Triply heavy baryon spectroscopy revisited

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  - Furthermore, the calculation in Ref. [16] neglected spin $S = 1/2$ and $S = 3/2$ state mixing.

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  - Furthermore, the calculation in Ref. [16] neglected spin $S = 1/2$ and $S = 3/2$ state mixing.
  - Additionally, Ref. [17] employs a quark-diquark model, whereas Ref. [12] uses the hyperspherical approximation. Finally, Ref. [19] utilized only a single incomplete set of Jacobi coordinates and omitted decay studies.

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Hamiltonian

The Hamiltonian of triply heavy baryons $\Omega_{ccc}, \Omega_{bcc}, \Omega_{bbc}, \Omega_{bbb}$ is derived from the nonrelativistic limit of the formalism presented in Refs. [56, 57].

$$H = \sum_i \left(m_i + \frac{p_i^2}{2m_i} \right) + \sum_{i<j} \left(V_{ij}^{\mathrm{conf}} + V_{ij}^{\mathrm{hyp}} + V_{ij}^{\mathrm{so(cm)}} + V_{ij}^{\mathrm{so(tp)}} \right), \tag{1}$$

$$V_{ij}^{\mathrm{conf}} = -\frac{2}{3}\frac{\alpha_s}{r_{ij}} + \frac{b}{2} r_{ij} + \frac{1}{2} C, \tag{2}$$

$$V_{ij}^{\mathrm{hyp}} = \frac{2\alpha_s}{3m_i m_j} \left[\frac{8\pi}{3} \tilde{\delta}(r_{ij}) \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \frac{1}{r_{ij}^3} S(\boldsymbol{r}_{ij}, \boldsymbol{S}_i, \boldsymbol{S}_j) \right], \tag{3}$$

$$V_{ij}^{\mathrm{so(cm)}} = \frac{2\alpha_s}{3r_{ij}^3} \left(\frac{\boldsymbol{r}_{ij} \times \boldsymbol{p}_i \cdot \boldsymbol{S}_i}{m_i^2} - \frac{\boldsymbol{r}_{ij} \times \boldsymbol{p}_j \cdot \boldsymbol{S}_j}{m_j^2} - \frac{\boldsymbol{r}_{ij} \times \boldsymbol{p}_j \cdot \boldsymbol{S}_i - \boldsymbol{r}_{ij} \times \boldsymbol{p}_i \cdot \boldsymbol{S}_j}{m_i m_j} \right), \tag{4}$$

$$V_{ij}^{\mathrm{so(tp)}} = -\frac{1}{2r_{ij}} \frac{\partial V_{ij}^{\mathrm{conf}}}{\partial r_{ij}} \left(\frac{\boldsymbol{r}_{ij} \times \boldsymbol{p}_i \cdot \boldsymbol{S}_i}{m_i^2} - \frac{\boldsymbol{r}_{ij} \times \boldsymbol{p}_j \cdot \boldsymbol{S}_j}{m_j^2} \right). \tag{5}$$

$$\tilde{\delta}(r_{ij}) = \frac{\sigma^3}{\pi^{3/2}} \mathrm{e}^{-\sigma^2 r_{ij}^2}, \ S(\boldsymbol{r}_{ij}, \boldsymbol{S}_i, \boldsymbol{S}_j) = \frac{3\boldsymbol{S}_i \cdot \boldsymbol{r}_{ij} \boldsymbol{S}_j \cdot \boldsymbol{r}_{ij}}{r_{ij}^2} - \boldsymbol{S}_i \cdot \boldsymbol{S}_j. \tag{6}$$

The parameters of the model include:

- α_s (one-gluon-exchange coupling constant),
- b (linear confinement strength),
- C (renormalized mass constant),
- σ (smearing parameter),
- m_c (charm quark mass),
- m_b (bottom quark mass).

Table: The parameters involved in the adopted potential model.

Quarkonium	α_s	$b\,(\mathrm{GeV}^2)$	$C\,(\mathrm{GeV})$	$\sigma\,(\mathrm{GeV})$
$c\bar{c}$	0.470	0.165	−0.409	1.220
$c\bar{b}$	0.362	0.189	−0.555	1.586
$b\bar{b}$	0.333	0.203	−0.603	1.908
$m_c = 1.649\,\mathrm{GeV}$ $\quad m_b = 5.036\,\mathrm{GeV}$				

States	$M^{\mathrm{Exp.}}$ (MeV) [80]	$M^{\mathrm{The.}}$ (MeV)	$M^{\mathrm{Err.}}$ (MeV) [80]
$\eta_c(1S)$	2984.1	2988.9	0.4
$\eta_c(2S)$	3637.7	3642.7	0.9
$J/\psi(1S)$	3096.900	3096.9	0.006
$J/\psi(2S)$	3686.097	3686.1	0.011
$h_c(1P)$	3525.37	3514.4	0.14
$\chi_{c0}(1P)$	3414.71	3439.4	0.30
$\chi_{c1}(1P)$	3510.67	3509.9	0.05
$\chi_{c2}(1P)$	3556.17	3556.1	0.07
$\chi^2/n = 1686.4$			
$\eta_b(1S)$	9398.7	9405.1	2.0
$\Upsilon(1S)$	9460.40	9460.6	0.10
$\Upsilon(2S)$	10023.4	10006.6	0.5
$h_b(1P)$	9899.3	9886.3	0.8
$h_b(2P)$	10259.8	10251.9	1.2
$\chi_{b0}(1P)$	9859.44	9838.0	$\pm 0.42 \pm 0.31$
$\chi_{b0}(2P)$	10232.5	10211.4	$\pm 0.4 \pm 0.5$
$\chi_{b1}(1P)$	9892.78	9883.7	$\pm 0.26 \pm 0.31$
$\chi_{b1}(2P)$	10255.46	10252.3	$\pm 0.22 \pm 0.50$
$\chi_{b2}(1P)$	9912.21	9907.1	$\pm 0.26 \pm 0.31$
$\chi_{b1}(2P)$	10268.65	10272.5	$\pm 0.22 \pm 0.50$
$\chi^2/n = 453.1$			
$B_c(1S)$	6274.47	6274.5	0.32
$B_c(2S)$	6871.2	6871.2	1.0
$\chi^2/n = 0.0407$			

Wave function

For the angular momentum components, we naturally select operators corresponding to L-S coupling. The operator set is

$$\{l_c^2, L_c^2, L^2, S_1^2, S_2^2, S_3^2, S_{ij}^2, S^2, J^2, J_z\}, \tag{7}$$

where $\boldsymbol{l}_c$ and $\boldsymbol{L}_c$ represent the orbital angular momenta associated with the c-th Jacobi coordinates $\boldsymbol{r}_c$ and $\boldsymbol{R}_c$, respectively, while $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, and $\boldsymbol{S}_3$ denote the spins of the constituent quarks m_1, m_2, and m_3, as illustrated in the figure below.

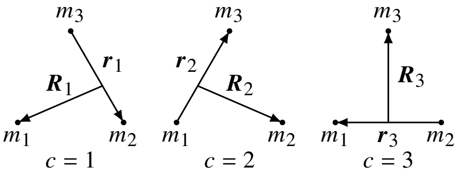

Figure: Three Jacobian coordinates of three-body system.

The composite angular momenta are defined as:

- $\boldsymbol{L} = \boldsymbol{l}_c + \boldsymbol{L}_c$ (total orbital angular momentum),
- $\boldsymbol{S}_{ij} = \boldsymbol{S}_i + \boldsymbol{S}_j$ (pair spin in $\boldsymbol{r}_c$-degree-of-freedom),
- $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2 + \boldsymbol{S}_3$ (total spin),
- $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ (total angular momentum).

