Lattice QCD study of the $K^*(892)$ resonance at the physical point

李衢智

四川大学

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Base on arxiv:2603.16266v1

Collaborate with CLQCD

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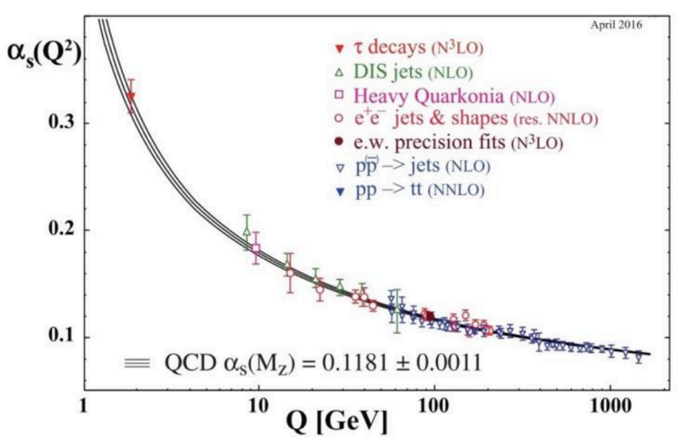
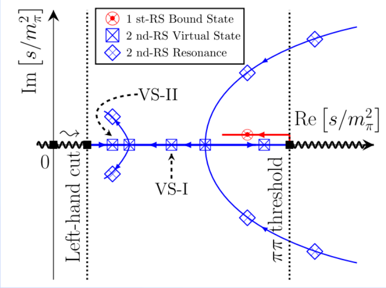
1. Introduction

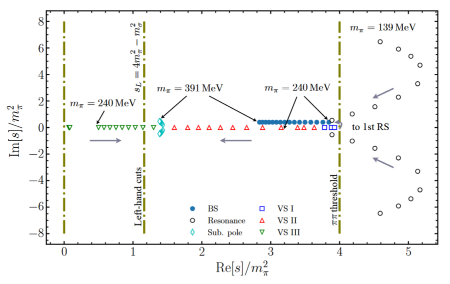
- QCD is non-perturbative at low energies.
- Lattice QCD enables first-principles hadron studies.
- **Unphysical mass** hadron scatterings: providing **new insight** into studying strong interaction.





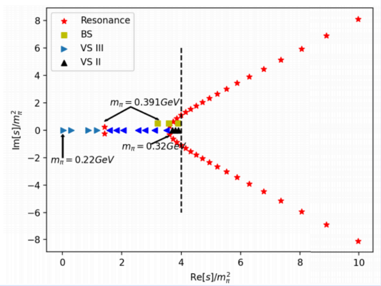
Roy equation + lattice data

X.H. Cao, et al., PHYS. REV. D 108, 034009 (2023)



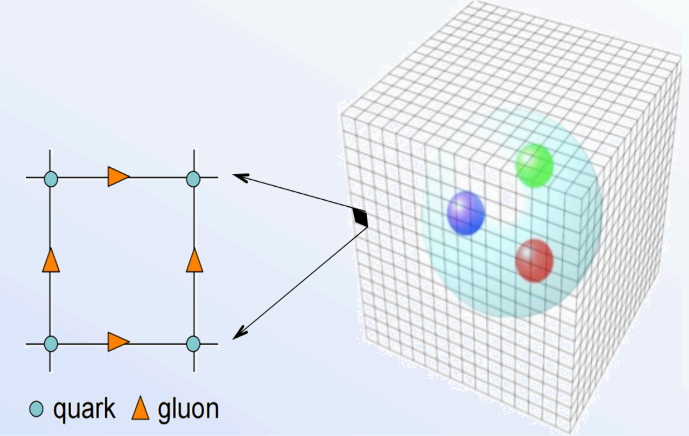
solvable $O(N)\sigma$ model + $\frac{N}{D}$ method

Y.L. Lyu, et al., PHYSICAL REVIEW D 109, 094026 (2024)



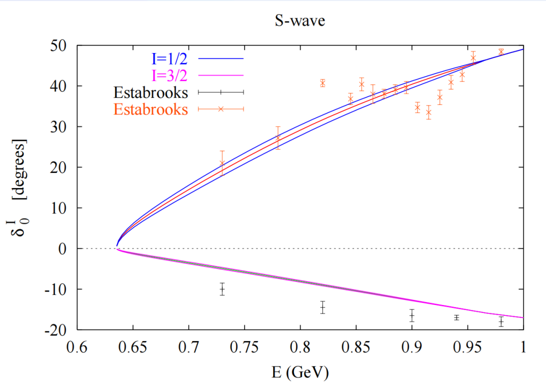
Linear σ model + Pade

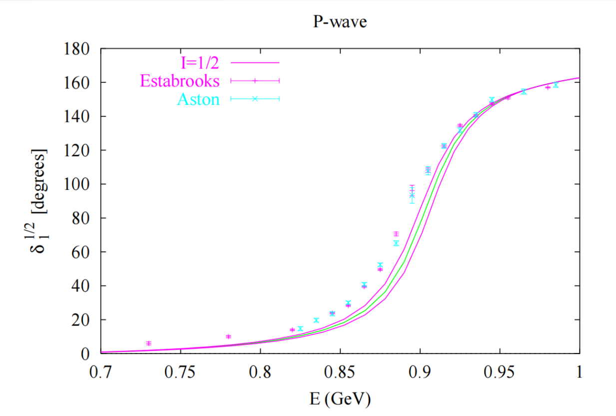
QZL, et al., Chin. Phys. C 49, 123103 (2025)



πK scatterings

- The simplest scattering process involving unequal mass particles carrying strangeness.





