重味夸克半轻衰变过程 @N^{3}LO 微扰 QCD

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Why Precision Study of Heavy Quark Decays

- Heaviest fundamental particle in SM

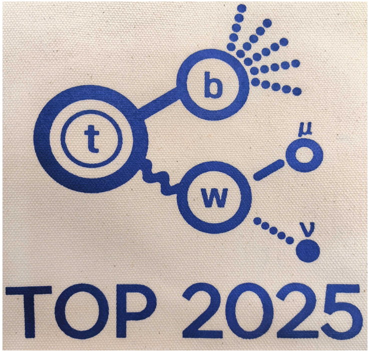
$$\boxed{m_t = 172.56 \pm 0.31\,\text{GeV}}$$

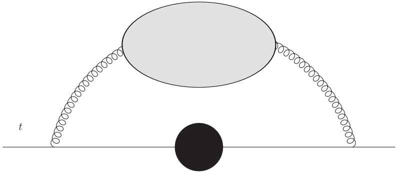
- Precision test of SM Higgs and CKM mechanism, and probes for BSM physics

- Decay exclusively to $b + W$ before hadronization:

$$\boxed{\Gamma_t = 1.4\,\text{GeV} \gg \Lambda_{\text{QCD}}}$$

- Convergence of perturbative QCD series (e.g. renormalon in pole-mass definition)





$B \to X_u \ell \bar{\nu}_\ell$ @ Belle II

- Measurement of $|V_{ub}|$ (and $|V_{cb}|$) at the unprecedented $\mathcal{O}(0.01)$ level
- Clarifying the long-standing discrepancy between the *inclusive* and *exclusive* measurements of $|V_{ub}|$
- Precision determination of the (global) non-perturbative parameters and *shape* functions for B-mesons
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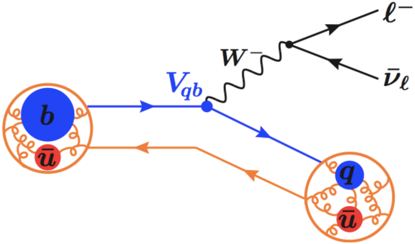


[→See talk by Q.D. Zhou]

	Statistical	Systematic (reducible, irreducible)	Total expt.	Theory	Total
1 ab^{-1}	2.5	(2.9, 1.6)	4.1	2.5 – 4.5	4.8 – 6.1
5 ab^{-1}	1.1	(1.3, 1.6)	2.3	2.5 – 4.5	3.4 – 5.1
50 ab^{-1}	0.4	(0.4, 1.6)	1.7	2.5 – 4.5	3.0 – 4.8

Precise measurements of both **inclusive and differential** observables of $B \to X_u \ell \bar{\nu}_\ell$ are crucial for these goals.

Why Precision Study of Semileptonic D-meson Decay

Why precision study of (inc.) semi-leptonic decay of D-meson? [→See also talks by Li and Qin]

- Test of CKM mechanism, and check our understanding of hadron dynamics

  (e.g. heavy-quark flavor-spin symmetry)

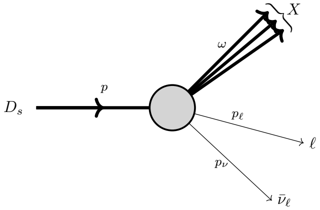
- Interesting interplay with, and complementary to, B-meson physics

  (e.g. shed light to the tension between inc. and excl. values of V_{cb} , V_{ub})

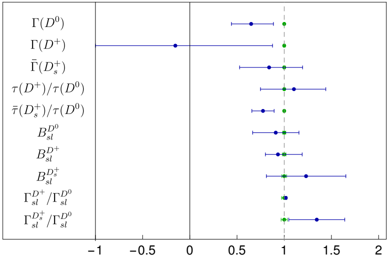
- Unique opportunity and stress-test for both pert. and non-pert. QCD methods @ $m_c \sim 1 \sim 2\,\mathrm{GeV}$

  (e.g. Heavy-Quark Expansion, Lattice-QCD ...)

- Considerable truncation errors and imprecise theory parameters in decays of D^0 , D^+ , D_s^+ to be improved



[De Santis et al. 25]



[King et al. 21]

Semileptonic Decays of $D^0\,, D^+\,, D_s^+$

Experimental precision of the semileptonic Γ_D and electron-energy moments for D_s^+ now reaches the level of a few $(2 \sim 3)$ %

- From CLEO: [Asner et al. 10]

$$\mathcal{B}(D_s^+ \to X e^+ \nu_e) \;=\; (6.52 \pm 0.39 \pm 0.15)\%$$

- From BES-III: [Ablikim et al. 21]

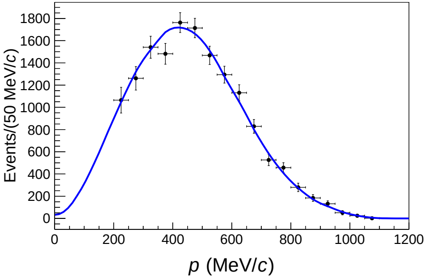
$$\mathcal{B}\left(D_s^+ \to X e^+ \nu_e\right) = (6.30 \pm 0.13\,(\text{stat.}) \pm 0.10\,(\text{syst.}))\,\%$$

