



第五届强子与重味物理理论与实验联合研讨会

Investigation of the $D^0 \rightarrow K_S^0 \pi^0 \eta$ and $K_S^0 \pi^0 \pi^0$ decays

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arXiv: 2603.16082

2026.3. 石家庄



Outline

1. Introduction
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§ 1. Introduction

$a_0(980)$

$$I^G(J^{PC}) = 1^-(0^{++})$$



VALUE (MeV)

980 ± 20 OUR ESTIMATE

$\eta\pi$ FINAL STATE ONLY

| VALUE (MeV) | EVTS | DOCUMENT ID | TECN | CHG | COMMENT |
|--------------------------------------|------|---------------------|------|-----|--|
| <u>50 to 100 OUR ESTIMATE</u> | | Width determination | | | very model dependent. Peak width in $\eta\pi$ is about 60 MeV, but decay width can be much larger. |



- A. Astier, L. Montanet, M. Baubillier and J. Duboc, Phys. Lett. B 25, 294 (1967).
- R. Ammar et al., Phys. Rev. Lett. 21, 1832 (1968).
- C. Defoix, P. Rivet, J. Siaud, B. Conforto, M. Widgoff and F. Shively, Phys. Lett. B 28, 353 (1968).



$f_0(980)$

$$I^G(J^{PC}) = 0^+(0^{++})$$

See the review on "Scalar Mesons below 1 GeV."

T-matrix pole $\sqrt{s} = (980-1010) - i(20-35)$ MeV [h]

Mass (Breit-Wigner) = 990 ± 20 MeV [h]

Full width (Breit-Wigner) = 10 to 100 MeV [h]

$f_0(500)$

$$I^G(J^{PC}) = 0^+(0^{++})$$

also known as σ ; was $f_0(600)$

See the review on "Scalar Mesons below 1 GeV."

Mass (T-Matrix Pole \sqrt{s}) = $(400-550) - i(200-350)$ MeV

Mass (Breit-Wigner) = 400 to 800 MeV

Full width (Breit-Wigner) = 100 to 800 MeV

$$|a_0(980)\rangle = |us\bar{d}\bar{s}\rangle, \frac{1}{\sqrt{2}}|(u\bar{u} - d\bar{d})s\bar{s}\rangle, |\bar{u}\bar{s}ds\rangle,$$

$$|f_0(980)\rangle = \cos \varphi |s\bar{s}\rangle + \sin \varphi |n\bar{n}\rangle,$$

$$|f_0(980)\rangle = \frac{1}{\sqrt{2}}|(u\bar{u} + d\bar{d})s\bar{s}\rangle,$$

$$|f_0(500)\rangle = -\sin \varphi |s\bar{s}\rangle + \cos \varphi |n\bar{n}\rangle.$$

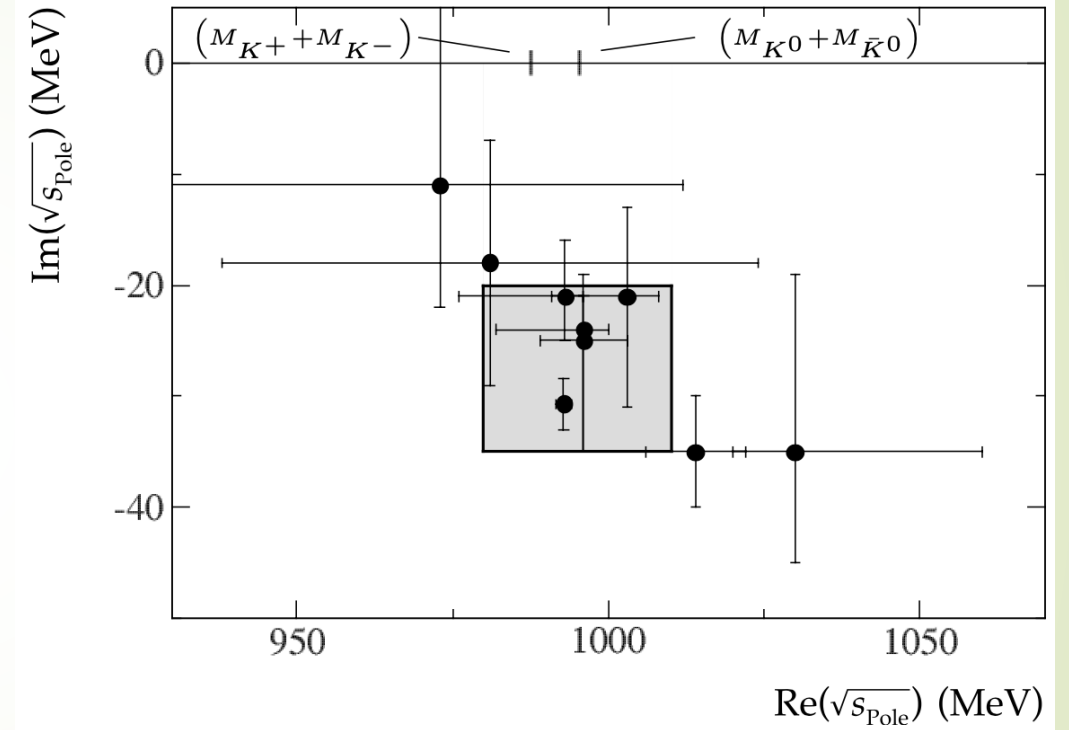
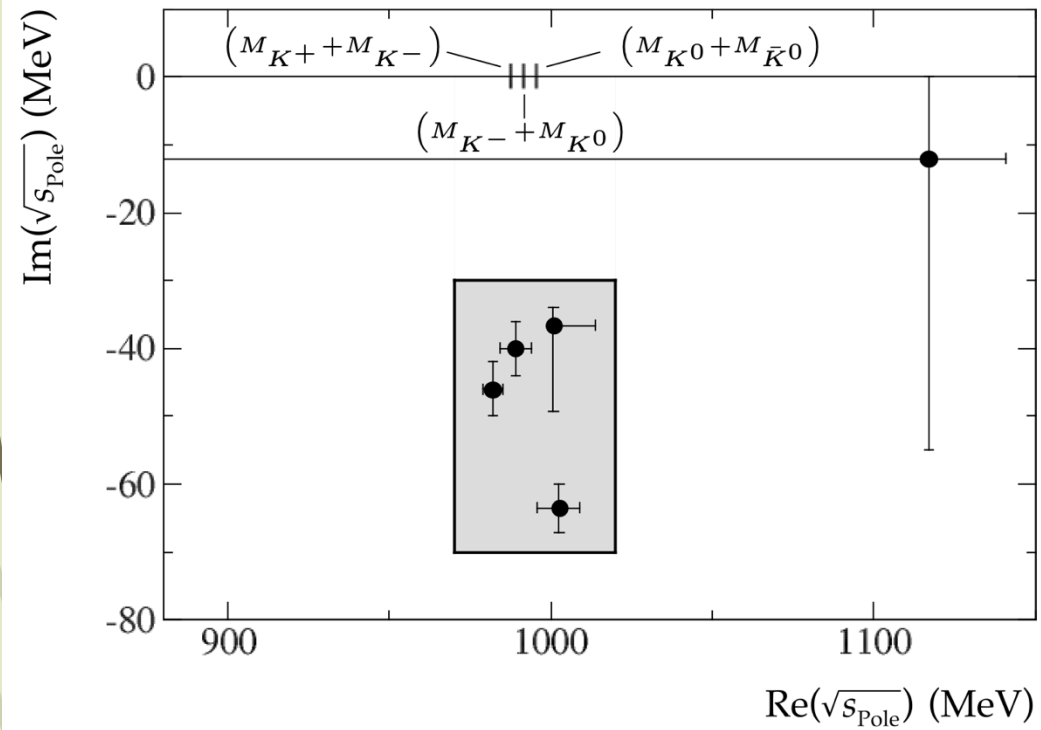
$$|f_0(500)\rangle = |\bar{u}\bar{d}ud\rangle,$$