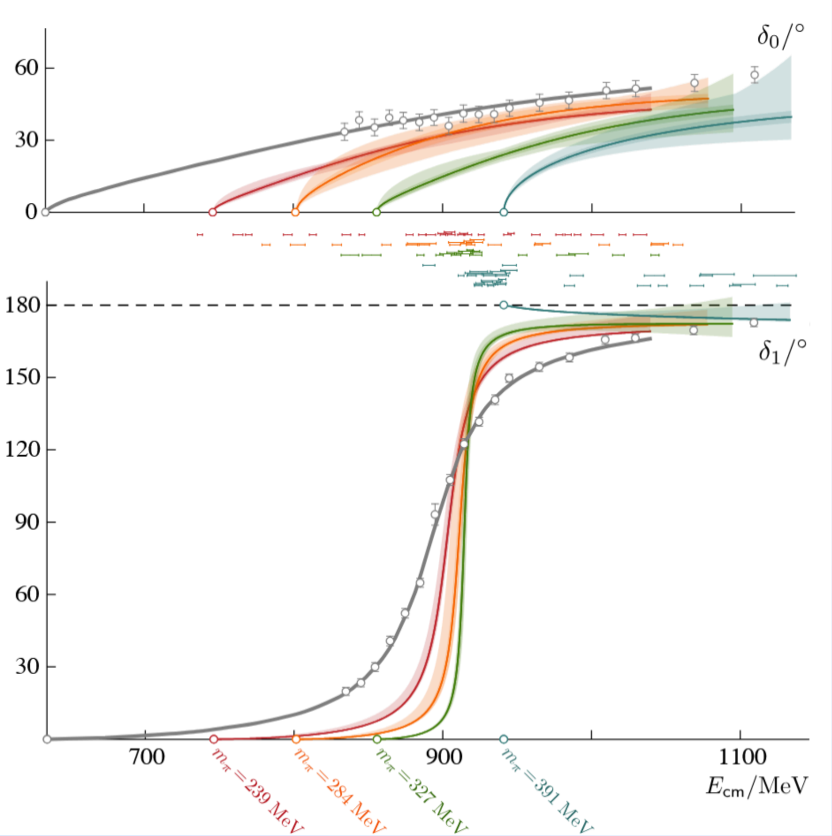
P. Büttiker, et al., Eur.Phys.J.C 33 (2004) 409-432

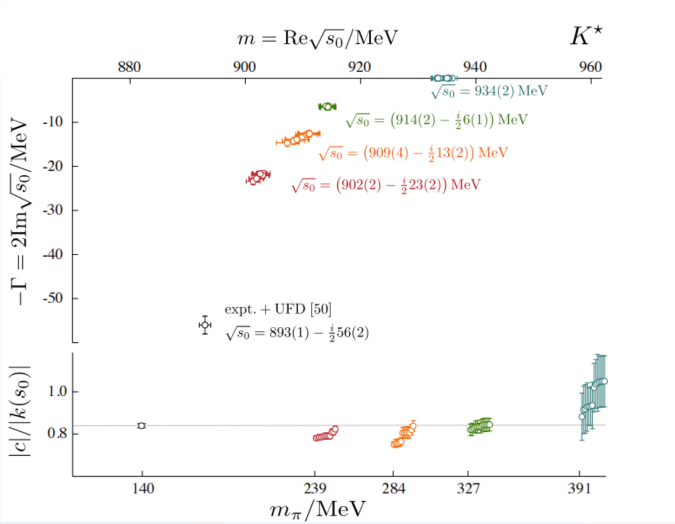
- $\kappa/K_0^*(700)\left(IJ=\frac{1}{2}0\right): \sqrt{s}=(658\pm 13)-i(279\pm 12)$ MeV

- $K^*(892)\left(IJ=\frac{1}{2}1\right): \sqrt{s}=(891\pm 2)-i(27\pm 1)$ MeV

πK scatterings

- Lattice researches on πK scatterings





been debated [59, 65–67]. When the UχPT amplitudes have their parameter freedom constrained by the finite-volume spectra presented above, a complex pole is found with a real energy around $m_\pi + m_K$ and a large imaginary part, not dissimilar to the experimental κ resonance. In addition, many of the K-matrix forms we implement, which lack any explicit left-hand cut behavior, also feature poles at similar energies; however, some do not and many have other nearby poles. Even with precise information about the amplitude for real energies, the analytic continuation required to reach any pole is sufficiently large that a unique result is not found.

D. J. Wilson et al., Phys. Rev. Lett. 123, 042002 (2019).

Motivation

configuration	$L^3 \times T$	a(fm)	m_π(MeV)	m_K(MeV)	$m_\pi L$	N_{cfgs}	N_{ev}
C48P14	$48^3 \times 96$	0.10530(18)	134.2(1.6)	508.9(3.0)	3.56	261	150
C48P23	$48^3 \times 96$	0.10530(18)	227.0(1.4)	483.7(2.9)	5.79	265	150
C32P29	$32^3 \times 64$	0.10530(18)	293.6(1.8)	511.0(3.0)	5.01	984	100
F32P21	$32^3 \times 64$	0.07746(18)	208.4(1.9)	491.8(1.3)	2.60	459	100
F48P21	$48^3 \times 96$	0.07746(18)	207.35(86)	491.3(1.2)	3.91	267	100
F48P30	$48^3 \times 96$	0.07746(18)	304.91(81)	524.3(1.3)	5.72	359	100
F32P30	$32^3 \times 96$	0.07746(18)	304.2(1.5)	524.5(1.5)	3.81	775	100
H48P32	$48^3 \times 144$	0.05187(26)	318.4(1.7)	539.8(2.7)	4.06	453	100