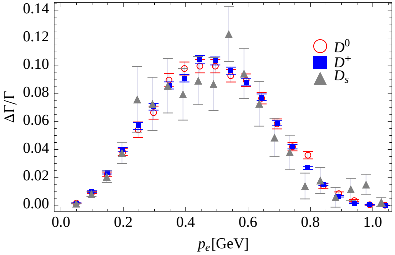
- From Lattice-QCD: [De Santis et al. 25] ([→see talk by Hou])

$$\Gamma = 8.72(56) \times 10^{-14}\,\text{GeV}\;,$$

$$M_1 = 0.456(22)\,\text{GeV}\;,$$

$$M_2 = 0.227(10)\,\text{GeV}^2\;.$$



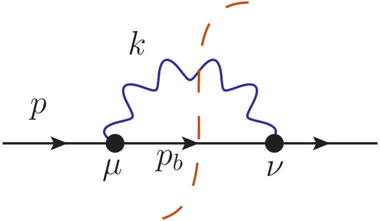


Cut Diagrams for Heavy-to-light Decay Width

Γ_{Q2q} in terms of the **semi-inclusive** $\mathcal{W}^{\mu\nu}_{Q2q}$

$$\Gamma_{Q2q} = \frac{1}{2\, m_Q} \int \frac{\mathrm{d}^{d-1}k}{(2\pi)^{d-1} 2E}\, \mathcal{W}^{\mu\nu}_{Q2q} \sum_{\lambda}^{L,R,0} \varepsilon^*_\mu(k,\lambda)\, \varepsilon_\nu(k,\lambda)\,,$$

$$\left| \mathcal{M}_{Q\to q+W} \right|^2 \Rightarrow$$

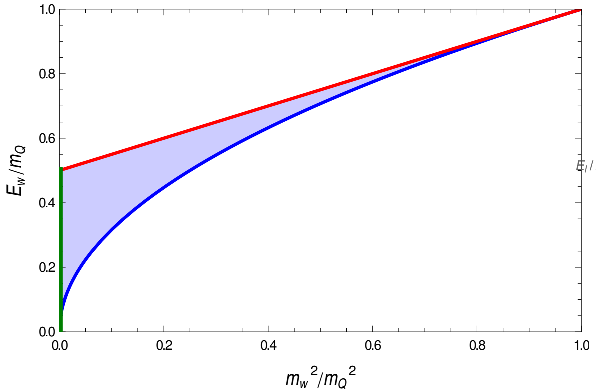


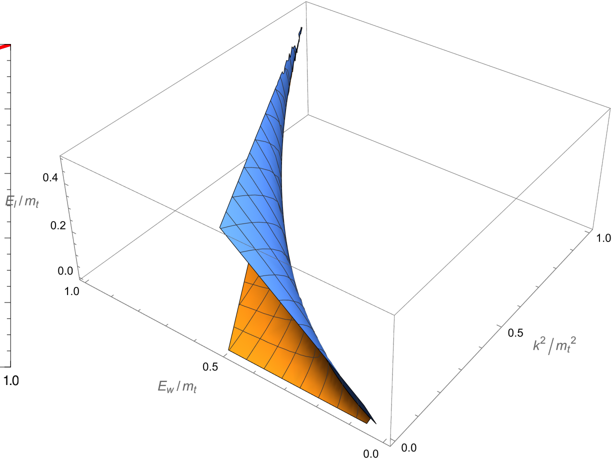
$$\mathcal{W}^{\mu\nu}_{Q2q}(p,k) = W_1\, g^{\mu\nu} + W_2\, p^\mu p^\nu + W_3\, k^\mu k^\nu + W_4 \left(p^\mu k^\nu + k^\mu p^\nu\right) + W_5\, i\epsilon^{\mu\nu\rho\sigma}\, p_\rho\, k_\sigma\,,$$

Selection Criteria: the cut diagrams of t-quark self-energy function with exactly one (cut) W propagator interacting with the *external* t-quark plus (up to 3) QCD loops

