



中国科学院大学  
University of Chinese Academy of Sciences

BESIII

# Multi-channel joint analysis of the exotic charmonium-like state $T_{c\bar{c}}(4020)$

Zhaoshen Xu (许昭燊)

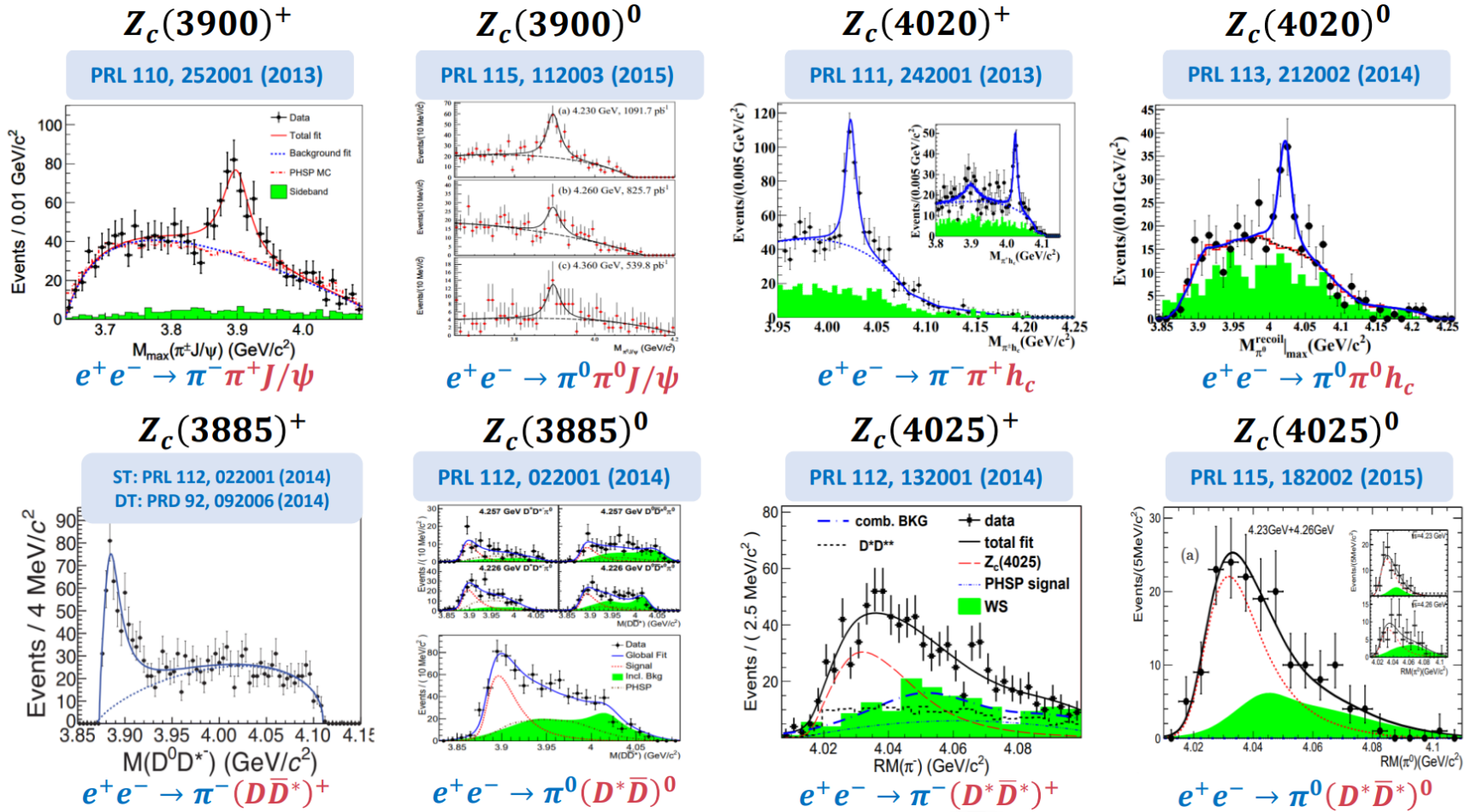
University of Chinese Academy of Sciences  
(on behalf of the BESIII collaboration)

[arXiv: 2603.05564](https://arxiv.org/abs/2603.05564)

March 30, 2026

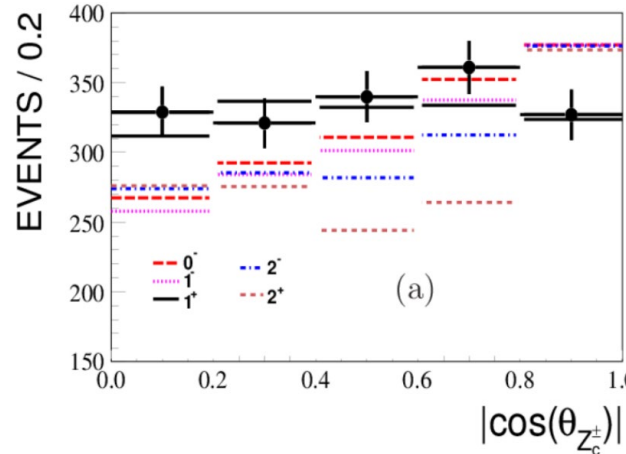
# Motivation

- The charmoniumlike iso-vector resonances  $T_{c\bar{c}1}(3900/3885)^{\pm(0)}$ ,  $T_{c\bar{c}}(4020/4025)^{\pm(0)}$  are observed at BESIII.

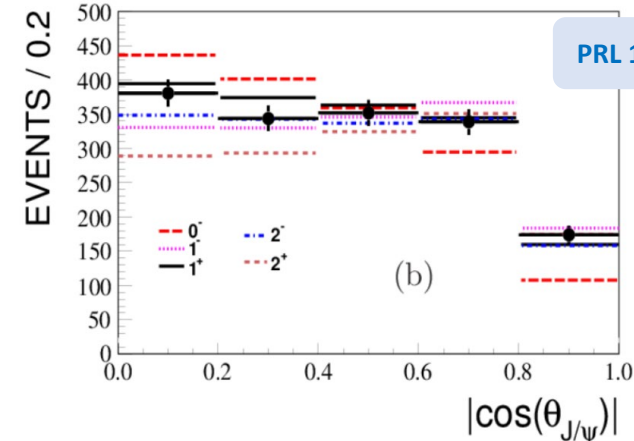


- Many theoretical explanations have been proposed.
- More experimental information is needed.

- The spin and parity of  $T_{c\bar{c}1}(3900)^\pm$  are determined as  $1^+$  with significance  $>7.5\sigma$  at BESIII.



$\theta_{Z_c} : \text{Polar angle of } T_{c\bar{c}1}(3900)^\pm$



$\theta_{J/\psi} : \text{Helicity angle of } J/\psi$

What is the  $J^P$  quantum number of  $T_{c\bar{c}}(4020/4025)$ ?

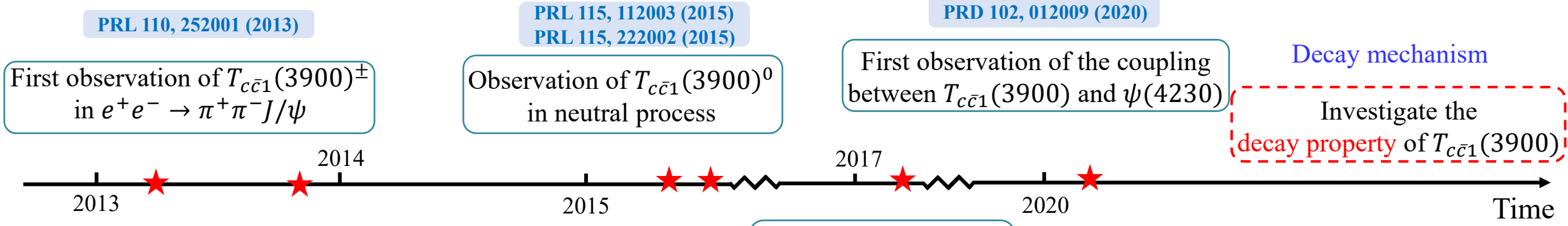
$T_{c\bar{c}}(4020) \quad I^G(J^{PC}) = 1^+(?^{?^-})$

From PDG

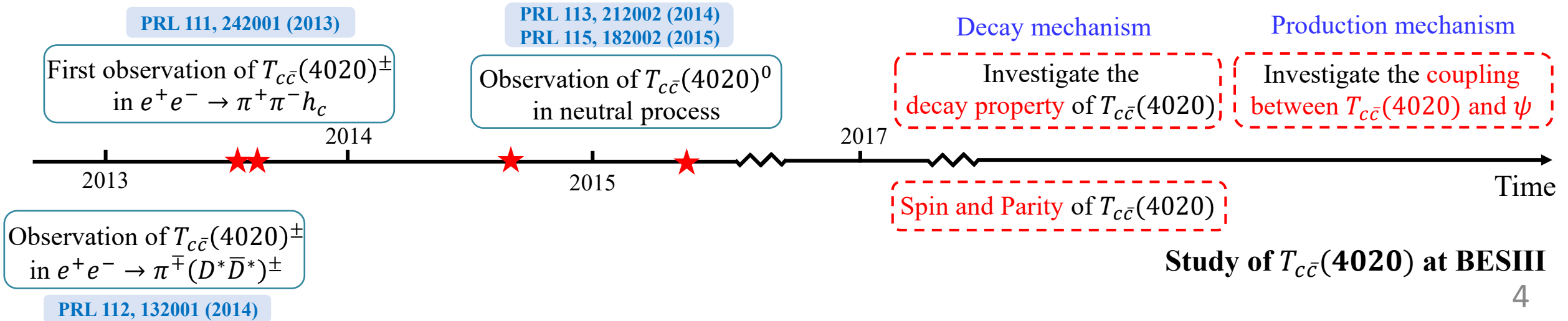
PDG naming:  $Z_c(3900/3885) = T_{c\bar{c}1}(3900)$ ,  $Z_c(4020/4025) = T_{c\bar{c}}(4020)$

# Motivation

Production mechanism



Study of  $T_{c\bar{c}1}(3900)$  at BESIII



Study of  $T_{c\bar{c}}(4020)$  at BESIII

# Multi-channel joint analysis for $T_{c\bar{c}}(4020)$

## Decay mechanism

Investigate the  
**decay property** of  $T_{c\bar{c}}(4020)$

**Spin and Parity** of  $T_{c\bar{c}}(4020)$

Three Channels:  $e^+e^- \rightarrow D^{*0}D^{*-}\pi^+, \pi^+\pi^-J/\psi, \pi^+\pi^-h_c$

PRL 112, 132001 (2014)

PRL 118, 092001 (2017)

PRL 111, 242001 (2013)

- **For  $D^{*0}D^{*-}\pi^+$ :** Double-tag method is used, details in Appendix. ✓ **The charge conjugated channels are implied.**  
 $D^{*0}D^{*-}\pi^+ \rightarrow [D^0\pi^0(\gamma)][\bar{D}^0\pi^-]\pi^+$