CLQCD, Phys.Rev.D 109 (2024)

- Systematically investigate the quark-mass dependence of κ and $K^*(892)$
- Push the way to further Roy-Steiner equation investigations

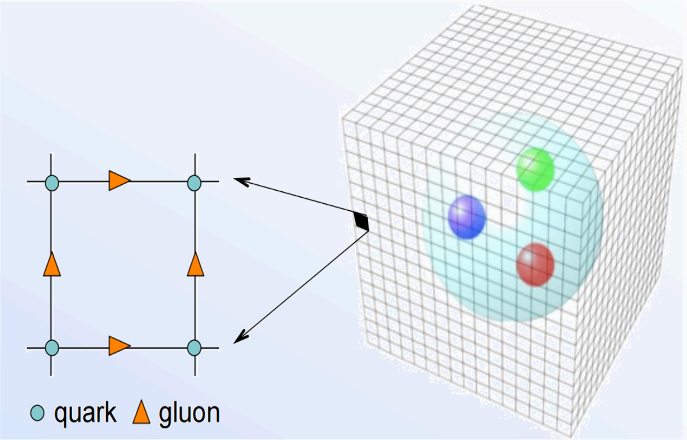
2. Method

Lattice QCD

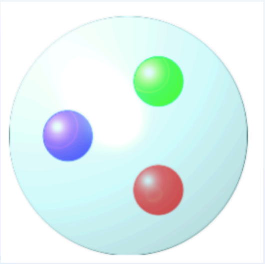
- QCD Lagrangian

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\left[\gamma_\mu(\partial_\mu - igA_\mu) + m_q\right] q$$

- Lattice QCD Key Features:
  - Non-perturbative approach to QCD
  - Discrete space-time lattice
  - Quarks on sites, gluons on links
  - Numerical simulations via Monte Carlo
  - Computes hadron masses, decay constants, etc.



meson

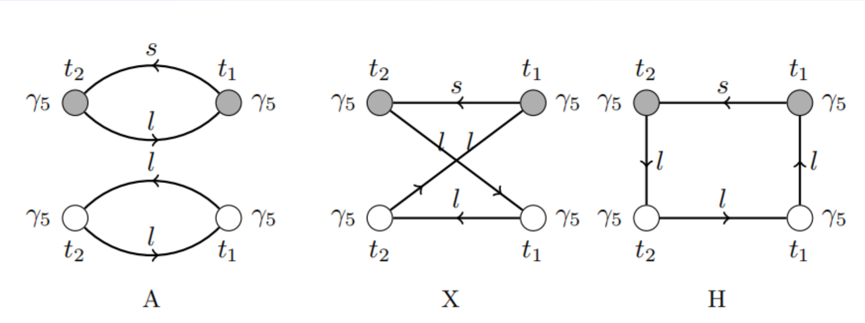


baryon

- Construction of operators: projection method H.B. Yan,et al., JHEP 10 (2025) 210

$$O_{K\pi}^{P,\Lambda,\lambda}(t,p_1,p_2)=\sum_{p_1,p_2}c_{\Lambda,\lambda}(t,p_1,p_2)\left[\sqrt{\frac{2}{3}}\pi^0(t,p_1)K^0(t,p_2)-\sqrt{\frac{1}{3}}\pi^0(t,p_1)K^+(t,p_2)\right],\quad P=p_1+p_2,$$

- Calculation of correlation matrix $C(t)$



$$\langle O_{\pi K}O^{\dagger}_{\pi K}\rangle = A+\frac{1}{2}X-\frac{3}{2}H$$

- Solving the generalized eigenvalue problem (GEVP)

Construction of operators $\{O_i\}$

$C_{ij}(t)=\langle O_i(t)O_j(0)\rangle$

GEVP

$\boldsymbol{C}(t)\boldsymbol{v}_n=\lambda_n(t)\boldsymbol{C}(t_0)\boldsymbol{v}_n$