PDG review

64. Scalar Mesons below 1 GeV



Revised August 2023 by T. Gutsche (Tübingen U.), C. Hanhart (FZ Jülich), R.E. Mitchell (Indiana U.) and S. Spanier (Tennessee U.).



$$\sqrt{s_{\text{Pole}}^{a_0(980)}} = (970 - 1020) - i(30 - 70) \text{ MeV}$$

$$\sqrt{s_{\text{Pole}}^{f_0(980)}} = (980 - 1010) - i(20 - 35) \text{ MeV}$$

$$\sqrt{s_{\text{Pole}}^\sigma} = (449^{+22}_{-16}) - i(275 \pm 12) \text{ MeV}$$



Conventional scalar $q\bar{q}$ states:

S. Godfrey and N. Isgur, Phys. Rev. D 32, 189-231 (1985).

D. Morgan and M. R. Pennington, Phys. Rev. D 48, 1185-1204 (1993).

N. A. Tornqvist and M. Roos, Phys. Rev. Lett. 76, 1575-1578 (1996).

Tetraquarks, composed of four quarks $qq\bar{q}\bar{q}$ states:

R. L. Jaffe, Phys. Rev. D 15, 267 (1977).

J. D. Weinstein and N. Isgur, Phys. Rev. Lett. 48, 659 (1982).

N. N. Achasov, Nucl. Phys. A 728, 425-438 (2003).

Molecular state, composed of $K\bar{K}$:

B. S. Zou and D. V. Bugg, Phys. Rev. D 50, 591-594 (1994).

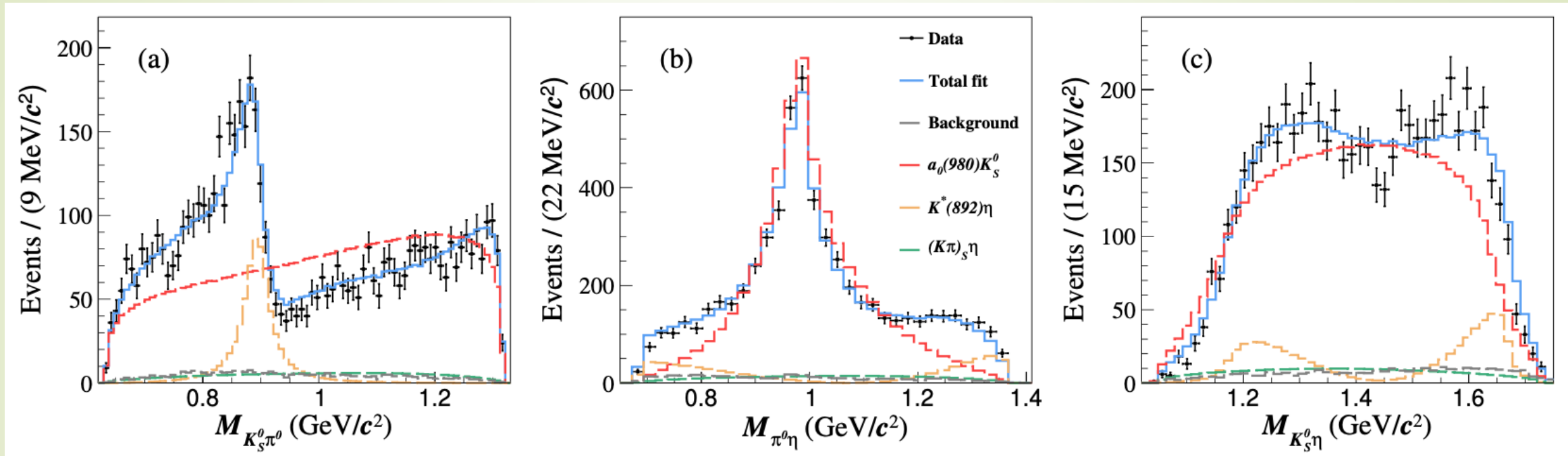
G. Janssen, B. C. Pearce, K. Holinde and J. Speth, Phys. Rev. D 52, 2690-2700 (1995).