- **For  $\pi^+\pi^-J/\psi$ :** Full reconstruction method is used.

$$J/\psi \rightarrow l^+l^- \quad (l = e, \mu)$$

- **For  $\pi^+\pi^-h_c$ :** Full reconstruction method is used.

$$h_c \rightarrow \gamma\eta_c, \eta_c \rightarrow 16 \text{ channels}$$

$$\eta_c \rightarrow p\bar{p}, 2(\pi^+\pi^-), 2(K^+K^-), \pi^+\pi^-K^+K^-, \pi^+\pi^-p\bar{p}, 3(\pi^+\pi^-), 2(\pi^+\pi^-)K^+K^-, K_S^0K^\pm\pi^\mp, K_S^0K^\pm\pi^\mp\pi^+\pi^-, K_S^0K^\pm\pi^\mp\pi^+\pi^-, K^+K^-\pi^0, p\bar{p}\pi^0, K^+K^-\eta, \pi^+\pi^-\eta, 2(\pi^+\pi^-)\eta, \pi^+\pi^-\pi^0\pi^0 \text{ and } 2(\pi^+\pi^-)\pi^0$$

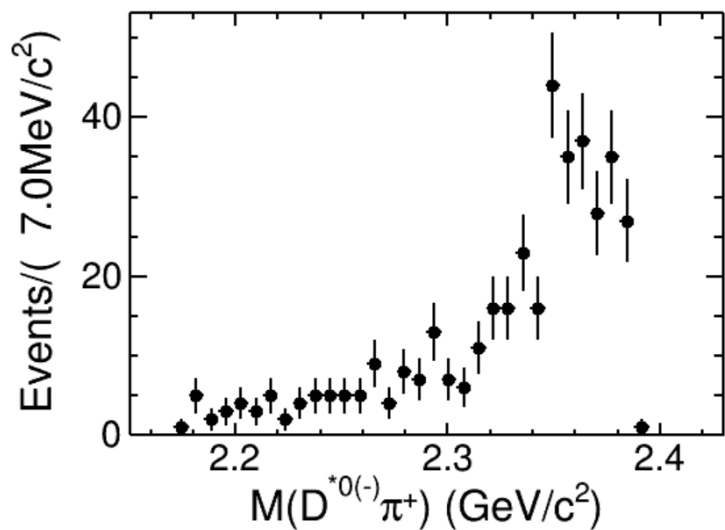
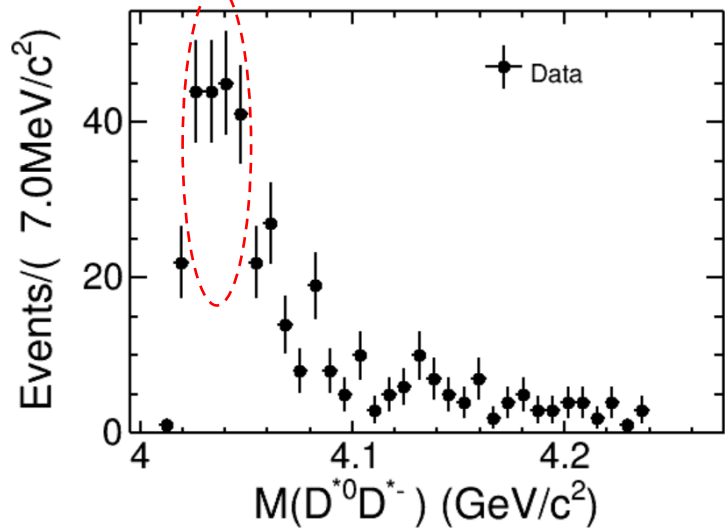
**The event selection criteria are consistent with published results.**

- Keep the balance of **integrated luminosity** and **3-body cross sections** of  $e^+e^- \rightarrow D^{*0}D^{*-}\pi^+, \pi^+\pi^-J/\psi, \pi^+\pi^-h_c$ .

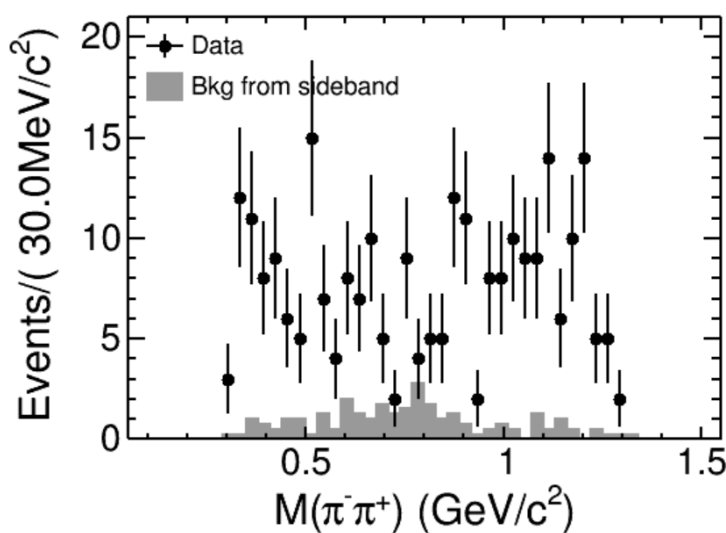
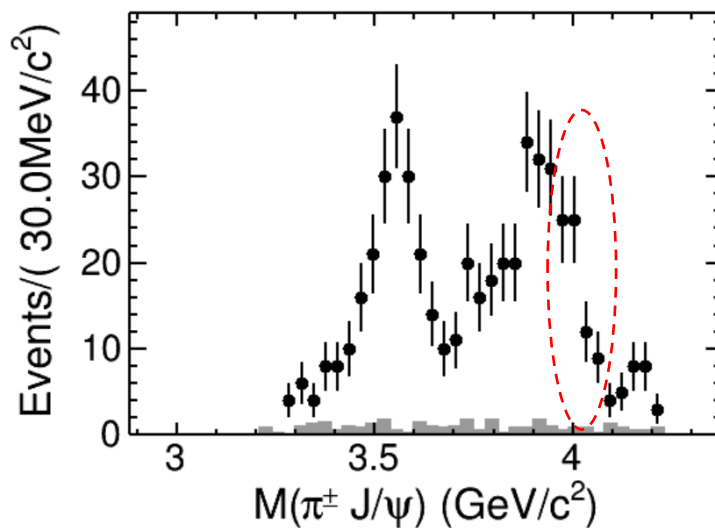
Two datasets:  $\sqrt{s} = 4.395 \text{ GeV}, L_{\text{int}} = 508.18 \text{ pb}^{-1}; \sqrt{s} = 4.416 \text{ GeV}, L_{\text{int}} = 1090.70 \text{ pb}^{-1}$ .

# Samples Summary ( $\sqrt{s} = 4.395 \text{ GeV}$ )

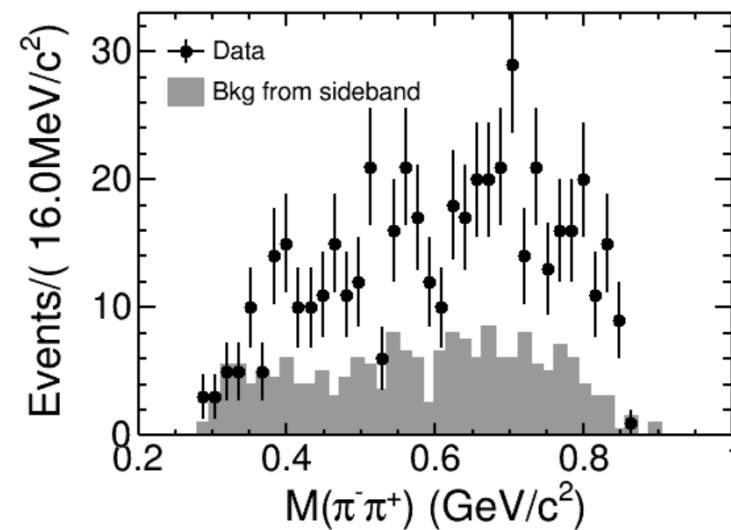
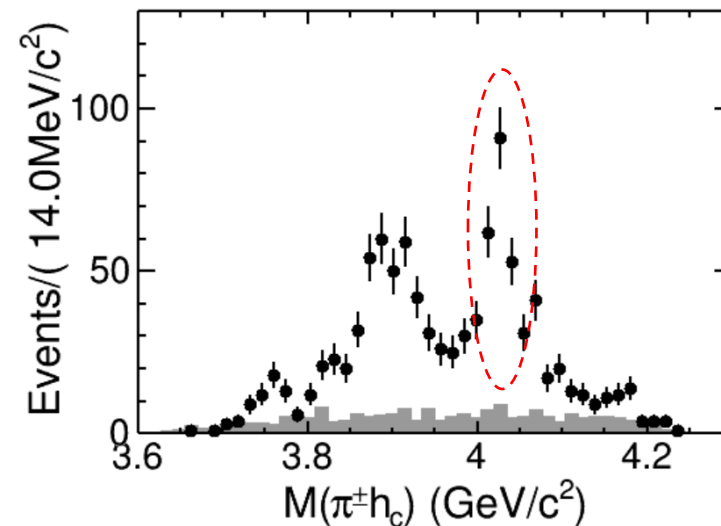
$D^{*0}D^{*-}\pi^+$



$\pi^+\pi^-J/\psi$

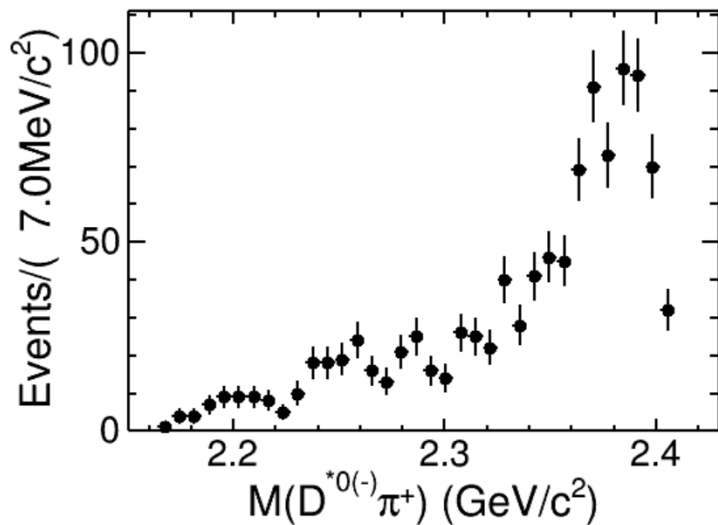
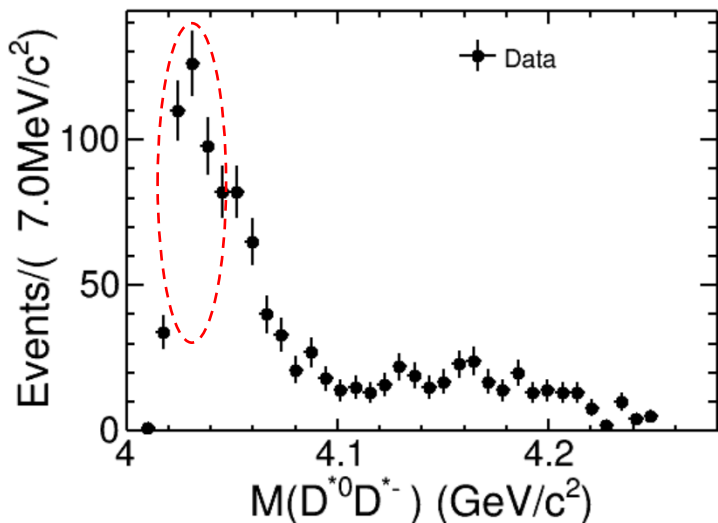