Gaussian Expansion Method

The system described by Hamiltonian H satisfies the stationary Schrödinger equation

$$H\Psi = E\Psi. \tag{8}$$

We expand the total wave function in terms of a complete set of L^2-integrable basis functions Φ_α, where α denotes the set of quantum numbers labeling each basis state:

$$\Psi = \sum_\alpha C_\alpha \Phi_\alpha. \tag{9}$$

Applying the Rayleigh-Ritz variational principle yields a generalized matrix eigenvalue problem:

$$HC = NCE, \tag{10}$$

where the Hamiltonian and overlap matrix elements are respectively given by

$$H_{\alpha'\alpha} = \langle \Phi_{\alpha'} | H | \Phi_\alpha \rangle, \tag{11}$$

$$N_{\alpha'\alpha} = \langle \Phi_{\alpha'} | \Phi_\alpha \rangle. \tag{12}$$

For Ω_{ccc} and Ω_{bbb} systems, accounting for the symmetry of three identical quarks, we construct the basis functions as:

$$\begin{aligned}\Phi_\alpha = &\Big[[\phi^{\mathrm{G}}_{nl}(\boldsymbol{r}_1)\phi^{\mathrm{G}}_{NL}(\boldsymbol{R}_1)]_{L_t}[s_1[s_2s_3]_{s_{23}}]_S\Big]_{JM} \\ &+\Big[[\phi^{\mathrm{G}}_{nl}(\boldsymbol{r}_2)\phi^{\mathrm{G}}_{NL}(\boldsymbol{R}_2)]_{L_t}[s_2[s_3s_1]_{s_{31}}]_S\Big]_{JM} \\ &+\Big[[\phi^{\mathrm{G}}_{nl}(\boldsymbol{r}_3)\phi^{\mathrm{G}}_{NL}(\boldsymbol{R}_3)]_{L_t}[s_3[s_1s_2]_{s_{12}}]_S\Big]_{JM}.\end{aligned} \tag{13}$$

For Ω_{bcc} and Ω_{bbc} systems, with two identical quarks positioned at m_2 and m_3 in Fig. 1, we use the symmetric basis:

$$\Phi_\alpha = \Big[[\phi^{\mathrm{G}}_{nl}(\boldsymbol{r}_1)\phi^{\mathrm{G}}_{NL}(\boldsymbol{R}_1)]_{L_t}[s_1[s_2s_3]_{s_{23}}]_S\Big]_{JM}, \tag{14}$$

where $\alpha = \{l, L, L_t, s_{ij}, S, n, N\}$. The quantum numbers used for each J^P state are listed in Table 2. Here we simplify notation by abbreviating l_c and L_c to l and L (due to identical particle symmetry) and denote the total orbital angular momentum as L_t to avoid confusion.

Table: LS coupling three-body angular-momentum space of triply heavy baryons. The units of $r_{\rm min}$, $r_{\rm max}$, $R_{\rm min}$ and $R_{\rm max}$ are fm.

J^P	l	L	L_t	s_{ij}	S	$n_{\rm max}$	$r_{\rm min}$	$r_{\rm max}$	$N_{\rm max}$	$R_{\rm min}$	$R_{\rm max}$
$\frac{1}{2}^+$	0	0	0	1	$\frac{1}{2}$	10	0.1	5.0	10	0.1	5.0
	1	1	0	0	$\frac{1}{2}$	10	0.1	5.0	10	0.1	5.0
	0	2	2	1	$\frac{3}{2}$	10	0.1	5.0	10	0.1	5.0
	2	0	2	1	$\frac{3}{2}$	10	0.1	5.0	10	0.1	5.0
	1	1	1	0	$\frac{1}{2}$	10	0.1	5.0	10	0.1	5.0
	2	2	1	1	$\frac{1}{2}$	10	0.1	5.0	10	0.1	5.0
	2	2	1	1	$\frac{3}{2}$	10	0.1	5.0	10	0.1	5.0
$\frac{3}{2}^+$	0	0	0	1	$\frac{3}{2}$	10	0.1	5.0	10	0.1	5.0
	2	2	0	1	$\frac{3}{2}$	10	0.1	5.0	10	0.1	5.0
	0	2	2	1	$\frac{3}{2}$	10	0.1	5.0	10	0.1	5.0
	2	0	2	1	$\frac{3}{2}$	10	0.1	5.0	10	0.1	5.0
	2	0	2	1	$\frac{1}{2}$	10	0.1	5.0	10	0.1	5.0
	1	1	2	0	$\frac{1}{2}$	10	0.1	5.0	10	0.1	5.0
	0	2	2	1	$\frac{1}{2}$	10	0.1	5.0	10	0.1	5.0
	1	1	1	0	$\frac{1}{2}$	10	0.1	5.0	10	0.1	5.0
	2	2	1	1	$\frac{1}{2}$	10	0.1	5.0	10	0.1	5.0
	2	2	1	1	$\frac{3}{2}$	10	0.1	5.0	10	0.1	5.0
	2	2	3	1	$\frac{3}{2}$	10	0.1	5.0	10	0.1	5.0

Radiative decay

The quark-photon electromagnetic interaction at tree level is described by

$$H_e = -\sum_j e_j \bar{\psi}_j \gamma_\mu^j A^\mu(\boldsymbol{k}, \boldsymbol{r}) \psi_j. \tag{15}$$

In the nonrelativistic limit, this interaction simplifies to

$$h_e \approx \sum_j \left[e_j \boldsymbol{r}_j \cdot \boldsymbol{\epsilon} - \frac{e_j}{2m_j} \boldsymbol{\sigma}_j \cdot (\boldsymbol{\epsilon} \times \hat{\boldsymbol{k}}) \right] \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_j}. \tag{16}$$

The corresponding helicity amplitude for the electromagnetic transition is given by

$$\mathcal{A} = -\mathrm{i}\sqrt{\frac{\omega_\gamma}{2}} \langle f | h_e | i \rangle. \tag{17}$$

The radiative decay width is then obtained from the helicity amplitude through

$$\Gamma = \frac{|\boldsymbol{k}|^2}{\pi} \frac{2}{2J_i + 1} \frac{M_f}{M_i} \sum_{J_{fz}, J_{iz}} |\mathcal{A}_{J_{fz}, J_{iz}}|^2, \tag{18}$$

where J_i is the total angular momentum of the initial baryon, and J_{iz} and J_{fz} are the z-components of the angular momenta for the initial and final states, respectively.

