Coupled-channel description for five near-threshold structures: $\psi(3770)$, $G(3900)$, $R(3760)$, $R(3780)$ and $R(3810)$ from e^+e^- annihilation

Ri-Qing Qian
(钱日庆)
Lanzhou University

Phys. Rev. D 112, L091502 (2025)

Collaborator: Xiang Liu

第五届强子与重味物理理论与实验联合研讨会
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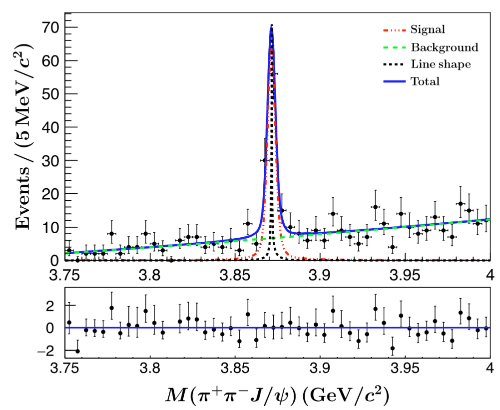
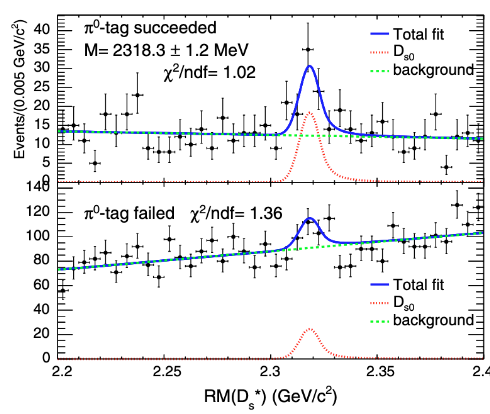
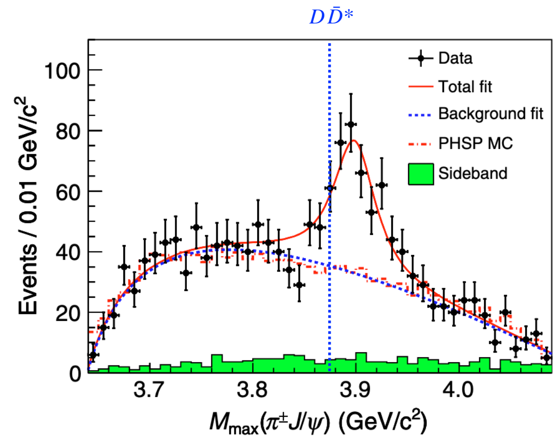
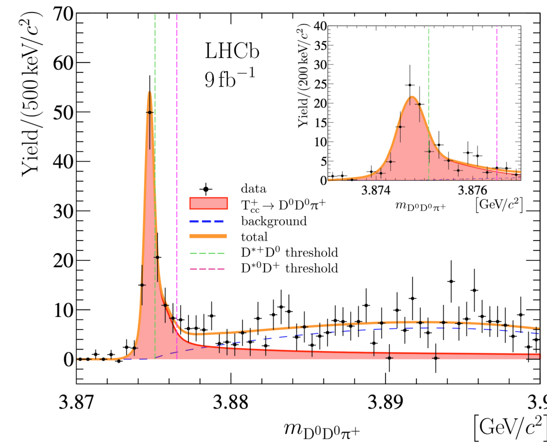
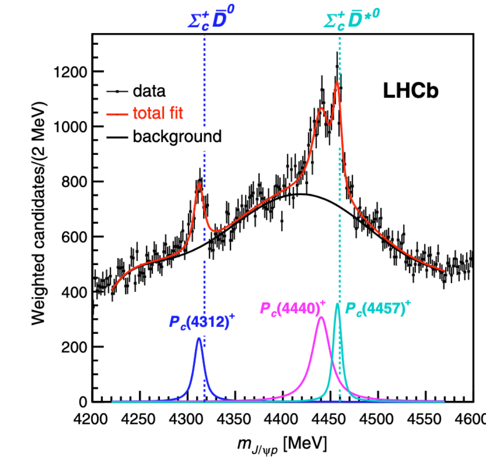
New hadron state & hadron-hadron threshold

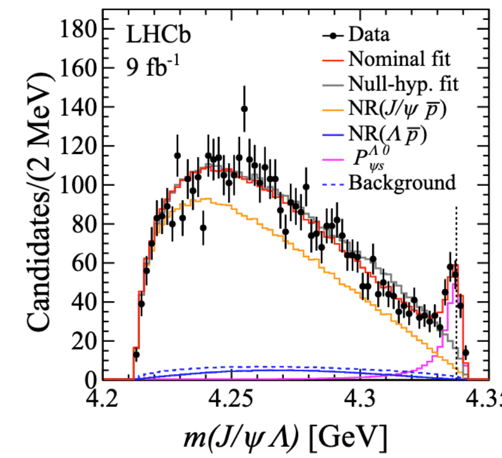












- Meson sector

$X(3872)$: $D\bar{D}^*$ threshold

$D_{s0}(2317)$: DK threshold

- Four quark sector

$Z_c(3900)$: $D\bar{D}^*$ thresholds

T_{cc}: DD^* threshold

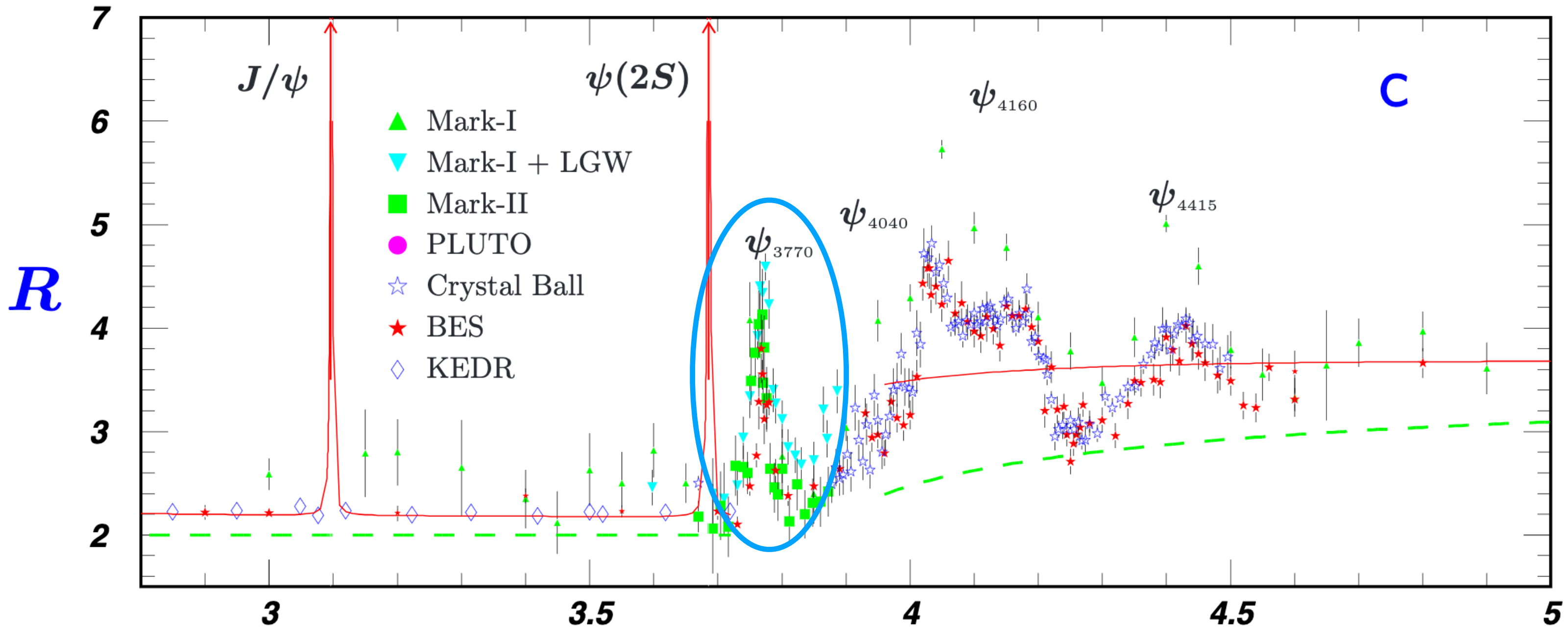
- Five quark sector

P_c states: $\Sigma_c\bar{D}^{(*)}$ threshold

P_{cs} : $\Xi_c^+ D^-$ threshold

Hadronic molecular state, cusp effect, coupled channel effect
…

$\psi(3770)$: the first $c\bar{c}$ above open charm threshold



- Identified as $\psi(1D)$ state after its observation in 1977 @SLAC

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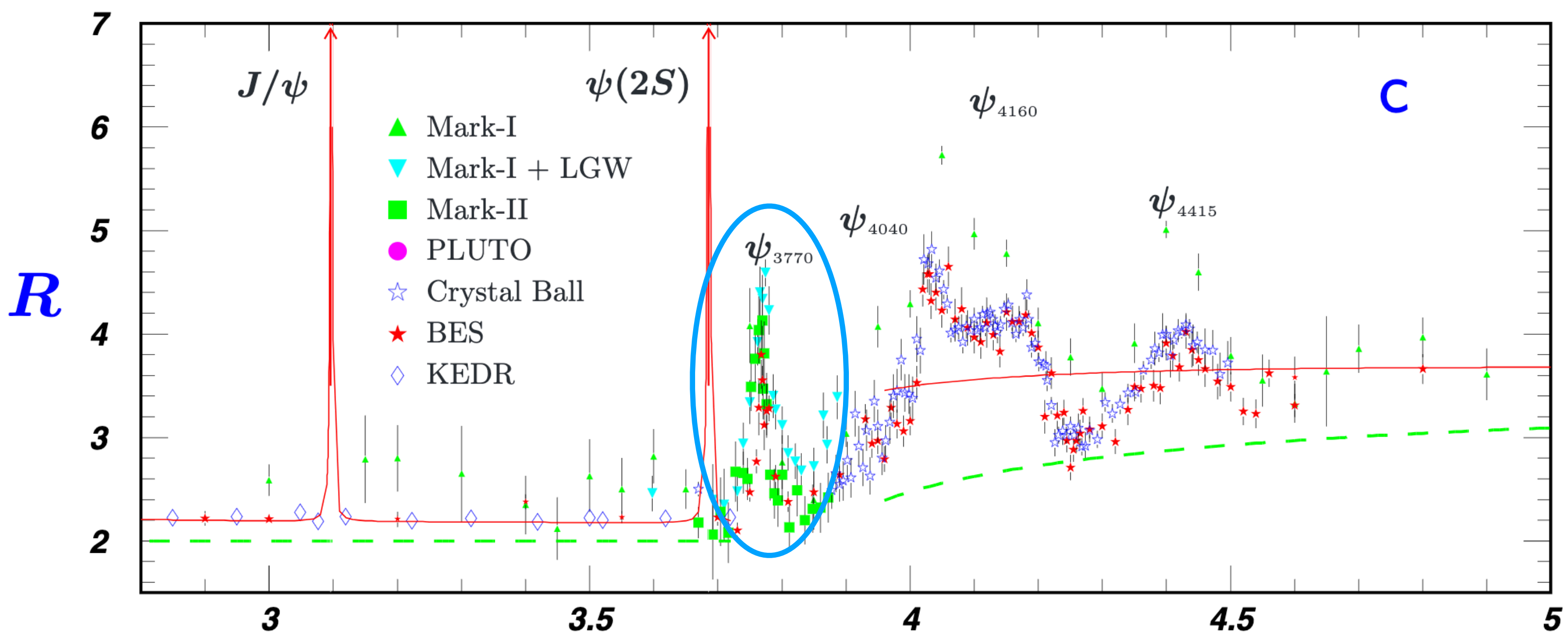


