Semileptonic decays $D_{(s)} \to \eta^{(\prime)} \ell^+ \nu_\ell$ from QCD Light-Cone Sum Rules

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Contents

Charmed Semileptonic Decays: A Light-Hadron Probe

Two key probes

- Charged currents ($D \to K\ell^+\nu$): precise CKM $|V_{cs}|$
- Flavor-changing neutral currents ($D \to \pi\ell^+\ell^-$, $\pi\pi\ell^+\ell^-$, etc.): new physics sensitivity

Complement to B anomalies

- B decays probe down-type sector; charmed decays probe up-type sector
- Independent test of lepton flavor universality

Ideal laboratory

- Clean separation of leptonic and hadronic parts

Light hadron structure

- Scalar f_0: traditionally studied in cascade decays $D_s \to (f_0 \to \pi\pi)e\nu$; now complemented by direct S-wave analysis using two-pion DAs
- η-η': $U_A(1)$ anomaly, gluonic content, mixing mechanism under debate

The Mixing Schemes

Singlet-Octet (SO) scheme[Leutwyler:1997yr]

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_8 & -\sin\theta_1 \\ \sin\theta_8 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}$$

- Requires two mixing angles (SU(3) breaking $\sim 10-20\%$)

Quark-Flavor (QF) scheme[Feldmann:1998sh,Feldmann:1999uf]

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$

- Only one mixing angle ϕ (OZI rule suppresses $q\bar{q} \leftrightarrow s\bar{s}$)

The flavor eigenstate masses are expressed via the pion and kaon masses as

$$m_{qq}^2 = \frac{\sqrt{2}}{f_{\eta_q}} \langle 0| m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d |\eta_q\rangle = m_\pi^2,$$
$$m_{ss}^2 = \frac{2}{f_{\eta_s}} \langle 0| m_s \bar{s} i\gamma_5 s |\eta_s\rangle = 2m_K^2 - m_\pi^2.$$

For the singlet decay constants, we have

$$f_\eta^1 = \sqrt{\frac{2}{3}} \cos\phi f_{\eta_q} - \sqrt{\frac{1}{3}} \sin\phi f_{\eta_s}, \qquad f_{\eta'}^1 = \sqrt{\frac{2}{3}} \sin\phi f_{\eta_q} + \sqrt{\frac{1}{3}} \cos\phi f_{\eta_s} .$$

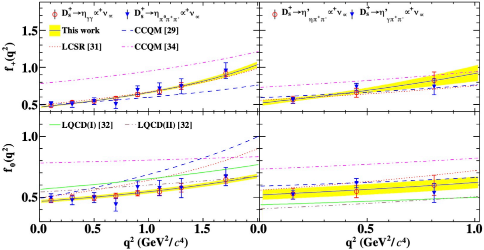
BESIII Data Resolves the Four Mixing Sets

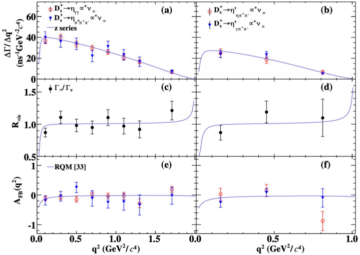
Four sets of parameters used in the QF mixing scheme:

	f_{η_q}	f_{η_s}	ϕ (Degree)
Set A[Bali:2021qem]	$(1.02^{+0.02}_{-0.05})f_\pi$	$(1.37^{+0.04}_{-0.06})f_\pi$	$39.6^{+1.2}_{-2.1}$
Set B[Feldmann:1998vh]	$(1.07 \pm 0.02)f_\pi$	$(1.34 \pm 0.06)f_\pi$	39.3 ± 1.0
Set C[Escribano:2005qq]	$(1.09 \pm 0.03)f_\pi$	$(1.66 \pm 0.06)f_\pi$	40.3 ± 1.8
Set D[Cao:2012nj]	$(1.08 \pm 0.04)f_\pi$	$(1.25 \pm 0.09)f_\pi$	37.7 ± 0.7

- Previous data: large uncertainties —unable to distinguish mixing schemes; now, precision BESIII data (2023–25) allow test of all four mixing sets.

[BESIII:2023gbn]





Physics Questions and Strategy

Key questions:

(Q1) Which mixing set (A–D) is favored by BESIII?

(Q2) How sensitive is η' to gluonic component (b_2^g)[Ball:2007hb]?

(Q3) Can our LCSR calculation describe current data?

