

# $B$ -meson LCDA and power corrections

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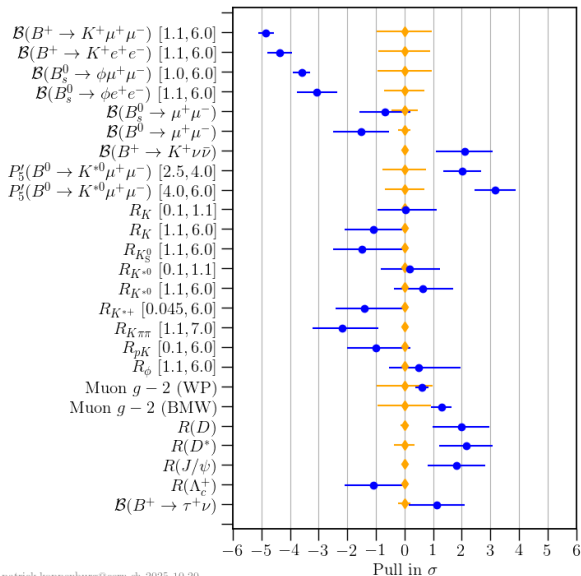
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# B Physics



- Multi-scale problem:

$$m_b > \sqrt{\Lambda m_b} > \Lambda$$

$$B \text{ LCDA} \sim \mathcal{O}(\Lambda)$$

- Power correction:

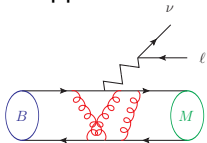
$$\frac{\Lambda}{m_b} \sim \mathcal{O}(20\%)$$





# HQET LCDA of heavy meson

HQET LCDA: appears in exclusive  $B$  decays

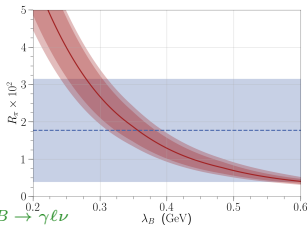


- $B \rightarrow \pi K \dots$
- $B \rightarrow D^{(*)} \ell \nu \dots$
- $B \rightarrow \gamma \ell \nu \dots$

The leading-twist heavy-meson HQET LCDA [Grozin and Neubert, 96']

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 q_s(tn_+) | 0 \rangle = -i\tilde{f}_H(\mu) n_+ \cdot v \int_0^\infty d\omega e^{i\omega tn_+ \cdot v} \varphi_+(\omega)$$

**First inverse momentum**  $\lambda_B \sim \mathcal{O}(\Lambda_{\text{QCD}})$ : NOT well determined



[Belle, 18']:  $B \rightarrow \gamma \ell \nu$



# Study of HQET LCDA

## HQET LCDA: non-perturbative physics

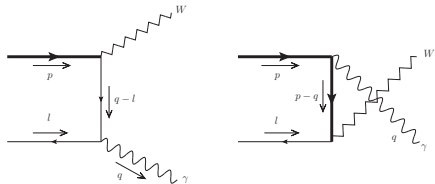
- Inverse moments: [Wang and Shen, 18'], [Beneke, Braun, Ji, and YBW, 18'], [Belle, 18'], [Galda, Neubert and Wang, 22'], [Han, et al., 24']
- EOM [Kawamura, Kodaira, Qiao and Tanaka, 02'], [Lü, Shen, Wang and YBW, 18']
- RG evolution properties: [Braun, Ji and Manashov, 19'], [Galda and Neubert, 20'], [Cui, Shen, Wang and YBW, 23']
- generalized LCDA [Beneke, Böer, Toelstede, and Vos, 22'], [Qin, Shen, Wang and Wang, 22'], [Huang, Ji, Shen, Wang and Wang, 23']