Phase-space for Structure functions $W_i(k^2, E_w)$

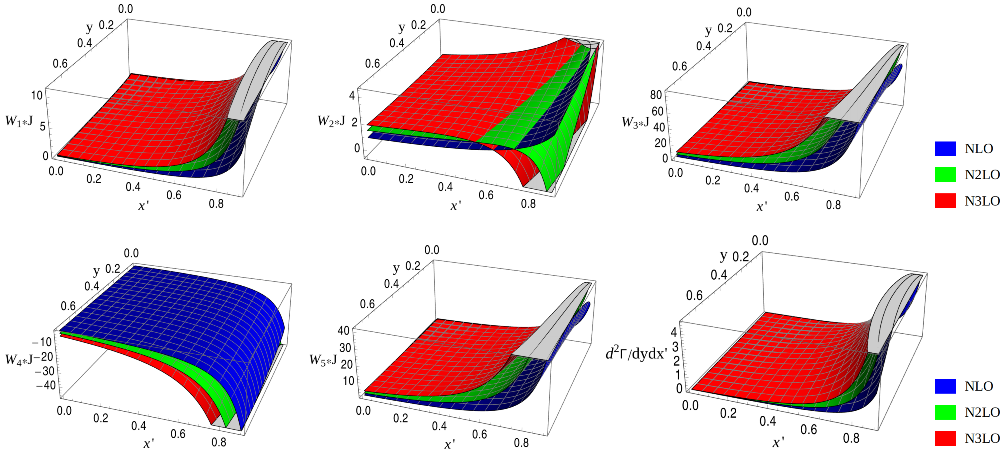
$$\frac{d^3\Gamma_{sl}}{dE_l\, dE_w\, dk^2}\bigg|_{m_W \gg m_q} \propto -k^2 W_1/m_q^2 - 1/2(4E_l^2 - 4E_l E_w + k^2) W_3 + (2E_l - E_w) k^2 W_5/m_q$$





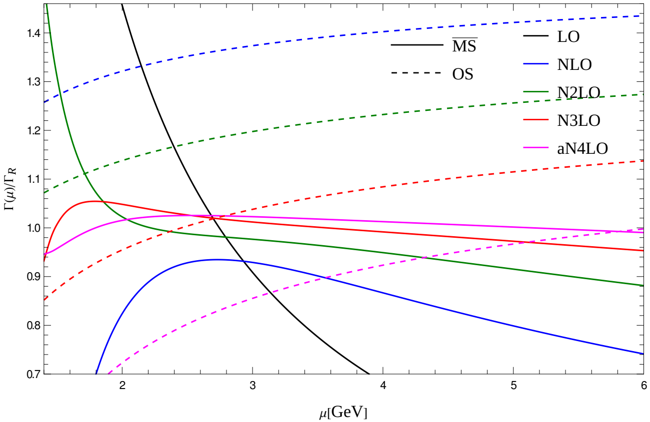
Heavy-to-light Structure Functions @ N3LO

- **First complete $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ corrections to all H2L-structure functions underlying the *triple-differential* distribution** $\frac{d^3\Gamma_{sl}}{dE_l\, dE_w\, dk^2}$, hence all precision observables for (inclusive) D and (charmless) B meson decays [**LC**, X.Chen, X.Guan ,Y.Q. Ma] [2602.11537; 2602.11879]



Results for Free b-quark Decay $\Gamma_{\mathrm{sl}}^{b\to u}$ in $\overline{\mathrm{MS}}$ scheme

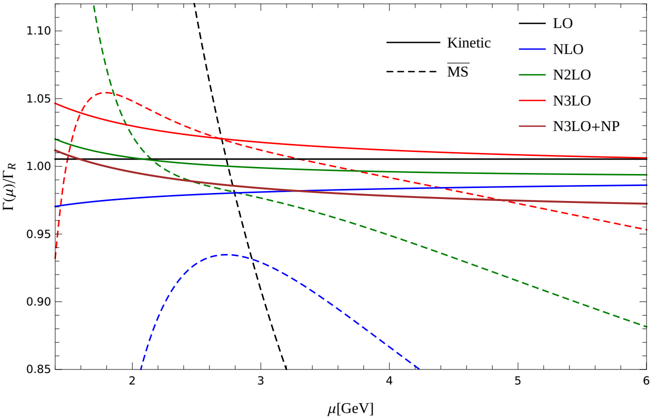
$$\Gamma_{\mathrm{sl}}^{b\to u} \equiv \Gamma_0(m_b)\mathbf{C}_{\mathrm{p}} = \overline{\Gamma}_0\left(1 + 0.3036075 + 0.1365820 + 0.06841766 + 0.034184\right)$$



- **Every one more perturbative order higher in the $\overline{\mathrm{MS}}$ result, the term is reduced roughly by 1/2**
- However, the scale uncertainty of $\bar{\Gamma}_{\mathrm{sl}}^{b\to u}$ at N3LO is still $[+6\%, -8\%]$ (!) for $\mu/\overline{m}_b \in [1/2, 2]$

Result for $\Gamma(B \to X_u \ell \bar{\nu}_\ell)$ in Kinetic-mass Scheme

In Heavy Quark Expansion theory $\Gamma(B \to X_u \ell \bar{\nu}_\ell) = \Gamma_0(m_b)\left[\mathbf{C}_{\mathrm{p}}\left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2\,m_b^2}\right) - 2\,\frac{\mu_G^2}{m_b^2} + \mathcal{O}(\Lambda_{\mathrm{QCD}}^3/m_b^3)\right]$.