$\pi^+\pi^-h_c$

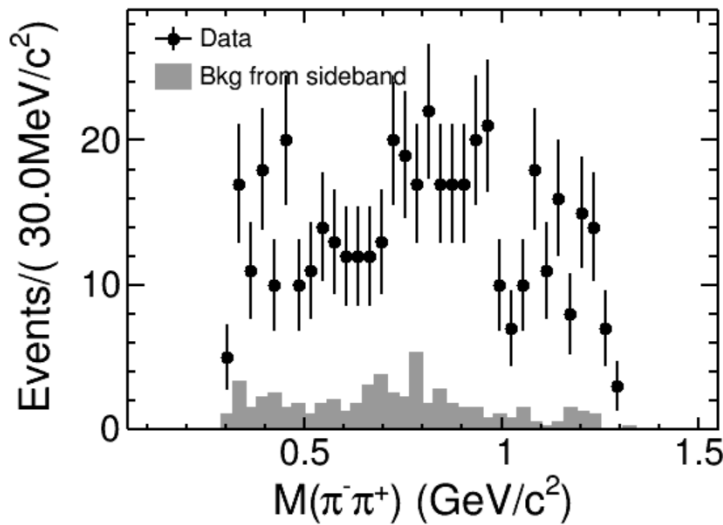
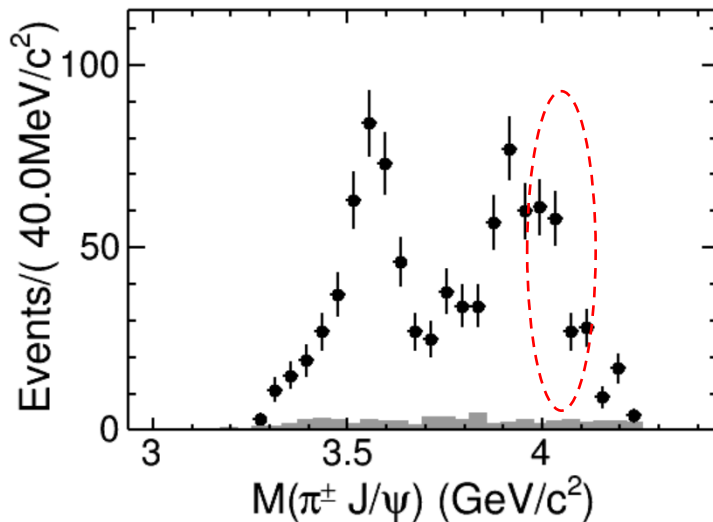


# Samples Summary ( $\sqrt{s} = 4.416 \text{ GeV}$ )

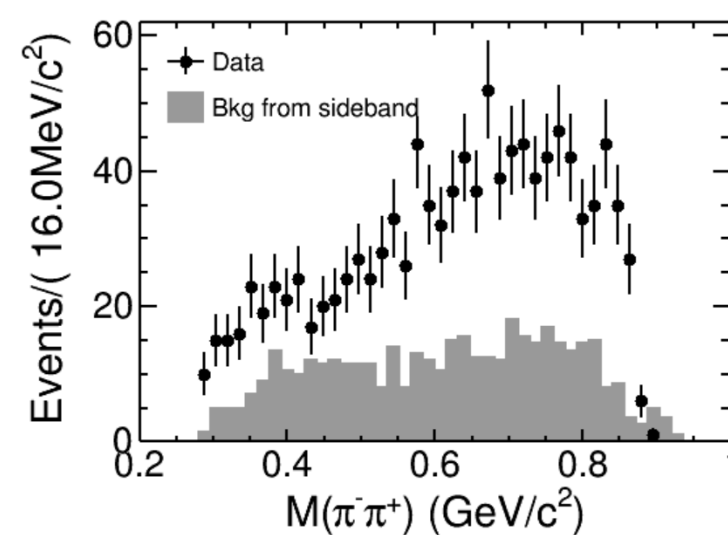
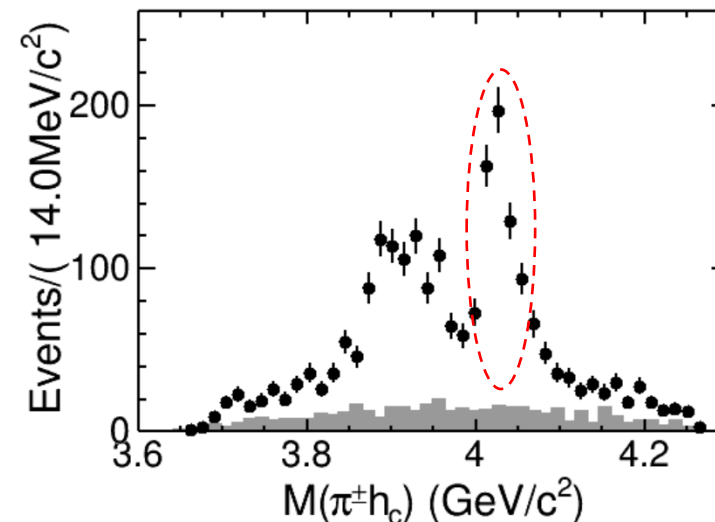
$D^{*0}D^{*-}\pi^+$



$\pi^+\pi^-J/\psi$



$\pi^+\pi^-h_c$

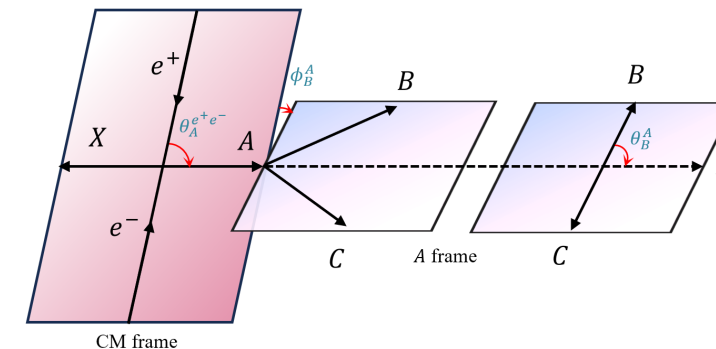


- Full procedure uses helicity amplitude formalism<sup>[1]</sup> based on [TF-PWA](#) package.

For decay:  $A [ (spin)J, (parity)\eta_J ] \rightarrow B [s_b, \eta_b] + C [s_c, \eta_c]$

The decay amplitudes are given by:  $A_{\lambda_B, \lambda_C}^J (\theta, \phi; M) \propto D_{M, \lambda_B - \lambda_C}^{J*} (\phi, \theta, 0) H_{\lambda_B, \lambda_C}^J$

- $\lambda_B, \lambda_C$ : Helicity of the daughter particle  $B$  and  $C$ .
- $\theta, \phi$ : Corresponding helicity angles.
- $M$ : Z-projection of spin- $J$ .
- $D$ : Wigner- $D$  function.  $[D_{a,b}^J(\phi, \theta, 0) = e^{-ia\phi} d_{a,b}^J(\theta)]$
- $H$ : Helicity-coupling amplitude.



In the  $LS$ -coupling scheme, we use the Chung's formula [\*]:

$$H_{\lambda_B, \lambda_C}^J = \sum_{LS} g_{LS} \left( \frac{2L+1}{2J+1} \right)^{\frac{1}{2}} \langle L0 S\delta | J\delta \rangle \langle s_b \lambda_B s_c -\lambda_C | S\delta \rangle r^L B'_L(r, r_0, d)$$

- $g_{LS}$ : Coupling constants (complex), free parameters, determined by fitting.

- Resonance  $f_0(1370)/f_2(1270)$  is parametrized by relativistic Breit-Wigner function<sup>[1]</sup>.

- Resonance  $\sigma$  for E791 type<sup>[2]</sup>:

$$R(m) = \frac{1}{m^2 - m_0^2 + im\Gamma_X(m)}, \Gamma_X(m) = \sqrt{1 - \frac{4m_\pi^2}{m^2}} \Gamma,$$

- $f_0(980)$  is parametrized by the Flatte-formula<sup>[3]</sup>:

$$R(m) = \frac{1}{m^2 - m_0^2 + i[g_1\rho_{\pi\pi}(m) + g_2\rho_{K\bar{K}}(m)]}$$

$\rho(s) = \frac{2k}{\sqrt{s}}$ , the  $k$  is the center of mass momentum of the  $\pi$  or  $K$  in the resonance rest frame. The  $g_1$  and  $g_2$  are coupling strength, which fixed to the BESII results [3]. ( $g_2/g_1 = 4.45$ )

- If assuming  $T_{c\bar{c}1}(3900)$  is dominated by the decay to  $\pi J/\psi$  and  $\bar{D}D^*$ , it also can be parametrized by the Flatte-formula:

$$R(m) = \frac{1}{m^2 - m_0^2 + i[g'_1\rho_{\pi J/\psi}(m) + g'_2\rho_{D^*\bar{D}}(m)]}$$

$\rho(s) = \frac{2k}{\sqrt{s}}$ , the  $k$  is the center of mass momentum of the  $\pi$  or  $D$  in the  $T_{c\bar{c}1}$  rest frame, and  $g'_1$  and  $g'_2$  are coupling strength, which fixed to the BESIII results [4]. ( $g'_2/g'_1 = 27.1$ )

- For non-resonance (NR):  $BW(m) = 1$

[1] Phys. Lett. B 607, 243 (2005)

[3] PLB 598, 149 (2004)

[2] Phys. Rev. Lett. 86, 770 (2001)

[4] PRL 119, 072001 (2017)

- For  $T_{c\bar{c}}(4020)^-$ , to take into consideration of multi-channel, its line-shape with Breit-Wigner is defined as<sup>[1]</sup>:

$$R(m) = \frac{1}{m^2 - M^2 + im[\Gamma_{D^{*0}D^{*-}}(m) + \Gamma_{J/\psi\pi}(m) + \Gamma_{h_c\pi}(m) + \Gamma_X(m)]}$$

$\Gamma_{D^{*0}D^{*-}}(m), \Gamma_{J/\psi\pi}(m), \Gamma_{h_c\pi}(m)$  and  $\Gamma_X(m)$  :

the  $T_{c\bar{c}}(4020)$  partial decay width for  $D^{*0}D^{*-}, J/\psi\pi, h_c\pi$  and missing decays, respectively.