- Identified as $\psi(1D)$ state after its observation in 1977 @SLAC

- $\psi(1D)$ mass in the quark model
  - Conventional potential model:
    - E. Eitchten, et al — 3.81 GeV
    - Godfrey & Isgur — 3.82 GeV
    - T. Barnes, et al — 3.785 GeV
  - Screened potential model:
    - Li & Zhao, — 3.787 GeV
    - Jun-Zhang Wang, et al — 3.83 GeV

In most quark model calculation: $m_{\psi(1D)} \sim 3.8$ GeV

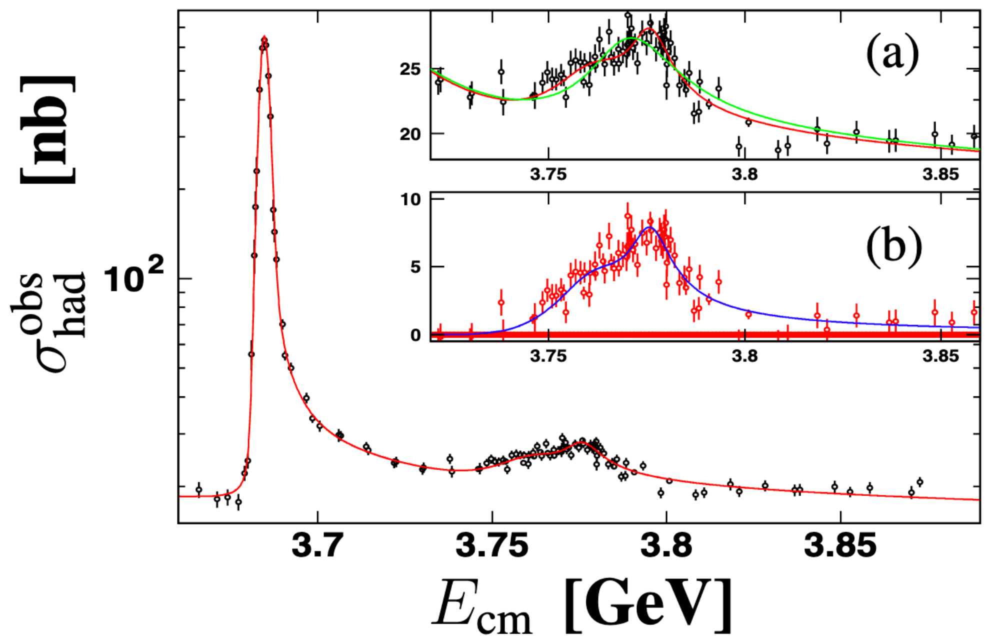
- Large non-$D\bar{D}$ decay $\sim 15\,\%$

$$\Gamma[\psi(3770) \to \text{non} - D\bar{D}] \sim 4 \text{ MeV}$$

In sharp contrast to $\psi(3686)$:
$\Gamma[\psi(3686) \to \text{hadrons}] \sim 0$

non-$D\bar{D}$ decay of $\psi(3770)$

BESIII Collaboration, PRL 101, 102004 (2008)



- Deviation from a Breit-Wigner distribution
  - initial state radiation (ISR)
  - the $D\bar{D}$ production threshold

➡ the slope at the high-energy side less steep

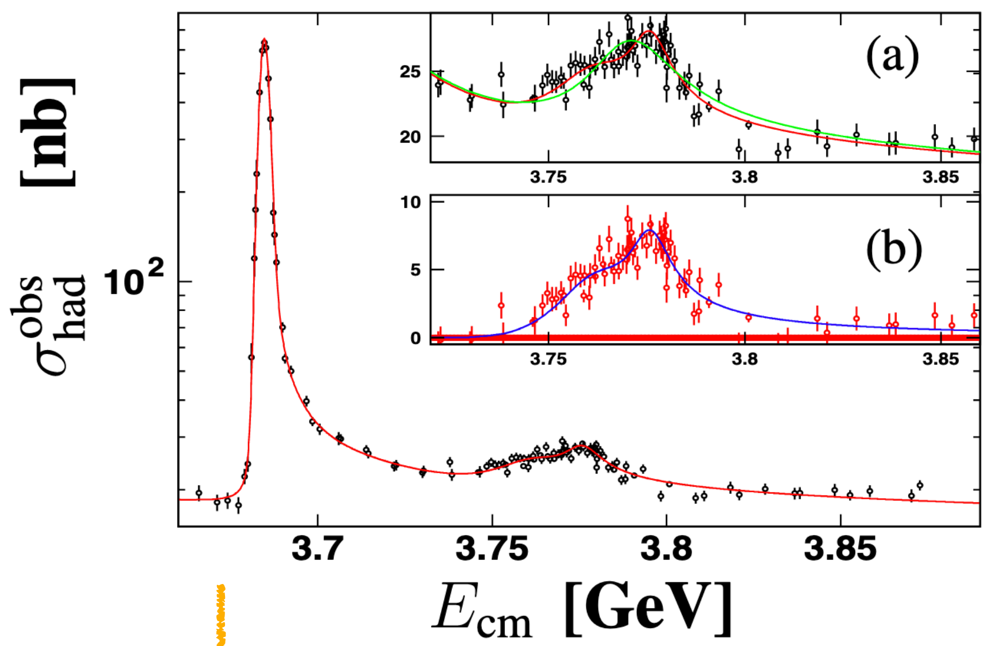
there exist two resonances ?

one dominantly decay to $D\bar{D}$

the other dominantly decay to non-$D\bar{D}$

non-$D\bar{D}$ decay $\rightarrow$ $R(3760), R(3780), R(3810)$

BESIII Collaboration, PRL 101, 102004 (2008)



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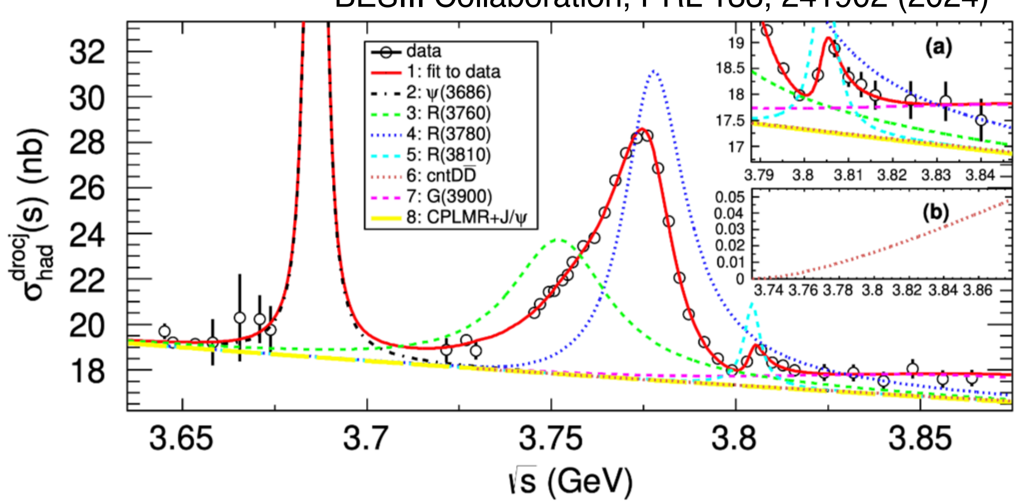
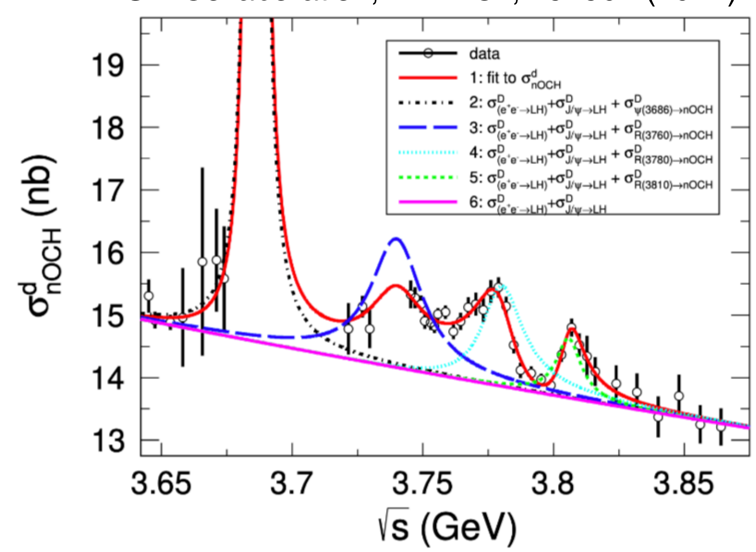
BESIII Collaboration, PRL 133, 241902 (2024)

