# Hadronic Enhancement in Axion Thermalization

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报告人：王金葆　合作者：郭志辉 周海清

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- General QCD Lagrangian $\mathcal{L}_{\rm QCD} = \bar{q}\left(i\not{D} - M_q e^{i\theta_q\gamma_5}\right)q - \frac{1}{4}G^c_{\mu\nu}G^{\mu\nu,c} + \boxed{\theta\frac{\alpha_s}{8\pi}G\widetilde{G}}$, $\quad G\widetilde{G} \equiv \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}G^c_{\mu\nu}G^c_{\rho\sigma}$

  $\theta$-term

- QCD $\theta$-vacuum: $|\theta\rangle = \sum_{n\in\mathbb{Z}} e^{i\theta n}|n\rangle$, $\theta \in [0,2\pi)$ ⇐ Equivalent to adding the $\theta$-term in QCD Lagrangian

  Topological charge: $\frac{\alpha_s}{8\pi}\int d^4x\, G\widetilde{G} \in \mathbb{Z}$ dependent on pure-gauge configuration $A_\mu = \frac{i}{g_s}U^\dagger(x)\partial_\mu U(x)$

- $\theta$-term violates P and CP  True observable CPV parameter: $\boxed{\bar{\theta} = \theta + \mathrm{Tr}(\theta_q)}$

  $q \to e^{-i\frac{\theta_q}{2}\gamma_5}q$ ⇒ $\mathcal{L}_{\rm QCD} = \bar{q}\left(i\not{D} - M_q\right)q - \frac{1}{4}G^c_{\mu\nu}G^{\mu\nu,c} + \bar{\theta}\frac{\alpha_s}{8\pi}G\widetilde{G}$

  $\bar{\theta}$ contributes to nEDM: $H_{\rm nEDM} = -d_n\vec{E}\cdot\hat{S}$

  $$\begin{cases} d_n = 2.4(1.0)\times 10^{-16}\bar{\theta}\ e\ \mathrm{cm} \\ |d_n| < 3.0\times 10^{-26}\ e\ \mathrm{cm}\ (90\%\ \mathrm{CL}) \end{cases}$$

  Pospelov, 2000

  Pendlebury, 2015

  ⇒ $|\bar{\theta}| < 10^{-10}$

- Strong CP problem: Why $\bar{\theta}$ is so small? ⇒
  - $m_u = 0$ (excluded)
  - PQ mechanism
  - …

- PQ mechanism  Additional global $\mathrm{U(1)_{PQ}}$ symm. $\begin{cases} \text{explicitly broken by the } SU(3)_C \text{ anomaly} \\ \text{spontaneously broken at the scale } v_a \end{cases}$ ⟹ Axion

  Peccei and Quinn, 1977                                    Weinberg, Wilczek, 1978

$$\mathcal{L}_{\rm QCD} \supset \bar\theta \frac{\alpha_s}{8\pi} G\widetilde{G} \quad \Rightarrow \quad \left(\frac{a}{f_a} + \bar\theta\right) \frac{\alpha_s}{8\pi} G\widetilde{G}$$

Axion decay constant $f_a \sim v_a$

The axion potential is minimized at $\frac{\langle a \rangle}{f_a} + \bar\theta = 0$, dynamically solves the strong CP problem.

Vafa and Witten, 1984

- General axion couplings below $v_{EW}$, up to dim-5

  Di Luzio, 2020

$$\mathcal{L}_a \supset \frac{\partial_\mu a}{v_a} J^\mu_{PQ} + \frac{a}{v_a} N \frac{\alpha_s}{4\pi} G\widetilde{G} + \frac{a}{v_a} E \frac{\alpha_{em}}{4\pi} F\widetilde{F} = \frac{\partial_\mu a}{2f_a} \frac{1}{N} J^\mu_{PQ} + \boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi} G\widetilde{G}} + \frac{1}{4} g^0_{a\gamma\gamma} a F\widetilde{F}$$

