

Threshold effects as the origin of $Y(4500)$ in $e^+e^- \rightarrow J/\psi K^+ K^-$

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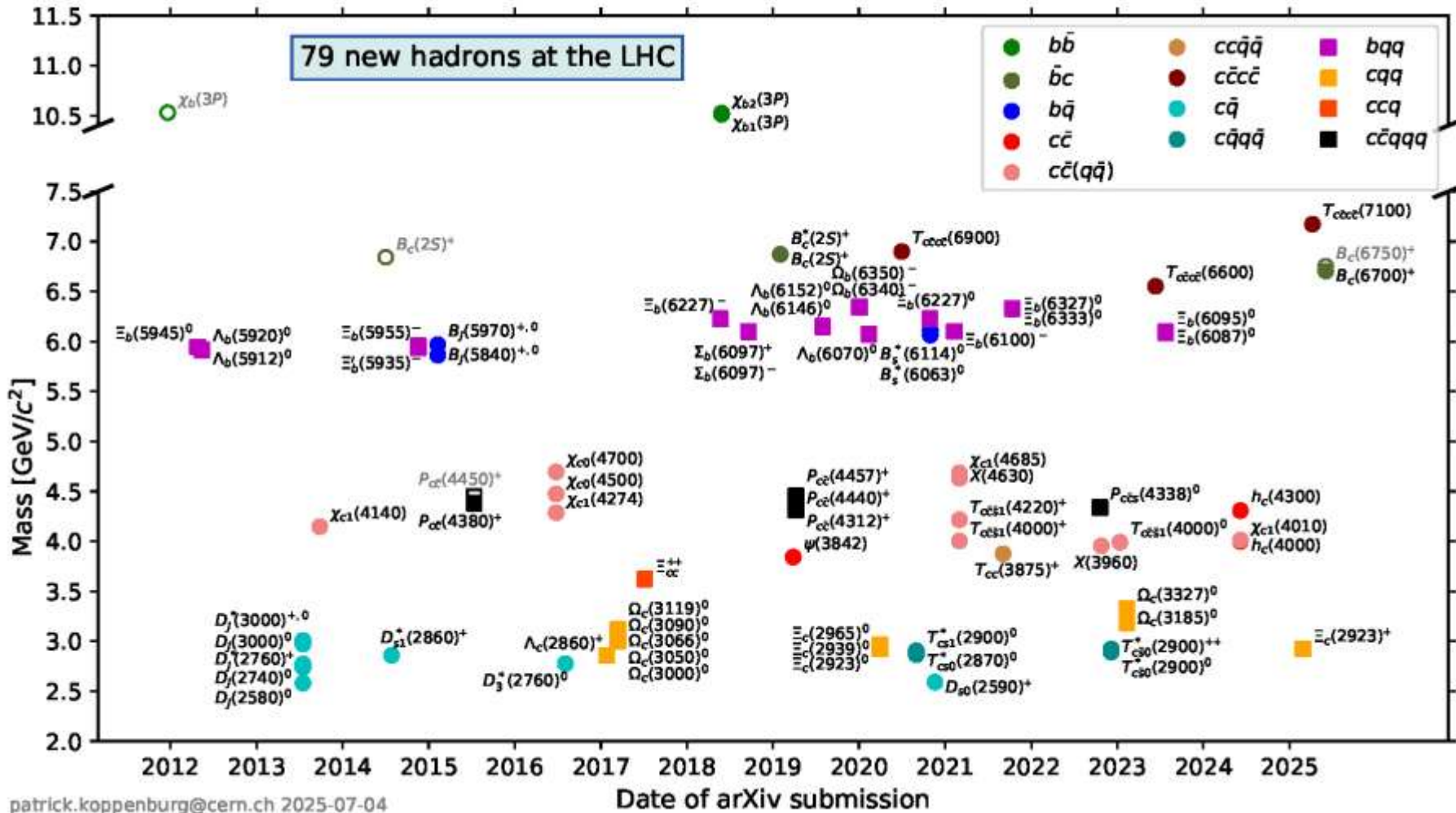
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2026.3.30

Outline

- **Introduction**
- **Theoretical framework**
- **Results and discussion**
- **Summary**

New particles discovered at the LHC

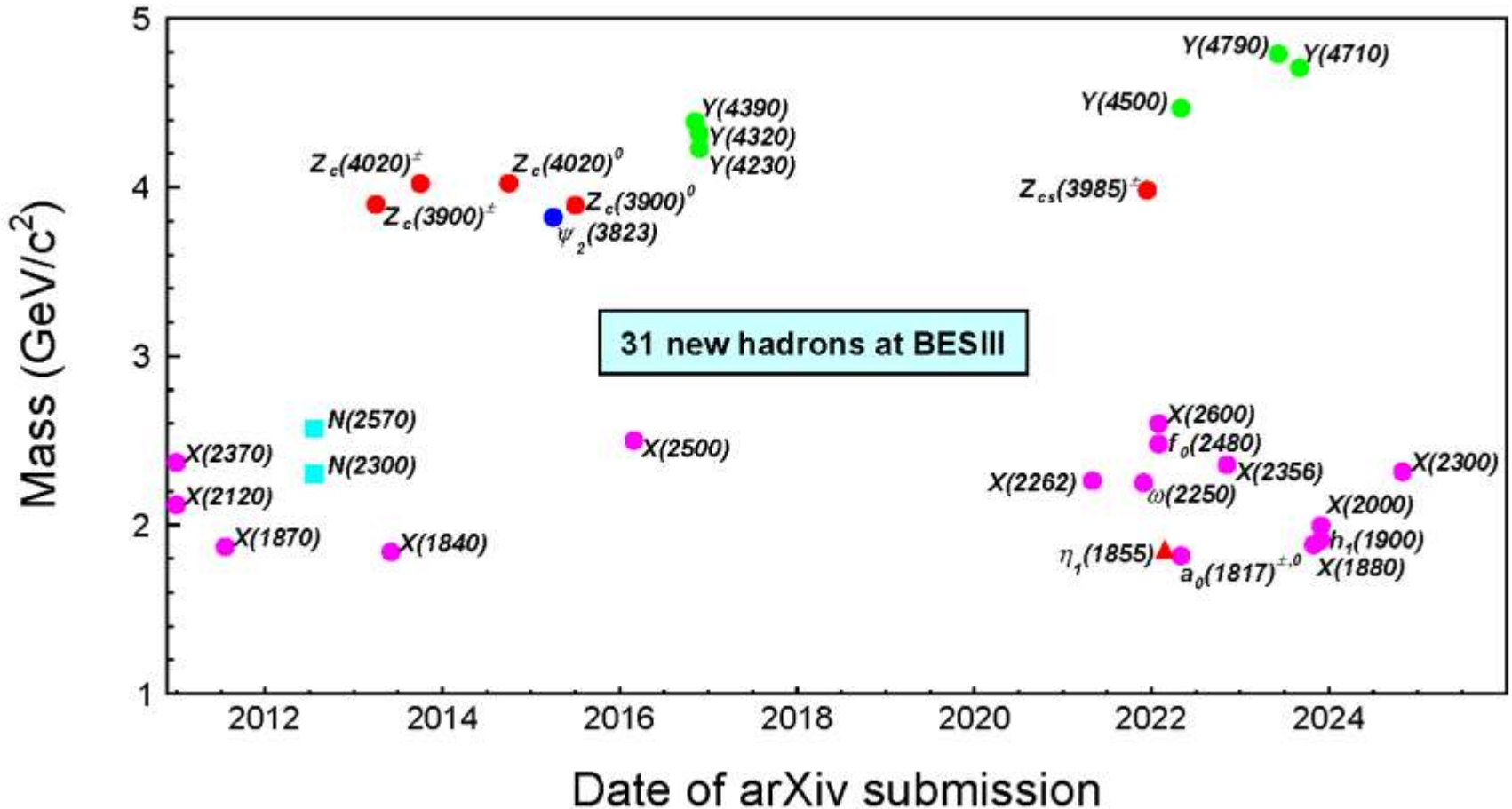
79 new hadrons



Renaissance of Hadron Spectroscopy!