$\lambda_n(t)\sim\exp(-E_nt)$

- Extracting the finite spectra by fitting

$$\lambda_n(t) = A\exp(-E_n\ t), t \in (t_i, t_f)$$

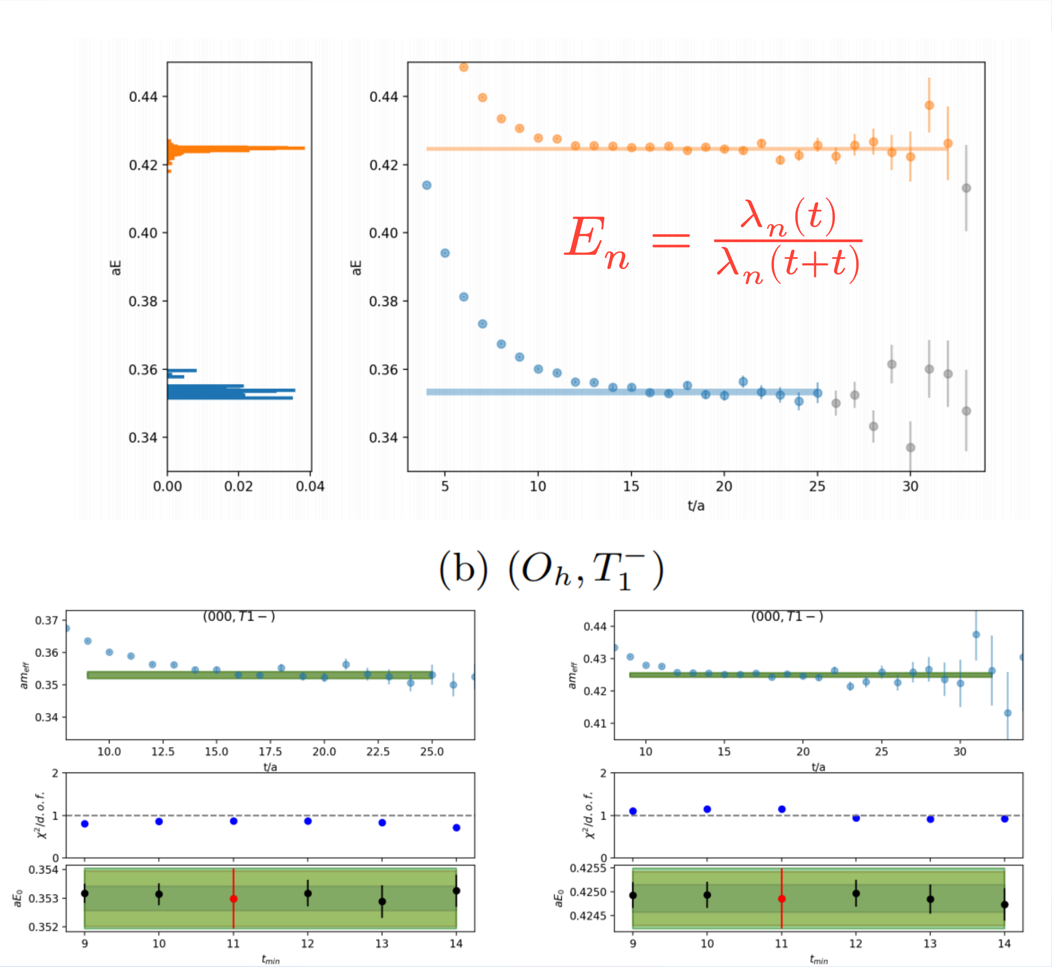
- Statistical uncertainties: **bootstrap resample method**
- Systematical uncertainties from fit windows: **AIC method**

$$\mathrm{AIC}^l = \chi^2_l + 2n^{\mathrm{para}} - n^l_{\mathrm{data}}$$

- Fit results is weighted by

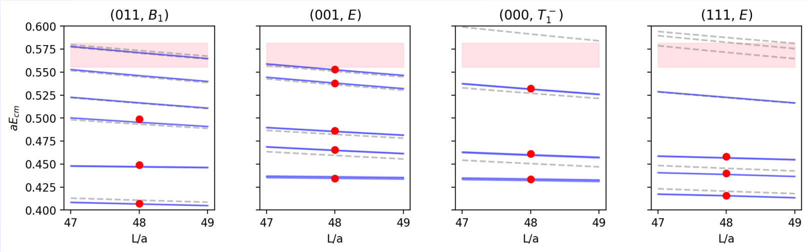
$$\omega^l \propto e^{-\frac{1}{2}\mathrm{AIC}^l}$$

P. Boyle et al., Phys. Rev. Lett. 134, 111901 (2025), Phys. Rev. D 111, 054510 (2025).

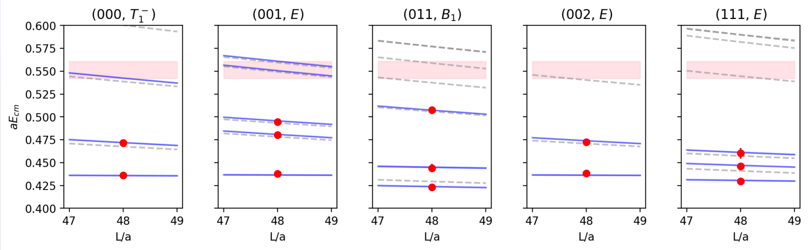


(b) (O_h, T_1^-)