## Determination of $\lambda_B$

- Extract from the experiments: clean process  $B \rightarrow \gamma \nu$
- Calculate with LQCD [LPC collaboration, 22']



$$B \rightarrow \gamma \ell \nu$$

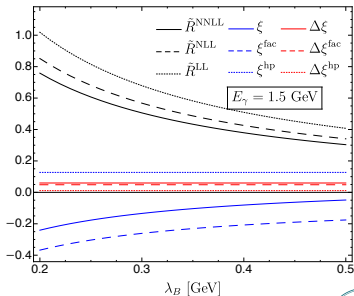


[Wang and Shen, 18'], [Beneke, Braun, Ji, and YBW, 18'], [Galda, Neubert and Wang, 22'], [Cui, Shen, Wang, and YBW, 23']

## Decay form factor

$$F_{V/A}(E_\gamma) = R(E_\gamma) + [\xi(E_\gamma) \pm \Delta\xi(E_\gamma)]$$

- Factorizable contributions: higher-twist  $B$ -meson LCDA, QCD  $\rightarrow$  HQET:  $b$  quark field
- Non-factorizable contributions: hadronic structure of the photon



[Cui, Shen, Wang, and YBW, 23']



$$B \rightarrow \gamma \ell \nu$$

Exclusive  $B$  decays:  $B$ -meson HQET decay constant  $\tilde{f}_B$ :

$$\tilde{f}_B(\mu) = K^{-1}(\mu) f_B$$

QCD decay constant  $f_B$  calculated with LQCD [FLAG].



$$B \rightarrow \gamma \ell \nu$$

Exclusive  $B$  decays:  $B$ -meson HQET decay constant  $\tilde{f}_B$ :

$$\tilde{f}_B(\mu) = K^{-1}(\mu) f_B$$

QCD decay constant  $f_B$  calculated with LQCD [FLAG].

The relation receive  
power corrections  
[Becher, Neubert and Petrov, 00']

$$f_B = K(\mu) \tilde{f}_B(\mu) + \text{NLP}$$

#### Two loop renormalization of heavy light currents at order $1/m(Q)$ in the heavy quark expansion

Thomas Becher (Cornell U., LNS), Matthias Neubert (Cornell U., LNS), Alexey A. Petrov (Cornell U., LNS)  
Dec, 2000

16 pages

Published in: *Nucl.Phys.B* 611 (2001) 367-382

e-Print: [hep-ph/0012183](https://arxiv.org/abs/hep-ph/0012183) [hep-ph]

DOI: [10.1016/S0550-3213\(01\)00341-8](https://doi.org/10.1016/S0550-3213(01)00341-8)

Report number: CLNS-00-1713

View in: [ADS Abstract Service](#)

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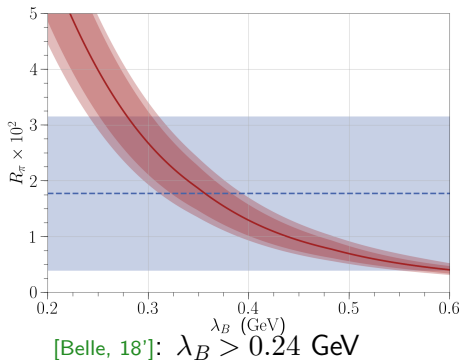
$$B \rightarrow \gamma \ell \nu$$

Power correction in  $f_B$

$\Rightarrow$  20% increase to  $F_{V/A}$

$\Rightarrow$  40% increase to decay width

[Liu, Su and YBW, in progress]



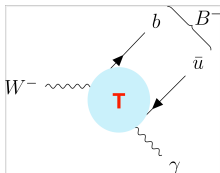
Also affect the exclusive extraction of  $|V_{ub}|$  and  $|V_{cb}|$ .



$$W \rightarrow B\gamma$$

Rare decay  $W^- \rightarrow B^- \gamma$ :  $B$ -meson QCD LCDA

[Grossman, König and Neubert, 15']



Match from QCD LCDA to HQET LCDA [Ishaq, Jia, Xiong and Yang, 19'],

[Zhao, 19'], [Beneke, Finauri, Vos and YBW, 23'], [Deng, Wang, YBW and Zeng, 24']

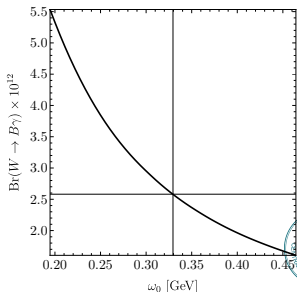
$$\phi_{\text{QCD}}(u) = \mathcal{J}(u, \omega) \otimes \varphi_{\text{HQET}}(\omega)$$