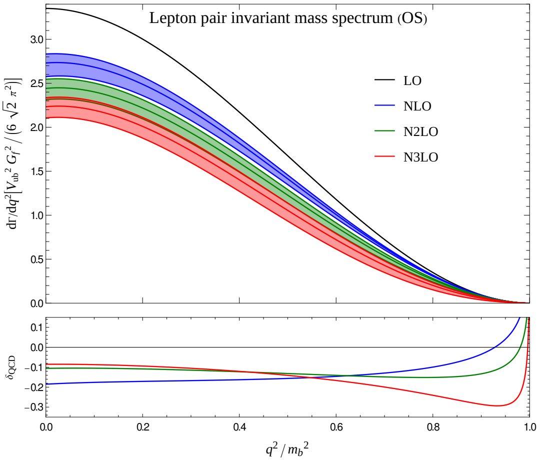


We obtain the **most accurate theoretical prediction for the inclusive** $B \to X_u \ell \bar{\nu}_\ell$ (kine-scheme):

$$\Gamma(B \to X_u \ell \bar{\nu}_\ell) = \frac{|V_{ub}|^2}{|3.82 \times 10^{-3}|^2}\left(6.53 \pm 0.12 \pm 0.13 \pm 0.03\right) \times 10^{-16}\,\mathrm{GeV}\,.$$

Lepton-pair Invariant-mass Spectrum $\mathrm{d}\Gamma_{\mathrm{sl}}^{b\to u}/\mathrm{d}q^2$

In the OS scheme, δ_{QCD} is large and does not converge quickly.



- $\mathcal{O}(\alpha_s^3)$ Leading-color **integrated** form agrees with [Chen, Li, Li, Wang, Wang, Wu 23] .
- $\mathcal{O}(\alpha_s^3)$ **integrated** fermionic part agrees with a recent evaluation [Fael, Usovitsch 23] .

Subtlety with Re-expanding non-inclusive Observables

Rewrite $\Gamma_{\rm sl}^{b\to u} = \int_0^{m^2} f(q^2, m^2)\,{\rm d}q^2$ using an alter. $\bar{m}$ is **NOT** the same as changing mass-renormalization schemes in the usual sense.

- Consistency for the integral:

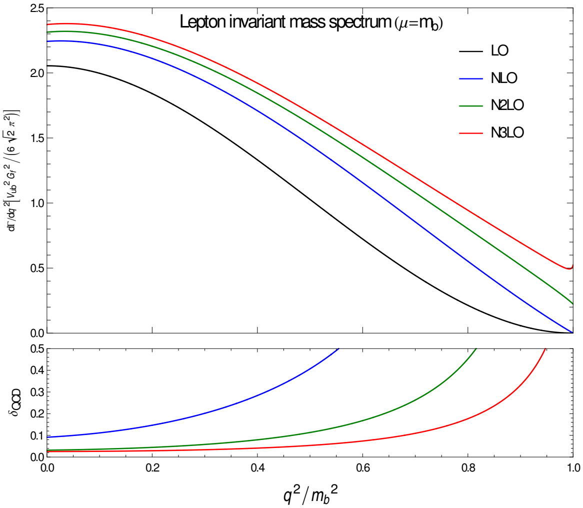
$$\int_0^{m^2} f(q^2, m^2)\,{\rm d}q^2 =$$

$$\int_0^{\bar{m}^2 + dm} f(q^2, \bar{m}^2 + dm)\,{\rm d}q^2$$

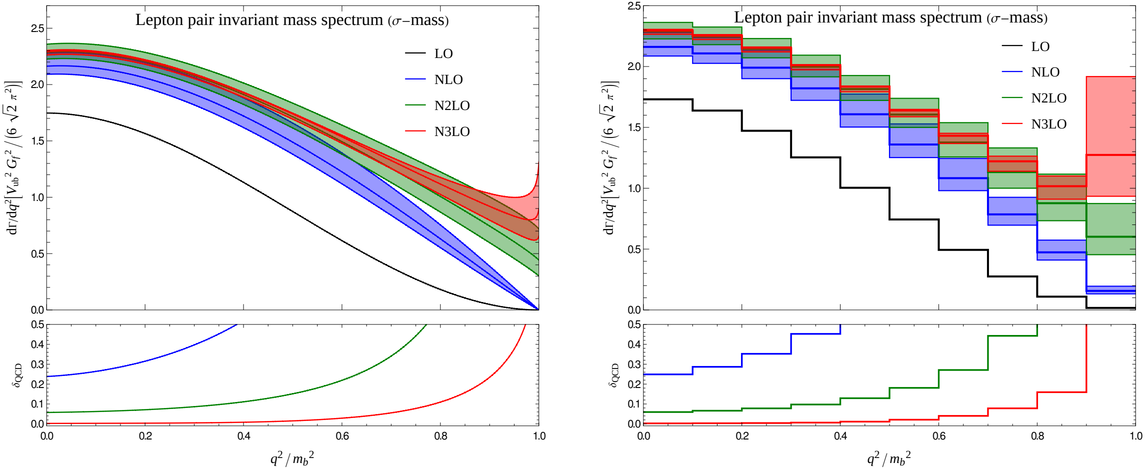
- q^2-range bounded by $\bar{m}(\mu)$ (phase-space expanded too)

- Not (necess.) 0 at **new** ends (non-0 high-order derivatives)

- Dirac-δ type boundary(!) term (histogramming in bins)