Calculation & Inputs

- LO: twist-2–4, factorizable twist-5–6[Rusov:2017chr]
- NLO: twist-2,3[Duplancic:2008ix]; gluonic (twist-2)[Duplancic:2015zna]
- Inputs: data-driven a_n, $b_2^g = 0 \pm 20$

Test & Extract

- Compare Sets A–D with BESIII data
- Extract: optimal mixing, gluonic sensitivity via b_2^g

$\Rightarrow$ Comprehensive test of η-η' mixing with modern LCSR + BESIII data

LCSR Method for $D_{(s)} \to \eta^{(\prime)}$ Form Factors

- **LCSR:** Connects perturbative QCD calculations with hadronic properties via dispersion relation.
- **Correlation function:** $\langle \eta_q | T\{J_\mu, J_5\} | 0 \rangle$, $J_\mu = \bar{q}\gamma_\mu c$, $J_5 = m_c \bar{c} i\gamma_5 q$.
- **Light-cone OPE:** For $q^2, \tilde{q}^2 \ll m_c^2$, expand c-propagator near $x^2 \approx 0$:

  $$T_{q\bar{q}}^{(t),\mathrm{LO}}(u, q^2, \tilde{q}^2)\, \phi_{q\bar{q}}^{(t)}(u)$$

  Factorization into hard kernels $\otimes$ light-meson DAs.
- **Dispersion relation:** Isolate $D_{(s)}$ ground state via Borel transform + quark-hadron duality.
- **Result:** Form factors $f_+^{(D_{(s)} \to \eta^{(\prime)})}(q^2)$, $f_0^{(D_{(s)} \to \eta^{(\prime)})}(q^2)$ = convolution of DAs with perturbative kernels.

FFs

Eventually, the physical transition FFs can be expressed in terms of those derived with flavor eigenstates:

$$f_i^{D\to\eta^{(\prime)}}(q^2) = \frac{f^q_{\eta^{(\prime)}}}{\sqrt{2}}\left[\sum_{t=2}^{6} f_{i,\text{LO}}^{D\to\eta_q,(t)}(q^2) + \sum_{t=2}^{3} f_{i,\text{NLO}}^{D\to\eta_q,(t)}(q^2)\right] + f^1_{\eta^{(\prime)}}\, f_{i,\text{NLO}}^{D\to\eta_g,(t=2)}(q^2),$$

$$f_i^{D_s\to\eta^{(\prime)}}(q^2) = f^s_{\eta^{(\prime)}}\left[\sum_{t=2}^{6} f_{i,\text{LO}}^{D_s\to\eta_s,(t)}(q^2) + \sum_{t=2}^{3} f_{i,\text{NLO}}^{D_s\to\eta_s,(t)}(q^2)\right] + f^1_{\eta^{(\prime)}}\, f_{i,\text{NLO}}^{D_s\to\eta_g,(t=2)}(q^2).$$

The twist-2, twist-3 contributions to the $D \to \eta_q$ transition FFs read

$$f_{+,\text{LO}}^{D\to\eta_q,(t=2)}(q^2) = \frac{e^{m_D^2/M^2}}{2m_D^2 f_D^2}\, m_c^2 \int_{u_0}^{1}\frac{du}{u}e^{-\frac{s(u)}{M^2}}\,\varphi_2(u),$$

$$\begin{aligned} f_{+,\text{LO}}^{D\to\eta_q,(t=3)}(q^2) = \frac{e^{m_D^2/M^2}}{2m_D^2 f_D^2}\int_{u_0}^{1} du\, e^{-\frac{s(u)}{M^2}}\Bigg\{ m_c\mu_{\eta_q}\Bigg(\phi_3^p(u) + \frac{1}{3}\frac{\phi_3^\sigma}{u} - \frac{m_c^2+q^2-u^2m_{\eta_q}^2}{\triangle(u,q^2)}\frac{d\phi_3^\sigma(u)}{du} \\ + \frac{4um_{\eta_q}^2 m_c^2}{[\triangle(u,q^2)]^2}\phi_3^\sigma(u)\Bigg) - m_c\frac{f_{3\eta_q}}{f_{\eta_q}}\Bigg(\frac{1}{\triangle(u,q^2)}\frac{dI_3}{du} - \frac{2um_{\eta_q}^2}{[\triangle(u,q^2)]^2}I_3\Bigg)\Bigg\}. \end{aligned}$$