- They are calculated with the standard formula for two-body decays :

$$\Gamma(m) = \frac{1}{8\pi} \sum_{\lambda_B, \lambda_C} \overline{|H_{\lambda_B, \lambda_C}^J|^2} \frac{|\mathbf{p}|}{m^2} \quad \text{Helicity amplitude: } H_{\lambda_B, \lambda_C}^J = H(g_{LS})$$

✓  $|\mathbf{p}|$ : the momentum magnitude of final state.  
 $i|\mathbf{p}|$  will be used for below threshold events.

✓  $\overline{\sum_{\lambda_B, \lambda_C}}$  : the average is taken over spin-J.

- For missing decay, the sum of helicity amplitude,  $H_X$ , is assumed as a constant, determined in the fit.  
Momentum dependence:  $\pi J/\psi$  mode.
- To further constrain the coupling parameters used in calculation of  $\Gamma(m)$ . Extended likelihood method<sup>[\*]</sup> is used.

[1] Phys. Rev. D 94, 114019 (2016).

[\*] In Appendix

- Four-vector momentum as input :  $x_i = (p_1, p_2, p_3)$

- The signal probability density function can be written as:

$$p^{(i)} = \epsilon(x_i) |A(x_i)|^2,$$

$$|A|^2 = \begin{cases} \sum_{\lambda_{\gamma^*}, \lambda_{\pi^0(\gamma)}} |A_D(\lambda_{\gamma^*}, \lambda_{\pi^0(\gamma)})|^2, \\ \sum_{\lambda_{\gamma^*}, \lambda_{l^+}, \lambda_{l^-}} |A_J(\lambda_{\gamma^*}, \lambda_{l^+}, \lambda_{l^-})|^2, \\ \sum_{\lambda_{\gamma^*}, \lambda_{\gamma}} |A_H(\lambda_{\gamma^*}, \lambda_{\gamma})|^2. \end{cases}$$

- The extended negative log likelihood (NLL) is formed using the background subtraction method:

$$-\ln \mathcal{L}_n = -\alpha \left( \sum_{i=1}^{N_{\text{data}}} \ln p_{\text{data}}^{(i)} - \mathcal{W}_{\text{bkg}} \sum_{i=1}^{N_{\text{bkg}}} \ln p_{\text{bkg}}^{(i)} - \mu \right) \quad \text{normalization factor: } \alpha = \frac{N_{\text{data}} - N_{\text{bkg}} w_{\text{bkg}}}{N_{\text{data}} + N_{\text{bkg}} w_{\text{bkg}}^2}$$

- The  $\mu$  will converge to the signal events in each channel, where  $\mu = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} \left( \frac{d\sigma}{d\Phi} \right)_i \cdot L_{\text{int}} \cdot \epsilon \cdot B \cdot V_{PHSP} (\times 2)$

✓  $-\ln L$  is minimized by **MINUIT**.

- Fitting data to obtain the complex parameters  $c_i$  and  $g_{LS}$ .

$$H_{\lambda_B, \lambda_C}^J = \sum_{LS} g_{LS} \left( \frac{2L+1}{2J+1} \right)^{\frac{1}{2}} \langle L0 S\delta | J\delta \rangle \langle s_b \lambda_B s_c - \lambda_C | S\delta \rangle r^L B'_L(r, r_0, d)$$

- The **fit fraction** (FF) of each intermediate resonance is calculated by:

$$FF_i = \frac{\int |A_i|^2 d\Phi}{\int |\sum_k A_k|^2 d\Phi} \propto \frac{\sum_{j \in \text{PHSP}} |A_i(x_j)|^2}{\sum_{j \in \text{PHSP}} |\sum_k A_k(x_j)|^2}, \quad FF_{i,j} = \frac{\int |A_i + A_j|^2 d\Phi'}{\int |\sum_k A_k|^2 d\Phi'} = FF_i + FF_j$$

- Test baseline solution based on the significance.

$$D^{*0}D^{*-}\pi^+ = T_{c\bar{c}}(4020)^-\pi^+, D^*\bar{D}_1(2420), NRD^{*0}$$

$$\pi^+\pi^-J/\psi = T_{c\bar{c}}(4020)^{\mp}\pi^{\pm}, T_{c\bar{c}1}(3900)^{\mp}\pi^{\pm}, \underbrace{\sigma J/\psi, f_0(980)J/\psi, f_0(1370)J/\psi, f_2(1270)J/\psi}_{(\pi^+\pi^-)_{S\text{ wave}} = \sigma + f_0(980) + f_0(1370)}$$

$$\pi^+\pi^-h_c = T_{c\bar{c}}(4020)^{\mp}\pi^{\pm}, \underbrace{\sigma h_c, f_0(980)h_c}_{(\pi^+\pi^-)_{S\text{ wave}} = \sigma + f_0(980)}$$

$$(\pi^+\pi^-)_{S\text{ wave}} = \sigma + f_0(980)$$

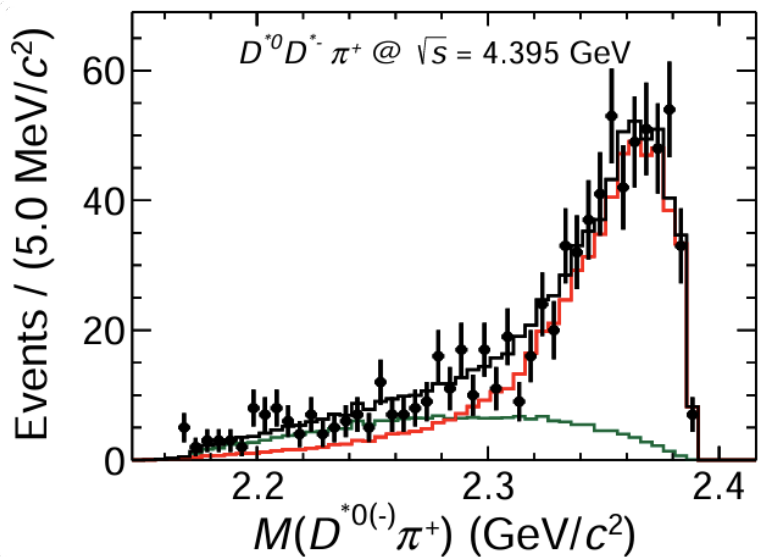
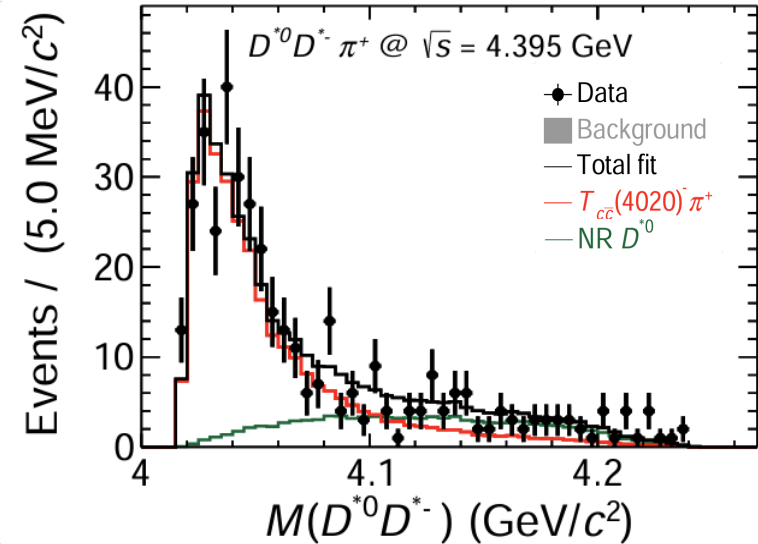
- Simultaneously fit on  $\sqrt{s} = 4.395, 4.416$  GeV.
- The shared parameters between two energy points are decaying  $g_{LS}$ .

$$A = A_{e^+e^- \rightarrow T_{c\bar{c}}^-\pi^+} \cdot R(T_{c\bar{c}}^-) \cdot A_{T_{c\bar{c}}^- \rightarrow X}$$

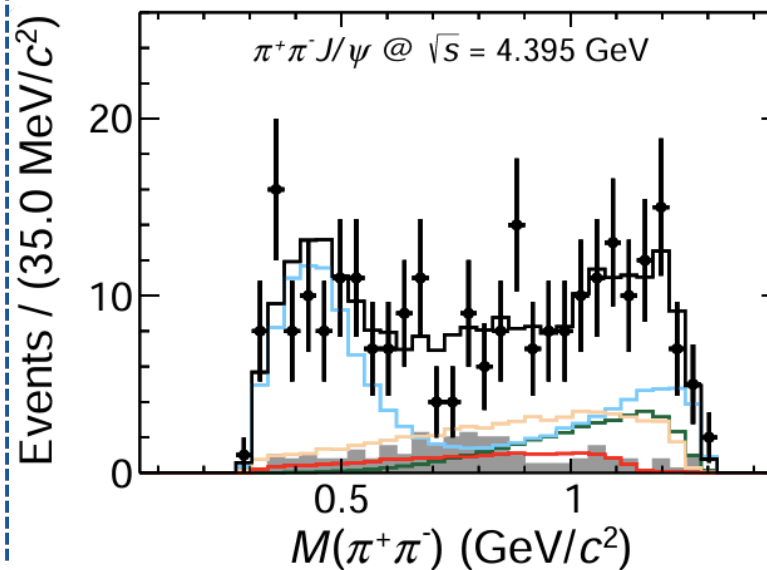
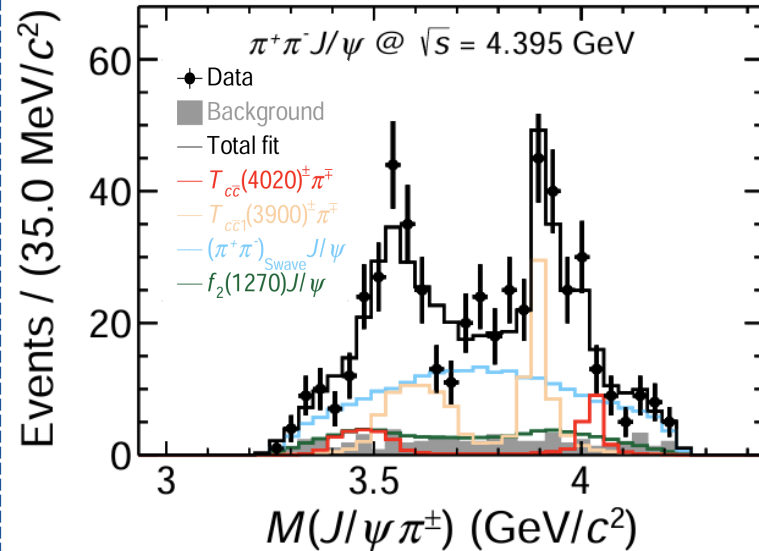
- The insignificant processes will be considered in studying of the systematic uncertainty.