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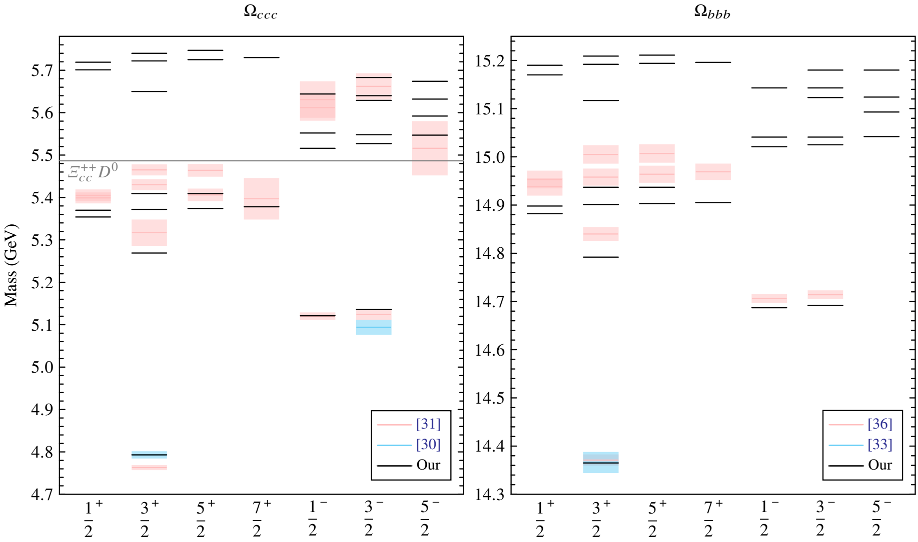
The Ω_{ccc} and Ω_{bbb} baryons



		Ω_{ccc}								Ω_{bbb}							
J^P	(L_t, S)	5354	5370	5701	5719	5784	5851	5885	6011	14882	14898	15170	15190	15257	15324	15349	15424
$\frac{1}{2}^+$	$(0,\frac{1}{2})$	**97.34**	2.59	**98.11**	1.80	**99.43**	0.55	0.03	**98.55**	**99.83**	0.17	**99.90**	0.09	**99.99**	0.01	0.01	**99.92**
	$(2,\frac{3}{2})$	2.66	**97.38**	1.86	**98.09**	0.14	3.58	**96.35**	1.38	0.17	**99.83**	0.10	**99.90**	0.01	0.08	**99.91**	0.07
	$(1,\frac{1}{2})$	0.00	0.02	0.04	0.10	0.43	**95.87**	3.61	0.06	0.00	0.00	0.00	0.01	0.01	**99.91**	0.08	0.00
	$(1,\frac{3}{2})$	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
J^P	(L_t, S)	4793	5269	5372	5409	5650	5722	5740	5796	14365	14792	14901	14937	15117	15192	15209	15266
$\frac{3}{2}^+$	$(0,\frac{3}{2})$	**99.78**	**99.79**	0.06	0.03	**99.71**	0.11	0.05	0.02	**99.98**	**99.98**	0.00	0.00	**99.98**	0.01	0.00	0.00
	$(2,\frac{3}{2})$	0.12	0.13	**97.41**	2.48	0.19	**96.44**	3.17	0.31	0.01	0.01	**99.93**	0.06	0.01	**99.94**	0.04	0.01
	$(2,\frac{1}{2})$	0.09	0.08	2.51	**97.47**	0.08	3.39	**96.71**	**99.31**	0.01	0.01	0.06	**99.94**	0.01	0.04	**99.96**	**99.98**
	$(1,\frac{1}{2})$	0.00	0.00	0.01	0.00	0.02	0.04	0.05	0.35	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
	$(1,\frac{3}{2})$	0.00	0.00	0.01	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$(3,\frac{3}{2})$	0.00	0.00	0.01	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
J^P	(L_t, S)	5374	5409	5725	5747	5801	5852	5883	5887	14903	14937	15194	15211	15267	15327	15351	15354
$\frac{5}{2}^+$	$(2,\frac{3}{2})$	**90.14**	9.82	**87.88**	11.53	0.54	0.90	48.87	**50.24**	**99.66**	0.33	**99.53**	0.43	0.03	0.05	**99.87**	0.08
	$(2,\frac{1}{2})$	9.83	**90.17**	12.09	**88.45**	**99.42**	**98.99**	0.78	0.25	0.34	**99.67**	0.46	**99.56**	**99.97**	**99.95**	0.05	0.00
	$(1,\frac{3}{2})$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$(3,\frac{3}{2})$	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$(3,\frac{1}{2})$	0.02	0.00	0.02	0.01	0.02	0.10	**50.35**	49.50	0.00	0.00	0.00	0.00	0.00	0.00	0.08	**99.91**
J^P	(L_t, S)	5378	5730	5879	5891	6041	6131	6186	6200	14905	15196	15352	15356	15448	15538	15600	15608
$\frac{7}{2}^+$	$(2,\frac{3}{2})$	**99.99**	**99.96**	**59.87**	40.14	**99.93**	**99.83**	9.28	**85.18**	**100.0**	**100.0**	**95.09**	4.91	**100.0**	**100.0**	0.67	**99.28**
	$(3,\frac{3}{2})$	0.01	0.00	0.00	0.01	0.00	0.01	0.47	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.01
	$(3,\frac{1}{2})$	0.01	0.03	40.13	**59.85**	0.06	0.16	**90.25**	14.32	0.00	0.00	4.91	**95.09**	0.00	0.00	**99.29**	0.71
J^P	(L_t, S)	5121	5516	5552	5644	5854	5872	5958	5983	14687	15021	15041	15143	15299	15304	15398	15422
$\frac{1}{2}^-$	$(1,\frac{1}{2})$	**99.93**	**99.76**	0.30	**99.87**	**98.93**	1.10	**99.93**	**99.78**	**99.99**	**99.94**	0.06	**99.99**	**99.03**	0.97	**99.99**	**99.99**
	$(1,\frac{3}{2})$	0.04	0.22	**99.67**	0.12	1.05	**98.86**	0.05	0.16	0.00	0.06	**99.93**	0.01	0.97	**99.02**	0.00	0.01
	$(2,\frac{3}{2})$	0.02	0.02	0.03	0.01	0.02	0.03	0.02	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
J^P	(L_t, S)	5136	5527	5548	5629	5640	5683	5862	5868	14692	15025	15041	15123	15143	15180	15302	15304
$\frac{3}{2}^-$	$(1,\frac{1}{2})$	**99.90**	**99.00**	1.21	5.11	**93.61**	0.98	**87.24**	12.79	**99.99**	**99.92**	0.08	0.05	**99.92**	0.02	**96.91**	3.09
	$(1,\frac{3}{2})$	0.00	0.87	**98.67**	0.01	0.34	0.05	12.58	**87.09**	0.00	0.07	**99.90**	0.00	0.01	0.01	3.08	**96.90**
	$(3,\frac{3}{2})$	0.08	0.08	0.02	**93.12**	5.58	1.27	0.09	0.02	0.01	0.01	0.00	**99.90**	0.05	0.04	0.01	0.00
	$(2,\frac{1}{2})$	0.00	0.03	0.07	1.76	0.46	**97.69**	0.07	0.07	0.00	0.00	0.01	0.04	0.02	**99.92**	0.00	0.01
	$(2,\frac{3}{2})$	0.02	0.01	0.03	0.00	0.01	0.01	0.02	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
J^P	(L_t, S)	5547	5592	5632	5674	5868	5910	5930	5997	15042	15093	15124	15180	15305	15352	15364	15436
$\frac{5}{2}^-$	$(1,\frac{3}{2})$	**99.80**	0.01	0.05	0.05	**99.77**	0.01	0.09	0.04	**99.99**	0.00	0.00	0.00	**99.98**	0.00	0.00	0.00
	$(3,\frac{1}{2})$	0.06	**99.27**	0.51	0.23	0.06	**98.09**	2.32	**49.69**	0.01	**99.99**	0.00	0.01	0.00	**99.96**	0.07	**95.30**
	$(3,\frac{3}{2})$	0.07	0.40	**97.69**	1.84	0.10	1.67	**97.34**	20.95	0.01	0.00	**99.94**	0.05	0.01	0.03	**99.92**	4.50
	$(2,\frac{1}{2})$	0.06	0.32	1.74	**97.87**	0.06	0.22	0.24	29.29	0.00	0.01	0.05	**99.93**	0.00	0.01	0.01	0.20
	$(2,\frac{3}{2})$	0.01	0.01	0.00	0.01	0.01	0.01	0.02	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Most states are dominated by single total orbital angular momentum excitations.