J. A. Oller and E. Oset, Nucl. Phys. A 620, 438-456 (1997) [erratum: Nucl. Phys. A 652, 407-409 (1999)].

M. P. Locher, V. E. Markushin and H. Q. Zheng, Eur. Phys. J. C 4, 317-326 (1998).

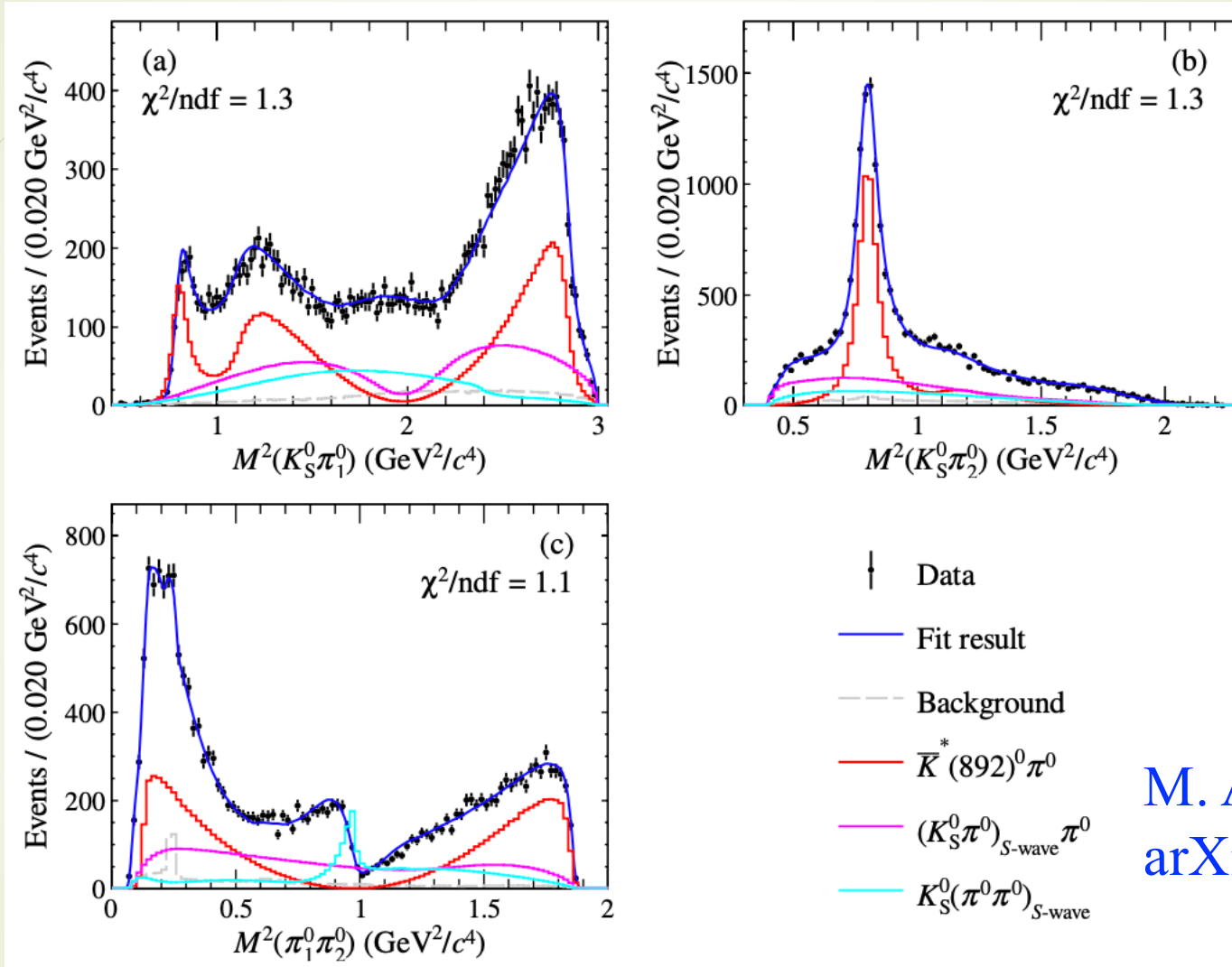
J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. Lett. 80, 3452-3455 (1998).

$$D^0 \rightarrow K_S^0 \pi^0 \eta$$



| Amplitude | Phase ϕ (rad) | FF (%) | BF (10^{-3}) |
|---|---------------------------|------------------------|--------------------------|
| $D^0 \rightarrow K_S^0 a_0(980)^0, a_0(980)^0 \rightarrow \pi^0 \eta$ | 0.0(fixed) | $93.2 \pm 3.3 \pm 3.2$ | $9.88 \pm 0.37 \pm 0.42$ |
| $D^0 \rightarrow K_S^0 a_2(1320)^0, a_2(1320)^0 \rightarrow \pi^0 \eta$ | $-5.41 \pm 0.24 \pm 0.22$ | $0.8 \pm 0.1 \pm 0.1$ | $0.08 \pm 0.11 \pm 0.11$ |
| $D^0 \rightarrow \bar{K}^*(892)^0 \eta, \bar{K}^*(892)^0 \rightarrow K_S^0 \pi^0$ | $-2.70 \pm 0.09 \pm 0.06$ | $11.5 \pm 0.8 \pm 1.0$ | $1.22 \pm 0.09 \pm 0.11$ |

$$D^0 \rightarrow K_S^0 \pi^0 \pi^0$$



M. Ablikim et al. [BESIII],
arXiv: 2510.25111

What can we learn from the invariant mass distributions of the experimental measurement?

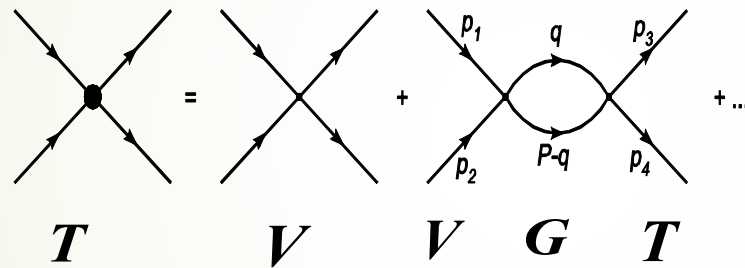


§2. Formalism

(1) Coupled channel interaction from the chiral unitary approach

► **Chiral Unitary Approach**: solving Bethe-Salpeter equations, which take on-shell approximation for the loops.

$$T = V + V G T, T = [1 - V G]^{-1} V$$



D. L. Yao, L. Y. Dai, H. Q. Zheng and Z. Y. Zhou, Rept. Prog. Phys. 84, 076201 (2021)

where **V matrix (potentials)** can be evaluated from the interaction Lagrangians.