New particles discovered at BESIII

31 new hadrons



<https://english.ihep.cas.cn/bes/re/pu/NewParticles/>

Renaissance of Hadron Spectroscopy!

Theoretical Interpretation

✓ Hadronic molecule

✓ Compact multiquark state

✓ Hybrid

✓ Hadrocharmonium

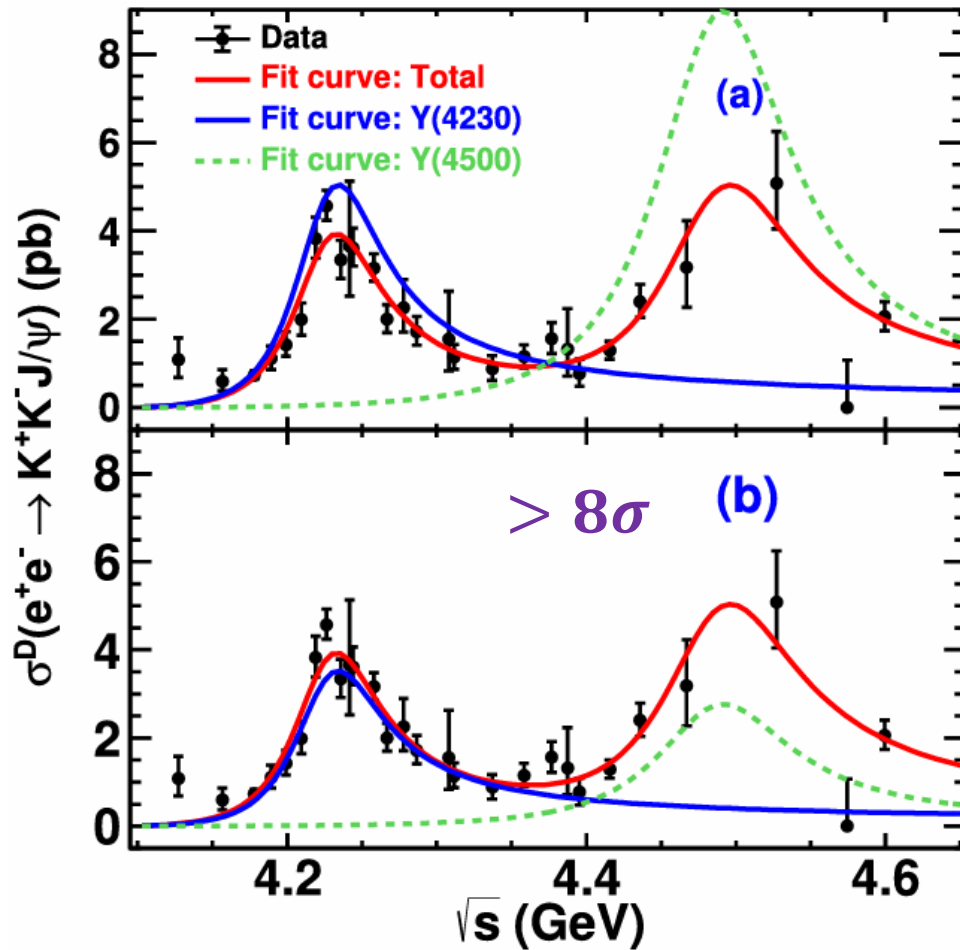
Genuine resonance interpretations

✓ Threshold effect (cusp, triangle singularity, ...) (Non-resonance interpretation)

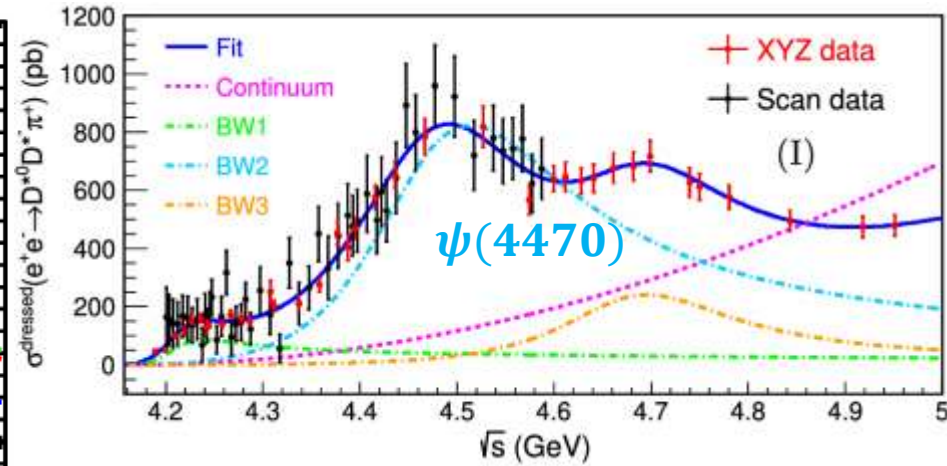
Many new particles are located precisely near the thresholds of certain hadron pairs.

“Resonance-like” structure $\stackrel{?}{=}$ Genuine particle

Y(4500) in $e^+e^- \rightarrow J/\psi K^+ K^-$



$e^+e^- \rightarrow D^{*0} D^{*-} \pi^+$



M_i (MeV)	Γ_i (MeV)
$4209 \pm 4.7 \pm 5.9$	$81.6 \pm 17.8 \pm 9.0$
$4469.1 \pm 26.2 \pm 3.6$	$246.3 \pm 36.7 \pm 9.4$
$4675.3 \pm 29.5 \pm 3.5$	$218.3 \pm 72.9 \pm 9.3$

BESIII, PRL130(2023)121901

$M = 4484.7 \pm 13.3 \pm 24.1$ MeV

$\Gamma = 111.1 \pm 30.1 \pm 15.2$ MeV

BESIII, CPC46(2022)111002

Theoretical Model

✓ highly excited charmonium:

- $\psi(4^3D_1)$ $M=4484$ MeV Wang,Chen,Liu,Matsuki, PRD99(2019)114003
- $\psi(3^3D_1)$ $M=4486/4463$ MeV Deng,Ni,Li,Zhong, PRD110(2024)056034
- 5S-4D mixture J.Z.Wang, X.Liu, PRD107(2023)054016
- ...

✓ Compact multiquark state: $c\bar{c}q\bar{q}$

Z.G.Wang, NPB1005(2024)116580;
NPB993(2023)116265

$c\bar{c}s\bar{s}$

Chiu, Hsieh, PRD73(2006)094510

✓ $D_s D_{s1}(2536)$ molecule

Dong, Guo, Zou, Progr.Phys.41, 65(2021)