The Ω_{bcc} and Ω_{bbc} baryons

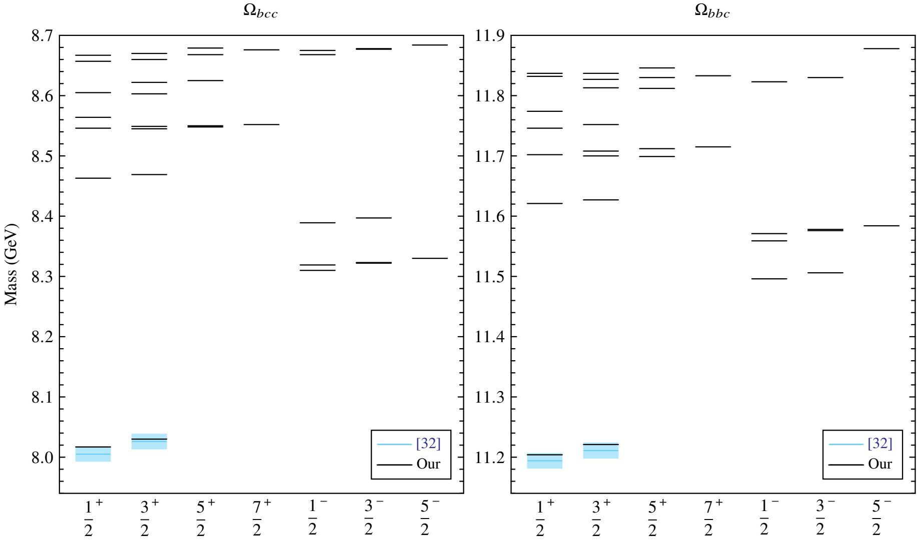


TABLE V. The mass spectra of the Ω_{bcc} and Ω_{bbc} baryons and the proportion of (l, L, L_t, S) in each state.

		Ω_{bcc}								Ω_{bbc}							
J^P	(l, L, L_t, S)	8017	8463	8546	8564	8605	8657	8667	8825	11204	11621	11702	11746	11774	11832	11837	11948
$\frac{1}{2}^+$	$(0, 0, 0, \frac{1}{2})$	**99.91**	**99.91**	0.01	0.20	**99.32**	0.18	0.29	**99.88**	**99.98**	**99.95**	0.07	**99.16**	0.78	0.04	0.01	**99.83**
	$(1, 1, 0, \frac{1}{2})$	0.01	0.01	0.38	**99.40**	0.20	0.01	0.00	0.01	0.01	0.03	0.03	0.74	**99.11**	0.06	0.07	0.06
	$(0, 2, 2, \frac{3}{2})$	0.00	0.00	**98.56**	0.37	0.00	0.07	1.41	0.00	0.00	0.00	0.27	0.00	0.07	26.76	**72.99**	0.00
	$(2, 0, 2, \frac{3}{2})$	0.07	0.06	1.00	0.02	0.35	0.51	**97.76**	0.06	0.01	0.02	**99.57**	0.07	0.02	0.04	0.21	0.09
	$(1, 1, 1, \frac{1}{2})$	0.01	0.02	0.03	0.01	0.12	**99.22**	0.53	0.05	0.00	0.00	0.05	0.02	0.02	**73.09**	26.72	0.01
	$(2, 2, 1, \frac{1}{2})$	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$(2, 2, 1, \frac{3}{2})$	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
J^P	(l, L, L_t, S)	8030	8469	8545	8549	8603	8622	8660	8670	11221	11627	11700	11708	11752	11813	11827	11837
$\frac{3}{2}^+$	$(0, 0, 0, \frac{3}{2})$	**99.70**	**99.38**	0.05	0.02	**98.35**	0.08	0.18	0.08	**99.89**	**99.28**	0.07	0.01	**99.76**	0.06	0.00	0.04
	$(2, 2, 0, \frac{3}{2})$	0.20	0.53	0.00	0.01	1.19	0.00	0.00	0.00	0.09	0.69	0.00	0.00	0.07	0.00	0.00	0.00
	$(0, 2, 2, \frac{3}{2})$	0.01	0.00	**53.37**	44.20	0.08	0.95	0.06	0.67	0.01	0.01	0.07	0.39	0.00	0.36	21.69	9.58
	$(2, 0, 2, \frac{3}{2})$	0.04	0.04	0.67	0.66	0.03	0.07	8.31	39.60	0.01	0.01	32.86	**66.58**	0.06	0.03	0.15	0.01
	$(0, 2, 2, \frac{1}{2})$	0.00	0.00	43.57	**53.77**	0.00	0.60	0.05	1.45	0.00	0.00	0.44	0.44	0.00	0.21	**77.54**	2.74
	$(1, 1, 2, \frac{1}{2})$	0.00	0.00	1.50	0.00	0.06	**98.29**	0.01	0.10	0.00	0.00	0.12	0.00	0.05	**99.13**	0.01	0.02
	$(2, 0, 2, \frac{1}{2})$	0.03	0.03	0.79	1.18	0.15	0.00	2.13	**57.54**	0.00	0.01	**66.43**	32.57	0.02	0.08	0.61	0.01
	$(1, 1, 1, \frac{1}{2})$	0.01	0.02	0.01	0.15	0.12	0.00	**89.25**	0.55	0.00	0.00	0.01	0.00	0.02	0.13	0.00	**87.59**
	$(2, 2, 1, \frac{1}{2})$	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$(2, 2, 1, \frac{3}{2})$	0.00	0.00	0.01	0.01	0.02	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01
	$(2, 2, 3, \frac{3}{2})$	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
J^P	(l, L, L_t, S)	8548	8550	8625	8668	8679	8862	8863	8957	11699	11712	11812	11830	11846	11986	11996	12091
$\frac{5}{2}^+$	$(0, 2, 2, \frac{3}{2})$	18.23	**78.37**	1.68	0.57	1.85	28.19	**70.13**	1.12	0.03	0.70	0.69	**54.28**	44.48	0.06	0.74	0.03
	$(2, 0, 2, \frac{3}{2})$	0.34	1.52	0.20	19.41	**77.78**	0.24	0.59	0.44	13.23	**86.04**	0.01	0.40	0.13	13.60	**85.60**	0.00
	$(2, 0, 2, \frac{1}{2})$	2.02	0.48	0.37	**77.22**	19.02	0.82	0.32	0.47	**86.16**	13.05	0.08	0.32	0.18	**85.67**	13.46	0.03
	$(1, 1, 2, \frac{1}{2})$	1.65	0.55	**96.86**	0.15	0.79	0.96	0.20	**97.25**	0.04	0.01	**98.73**	1.22	0.00	0.03	0.01	**99.88**
	$(0, 2, 2, \frac{1}{2})$	**77.74**	19.06	0.88	2.64	0.56	**69.75**	28.71	0.69	0.54	0.21	0.49	43.77	**55.21**	0.64	0.19	0.03
	$(2, 2, 1, \frac{3}{2})$	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$(2, 2, 3, \frac{3}{2})$	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$(1, 3, 3, \frac{1}{2})$	0.00	0.01	0.00	0.00	0.00	0.02	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$(2, 2, 3, \frac{1}{2})$	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$(3, 1, 3, \frac{1}{2})$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02
J^P	(l, L, L_t, S)	8552	8676	8866	8985	9029	9033	9080	9087	11715	11833	11998	12160	12174	12189	12198	12236
$\frac{7}{2}^+$	$(0, 2, 2, \frac{3}{2})$	**97.70**	3.16	**98.78**	**73.97**	22.35	4.45	0.02	0.03	0.97	**99.25**	0.97	**97.20**	0.05	25.72	**76.22**	0.00
	$(2, 0, 2, \frac{3}{2})$	2.28	**96.84**	1.15	25.52	**68.46**	5.22	1.05	0.99	**99.03**	0.74	**99.03**	2.80	0.65	**73.50**	23.67	0.04
	$(2, 2, 3, \frac{3}{2})$	0.00	0.00	0.01	0.00	0.04	1.82	30.45	**64.35**	0.00	0.00	0.00	0.00	0.04	0.11	0.03	**59.84**
	$(1, 3, 3, \frac{1}{2})$	0.01	0.00	0.06	0.49	8.96	**87.47**	0.02	2.62	0.00	0.00	0.00	0.00	0.19	0.00	0.00	0.10
	$(2, 2, 3, \frac{1}{2})$	0.00	0.00	0.00	0.00	0.07	0.47	**68.46**	30.35	0.00	0.00	0.00	0.00	0.01	0.02	0.01	39.98
	$(3, 1, 3, \frac{1}{2})$	0.00	0.00	0.00	0.00	0.11	0.58	0.00	1.66	0.00	0.00	0.00	0.00	**99.05**	0.65	0.06	0.04