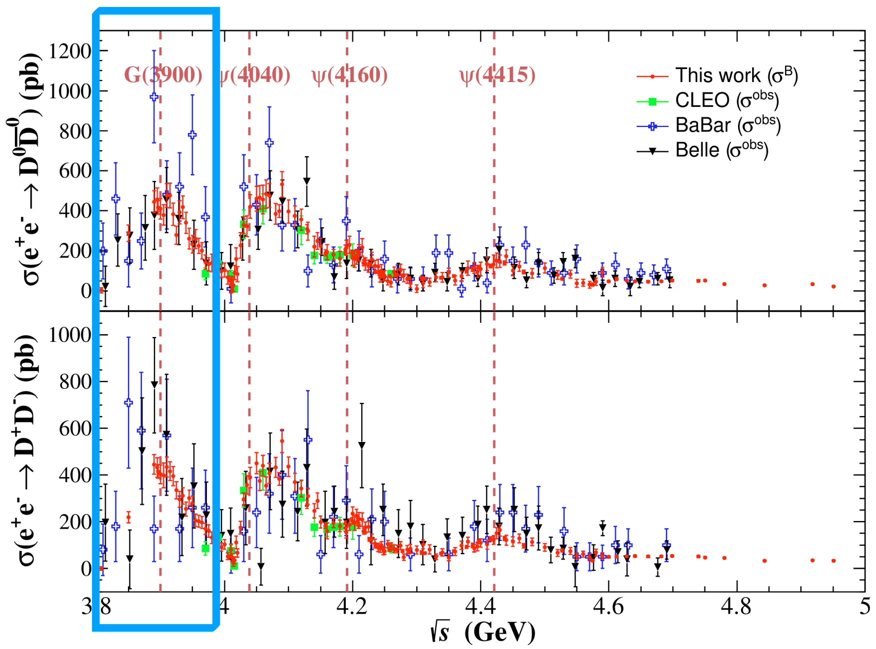


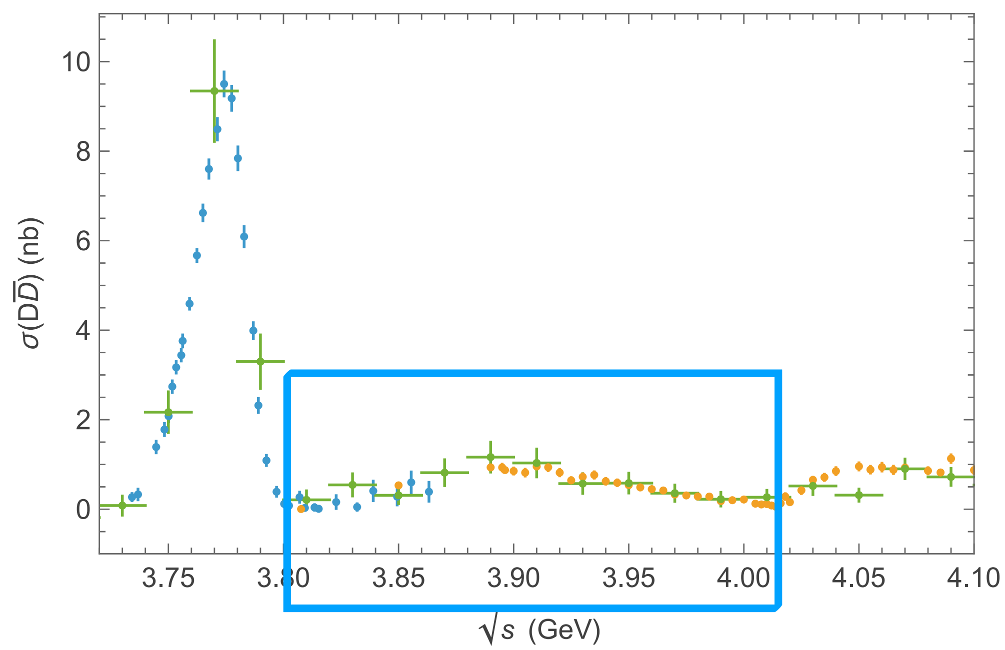
BESIII Collaboration, PRL 132, 191902 (2024)



- $R(3760)$: near the $D\bar{D}$ threshold & apparent at nOCH channel
- $R(3780)$: the previous $\psi(3770)$
- $R(3810)$: just above the $h_c\pi\pi$ threshold & appear in nOCH channel

$D\bar{D}$ channel: $G(3900)$





- P-wave $D\bar{D}^*/D^*\bar{D}$ resonance
  - Z.-Y. Lin, J.-Z. Wang, J.-B. Cheng, L. Meng, and S.-L. Zhu, Phys. Rev. Lett. 133, 241903 (2024)

- Coupled channel fits

  Pole find near 3.9 GeV

  - M.-L. Du, U.-G. Meißner, and Q. Wang, PRD 94, 096006 (2016)
  - Q. Ye, Z. Zhang, M.-L. Du, U.-G. Meißner, P.-Y. Niu, and Q. Wang, PRD 112, 016015 (2025)
  - S. X. Nakamura, X. H. Li, H. P. Peng, Z. T. Sun, and X. R. Zhou, arXiv:2312.17658 [hep-ph].

  No poles near 3.9 GeV

  - T. V. Uglov, Y. S. Kalashnikova, A. V. Nefediev, G. V. Pakhlova, and P. N. Pakhlov, JETP Lett. 105, 1(2017)
  - N. H¨usken, R. F. Lebed, R. E. Mitchell, E. S. Swanson, Y.-Q. Wang, and C.-Z. Yuan, Phys. Rev. D 109, 114010 (2024)

A short summery

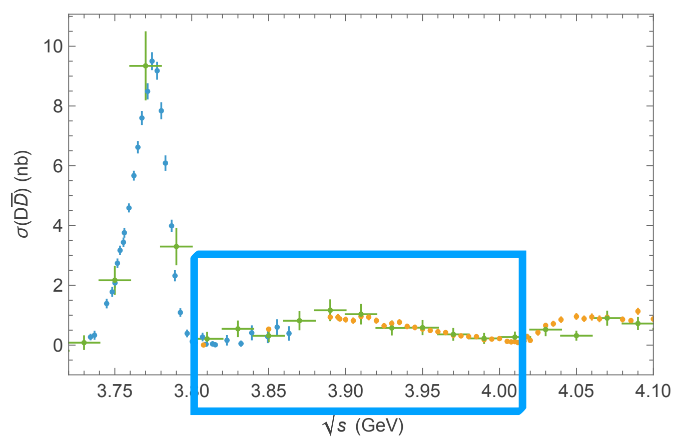
$\psi(3770)$ as $^3D_1$ charmonium

- Mass in potential model > experimental value

  Coupled channel effect

- Large non-$D\bar{D}$ decay width

$G(3900)$ in open charm channel

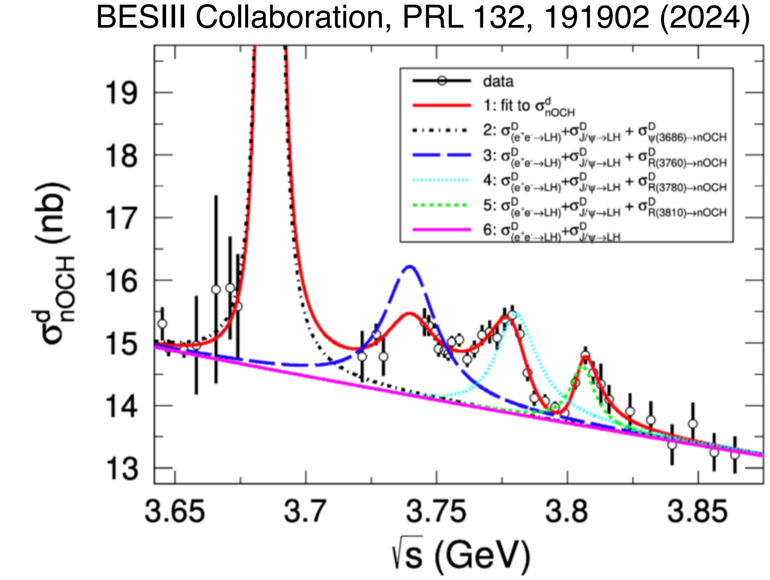


Coupled channel effect ?

$D\bar{D}^*$ interaction?

$R(3760)$, $R(3780)$, $R(3810)$ in non-open charm channel

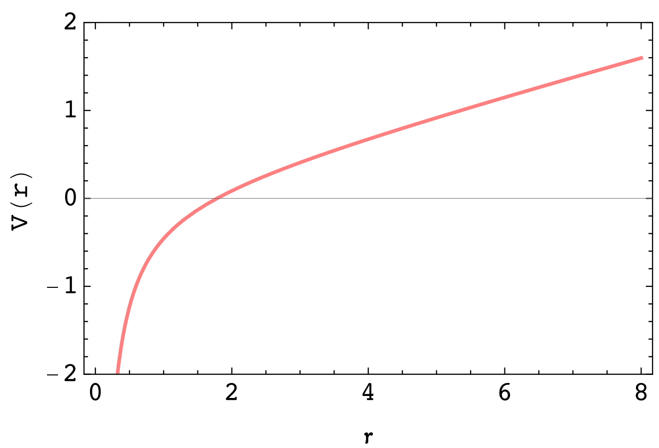
BESIII Collaboration, PRL 132, 191902 (2024)



- Coupled-channel framework
  to open charm channels

Potential model of meson

Potential



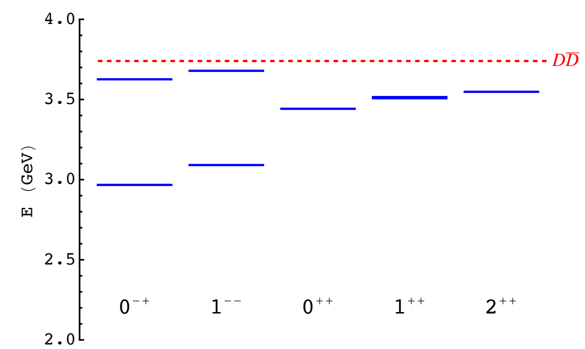
$$H_{c\bar{c}} = 2m_c + \frac{p^2}{2\mu} + V_{c\bar{c}}(r)$$

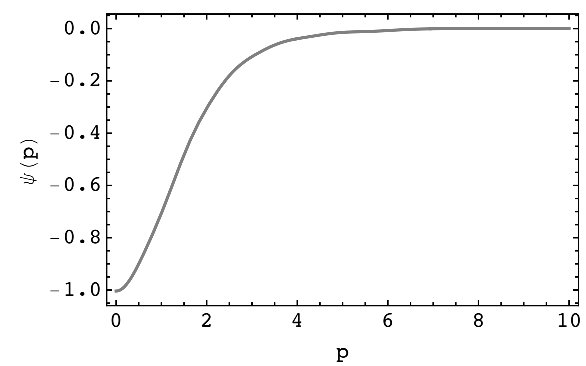
Phenomenological potential $V_{c\bar{c}}$

Spin independent part: $V(r) = \frac{\alpha}{r} + br$

Spin dependent part: $\hat{S}_c \cdot \hat{S}_{\bar{c}}, \hat{L} \cdot \hat{S}_i, \hat{S}_{12}$