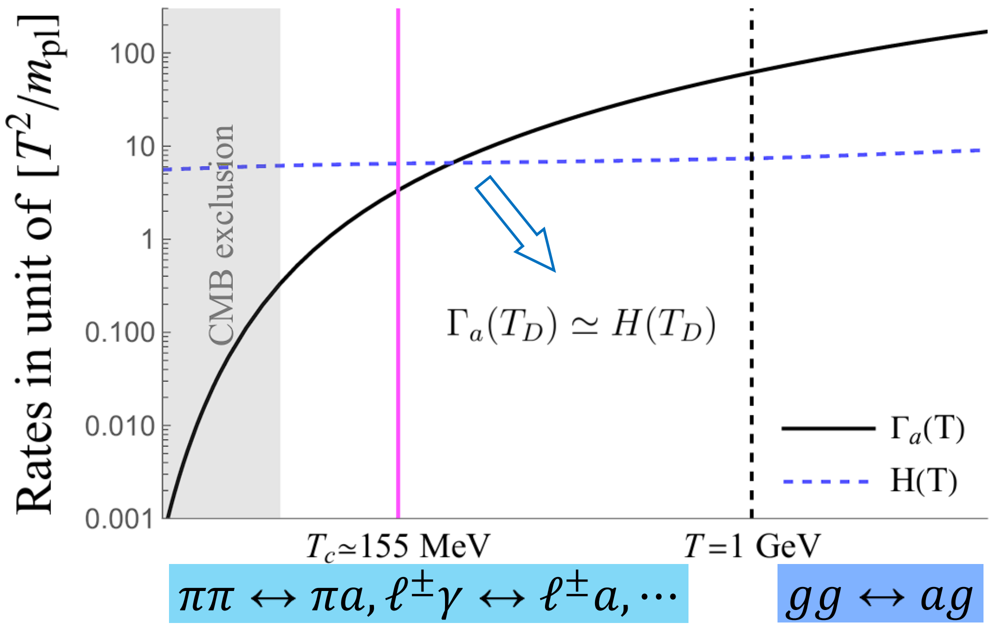
Model-independent operator

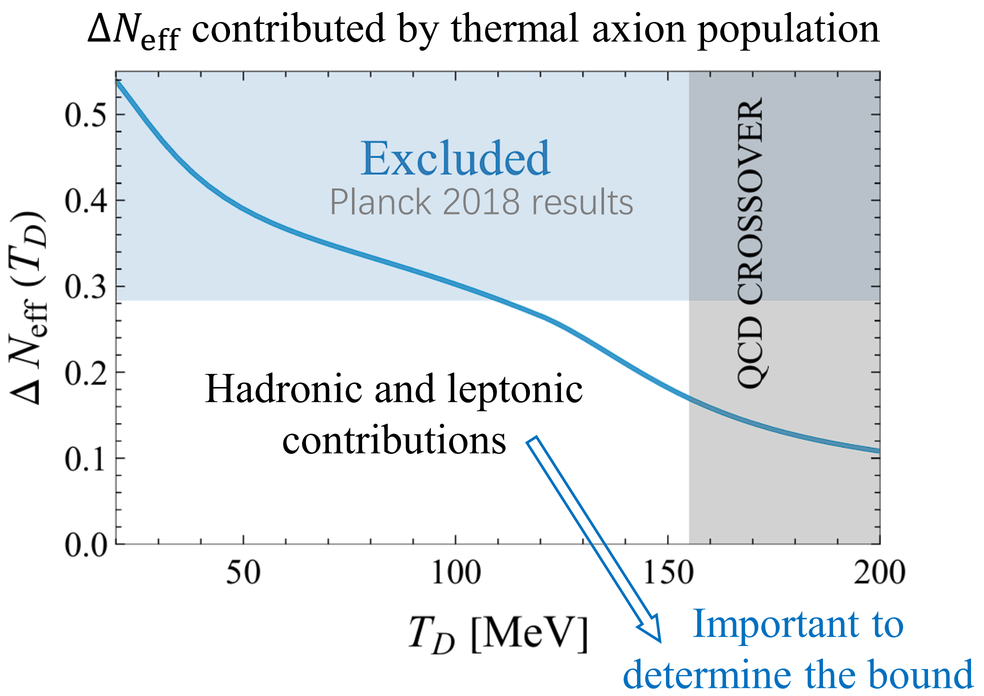
$$g^0_{a\gamma\gamma} = \frac{E}{N}\frac{\alpha_{em}}{2\pi f_a}$$

PQ current $J^\mu_{PQ}$ is composed of PQ-charged SM d.o.f.

