核子及其共振态电磁形状因子的夸克-双夸克描述

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Based on:

arXiv:2507.13484, 2512.09160

第五届强子与重味物理理论与实验联合研讨会
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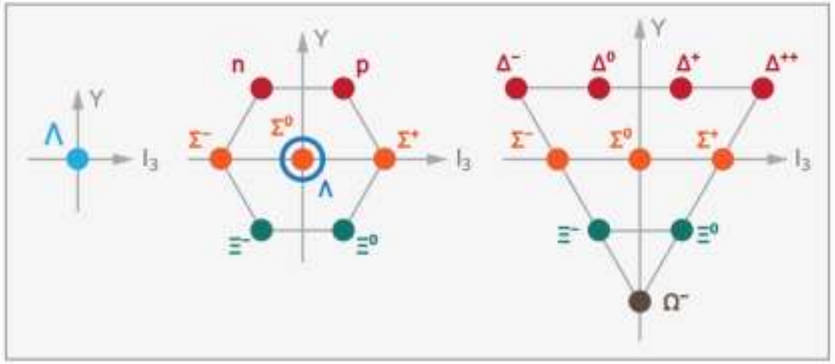
Content

- Motivation
- Dyson-Schwinger Equations
- EMFFs of Nucleon
- EMTFFs of $\Delta(1700)3/2^-$
- Summary and Outlook

Motivations

$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^+$	$\frac{7}{2}^-$	$\frac{9}{2}^+$	$\frac{9}{2}^-$	$\frac{11}{2}^+$	$\frac{11}{2}^-$	$\frac{13}{2}^+$	$\frac{13}{2}^-$	$\frac{15}{2}^+$
N N(1440) N(1710) N(1880) N(2100) N(2300)	N(1535) N(1650) N(1895)	N(1720) N'(1720) N(1900) N(2040)	N(1520) N(1700) N(1875) N(2120)	N(1680) N(1860) N(2000)	N(1675) N(2060) N(2570)	N(1990)	N(2190)	N(2220)	N(2250)		N(2600)	N(2700)		
Δ(1750) Δ(1910)	Δ(1620) Δ(1900) Δ(2150)	Δ(1232) Δ(1600) Δ(1920)	Δ(1700) Δ(1940)	Δ(1905) Δ(2000)	Δ(1930) Δ(2350)	Δ(1950) Δ(2390)	Δ(2200)	Δ(2300)	Δ(2400)	Δ(2420)			Δ(2750)	Δ(2950)
Λ(1116) Λ(1600) Λ(1710) Λ(1810)	Λ(1380) Λ(1405) Λ(1670) Λ(1800) Λ(2000)	Λ(1890) Λ(2070)	Λ(1520) Λ(1690) Λ(2050) Λ(2325)	Λ(1820) Λ(2110)	Λ(1830) Λ(2080)	Λ(2085)	Λ(2100)	Λ(2350)						
Σ(1189) Σ(1660) Σ(1880)	Σ(1620) Σ(1750) Σ(1900) Σ(2110)	Σ(1385) Σ(1780) Σ(1940) Σ(2080) Σ(2230)	Σ(1580) Σ(1670) Σ(1910) Σ(2010)	Σ(1915) Σ(2070)	Σ(1775)	Σ(2030)	Σ(2100)							
Ξ(1315)		Ξ(1530)	Ξ(1820)											
		Ω(1672)												



The observed resonances in experiment differ from those predicted by the quark model!

Motivations

➢Electromagnetic form factors(EMFFs)

✓The EMFFs are important quantities for the understanding of hadron's electromagnetic structure;

charge radius, magnetic momentum, charge and magnetisation densities

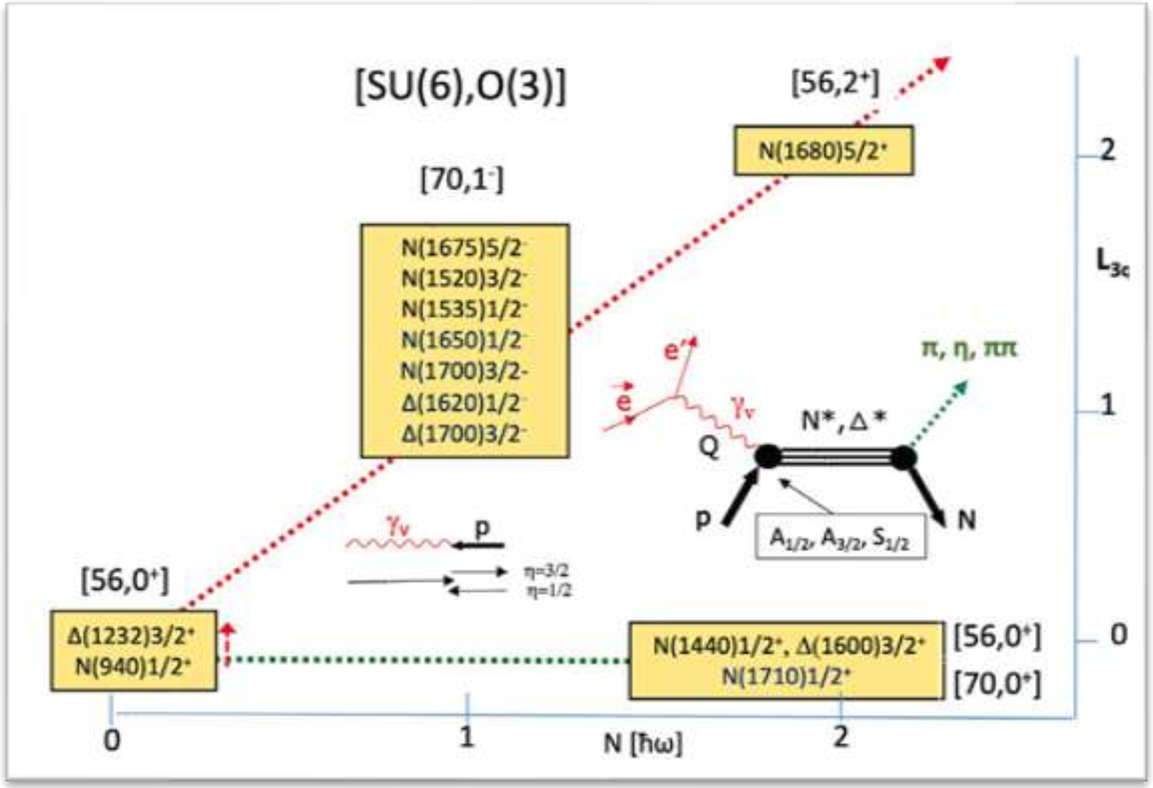
✓The EMFFs are sensitive to the internal structure and are helpful for distinguishing the internal structure;