The Ω_{ccc} baryons

Through our calculations and comparison with previous studies, we have identified two fundamental patterns governing radiative decays in our model:

1. **Three identical quarks**: Radiative transitions between states with conserved parity ($P_i = P_f$) and total spin ($S_i = S_f$) typically exhibit larger partial widths.
2. **Two identical quarks**: The dominant transitions occur between states with conserved total spin ($S_i = S_f$) but flipped parity ($P_i = -P_f$), yielding larger partial widths.

TABLE VIII. Radiative decay widths (in keV) for Ω_{ccc} baryons with masses below the $\Xi_{cc}^{++}D^0$ threshold. Residual radiative decay widths for Ω_{ccc} baryons not tabulated are all below 0.005 keV.

		$\Omega_{ccc}(4793)\ 3/2^+\gamma$		$\Omega_{ccc}(5121)\ 1/2^-\gamma$		$\Omega_{ccc}(5136)\ 3/2^-\gamma$	
(L_t, S)	Initial state	Our	Ref. [26]	Our	Ref. [26]	Our	Ref. [26]
$(0, \frac{1}{2})$	$\Omega_{ccc}(5354)\ 1/2^+$	4.99	-	0.18	20.14	0.50	27.43
$(0, \frac{3}{2})$	$\Omega_{ccc}(5269)\ 3/2^+$	0.68	-	0.02	0.002	0.03	0.010
$(2, \frac{1}{2})$	$\Omega_{ccc}(5409)\ 3/2^+$	9.96	-	0.43	106.88	1.33	33.58
	$\Omega_{ccc}(5409)\ 5/2^+$	24.68	-	0.47	0.25	0.94	122.10
$(2, \frac{3}{2})$	$\Omega_{ccc}(5370)\ 1/2^+$	176.94	-	0.14	<0.001	0.07	0.04
	$\Omega_{ccc}(5372)\ 3/2^+$	162.60	-	0.58	0.38	0.02	0.02
	$\Omega_{ccc}(5374)\ 5/2^+$	132.83	-	0.36	0.22	0.22	0.39
	$\Omega_{ccc}(5378)\ 7/2^+$	138.47	-	0.00	<0.001	0.39	0.80
$(1, \frac{1}{2})$	$\Omega_{ccc}(5121)\ 1/2^-$	2.52	3.10	-	-	-	-
	$\Omega_{ccc}(5136)\ 3/2^-$	3.05	4.07	0.00	-	-	-

	$\Omega_{ccc}(5269)\ 3/2^+\gamma$
$\Omega_{ccc}(5370)\ 1/2^+$	0.15
$\Omega_{ccc}(5372)\ 3/2^+$	0.18
$\Omega_{ccc}(5374)\ 5/2^+$	0.18
$\Omega_{ccc}(5378)\ 7/2^+$	0.24
$\Omega_{ccc}(5409)\ 3/2^+$	0.02
$\Omega_{ccc}(5409)\ 5/2^+$	0.09
$\Omega_{ccc}(5409)\ 3/2^+ \to \Omega_{ccc}(5354)\ 1/2^+\gamma$	0.01
$\Omega_{ccc}(5409)\ 5/2^+ \to \Omega_{ccc}(5354)\ 1/2^+\gamma$	0.01
$\Omega_{ccc}(5409)\ 5/2^+ \to \Omega_{ccc}(5372)\ 3/2^+\gamma$	0.01
$\Omega_{ccc}(5409)\ 5/2^+ \to \Omega_{ccc}(5378)\ 7/2^+\gamma$	0.01

It can be observed that our partial calculations differ significantly from those in Ref. [26]. We attribute this discrepancy to two main factors: (1) Ref. [26] does not account for the symmetry of three identical particles; (2) the potential models employed differ between the studies.

The Ω_{bbb} baryons

TABLE IX. The radiative decay widths for Ω_{bbb} baryons with masses below 15.25 GeV (in keV). Residual radiative decay widths for Ω_{bbb} baryons not tabulated are all below 0.005 keV.

(L_t, S)	Initial state	$\Omega_{bbb}(14365)\ 3/2^+\gamma$ Our	Ref. [26]
$(0, \frac{1}{2})$	$\Omega_{bbb}(14882)\ 1/2^+$	0.01	-
	$\Omega_{bbb}(15170)\ 1/2^+$	0.01	-
$(2, \frac{1}{2})$	$\Omega_{bbb}(14937)\ 3/2^+$	0.01	-
	$\Omega_{bbb}(14937)\ 5/2^+$	0.03	-
	$\Omega_{bbb}(15209)\ 3/2^+$	0.01	-
	$\Omega_{bbb}(15211)\ 5/2^+$	0.03	-
$(2, \frac{3}{2})$	$\Omega_{bbb}(14898)\ 1/2^+$	4.63	-
	$\Omega_{bbb}(14901)\ 3/2^+$	4.56	-
	$\Omega_{bbb}(14903)\ 5/2^+$	4.45	-
	$\Omega_{bbb}(14905)\ 7/2^+$	4.37	-
	$\Omega_{bbb}(15190)\ 1/2^+$	2.81	-
	$\Omega_{bbb}(15192)\ 3/2^+$	2.84	-
	$\Omega_{bbb}(15194)\ 5/2^+$	2.84	-
	$\Omega_{bbb}(15196)\ 7/2^+$	2.87	-
$(1, \frac{1}{2})$	$\Omega_{bbb}(14687)\ 1/2^-$	0.02	0.035
	$\Omega_{bbb}(14692)\ 3/2^-$	0.03	0.038
	$\Omega_{bbb}(15021)\ 1/2^-$	0.03	-
	$\Omega_{bbb}(15025)\ 3/2^-$	0.03	-
$(1, \frac{3}{2})$	$\Omega_{bbb}(15041)\ 1/2^-$	0.14	-
	$\Omega_{bbb}(15041)\ 3/2^-$	0.13	-
	$\Omega_{bbb}(15042)\ 5/2^-$	0.12	-
$(3, \frac{3}{2})$	$\Omega_{bbb}(15123)\ 3/2^-$	0.36	-
	$\Omega_{bbb}(15124)\ 5/2^-$	0.35	-