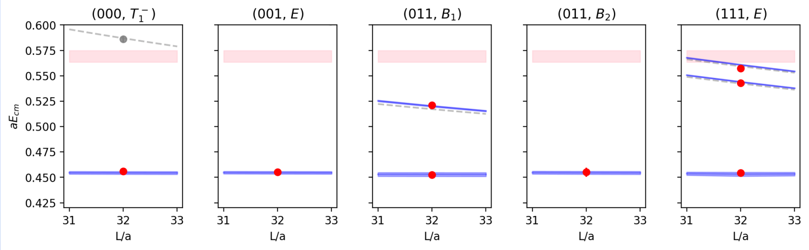
Finite-volume spectra



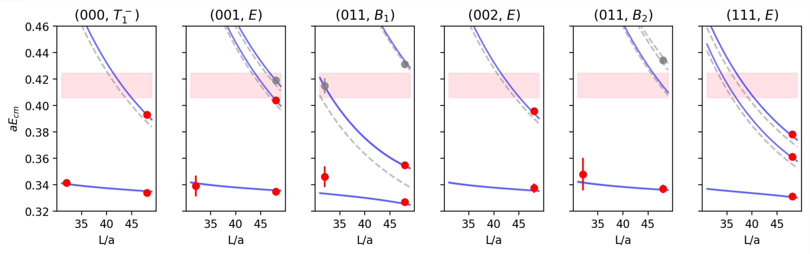
C48P14



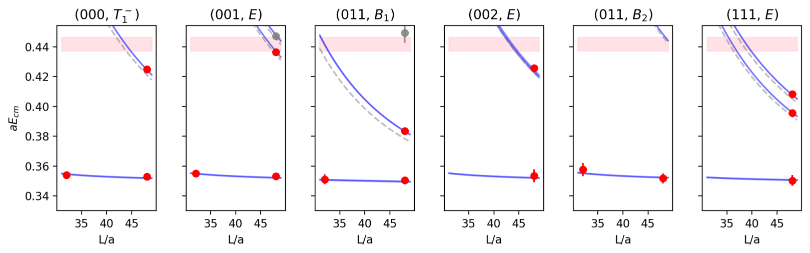
C48P23



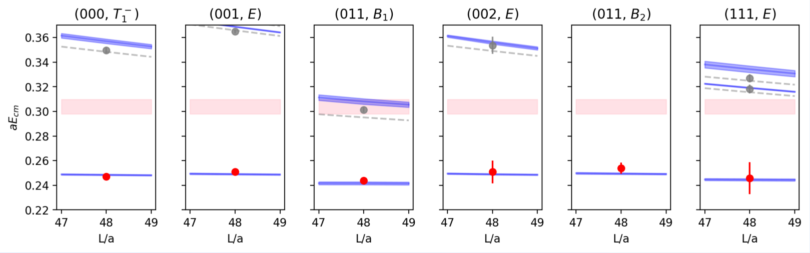
C32P29



F48P21 and F32P21



F48P30 and F32P30



H48P32

3. Numerical results

- Lüscher analysis

$$\cot \delta_1 = \frac{2}{\gamma L \sqrt{\pi}\, q^*} Z_{00}^{P}\left(1, q^* \frac{L}{2\pi}\right)$$

Parameterization of δ:

- K matrix
  - $K(s) = \frac{g^2}{m^2 - s}$ (BW)
  - effective range expansion (ERE)
- Product representation (PR)

$\boldsymbol{d}$	LG($\boldsymbol{d}$)	irrep	QCs
$(0,0,0)$	O_h	T_1^-	$\cot \delta_1 = \omega_{00}$
$(0,0,n)$	C_{4v}	E	$\cot \delta_1 = \omega_{00} - \omega_{20}$
$(n,n,0)$	C_{2v}	B_2	$\cot \delta_1 = \omega_{00} + 2\omega_{20}$
		B_1	$\cot \delta_1 = \omega_{00} - \omega_{20} - \sqrt{6}\,\mathrm{Im}\left[\omega_{22}\right]$
(n,n,n)	C_{3v}	E	$\cot \delta_1 = \omega_{00} + i\sqrt{6}\omega_{22}$

$$w_{\ell m} = w_{\ell m}^{P}(k, L) = \frac{Z_{\ell m}^{P}(1; (k\frac{L}{2\pi})^2)}{\gamma \pi^{3/2} \sqrt{2}\sqrt{2}\ell + 1 (k\frac{L}{2\pi})^{\ell+1}}$$

Product representation

- S matrix in terms of a product form:

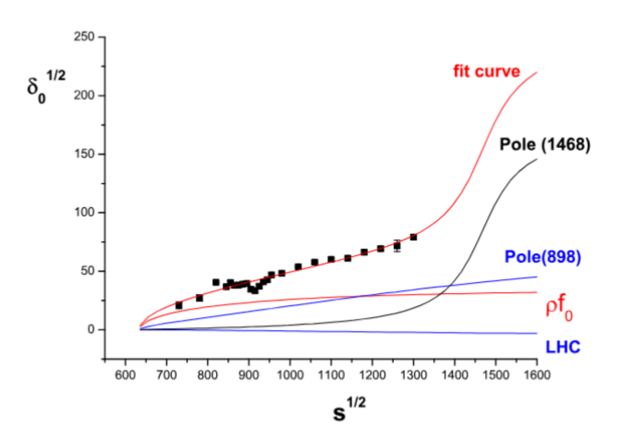
H. Q. Zheng et al., Nucl. Phys. A 733, 235 (2004)

left-hand cuts Construction

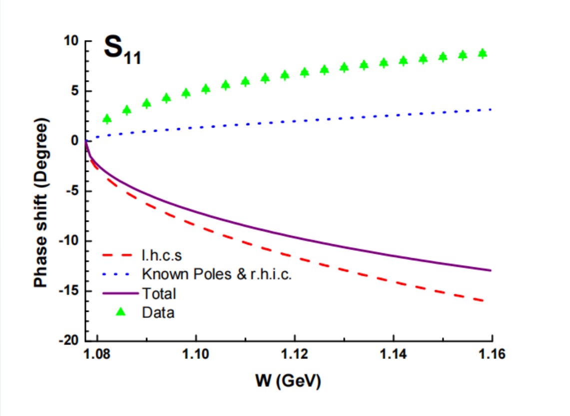
$$S(s) = S_v(s)S_b(s)S_r(s)\exp(2i\rho(s)f(s))$$

virtual state — bound state — resonance state

- Features
  - phase shifts are additive
  - virtual states and resonances: **positive**
  - bound states: **negative**
  - left-hand cuts: **usually negative**



