



南華大學

UNIVERSITY OF SOUTH CHINA

# FBA induced $CP$ asymmetry in $\bar{B}^0 \rightarrow K^- \pi^+ \pi^0$ in phase space around the resonances $\bar{K}^*(892)^0$ and $\bar{K}_0^*(700)$

Jian-Yu Yang (杨健宇)

Collaborators: 赵宇杰, 祁敬娟, 张振华

Based on: *Chin. Phys. C* 50, 053102 (2026)

第五届强子与重味物理理论与实验联合研讨会

河北师范大学, 石家庄

**1** Introduction

## 2 Theoretical Framework

## 3 Results and Analysis

## 4 Summary

# Motivation & Overview

## Motivation

- In multi-body decays, interference among resonances induces significant CP asymmetry.

## Target Channel

- **Decay Process:**  $\bar{B}^0 \rightarrow K^- \pi^+ \pi^0$
- **Dominant Resonances:**  $K^*(892)^0$  and  $K_0^*(700)$

## What We Did

- **Key Observables:** Investigated Forward-Backward Asymmetry (FBA) and FB-CPA to study the interference effects.
- **Main Results:** Analyzed the **non-trivial correlation** between FBA and FB-CPA.

FB-CPAs can be found around  $K^*(892)^0$ .

- 1 Introduction
- 2 Theoretical Framework**
- 3 Results and Analysis
- 4 Summary

# Helicity Angle and FBA

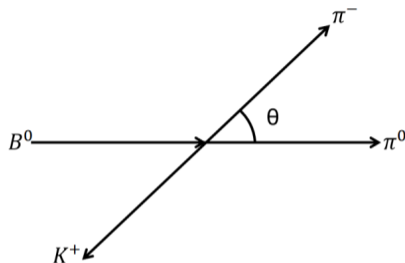


Figure 1: Definition of  $\theta$  in the  $\bar{B}^0 \rightarrow K^- \pi^+ \pi^0$  decay channel

- **FBA Definition:** The event yield difference of  $\pi^0$  flying forward ( $\cos \theta > 0$ ) versus backward ( $\cos \theta < 0$ ) in the  $K^+ \pi^-$  C.O.M. frame:

$$A^{FB} = \frac{\int_0^1 \langle |\mathcal{M}|^2 \rangle d \cos \theta - \int_{-1}^0 \langle |\mathcal{M}|^2 \rangle d \cos \theta}{\int_{-1}^1 \langle |\mathcal{M}|^2 \rangle d \cos \theta}$$

# Derivation of FBA Formula

- **Total Amplitude (S & P waves):**  $\mathcal{M} = \mathcal{A}_S + \mathcal{A}_P \cos \theta$  (Resonances:  $S = \bar{K}_0^*(700)$ ,  $P = \bar{K}^*(892)^0$ )

↓ *Modulus squared*

$$\langle |\mathcal{M}|^2 \rangle = \langle |\mathcal{A}_S|^2 \rangle + \langle |\mathcal{A}_P|^2 \rangle \cos^2 \theta + \underbrace{2\Re(\langle \mathcal{A}_P \mathcal{A}_S^* \rangle)}_{\text{Interference Term}} \cos \theta$$

↓ *Integrate over cos θ*

- **Denominator**  $\left( \int_{-1}^1 \right) \Rightarrow 2\langle |\mathcal{A}_S|^2 \rangle + \frac{2}{3}\langle |\mathcal{A}_P|^2 \rangle + 0$  *(Odd interference term vanishes)*

- **Numerator**  $\left( \int_0^1 - \int_{-1}^0 \right) \Rightarrow 0 + 0 + 2\Re(\langle \mathcal{A}_P \mathcal{A}_S^* \rangle)$  *(Even terms cancel out)*

↓ *Substitute back & simplify*

$$A^{FB} = \frac{\Re(\langle \mathcal{A}_P \mathcal{A}_S^* \rangle)}{\langle |\mathcal{A}_S|^2 \rangle + \langle |\mathcal{A}_P|^2 \rangle / 3}$$