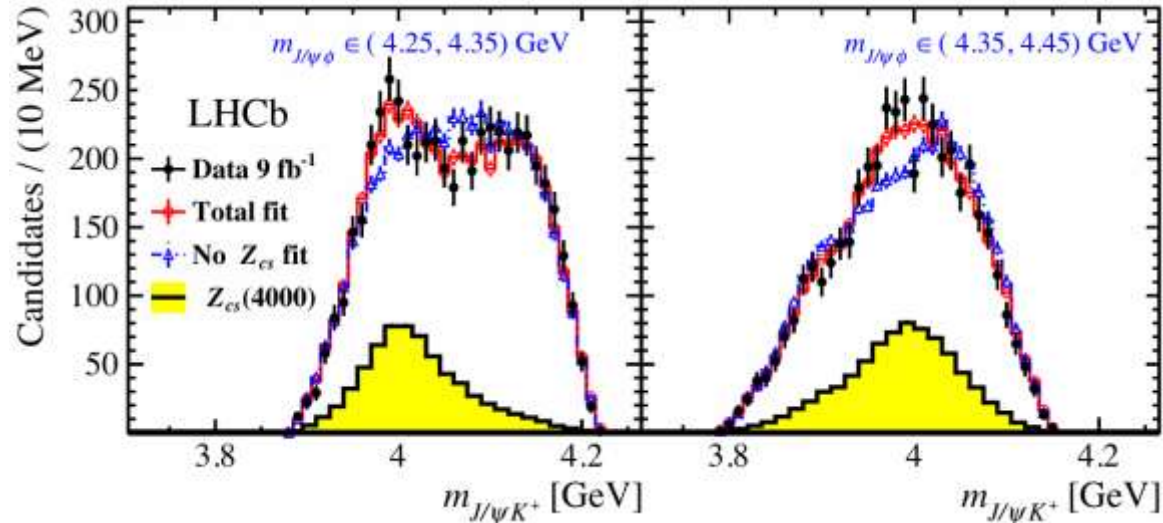
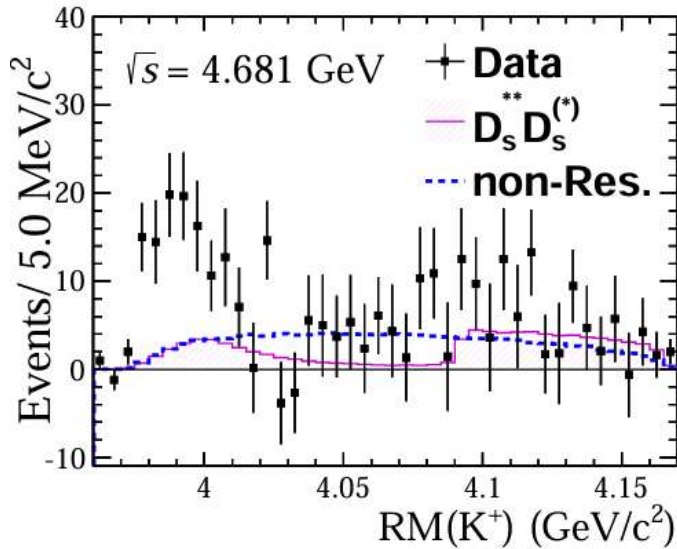
$Z_{cs}(3985)$ and $Z_{cs}(4000)$

BESIII, 2011.07855

LHCb, 2103.01803

$$e^+e^- \rightarrow K^+(D_s^- D^{*0} + D_s^{*-} D^0)$$

$$B^+ \rightarrow J/\psi \phi K^+$$



$$m_{\text{pole}}(Z_{cs}(3985)^-) = (3982.5_{-2.6}^{+1.8} \pm 2.1) \text{ MeV}/c^2,$$

$$Z_{cs}(4000)$$

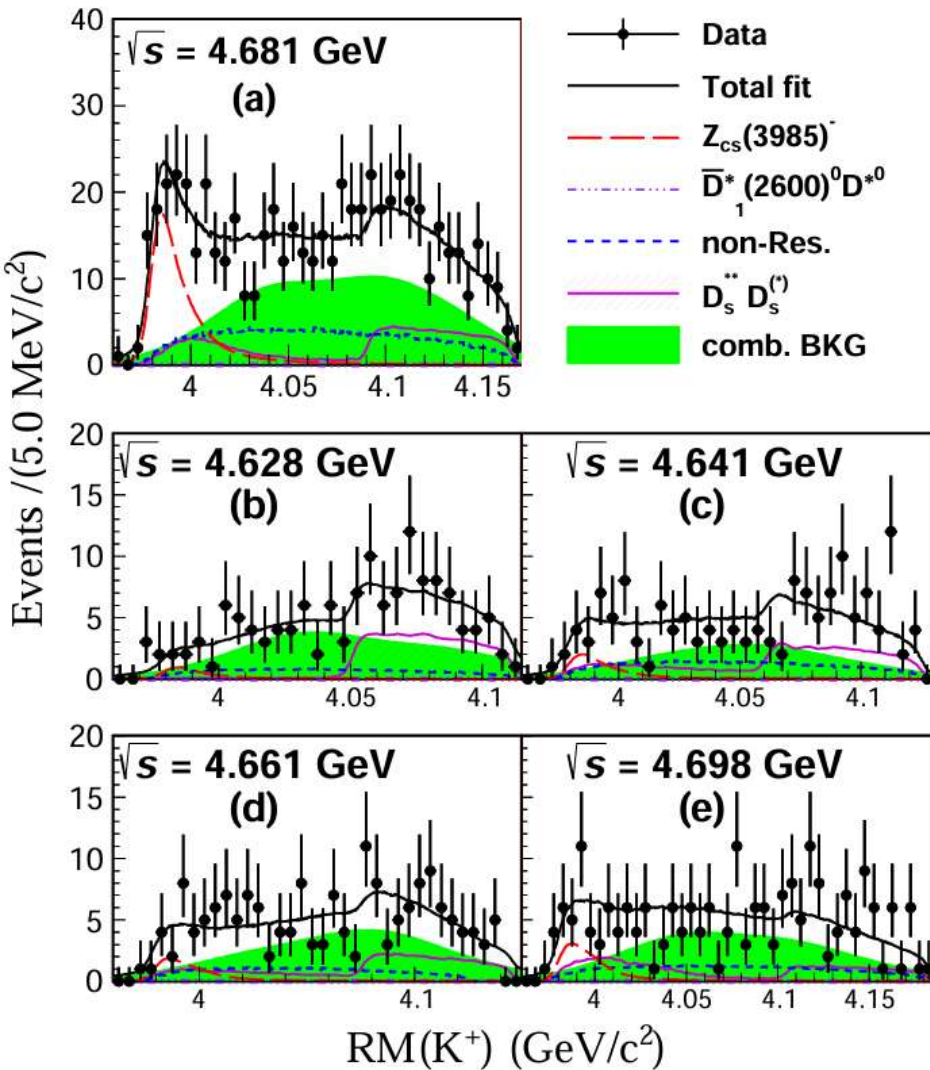
$$\Gamma_{\text{pole}}(Z_{cs}(3985)^-) = (12.8_{-4.4}^{+5.3} \pm 3.0) \text{ MeV}.$$

$$M = 4003 \pm 6_{-14}^{+4} \text{ MeV},$$

$$\Gamma = 131 \pm 15 \pm 26 \text{ MeV}$$

Widths are quite different

$Z_{cs}(3985)$ and $Z_{cs}(4000)$



Theoretical interpretations:

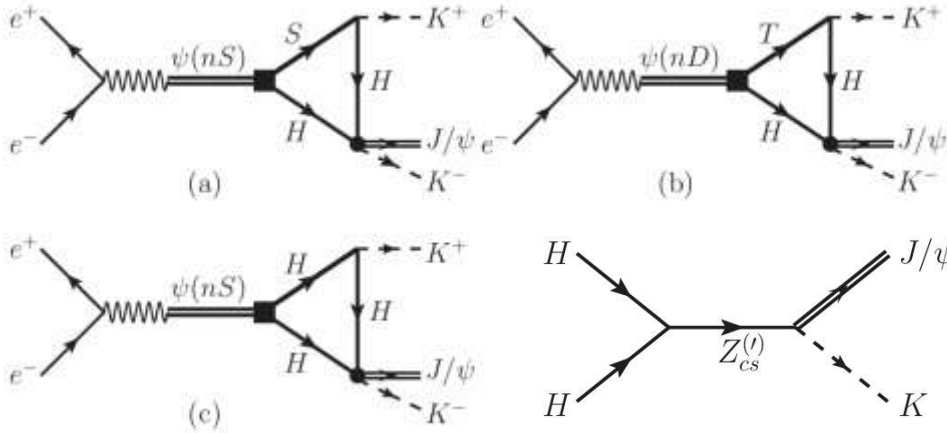
- $D_s D^{*+} + D_s^{*+} D$
- Virtual state [Ortega, Entem, Fernandez, 2103.07871](#)
- Compact tetraquark
- Threshold effects [Yang, Cao, Guo, Nieves, Valderrama 2011.08725](#)
- Reflection effects

