.1	-131.4	10055.9
Ω_{cc}	-94.4	3642.8	-134.0	3603.2	-95.0	3660.1
Ω_{cc}^*	-94.4	3711.6	-134.0	3672.0	-95.0	3691.8
Ω_{bb}	-202.3	10151.2	-292.6	10060.9	-223.7	10133.3
Ω_{bb}^*	-202.3	10172.0	-292.6	10081.7	-223.7	10142.6

$cc\overline{nn}$:QQ in color $\overline{\mathbf{3}}$



- Small heavy-quark masses：

  GEM～BOA-STFs～BOA-GTFs

- Large heavy-quark masses:

  BOA-STFs>GEM>BOA-GTFs

GTFs perform better than STFs, while simplified STFs tend to overestimate the binding energy due to their poor description of the long-range confining potential.

Doubly heavy tetraquark

$I(J^P)$	BOA-STFs		BOA-GTFs		GEM	
	E_{BOA}	Mass	E_{BOA}	Mass	E	Mass
			$cc\bar{n}\bar{n}$ system			
$0(1^+)$	233.7	4088.4	188.5	4043.2	195.8	4051.1
$1(0^+)$	233.7	4213.2	188.5	4168.0	195.8	4184.7
$1(1^+)$	233.7	4248.4	188.5	4203.2	195.8	4193.3
$1(2^+)$	233.7	4318.8	188.5	4273.6	195.8	4208.8
			$bb\bar{n}\bar{n}$ system			
$0(1^+)$	128.9	10568.0	27.7	10466.7	84.4	10615.4
$1(0^+)$	128.9	10740.8	27.7	10639.5	84.4	10668.7
$1(1^+)$	128.9	10752.0	27.7	10650.7	84.4	10670.9
$1(2^+)$	128.9	10774.4	27.7	10673.1	84.4	10675.1

J^P	BOA-STFs		BOA-GTFs		GEM	
	E_{BOA}	Mass	E_{BOA}	Mass	E	Mass
			$cc\bar{s}\bar{s}$ system			
0^+	61.6	4366.6	18.3	4323.3	38.6	4384.1
1^+	61.6	4402.4	18.3	4359.1	38.6	4392.3
2^+	61.6	4473.8	18.3	4430.5	38.6	4407.0
			$bb\bar{s}\bar{s}$ system			
0^+	-38.4	10897.9	-136.8	10799.5	-76.2	10864.3
1^+	-38.4	10910.1	-136.8	10811.8	-76.2	10866.5
2^+	-38.4	10934.7	-136.8	10836.3	-76.2	10870.8

Summary

- How reliable is the BOA in potential models for doubly heavy hadrons?

  For doubly heavy systems with charm quarks, BOA can provide reasonable estimates at the qualitative level.

- Which basis functions are more suitable for describing hadronic systems: STFs or GTFs?

  GTFs perform better than STFs, mainly because that STFs are not well suited to describe the long-range behavior induced by the confining interaction.

- Does the confinement term and the attractive interaction between two heavy quarks modify the applicability of the BOA in hadrons?

  The confinement term and the attractive interaction between two heavy quarks lead to similar binding energies when the heavy-quark mass is small.

Thank you for your attention !