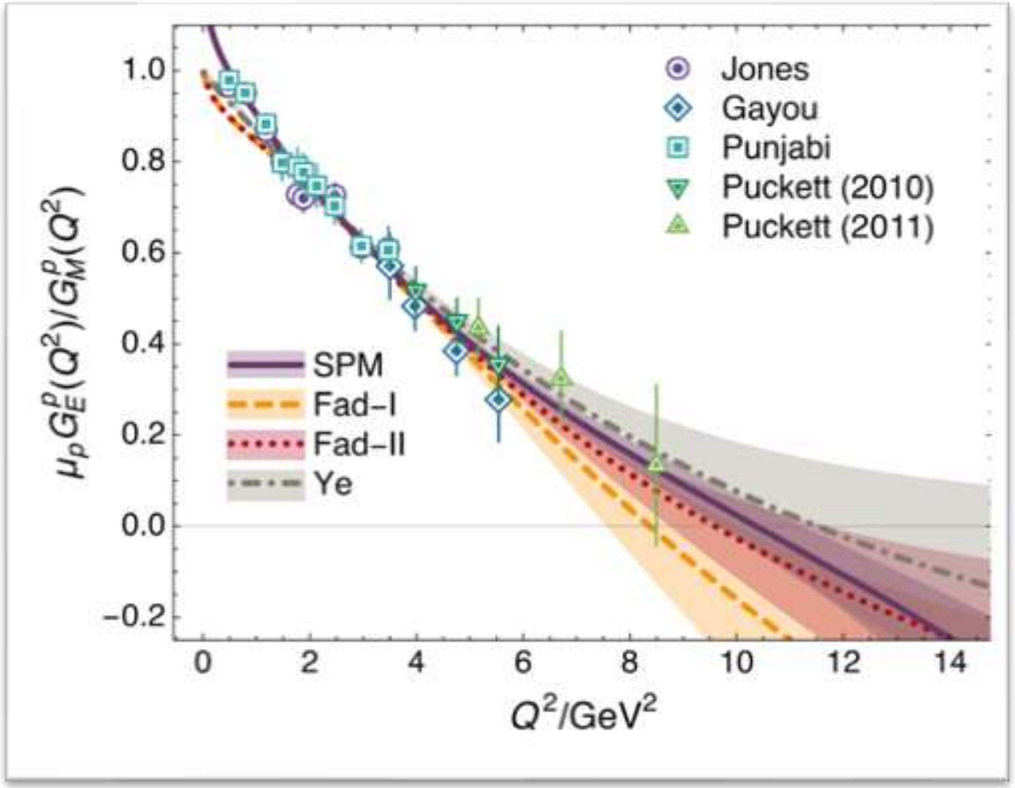
three quark, quark-diquark, multiquark,hadronic molecules

✓The evolution behavior of the EMFFs with Q^2 can reveal the scale dependence of effective degrees of freedom.

meson-baryon, dressed quark, current quark

Motivations





V. Burkert, G. Eichmann, E. Klempt, Prog. Part. Nucl. Phys. 146 (2026) 104214

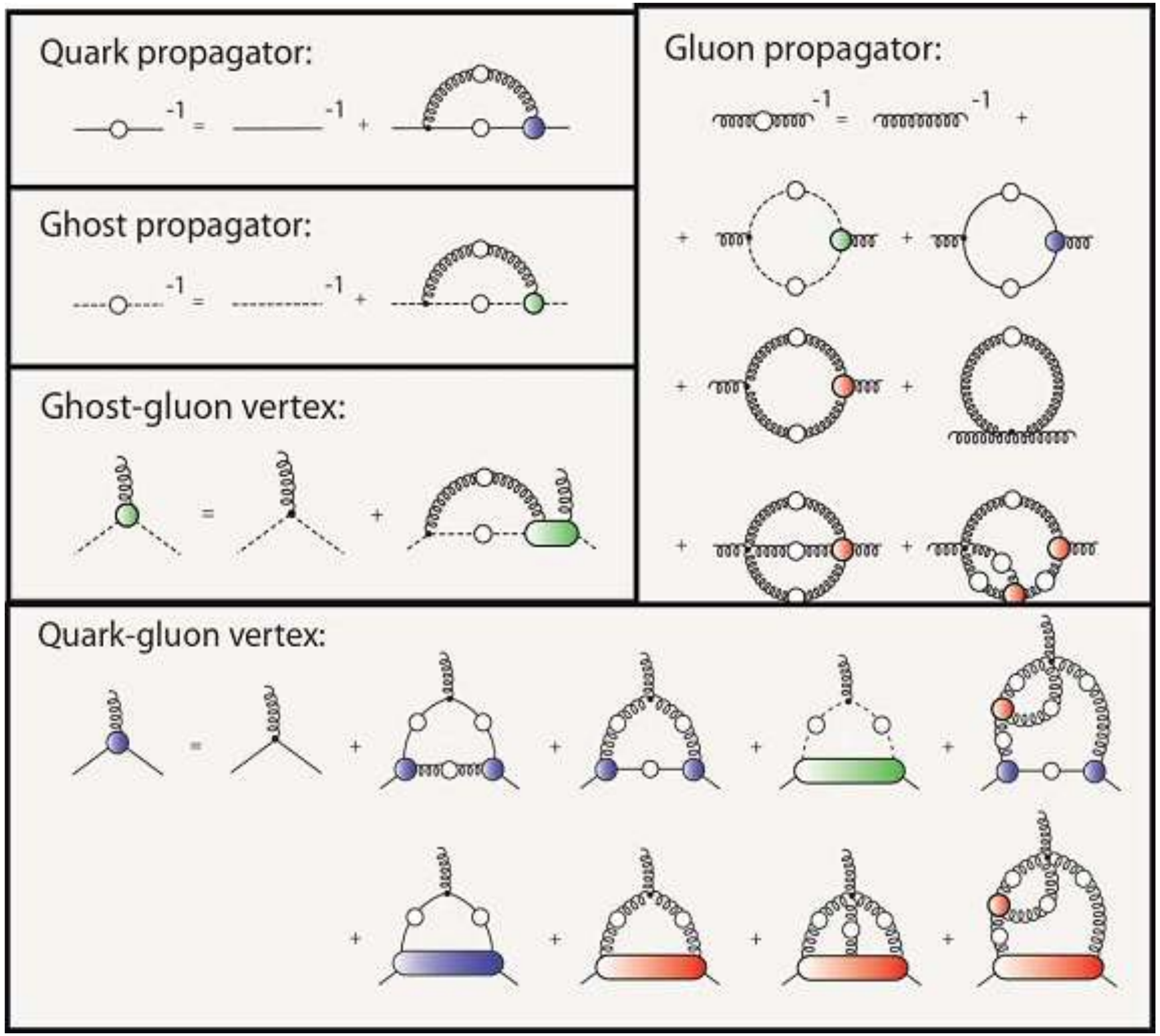
D. Binosi, C. D. Roberts, Z.-Q.Yao, arXiv: 2503.05984

JLab: CLAS、CLAS12 and planned CLAS22

I. G. Aznauryan et al., Int. J. Mod. Phys. E 22 (2013) 1330015
D. S. Carman, K. Joo, V. I. Mokeev, Few Body Syst. 61 (2020) 3, 29
A. Accardi et al., Eur.Phys.J.A 60 (2024) 9, 173

Dyson-Shwinger Equations(DSEs)

- Continuum Schwinger function methods
- ✓ nonperturbative
- ✓ symmetry-preserving
- Owing to the infinite coupling feature, it is necessary to truncate the DSEs at a certain level for practical calculations.



C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. 33 (1994) 477-575.
C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. 45 (2000) S1-S103.
P. Maris and C. D. Roberts, Int. J. Mod. Phys. E 12 (2003) 297-365.