Initial state	$\Omega_{bbb}(14687)\ 1/2^-\gamma$
$\Omega_{bbb}(15170)\ 1/2^+$	0.02
$\Omega_{bbb}(15209)\ 3/2^+$	0.03
$\Omega_{bbb}(15211)\ 5/2^+$	0.02
$\Omega_{bbb}(15180)\ 3/2^-$	0.88
$\Omega_{bbb}(15143)\ 3/2^-$	0.68
$\Omega_{bbb}(15025)\ 3/2^-$	0.42
$\Omega_{bbb}(15180)\ 5/2^-$	0.33
$\Omega_{bbb}(15093)\ 5/2^-$	3.69

Initial state	$\Omega_{bbb}(14692)\ 3/2^-\gamma$ Our	Ref. [26]
$\Omega_{bbb}(15170)\ 1/2^+$	0.02	-
$\Omega_{bbb}(14937)\ 3/2^+$	0.01	2.75
$\Omega_{bbb}(15209)\ 3/2^+$	0.03	-
$\Omega_{bbb}(15211)\ 5/2^+$	0.09	-
$\Omega_{bbb}(15143)\ 1/2^-$	1.43	-
$\Omega_{bbb}(15021)\ 1/2^-$	0.76	-
$\Omega_{bbb}(15180)\ 3/2^-$	0.98	-
$\Omega_{bbb}(15143)\ 3/2^-$	0.64	-
$\Omega_{bbb}(15025)\ 3/2^-$	0.40	-
$\Omega_{bbb}(15180)\ 5/2^-$	1.37	-
$\Omega_{bbb}(15093)\ 5/2^-$	1.00	-

Initial state	$\Omega_{bbb}(14792)\ 3/2^+\gamma$
$\Omega_{bbb}(14898)\ 1/2^+$	0.01
$\Omega_{bbb}(14901)\ 3/2^+$	0.01
$\Omega_{bbb}(14903)\ 5/2^+$	0.01
$\Omega_{bbb}(14905)\ 7/2^+$	0.01
$\Omega_{bbb}(15190)\ 1/2^+$	2.49
$\Omega_{bbb}(15192)\ 3/2^+$	2.44
$\Omega_{bbb}(15194)\ 5/2^+$	2.35
$\Omega_{bbb}(15196)\ 7/2^+$	2.27
$\Omega_{bbb}(15211)\ 5/2^+$	0.01
$\Omega_{bbb}(15021)\ 1/2^-$	0.01
$\Omega_{bbb}(15025)\ 3/2^-$	0.01
$\Omega_{bbb}(15123)\ 3/2^-$	0.02
$\Omega_{bbb}(15124)\ 5/2^-$	0.02

Initial state	$\Omega_{bbb}(14882)\ 1/2^+\gamma$
$\Omega_{bbb}(15143)\ 1/2^-$	0.01
$\Omega_{bbb}(15209)\ 3/2^+$	1.02
$\Omega_{bbb}(15211)\ 5/2^+$	1.02

Initial state	$\Omega_{bbb}(14898)\ 1/2^+\gamma$
$\Omega_{bbb}(15117)\ 3/2^+$	0.05
$\Omega_{bbb}(15192)\ 3/2^+$	0.20
$\Omega_{bbb}(15194)\ 5/2^+$	0.06

Initial state	$\Omega_{bbb}(14901)\ 3/2^+\gamma$
$\Omega_{bbb}(15117)\ 3/2^+$	0.10
$\Omega_{bbb}(15190)\ 1/2^+$	0.41
$\Omega_{bbb}(15194)\ 5/2^+$	0.20
$\Omega_{bbb}(15196)\ 7/2^+$	0.03

Initial state	$\Omega_{bbb}(14903)\ 5/2^+\gamma$
$\Omega_{bbb}(15117)\ 3/2^+$	0.15
$\Omega_{bbb}(15190)\ 1/2^+$	0.18
$\Omega_{bbb}(15192)\ 3/2^+$	0.32
$\Omega_{bbb}(15194)\ 5/2^+$	0.06
$\Omega_{bbb}(15196)\ 7/2^+$	0.19

Initial state	$\Omega_{bbb}(14905)\ 7/2^+\gamma$
$\Omega_{bbb}(15117)\ 3/2^+$	0.20
$\Omega_{bbb}(15192)\ 3/2^+$	0.07
$\Omega_{bbb}(15194)\ 5/2^+$	0.27
$\Omega_{bbb}(15196)\ 7/2^+$	0.37

Initial state	$\Omega_{bbb}(14937)\ 3/2^+\gamma$
$\Omega_{bbb}(15170)\ 1/2^+$	0.22
$\Omega_{bbb}(15209)\ 3/2^+$	0.27
$\Omega_{bbb}(15211)\ 5/2^+$	0.08

Initial state	$\Omega_{bbb}(14937)\ 5/2^+\gamma$
$\Omega_{bbb}(15170)\ 1/2^+$	0.34
$\Omega_{bbb}(15209)\ 3/2^+$	0.12
$\Omega_{bbb}(15211)\ 5/2^+$	0.32

Initial state	$\Omega_{bbb}(15117)\ 3/2^+\gamma$
$\Omega_{bbb}(15194)\ 5/2^+$	0.01
$\Omega_{bbb}(15196)\ 7/2^+$	0.01

The Ω_{bcc} baryons

TABLE X. The radiative decay widths of the Ω_{bcc} baryons below 8.58 GeV and the Ω_{bbc} baryons below 11.72 GeV in units of keV. All unlisted decay widths for these states are below 0.05 keV.

(L_t, S)	Initial state	$\Omega_{bcc}(8017)\ 1/2^+\gamma$	$\Omega_{bcc}(8030)\ 3/2^+\gamma$
$(0, \frac{1}{2})$	$\Omega_{bcc}(8463)\ 1/2^+$	0.2	0.0
	$\Omega_{bcc}(8564)\ 1/2^+$	0.8	1.1
$(0, \frac{3}{2})$	$\Omega_{bcc}(8469)\ 3/2^+$	0.2	0.2
$(2, \frac{1}{2})$	$\Omega_{bcc}(8549)\ 3/2^+$	22.9	15.0
	$\Omega_{bcc}(8548)\ 5/2^+$	19.1	6.0
$(2, \frac{3}{2})$	$\Omega_{bcc}(8546)\ 1/2^+$	0.0	31.0
	$\Omega_{bcc}(8545)\ 3/2^+$	12.7	15.0
	$\Omega_{bcc}(8550)\ 5/2^+$	4.7	22.6
	$\Omega_{bcc}(8552)\ 7/2^+$	0.0	27.7
$(1, \frac{1}{2})$	$\Omega_{bcc}(8319)\ 1/2^-$	141.5	37.2
	$\Omega_{bcc}(8322)\ 3/2^-$	135.9	24.2
$(1, \frac{3}{2})$	$\Omega_{bcc}(8310)\ 1/2^-$	32.4	122.6
	$\Omega_{bcc}(8323)\ 3/2^-$	19.3	134.8
	$\Omega_{bcc}(8330)\ 5/2^-$	0.1	150.9
$(1, \frac{1}{2})$	$\Omega_{bcc}(8389)\ 1/2^-$	1.9	1.9
	$\Omega_{bcc}(8397)\ 3/2^-$	2.9	6.1
$\Omega_{bcc}(8545)\ 3/2^+ \to \Omega_{bcc}(8389)\ 1/2^-\gamma$			0.5
$\Omega_{bcc}(8546)\ 1/2^+ \to \Omega_{bcc}(8389)\ 1/2^-\gamma$			0.2
$\Omega_{bcc}(8564)\ 1/2^+ \to \Omega_{bcc}(8389)\ 1/2^-\gamma$			13.6
$\Omega_{bcc}(8546)\ 1/2^+ \to \Omega_{bcc}(8397)\ 3/2^-\gamma$			0.2
$\Omega_{bcc}(8550)\ 5/2^+ \to \Omega_{bcc}(8397)\ 3/2^-\gamma$			0.1
$\Omega_{bcc}(8564)\ 1/2^+ \to \Omega_{bcc}(8397)\ 3/2^-\gamma$			25.6
$\Omega_{bbc}(11700)\ 3/2^+ \to \Omega_{bbc}(11496)\ 1/2^-\gamma$			0.1