# Interference Observables and CP Asymmetries

- The FBA precisely isolates the real part of the S-P wave interference:

$$A^{FB} = \frac{\Re(\langle \mathcal{A}_P \mathcal{A}_S^* \rangle)}{\langle |\mathcal{A}_S|^2 \rangle + \langle |\mathcal{A}_P|^2 \rangle / 3}$$

- **CP-Averaged FBA** (Describes the overall interference effect):

$$A_{ave}^{FB} = \frac{1}{2} \left( A^{FB} + \overline{A^{FB}} \right)$$

- **FB-CPA** (Extracts the pure CP-violating asymmetry):

$$A_{CP}^{FB} = \frac{1}{2} \left( A^{FB} - \overline{A^{FB}} \right)$$

# Amplitude Parameterization

- In the resonance region, the decay is dominated by  $\bar{B}^0 \rightarrow \bar{K}_0^*(700)(\rightarrow K^- \pi^+) \pi^0$  and  $\bar{B}^0 \rightarrow \bar{K}^*(892)^0(\rightarrow K^- \pi^+) \pi^0$  cascade processes, allowing the amplitude to be parameterized as:

$$\mathcal{M}_{\bar{B}^0 \rightarrow K^- \pi^+ \pi^0} = \mathcal{A}_S + \mathcal{A}_P e^{i\delta} \cos \theta$$

- where  $\delta$  is the relative phase between them. We can further isolate the Breit-Wigner factor,

$$\mathcal{A}_X = \frac{\tilde{\mathcal{A}}_X}{s - m_X^2 + im_X \Gamma_X}$$

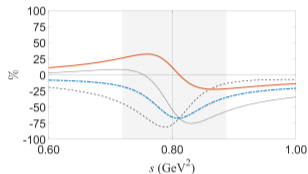
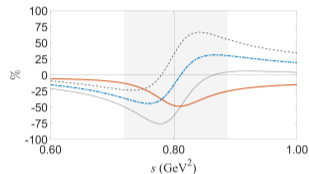
- for  $X = S$  and  $P$ ,

$$\begin{aligned} \tilde{\mathcal{A}}_P &= \sqrt{2} m_{K^*0} g_{K^*0 K \pi} \left( \frac{s_{\pi^0 \pi^-, \max} - s_{\pi^0 \pi^-, \min}}{2} \right) \cdot \left\{ V_{ub} V_{us}^* a_2 f_\pi A_0^{B \rightarrow K^*0} \right. \\ &\quad \left. + V_{tb} V_{ts}^* \left[ \left( a_4 - \frac{1}{2} a_{10} \right) f_{K^*(892)^0} F_1^{B \rightarrow \pi} + \frac{3}{2} (a_7 - a_9) f_\pi A_0^{B \rightarrow K^*0} \right] \right\} \\ \tilde{\mathcal{A}}_S &= \sqrt{2} g_{K_0^* K \pi} \cdot \left( - V_{ub} V_{us}^* a_2 (m_B^2 - m_{K_0^*}^2) f_\pi F_0^{B \rightarrow K_0^*} \right. \\ &\quad \left. + V_{tb} V_{ts}^* \left\{ \left[ a_4 - \frac{1}{2} a_{10} - \frac{2m_{K_0^*}^2}{m_b m_s} \left( a_6 - \frac{1}{2} a_8 \right) \right] (m_B^2 - m_\pi^2) f_{K_0^*} F_0^{B \rightarrow \pi} \right. \right. \\ &\quad \left. \left. - \frac{3}{2} (a_7 - a_9) (m_B^2 - m_{K_0^*}^2) f_\pi F_0^{B \rightarrow K_0^*} \right\} \right) \end{aligned}$$

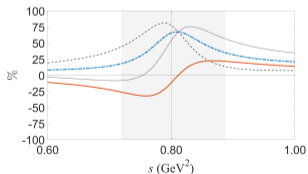
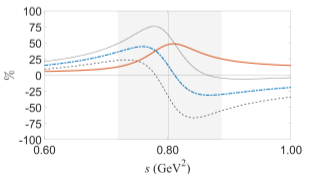
- 1 Introduction
- 2 Theoretical Framework
- 3 Results and Analysis**
- 4 Summary