$I = 0$ sector

$$\pi^+ \pi^-, \pi^0 \pi^0, K^+ K^-, K^0 \bar{K}^0, \eta \eta$$

$I = 1$ sector

$$K^+ K^-, K^0 \bar{K}^0, \pi^0 \eta$$

$I = 1/2$ sector

$$K^+ \pi^-, K^0 \pi^0, K^0 \eta$$

J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438

E. Oset and A. Ramos, Nucl. Phys. A 635 (1998) 99

J. A. Oller and U. G. Meißner, Phys. Lett. B 500 (2001) 263



G is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(P-q)^2 - m_{l1}^2 + i\varepsilon} \frac{1}{q^2 - m_{l2}^2 + i\varepsilon}$$

The coupled channel scattering amplitudes **T matrix satisfy the unitary** :

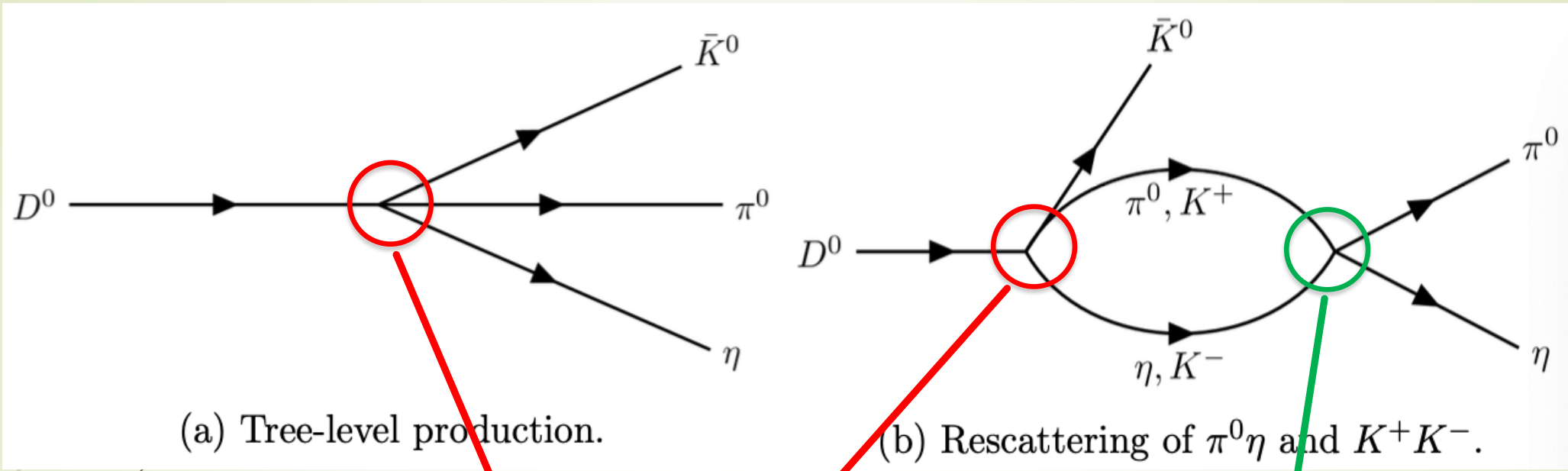
$$\text{Im } T_{ij} = T_{in} \sigma_{nn} T_{nj}^*$$

$$\sigma_{nn} \equiv \text{Im } G_{nn} = - \frac{q_{cm}}{8\pi\sqrt{s}} \theta(s - (m_1 + m_2)^2)$$

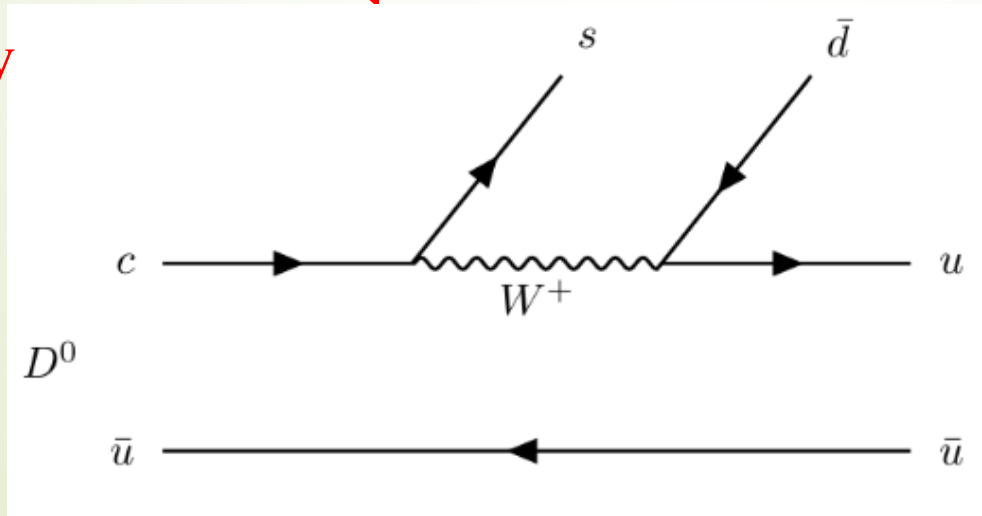
To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G_{ll}^{II}(s) = G_{ll}^I(s) + i \frac{q_{cm}}{4\pi\sqrt{s}}$$

(2) Final state interaction



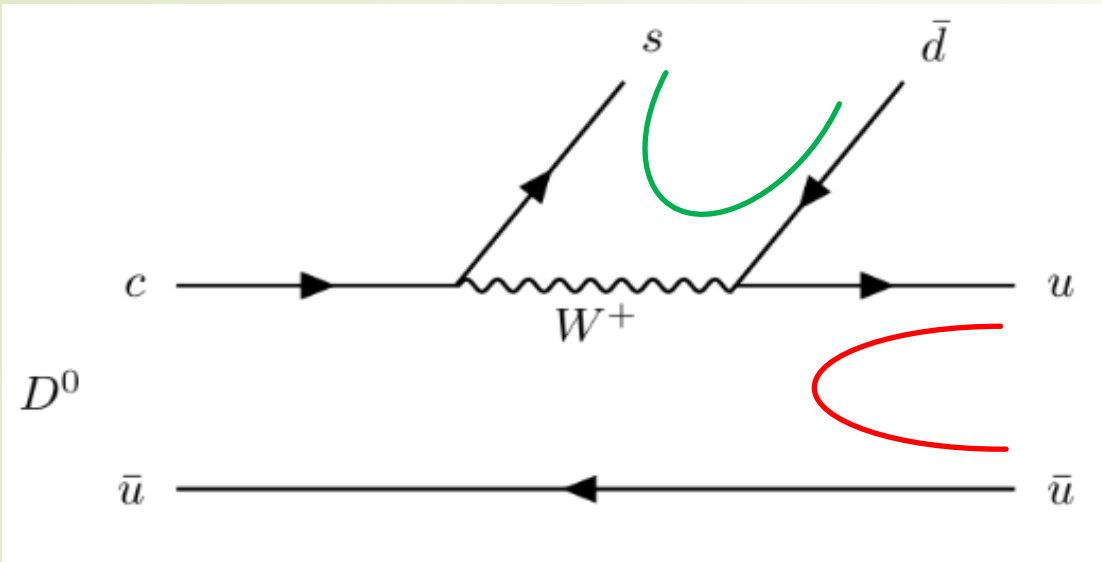
The weak decay process at the quark level