Dyson-Shwinger Equations(DSEs)

- 2n-point Green function G^n obeys Dyson's equation:

$$G^n = G_0^n + G_0^n K G^n$$

K is the n-quark scattering kernel, G_0^n is the disconnected n-quark propagator.

- A bound state of mass M with wave function ψ show up as a pole in the 2n-point function, i.e. in the vicinity of the pole G^n becomes(P is the total momentum):

$$G^n \sim \frac{\psi(k_1, k_2, ...k_n)\overline{\psi}(p_1, p_2, ...p_n)}{P^2 + M^2}$$

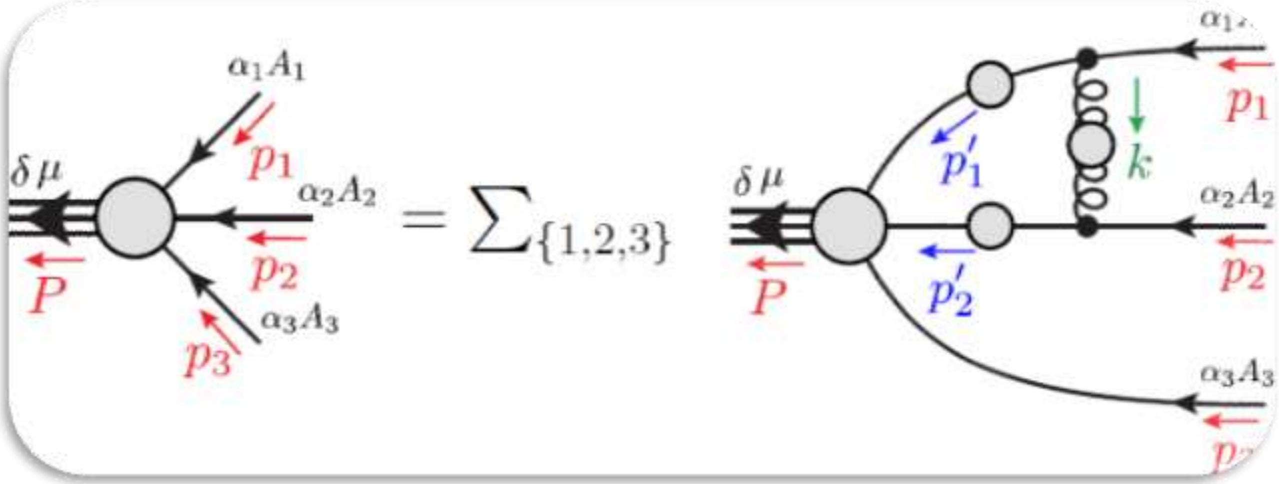
- Comparing residues, one finds the homogeneous bound state equation:

$$\psi = G_0 K \psi$$

- For the Faddeev equation of three quark system, the kernel consists of three terms:

$$K = K_1 + K_2 + K_3$$

Dyson-Shwinger Equations(DSEs)



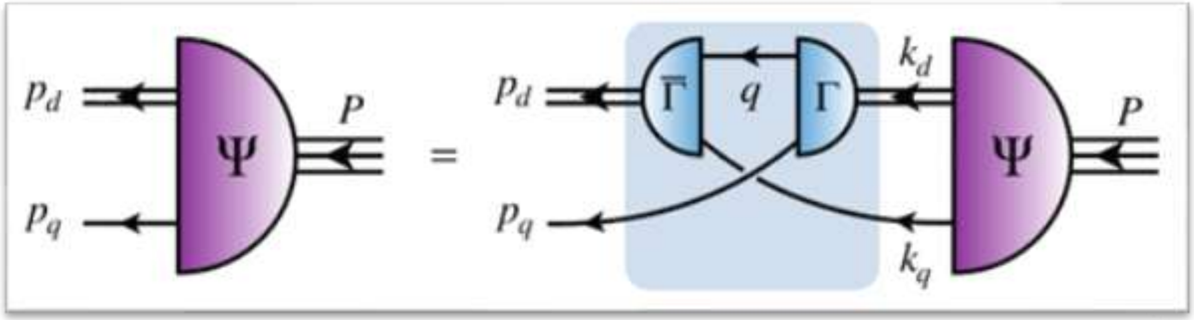
- ✓ Nucleon mass from a covariant three-quark Faddeev equation, G. Eichmann et al., Phys. Rev. Lett. 104 (2010) 201601
- ✓ Poincaré-covariant analysis of heavy-quark baryons, S.-X. Qin (秦思学) et al., Phys.Rev. D 97 (2018) 114017/1-13
- ✓ Nucleon Gravitational Form Factors, Z.-Q. Yao (姚照千) et al., Eur. Phys. J. A 61 (2025) 92/1-13

- Baryons appear as poles in the six-point Schwinger function, and their amplitude satisfy a homogeneous integral equation-Faddeev equation.

- Direct solution of Faddeev equation using rainbow-ladder truncation is now possible, but remains challenging problem – algorithms and numerical analysis.

Dyson-Shwinger Equations(DSEs)

- **Faddeev Equation(Quark+Diuark Framework)**
  - For many applications, diquark approximation to quark+quark scattering kernel is used
  - ***Prediction***: owing to DCSB, *diquark correlations may exist within baryons*



Diquark correlations in hadron physics: Origin, impact and evidence
M.Yu.Barabanov, Craig D.Roberts et al. Prog.Part.Nucl.Phys. 116 (2021) 103835

- Diquark correlation
  - (I=0,J^P=0^+):isoscalar-scalar diquark
  - (I=1,J^P=1^+):isovector-axialvector diquark
  - (I=0,J^P=0^-):isoscalar-pesudoscalar diquark
  - (I=0,J^P=1^-):isoscalar-vector diquark
  - (I=1,J^P=1^-):isovector-vector diquark

✓ G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, Prog.Part.Nucl.Phys. 91 (2016) 1- 100
✓ ChenChen, B. El-Bennich, C. D. Roberts, S. M. Schmidt, J. Segovia, S-L. Wan, Phys.Rev. D97 (2018) no.3, 034016

EMFFs of Nucleon

- The nucleon electromagnetic interaction current:

$$J_\mu(P_f,P_i) = ie\,\Lambda_+(P_f)\,\Lambda_\mu(Q,P)\,\Lambda_+(P_i)\,,$$
$$= ie\,\Lambda_+(P_f)\,\left[\gamma_\mu F_1(Q^2) + \frac{1}{2m_N}\sigma_{\mu\nu}\,Q_\nu\,F_2(Q^2)\right]\,\Lambda_+(P_i)\,,$$

where $Q = P_f - P_i$ is the transferred photon momentum. $F_{1,2}$ are the Dirac and Pauli form factors.