Mass spectrum



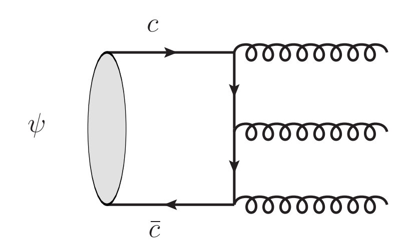


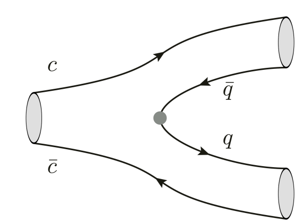
State labeled by $|^{2S+1}L_J\rangle$

Quantum number

$$P = (-1)^L + 1$$
$$C = (-1)^{L+S}$$

Decay model





Radiative decay

Annihilation decay

Open charm decay

Work good for $c\bar{c}$ below open charm threshold

The coupled channel effect

$H_{c\bar{c}}$: the bare $c\bar{c}$ spectrum, bound state of $c\bar{c}$

$$H_{c\bar{c}}|\psi_n\rangle = E_n|\psi_n\rangle$$

$H_{c\bar{c}\leftrightarrow hadrons}$: the strong decay of $c\bar{c}$

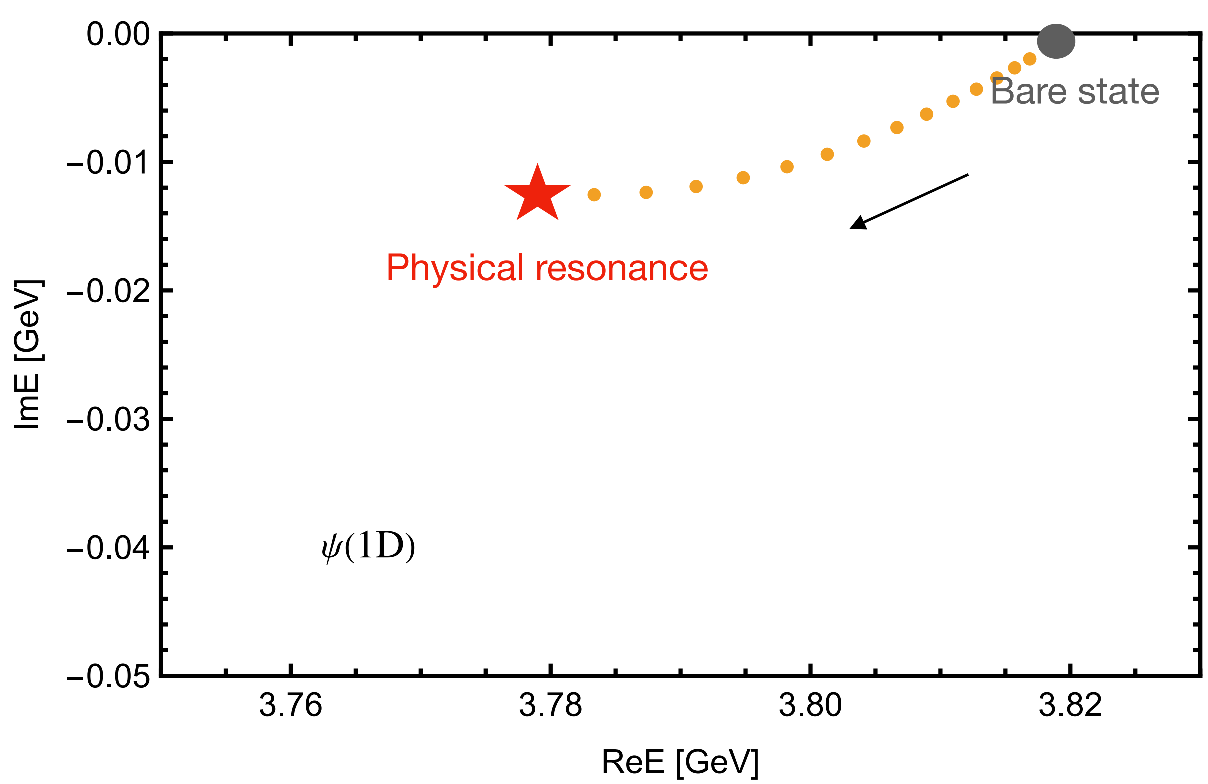
$$\langle\psi_n|H_{c\bar{c}\leftrightarrow hadrons}|i;\boldsymbol{p}\rangle = f_{n,i}(\boldsymbol{p})$$

Strong decay width comparable to energy level spacing

$$H = H_{c\bar{c}} + H_{c\bar{c}\leftrightarrow hadrons} + H_{hadrons}$$

Physical state as superposition of bare state and continuum state:

$$|\psi_E\rangle = c_0|\psi_0\rangle + \int d^3p\chi_E(\boldsymbol{p})|M_1M_2(\boldsymbol{p})\rangle\,.$$



The coupled channel dynamics

- The $c\bar{c}$-M_1M_2 coupled channel system $H = H_{c\bar{c}} + H_{c\bar{c}\leftrightarrow M_1M_2} + H_{M_1M_2}$

$$\left(\begin{array}{c|c} H_{c\bar{c}} & H_{c\bar{c}\leftrightarrow M_1M_2} \\ \hline H_{c\bar{c}\leftrightarrow M_1M_2} & H_{M_1M_2} \end{array}\right)\begin{pmatrix} c_1|\psi_1\rangle \\ \vdots \\ c_n|\psi_n\rangle \\ \vdots \\ \int d^3p\,\chi_i(\boldsymbol{p})|i;\boldsymbol{p}\rangle \\ \vdots \end{pmatrix} = E\begin{pmatrix} c_1|\psi_1\rangle \\ \vdots \\ c_n|\psi_n\rangle \\ \vdots \\ \int d^3p\,\chi_i(\boldsymbol{p})|i;\boldsymbol{p}\rangle \\ \vdots \end{pmatrix}$$

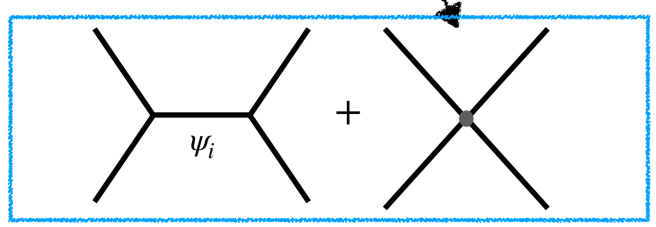
$$\langle\psi_n|H_{c\bar{c}\leftrightarrow M_1M_2}|i;\boldsymbol{p}\rangle = f_{n,i}(\boldsymbol{p})$$

$$H_{c\bar{c}}|\psi_n\rangle = E_n|\psi_n\rangle$$

restrict to M_1M_2 space

- The meson-meson scattering: $\dfrac{p_i^2}{2\mu_i}\chi_i(\boldsymbol{p}) + \int d^3p'\left(V_{ij}^s(\boldsymbol{p},\boldsymbol{p}') + V_{ij}^{direct}(\boldsymbol{p},\boldsymbol{p}')\right)\chi_j(\boldsymbol{p}') = (E - m_i^{th})\chi_i(\boldsymbol{p})$

$$V_{ij}^s(\boldsymbol{p},\boldsymbol{p}') = \sum_n \frac{f_{in}(\boldsymbol{p})f_{nj}(\boldsymbol{p}')}{E - E_n}$$



- Ignore the direct meson-meson interaction:

$$T_{ij}(E;\boldsymbol{p},\boldsymbol{p}') = f_i^\dagger(\boldsymbol{p})\frac{\lambda}{I - \lambda\sum_k I_k(E)}f_j(\boldsymbol{p}'),$$

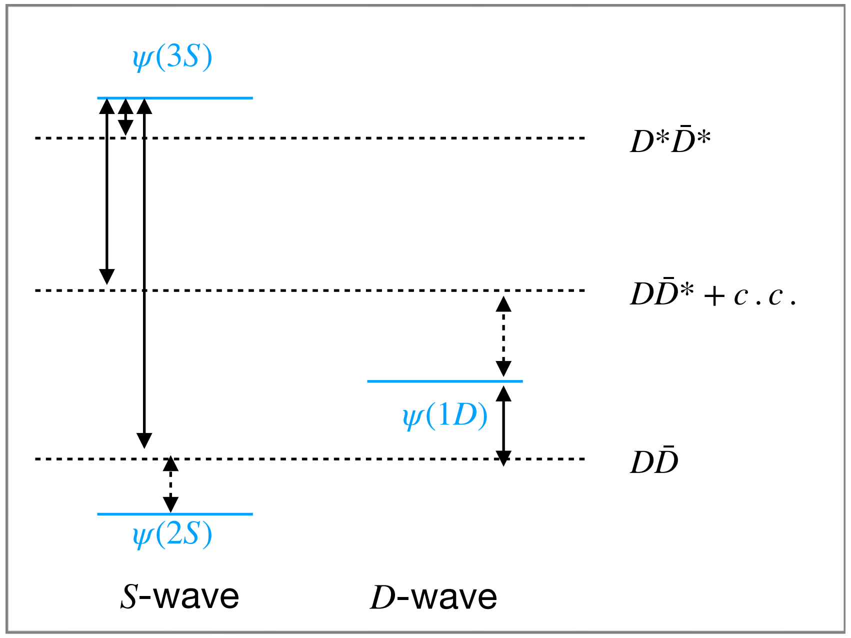
Reduce to Flatté formula: $T \sim \dfrac{1}{E - E_0 - \sum_i I_i(E)} \sim \dfrac{1}{E - m_R + im_R\sum_i g_i\rho_i}$

$$f_i = (f_{1,i}, f_{2,i}, \ldots) \qquad \lambda = diag\left(\frac{1}{E-E_1}, \frac{1}{E-E_2}, \ldots\right)$$

$$I_k(E) = \int d^3q \frac{f_k(\boldsymbol{q})f_k^\dagger(\boldsymbol{q})}{E - m_k^{\rm th} - q^2/(2\mu_k)}.$$

determine the pole positions

The coupled channel dynamics

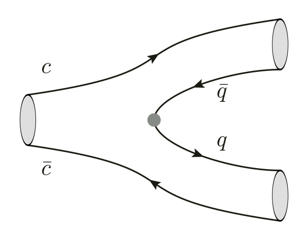


- $H_{c\bar{c}}$: Godfrey-Isgur (GI) model

Three bare state

$\psi(2S)$	$\psi(1D)$	$\psi(3S)$
$E_1 = 3.687\ GeV.$	$E_2 = 3.82\ GeV.$	$E_3 = 4.10\ GeV$

- $H_{c\bar{c}\leftrightarrow hadrons}$: quark pair creation (QPC) model



$$T = -3\gamma \sum_m \langle 1m\, 1{-m}|00\rangle \int \mathrm{d}^3\vec{p}_3\, \mathrm{d}^3\vec{p}_4\, \delta^3(\vec{p}_3+\vec{p}_4)\, \mathcal{Y}_1^m\left(\tfrac{\vec{p}_3-\vec{p}_4}{2}\right) \chi^{34}_{1-m}\, \phi^{34}_0\, \omega^{34}_0\, b^\dagger_3(p_3)\, d^\dagger_4(p_4).$$

$$f_{i,n}(\boldsymbol{p}) = \gamma_n \mathcal{M}^{QPC}(\boldsymbol{p}) e^{-\frac{\boldsymbol{p}^2}{2\Lambda}}$$

$e^{-\frac{p^2}{2\Lambda}}$: suppress the high momentum contribution

- the e^+e^- coupling.

$$f_{n,ee} = \langle \psi_n | H_{em} | e^+e^- \rangle$$

$f_{n,ee}$ Constant, can be related to the dilepton width in potential model

$$\Gamma_n^{ee} = 8\pi^2 \frac{|\boldsymbol{p}^{cm}|}{m_n} E_e^2 \frac{1}{2J_n+1} \sum_{spin} |f_{n,ee}|^2 ,$$

$$T_{D\bar{D},ee}(E+i\epsilon; \boldsymbol{p}_D^{cm}) = \frac{f_{D\bar{D}}(\boldsymbol{p}_D^{cm})\lambda f_{ee}}{I - \lambda(I_{D\bar{D}} + I_{D\bar{D}^*} + I_{D^*\bar{D}^*}) + i\epsilon} .$$

Morel and S. Capstick, (2002), arXiv:nucl-th/020401
P. G. Ortega, J. Segovia, D. R. Entem, and F. Fernandez, Phys.Rev. D 94, 074037 (2016)

The coupled channel dynamics: results

- (a) The $D\bar{D}$ cross section

$$T_{D\bar{D},ee}(E+i\epsilon;p_D^{cm}) = \frac{f_{D\bar{D}}(p_D^{cm})\lambda f_{ee}}{I-\lambda(I_{D\bar{D}}+I_{D\bar{D}^*}+I_{D^*\bar{D}^*})+i\epsilon}.$$