S-wave

$$T = [1 - VG]^{-1}V$$

The final state interaction at the hadron level



$$\underline{u(\bar{u}u + \bar{d}d + \bar{s}s)\bar{u} = (M \cdot M)_{11},}$$

$$\underline{s(\bar{u}u + \bar{d}d + \bar{s}s)\bar{d} = (M \cdot M)_{32}.}$$

$$H = V_p V_{cs} V_{ud} \left[\underline{(s\bar{d} \rightarrow \bar{K}^0) [M_{11} \rightarrow (M \cdot M)_{11}]} + \left(u\bar{u} \rightarrow \frac{1}{\sqrt{2}}\pi^0 \right) [M_{32} \rightarrow (M \cdot M)_{32}] \right. \\ \left. + \left(u\bar{u} \rightarrow \frac{1}{\sqrt{3}}\eta \right) [M_{32} \rightarrow (M \cdot M)_{32}] \right]$$



$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$



$$(M \cdot M)_{11} = \frac{1}{2}\pi^0\pi^0 + \frac{1}{3}\eta\eta + \frac{2}{\sqrt{6}}\pi^0\eta + \pi^+\pi^- + K^+K^-,$$

$$(M \cdot M)_{32} = K^-\pi^+ - \frac{1}{\sqrt{2}}\bar{K}^0\pi^0,$$



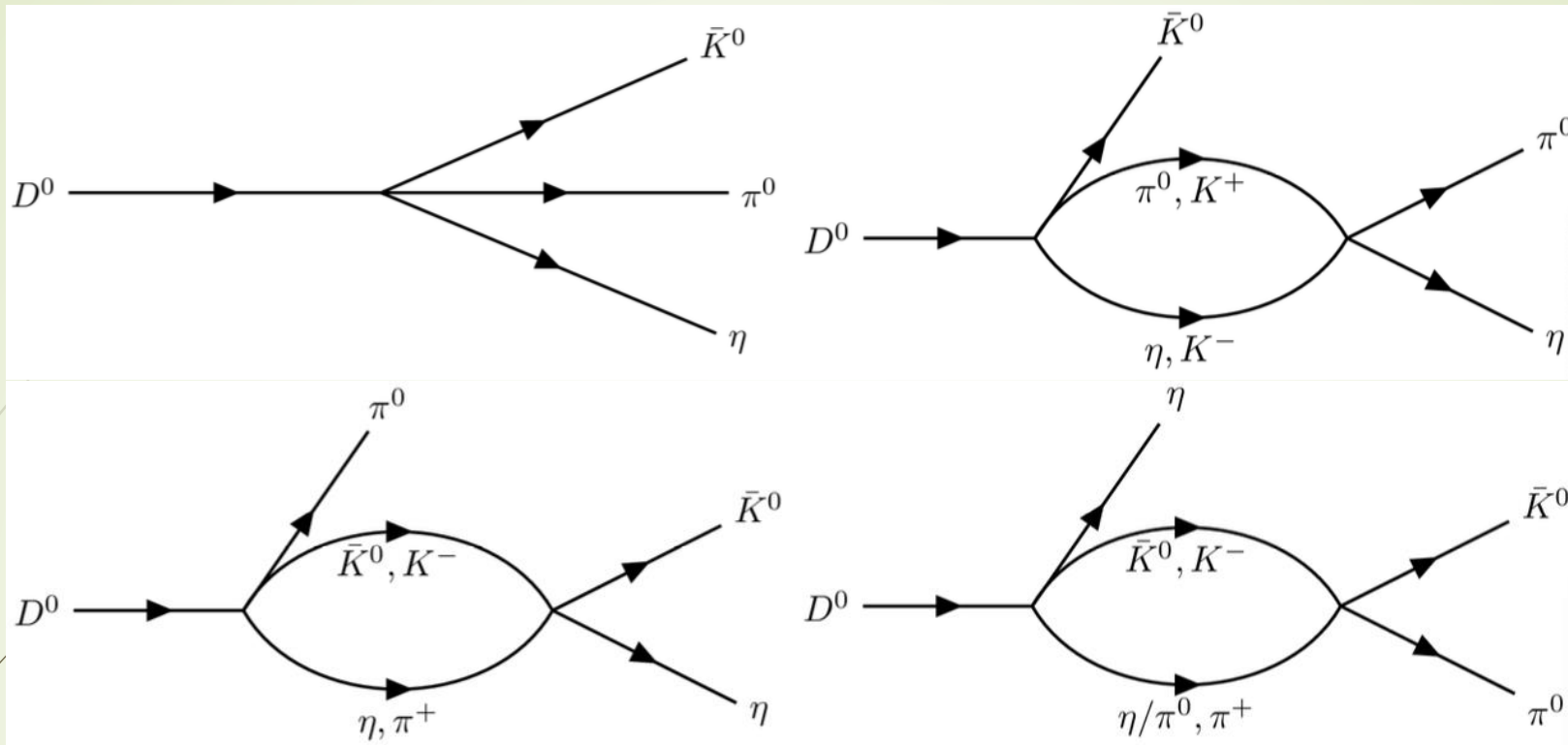
$$H = V_p V_{cs} V_{ud} \left[(s\bar{d} \rightarrow \bar{K}^0) [M_{11} \rightarrow (M \cdot M)_{11}] + \left(u\bar{u} \rightarrow \frac{1}{\sqrt{2}}\pi^0 \right) [M_{32} \rightarrow (M \cdot M)_{32}] \right. \\ \left. + \left(u\bar{u} \rightarrow \frac{1}{\sqrt{3}}\eta \right) [M_{32} \rightarrow (M \cdot M)_{32}] \right]$$