(L_t, S)	Initial state	$\Omega_{bbc}(11204)\ 1/2^+\gamma$	$\Omega_{bbc}(11221)\ 3/2^+\gamma$
$(0, \frac{3}{2})$	$\Omega_{bbc}(11627)\ 3/2^+$	0.1	0.0
$(2, \frac{1}{2})$	$\Omega_{bbc}(11700)\ 3/2^+$	0.7	0.4
	$\Omega_{bbc}(11699)\ 5/2^+$	0.9	0.1
$(2, \frac{3}{2})$	$\Omega_{bbc}(11702)\ 1/2^+$	0.0	1.2
	$\Omega_{bbc}(11708)\ 3/2^+$	0.3	0.7
	$\Omega_{bbc}(11712)\ 5/2^+$	0.1	0.7
	$\Omega_{bbc}(11715)\ 7/2^+$	0.0	0.8
$(1, \frac{1}{2})$	$\Omega_{bbc}(11571)\ 1/2^-$	131.7	12.8
	$\Omega_{bbc}(11578)\ 3/2^-$	126.3	65.0
$(1, \frac{3}{2})$	$\Omega_{bbc}(11559)\ 1/2^-$	16.7	147.6
	$\Omega_{bbc}(11576)\ 3/2^-$	45.7	98.5
	$\Omega_{bbc}(11584)\ 5/2^-$	0.6	145.3
$(1, \frac{1}{2})$	$\Omega_{bbc}(11496)\ 1/2^-$	0.3	0.4
	$\Omega_{bbc}(11506)\ 3/2^-$	0.1	0.0
$\Omega_{bbc}(11627)\ 3/2^+ \to \Omega_{bbc}(11559)\ 1/2^-\gamma$			0.1
$\Omega_{bbc}(11621)\ 1/2^+ \to \Omega_{bbc}(11571)\ 1/2^-\gamma$			0.1
$\Omega_{bbc}(11700)\ 3/2^+ \to \Omega_{bbc}(11571)\ 1/2^-\gamma$			0.1
$\Omega_{bbc}(11708)\ 3/2^+ \to \Omega_{bbc}(11571)\ 1/2^-\gamma$			0.1
$\Omega_{bbc}(11708)\ 3/2^+ \to \Omega_{bbc}(11576)\ 3/2^-\gamma$			0.1
$\Omega_{bbc}(11712)\ 5/2^+ \to \Omega_{bbc}(11576)\ 3/2^-\gamma$			0.2
$\Omega_{bbc}(11621)\ 1/2^+ \to \Omega_{bbc}(11578)\ 3/2^-\gamma$			0.1
$\Omega_{bbc}(11699)\ 5/2^+ \to \Omega_{bbc}(11578)\ 3/2^-\gamma$			0.1
$\Omega_{bbc}(11627)\ 3/2^+ \to \Omega_{bbc}(11584)\ 5/2^-\gamma$			0.1
$\Omega_{bbc}(11715)\ 7/2^+ \to \Omega_{bbc}(11584)\ 5/2^-\gamma$			0.2

(L_t, S)	Initial state	$\Omega_{bcc}(8310)\ 1/2^-\gamma$	$\Omega_{bcc}(8319)\ 1/2^-\gamma$	$\Omega_{bcc}(8322)\ 3/2^-\gamma$	$\Omega_{bcc}(8323)\ 3/2^-\gamma$	$\Omega_{bcc}(8330)\ 5/2^-\gamma$
$(0, \frac{1}{2}/\frac{3}{2})$	$\Omega_{bcc}(8463)\ 1/2^+$	5.0	16.0	36.8	6.4	0.0
	$\Omega_{bcc}(8564)\ 1/2^+$	0.2	0.5	0.7	1.2	0.1
	$\Omega_{bcc}(8469)\ 3/2^+$	9.6	2.1	2.8	18.6	32.1
$(2, \frac{1}{2}/\frac{3}{2})$	$\Omega_{bcc}(8546)\ 1/2^+$	146.3	32.3	4.4	29.8	0.0
	$\Omega_{bcc}(8545)\ 3/2^+$	98.1	19.3	0.8	68.3	5.4
	$\Omega_{bcc}(8549)\ 3/2^+$	1.1	129.3	45.9	23.8	5.3
	$\Omega_{bcc}(8548)\ 5/2^+$	0.0	0.2	170.4	0.0	11.5
	$\Omega_{bcc}(8550)\ 5/2^+$	0.1	0.0	2.4	133.3	44.2
	$\Omega_{bcc}(8552)\ 7/2^+$	0.0	0.0	0.1	0.0	172.1

The Ω_{bbc} baryons

TABLE X. The radiative decay widths of the Ω_{bcc} baryons below 8.58 GeV and the Ω_{bbc} baryons below 11.72 GeV in units of keV. All unlisted decay widths for these states are below 0.05 keV.

(L_t, S)	Initial state	$\Omega_{bcc}(8017)\ 1/2^+\gamma$	$\Omega_{bcc}(8030)\ 3/2^+\gamma$
$(0, \frac{1}{2})$	$\Omega_{bcc}(8463)\ 1/2^+$	0.2	0.0
	$\Omega_{bcc}(8564)\ 1/2^+$	0.8	1.1
$(0, \frac{3}{2})$	$\Omega_{bcc}(8469)\ 3/2^+$	0.2	0.2
$(2, \frac{1}{2})$	$\Omega_{bcc}(8549)\ 3/2^+$	22.9	15.0
	$\Omega_{bcc}(8548)\ 5/2^+$	19.1	6.0
$(2, \frac{3}{2})$	$\Omega_{bcc}(8546)\ 1/2^+$	0.0	31.0
	$\Omega_{bcc}(8545)\ 3/2^+$	12.7	15.0
	$\Omega_{bcc}(8550)\ 5/2^+$	4.7	22.6
	$\Omega_{bcc}(8552)\ 7/2^+$	0.0	27.7
$(1, \frac{1}{2})$	$\Omega_{bcc}(8319)\ 1/2^-$	141.5	37.2
	$\Omega_{bcc}(8322)\ 3/2^-$	135.9	24.2
$(1, \frac{3}{2})$	$\Omega_{bcc}(8310)\ 1/2^-$	32.4	122.6
	$\Omega_{bcc}(8323)\ 3/2^-$	19.3	134.8
	$\Omega_{bcc}(8330)\ 5/2^-$	0.1	150.9
$(1, \frac{1}{2})$	$\Omega_{bcc}(8389)\ 1/2^-$	1.9	1.9
	$\Omega_{bcc}(8397)\ 3/2^-$	2.9	6.1
	$\Omega_{bcc}(8545)\ 3/2^+ \rightarrow \Omega_{bcc}(8389)\ 1/2^-\gamma$		0.5
	$\Omega_{bcc}(8546)\ 1/2^+ \rightarrow \Omega_{bcc}(8389)\ 1/2^-\gamma$		0.2
	$\Omega_{bcc}(8564)\ 1/2^+ \rightarrow \Omega_{bcc}(8389)\ 1/2^-\gamma$		13.6
	$\Omega_{bcc}(8546)\ 1/2^+ \rightarrow \Omega_{bcc}(8397)\ 3/2^-\gamma$		0.2
	$\Omega_{bcc}(8550)\ 5/2^+ \rightarrow \Omega_{bcc}(8397)\ 3/2^-\gamma$		0.1
	$\Omega_{bcc}(8564)\ 1/2^+ \rightarrow \Omega_{bcc}(8397)\ 3/2^-\gamma$		25.6
	$\Omega_{bbc}(11700)\ 3/2^+ \rightarrow \Omega_{bbc}(11496)\ 1/2^-\gamma$		0.1