$d\Gamma_{sl}^{b\to u}/dq^2$ in σ-mass Scheme



- The curve at N3LO does not vanish at $q^2 \to (m_b^\sigma)^2$, even **without** the boundary-term included in the continuous plot。

Composition of Perturbative Pole Mass in Gauge Theories

Take-home messages: [LC, Li, Niggetiedt 25; LC, Zhao 25]

- A novel **diagrammatic proof of the *identity***

$$\langle \vec{\mathbf{p}} | -\frac{\epsilon}{2}\left[F^a_{\rho\sigma} F^{a\,\rho\sigma}\right]_B + \sum_f \left[m_f \bar{\psi}_f \psi_f\right]_B | \vec{\mathbf{p}} \rangle = \bar{u}(\vec{\mathbf{p}})\, m_{\rm os}\, u(\vec{\mathbf{p}})$$

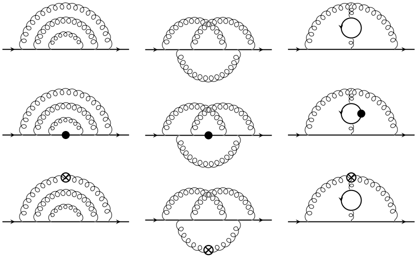
to any loops in perturbative gauge theories (and all orders in ϵ) without any ref. to prelaid *operator* renormalization conditions.

- The proof is based on:
  - equation of mass-dimensional analysis in DR
  - topological properties of contributing diagrams
  - the pole-mass definition

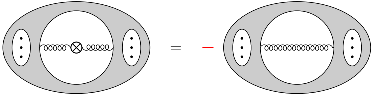
- The role of the trace anomaly is clarified

(crucial for massive fermions, albeit vanishing for gauge bosons)

[arXiv: 2509.03580]



Reduction of diagrams with degree-2 vertex



For arbitrary vacuum diagrams in gauge theories:

$$N_g^{\rm eff} = N_g - V_3 - V_4 = N_L - 1$$

A scheme/scale-invariant trace-anomaly-subtracted σ-mass

Take-home messages: [LC, Zhao 25]

- A discovery: Leading IR-renormalon div. in $m_{\rm os}$ of massive quarks resides entirely in the EMT-trace anomaly contribution!
- A trace-anomaly-subtracted σ-mass definition for heavy quarks, scheme/scale-invariant and free from the leading IR-renorm. ambiguity
- A formula

$$Z_\sigma = \frac{m_\sigma}{m_{\rm os}} = \frac{1 + 2\beta \frac{\partial \ln\left(C_{\overline{m}}\left(\alpha_s, \mu/\overline{m}\right)\right)}{\partial \ln\left(\alpha_s\right)}}{1 - 2\gamma_m},$$

i.t.o. AMDs of α_s and $\overline{m}$ and the pole-to-$\overline{\rm MS}$ ratio $C_{\overline{m}}$, useful for deriving 5-loop Z_σ with 4-loop $C_{\overline{m}}$.

[arXiv:2509.10786]

- Portions $Z_{\rm TA}$ of the trace-anomaly contribution to $m_{\rm os}$ and m_σ of elementary fermions: [LC, Li, Niggetiedt 25]

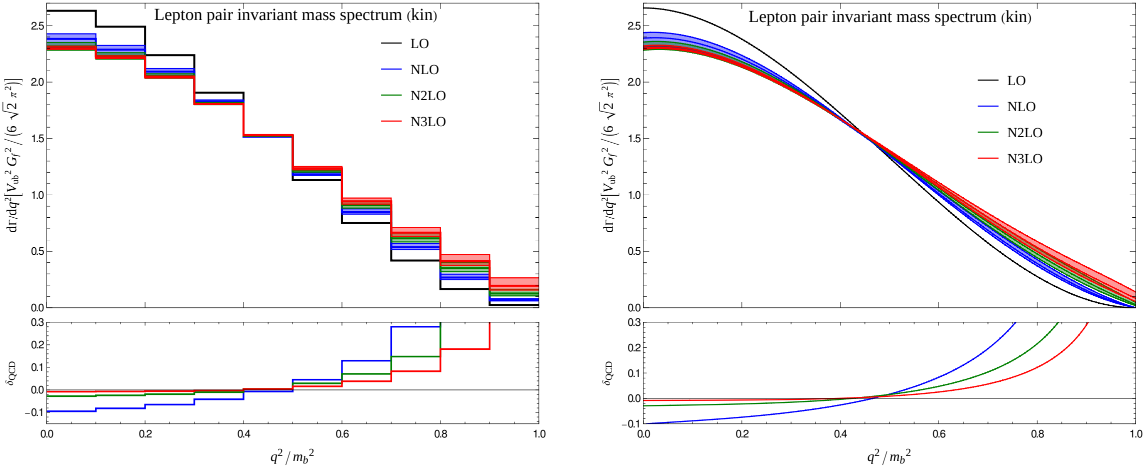
	electron	t-quark	b-quark	c-quark
$Z_{\rm TA}$	0.347 %	7.9 %	20.4 %	34.3 %
m_σ	0.509 MeV	159.0 GeV	3.96 GeV	1.17 GeV