# Numerical Results: $s$ -dependence for $N_c^{eff} = 1$

$$a_i = c_i + c_{i+1}/N_c^{eff} \text{ for odd } i \text{ and } a_i = c_i + c_{i-1}/N_c^{eff} \text{ for even } i$$

(a)  $\delta = 0$ (b)  $\delta = \pi/2$ 

The sign behaviors of  $A_{ave}^{FB}$  and  $A_{CP}^{FB}$  are **correlated!**

(c)  $\delta = \pi$ (d)  $\delta = 3\pi/2$ 

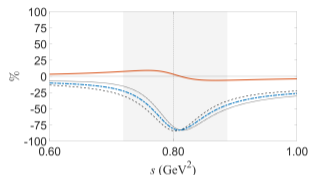
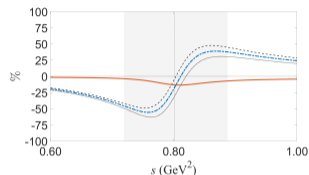
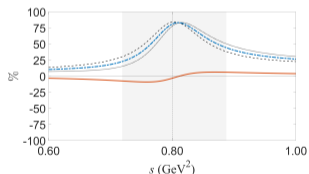
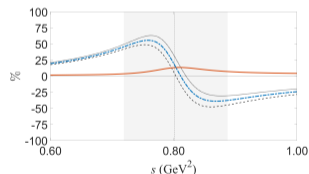
$$\delta \sim 0, \pi \implies A_{CP}^{FB} \text{ changes sign}$$

$$\delta \sim 0, \pi \implies A_{ave}^{FB} \text{ NO sign change}$$

$$\delta \sim \pi/2, 3\pi/2 \implies A_{CP}^{FB} \text{ NO sign change}$$

$$\delta \sim \pi/2, 3\pi/2 \implies A_{ave}^{FB} \text{ changes sign}$$

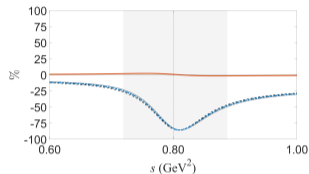
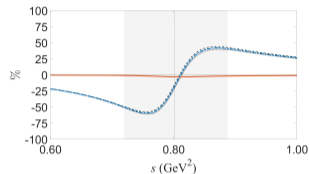
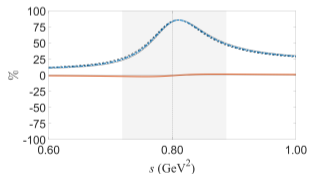
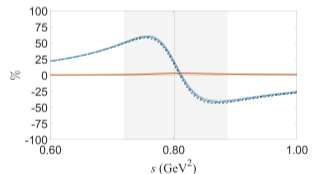
# Numerical Results: $s$ -dependence for $N_c^{eff} = 2$

(a)  $\delta = 0$ (b)  $\delta = \pi/2$ (c)  $\delta = \pi$ (d)  $\delta = 3\pi/2$ 

—  $A_{CP}^{FB}$   
- - -  $A_{ave}^{FB}$

- The CP violation observables FB-CPAs are much more sensitive to the values of  $N_c^{eff}$
- A **non-zero** FB-CPAs lies in the regions of  $\bar{K}^*(892)^0$  and  $\bar{K}_0^*(700)$  for  $N_c^{eff} = 1$  and 2

# Numerical Results: $s$ -dependence for $N_c^{eff} = 3$

(a)  $\delta = 0$ (b)  $\delta = \pi/2$ (c)  $\delta = \pi$ (d)  $\delta = 3\pi/2$ 

—  $A_{CP}^{FB}$   
- - -  $A_{ave}^{FB}$

- The FB-CPAs is negligibly **small** for  $N_c^{eff} = 3$
- $A_{CP}^{FB}$  strongly depends on non-factorizable effects

# Key Points from Numerical Results

The numerical results highlight two critical phenomena:

- **Sign-Change mechanism:** The underlying dynamics that drive  $A_{CP}^{FB}$  to **change sign (or remain unchanged)** when  $s$  passes through the resonance peak  $m_P^2$ .
- **Observational correlation:** The physical origin of the approximately **correlated behavior** between the two observables.  
( $A_{ave}^{FB}$  changes sign  $\iff A_{CP}^{FB}$  remains unchanged, and vice versa)

## Underlying Mechanism

# Analysis

By isolating the Breit-Wigner factors, the interference term in FBA can be expressed as

$$\Re(\mathcal{A}_S^* \mathcal{A}_P e^{i\delta}) = \lambda \left\{ \left[ 1 + \frac{(s - m_P^2)(s - m_S^2)}{m_P \Gamma_P m_S \Gamma_S} \right] \Re(\tilde{\mathcal{A}}_S^* \tilde{\mathcal{A}}_P e^{i\delta}) + \left[ -\frac{s - m_P^2}{m_P \Gamma_P} + \frac{s - m_S^2}{m_S \Gamma_S} \right] \Im(\tilde{\mathcal{A}}_S^* \tilde{\mathcal{A}}_P e^{i\delta}) \right\}$$

The normalization factor:

$$\lambda = \frac{m_P \Gamma_P m_S \Gamma_S}{\left[ (s - m_P^2)^2 + (m_P \Gamma_P)^2 \right] \left[ (s - m_S^2)^2 + (m_S \Gamma_S)^2 \right]}$$

$\Gamma_S$  is considerably larger than  $\Gamma_P$ .

Since we focus mainly on the region when  $s$  is around the mass-squared of  $S$  and  $P$ , we have  $\Gamma_s \gg \sqrt{|s - m_S^2|}$  and

$$\Gamma_s \gg \sqrt{|s - m_P^2|}.$$

# Analysis

After the approximation:

$$\Re(\mathcal{A}_S^* \mathcal{A}_P e^{i\delta}) = \lambda \left[ \Re(\tilde{\mathcal{A}}_S^* \tilde{\mathcal{A}}_P e^{i\delta}) - \frac{s - m_P^2}{m_P \Gamma_P} \Im(\tilde{\mathcal{A}}_S^* \tilde{\mathcal{A}}_P e^{i\delta}) + \mathcal{O}(\epsilon) \right]$$

$$\left( \frac{\Gamma_P}{\Gamma_S}, \frac{\sqrt{|s - m_S^2|}}{\Gamma_S}, \frac{\sqrt{|s - m_P^2|}}{\Gamma_S} \sim \mathcal{O}(\epsilon) \ll 1 \right)$$

Up to the order  $\mathcal{O}(\epsilon)$ , the interference term can be strictly splitted into two parts:

$$\Re(\mathcal{A}_S^* \mathcal{A}_P e^{i\delta}) = \lambda (\Delta + \Sigma)$$

Where

$$\Delta = \left( -\sin \delta - \frac{s - m_P^2}{m_P \Gamma_P} \cos \delta \right) \Im(\tilde{\mathcal{A}}_S^* \tilde{\mathcal{A}}_P)$$

$$\Sigma = \left( -\frac{s - m_P^2}{m_P \Gamma_P} \sin \delta + \cos \delta \right) \Re(\tilde{\mathcal{A}}_S^* \tilde{\mathcal{A}}_P)$$

## Answer

- CP-Conjugation Properties (If  $\delta$  is the dominant strong phase):

$$\Re(\tilde{\mathcal{A}}_S^* \tilde{\mathcal{A}}_P) \approx \Re(\tilde{\mathcal{A}}_S^{CP*} \tilde{\mathcal{A}}_P^{CP}), \quad \Im(\tilde{\mathcal{A}}_S^* \tilde{\mathcal{A}}_P) \approx -\Im(\tilde{\mathcal{A}}_S^{CP*} \tilde{\mathcal{A}}_P^{CP}) \implies \Delta \approx -\Delta^{CP}, \quad \Sigma \approx \Sigma^{CP}$$

- Simplification:

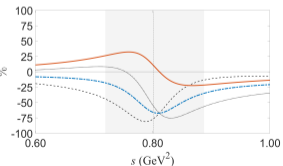
$$A_{CP}^{FB} \sim \Re(\mathcal{A}_S^* \mathcal{A}_P e^{i\delta}) - \Re(\mathcal{A}_S^{CP*} \mathcal{A}_P^{CP} e^{i\delta}) \sim (\Delta + \Sigma) - (\Delta^{CP} + \Sigma^{CP}) \approx \Delta - \Delta^{CP} \approx 2\Delta$$

$$A_{ave}^{FB} \sim \Re(\mathcal{A}_S^* \mathcal{A}_P e^{i\delta}) + \Re(\mathcal{A}_S^{CP*} \mathcal{A}_P^{CP} e^{i\delta}) \sim (\Delta + \Sigma) + (\Delta^{CP} + \Sigma^{CP}) \approx \Sigma + \Sigma^{CP} \approx 2\Sigma$$

- Where

$$\Delta = \left( -\sin \delta - \frac{s - m_P^2}{m_P \Gamma_P} \cos \delta \right) \Im(\tilde{\mathcal{A}}_S^* \tilde{\mathcal{A}}_P)$$

$$\Sigma = \left( -\frac{s - m_P^2}{m_P \Gamma_P} \sin \delta + \cos \delta \right) \Re(\tilde{\mathcal{A}}_S^* \tilde{\mathcal{A}}_P)$$



$$\delta = 0, N_c^{eff} = 1$$

# Correlation: Strong Phase and Sign-Change Behavior

The behaviors of the two observables across the resonance mass are correlated:

Strong Phase ( $\delta$ )	Sign Change Across Resonance?	
	$A_{ave}^{FB} (\sim 2\Sigma)$	$A_{CP}^{FB} (\sim 2\Delta)$
$\delta \sim 0, \pi$	No	Yes
$\delta \sim \pi/2, 3\pi/2$	Yes	No

- Exactly one observable experiences a sign change, driven by the dominance of either the  $\Delta$  or  $\Sigma$  term.
- Universality:** Crucially, this correlated sign-change behavior is **model-independent** and **channel-independent**.

# Potential FB-CPA Values & Experimental Viability

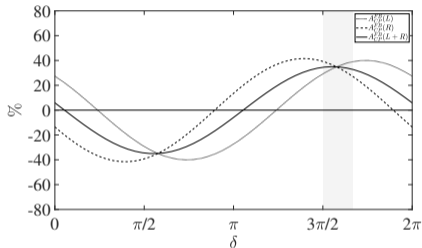


Figure 2: FB-CPAs vs. strong phase  $\delta$

## Model Predictions:

- Dalitz analysis by BaBar [arXiv:0711.4417] favors:  $\delta \in [3\pi/2, 5\pi/3]$ .
- Assuming effective color  $N_c^{eff} = 1$ :
  - FB-CPA around  $\overline{K}^*(892)^0$  reaches  $\sim$  **35%**.
  - Likely accessible for **Belle-II**.

## Connection to LHCb's $A_{CP|S}$ :

- LHCb searches for CPV in  $D \rightarrow KK\pi$  near  $\phi(1020)$  using the  $A_{CP|S}$  observable [arXiv:2409.01414]. Our Sign Change Rule provides the physical mechanism for this: it maps phase-space sign flips, dictating exactly how to prevent CP asymmetry cancellation.

- ① Introduction
- ② Theoretical Framework
- ③ Results and Analysis
- ④ Summary

# Summary

## Theoretical Mechanism

- **Interference Extraction:** FBA precisely isolates the S-P wave interference.
- **Non-trivial Correlation:** The sign behaviors of  $A_{ave}^{FB}$  and  $A_{CP}^{FB}$  are **correlated**.  
(1) Model-independent (2) Channel-independent
- **Underlying Mechanism:** Driven by the different behaviors of the  $\Sigma$  and  $\Delta$  terms depending on the strong phase  $\delta$ .

## Experimental Impacts

- **Potential Signal:** Obtained a CP asymmetry up to  $\sim 35\%$ .
- **Future Prospect:** Potentially accessible by **Belle-II** collaborations.

Thanks for your attention !