H. Q. Zheng et al., Nucl. Phys. A 733, 235 (2004)

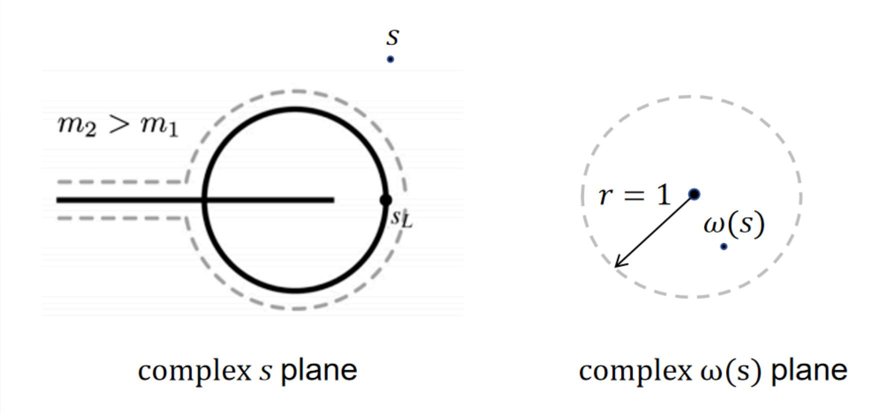


Y.F. Wang et al., Chin. Phys. C 43, 064110 (2019),

Conformal mapping

- $f(s)$ satisfies a dispersion relation:

$$f(s) = f(s_0) + \frac{1}{2\pi i}\int_L \frac{\mathrm{disc}_L f(z)}{(z-s)(z-s_0)} dz$$



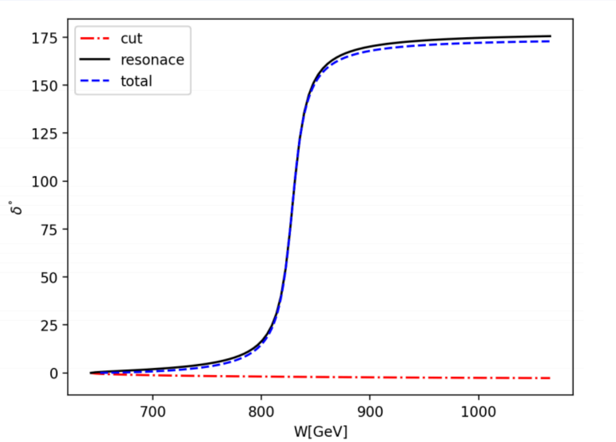
conformal mapping

- Expanding the $f(s)$ in terms of **conformal mapping variable $\omega(s)$**

$$f(s) = \sum_n C_n \omega^n(s)$$

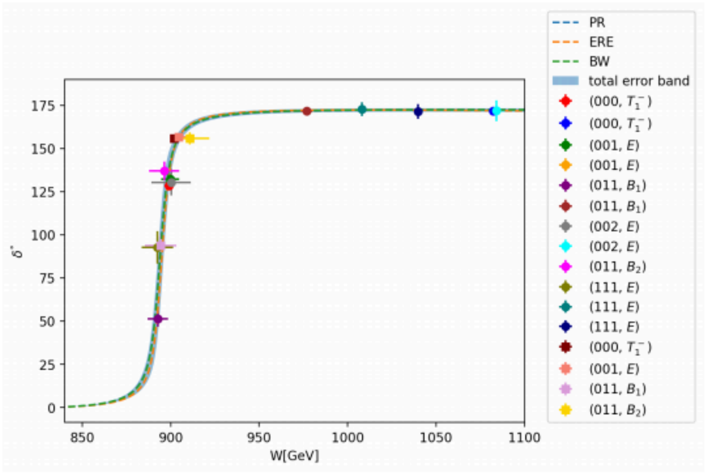
W. R. Frazer, Phys. Rev. 123, 2180 (1961).

$$\omega(s) = -\frac{(\sqrt{s}-\sqrt{s_E})(\sqrt{s}\sqrt{s_E}+s_-)}{(\sqrt{s}+\sqrt{s_E})(\sqrt{s}\sqrt{s_E}-s_-)}, \quad s_- = m_2^2 - m_1^2.$$

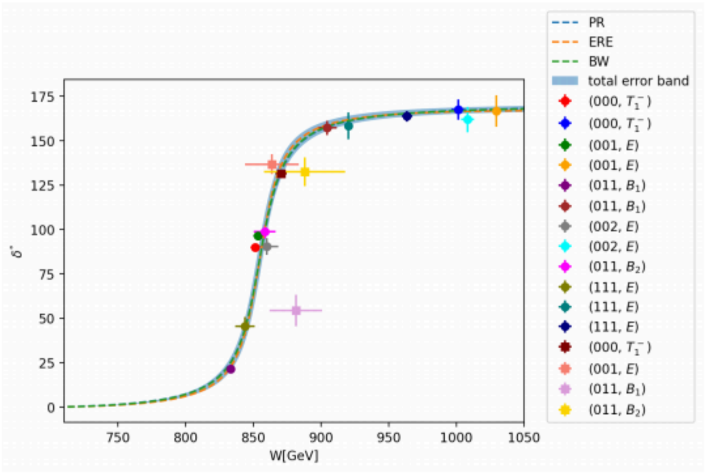


The PR decomposition.