[Y.H. Ge, XHL, H.W. Ke, 2103.05282](#)

[Wang, Zhou, Liu, Matsuki, 2011.08628](#)

$D_s^{*-} D_{s2}^{*+}(2573)^+$ threshold ~ 4681 MeV

Rescattering contributions in $e^+ e^- \rightarrow J/\psi K^+ K^-$



- ✓ Open-charm rescatterings
- ✓ No genuine resonance ($Y(4500)$) introduced
- ✓ The nearby $\psi(4415)$ will contribute
- ✓ Consider $\psi(4415)$ as a conventional charmonium state
- ✓ $Z_{cs}^{(\prime)}$ exist or not

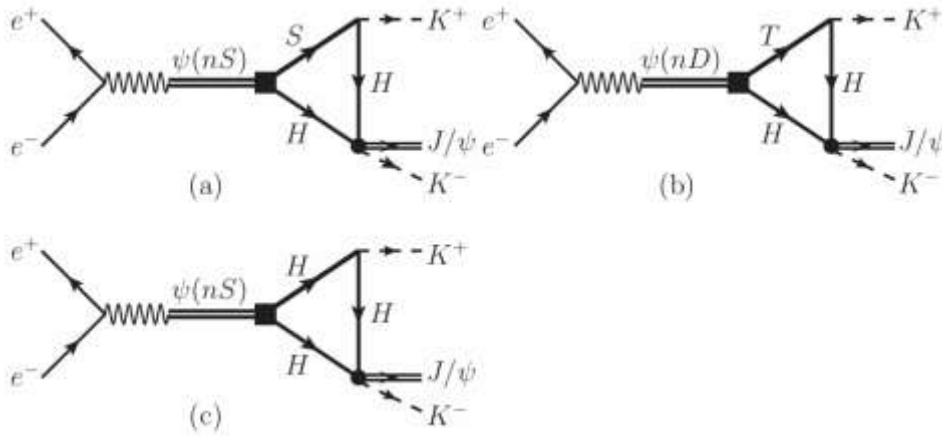
$$H_{1a} = \frac{1 + \not{v}}{2} [\mathcal{D}_{a\mu}^* \gamma^\mu - \mathcal{D}_a \gamma_5],$$

$$S_{1a} = \frac{1 + \not{v}}{2} [\mathcal{D}_{1a}^{\prime\mu} \gamma_\mu \gamma_5 - \mathcal{D}_{0a}], (D_{s0}^*(2317), D_{s1}(2460))$$

$$T_{1a}^\mu = \frac{1 + \not{v}}{2} [\mathcal{D}_{2a}^{\mu\nu} \gamma_\nu - \sqrt{\frac{3}{2}} \mathcal{D}_{1a\nu} \gamma_5 [g^{\mu\nu} - \frac{1}{3} \gamma^\nu (\gamma^\mu - v^\mu)]], (D_{s1}(2536), D_{s2}^*(2573))$$

Diagram	Triangle loop
Fig. 1(a)	$D_{s0}^* D_s^*[D], D_{s1}' D_s[D^*], D_{s1}' D_s^*[D^*]$
Fig. 1(b)	$D_{s1} D_s[D^*], D_{s1} D_s^*[D^*], D_{s2} D_s^*[D], D_{s2} D_s^*[D^*]$
Fig. 1(c)	$D_s D_s[D^*], D_s D_s^*[D^*], D_s^* D_s^*[D], D_s^* D_s^*[D^*]$

Effective couplings



ψSH and ψTH coupling [GeV]

ψ	Channel	Coupling	Channel	Coupling
$\psi(4S)$	$D_s^* D_{s0}^*$	5.31	$D_s D_{s1}'$	1.05
	$D_s^* D_{s1}'$	1.09		
$\psi(5S)$	$D_s^* D_{s0}^*$	2.69	$D_s D_{s1}'$	0.54
	$D_s^* D_{s1}'$	0.55		
$\psi(3D)$	$D_s D_{s1}$	3.26	$D_s^* D_{s1}$	3.38
	$D_s^* D_{s2}$	4.19		
$\psi(4D)$	$D_s D_{s1}$	1.47	$D_s^* D_{s1}$	1.02
	$D_s^* D_{s2}$	1.87		

HQSS involved

$$\mathcal{L}_\psi = \frac{g_{TH}}{\sqrt{2}} \langle J^{\mu\nu} \bar{H}_{2a} \gamma_\nu \bar{T}_{1a\mu} - J^{\mu\nu} \bar{T}_{2a\mu} \gamma_\nu \bar{H}_{1a} \rangle$$

$$+ C_S \langle J \bar{H}_{2b} \gamma_\mu \gamma_5 \bar{H}_{1a} \mathcal{A}_{ba}^\mu \rangle + ig_{HH} \langle J^{\mu\nu} \bar{H}_{2a} \gamma_\mu \overset{\leftrightarrow}{\partial}_\nu \bar{H}_{1a} \rangle$$

$$+ g_{SH} [\langle J \bar{S}_{2a} \bar{H}_{1a} + J \bar{H}_{2a} \bar{S}_{1a} \rangle] + \text{H.c.},$$

$$T = -3\gamma \sum_m \langle 1m1 - m | 00 \rangle \sqrt{96\pi} \int d^3 \vec{p}_q d^3 \vec{p}_{\bar{q}} \delta^3(\vec{p}_q + \vec{p}_{\bar{q}})$$

$$\times \mathcal{Y}_{1m} \left(\frac{\vec{p}_q - \vec{p}_{\bar{q}}}{2} \right) \chi_{1-m} \phi_0 \omega_0 b_q^\dagger(\vec{p}_q) d_{\bar{q}}^\dagger(\vec{p}_{\bar{q}}), \quad {}^3P_0$$

$$\psi_{nLM_L}(\vec{p}) = R_{nL}(p) Y_{LM_L}(\theta_p, \phi_p),$$

$$\mathcal{L}_D = i \frac{h'}{\Lambda_\chi} \langle \bar{H}_{1a} T_{1b}^\mu \gamma_\nu \gamma_5 (D_\mu \mathcal{A}_\nu + D_\nu \mathcal{A}_\mu)_{ba} \rangle$$

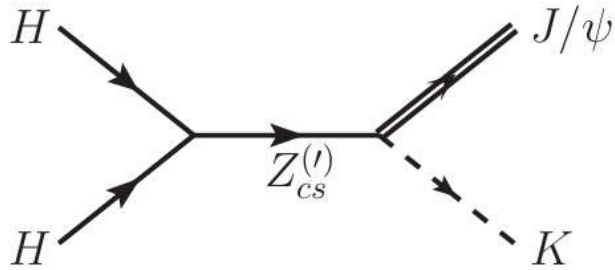
$$+ ig \langle H_{2b} \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu \bar{H}_{1a} \rangle + ih \langle S_{1b} \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu \bar{H}_{1a} \rangle,$$

$$R_{nL}(p) = \frac{(-1)^n (-i)^L}{\beta^{\frac{3}{2}}} \sqrt{\frac{2n!}{\Gamma(n+L+\frac{3}{2})}} \left(\frac{p}{\beta} \right)^L L_n^{L+\frac{1}{2}} \left(\frac{p^2}{\beta^2} \right) e^{-p^2/(2\beta^2)},$$

$$\mathcal{L}_{V\gamma} = \frac{eM_V^2}{f_V} V_\mu A^\mu, \quad \frac{e}{f_V} = \left[\frac{3\Gamma_{V \rightarrow e^+e^-}}{\alpha M_V} \right]^{1/2} \quad \mathbf{VMD}$$

$$\gamma = 0.3 \quad \beta = 0.5 \text{ GeV}$$

Effective couplings



Both Z_{cs} and Z'_{cs} ($D^* D_s^*$ molecule) are included

$$\text{BW}[Z'_{cs}] = (s - M_{Z'_{cs}}^2 + iM_{Z'_{cs}}\Gamma_{Z'_{cs}})^{-1} X = \left(\frac{2r}{a} - 1\right)^{-1/2} \leftarrow \text{Resonance pole at } E_R = M_R - i\Gamma_R/2$$