$N$ and $E$ are model-dependent anomaly coefficients, $N \neq 0$ demanded by solving strong CP problem.

- Axion thermalization and decoupling Chang and Choi, 1993




- Axion thermalization rate contributed by $\underbrace{a + \cdots}_{i} \leftrightarrow \underbrace{1 + 2 + 3 + \cdots}_{j}$

$$\Gamma_a(T) = \frac{1}{n_a^{eq}} \int \left[\prod_i \frac{d^3 p_i}{(2\pi)^3 2E_i}\right] \left[\prod_j \frac{d^3 p_j}{(2\pi)^3 2E_j}\right] (2\pi)^4 \delta^4 \left(\sum_i p_i - \sum_j p_j\right) \boxed{|\mathcal{M}_{\text{reaction}}|^2} \prod_i f_i(x_i) \prod_j [1 + \eta_j f_j(x_j)]$$

$$f_i(x_i) = \frac{1}{e^{x_i} - \eta_i}, \quad x_i = \frac{E_i}{T}, \quad \eta_i = \begin{cases} +1 & \text{for bosonic} \\ -1 & \text{for fermic} \end{cases}$$

$aG\tilde{G}$ coupling induces model-independent hadronic contribution,

- dominated by $a\pi \leftrightarrow \pi\pi$ reaction below $T_c$ ⟹ Questioning in the presence of **dynamical enhancement**
  assumed by all previous calculation

- $\Gamma_a(T)$ contributed by $aK \leftrightarrow \pi K$ reaction?

  Boltzmann suppressed compared to the $a\pi$ channel by $\left[\dfrac{n_B(m_\pi)}{n_B\left(\frac{m_\pi+m_K}{2}\right)}\right]^2 \simeq 2\%$ (at $T \simeq 110$ MeV)     **Dynamical enhancement?**

- Effective Lagrangian $(N_f = 3)$

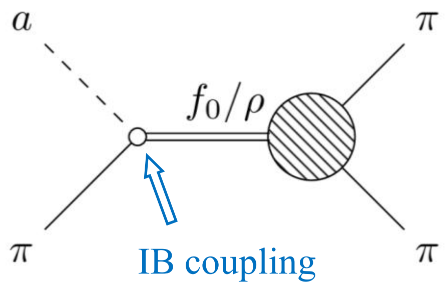
$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\widetilde{G} + \mathcal{L}_{\rm QCD} \xrightarrow[Q_a = M^{-1}/\langle M^{-1}\rangle]{q \to e^{i\frac{a}{2f_a}Q_a\gamma_5}q} \mathcal{L}' \supset \underbrace{-\frac{\partial_\mu a}{2f_a}C_3\bar{q}\gamma^\mu\gamma_5\frac{\lambda_3}{2}q}_{\text{IB part}} \underbrace{-\frac{\partial_\mu a}{2f_a}\sum_{i=0,8}C_i\bar{q}\gamma^\mu\gamma_5\frac{\lambda_i}{2}q - C_S\frac{a}{f_a}\frac{i}{B_0}\bar{q}\gamma_5 q}_{\text{IC part}}$$

$$z = \frac{m_u}{m_d}, \quad r = \frac{m_s}{\hat{m}}, \quad \hat{m} = \frac{1}{2}(m_u + m_d)$$

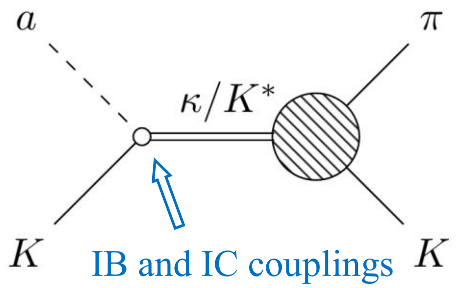
$$C_0 = \sqrt{\frac{2}{3}}, \quad C_3 = \frac{r(1-z^2)}{2z + r(1+z)^2} \overset{r\to\infty}{\simeq} \frac{1}{3}, \quad C_8 = \frac{r(1+z)^2 - 4z}{\sqrt{3}\,[2z + r(1+z)^2]} \overset{r\to\infty}{=} \frac{1}{\sqrt{3}}, \quad C_S = \frac{2B_0\hat{m}}{\frac{(1+z)^2}{z} + \frac{2}{r}}.$$