# Fit Result at $\sqrt{s} = 4.395$ GeV

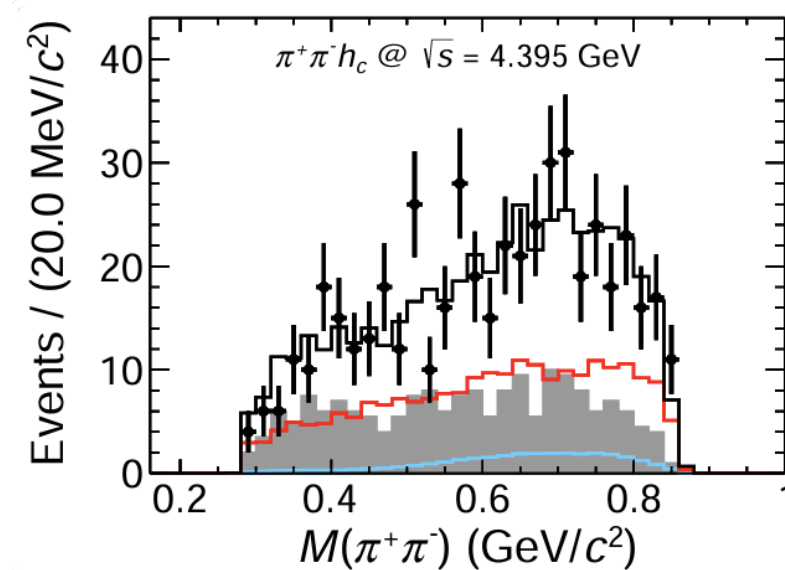
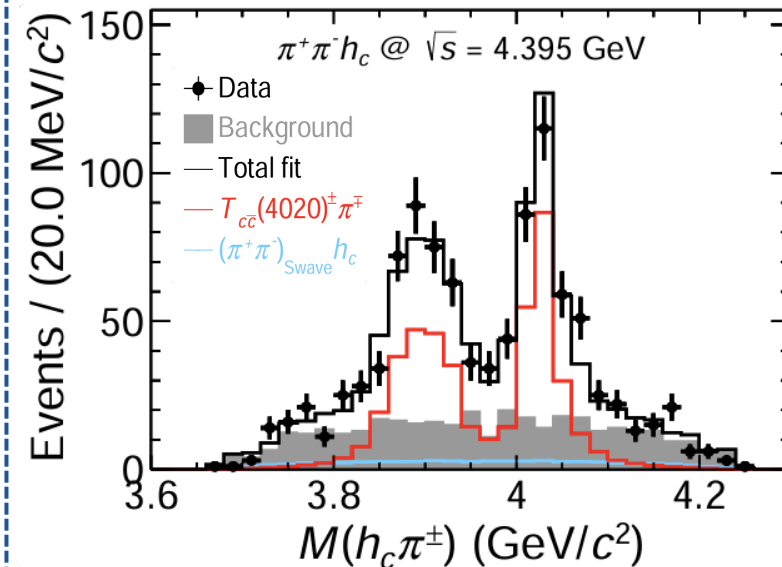
$D^{*0}D^{*-}\pi^+$



$\pi^+\pi^-\psi$

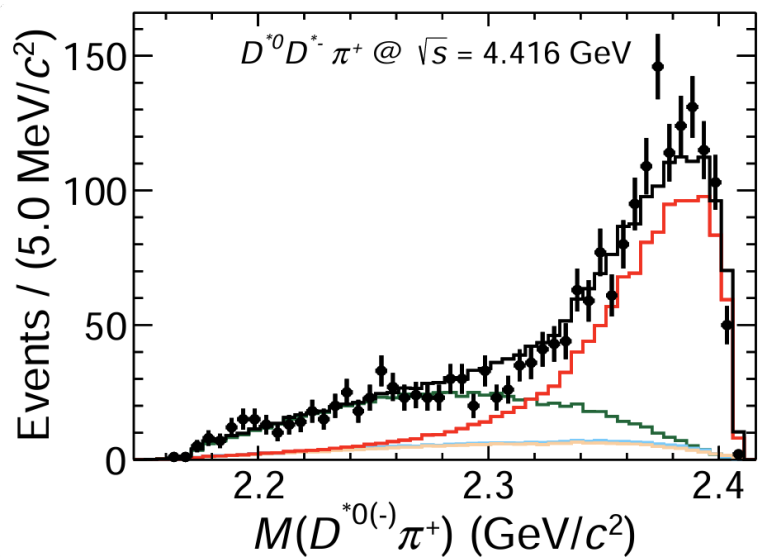
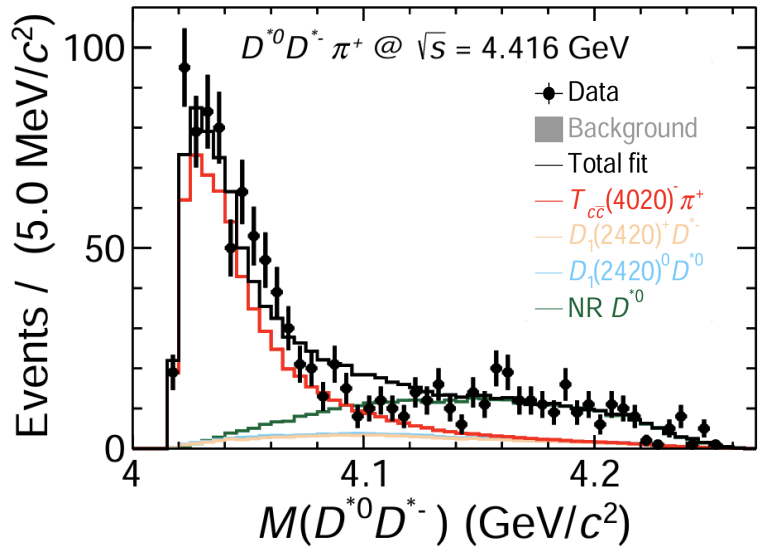


$\pi^+\pi^-h_c$

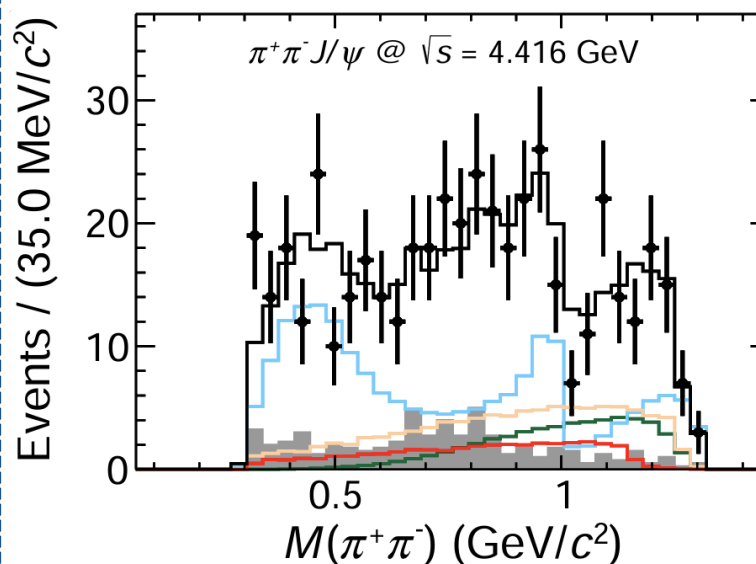
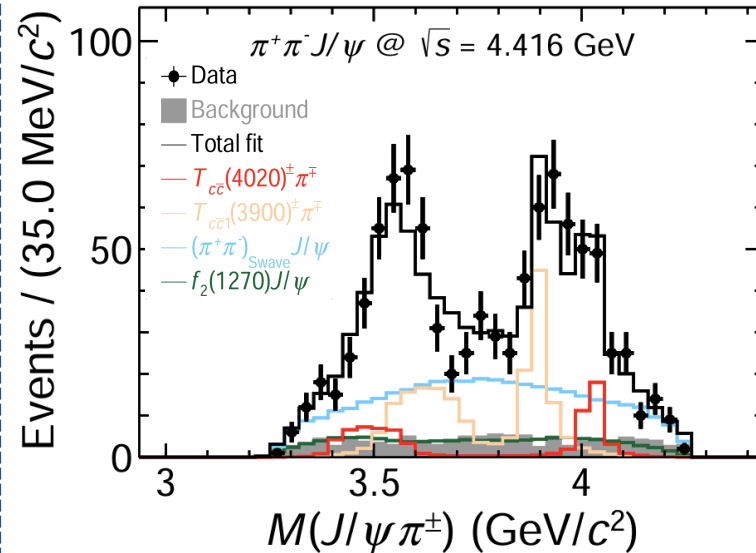


# Fit Result at $\sqrt{s} = 4.416$ GeV

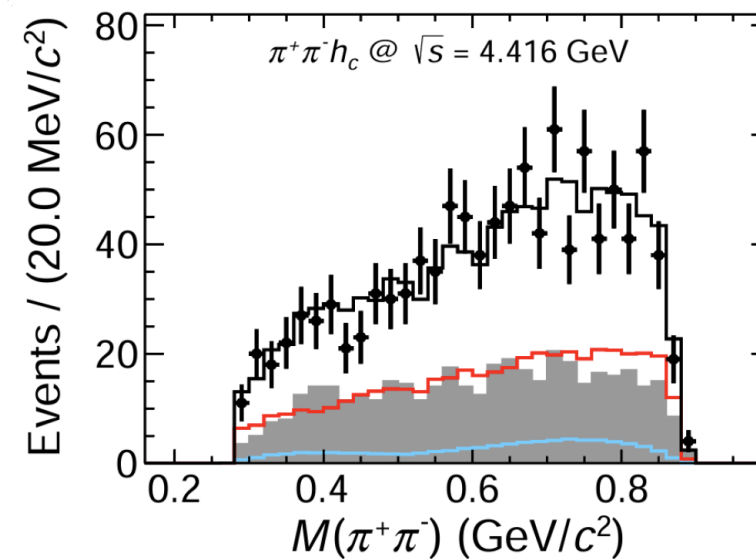
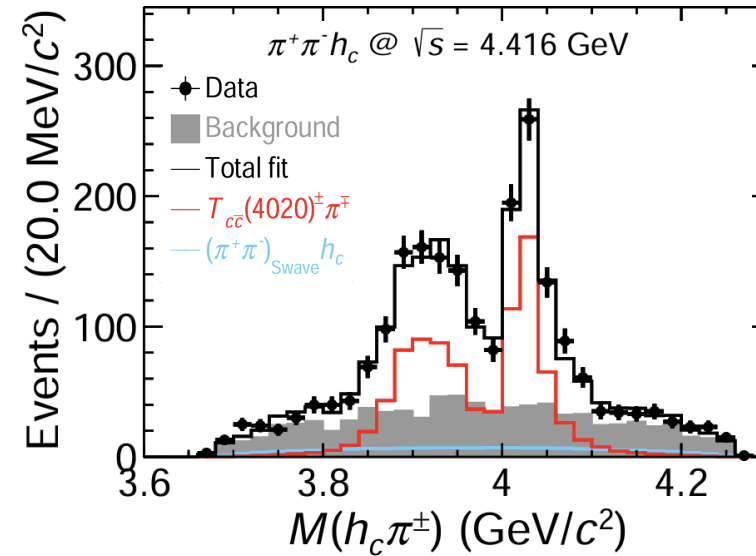
$D^{*0}D^{*-}\pi^+$