LCSR Inputs and BCL Extrapolation

Key sources of uncertainty in LCSR

- M^2: controls OPE convergence vs continuum suppression
- s_0: separates ground state from higher resonances
- Stability:

$$\frac{d}{d(1/M^2)} \ln f_i(q^2) \approx 0$$

- Input[Offen:2013nma]:

$$M^2 = 4.4 \pm 1.1 \text{ GeV}^2, \quad s_0 = 7.0 \pm 0.6 \text{ GeV}^2$$

Validity and extrapolation

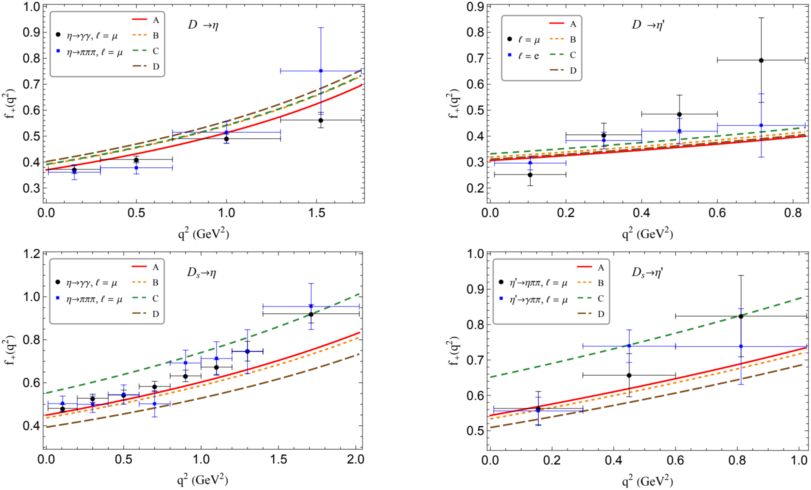
- LCSR valid for $0 \leq |q^2| \lesssim 0.4$ GeV^2, requiring extension to full kinematics

BCL parametrization

- $$f_i(q^2) = \frac{1}{1 - q^2/M_{R,i}^2} \sum_{k \leq 2} \alpha_i^k [z(q^2) - z(0)]^k$$
- Fast convergence and explicit pole structure

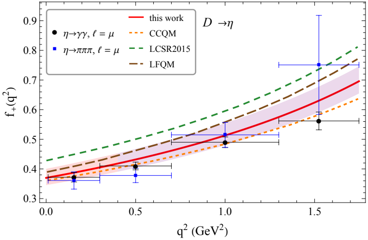
Comparison of mixing scenarios via f_+

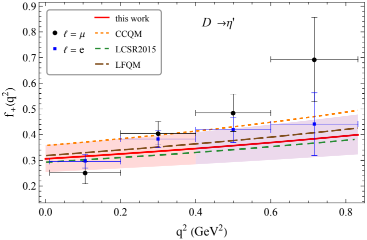
Set A best fits the BESIII data[BESIII:2025hjc,BESIII:2023gbn,BESIII:2024njj], favoring smaller decay constants and a larger mixing angle.

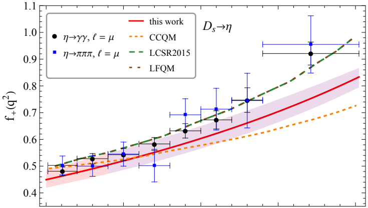


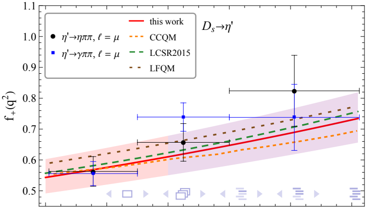
Comparison of FFs f_+ with other approaches

The LCSR predictions (low q^2 combined with BCL extrapolation) are consistent with results from BESIII, CCQM[Ivanov:2019nqd], LFQM[Verma:2011yw], lattice QCD[Bali:2014pva], and previous LCSR[Duplancic:2015zna] calculations across the full kinematic range.



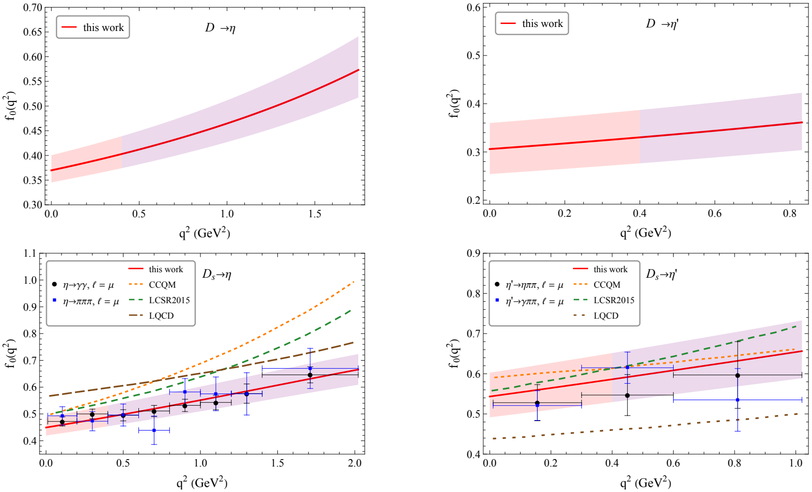






Comparison of FFs f_0 with other approaches

At high q^2 (low recoil), a notable discrepancy is observed between the present LCSR results and other approaches.