- $a\pi \to \pi\pi$ violates G-parity

$a\pi^0 \to \pi^+\pi^-, \pi^0\pi^0$
$a\pi^\pm \to \pi^\pm\pi^0$




$aK^+ \to \pi^+K^0, \pi^0K^+$
$aK^- \to \pi^-\bar{K}^0, \pi^0K^-$
$aK^0 \to \pi^0K^0, \pi^-K^+$
$a\bar{K}^0 \to \pi^0\bar{K}^0, \pi^+K^-$



- Lowest order chiral Lagrangian for axion and mesons

$$\mathcal{L}_\chi = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{F_\pi^2}{4}\left\langle \partial_\mu U \partial^\mu U^\dagger + 2B_0\left[M(a)U^\dagger + UM^\dagger(a)\right]\right\rangle - \frac{\partial_\mu a}{2f_a}\sum_{i=3,8} C_i\left(i\frac{F_\pi^2}{4}\left\langle \lambda_i\{\partial^\mu U, U^\dagger\}\right\rangle\right)$$

$$U = \exp\left(\frac{i\phi}{F}\right), \quad \phi = \sum_{i=1}^{8}\lambda_i\phi_i = \begin{pmatrix} \pi^3 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^3 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 \end{pmatrix}. \quad M(a) = Me^{-i\frac{a}{f_a}Q_a}$$

- Leading order scattering amplitudes

$$\mathcal{M}^{(2)}_{a\pi^0\to\pi^+\pi^-} = \frac{C_3\left(s-m_\pi^2\right)}{2f_aF_\pi} - \frac{2C_S\delta_I x_{\pi\eta}}{\sqrt{3}f_aF_\pi}, \quad \mathcal{M}^{(2)}_{a\pi^0\to\pi^0\pi^0} = -\frac{2\sqrt{3}C_S\delta_I x_{\pi\eta}}{f_aF_\pi}, \qquad x_{\pi\eta} = -\frac{1}{\sqrt{3}\left(m_\eta^2-m_\pi^2\right)}$$

$$\mathcal{M}^{(2)}_{aK^+\to\pi^+K^0} = \frac{C_3\left(m_{K^0}^2 - m_{K^\pm}^2 + s - u\right)}{4\sqrt{2}f_aF_\pi} + \frac{\sqrt{3}C_8\left(t-m_\pi^2\right)}{4\sqrt{2}f_aF_\pi} + \frac{\sqrt{2}C_S}{f_aF_\pi},$$

$$\mathcal{M}^{(2)}_{aK^+\to\pi^0K^+} = \frac{t-m_\pi^2}{8f_aF_\pi}\left[C_3(1-\sqrt{3}\delta_I x_{\pi\eta}) + C_8(\sqrt{3}-3\delta_I x_{\pi\eta})\right] + \frac{C_S\left(3+\sqrt{3}\delta_I x_{\pi\eta}\right)}{3f_aF_\pi},$$

$$\mathcal{M}^{(2)}_{aK^0\to\pi^0K^0} = \frac{t-m_\pi^2}{8f_aF_\pi}\left[C_3(1+\sqrt{3}\delta_I x_{\pi\eta}) - C_8(\sqrt{3}+3\delta_I x_{\pi\eta})\right] - \frac{C_S\left(3-\sqrt{3}\delta_I x_{\pi\eta}\right)}{3f_aF_\pi}.$$