  - $Z_{\rm TA}$ increase for lighter heavy quarks
  - $Z_{\rm TA}$ will vanish in chiral limit $m_\sigma = 0$

- The t-**quark decay width in** m_σ **scheme**:

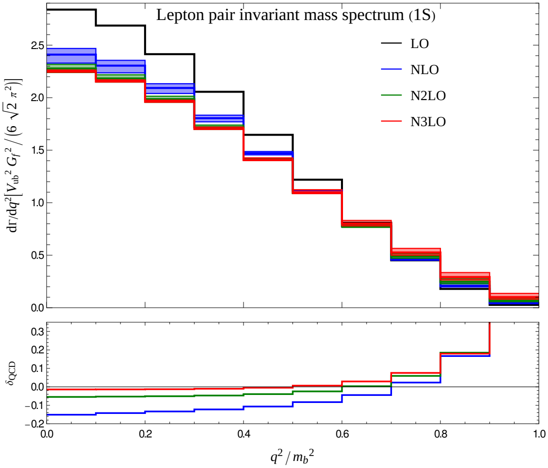
$$\Gamma_t^\sigma(\mu = m_t^\sigma/2) = 1.10470 + 0.20790 + 0.01341 - 0.00401$$
$$= 1.3220(33)\ \text{GeV}$$

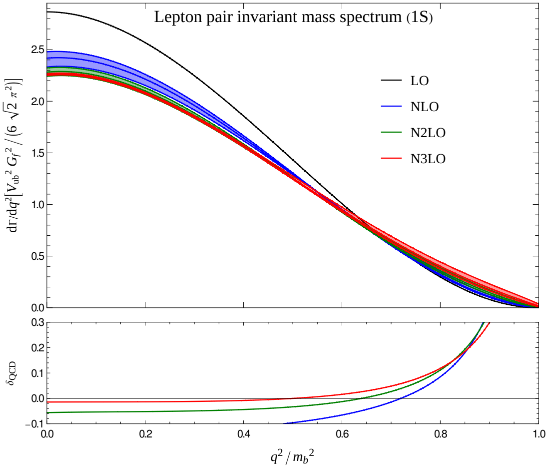
$d\Gamma_{sl}^{b\to u} / dq^2$ in Kinetic-mass Scheme



- δ_{QCD} exhibits the usual expected (regular) behavior
- The particular (crossing) **pattern** helps to "understand" the puzzle (scale-uncertainty of the total inclusive result not reduced from N2LO to N3LO)
- The boundary-term is not included in the continuous plot (changing the last of 10-bin histograms by $3 \sim 5\%$)

$\mathrm{d}\Gamma_{\mathrm{sl}}^{b\to u}/\mathrm{d}q^2$ in 1S Scheme





- δ_{QCD} exhibits the usual expected (regular) behavior
- Similar (crossing) **pattern** also helps to "understand" the puzzle (scale-uncertainty of the total inclusive result not reduced from N2LO to N3LO)
- The boundary-term is not included in the continuous plot (smaller than in the kinetic-mass case)

A Tentative Application to $\Gamma_{\rm sl}^{c\to q}$ (3+1)

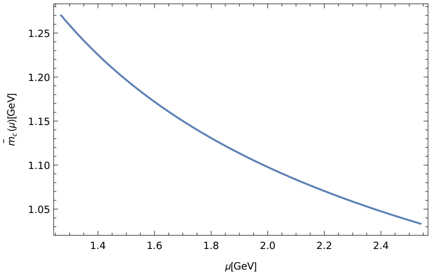
- Large QCD coupling at $\bar{m}_c$:

$$\alpha_s^{[4]}(\bar{m}_c) \approx 0.38 \approx 2\,\alpha_s^{[5]}(\bar{m}_b) \approx 4\,\alpha_s^{[6]}(m_t)$$

- Significant running-mass effect:

$$\bar{m}_c(\bar{m}_c) = 1.27\,\text{GeV};\ \bar{m}_c(\mu = 2\,\text{GeV}) = 1.10\text{GeV}$$

$$(\bar{m}_c + \Delta m)^5 \approx \bar{m}_c^5\,(1 + 5\,\frac{\Delta m}{\bar{m}_c} + \cdots)$$



To get an idea of the sizes of the perturbative QCD correction factors:

$$\tilde{\Gamma}_0\,\bar{m}_c^5(2\,\text{GeV})\,\Big(1 + 0.832 + 0.710 + 0.606\Big) = 5.01\,\tilde{\Gamma}_0$$

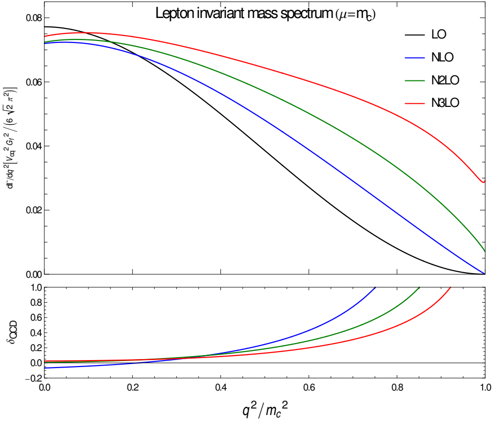
$$\tilde{\Gamma}_0\,\bar{m}_c^5(2\times 1.27\,\text{GeV})\,\Big(1 + 0.953 + 0.850 + 0.725\Big) = 4.16\,\tilde{\Gamma}_0$$