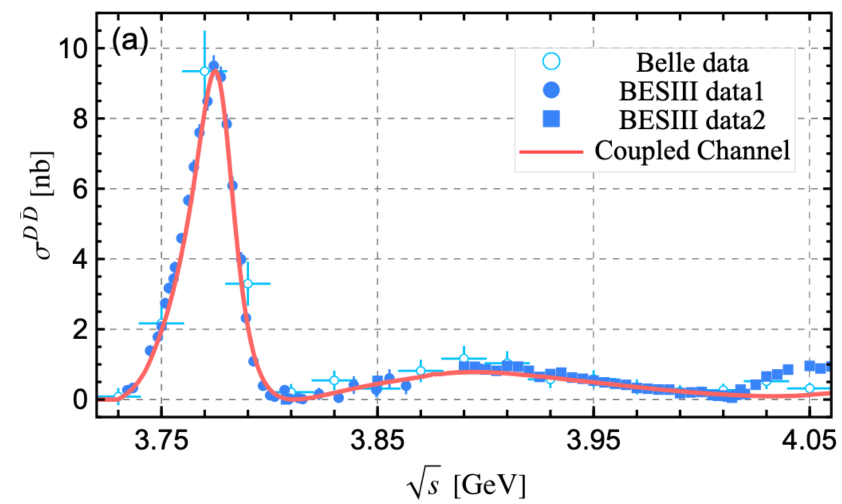
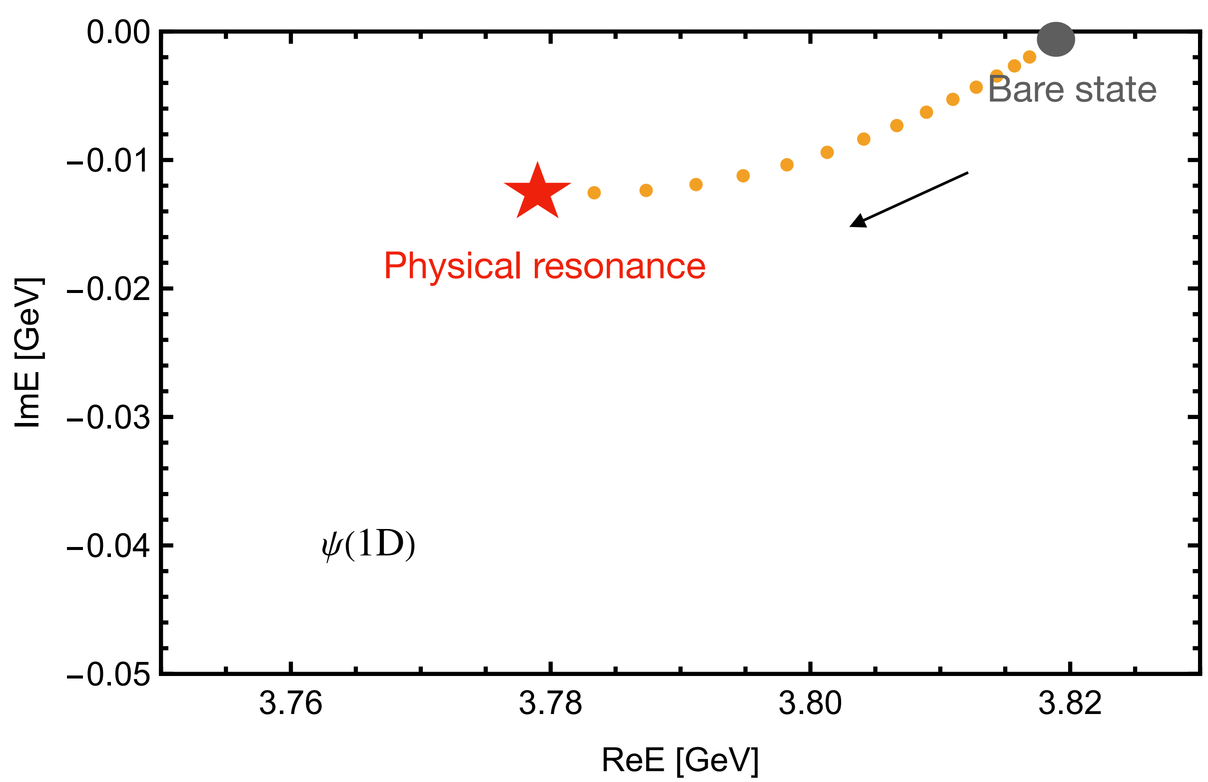


TABLE II. The parameters in the coupled-channel model. The coupling f_{n,e^+e^-} are given in units of $\mathrm{GeV}^{-\frac{1}{2}}$.

γ_{2S}	γ_{1D}	γ_{3S}	$f_{\psi(2S),e^+e^-}$	$f_{\psi(1D),e^+e^-}$	$f_{\psi(3S),e^+e^-}$	Λ (GeV)
3.54	6.51	6.15	2.48×10^{-4}	0.15×10^{-4}	2.11×10^{-4}	0.65

$$\Gamma_{2S}^{e^+e^-}=5.50\ \mathrm{keV},\quad \Gamma_{1D}^{e^+e^-}=0.02\ \mathrm{keV},\quad \Gamma_{3S}^{e^+e^-}=4.91\ \mathrm{keV}.$$

In GI model: $\Gamma_{2S}^{ee}=3.27$ keV, $\Gamma_{1D}^{ee}=0.10$ keV, $\Gamma_{3S}^{ee}=1.94$ keV



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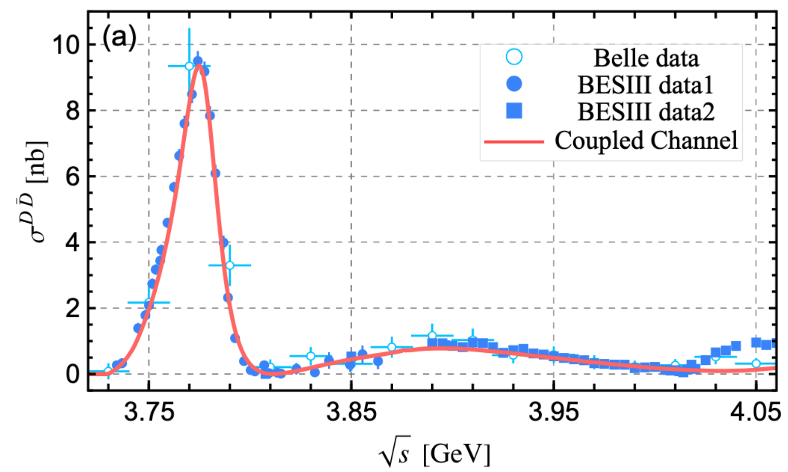


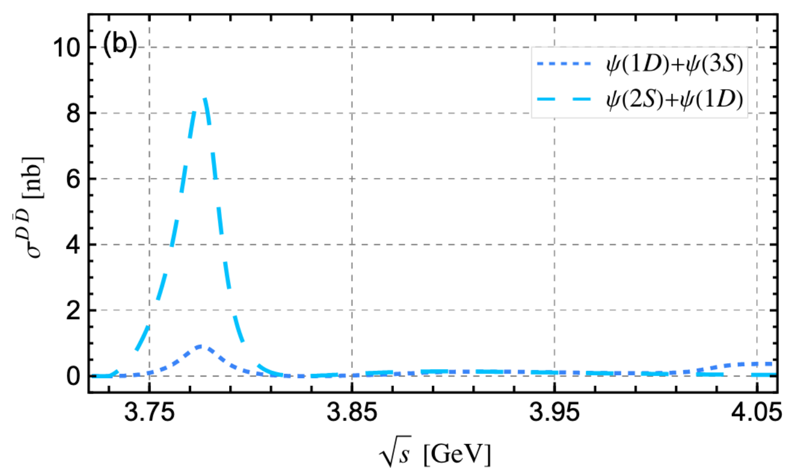
- (b) $G(3900)$ can be produced via coupled channel dynamics

1. Removing $\psi(2S)$ in the model → small $\psi(3770)$ signal

   Mixing via $\psi(2S)-D\bar{D}-\psi(1D)$

2. Removing $\psi(3S)$ in the model → no $G(3900)$ signal

   $\psi(1D)-D\bar{D}^*-\psi(3S)$



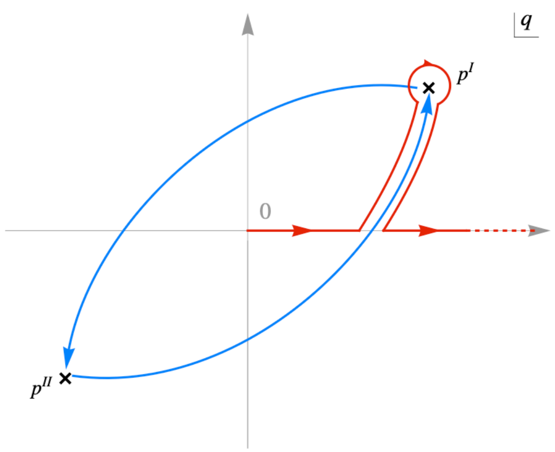
The coupled channel dynamics: results

- Poles of scattering amplitude $T_{D\bar{D},e^+e^-}$

$$T_{D\bar{D},ee}(E+i\epsilon;\boldsymbol{p}_D^{cm}) = \frac{f_{D\bar{D}}(\boldsymbol{p}_D^{cm})\lambda f_{ee}}{I-\lambda(I_{D\bar{D}}+I_{D\bar{D}^*}+I_{D^*\bar{D}^*})+i\epsilon}\,.$$

$$I_k(E) = \int d^3q \frac{f_k(\boldsymbol{q})f_k^\dagger(\boldsymbol{q})}{E-m_k^{\text{th}}-q^2/(2\mu_k)}\,.$$

- Analytical continuation & contour deformation



$$I(E) = \int_0^\infty dq \frac{f(q)}{E-q^2/(2\mu)} = -2\mu\int_0^\infty dq \frac{f(q)}{(q-p^I)(q-p^{II})}\,,$$

$$I(E^{II}) - I(E^I) = -2\pi i \frac{\mu}{p^{II}} f(p^{II})\,.$$

The coupled channel dynamics: results

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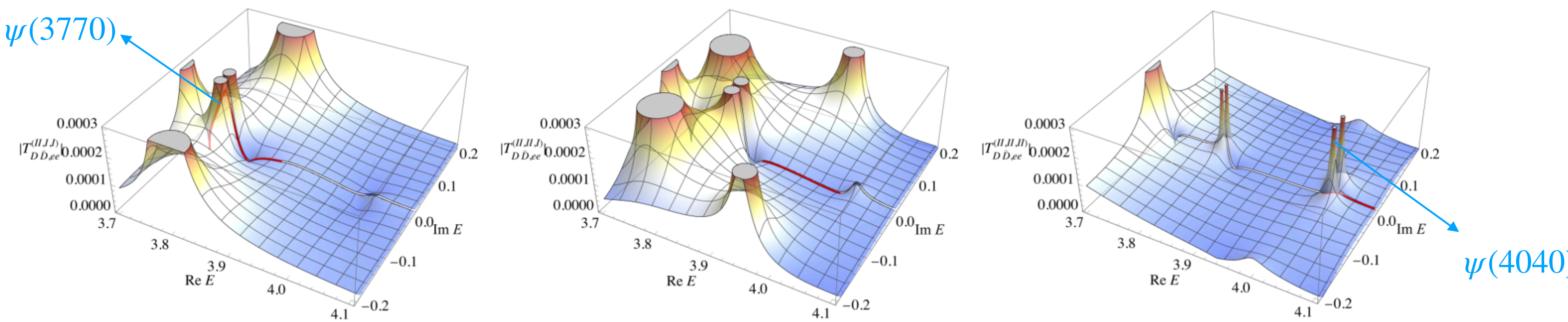


Only three poles near the physical region

Riemann sheet	(I,I,I)	(II,I,I)	(II,II,II)
Pole (GeV)	3.686	$3.778-0.012i$	$4.031-0.024i$
Physical state	$\psi(3686)$	$\psi(3770)$	$\psi(4040)$

- The non-open charm hadron (nOCH) channels

The non-open charm hadron(nOCH) cross section

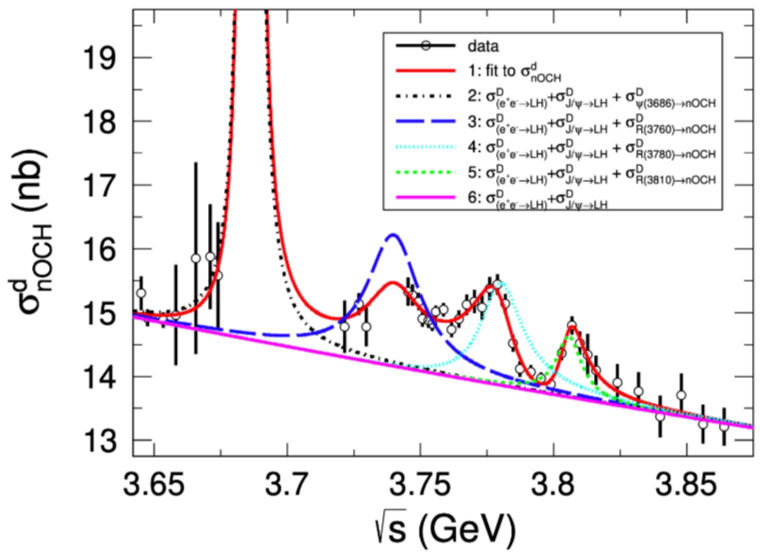
- $R(3780)$

  The signal of $\psi(3770)$

- $R(3760)$

  Structure around $m_{D\bar{D}} = 3.730$ GeV

  Threshold effect?