(L_t, S)	Initial state	$\Omega_{bbc}(11204)\ 1/2^+\gamma$	$\Omega_{bbc}(11221)\ 3/2^+\gamma$
$(0, \frac{3}{2})$	$\Omega_{bbc}(11627)\ 3/2^+$	0.1	0.0
$(2, \frac{1}{2})$	$\Omega_{bbc}(11700)\ 3/2^+$	0.7	0.4
	$\Omega_{bbc}(11699)\ 5/2^+$	0.9	0.1
$(2, \frac{3}{2})$	$\Omega_{bbc}(11702)\ 1/2^+$	0.0	1.2
	$\Omega_{bbc}(11708)\ 3/2^+$	0.3	0.7
	$\Omega_{bbc}(11712)\ 5/2^+$	0.1	0.7
	$\Omega_{bbc}(11715)\ 7/2^+$	0.0	0.8
$(1, \frac{1}{2})$	$\Omega_{bbc}(11571)\ 1/2^-$	131.7	12.8
	$\Omega_{bbc}(11578)\ 3/2^-$	126.3	65.0
$(1, \frac{3}{2})$	$\Omega_{bbc}(11559)\ 1/2^-$	16.7	147.6
	$\Omega_{bbc}(11576)\ 3/2^-$	45.7	98.5
	$\Omega_{bbc}(11584)\ 5/2^-$	0.6	145.3
$(1, \frac{1}{2})$	$\Omega_{bbc}(11496)\ 1/2^-$	0.3	0.4
	$\Omega_{bbc}(11506)\ 3/2^-$	0.1	0.0
	$\Omega_{bbc}(11627)\ 3/2^+ \rightarrow \Omega_{bbc}(11559)\ 1/2^-\gamma$		0.1
	$\Omega_{bbc}(11621)\ 1/2^+ \rightarrow \Omega_{bbc}(11571)\ 1/2^-\gamma$		0.1
	$\Omega_{bbc}(11700)\ 3/2^+ \rightarrow \Omega_{bbc}(11571)\ 1/2^-\gamma$		0.1
	$\Omega_{bbc}(11708)\ 3/2^+ \rightarrow \Omega_{bbc}(11571)\ 1/2^-\gamma$		0.1
	$\Omega_{bbc}(11708)\ 3/2^+ \rightarrow \Omega_{bbc}(11576)\ 3/2^-\gamma$		0.1
	$\Omega_{bbc}(11712)\ 5/2^+ \rightarrow \Omega_{bbc}(11576)\ 3/2^-\gamma$		0.2
	$\Omega_{bbc}(11621)\ 1/2^+ \rightarrow \Omega_{bbc}(11578)\ 3/2^-\gamma$		0.1
	$\Omega_{bbc}(11699)\ 5/2^+ \rightarrow \Omega_{bbc}(11578)\ 3/2^-\gamma$		0.1
	$\Omega_{bbc}(11627)\ 3/2^+ \rightarrow \Omega_{bbc}(11584)\ 5/2^-\gamma$		0.1
	$\Omega_{bbc}(11715)\ 7/2^+ \rightarrow \Omega_{bbc}(11584)\ 5/2^-\gamma$		0.2

(L_t, S)	Initial state	$\Omega_{bcc}(8310)\ 1/2^-\gamma$	$\Omega_{bcc}(8319)\ 1/2^-\gamma$	$\Omega_{bcc}(8322)\ 3/2^-\gamma$	$\Omega_{bcc}(8323)\ 3/2^-\gamma$	$\Omega_{bcc}(8330)\ 5/2^-\gamma$
$(0, \frac{1}{2}/\frac{3}{2})$	$\Omega_{bcc}(8463)\ 1/2^+$	5.0	16.0	36.8	6.4	0.0
	$\Omega_{bcc}(8564)\ 1/2^+$	0.2	0.5	0.7	1.2	0.1
	$\Omega_{bcc}(8469)\ 3/2^+$	9.6	2.1	2.8	18.6	32.1
$(2, \frac{1}{2}/\frac{3}{2})$	$\Omega_{bcc}(8546)\ 1/2^+$	146.3	32.3	4.4	29.8	0.0
	$\Omega_{bcc}(8545)\ 3/2^+$	98.1	19.3	0.8	68.3	5.4
	$\Omega_{bcc}(8549)\ 3/2^+$	1.1	129.3	45.9	23.8	5.3
	$\Omega_{bcc}(8548)\ 5/2^+$	0.0	0.2	170.4	0.0	11.5
	$\Omega_{bcc}(8550)\ 5/2^+$	0.1	0.0	2.4	133.3	44.2
	$\Omega_{bcc}(8552)\ 7/2^+$	0.0	0.0	0.1	0.0	172.1

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4 SUMMARY

SUMMARY

Within the framework of the nonrelativistic quark model, we systematically investigate the **mass spectra** and **radiative decay widths** of the triply heavy baryons up to D-wave states using the Gaussian expansion method.

- **Ω_{ccc} and Ω_{bbb} baryons:**
  1. The predicted masses for low-lying states agree with lattice QCD results, while those for excited states are generally lowera trend consistent with most potential model predictions.
  2. We establish that $J^P = 1/2^+$ baryons composed of three identical quarks still possess S-wave bound states, correcting a misinterpretation in earlier literature.
- **Ω_{bcc} and Ω_{bbc} baryons:**
  1. Our calculated mass spectra are consistent with lattice results within their respective uncertainties.
  2. We demonstrate the existence of all excitation modes for baryons with two identical quarks, with certain modes exhibiting mixing effects.

For radiative decays, the principal conclusions are:

1. Radiative decay patterns differ fundamentally between baryons with three identical quarks and those with two identical quarks, governed by distinct underlying mechanisms.
2. Our calculated radiative decay widths for Ω_{ccc} and Ω_{bbb} baryons exhibit discrepancies with Ref. [16], likely attributable to its omission of identical particle symmetry considerations.
3. Mixing between the total spin states $S = 1/2$ and $S = 3/2$ plays a crucial role in electric dipole transitions. Consequently, accounting for this mixing is essential for accurate radiative decay width calculations.

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