Our result: with  $\ln \Lambda_{\text{QCD}}/m_Q$  resummation

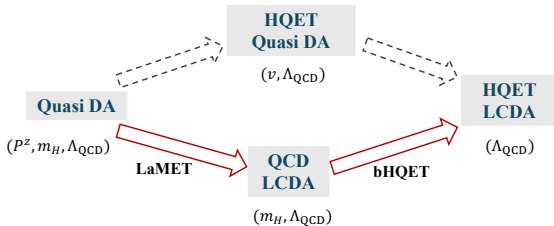
[Beneke, Finauri, Vos and YBW, 23']

$$\text{Br} = (2.58 \pm 0.21_{\text{in}} \begin{matrix} +0.05 \\ -0.08 \end{matrix} \begin{matrix} +2.95 \\ -0.98 \end{matrix} \mu_h \lambda_B) \cdot 10^{-12}$$

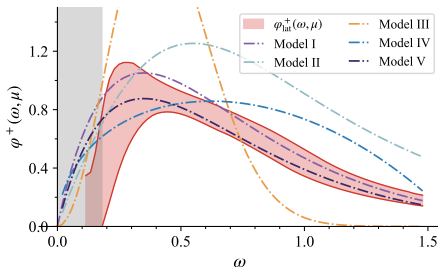
30% increase



LaMET [Ji, 13']



From QCD LCDA to HQET LCDA [Beneke, Finauri, Vos and YBW, 23']



First inverse moment [Han, et al. 24']  
 $\lambda_B = 0.376(63) \text{ GeV}$



# Indirect study of HQET LCDA

**Indirect determination:**  $\lambda_B$

From **QCD LCDA** to **HQET LCDA** [Ishaq, Jia, Xiong and Yang, 19'], [Zhao, 19'],  
[Beneke, Finauri, Vos and **YBW**, 23'], [Deng, Wang, **YBW** and Zeng, 24']

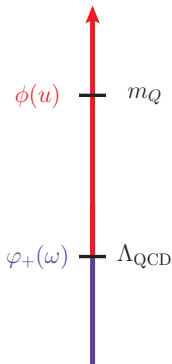
$$\phi(u) = \mathcal{J}(u, \omega) \otimes \varphi_+(\omega)$$

The heavy meson **QCD LCDA** [Braun and Filyanov, 89']

$$\langle H(p_H) | \bar{Q}(0) \not{n}_+ \gamma^5 q(tn_+) | 0 \rangle = -i f_H n_+ \cdot p_H \int_0^1 du e^{iutn_+ \cdot p_H} \phi(u)$$

The leading-twist heavy-meson **HQET LCDA** [Grozin and Neubert, 96']

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 q_s(tn_+) | 0 \rangle = -i \tilde{f}_H(\mu) n_+ \cdot v \int_0^\infty d\omega e^{i\omega tn_+ \cdot v} \varphi_+(\omega)$$



# Systematic power counting

QCD LCDA  $\Rightarrow$   $B$ -meson production: **highly boosted**

※① **Systematic power counting**: boosted frame

The LP operators

$$\hat{\mathcal{O}}_k = \frac{1}{n_+ v} \sqrt{\frac{n_+ v}{2}} \bar{h}_n \not{v}_+ \left( n_+ v \frac{i \not{D}_\perp}{i n_+ D} \right)^k \gamma^5 \xi_{sc}$$

Only operators  $\hat{\mathcal{O}}_0$  and  $\hat{\mathcal{O}}_1$  will appear at LP [Beneke, Finauri, Vos and **YBW**, 23']

For heavy particle with large energy

- **Top-quark** production [Fleming, Hoang, Mantry and Stewart, 07']
- $B$ -meson leptonic decay with  $\tau$  **lepton** [Huang, Shen, Zhao and Zhou, 23'], [Shen, Wang, **YBW** and Zhou, in progress]