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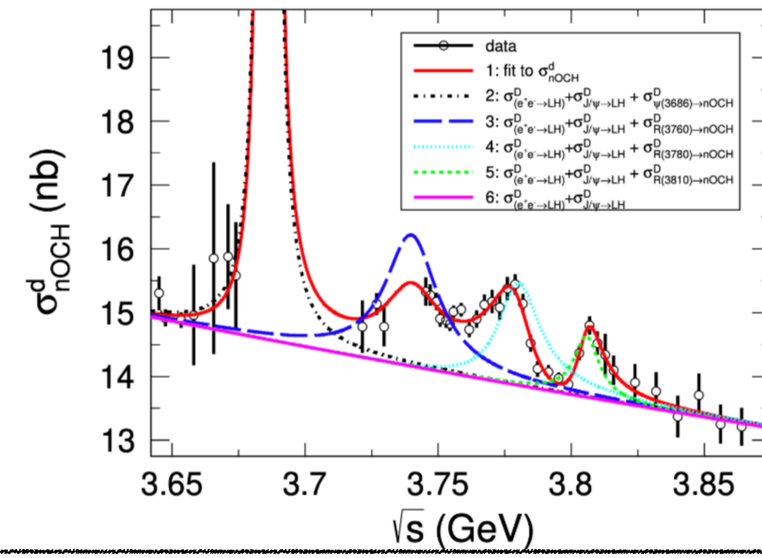
  Structure around $m_{D\bar{D}} = 3.730$ GeV

  Threshold effect?



- The possibility of direct $\psi(1D)-\mathrm{nOCH}$ coupling

$$T_{ij}(E+i\epsilon) = f_i \frac{\lambda}{I-\lambda\sum_k I_k(E)} f_j \approx f_i \frac{\lambda}{I-\lambda I_{D\bar{D}}(E)-\lambda I_{nOC}(m^{\rm th}_{D\bar{D}})} f_j\,, \qquad \lambda = \frac{1}{E-E_i}$$

$$T_{nOCH,ee}(E+i\epsilon) \approx \frac{f_{nOCH}(m^{\rm th}_{D\bar{D}})\lambda f_{ee}}{I-\lambda\left(I_{D\bar{D}}(E+i\epsilon)-\sum_k I_{k\neq D\bar{D}}(m^{\rm th}_{D\bar{D}})\right)}\,,$$

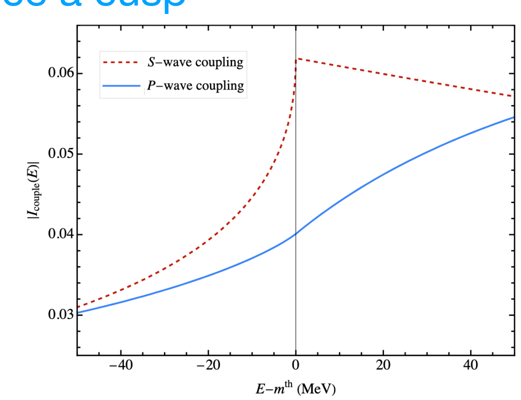
In general the $I_{D\bar{D}}(E+i\epsilon)$ can produce a cusp at $D\bar{D}$ mass threshold

$$\begin{aligned} I_{D\bar{D}}(E+i\epsilon) \;\approx\; & -2\mu_{D\bar{D}}\int dq (f^\dagger_{D\bar{D}} f_{D\bar{D}})(q) \\ & -i\pi\mu_{D\bar{D}} k_{D\bar{D}}\,(f^\dagger_{D\bar{D}} f_{D\bar{D}})(k_{D\bar{D}})\,, \end{aligned} \qquad I_k(E) = \int d^3q \frac{f_k(\boldsymbol{q}) f^\dagger_k(\boldsymbol{q})}{E-m^{\rm th}_k - q^2/(2\mu_k)}\,.$$

$k_{D\bar{D}} = \sqrt{2\mu_{D\bar{D}} E_{D\bar{D}}}$. The branch point of square root function produce a cusp

If the $\psi - D\bar{D}$ coupling in S-wave, $f_{D\bar{D}}(k) \approx f(0)$

the $\psi - D\bar{D}$ coupling in P-wave, $f_{D\bar{D}}(k) \approx 2\mu_{D\bar{D}} E_{D\bar{D}}$



A direct nOCH coupling $\not\Rightarrow$ near $D\bar{D}$ structure

The recattering mechanism

- How to model the non-open charm hadron (nOCH) channels

$$D\bar{D} \to J/\psi X$$
$$D\bar{D} \to \text{light hadrons}$$
$$\cdots$$

Near the $D\bar{D}$ mass threshold, only the relative momentum between $D\bar{D}$ matters

Assuming a two body nOCH threshold

$$\mu_{nOCH} = 0.7 \text{ GeV } \& \ m^{\text{th}}_{nOCH} = 3.0 \text{ GeV}$$

Ignore momentum dependence of nOCH channel

- The rescattering coupling $V_{D\bar{D}\leftrightarrow nOCH}$

$$V_{nOCH,D\bar{D}}(\boldsymbol{q}) = g_{nOCH} q e^{-q/\Lambda_{nOCH}} Y_1^1(\Omega_q)$$

$qY_1^m(\Omega_q)$: reflect the P-wave coupling

Λ_{nOCH}: the characteristic momentum scale of rescattering process

$$T_{nOCH,ee}(E+i\epsilon) = V_{nOCH,D\bar{D}} G^{D\bar{D}}(E+i\epsilon) T_{D\bar{D},ee}(E+i\epsilon)$$
$$= \int d^3q \frac{V_{nOCH,D\bar{D}}(\boldsymbol{q}) T_{D\bar{D},ee}(E+i\epsilon;\boldsymbol{q})}{E - m^{\text{th}}_{D\bar{D}} - q^2/(2\mu_{D\bar{D}}) + i\epsilon}$$

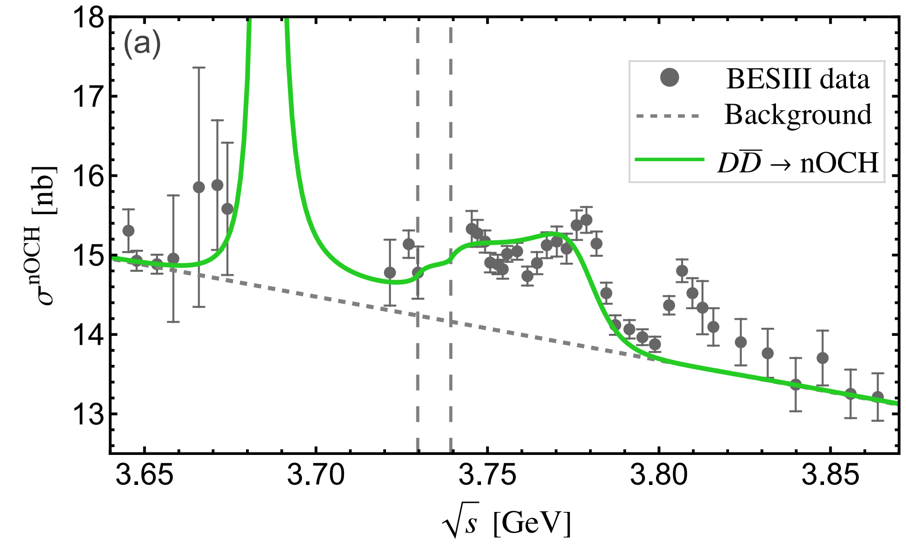
The result of recattering mechanism

- The rescattering coupling $V_{nOCH \leftrightarrow D\bar{D}}$

$$V_{nOCH,D\bar{D}}(\boldsymbol{q}) = g_{nOCH} q e^{-q/\Lambda_{nOCH}} Y_1^1(\Omega_q)$$

$$g_{nOCH} = 11.5\,, \quad \Lambda_{nOCH} = 0.09 \text{ GeV}$$

- Conclusion
  - No direct $\psi(1D)$-nOCH coupling
    just as the negligible small width of $\psi(3686)$.
  - $\Lambda_{nOCH} \sim 100$ MeV
    The mass of $\psi(3770)$ is very close to the $D\bar{D}$ threshold
    $\rightarrow$ Large nOCH branching fraction of $\psi(3770)$
    the rescattering $D\bar{D} \rightarrow nOCH$ contribute its nOCH decays.