- The nucleon charge and magnetization distributions ($\tau = Q^2/[4m_N^2]$):

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)\,,$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)\,.$$

The $SU(2)_F$ isospin limit is used： $m_u = m_d$.

$$J_\mu(P_f,P_i) = \int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} \sum_{i=1}^{6} \bar\Psi(p,-P_f)\Lambda^i_\mu(p,P_f;k,P_i)\Psi(k;P_i)$$

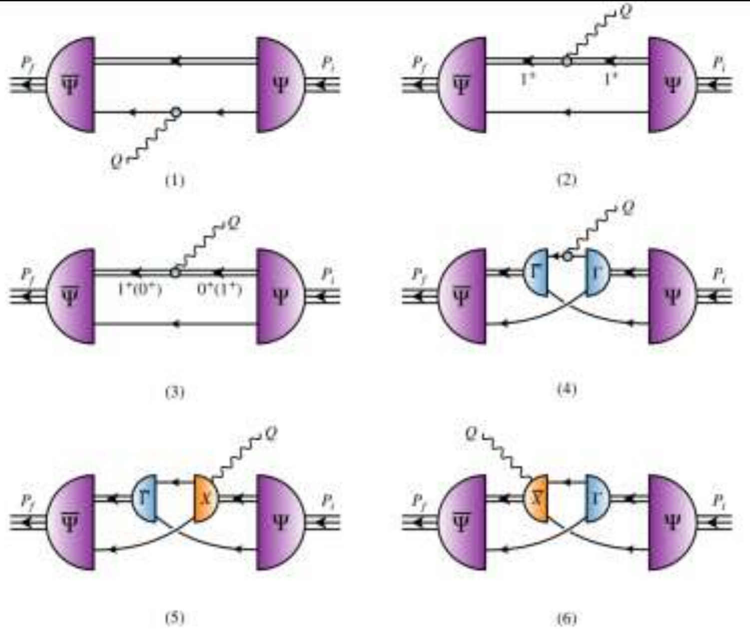
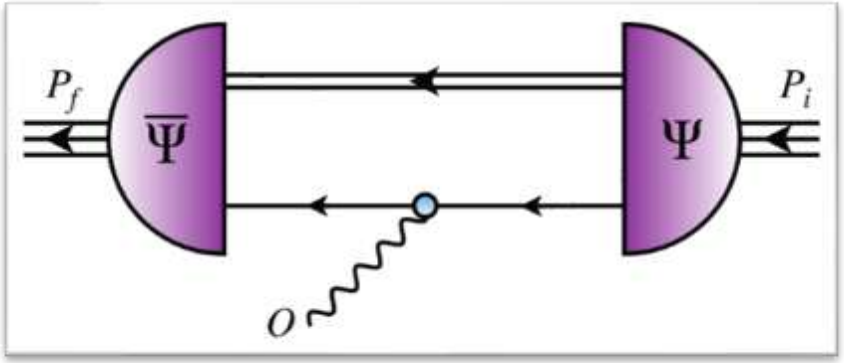


Fig. 2 The collection of electromagnetic interaction diagrams that ensures a conserved current for on-shell nucleons described by the Faddeev amplitude, Ψ, calculated as described in Sect. 2. Legend. *single line*, dressed-quark propagator; *undulating line*, electromagnetic probe; Γ, diquark correlation amplitude; *double line*, diquark propagator; and χ, seagull terms.

EMFFs of Nucleon

➢ The unamputated photon-quark vertex satisfy a Ward-Takahashi identity:

$$i(\ell_1-\ell_2)_\mu \chi_\mu(\ell_1,\ell_2) = S(\ell_2) - S(\ell_1)$$



the following Ansatz for the vertex, expressed in terms of quantities already specified in dress quark propagator:

$$\begin{aligned}\chi_\mu(\ell_1,\ell_2) = {} & \gamma_\mu \Sigma_{\sigma_V} + 2\ell_\mu[\gamma\cdot\ell \Delta_{\sigma_V} + i\Delta_{\sigma_S}] \\ & + \tfrac{1}{2}[s_1-\bar{s}_1][\gamma\cdot\check{\ell}\gamma_\mu\gamma\cdot\ell - \gamma\cdot\ell\gamma_\mu\gamma\cdot\check{\ell}]\Delta_{\sigma_V} \\ & - [s_2-\bar{s}_2]\sigma_{\mu\nu}\check{\ell}_\nu\Delta_{\sigma_S} \\ & + \tfrac{i}{2}[1+s_3]\gamma\cdot\check{\ell}\sigma_{\mu\nu}\check{\ell}_\nu\Delta_{\sigma_V},\end{aligned}$$

M. Ding, K. Raya, A. Bashir, D. Binosi, L. Chang, M. Chen, C.D. Roberts, Phys. Rev. D 99, 014014 (2019)

$\ell = [\ell_1+\ell_2]/2$, $\check{\ell} = \ell_1 - \ell_2$, and, $\bar{s}_{1,2} = 1 - s_{1,2}$

$$\Sigma_F = \tfrac{1}{2}[F(\ell_1^2)+F(\ell_2^2)],$$
$$\Delta_F = [F(\ell_1^2)-F(\ell_2^2)]/[\ell_1^2-\ell_2^2],$$

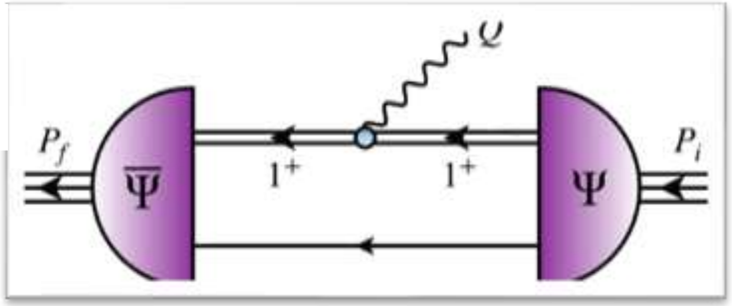
$$s_i = a_i + b_i \exp[-\tfrac{1}{4}\mathcal{E}(\check{\ell})/M_q^E],$$

$$\mathcal{E}(\check{\ell})/M_q^E = (1+\check{\ell}^2/[2M_q^E]^2)^{1/2} - 1.$$

EMFFs of Nucleon

➢ The photon-diquark vertex satisfy a Ward identity:

$$Q_\mu \chi_\mu^{0^+}(\ell_1,\ell_2) = \left[\Delta^{0^+}(\ell_2^2) - \Delta^{0^+}(\ell_1^2)\right] F_{sc}(Q^2) \quad F_{sc}(Q^2) = \frac{1}{1+\frac{1}{6}r_{sc}^2 Q^2}$$



➢ The photon- scalar diquark vertex:

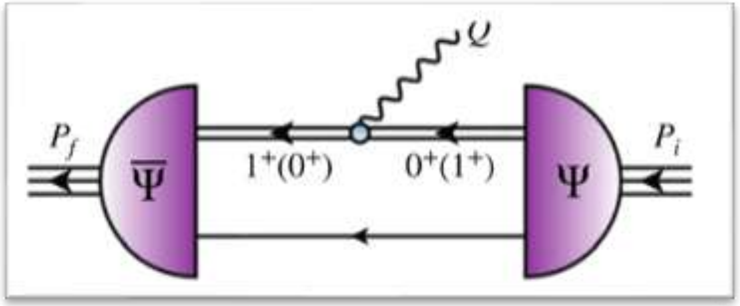
$$\chi_\mu^{0^+}(\ell_1,\ell_2) = -2k_\mu \, \Delta_{\Delta^{0+}}(\ell_1^2,\ell_2^2)\, F_{sc}(Q^2)$$