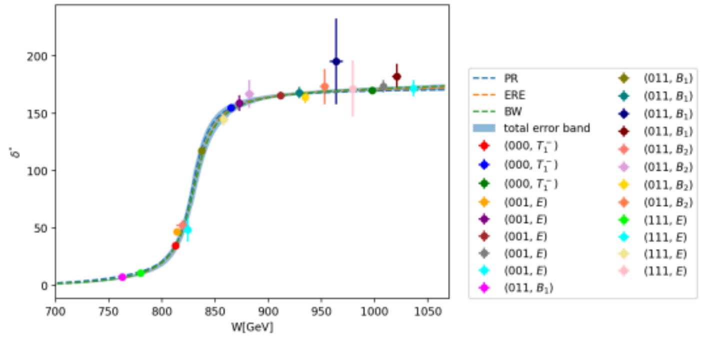
phase shifts and coupling constant



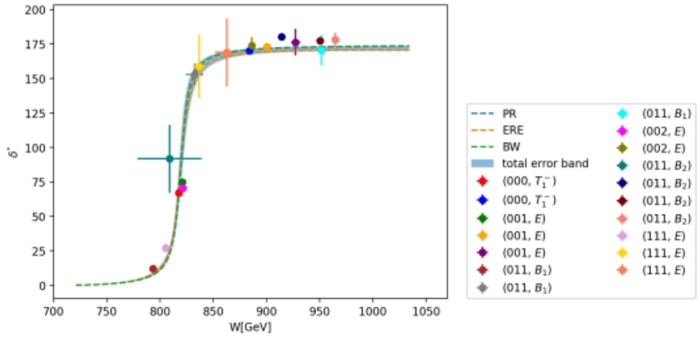
(a) F48P30 and F32P30



(b) F48P21 and F32P21

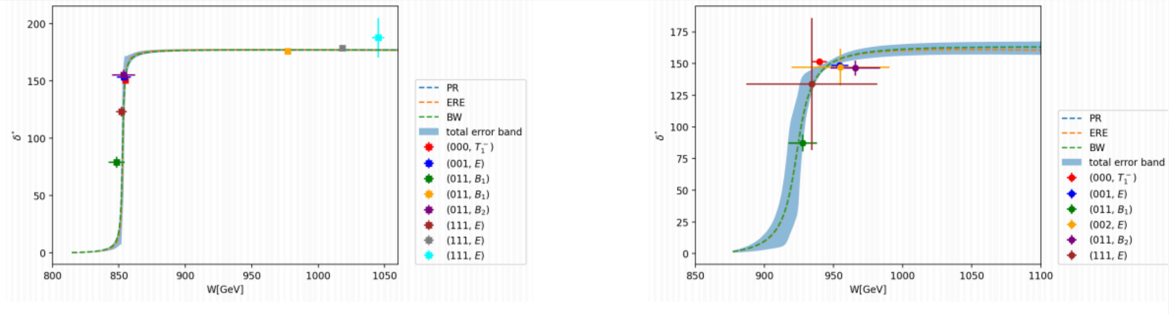


(c) C48P14

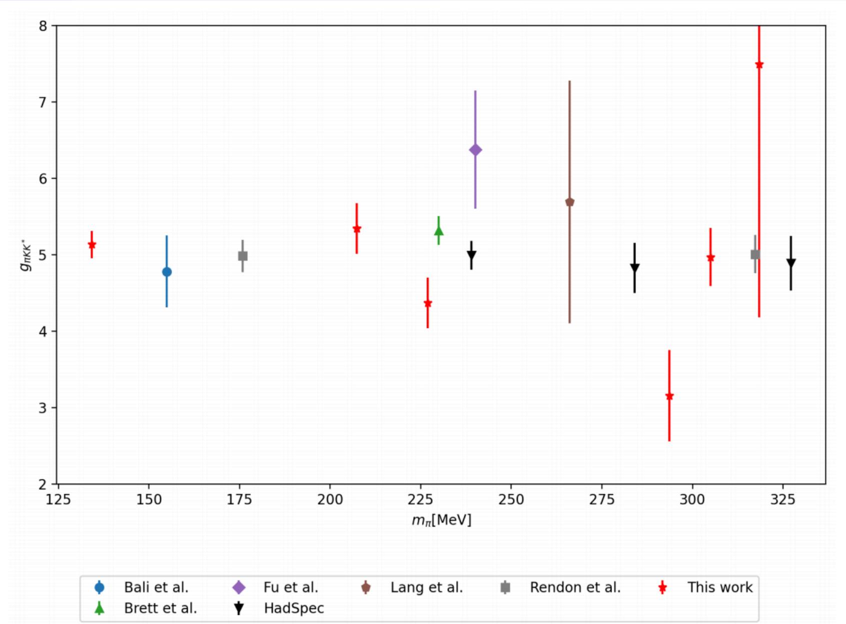


(d) C48P23

phase shifts and coupling constant



(e) C32P29 (f) H48P32



$$\Gamma = \frac{g_{\pi K K^*}}{6\pi} \frac{k_*^3}{m_R^2}$$

Extrapolation

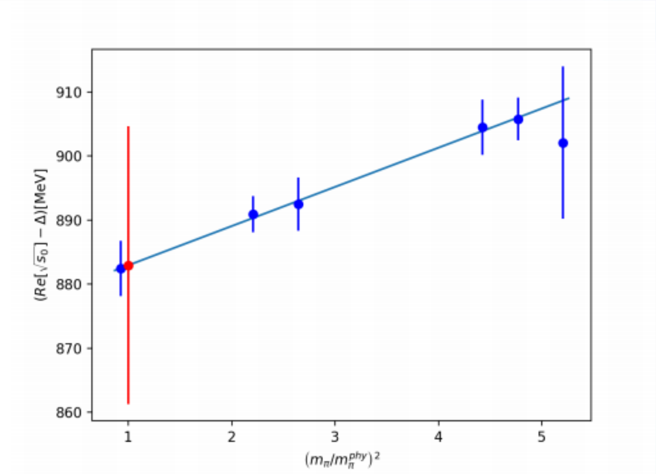
ensemble	pole postion (MeV)
F48P30	$894.2(1.8) + i3.50(47)$
F48P21	$853.6(1.9) + i12.9(1.6)$
C48P23	$819.4(1.6) + i5.52(96)$
C48P14	$828.6(1.8) + i12.90(94)$
C32P29	$852.2(1.8) + i0.94(33)$
H48P32	$923(12) + i7.4(2.4)$