$$D^0 \rightarrow K_S^0 \pi^0 \eta$$

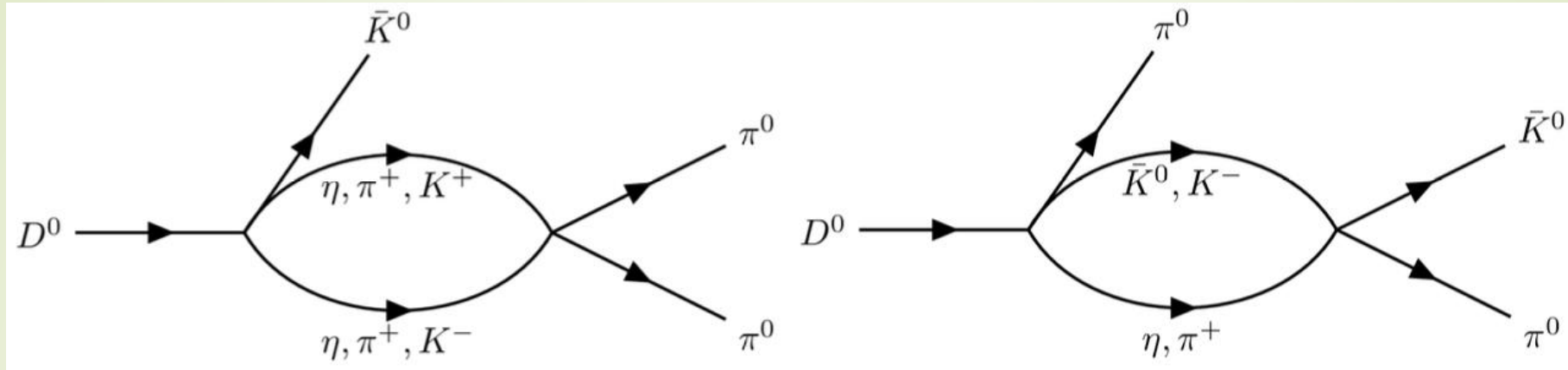
$$= V_p V_{cs} V_{ud} \left[\frac{1}{3}\bar{K}^0\eta\eta + \frac{1}{\sqrt{6}}\bar{K}^0\pi^0\eta + \pi^+\pi^-\bar{K}^0 + K^+K^-\bar{K}^0 + \frac{1}{\sqrt{2}}\pi^0 K^-\pi^+ + \frac{1}{\sqrt{3}}\eta K^-\pi^+ \right]$$

$$D^0 \rightarrow K_S^0 \pi^0 \pi^0$$

$$= C_1 \left[\frac{1}{3}\bar{K}^0\eta\eta + \frac{1}{\sqrt{6}}\bar{K}^0\pi^0\eta + \pi^+\pi^-\bar{K}^0 + K^+K^-\bar{K}^0 + \frac{1}{\sqrt{2}}\pi^0 K^-\pi^+ + \frac{1}{\sqrt{3}}\eta K^-\pi^+ \right],$$

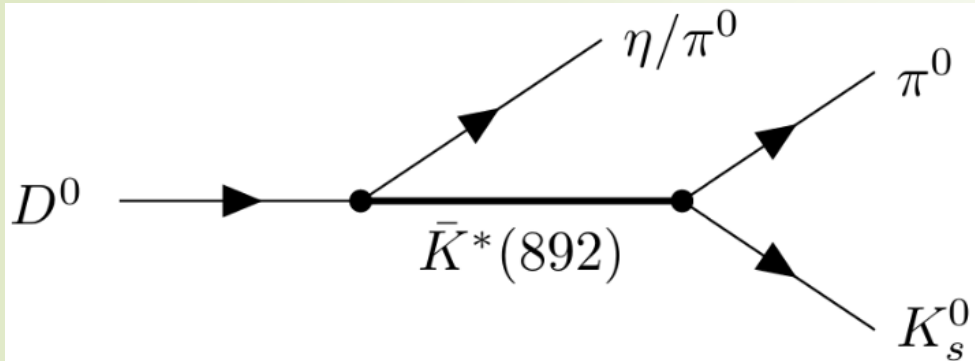


$$\begin{aligned}
 t(s_{12}, s_{23})_{D^0 \rightarrow \bar{K}^0 \pi^0 \eta} &= \frac{1}{\sqrt{6}} C_1 + \frac{1}{\sqrt{6}} C_1 G_{\pi^0 \eta}(s_{23}) T_{\pi^0 \eta \rightarrow \pi^0 \eta}(s_{23}) + C_1 G_{K^+ K^-}(s_{23}) T_{K^+ K^- \rightarrow \pi^0 \eta}(s_{23}) \\
 &+ \frac{1}{\sqrt{6}} C_1 G_{\bar{K}^0 \eta}(s_{23}) T_{\bar{K}^0 \eta \rightarrow \bar{K}^0 \eta}(s_{13}) + \frac{1}{\sqrt{2}} C_1 G_{K^- \pi^+}(s_{13}) T_{K^- \pi^+ \rightarrow \bar{K}^0 \eta}(s_{13}) \\
 &+ \frac{1}{3} C_1 G_{\bar{K}^0 \eta}(s_{12}) T_{\bar{K}^0 \eta \rightarrow \bar{K}^0 \pi^0}(s_{12}) + \frac{1}{\sqrt{6}} C_1 G_{\bar{K}^0 \pi^0}(s_{12}) T_{\bar{K}^0 \pi^0 \rightarrow \bar{K}^0 \pi^0}(s_{12}) \\
 &+ \frac{1}{\sqrt{3}} C_1 G_{K^- \pi^+}(s_{12}) T_{K^- \pi^+ \rightarrow \bar{K}^0 \pi^0}(s_{12}),
 \end{aligned}$$



$$\begin{aligned}
 \underline{t}(s_{12}, s_{23})_{D^0 \rightarrow \bar{K}^0 \pi^0 \pi^0} &= \frac{1}{2} \times \frac{1}{3} C_1 G_{\eta\eta}(s_{23}) T_{\eta\eta \rightarrow \pi^0 \pi^0}(s_{23}) + C_1 G_{\pi^+ \pi^-}(s_{23}) T_{\pi^+ \pi^- \rightarrow \pi^0 \pi^0}(s_{23}) \\
 &+ C_1 G_{K^+ K^-}(s_{23}) T_{K^+ K^- \rightarrow \pi^0 \pi^0}(s_{23}) + \frac{1}{\sqrt{6}} C_1 G_{\bar{K}^0 \eta}(s_{12}) T_{\bar{K}^0 \eta \rightarrow \bar{K}^0 \pi^0}(s_{12}) \\
 &+ \frac{1}{\sqrt{2}} C_1 G_{K^- \pi^+}(s_{12}) T_{K^- \pi^+ \rightarrow \bar{K}^0 \pi^0}(s_{12}).
 \end{aligned}$$