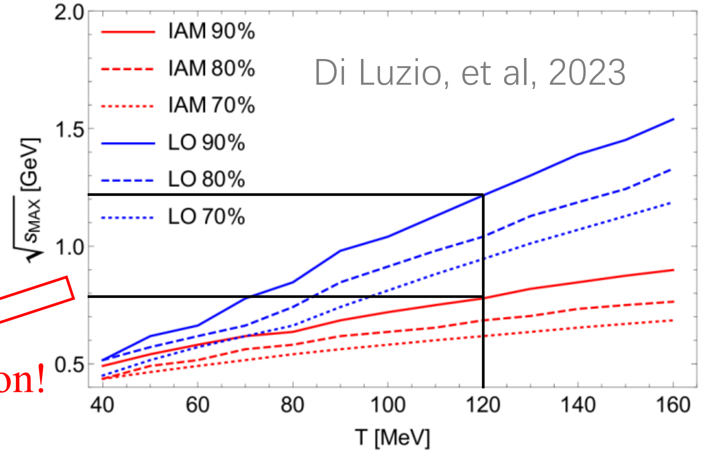
other amplitudes by crossing…

- Axion rate from $a\pi \leftrightarrow \pi\pi$ reaction

$$\Gamma_a(T) = \frac{1}{n_a^{eq}} \int \mathrm{d}\widetilde{\Gamma} \sum |\mathcal{M}_{a\pi\to\pi\pi}|^2 n_B(E_1) n_B(E_2) \left[1 + n_B(E_3)\right] \left[1 + n_B(E_4)\right]$$

$$\mathrm{d}\widetilde{\Gamma} = \left[\prod_{i=1}^{4} \frac{\mathrm{d}^3 p_i}{(2\pi)^3} \frac{1}{2E_i}\right] (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)$$

covers broad energy region




Requires going beyond the chiral expansion!

- Coupled-channel unitarized amplitude

  - Partial waves $\mathcal{M}^J_{i\to f}(s) = \frac{1}{2(\sqrt{2})^{N_i+N_f}} \int_{-1}^{+1} \mathrm{d}\cos\theta\, P_J(\cos\theta) \mathcal{M}_{i\to f}(s,\cos\theta)$

  - Unitarized amplitudes including FSIs   Oller J. A., Oset E. and Pelaez J. R, 1999

$$T^{uni} = T^{(2)} \cdot \left[T^{(2)} - T^{(4)\mathrm{LECs}} - T^{(2)} \cdot \mathcal{G} \cdot T^{(2)}\right]^{-1} \cdot T^{(2)},$$

$$\vec{\mathcal{M}}^{uni} = T^{(2)} \cdot \left[T^{(2)} - T^{(4)\mathrm{LECs}} - T^{(2)} \cdot \mathcal{G} \cdot T^{(2)}\right]^{-1} \cdot \vec{\mathcal{M}}^{(2)},$$