PDG average value for Z_{cs} (4000)

$$M = 3988 \pm 5 \text{ MeV}$$

$$\Gamma = 14 \pm 4 \text{ MeV}$$

In a coupled-channel model

Two virtual state pole: 3970 & 4110 MeV

Ortega, Entem, Fernandez, 2103.07871

Compositeness relationship

$$X = X_1 + X_2 = |g_1|^2 \left| \frac{\partial G_1^{II}(s_R)}{\partial s} \right| + |g_2|^2 \left| \frac{\partial G_2^{II}(s_R)}{\partial s} \right|,$$

$$\Gamma_R = \Gamma_1 + \Gamma_2$$

$$= |g_1|^2 \frac{q_1(M_R^2)}{8\pi M_R^2} + |g_2|^2 \int_{m_{th}}^{M_R+2\Gamma_R} dE \frac{q_2(E^2)}{16\pi^2 E^2} \frac{\Gamma_R}{(M_R - E)^2 + \frac{\Gamma_R^2}{4}}$$

Two channels saturate

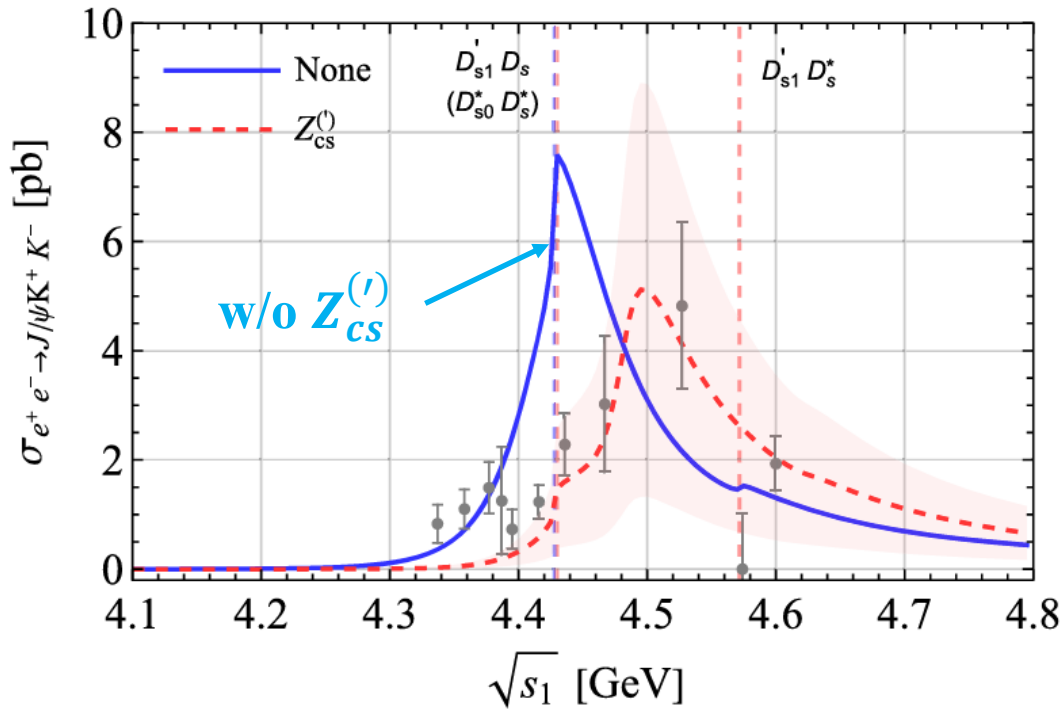
$$a = -\frac{2k_i}{k_r^2 + k_i^2}, \quad r = -\frac{1}{k_i}, \quad M_R > M_{th}$$

$$k_r + ik_i \equiv \sqrt{\frac{2m_1 m_2}{m_1 + m_2} (E_R - m_1 - m_2)},$$

Guo&Oller, PRD103(2021)054021

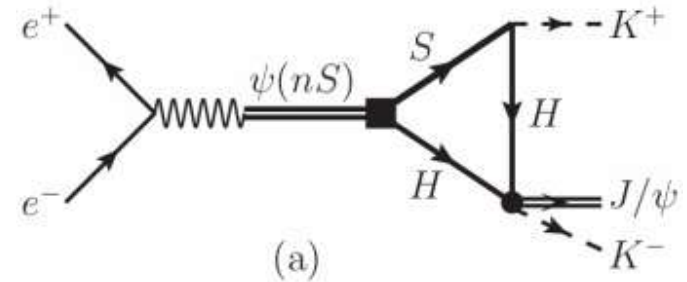
Kang,Guo&Oller, PRD94(2016)014012

$e^+ e^- \rightarrow J/\psi K^+ K^-$ line shape



$$\sigma = L^{\mu\nu}(k_1, k_2) H_{\mu\nu}(Q),$$

Leptonic part



Via $\psi(4S)$ and SHH loops

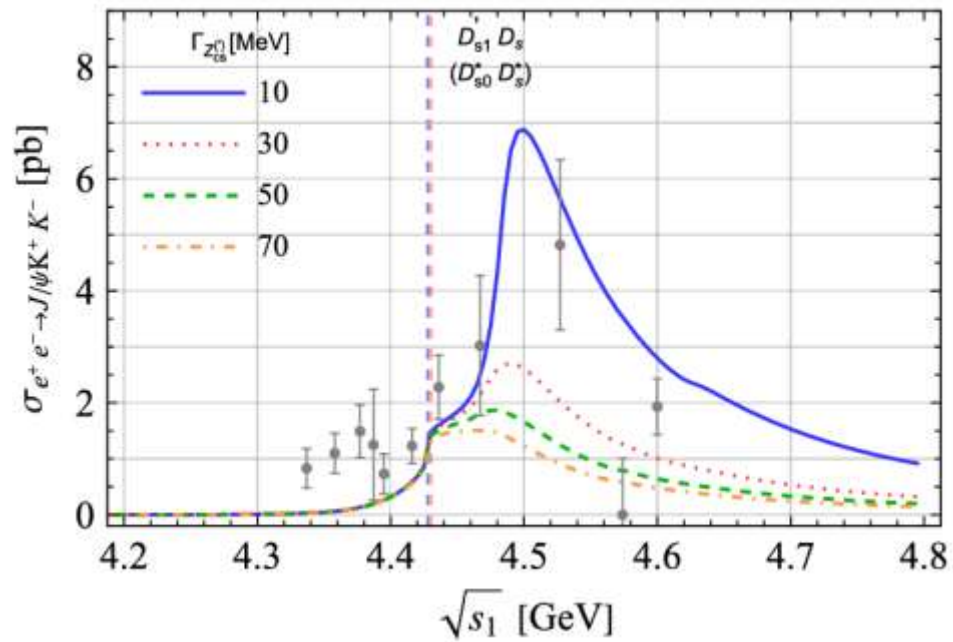
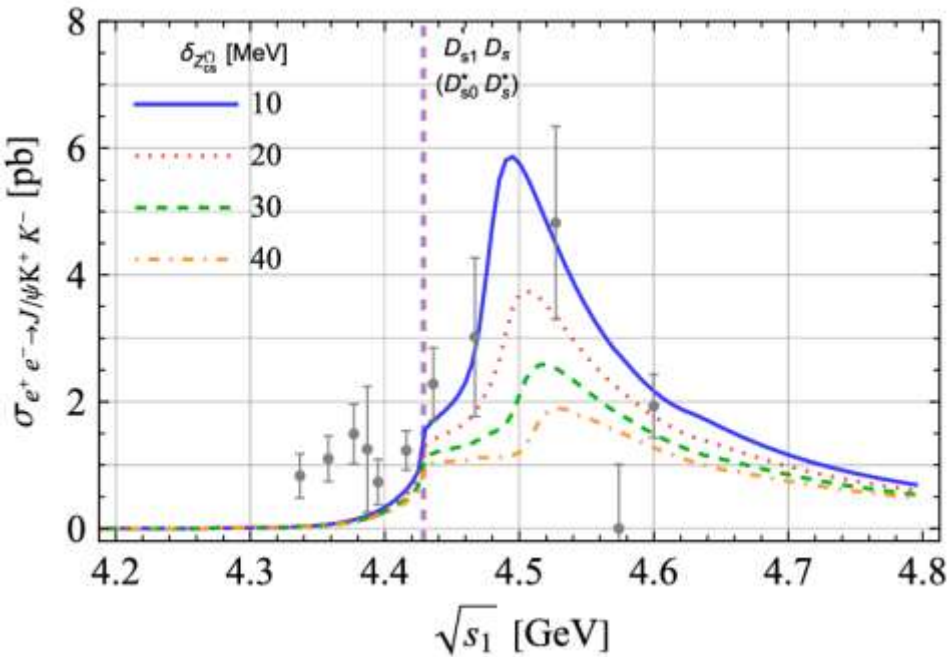
$$D_{s0}^* D_s^*[D], D_{s1}^* D_s[D^*], D_{s1}^* D_s^*[D^*]$$