$\pi^+\pi^-\psi$



$\pi^+\pi^-h_c$



- Statistic uncertainty only

$$\sigma_{e^+e^- \rightarrow T_{c\bar{c}}(4020)^-\pi^+ + c.c.}^B \cdot Br[T_{c\bar{c}}(4020)^\pm \rightarrow D^{*0}D^{*-}(c.c.)] = FF(T_{c\bar{c}}^-\pi^+) \text{ in } D^{*0}D^{*-}\pi^+ \cdot \sigma_{e^+e^- \rightarrow D^{*0}D^{*-}\pi^+ + c.c.}^B$$

Simultaneously Fit		$\sqrt{s} = 4.395 \text{ GeV}$	$\sqrt{s} = 4.416 \text{ GeV}$
$D^{*0}D^{*-}\pi^+$	FF( $T_{c\bar{c}}^-\pi^+$ ) (%)	$75.1 \pm 4.3$	$61.4 \pm 4.6$
	FF( $T_{c\bar{c}}^+\pi^-$ ) (%)	$4.4 \pm 1.4$	$5.1 \pm 1.5$
$\pi^+\pi^-J/\psi$	FF( $T_{c\bar{c}}^-\pi^+$ ) (%)	$4.4 \pm 1.4$	$5.1 \pm 1.5$
	FF( $T_{c\bar{c}}^+\pi^-$ ) (%)	$35.0 \pm 6.4$	$33.5 \pm 4.7$
$\pi^+\pi^-h_c$	FF( $T_{c\bar{c}}^+\pi^-$ ) (%)	$35.0 \pm 6.4$	$33.5 \pm 4.7$
	FF( $T_{c\bar{c}}^-\pi^+$ ) (%)		
$\sigma_{2body}^B \cdot Br[T_{c\bar{c}}(4020)^\pm \rightarrow D^{*0}D^{*-}(c.c.)]$ (pb)		$335.2 \pm 21.0$	$336.4 \pm 25.7$
$\sigma_{2body}^B \cdot Br[T_{c\bar{c}}(4020)^\pm \rightarrow \pi^\pm J/\psi]$ (pb)		$1.1 \pm 0.3$	$1.4 \pm 0.3$
$\sigma_{2body}^B \cdot Br[T_{c\bar{c}}(4020)^\pm \rightarrow \pi^\pm h_c]$ (pb)		$30.4 \pm 4.5$	$29.6 \pm 3.3$

↙  
(From PRL 130, 121901)

- The relative branching fractions of  $T_{c\bar{c}}(4020)^\pm$  combining two datasets (weighted by yields) :

$$Br[T_{c\bar{c}}(4020)^\pm \rightarrow D^{*0}D^{*-}(c.c.)] : Br[T_{c\bar{c}}(4020)^\pm \rightarrow \pi^\pm J/\psi] : Br[T_{c\bar{c}}(4020)^\pm \rightarrow \pi^\pm h_c] = 1.000(48) : 0.004(1) : 0.090(8)$$

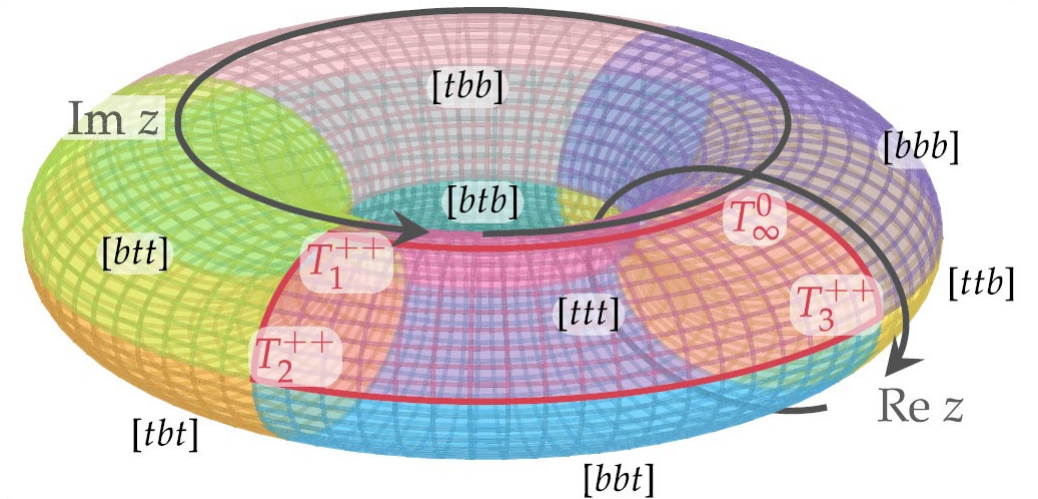
- Eight poles on Riemann-sheets:

$$R(m) = \frac{1}{m^2 - M^2 + im[\Gamma_{D^{*0}D^{*-}}(m) + \Gamma_{J/\psi\pi}(m) + \Gamma_{h_c\pi}(m) + \Gamma_X(m)]}$$

$$\Gamma(m) = \frac{1}{8\pi} \sum_{\lambda_B, \lambda_C} \overline{|H_{\lambda_B, \lambda_C}^J|^2} \frac{|\mathbf{p}|}{m^2}$$

$$\text{Imag.}(p) = (J/\psi, h_c, D^*)$$

Pole	$M_{\text{pole}} \text{ (MeV}/c^2)$	$\Gamma_{\text{pole}} \text{ (MeV)}$	$\text{Imag.}(p)$
1	$4022.44 \pm 1.55$	$38.54 \pm 2.94$	(-, -, -)
2	$4023.01 \pm 1.35$	$35.02 \pm 2.20$	(-, +, -)
3	$4029.39 \pm 0.86$	$0.03 \pm 15.66$	(-, +, +)
4	$4029.87 \pm 0.95$	$2.85 \pm 14.56$	(-, -, +)
5	$4022.44 \pm 1.55$	$-38.54 \pm 2.94$	(-, -, -)
6	$4023.01 \pm 1.35$	$-35.02 \pm 2.20$	(-, +, -)
7	$4029.39 \pm 0.86$	$-0.03 \pm 15.66$	(-, +, +)
8	$4029.87 \pm 0.95$	$-2.85 \pm 14.56$	(-, -, +)



- Conservation of spin and parity:  $\left\{ \begin{array}{l} \gamma^* \rightarrow \pi T_{c\bar{c}}(1^- \rightarrow 0^- J^P) : J^P \neq 0^+ \\ T_{c\bar{c}} \rightarrow \pi h_c (J^P \rightarrow 0^- 1^+) : J^P \neq 0^- \end{array} \right.$
- Only :  $J^P = 1^+, 1^-, 2^+, 2^-$ .

Two hypotheses:

- $H_0$  stands the  $T_{c\bar{c}}(4020)^-$  component in data **only** includes  $J^P = 1^- / 2^+ / 2^-$ .
- $H_1$  stands the  $T_{c\bar{c}}(4020)^-$  component in data includes  $J^P = 1^- / 2^+ / 2^-$  but adding a more component with  $J^P = 1^+$ .

Using two hypotheses to fit data.

Checking the significance for  $J^P = 1^+$  over the  $J^P = 1^- / 2^+ / 2^-$ .

Table 1. Significance to distinguish the  $T_{c\bar{c}}(4020)^-$  quantum number  $1^+$  over other quantum numbers.

$H_0$	$H_1$	$\Delta(-2 \ln \mathcal{L})$	$\Delta(\text{ndof})$	Statistical significance
only $1^-$	$1^-$ with additional $1^+$	409.2	23	$18.0\sigma$
only $2^+$	$2^+$ with additional $1^+$	435.4	23	$18.7\sigma$
only $2^-$	$2^-$ with additional $1^+$	329.4	23	$15.8\sigma$
only $1^+$	$1^+$ with additional $1^-$	27.8	21	$1.5\sigma$
only $1^+$	$1^+$ with additional $2^+$	30.2	23	$1.5\sigma$
only $1^+$	$1^+$ with additional $2^-$	56.4	25	$3.6\sigma$

## Part 1. Uncertainties from input cross section

Part 1 is used in the cross section measurement for two body process.

## Part 2. Uncertainties from PWA Fit

Table 1. The relative systematic uncertainties (%) of FFs in different  $T_{c\bar{c}}(4020)^-$  decay channels.

Data set	4.395 GeV			4.416 GeV		
	$D^{*0}D^{*-}$	$\pi^- J/\psi$	$\pi^- h_c$	$D^{*0}D^{*-}$	$\pi^- J/\psi$	$\pi^- h_c$
$\pi\pi$ S-wave	1.0	19.0	0.4	0.8	12.3	2.0
Background	0.7	8.0	6.6	0.4	6.8	6.3
Fit model	7.0	20.0	12.3	3.5	16.5	3.5
$T_{c\bar{c}1}(3900)$ propagator	0.1	13.4	2.5	0.9	12.0	3.7
Other BW parameters	1.2	29.9	12.3	1.9	28.0	11.8
Fit strategy	2.5	8.2	2.5	1.3	10.7	0.2
Total	7.6	44.3	19.0	4.4	38.9	14.4

Table 2. The absolute systematic uncertainties of the pole mass ( $m_{\text{pole}}$ , in MeV/ $c^2$ ) and width ( $\Gamma_{\text{pole}}$ , in MeV) for the  $T_{c\bar{c}}(4020)^-$  state.

Source	Pole 1		Pole 2		Pole 3		Pole 4	
	$m_{\text{pole}}$	$\Gamma_{\text{pole}}$	$m_{\text{pole}}$	$\Gamma_{\text{pole}}$	$m_{\text{pole}}$	$\Gamma_{\text{pole}}$	$m_{\text{pole}}$	$\Gamma_{\text{pole}}$
$\pi\pi$ S-wave	0.38	0.77	0.29	0.51	0.08	1.95	0.00	1.83
Background	0.05	0.17	0.06	0.21	0.10	0.08	0.09	0.05
Fit model	0.40	1.09	0.18	1.60	0.79	7.04	0.55	1.07
$T_{c\bar{c}}(3900)$ propagator	0.03	1.08	0.14	0.52	0.38	5.57	0.53	5.16
Other BW parameters	0.36	1.43	0.28	1.23	0.70	10.98	0.70	9.15
Fit strategy	1.07	1.33	0.86	0.80	0.09	9.07	0.14	8.49
Mass resolution	-	1.37	-	1.37	-	1.37	-	1.37
Total	1.26	2.95	0.98	2.68	1.13	17.00	1.05	13.74

- Considering the systematic uncertainties from PWA Fit.