➢ The photon- axialvector diquark vertex:

$$\chi_{\mu\alpha\beta}^{1^+}(\ell_1,\ell_2) = -\Big[2\delta_{\alpha\beta}\ell_\mu \Delta_{\Delta^1}(\ell_1^2,\ell_2^2) + \frac{1}{m_{1^+}^2}\left(\delta_{\mu\alpha}\ell_{1\beta}\Delta^1(\ell_1^2) + \delta_{\mu\beta}\ell_{2\alpha}\Delta^1(\ell_2^2)\right)$$
$$+\frac{2}{m_{1^+}^2}\ell_{2\alpha}\ell_{1\beta}\ell_\mu \Delta_{\Delta^1}(\ell_1^2,\ell_2^2)\Big] F_{av}(Q^2) + \kappa(Q_\beta\delta_{\mu\alpha} - Q_\alpha\delta_{\mu\beta})\Delta_{\Delta^1}(\ell_1^2,\ell_2^2)$$

➢ The photon-scalar-axialvector diquark vertex:

$$\Gamma_{SA}^{\gamma\alpha}(\ell_1,\ell_2) = -\Gamma_{AS}^{\gamma\alpha}(\ell_1,\ell_2) = \frac{i}{M_N}\,\kappa_{sa}F_{sa}(Q^2)\,\varepsilon_{\gamma\alpha\rho\lambda}\ell_{1\rho}\ell_{2\lambda}$$

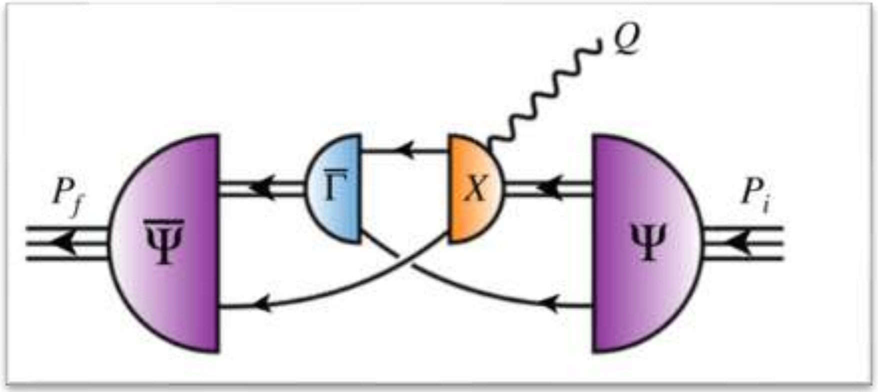


pcheng@mail.ahnu.edu.cn

EMFFs of Nucleon

➢The coupling of photon to diquark amplitude:

$$X_{\mu}^{J^P}(k,Q) = e_{\rm by}\frac{4k_\mu - Q_\mu}{4k\cdot Q - Q^2}\left[\Gamma^{J^P}(k-Q/2) - \Gamma^{J^P}(k)\right] + e_{\rm ex}\frac{4k_\mu + Q_\mu}{4k\cdot Q + Q^2}\left[\Gamma^{J^P}(k+Q/2) - \Gamma^{J^P}(k)\right]$$



➢The parameters are chosen by reproducing selected results from 3-body analyses of nucleon elastic electromagnetic form factors:

a_1	a_2	a_3	b_1	b_2	b_3
0.17	1.47	−0.93	1.03	−0.17	1.61
$r_{\rm sc}$/fm	$r_{\rm av}$/fm				
0.31	1.05				

Z.-Q. Yao, D. Binosi, Z.-F. Cui, C. D. Roberts, arXiv:2403.08088

EMFFs of Nucleon

Table 2 Calculated results for $Q^2 \simeq 0$ (static) properties of nucleon electromagnetic form factors. Also listed, for comparison, results obtained using the *ab initio* three-body approach [25] and experiment [74, PDG].

		r_E^2/fm^2	$r_E^2 M^2$	r_M^2/fm^2	$r_M^2 M^2$	μ
herein	p	0.67^2	4.02^2	0.67^2	4.02^2	2.80
	n	-0.29^2	-1.73^2	0.72^2	4.30^2	−1.86
[25]	p	0.89^2	4.23^2	0.82^2	3.91^2	2.23
	n	-0.25^2	-1.19^2	0.81^2	3.87^2	−1.33
[74, PDG]	p	0.84^2	4.00^2	0.85^2	4.05^2	2.79
	n	-0.34^2	-1.62^2	0.86^2	4.11^2	−1.91

Magnetic moments:

$$\mu_N = G_M^N(Q^2 = 0).$$

Radii:

$$\langle r_{E,M}^2 \rangle^N = -6 \frac{d \ln G_{E,M}^N(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

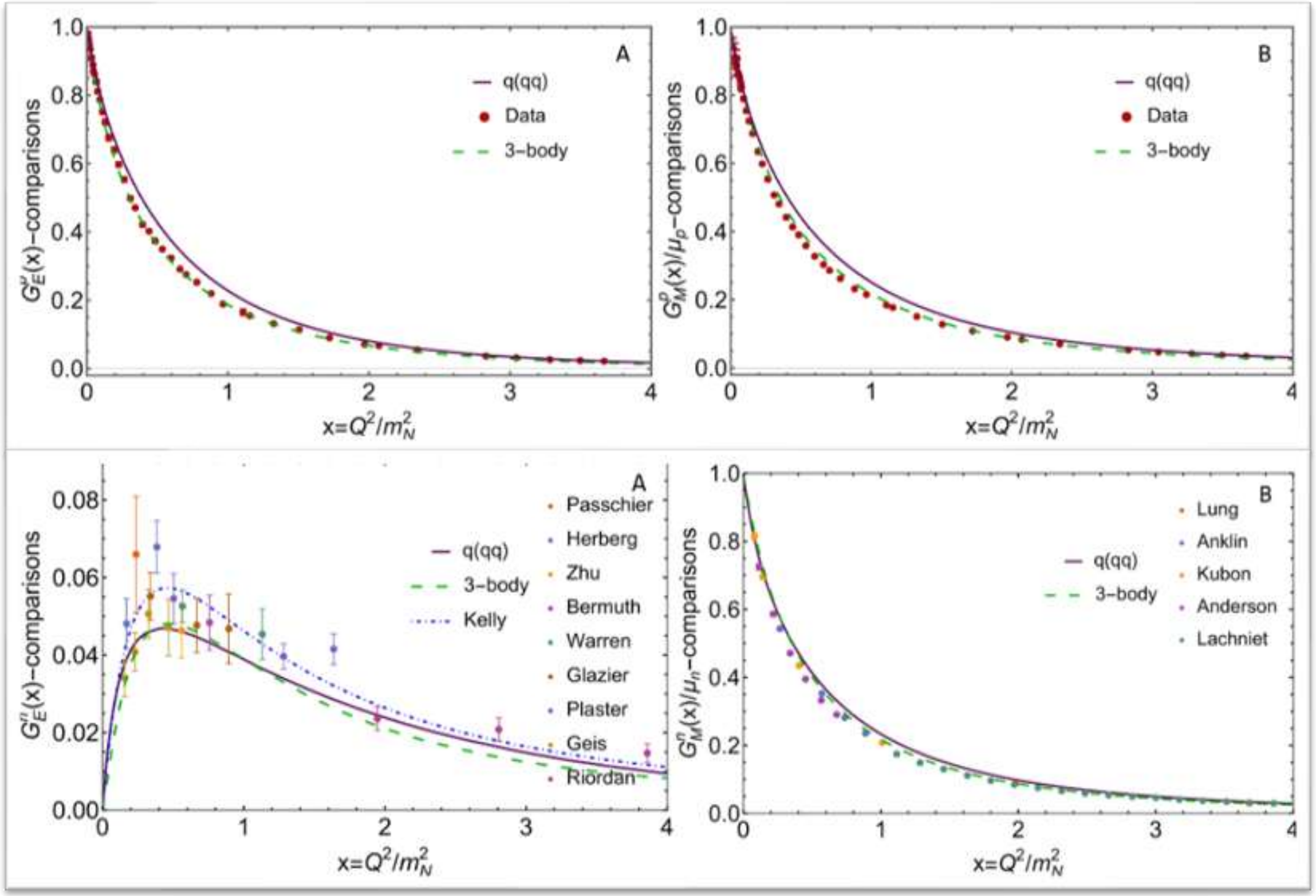
$$\langle r_E^2 \rangle^n = -6 G_E^{n\prime}(Q^2)|_{Q^2=0}$$