$\psi(4415)$ is pure $\psi(4S)$

Existence of $Z_{cs}^{(l)}$ plays a crucial role in the production of the $Y(4500)$ signal

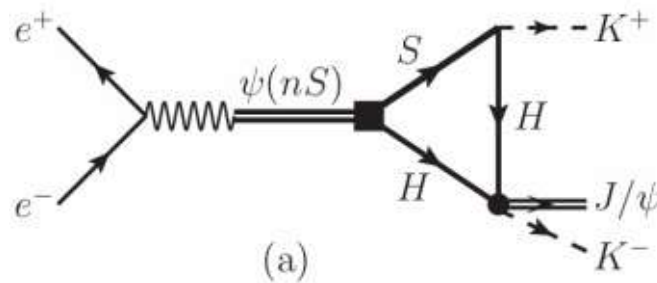
Data from [BESIII, CPC46(2022)111002]

Mass and width dependence

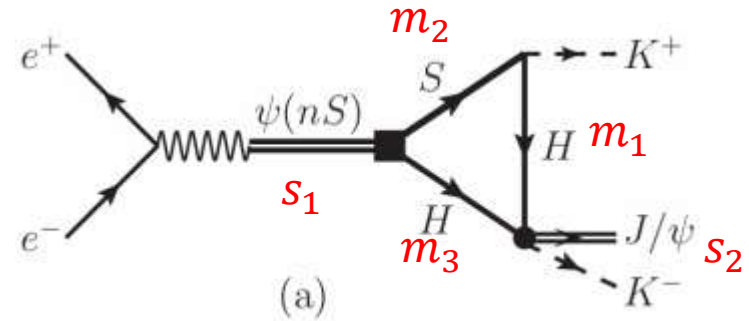
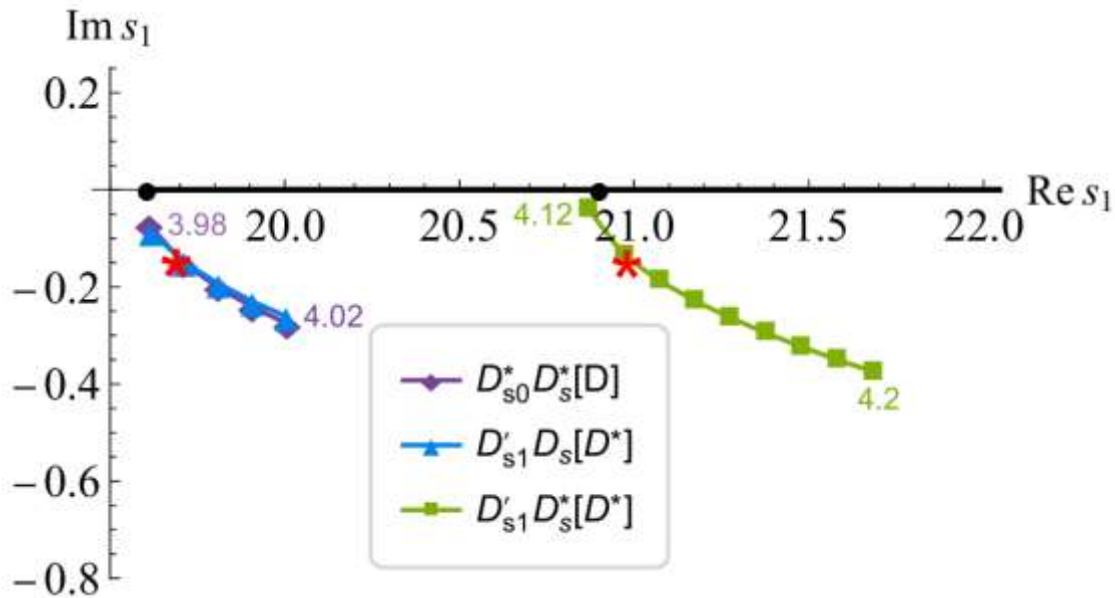


$$\delta \equiv M_R - M_{th}$$

For Z_{cs} and Z'_{cs} , take the same width



$e^+e^- \rightarrow J/\psi K^+ K^-$ line shape



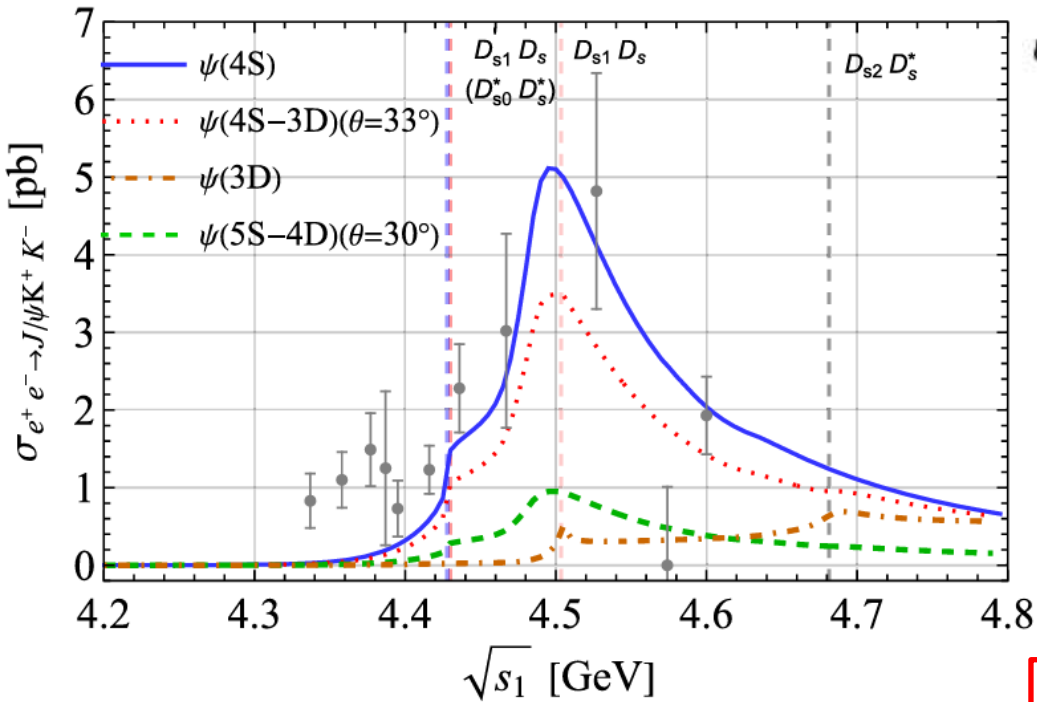
$$s_1^- = (m_2 + m_3)^2 + \frac{1}{2m_1^2} [(m_1^2 + m_2^2 - s_3)(s_2 - m_1^2 - m_3^2) - 4m_1^2 m_2 m_3 - \lambda^{1/2}(s_2, m_1^2, m_3^2) \lambda^{1/2}(s_3, m_1^2, m_2^2)]$$