Two hypotheses:

- $H_0$  stands the  $T_{c\bar{c}}(4020)^-$  component in data **only** includes  $J^P = 1^- / 2^+ / 2^-$ .
- $H_1$  stands the  $T_{c\bar{c}}(4020)^-$  component in data includes  $J^P = 1^- / 2^+ / 2^-$  but adding a more component with  $J^P = 1^+$ .

Using two hypotheses to fit data.

Checking the significance for  $J^P = 1^+$  over the  $J^P = 1^- / 2^+ / 2^-$ .

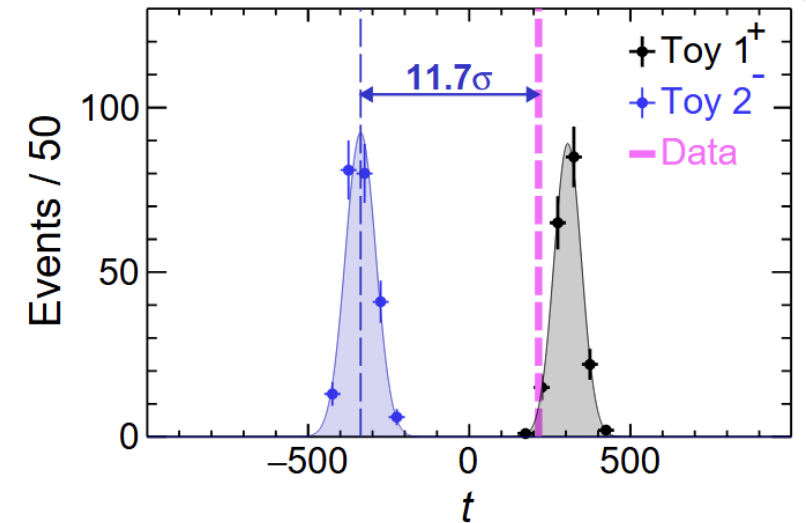
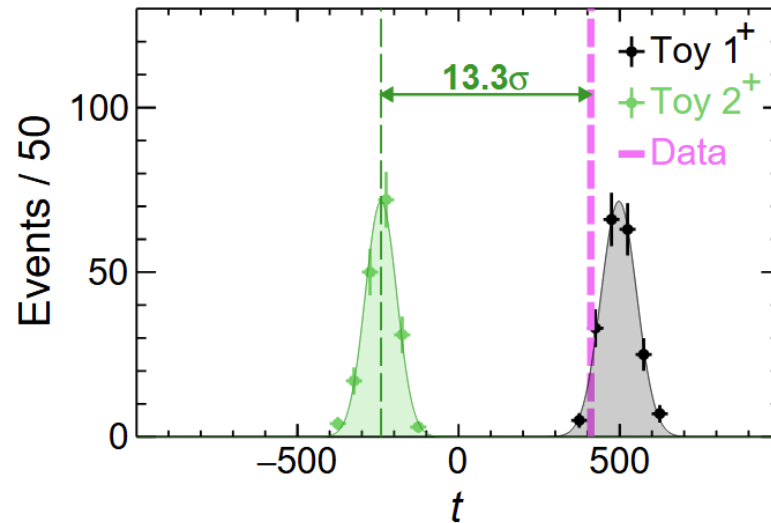
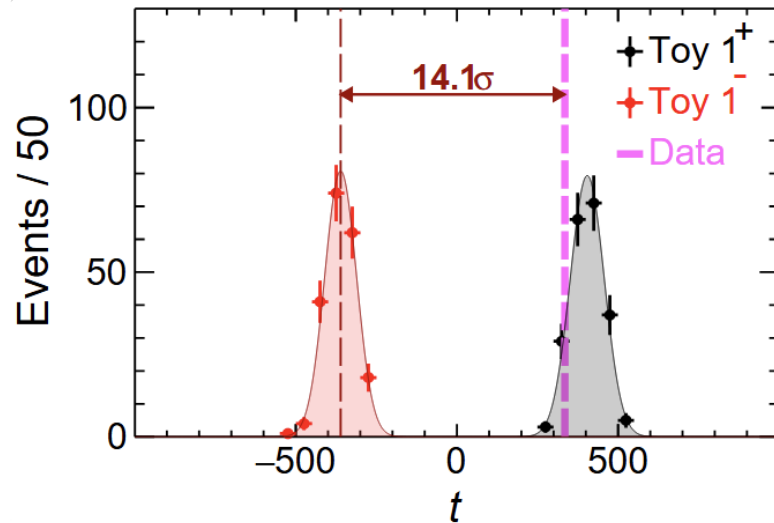
Table 1. Confidence levels of  $H_1$  hypothesis versus  $H_0$  hypothesis for the  $T_{c\bar{c}}(4020)^-$  spin-parity assignments.

Source	$1^+$ over $1^-$	$1^+$ over $2^+$	$1^+$ over $2^-$	$1^-$ over $1^+$	$2^+$ over $1^+$	$2^-$ over $1^+$
Default	$18.0\sigma$	$18.7\sigma$	$15.8\sigma$	$1.5\sigma$	$1.5\sigma$	$3.6\sigma$
$\pi^+\pi^-$ S-wave	$18.7\sigma$	$19.2\sigma$	$18.3\sigma$	$0.5\sigma$	$0.0\sigma$	$3.9\sigma$
Background	$17.2\sigma$	$17.8\sigma$	$15.7\sigma$	$1.6\sigma$	$1.4\sigma$	$3.8\sigma$
Fit model	$17.2\sigma$	$18.5\sigma$	$14.1\sigma$	$3.1\sigma$	$0.5\sigma$	$3.9\sigma$
$T_{c\bar{c}1}(3900)$ BW	$17.8\sigma$	$18.8\sigma$	$15.4\sigma$	$1.9\sigma$	$2.0\sigma$	$3.2\sigma$
Others BW parameters	$17.5\sigma$	$18.7\sigma$	$15.4\sigma$	$1.1\sigma$	$1.6\sigma$	$3.7\sigma$

Perform Toy MC test when consider this.

- Generate two Toy MC samples, repeating 500 times (Poisson sampling).
  - a. MC is based on fit result with  $1^+$  model.
  - b. MC is based on fit result **with  $1^-$  model**.
- Fit the same MC sample, repeating 1000 times.
  - a. MC is fitted with  $1^+$  model.
  - b. MC is fitted **with  $1^-$  model**.

$$t \equiv \Delta(-2 \ln L) = -2 [-\ln L^{1^+} - (-\ln L^{J^P})]$$



- Multi-channel joint analysis of the exotic charmonium-like state  $T_{c\bar{c}}(4020)$  at  $\sqrt{s} = 4.395$  and  $4.416$  GeV.
  - Determine spin-parity of  $T_{c\bar{c}_1}(4020)$ ,  $J^P = 1^+$  with  $> 11.7\sigma$
  - Relative decay BF of  $T_{c\bar{c}_1}(4020)^-$ 
$$Br[T_{c\bar{c}_1}(4020)^- \rightarrow \pi^- J/\psi] / Br[T_{c\bar{c}_1}(4020)^- \rightarrow D^{*0} D^{*-}] = (3.6 \pm 0.6 \pm 1.6) \times 10^{-3}$$
$$Br[T_{c\bar{c}_1}(4020)^- \rightarrow \pi^- h_c] / Br[T_{c\bar{c}_1}(4020)^- \rightarrow D^{*0} D^{*-}] = (8.9 \pm 1.3 \pm 2.3) \times 10^{-2}$$
  - Pole mass and width

Thanks for your attention!

# Backup

Data samples	$\sqrt{s}$ (GeV)	Integrated luminosity ( $\text{pb}^{-1}$ )
4400	4.395	508.18
4420	4.416	1090.70

## For signal MC samples

3-body processes --- PHSP model --- Cross section lineshapes measured in previous works.

## For background MC samples

Hadrons productions, bhabha productions, continuum process  $e^+e^- \rightarrow \gamma\gamma, \mu\mu$  and  $\tau\tau\dots$

# Determine Baseline Solution

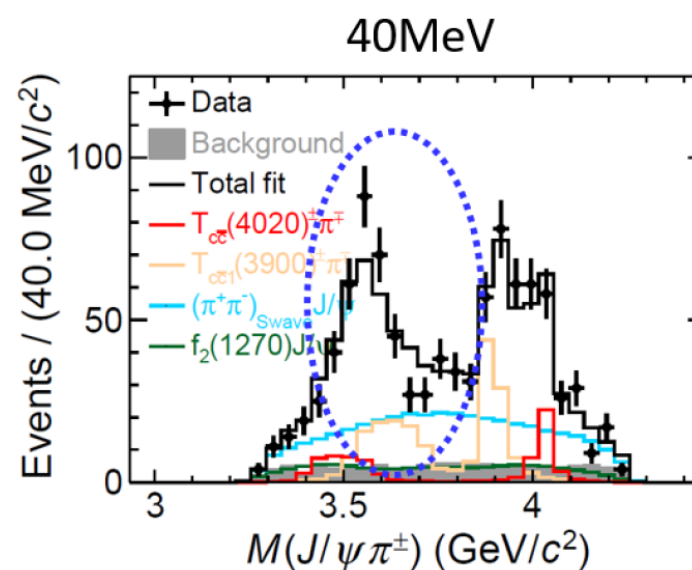
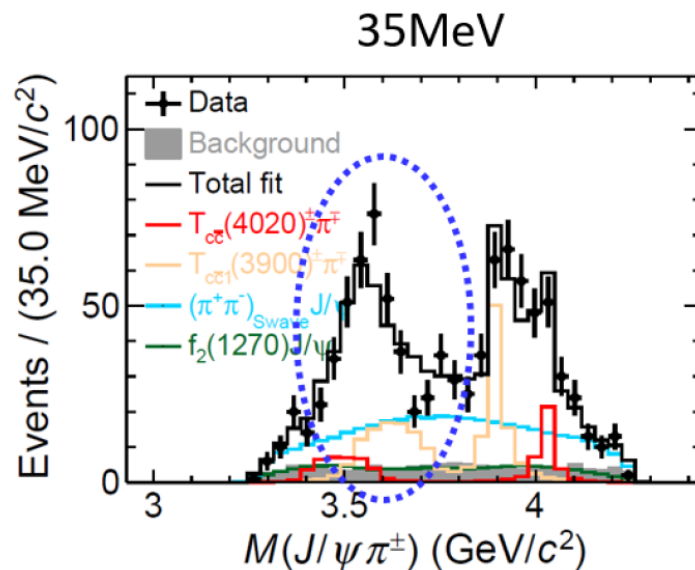
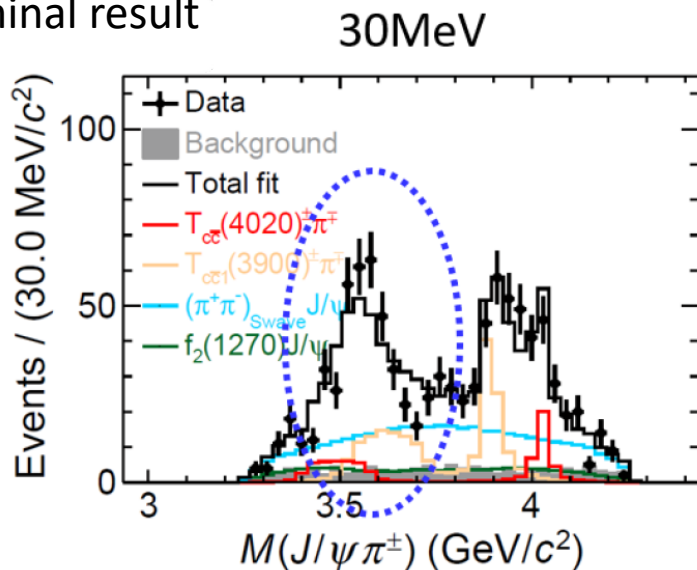
- We first consider all possible processes into the fitting for each channel.