$$r_E^p \approx r_M^p$$

[25] Z.-Q. Yao, D. Binosi, Z.-F. Cui, C. D. Roberts, arXiv: 2403. 08088.
[74] S. Navas, et al., Review of particle physics, Phys. Rev. D 110 (3) (2024) 030001.

P. Cheng, et al., arXiv: 2507.13484

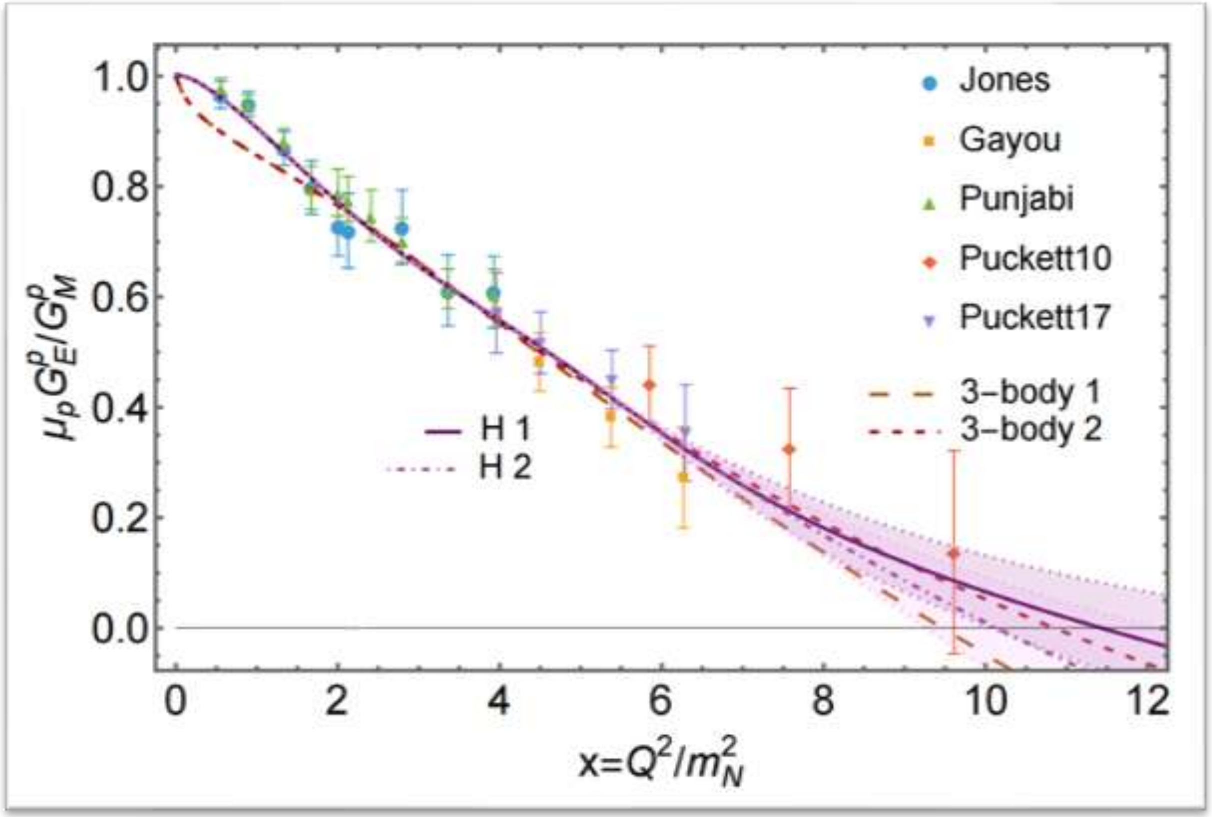
EMFFs of Nucleon



- ✓ The agreement is good for $x > 2$.
- ✓ For $x > 2$, the differences may stem from the contribution of the meson cloud.