TS location in the complex plane

$M_{Z_{cs}}$ 3980~4020 MeV

$M_{Z'_{cs}}$ 4120~4200 MeV

S-D mixing

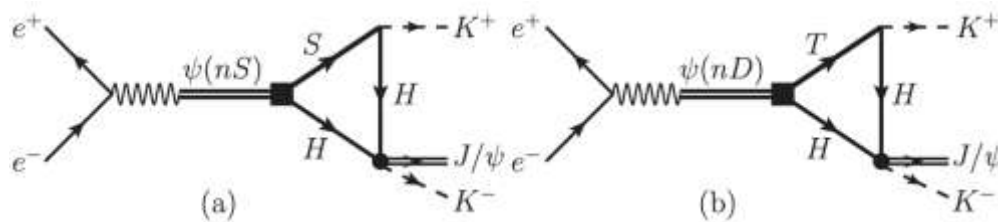


$$\psi(4415) = |n^3 S_1\rangle \cos \theta + |(n-1)^3 D_1\rangle \sin \theta.$$

Mixing angle

4S-3D $\theta = 33^\circ$ Badalian&Bakker,
PRD96(2017)014030

5S-4D $\theta = 30^\circ$
Wang,Chen,Liu,Matsuki,
PRD99(2019)114003



- Contribution from $\psi(nD)$ and THH loop is lower, suppressed by higher partial wave
- $\psi(4S)$ is more favorable for $\psi(4415)$

Summary

- TS (unphysical Riemann sheet) can induce the $Y(4500)$ structure in $e^+ e^- \rightarrow J/\psi K^+ K^-$
- The existence of $Z_{cS}^{(\prime)}$ resonance pole is correlated with the production of $Y(4500)$ signal
- The SHH loops play a crucial role in the rescattering process
- $\psi(4415)$ is compatible with the conventional assignment $\psi(4S)$ state
- More studies on the coupled channel dynamics

Thanks!

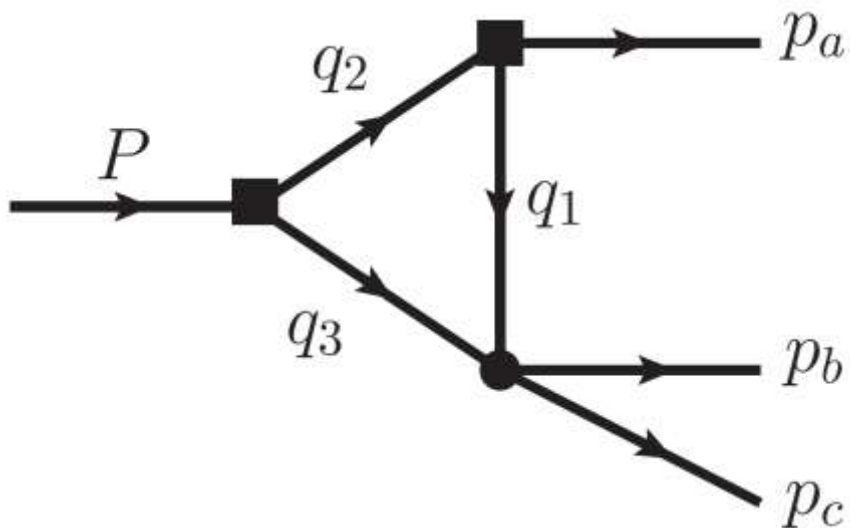
Backup

Triangle Singularity Mechanism

“The kinematic conditions for the existence of singularities on the physical boundary are equivalent to the condition that the relevant Feynman diagram be interpretable as a picture of an energy and momentum-conserving process occurring in space-time, with all internal particles real, on the mass shell and moving forward in time.” –Coleman-Norton theorem

Coleman&Norton, Nuovo Cimento 38,5018 (1965)

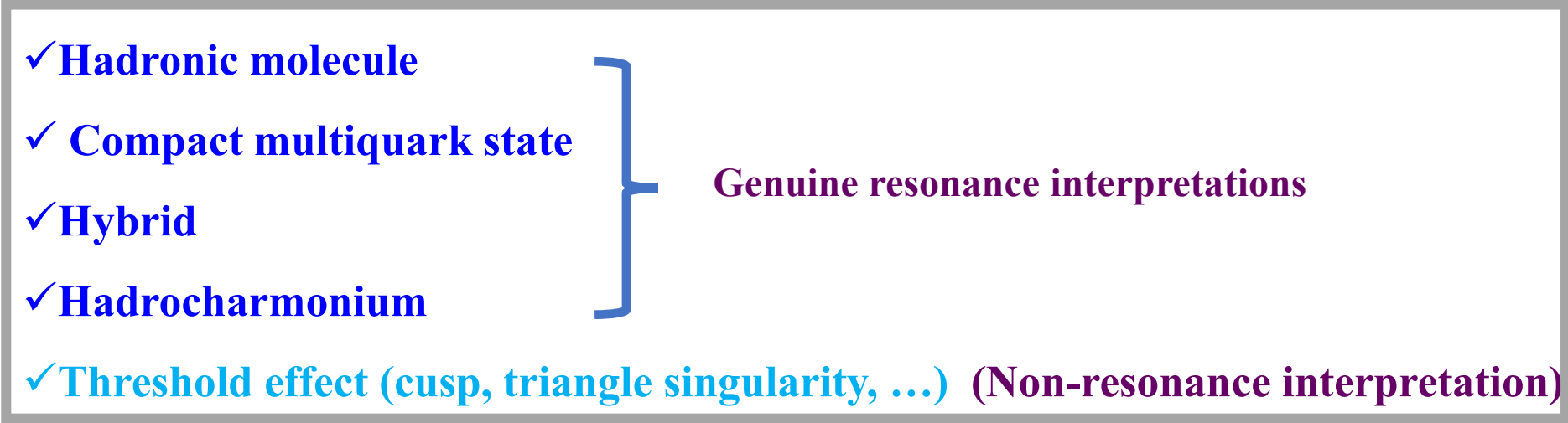
Fronsdal&Norton, J.Math.Phys.5, 100(1964)