FFs and Error Budget at $q^2 = 0$

Convergence: Higher-order terms suppressed by $\mathcal{O}(f_{3\eta_{q,s}}/m_c)$, $\mathcal{O}(\delta^2_{\eta_{q,s}})$, $\mathcal{O}(\langle\bar{q}q\rangle/m_c^3)$; NLO reduces $f_{+,0}$ by $2\% - 3\%$ via destructive interference between twist-2 and twist-3.

Twist and Order Contributions

	Twist-2 LO	Twist-2 NLO	Twist-3 LO	Twist-3 NLO	Twist-(4+5+6) LO
$f_{+,0}^{D\to\eta}(0)$	0.127	0.025	0.254	-0.035	-4.86×10^{-4}
$f_{+,0}^{D\to\eta'}(0)$	0.105	0.021	0.210	-0.029	-4.02×10^{-4}
$f_{+,0}^{D_s\to\eta}(0)$	0.153	0.031	0.317	-0.051	-1.05×10^{-4}
$f_{+,0}^{D_s\to\eta'}(0)$	0.185	0.037	0.383	-0.061	-1.27×10^{-4}

Error budget: Total uncertainty $\approx \pm10\%$, dominated by chiral masses $m_0^{\eta_{q,s}}$ (η channel) and gluon DA parameter b_2^g (η' channel). η' uncertainty amplified by $f^1_{\eta'} \gg f^1_\eta$.

Error Budget

Channels	$f_{+,0}(0)$	M^2	s_0	$m_0^{\eta_{q,s}}$	a_n^π	b_2^g
$D\to\eta$	$0.370^{+0.031}_{-0.024}$	${}^{+0.003}_{-0.001}$	${}^{+0.005}_{-0.006}$	${}^{+0.029}_{-0.023}$	±0.002	±0.006
$D\to\eta'$	$0.306^{+0.054}_{-0.052}$	${}^{+0.002}_{-0.001}$	${}^{+0.004}_{-0.005}$	${}^{+0.024}_{-0.019}$	±0.002	±0.048
$D_s\to\eta$	$0.449^{+0.038}_{-0.030}$	${}^{+0.011}_{-0.006}$	${}^{+0.006}_{-0.009}$	${}^{+0.036}_{-0.028}$	±0.003	±0.005
$D_s\to\eta'$	$0.543^{+0.059}_{-0.052}$	${}^{+0.013}_{-0.007}$	${}^{+0.008}_{-0.011}$	${}^{+0.043}_{-0.033}$	±0.003	±0.037

Differential decay rates and branching ratios

The differential decay rate of the semileptonic decay $D \to \eta^{(\prime)}\ell^+\nu_\ell$ takes the form

$$\left.\frac{d\Gamma}{dq^2}\right|_{D\to\eta^{(\prime)}\ell^+\nu_\ell} = \frac{G_F^2|V_{cd}|^2}{24\pi^3}\frac{(q^2-m_\ell^2)^2 p_{\eta^{(\prime)}}}{q^4 m_D^2}\Bigg[m_D^2 p_{\eta^{(\prime)}}^2|f_+(q^2)|^2 + \frac{m_\ell^2}{2q^2}\left(m_D^2 p_{\eta^{(\prime)}}^2|f_+(q^2)|^2 + \frac{3}{4}(m_D^2 - m_{\eta^{(\prime)}}^2)^2|f_0(q^2)|^2\right)\Bigg],$$

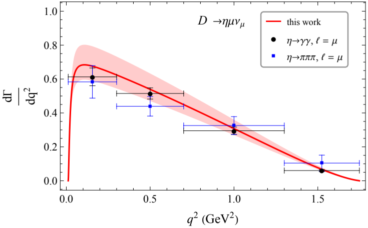
For the $D_s \to \eta^{(\prime)}\ell\nu$ decays, the corresponding formulae are obtained by replacing the mass $m_D \to m_{D_s}$ and the CKM matrix element $V_{cd} \to V_{cs}$, and by adapting the form factors appropriately.