(3) Contributions from P-wave



$$\mathcal{M}_{\bar{K}^*(892)}(s_{12}, s_{23}) = \frac{\mathcal{D}e^{i\alpha_{\bar{K}^*(892)}}}{s_{12} - M_{\bar{K}^*(892)}^2 + iM_{\bar{K}^*(892)}\Gamma_{\bar{K}^*(892)}} \times \left[\frac{(m_{D^0}^2 - m_{\eta/\pi^0}^2)(m_{K^0}^2 - m_{\pi^0}^2)}{M_{\bar{K}^*(892)}^2} - s_{13} + s_{23} \right]$$

$$t'(s_{12}, s_{23})_{D^0 \rightarrow K_S^0 \pi^0 \eta} = t(s_{12}, s_{23})_{D^0 \rightarrow K_S^0 \pi^0 \eta} + \mathcal{M}_{\bar{K}^*(892)}(s_{12}, s_{23})$$

$$t'(s_{12}, s_{23})_{D^0 \rightarrow K_S^0 \pi^0 \pi^0} = t(s_{12}, s_{23})_{D^0 \rightarrow K_S^0 \pi^0 \pi^0} + \mathcal{M}_{\bar{K}^*(892)}(s_{12}, s_{23}) + (2 \leftrightarrow 3)$$

$$\frac{d^2\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3} \frac{1}{32m_{D^0}^3} \left| t'(s_{12}, s_{23})_{D^0 \rightarrow K_S^0 \pi^0 \eta} \right|^2,$$



$$\frac{d^2\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3} \frac{1}{32m_{D^0}^3} \frac{1}{2} \left| t'(s_{12}, s_{23})_{D^0 \rightarrow K_S^0 \pi^0 \pi^0} \right|^2,$$

$$G(s)T(s) = G(s_{cut})T(s_{cut})e^{-\alpha(\sqrt{s}-\sqrt{s_{cut}})}, \quad \text{for } \sqrt{s} > \sqrt{s_{cut}}, \quad \sqrt{s_{cut}} = 1.1 \text{ GeV}$$

• free parameters

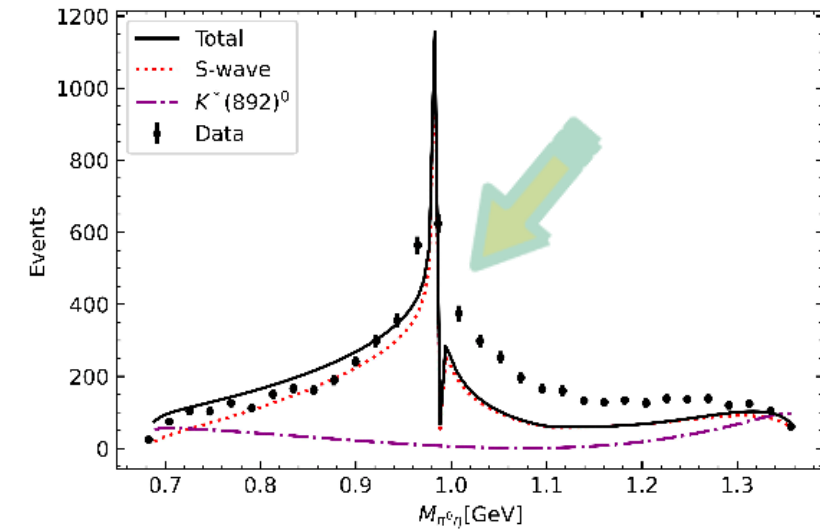
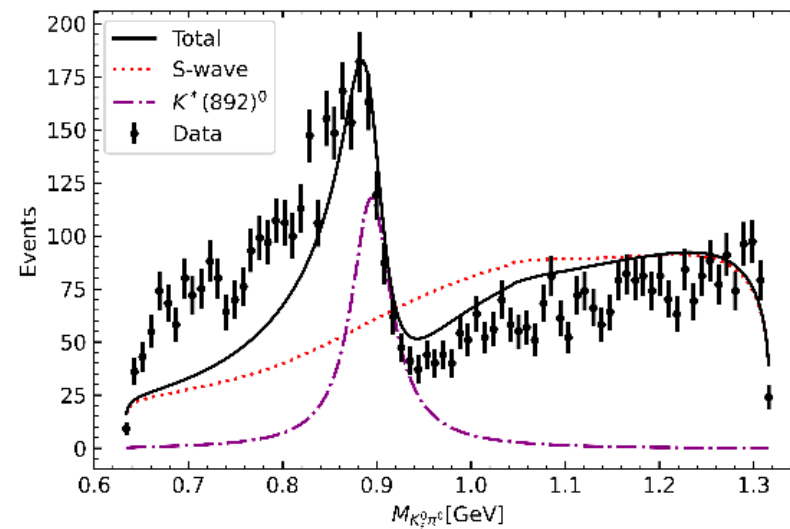
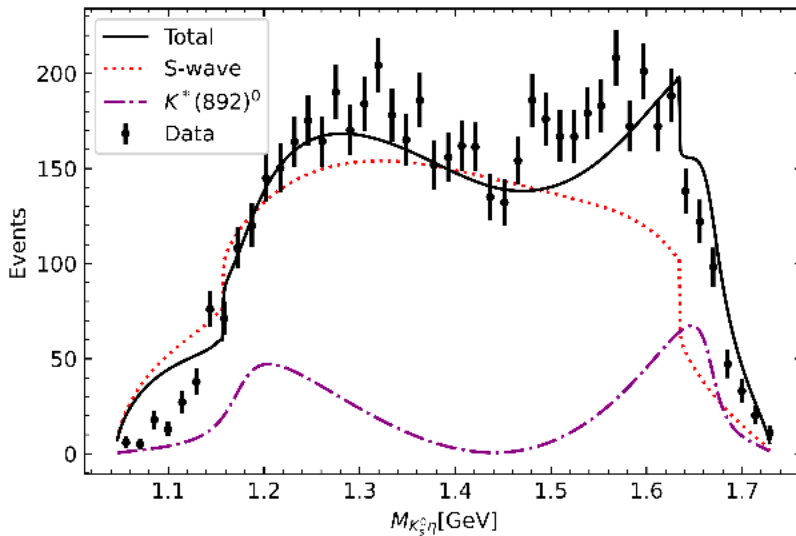
S-wave: $C_1, \alpha,$