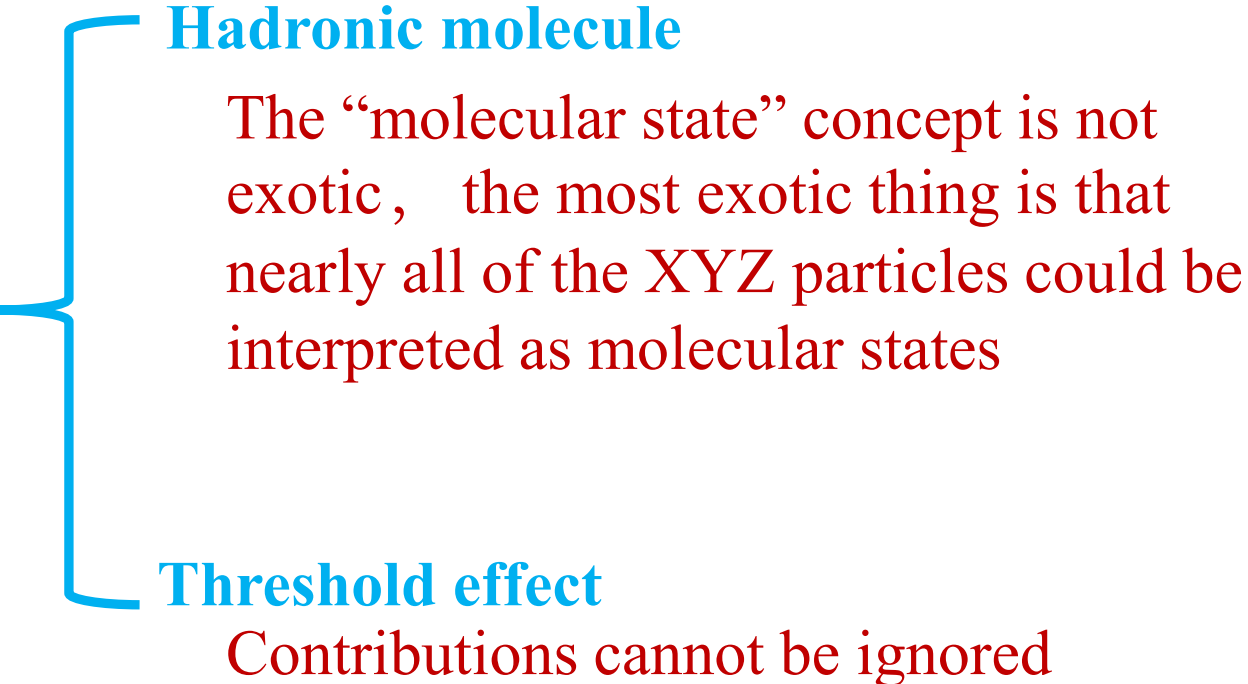
Width effect

Complex mass scheme: $m_2 = m - i\Gamma/2$

Theoretical Interpretation



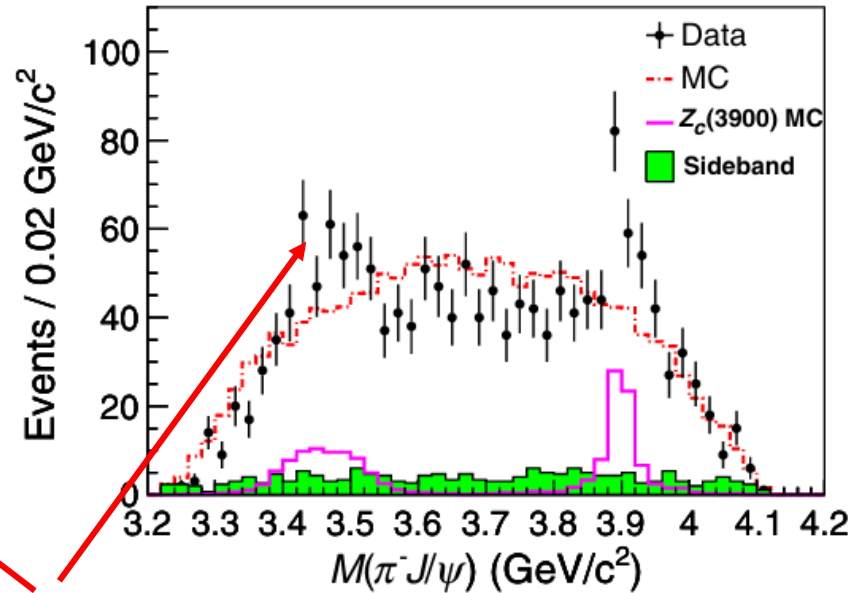
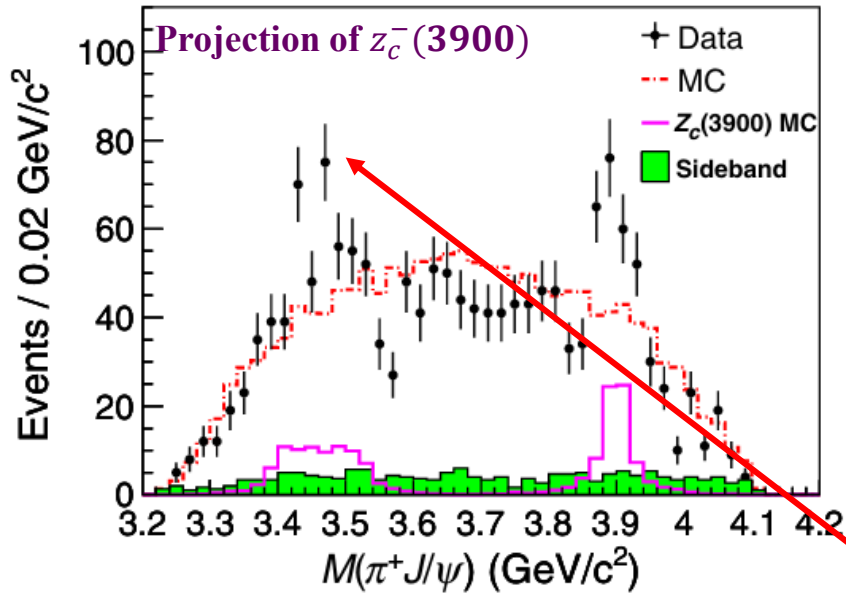
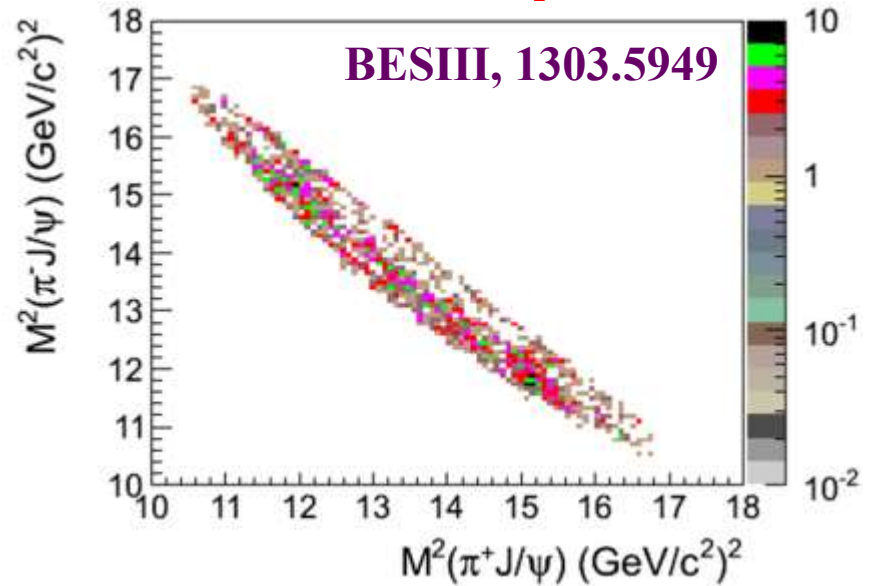
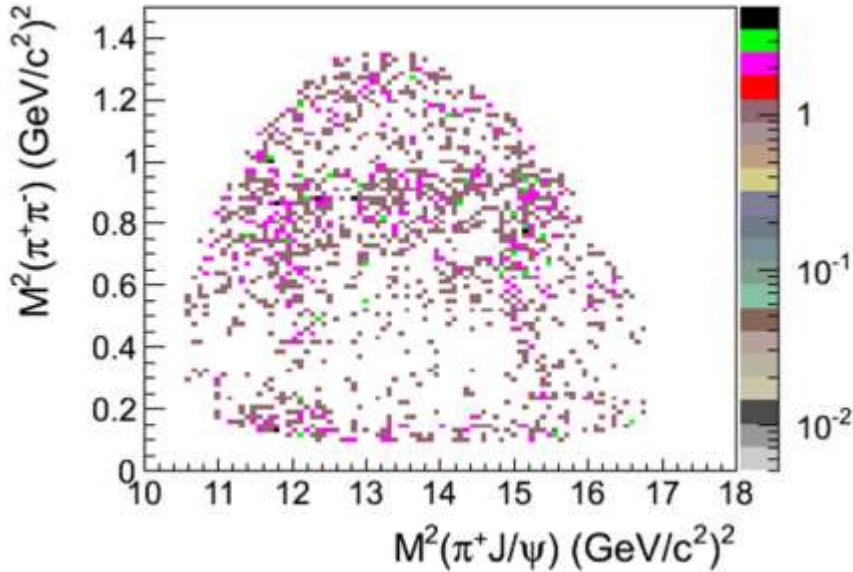
XYZ particle
Near threshold
characteristic



“Resonance-like” structure

?

Genuine particle



“Reflection” in Dalitz plot