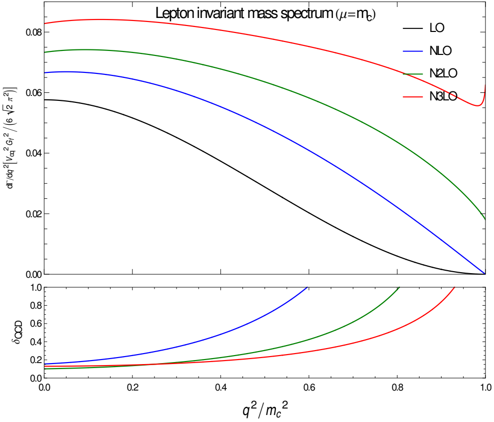
At the ref. scale $\mu = 2\,\text{GeV}$ (typically chosen for $\bar{m}_c$), the $\mathcal{O}(\alpha_s^3)$ correction could increase the perturbative NNLO correction by $\sim$24%.

The non-perturbative corrections (to higher twists) are important

$d\Gamma_{sl}^{c\to q}/dq^2$ in $\overline{\text{MS}}$ and Kinetic Scheme

Neither $\overline{\text{MS}}$ nor kinetic mass, unfortunately, works well for $d\Gamma_{sl}^{c\to q}/dq^2$

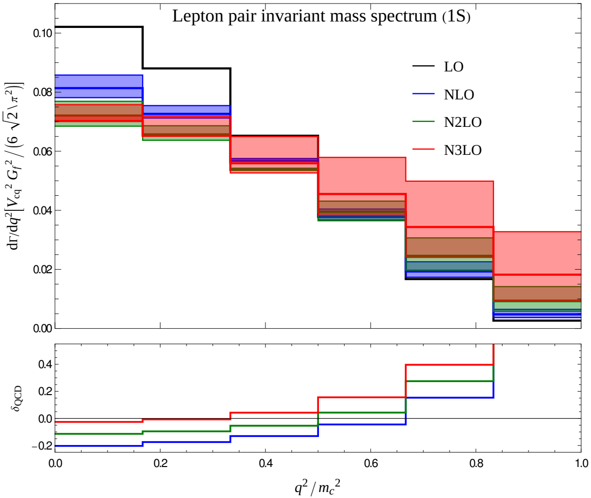


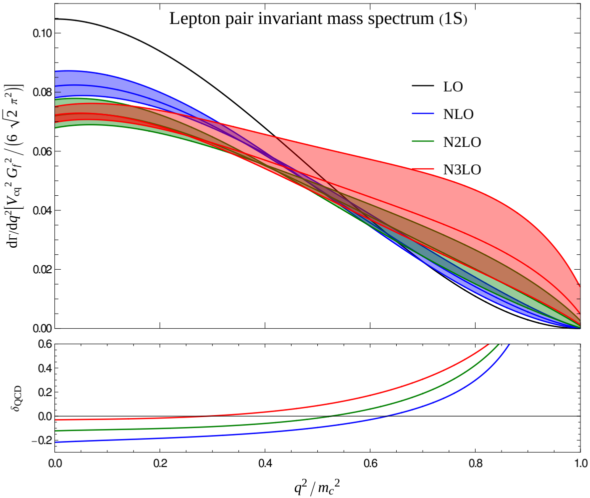


The left is for the result with kinetic-mass, slightly better than the right with $\overline{\text{MS}}$-mass, obtained using respectively standard parameter-settings employed in literature.

$d\Gamma_{sl}^{c\to q}/dq^2$ in 1S Scheme

But, there is still **hope**!





(To estimate the uncertainty, we used $[1\,\text{GeV}, 2\,m_{1S}]$ as the usual choice $\mu = m_{1S}/2$ is too low)

$\Gamma_{\text{sl}}^{D^i}$ and e-energy Moments in 1S Scheme

[Shao, Huang, Qin 25 ; K. K. Shao, H. L. Feng, X. Y. Liu, Q. Qin, L. Sun, F. S. Yu 25]

- $\Gamma_{\text{sl}}^{c\to q}$ and e-energy moments for a *free* c-quark decay in $\overline{\text{MS}}$ scheme:

$$\Gamma_{D_i} = \hat{\Gamma}_0 \sum_{q=d,s} |V_{cq}|^2\, \overline{m}_c^5(\mu) \left[1 + \frac{\alpha_s}{\pi}\left(5\bar{\ell} + 4.2536\right) + \frac{\alpha_s^2}{\pi^2}\left(\frac{425}{24}\bar{\ell}^2 + 38.3935\bar{\ell} + 29.8447\right)\right],$$

$$\langle E_e \rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2\, \overline{m}_c^6(\mu) \left[\frac{3}{10} + \frac{\alpha_s}{\pi}\left(\frac{9}{5}\bar{\ell} + 1.64052\right) + \frac{\alpha_s^2}{\pi^2}\left(\frac{291}{40}\bar{\ell}^2 + 16.2359\bar{\ell} + 12.7981\right)\right]$$