P-wave: $D, \alpha_{\bar{K}^*(892)}$

§3. Results

(1) $D^0 \rightarrow K^0_s \pi^0 \eta$ results (BESIII)

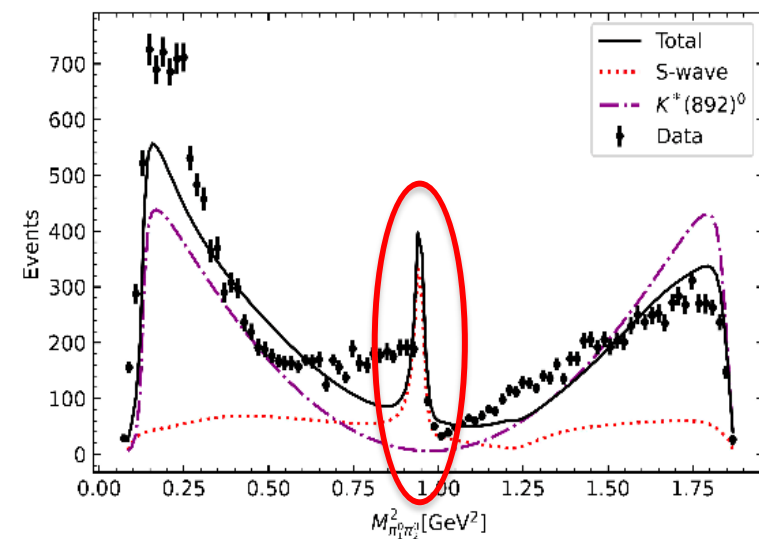
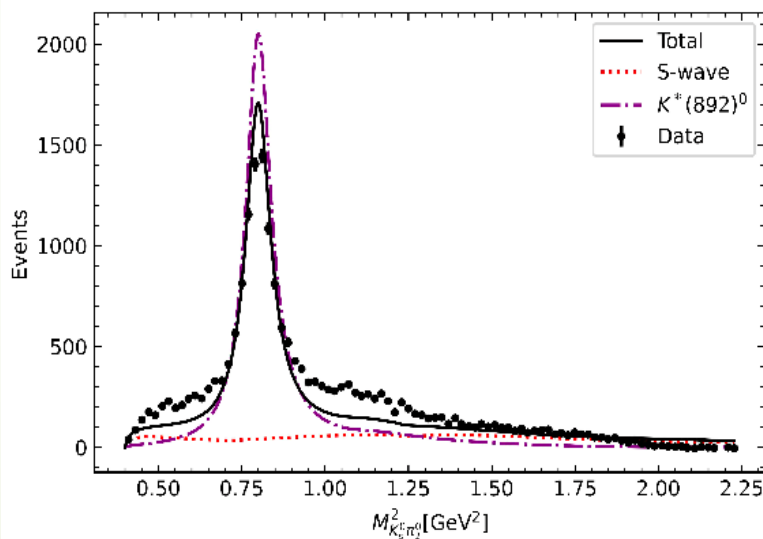
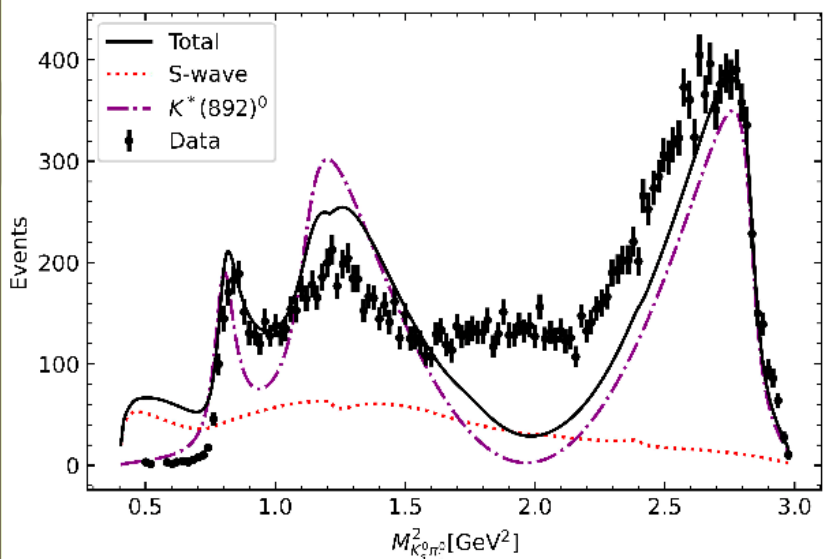
| Parameters | C_1 | α | D | $\alpha_{\bar{K}^*(892)}$ | $\chi^2/dof.$ |
|------------|---------|----------|--------|---------------------------|---------------|
| Fit | 3342.04 | -3.38 | 138.31 | 2.34 | 11.59 |



where we have taken into account the bin sizes

(2) $D^0 \rightarrow K^0_s \pi^0 \pi^0$ results (BESIII)

| Parameters | C_1 | α | D | $\alpha_{\bar{K}^*(892)}$ | D_1 | $\alpha'_{\bar{K}^*(892)}$ | $\chi^2/dof.$ |
|------------|---------|----------|--------|---------------------------|--------|----------------------------|---------------|
| Fit | 6040.13 | 31.36 | 122.10 | 1.09 | 522.41 | 1.47×10^{-2} | 27.94 |





§4. Summary

- We use the chiral unitary approach to dynamically generate the resonances $a_0(980), f_0(980), f_0(500)$
- Taking the data from BESIII, we fit the invariant mass distributions of the decays $D^0 \rightarrow K_S^0 \pi^0 \eta, D^0 \rightarrow K_S^0 \pi^0 \pi^0$ with the final state interaction formalism
- Indicating the molecular nature of these resonances



Thanks for your attention!

感谢大家的聆听！