P. Cheng, et al., arXiv: 2507.13484

EMFFs of Nucleon

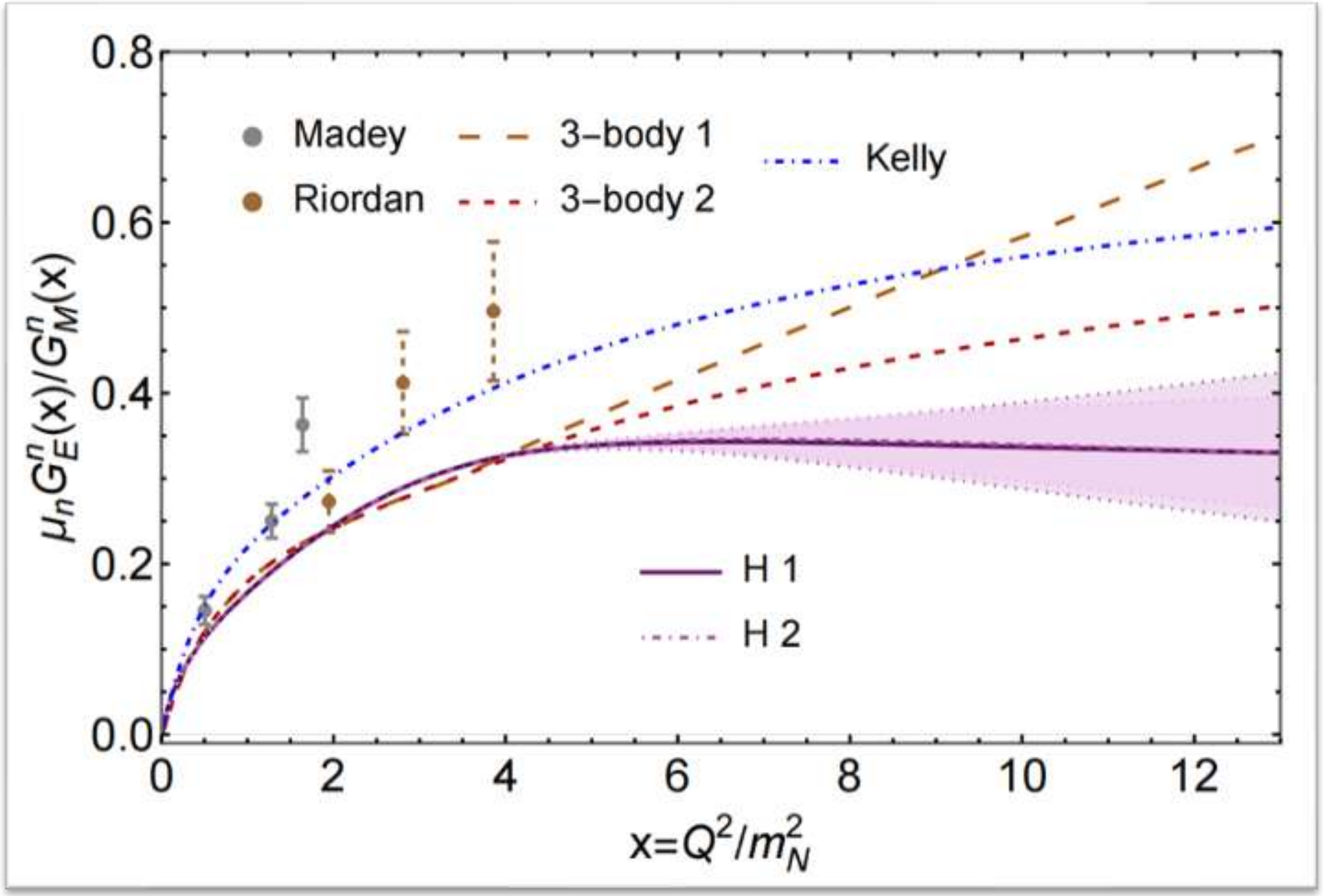


✓ A zero in G_E^p:

	$q(qq)$ herein	3 − body
SPM 1	$11.44^{+3.35}_{-1.37}$	$9.47^{+1.90}_{-0.92}$
SPM 2	$10.14^{+1.99}_{-0.88}$	$10.85^{+2.37}_{-0.96}$

P. Cheng, et al., arXiv: 2507.13484

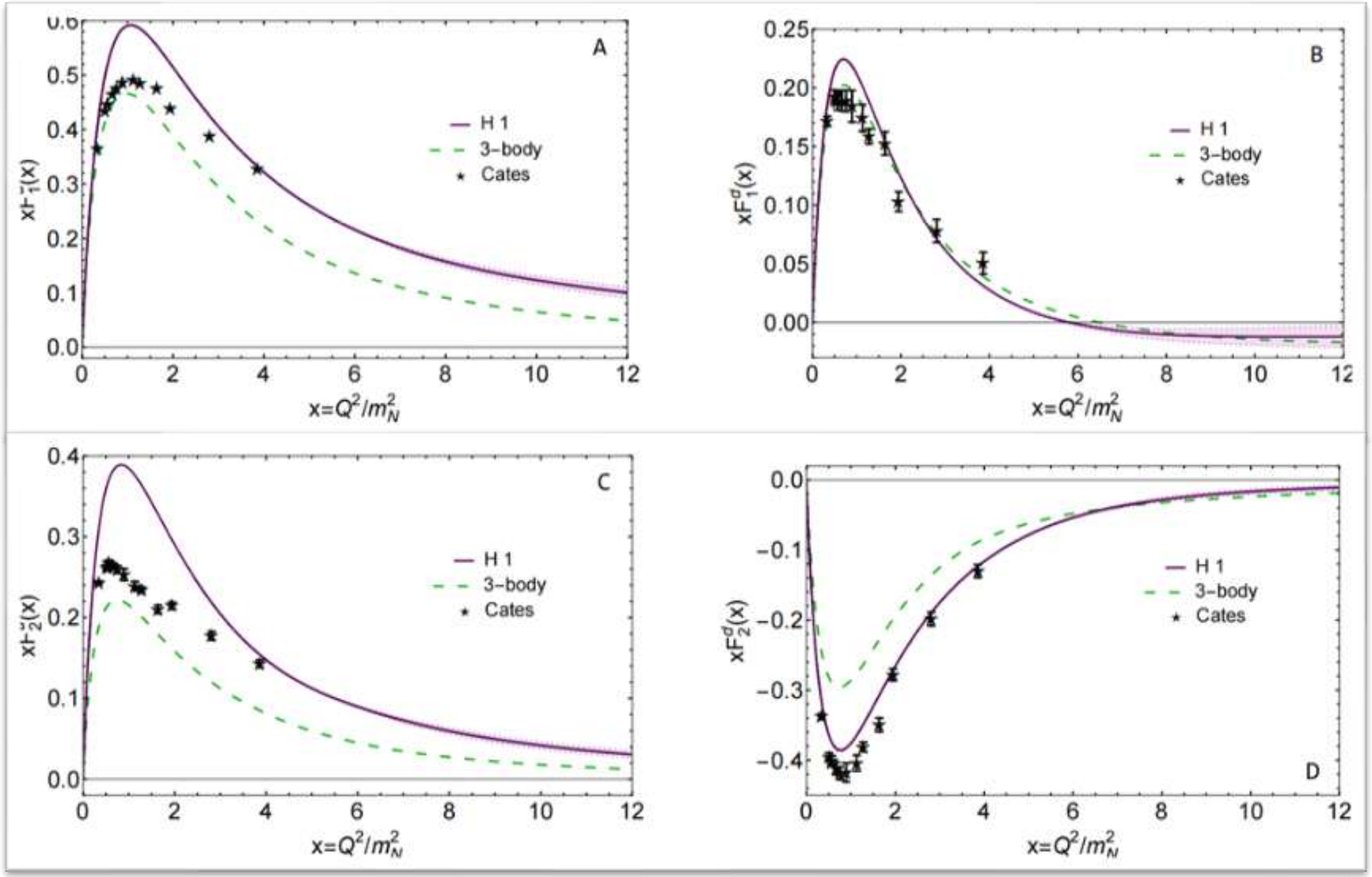
EMFFs of Nucleon



- ✓ The results are consistent with the trend of the experimental data.
- ✓ Regarding comparison with the 3-body analyses, deviations become evident on $x \gtrsim 5$.
- ✓ Finding no zero crossing in G_E^n on $x \lesssim 15$.

P. Cheng, et al., arXiv: 2507.13484

EMFFs of Nucleon



$$F_i^u = 2F_i^p + F_i^n,$$

$$F_i^d = F_i^p + 2F_i^n,$$

- ✓ The analysis predicts that F_1^d exhibits a zero:

$$x = 5.80^{+0.20}_{-0.14}$$

close to 3-body's results:

$$x \approx 6.5$$

- ✓ The analysis predicts that F_2^d doesn't exhibits a zero.