Round1	Round2	Round3	Round4
(Physical)			
$Z_c(4020)^-\pi^+$ (35.1 $\sigma$ )	$Z_c(4020)^-\pi^+$	$Z_c(4020)^-\pi^+$	$Z_c(4020)^-\pi^+$
$D^*D_1(2420)$ (9.3 $\sigma$ )	$D^*D_1(2420)$	$D^*D_1(2420)$	$D^*D_1(2420)$
$Z_c(3900)^\pm_{J/\psi}\pi^\mp$ (9.9 $\sigma$ )	$Z_c(3900)^\pm_{J/\psi}\pi^\mp$	$Z_c(3900)^\pm_{J/\psi}\pi^\mp$	$Z_c(3900)^\pm_{J/\psi}\pi^\mp$
$(\pi^+\pi^-)_{S\text{-wave}}J/\psi$ (14.3 $\sigma$ )	$(\pi^+\pi^-)_{S\text{-wave}}J/\psi$	$(\pi^+\pi^-)_{S\text{-wave}}J/\psi$	$(\pi^+\pi^-)_{S\text{-wave}}J/\psi$
$f_2J/\psi$ (4.5 $\sigma$ )	$f_2J/\psi$	$f_2J/\psi$	$f_2J/\psi$
$Z_c(3900)^\pm_{h_c}\pi^\mp$ (2.4 $\sigma$ )	$(\pi^+\pi^-)_{S\text{-wave}}h_c$	$(\pi^+\pi^-)_{S\text{-wave}}h_c$	$(\pi^+\pi^-)_{S\text{-wave}}h_c$
$(\pi^+\pi^-)_{S\text{-wave}}h_c$ (4.7 $\sigma$ )	$NR2^-D^{*0}$	$NR2^-D^{*0}$	$NR2^-D^{*0}$
		$NR0^-D^{*0}$	$NR0^-D^{*0}$
			$NR1^-D^{*0}$
(NR)			
$NR0^-D^{*0}$ (6.6 $\sigma$ )	$NR0^-D^{*0}$ (6.6 $\sigma$ )	-	
$NR1^-D^{*0}$ (6.5 $\sigma$ )	$NR1^-D^{*0}$ (6.5 $\sigma$ )	$NR1^-D^{*0}$ (5.6 $\sigma$ )	
$NR2^-D^{*0}$ (9.3 $\sigma$ )	-	-	
$NR2^+D^{*0}$ (4.5 $\sigma$ )	$NR2^+D^{*0}$ (4.5 $\sigma$ )	$NR2^+D^{*0}$ (3.6 $\sigma$ )	$NR2^+D^{*0}$ (1.5 $\sigma$ )

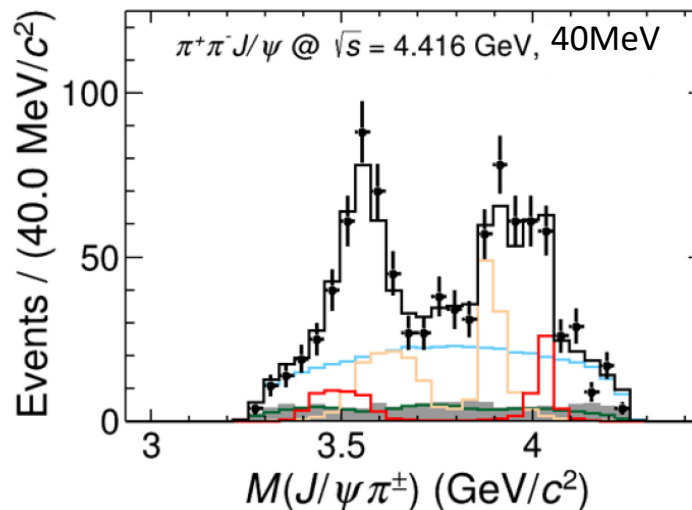
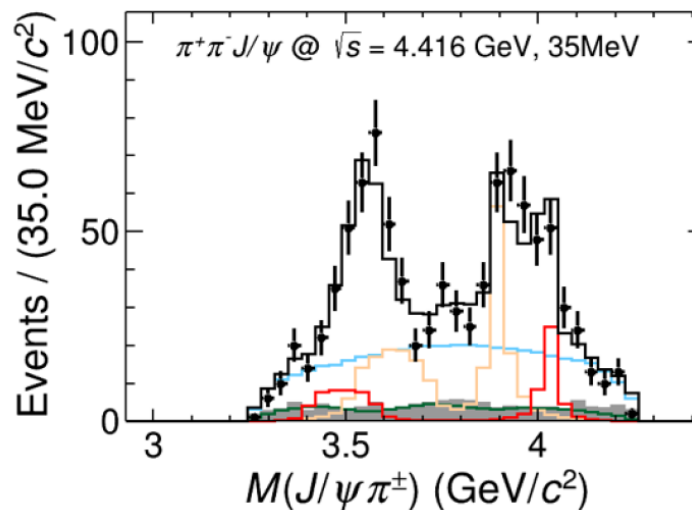
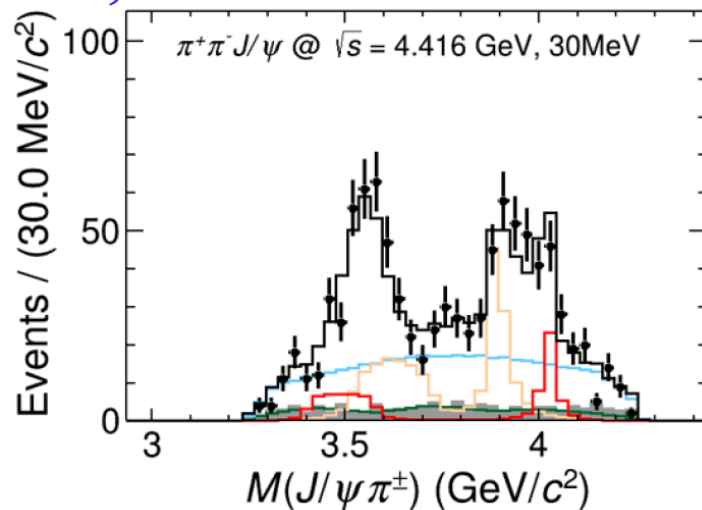
- The spin and parity of  $Z_c(3900)$  and  $Z_c(4020)$  are assigned at  $J^P = 1^+$ .
- The baseline solutions for other spin-parity *i. e.*  $J^P = 1^-, 2^+, 2^-$  are also checked.
  - All of them can reach the same conclusion: data sets favor  $1^+Z_c(4020)$ .

# $(\pi^+\pi^-)$ *S* wave

Nominal result



$(\pi^+\pi^-)$  *S* wave



$$R(m) = \frac{1}{m^2 - M^2 + im[\Gamma_{D^*0D^{*-}}(m) + \Gamma_{J/\psi\pi^-}(m) + \Gamma_{h_c\pi^-}(m) + \Gamma_X(m)]}$$

$$I_i = \int_{\Omega} |A_{e^+e^- \rightarrow Z_c^- \pi^+}|^2 \cdot |BW|^2 \cdot |A_{Z_c^- \rightarrow X_i}|^2 d\Omega$$

$$= \int_{threshold_i}^{E_{cms} - m_{\pi^+}} |A_{e^+e^- \rightarrow Z_c^- \pi^+}|^2 \cdot |BW(m)|^2 \cdot |A_{Z_c^- \rightarrow X_i}|^2 \cdot |q||p| dm$$

$$= \int_{threshold_i}^{E_{cms} - m_{\pi^+}} |A_{e^+e^- \rightarrow Z_c^- \pi^+}|^2 \cdot |BW(m)|^2 \cdot C_0 |\Gamma_i(m)| \cdot m^2 \cdot |q| dm$$

$$\Gamma(m) = \frac{1}{8\pi} \sum_{\lambda_B, \lambda_C} |H_{\lambda_B, \lambda_C}^J|^2 \frac{|\mathbf{p}|}{m^2}$$

$$I_i = \int_{threshold_i}^{E_{cms} - m_{\pi^+}} |A_{e^+e^- \rightarrow Z_c^- \pi^+}|^2 |BW(m)|^2 C_0 |\Gamma_i(m)| \cdot m^2 \cdot |q| dm$$

$$\frac{I_1}{I_1 + I_2 + I_3 + I_X} = 0.4097 \pm 0.2050$$

$$\frac{I_2}{I_1 + I_2 + I_3 + I_X} = 0.0022 \pm 0.0011$$

$$\frac{I_3}{I_1 + I_2 + I_3 + I_X} = 0.0433 \pm 0.0179$$

$$\frac{I_X}{I_1 + I_2 + I_3 + I_X} = 0.5447 \pm 0.2164$$

$$I_1 : I_2 : I_3 = 1.000(500) : 0.005(4) : 0.106(68)$$

$$\Gamma_{X_i}(m) = \frac{1}{8\pi} \sum_{\lambda_1, \lambda_2} \left| H_{\lambda_1, \lambda_2}^{T_{c\bar{c}} \rightarrow X_i} \right|^2 \frac{|\mathbf{p}|}{m^2},$$

TABLE I: Comparison of the different momentum assumptions on the missing decay.

Momentum dependence	Momentum $ \mathbf{p} $ (GeV/c)	Fitting ( $\sum  H_x ^2$ )	$ \mathbf{p}  \cdot (\sum  H_x ^2)$	$Br(X)$
$\pi^- J/\psi$	0.815	$9.5 \pm 3.7$	$7.7 \pm 3.0$	$0.530 \pm 0.222$
$\pi^- h_c$	0.453	$17.3 \pm 6.7$	$7.8 \pm 3.0$	$0.536 \pm 0.224$
$\pi^- \psi(3686)$	0.300	$26.5 \pm 10.4$	$8.0 \pm 3.1$	$0.538 \pm 0.226$
$(D\bar{D}^*)^-$	0.552	$14.2 \pm 5.5$	$7.9 \pm 3.1$	$0.528 \pm 0.223$
Non (set $ \mathbf{p}  = 1$ )	1.000	$7.6 \pm 2.9$	$7.6 \pm 2.9$	$0.542 \pm 0.210$