# Factorization of the QCD LCDA

※② **Cut  $\delta$**  to QCD LCDA: avoid end-point singularity

The factorization formula

$$\phi(u) = \begin{cases} u \sim \delta: & \mathcal{J}_p(u, \omega) \otimes \varphi_+(\omega), \\ u \sim 1: & \mathcal{J}_{\text{tail}}(u), \end{cases} \quad \begin{aligned} \mathcal{O}_{\text{QCD}} &= \mathcal{J}_p \otimes \mathcal{O}_0, \\ \mathcal{O}_{\text{QCD}} &= \mathcal{J}_0 \mathcal{O}_0 + \mathcal{J}_1 \mathcal{O}_1, \end{aligned}$$

**End-point singularity:** refactorization-based subtraction (RBS)

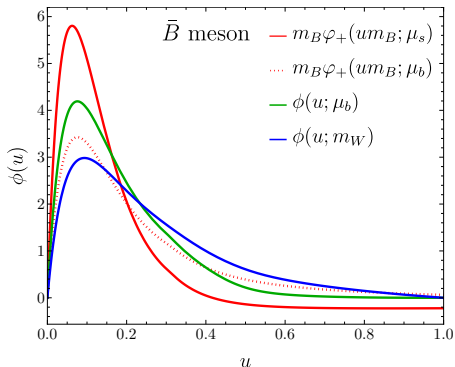
- NLP in  $h \rightarrow \gamma\gamma$ : [Liu and Neubert, 19']
- Off-diagonal parton scattering, NLP in gluon thrust: [Beneke et. al., 20', 22']
- QED correction to  $B \rightarrow \mu^+ \nu_\mu$ : [Cornella, König and Neubert, 22', 26']



# Evolution of the LCDAs: $\varphi_+$ and $\phi$

$\phi(m_Q) \rightarrow \phi(Q)$ :  
evolve of the Gegenbauer moments

$$\frac{a_n(\mu_h)}{a_n(\mu)} = \left( \frac{\alpha_s(\mu_h)}{\alpha_s(\mu)} \right)^{\frac{\gamma_n}{2\beta_0}}$$



# Heavy-quark spin symmetry

**Heavy-quark symmetry** [Grozin and Neubert, 97']: relates LCDAs of P and V heavy mesons

Factorization formulas

$$\phi_i(u) = \frac{\tilde{f}_H}{f_i} m_H \mathcal{J}_{\text{peak}}^i(m_H) \varphi_+(um_H), \quad i = P, \parallel, \perp$$

**EOM of the quarks, method of region** [Deng, Wang, **YBW** and Zeng, 24']

$$\mathcal{J}_{\text{peak}}^i(u, \omega) = 1 + \frac{\alpha_s C_F}{4\pi} \left( \frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2 \right) + \mathcal{O}(\alpha_s^2)$$

**Estimate NLP corrections**



# Summary

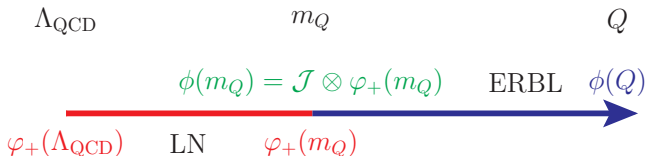
- ✧ **Power correction** in  $f_B$  matching and application to  $B \rightarrow \gamma l \nu$ 
  - 40% increase of decay width
  
- ✧ Match QCD LCDA to the HQET LCDA
  - Application in  $W \rightarrow B \gamma$  decay
  - LaMET quasi DA:  $\lambda_B = 0.376(63)$  GeV

**Thank you!**



# Evolution of the LCDAs: $\varphi_+$ and $\phi$

Two-step evolutions:  $\Lambda_{\text{QCD}} \rightarrow m_Q \rightarrow Q$



$\phi(m_Q) \rightarrow \phi(Q)$ :  
evolve of the Gegenbauer moments

$$\frac{a_n(\mu_h)}{a_n(\mu)} = \left( \frac{\alpha_s(\mu_h)}{\alpha_s(\mu)} \right)^{\frac{\gamma_n}{2\beta_0}}$$