The result of recattering mechanism

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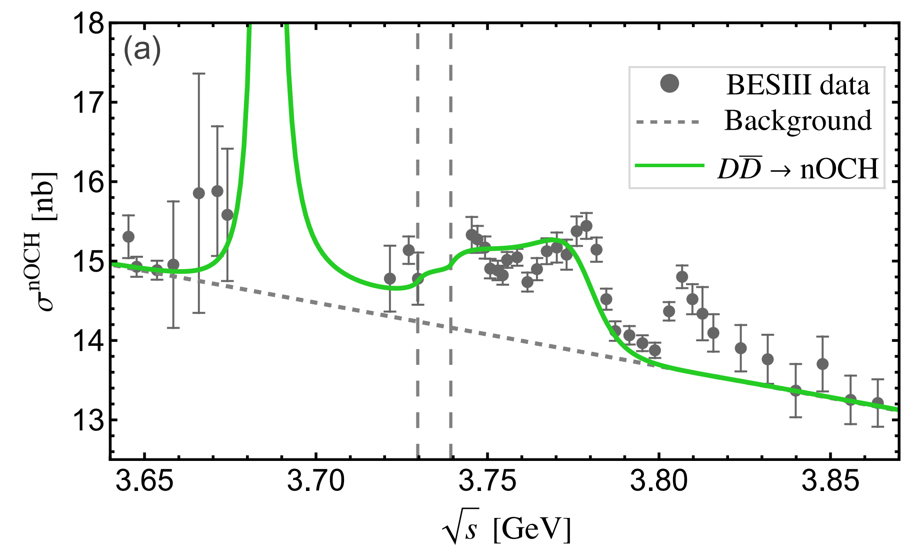

- Conclusion
  - No direct $\psi(1D)$-nOCH coupling
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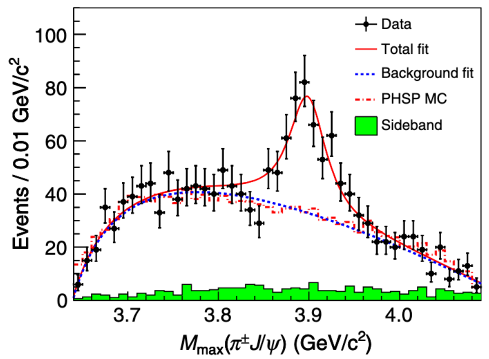
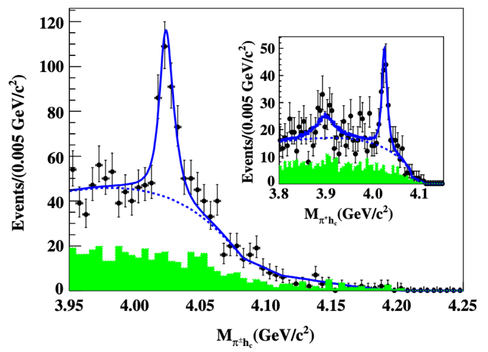
    $\to$ Large nOCH branching fraction of $\psi(3770)$
    the rescattering $D\bar{D} \to nOCH$ contribute its nOCH decays.
  - Other effect
    - Similar threshold structure in $D\bar{D}^*$ and $D^*\bar{D}^*$ threshold in nOCH channel?
      The $\psi(4040)$ may have large nOCH decay fraction ($m_{\psi(4040)} - m_{D^*\bar{D}^*} \approx 20$ MeV)
    - Near $D^{(*)}\bar{D}^{(*)}$ threshold structure can occur in charmonium(-like) decays: Z_c

$Y \to D^{(*)}\bar{D}^{(*)}\pi$

$D^{(*)}\bar{D}^{(*)}$ in S-wave





$Z_c(3900)$ & $Z_c(4020)$

The non-open charm hadron(nOCH) cross section

- $R(3780)$

  The signal of $\psi(3770)$

- $R(3760)$

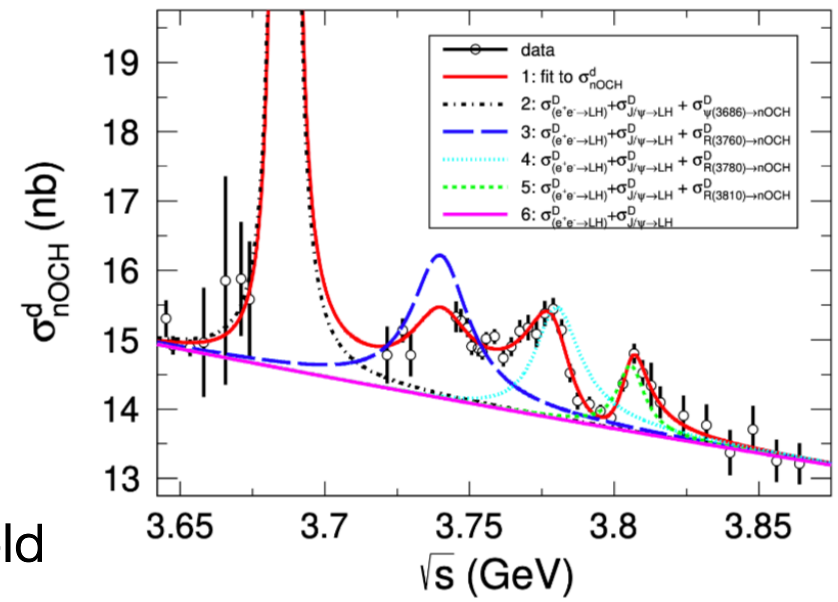
  Structure around $m_{D\bar{D}} = 3.730$ GeV

  Rescattering process $D\bar{D} \to \mathrm{nOCH}$

- $R(3810)$

  Structure just above the $h_c\pi\pi$ mass threshold

  don't have apparent signal in $D\bar{D}$ cross section



- The possibility of $\psi(1D) - h_c\pi\pi$ coupling

  Bare $\psi(2S)$ mass, 3.82 GeV in GI model

  Very close to $m_{h_c\pi\pi} = 3.80$ GeV

  Possible strong coupling between $\psi(1D) - h_c\pi\pi$

  Assume the relative angular momentum between $\pi\pi$ is $L_{\pi\pi} = 1$

$$f_{1D,h_c\pi\pi}(\boldsymbol{q},\boldsymbol{p}) = \frac{g_{h_c\pi\pi}}{\sqrt{4\pi}} p e^{-(q^2+p^2)/\Lambda_{h_c}^2} \frac{1}{\sqrt{2}} \left(Y_1^{*0}(\Omega_p) - Y_1^{*1}(\Omega_p)\right)$$

The result of $h_c\pi\pi$ coupling

$$f_{1D,h_c\pi\pi}(\boldsymbol{q},\boldsymbol{p}) = \frac{g_{h_c\pi\pi}}{\sqrt{4\pi}} p e^{-(q^2+p^2)/\Lambda_{h_c}^2} \frac{1}{\sqrt{2}} \left(Y_1^{*0}(\Omega_p) - Y_1^{*1}(\Omega_p)\right)$$

- The $R(3810)$ can be explained by $\psi(1D) - h_c\pi\pi$ coupling
  - Does not affect pole of $\psi(3770)$
  - $\Lambda_{h_c} = 0.04\ \text{GeV}$

  ➡ The momentum scale ~ 0.04 GeV

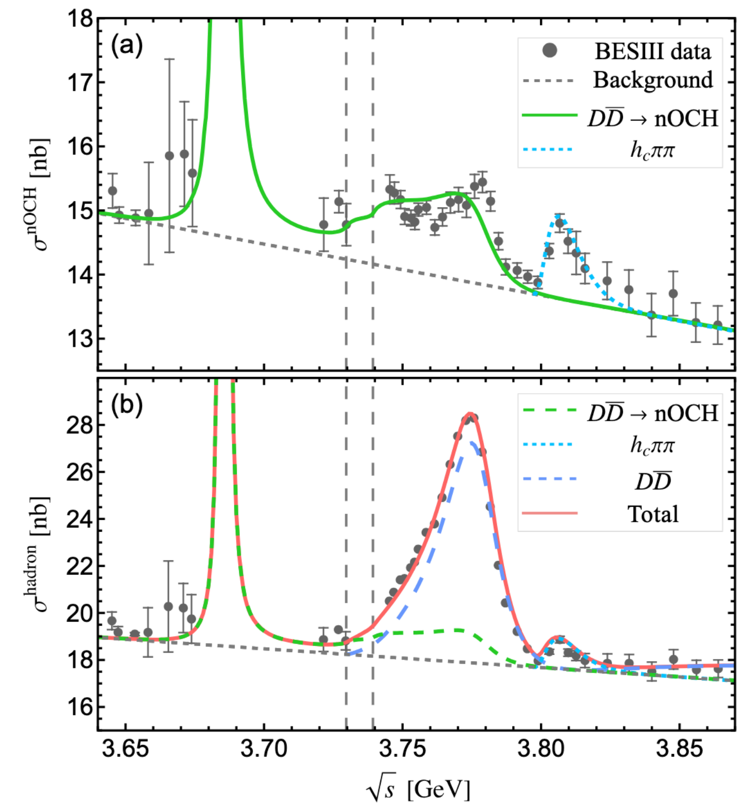
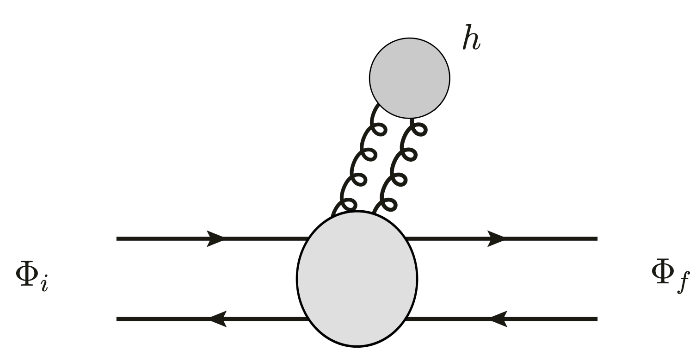
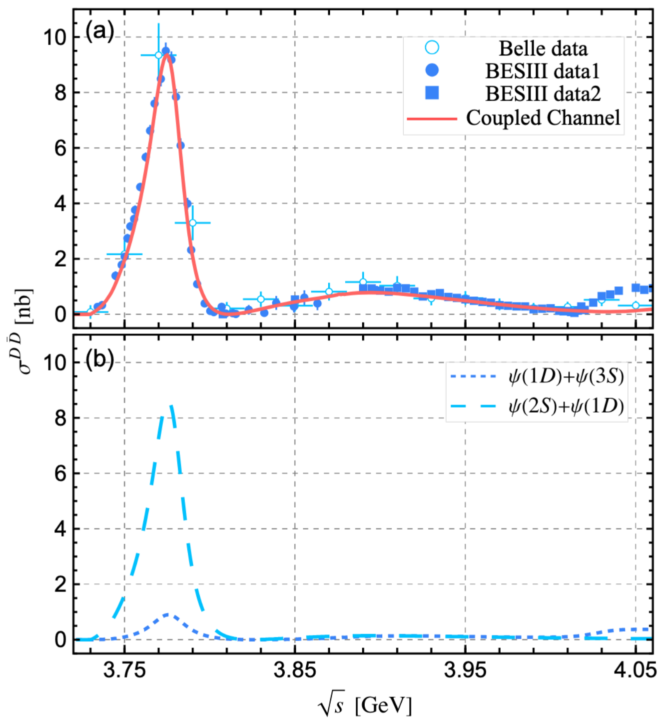


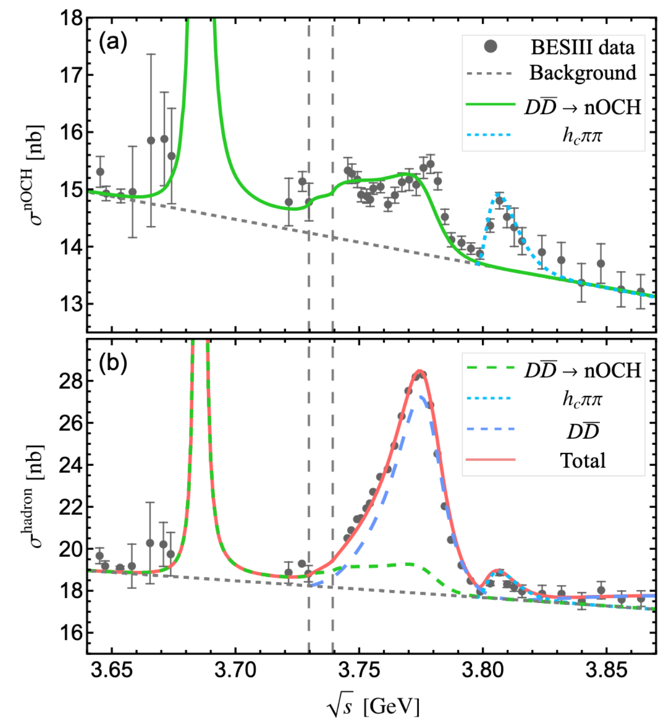
FIG. 2. Coupled-channel descriptions of (a) the $e^+e^- \to$ non-open-charm (nOCH) hadron cross section and (b) the inclusive hadronic cross sections. Vertical dashed lines indicate the $D^0\bar{D}^0$ and $D^+\bar{D}^-$ thresholds. Nonopen-charm hadron data are from Ref. [28], inclusive hadronic data are from Ref. [27].