P. Cheng, et al., arXiv: 2507.13484

EMTFFs of Δ(1700)3/2⁻

- The $\gamma^* N \to \Delta(1700)$ transition current:

$$J_{\mu\lambda}(P,Q) = \Lambda_+(P_f) R_{\lambda\alpha}(P_f) i\Gamma_{\alpha\mu}(K,Q)\Lambda_+(P_i),$$

$$\Gamma_{\alpha\mu}(K,Q) = \mathpalette\@k_- \left[-\frac{\lambda_m}{\lambda_-} G_M^* \gamma_5 \varepsilon_{\alpha\mu\gamma\delta} \check{K}_\gamma \check{Q}_\delta - \frac{1}{2}(G_M^* - G_E^*) \mathcal{T}^Q_{\alpha\gamma}\mathcal{T}^K_{\gamma\mu} - \frac{i\varsigma}{\lambda_m} G_C^* \check{Q}_\alpha \check{K}_\mu \right]$$

with $\check{K}_\mu := \check{P}_{T,\mu}$, $P_{T,\mu} = \mathcal{T}^Q_{\mu\nu} P_\nu = P_\mu - (P\cdot\check{Q})\check{Q}_\mu$, $\check{K}^2 = 1 = \check{Q}^2$, and $k_- = \sqrt{(3/2)}(m_\Delta/m_N - 1)$, $\varsigma = Q^2/[2\Sigma_{\Delta N}]$, $\lambda_\pm = \varsigma + t_\pm/[2\Sigma_{\Delta N}]$, $t_\pm = (m_\Delta \pm m_N)^2$, $\lambda_m = \sqrt{\lambda_+\lambda_-}$, $\Sigma_{\Delta N} = m_\Delta^2 + m_N^2$, $\Delta_{\Delta N} = m_\Delta^2 - m_N^2$.

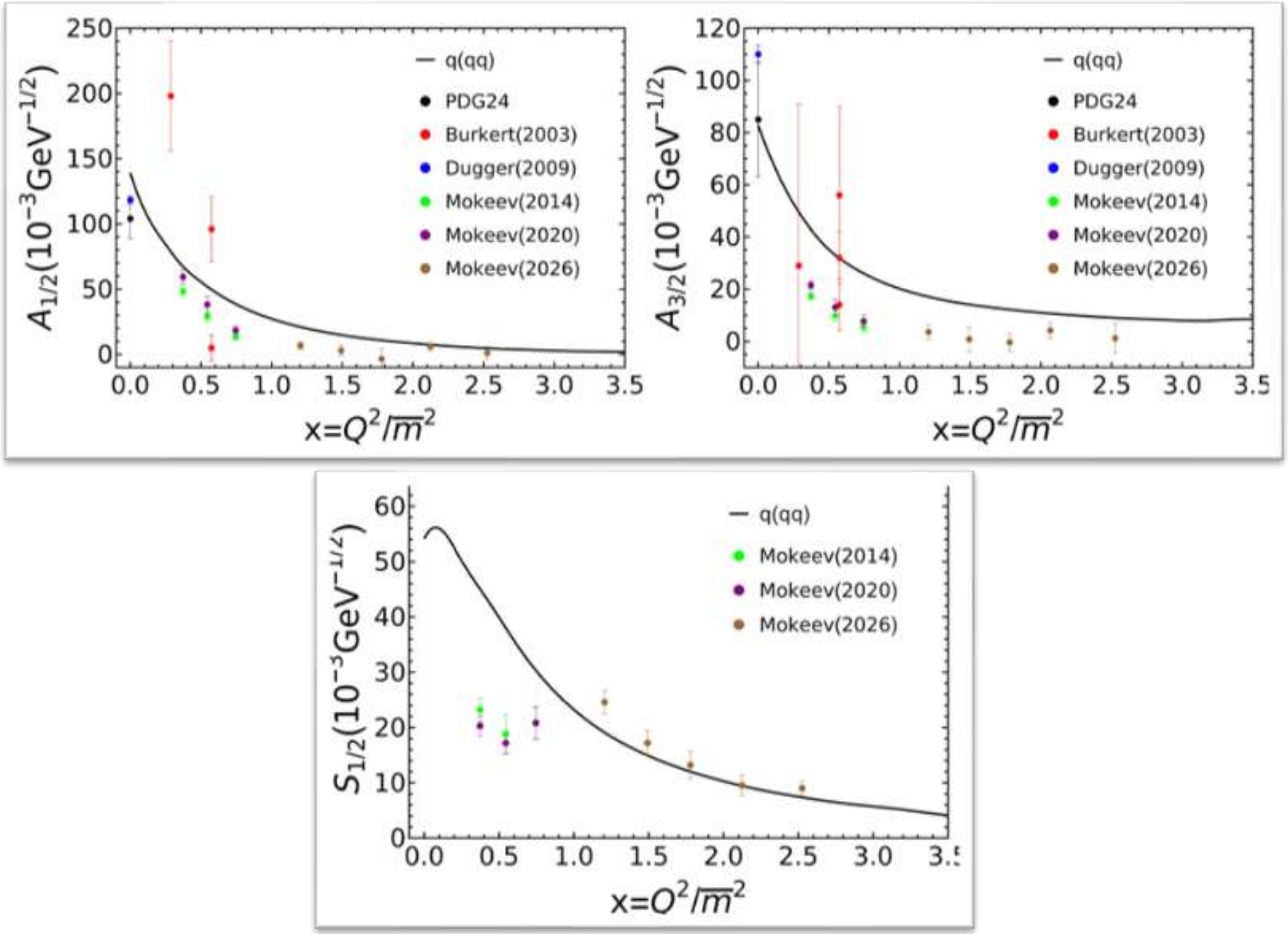
- The helicity amplitudes can be expressed in terms of the multipole form factors:

$$A_{1/2}(Q^2) = -\frac{1}{4F_{1-}}\left[G_E^*(Q^2) - 3G_M^*(Q^2)\right],\; S_{1/2}(Q^2) = -\frac{1}{\sqrt{2}F_{1-}}\frac{|\mathbf{q}|}{2m_\Delta} G_C^*(Q^2),$$

$$A_{3/2}(Q^2) = -\frac{\sqrt{3}}{4F_{1-}}\left[G_E^*(Q^2) + G_M^*(Q^2)\right],$$

with α the quantum electrodynamics fine structure constant, $K = (m_\Delta^2 - m_N^2)/(2m_\Delta)$, and the magnitude of the photon three-momentum being $|\mathbf{q}| = \frac{\sqrt{(Q^2+(m_\Delta+m_N)^2)(Q^2+(m_\Delta-m_N)^2)}}{2m_\Delta}$

EMTFFs of $\Delta(1700)3/2^-$



- ✓ On $x \gtrsim 2$, the predictions agree well with data, which indicates that there is also a significant meson cloud effect in the region $x < 2$.

P. Cheng, et al., arXiv: 2512.09160

Summary and Outlook

- Developing a refined symmetry preserving current for electron + nucleon elastic scattering, our $q(qq)$ picture agreement with available data;
- We predict a zero in G_E^p at $x = Q^2/{m_N}^2 \approx 11$; the absence of such a zero in G_E^n; a zero at $x \approx 5.8$ in the proton's *d*-quark Dirac form factor.
- Using the refined symmetry current to study the EMTFFs of $\Delta(1700)3/2^-$, the predictions agree well with data on *x* ≳ 2, which indicates there is also a significant meson cloud effect in the region x < 2.

- We plan to extend the above analysis to other nucleon resonances.
- Based on the above analysis, we will further consider the contribution of the meson cloud.

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- Using the refined symmetry current to study the EMTFFs of $\Delta(1700)3/2^-$, the predictions agree well with data on *x* ≳ 2, which indicates there is also a significant meson cloud effect in the region x < 2.

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Thank you for your attention!