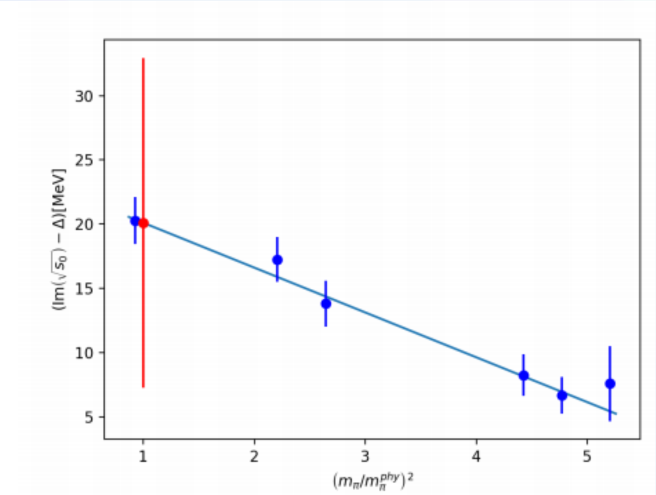
Pole positions for six ensembles

- Extrapolation formulas:

$$\mathrm{Re}(\sqrt{s_0}) = b_0^r + b_1^r m_{\pi,r}^2 + b_2^r m_{K,r}^2 + b_3^r a_r^2,$$

$$\mathrm{Im}(\sqrt{s_0}) = b_0^i + b_1^i m_{\pi,r}^2 + b_2^i m_{K,r}^2 + b_3^i a_r^2,$$





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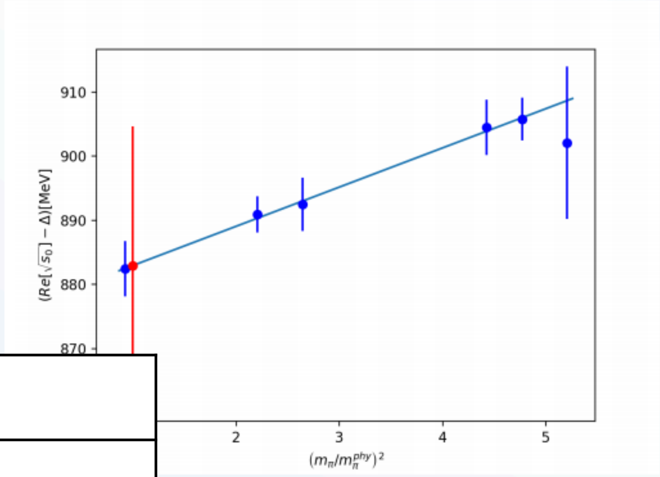
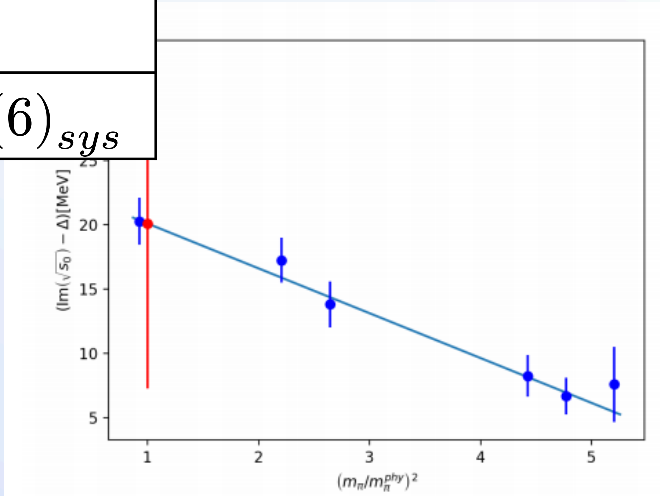
C48P23	$819.4(1.6) + i5.52(96)$
C48P14	[illegible]
C32P29	[illegible]
H48P32	[illegible]

Pole p

	pole position (MeV)
This work	$883(22) - i20(13)$
PDG	$890(14) - i26(6)$
Boyle et al.	$893(2)_{stat}(54)_{sys} - i26(1)_{stat}(6)_{sys}$





- Extrapolation formulas:

$$\mathrm{Re}(\sqrt{s_0}) = b_0^r + b_1^r m_{\pi,r}^2 + b_2^r m_{K,r}^2 + b_3^r a_r^2,$$

$$\mathrm{Im}(\sqrt{s_0}) = b_0^i + b_1^i m_{\pi,r}^2 + b_2^i m_{K,r}^2 + b_3^i a_r^2,$$

4. Conclusion and Outlook

- Conclusions:
  - Successful lattice study of $K^*(892)$.
  - Over **80** finite spectra from **6** pion mass and **3** lattice spacing ensembles.
  - Three amplitude models: BW, ERE, PR, yield consistent results.
  - Extrapolation: $883(22) - i20(13)$MeV, matches PDG.
  - Coupling $g_{\pi K K^*}$ constant vs. m_π.
- Outlooks:
  - Next: study broad κ resonance (S-wave).
  - Challenges: broad resonances.
  - Plan: Combine with unitarized χPT or Roy—Steiner equations.