In view of QCD multipole expansion, $\psi(1D) \to h_c\pi\pi$ via soft gluon emission small momentum scale expected



Summery





- The anomalous line shape of $\psi(3770)$ & $G(3900)$ is due to coupled channel dynamics

  $c\bar{c}$ – open charm

- The near $D\bar{D}$ structure in the nOCH channel is due to the $V_{nOCH \leftrightarrow D\bar{D}}$ coupling

  The $V_{nOCH,D\bar{D}}$ takes effect only at energy very near $D\bar{D}$ threshold.
  The large non-$D\bar{D}$ is $\psi(3770)$ is only because that its mass is very close to $m_{D\bar{D}}$

- The structure at 3.81 is from $e^+e^- \to \psi(1D) \to h_c\pi\pi$

  This coupling can produce a resonance like structure in nOCH while show no signals in $D\bar{D}$ channel

Thanks for your attention

Back up

The amplitude of QPC model

$$\mathcal{M}^{QPC}_{M_{J_A},M_{J_B}M_{J_C}}(\boldsymbol{p}) = \gamma \sum_{M_{L_A}} \langle L_A M_{L_A} S_A M_{S_A} | J_A M_A \rangle \langle L_B M_{L_B} S_B M_{S_B} | J_B M_B \rangle \langle L_C M_{L_C} S_C M_{S_C} | J_C M_C \rangle$$
$$\times \langle 1m1-m|00\rangle \langle \chi^{14}_{S_B M_{S_B}} \chi^{32}_{S_C M_{S_C}} | \chi^{12}_{S_A M_{S_A}} \chi^{34}_{1-m} \rangle \langle \phi^{14}_B \phi^{32}_C | \phi^{12}_A \phi^{34}_0 \rangle$$
$$\times Ip(\boldsymbol{p}),$$

$$Ip(\boldsymbol{p}) = \int d^3q\, \psi^*_{B,L_B,M_{L_B}}\left(\frac{m_q}{m_c+m_q}\boldsymbol{p}+\boldsymbol{q}\right) \psi^*_{C,L_C,M_{L_C}}\left(\frac{m_q}{m_{\bar{c}}+m_q}\boldsymbol{p}+\boldsymbol{q}\right) \psi_{A,L_A,M_{L_A}}(\boldsymbol{p}+\boldsymbol{q}) \mathcal{Y}^m_1(\boldsymbol{q}),$$

$$\mathcal{M}^{QPC}_{M_A,M_B,M_C}(\boldsymbol{p}) = \sum_{L,S} \mathcal{M}^{L,S}_{A\to BC}(p) \langle LM_L; SM_S | 1, 1\rangle \langle S_1 M_1; S_2 M_2 | S M_S \rangle Y^{M_L *}_L(\Omega).$$

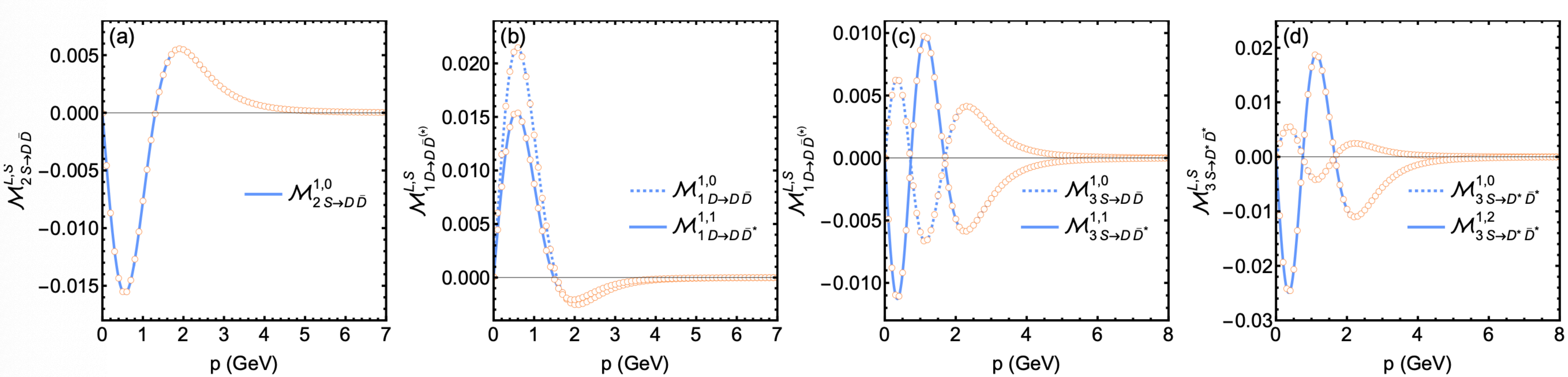
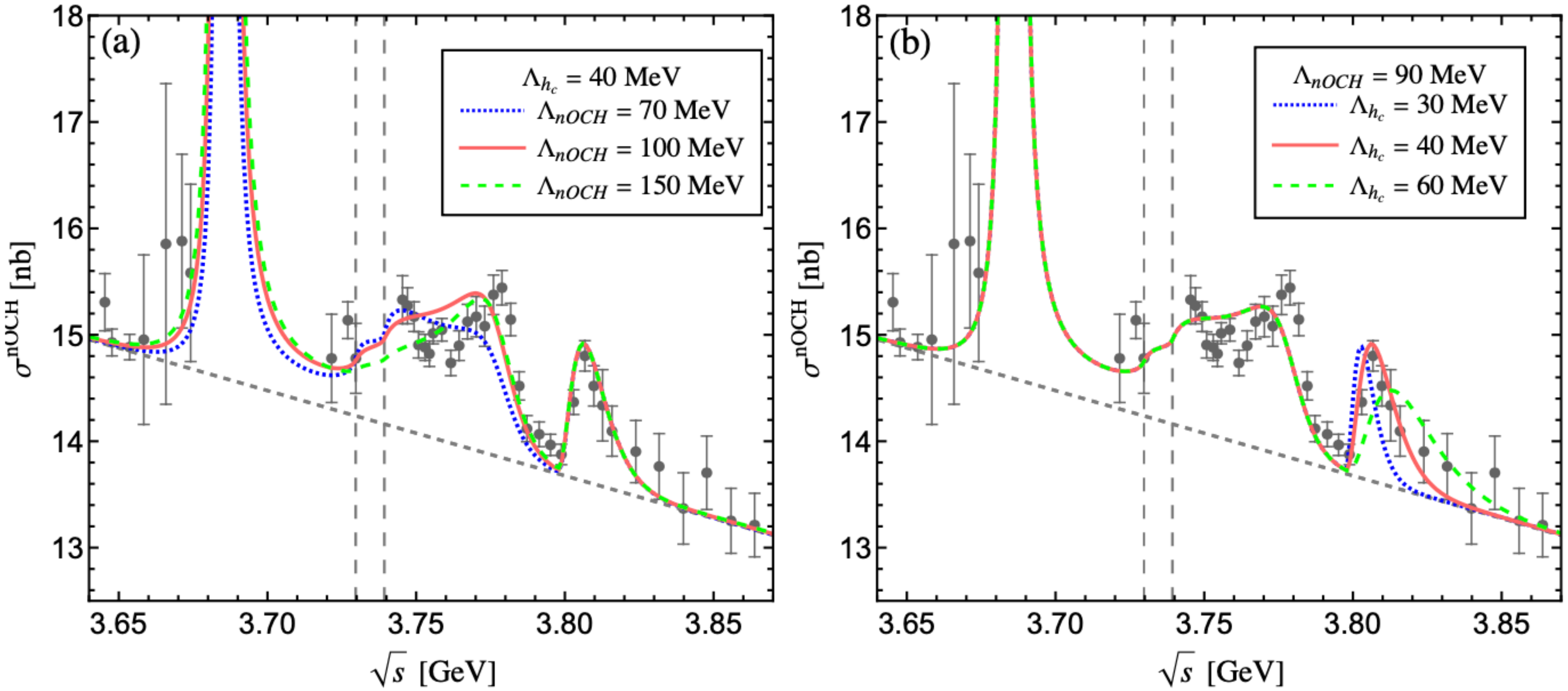


FIG. S1. The calculated QPC partial wave amplitudes (orange circle points) for $\gamma = 1$ and fits to them in terms of analytical function (blue lines).

$$\sum_i g_i p^L e^{-a_i p^2}$$

The parameter dependence on Λ_{nOCH} and Λ_{h_c}



$$70 \text{ MeV} \lesssim \Lambda_{nOCH} \lesssim 150 \text{ MeV}, \quad 30 \text{ MeV} \lesssim \Lambda_{h_c\pi\pi} \lesssim 60 \text{ MeV}.$$

The complete LS equation for nOCH channel

$$
\begin{aligned}
T_{D\bar{D},e^+e^-} &= V_{D\bar{D},e^+e^-} + V_{D\bar{D},D\bar{D}}G^{D\bar{D}}T_{D\bar{D},e^+e^-} + V_{D\bar{D},D\bar{D}^*}G^{D\bar{D}^*}T_{D\bar{D}^*,e^+e^-} + V_{D\bar{D},D^*\bar{D}^*}G^{D^*\bar{D}^*}T_{D^*\bar{D}^*,e^+e^-} \\
&\quad + V_{D\bar{D},nOCH}G^{nOCH}T_{nOCH,e^+e^-}\,, \\
T_{D^*\bar{D}^{(*)},e^+e^-} &= V_{D^*\bar{D}^{(*)},e^+e^-} + V_{D^*\bar{D}^{(*)},D\bar{D}}G^{D\bar{D}}T_{D\bar{D},e^+e^-} + V_{D^*\bar{D}^{(*)},D\bar{D}^*}G^{D\bar{D}^*}T_{D\bar{D}^*,e^+e^-} + V_{D^*\bar{D}^{(*)},D^*\bar{D}^*}G^{D^*\bar{D}^*}T_{D^*\bar{D}^*,e^+e^-} \\
T_{nOCH,e^+e^-} &= V_{nOCH,D\bar{D}}G^{D\bar{D}}T_{D\bar{D},e^+e^-}\,.
\end{aligned}
$$

$$
V_{nOCH,D\bar{D}}(\boldsymbol{p}_{nOCH},\boldsymbol{p}_{D\bar{D}}) = g_{nOCH,D\bar{D}}\,p_{nOCH}Y_1^1(\Omega_{nOCH})e^{-p_{nOCH}^2/\Lambda_1^2}\,p_{D\bar{D}}Y_1^{1*}(\Omega_{D\bar{D}})e^{-p_{D\bar{D}}^2/\Lambda_2^2}\,,
$$

$$
g_{nOCH} = 7.5\ \text{GeV}^{-3/2},\ \Lambda_1 = 1.58\ \text{GeV and } \Lambda_2 = 0.2\ \text{GeV}
$$