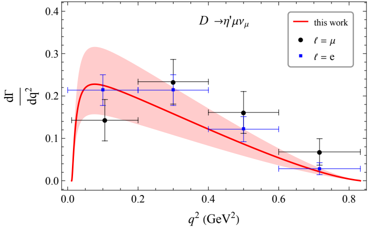
$$\Gamma(D_{(s)} \to \eta^{(\prime)}\ell^+\nu_\ell) = \int_{m_\ell^2}^{(m_{D_{(s)}}-m_{\eta^{(\prime)}})^2} dq^2 \left[\left.\frac{d\Gamma}{dq^2}\right|_{D_{(s)}\to\eta^{(\prime)}\ell^+\nu_\ell}\right],$$

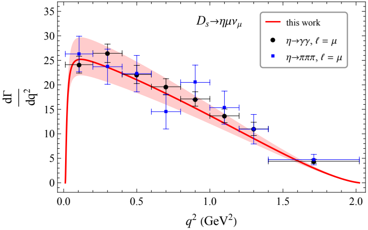
The branching fraction is defined by $\mathcal{B} = \Gamma(D_{(s)} \to \eta^{(\prime)}\ell^+\nu_\ell)\cdot\tau_{D_{(s)}}$.

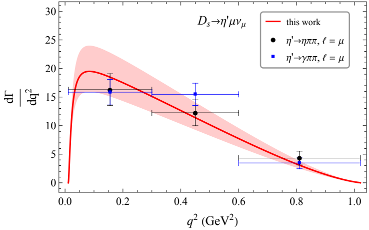
Differential decay rates

For $D \to \eta' \ell \nu_\ell$, the predictions lie below the BESIII data in the range $0.2 \leqslant q^2 \leqslant 0.6\ \mathrm{GeV}^2$. Future precise data will help clarify this tension and constrain the gluonic content of η and η'.









Branching fractions

Branching fractions (in unit of 10^{-3}) of the semileptonic decays $D, D_s^+ \to \eta^{(\prime)}\ell^+\nu_\ell$, together with the PDG averages, the BESIII measurements and the recent LCSR determination.

	$D^+ \to \eta e^+\nu_e$	$D^+ \to \eta\mu^+\nu_\mu$	$D^+ \to \eta' e^+\nu_e$	$D^+ \to \eta'\mu^+\nu_\mu$
This work	$0.99^{+0.17}_{-0.13}$	$0.97^{+0.17}_{-0.12}$	$0.16^{+0.06}_{-0.05}$	$0.15^{+0.06}_{-0.05}$
PDG	1.11 ± 0.07	1.04 ± 0.11	0.20 ± 0.04	—
BESIII	$0.98^{+0.29+0.28}_{-0.29-0.28}$	$0.91^{+0.35+0.23}_{-0.35-0.23}$	—	—
BESIII	—	—	$0.18^{+0.02+0.01}_{-0.02-0.01}$	$0.19^{+0.03+0.01}_{-0.03-0.01}$
	$D_s^+ \to \eta e^+\nu_e$	$D_s^+ \to \eta\mu^+\nu_\mu$	$D_s^+ \to \eta' e^+\nu_e$	$D_s^+ \to \eta'\mu^+\nu_\mu$
This work	$20.35^{+3.41}_{-2.52}$	$19.96^{+3.35}_{-2.48}$	$7.80^{+1.83}_{-1.45}$	$7.45^{+1.75}_{-1.38}$
LCSR	$23.46^{+4.18}_{-3.31}$	$23.20^{+4.13}_{-3.27}$	$7.92^{+1.41}_{-1.18}$	$7.73^{+1.38}_{-1.15}$
PDG	22.70 ± 0.60	22.40 ± 0.70	8.10 ± 0.60	8.00 ± 0.60
BESIII	—	$22.35^{+0.51+0.52}_{-0.51-0.52}$	—	$8.01^{+0.55+0.28}_{-0.55-0.28}$

Summary and Outlook

- BESIII data of seileptonic charm decay favor:

$$f_{\eta_q} = (1.02^{+0.02}_{-0.05}) f_\pi, \quad f_{\eta_s} = (1.37^{+0.04}_{-0.06}) f_\pi, \quad \phi = (39.6^{+1.2}_{-2.1})^\circ.$$

- Good OPE convergence; FFs dominated by twist-3 contribution (chiral enhancement). NLO corrections small (2%–3%); higher-twist and three-particle effects negligible.
- η' form factors show enhanced sensitivity to the gluonic DA parameter b_2^g, calling for future precise data to constrain the gluon content of η and η'.
- Outlook: Refined measurements and FFs needed to probe gluonic components.

Thank you very much for your patience.