- $\Gamma_{\text{sl}}^{c\to q}$ and e-energy moments for a *free* c-quark decay in $1S$ scheme (much improved convergence):

$$\Gamma_{D_i} = \hat{\Gamma}_0 \sum_{q=d,s} |V_{cq}|^2\, m_{c,1S}^5 \Big[1 + \epsilon\left(\frac{10\alpha_s^2}{9} - \frac{2\pi\alpha_s}{3} + \frac{25\alpha_s}{6\pi}\right)$$
$$+ \epsilon^2\left(2.9473\ell_\alpha\alpha_s^3 - 0.50936\ell_{1s}\alpha_s^2 + 0.74074\alpha_s^4 + 3.1352\alpha_s^3 - 2.3752\alpha_s^2\right)\Big],$$

$$\langle E_e \rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2\, m_{c,1S}^6 \Big[\frac{3}{10} + \epsilon\left(\frac{2\alpha_s^2}{5} - \frac{\pi\alpha_s}{5} + \frac{1093\alpha_s}{900\pi}\right)$$
$$+ \epsilon^2\left(-0.16031\ell_{1s}\alpha_s^2 + 1.061\ell_\alpha\alpha_s^3 + 0.31111\alpha_s^4 + 1.1136\alpha_s^3 - 0.78025\alpha_s^2\right)\Big]$$

- **First Inclusive $\Gamma_{\text{sl}}^{D^i}$ and e-energy moments in 1S scheme @ N3LO in ϵ:** [LC, Chen, Guan, Ma, 26]

$$\langle E_e^0 \rangle\big|_{unn.} = 8.947 - 1.187\,\epsilon - 0.447\,\epsilon^2 - 0.051\,\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$\langle E_e^1 \rangle\big|_{unn.} = 4.160 - 0.499\,\epsilon - 0.131\,\epsilon^2 + 0.086\,\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$\langle E_e^2 \rangle\big|_{unn.} = 2.149 - 0.231\,\epsilon - 0.0250\,\epsilon^2 + 0.111\,\epsilon^3 + \mathcal{O}(\epsilon^4),$$

Summary and Outlook

- ☑ **First complete $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ QCD corrections to all Heavy-to-light structure functions underlying the *triple-differential* distribution** $\frac{d^3\Gamma_{sl}}{dE_l\, dE_w\, dk^2}$

  - ☑ Combining our N^{3}LO perturbative and non-perturbative information (relevant at %-level), **an accurate theoretical prediction for the inclusive** $B \to X_u \ell \bar{\nu}_\ell$ (using kinetic-mass scheme):

$$\Gamma(B \to X_u \ell \bar{\nu}_\ell) = \frac{|V_{ub}|^2}{|3.82 \times 10^{-3}|^2} \left(6.53 \pm 0.12 \pm 0.13 \pm 0.03\right) \times 10^{-16}\,\text{GeV}\,.$$

  - ☑ Our N^{3}LO predictions for $d\Gamma_{\rm sl}^{c\to q}/dq^2$ and inclusive moments of lepton-energy spectrum (using 1S-mass scheme) could be of great help for **extracting (non-perturbative) HQE-parameters in Charm sector**.

  - ☒ The mixed QCD-EW corrections need to be completed for reliable sub-percent theoretical predictions.

- ☑ **Not only cutting-edge precision results but also theoretically novel observations and fresh insights are made regarding heavy-quark masses and decays!**

Thank you for listening!

Summary and Outlook

☑ **First complete $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ QCD corrections to all Heavy-to-light structure functions underlying the *triple-differential* distribution** $\frac{d^3\Gamma_{sl}}{dE_l\, dE_w\, dk^2}$

- ☑ Combining our N^{3}LO perturbative and non-perturbative information (relevant at %-level), **an accurate theoretical prediction for the inclusive** $B \to X_u \ell \bar{\nu}_\ell$ (using kinetic-mass scheme):

$$\Gamma(B \to X_u \ell \bar{\nu}_\ell) = \frac{|V_{ub}|^2}{|3.82 \times 10^{-3}|^2} \left(6.53 \pm 0.12 \pm 0.13 \pm 0.03\right) \times 10^{-16}\,\mathrm{GeV}\,.$$

- ☑ Our N^{3}LO predictions for $d\Gamma_{\mathrm{sl}}^{c\to q}/dq^2$ and inclusive moments of lepton-energy spectrum (using 1S-mass scheme) could be of great help for **extracting (non-perturbative) HQE-parameters in Charm sector**.

- ☒ The mixed QCD-EW corrections need to be completed for reliable sub-percent theoretical predictions.

☑ **Not only cutting-edge precision results but also theoretically novel observations and fresh insights are made regarding heavy-quark masses and decays!**

Thank you for listening!