$T^{(2)}$: $\mathcal{O}(p^2)$ meson scattering

$T^{(4)\mathrm{LECs}}$: $\mathcal{O}(p^4)$ LECs part

$\mathcal{G}$ $= \mathrm{diag}(G_1, G_2, \ldots)$, $\mathrm{Im}G = \rho$

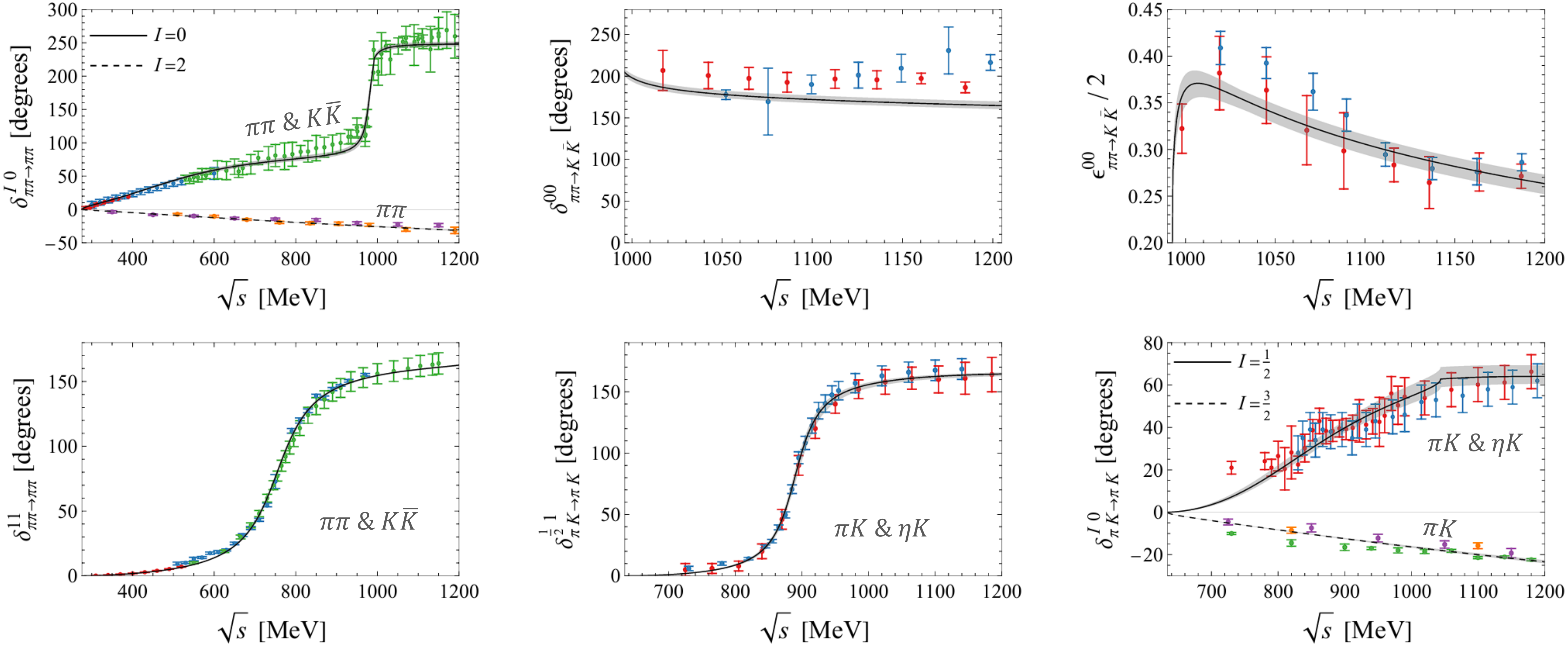
✓ Chiral symm. & unitarity

✓ Mesonic resonances generated dynamically

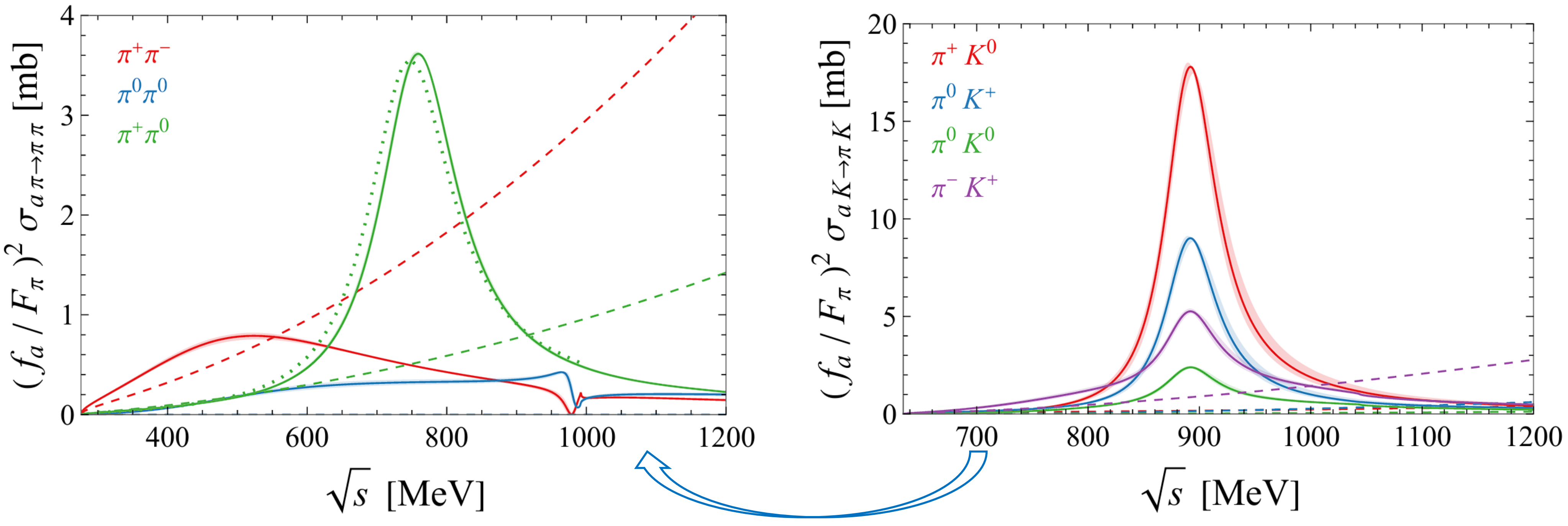
5 subtraction constants & 8 LECs fitted by meson scattering data

## ● Fitting results

Up to $\sqrt{s} = 1.2$ GeV



- Cross sections

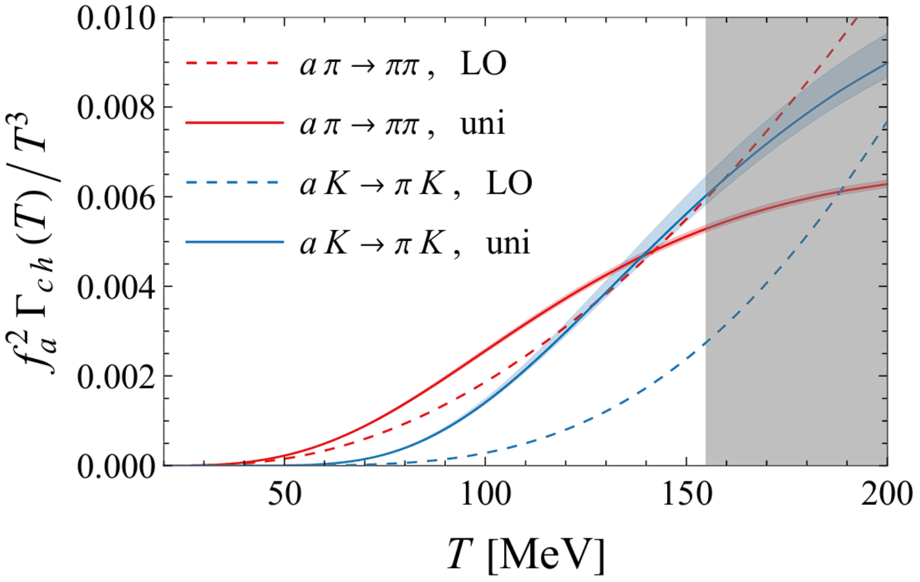


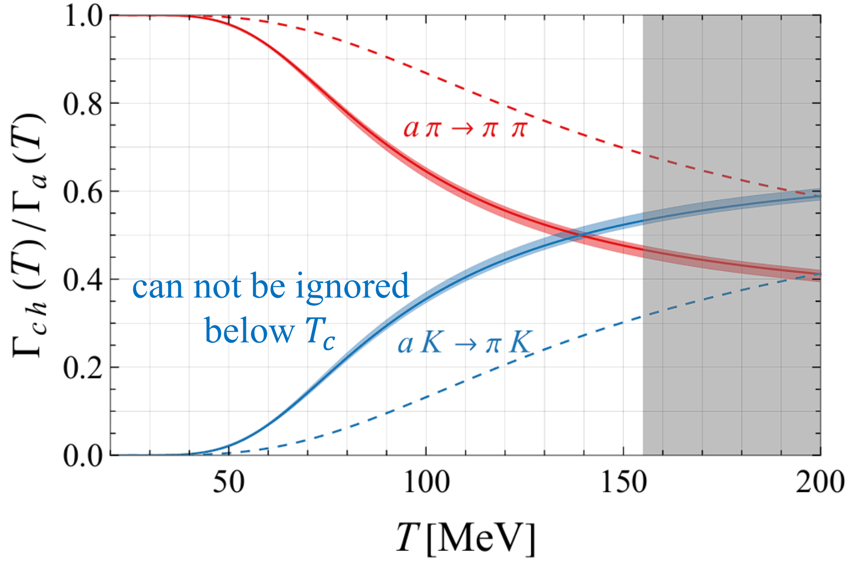

Four times larger at the squared-amplitude level

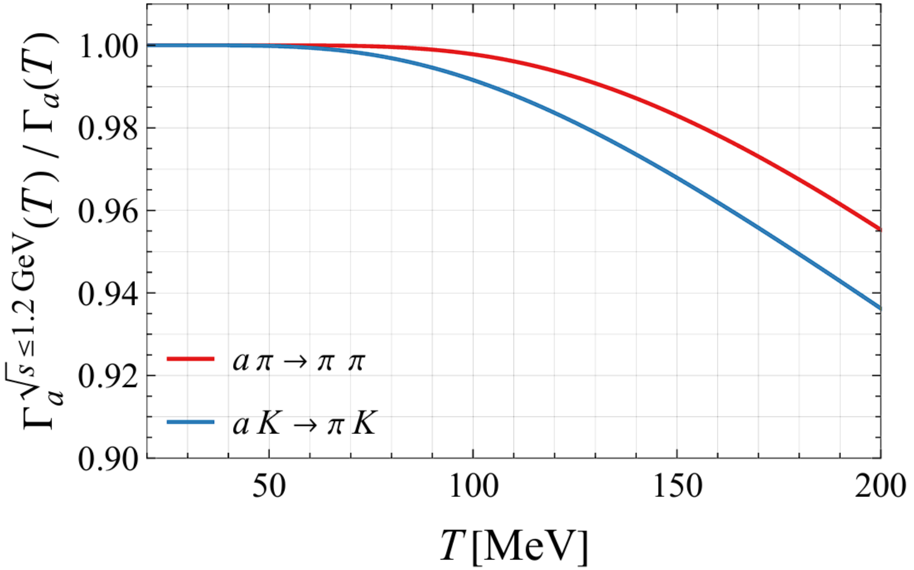
➢In $aK^+$ channels, $C_3$ and $C_8$ contributions interfere: constructively in $IJ = \frac{1}{2}1$ sector; destructively in $IJ = \frac{1}{2}0$ sector.

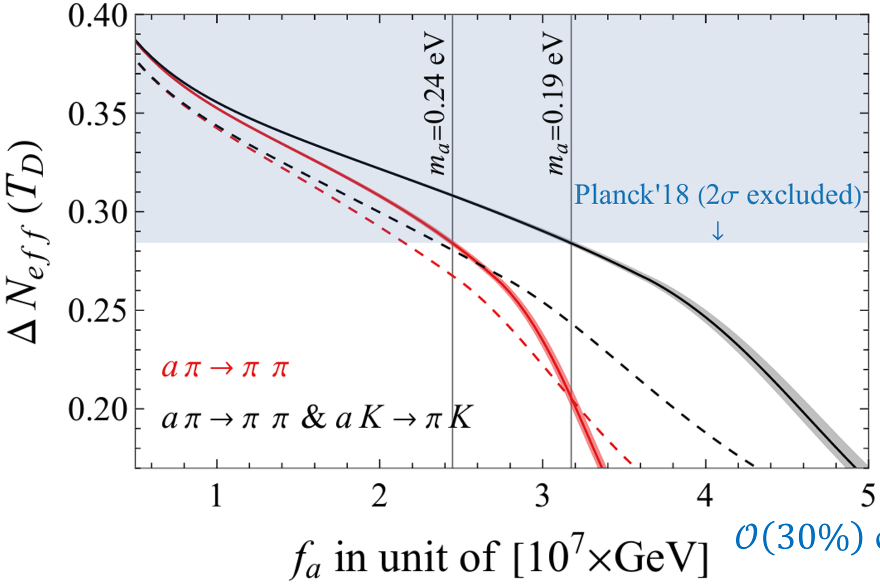
➢In $aK^0$ channels, $C_3$ and $C_8$ contributions interfere: constructively in $IJ = \frac{1}{2}0$ sector; destructively in $IJ = \frac{1}{2}1$ sector.

- Axion rate and bound




$\mathcal{O}(30\%)$ correction

# Summary

- Axion thermalization rate below $T_c$ constitutes a crucial input to determine the current cosmological bound on hot axion. Focusing model-independent contribution:

  - The $a\pi \to \pi\pi$ and $aK \to \pi K$ scattering amplitudes are calculated consistently in a coupled-channel unitarization framework.
  - Accounting for resonant enhancement properly, $aK \to \pi$K contribution becomes comparable to $a\pi \to \pi\pi$ at $T \simeq 110$ MeV and is therefore non-negligible.

Thank you for your listening