Evaluation of Muon g-2 from $\tau \to \pi\pi\nu$ Data

Reporter: Zhi-Xin Li(李志鑫)

Hebei Normal University (河北师范大学)

合作者：李澳、郝津、段春贵、郭志辉

March 30, 2026

1. Current status on muon g-2
2. The electromagnetic correction $G_{\mathsf{EM}}(t)$
3. Updated evaluation of a_μ from τ data
4. Summary

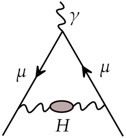
Current status on muon g-2

- The **Leading-order hadronic vacuum polarization** (LO HVP) contributes

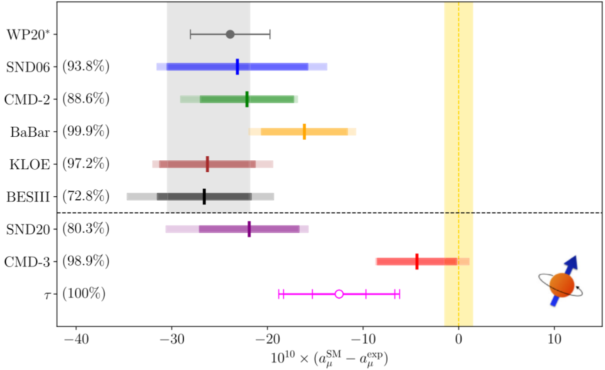
  [Gourdin and Rafael, NPB'69.] [R. Aliberti et al., Phys. Rept'25]

$$a_\mu^{\text{HVP,LO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} dt\, K(t)\, \sigma^0_{e^+e^-\to\text{hadrons}}(t)$$

e^+e^-hadrons cross sections $\Rightarrow \pi\pi > 70\%$



- Data-driven $e^+e^- \to \pi^+\pi^-$ inputs show a sizable spread.



Alternative way to address HVP from $\pi\pi$

- $e^+e^- \to \pi^+\pi^-$ cross section

$$a_\mu^{\text{HVP,LO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} dt\, K(t)\, \sigma^0_{e^+e^-\to\text{hadrons}}(t)$$

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \frac{\pi\alpha^2}{3t}\beta^3_{\pi^+\pi^-}(t)\left|F^{(0)}_{\pi\pi}(t)\right|^2$$

- $\pi\pi$ invariant-mass distribution of τ data

$$\frac{d\Gamma_{\tau_{\pi\pi[\gamma]}}}{dt} = \frac{G_F^2|V_{ud}|^2 m_\tau^3 S_{\text{EW}}}{384\pi^3}\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+\frac{2t}{m_\tau^2}\right)\beta^3_{\pi^-\pi^0}\left|F^{(-)}_{\pi\pi}(t)\right|^2 G_{\text{EM}}(t)$$

- **Conserved Vector Current (CVC)**

$$\left.\begin{array}{ll} e^+e^- \to \pi^+\pi^- : & \nu_0(t) \sim \beta^3_{\pi^+\pi^-}(t)\left|F^{(0)}_{\pi\pi}(t)\right|^2 \\ \tau^- \to \pi^-\pi^0\nu_\tau : & \nu_-(t) \sim \beta^3_{\pi^-\pi^0}(t)\left|F^{(-)}_{\pi\pi}(t)\right|^2 \end{array}\right\} \Rightarrow \nu_0(t) = \nu_-(t)(\text{CVC})$$

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \left[\frac{K_\sigma(t)}{K_\Gamma(t)}\frac{d\Gamma_{\tau_{\pi\pi[\gamma]}}}{dt}\right]\times\left[\frac{R_{\text{IB}}(t)}{S_{\text{EW}}}\right]$$

$$K_\sigma(t) = \frac{\pi\alpha^2}{3t}, \quad K_\Gamma(t) = \frac{G_F^2\,|V_{ud}|\,m_\tau^3}{384\pi^3}\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+2\frac{t}{m_\tau^2}\right).$$

Current status on muon g-2

- Key problem: isospin breaking (IB) effects

$$a_\mu^{\mathsf{HVP,LO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} dt\, K(t)\, \sigma^0_{e^+e^-\to \mathsf{hadrons}}(t)$$

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \left[\frac{K_\sigma(t)}{K_\Gamma(t)}\frac{d\Gamma_{\tau_{\pi\pi[\gamma]}}}{dt}\right] \times \left[\frac{R_{\mathsf{IB}}(t)}{S_{\mathsf{EW}}}\right]$$

- Correction term

$$R_{\mathsf{IB}}(t) = \frac{\mathsf{FSR}(t)}{G_{\mathsf{EM}}(t)}\frac{\beta^3_{\pi^+\pi^-}(t)}{\beta^3_{\pi^-\pi^0}(t)}\frac{\left|F^{(0)}_{\pi\pi}(t)\right|^2}{\left|F^{(-)}_{\pi\pi}(t)\right|^2}$$

- $\pi\pi$ form factors ($F^{(0,-)}_{\pi\pi}(t)$)
- kinematical factors ($\beta_{+,0}$)
- final-state radiation (FSR)
- electroweak short-distance correction (S_{EW})
- **long-distance electromagnetic correction (G_{EM})**

The electromagnetic correction function $G_{\text{EM}}(t)$

- The EM correction function $G_{\text{EM}}(t)$ for the $\tau \to \pi\pi\nu_\tau$

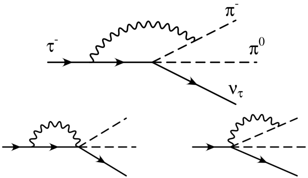
$$\frac{d\Gamma_{\tau_{\pi\pi[\gamma]}}}{dt} = \frac{G_F^2 |V_{ud}|^2 m_\tau^3 S_{\text{EW}}}{384\pi^3} \left(1 - \frac{t}{m_\tau^2}\right)^2 \left(1 + \frac{2t}{m_\tau^2}\right) \beta^3_{\pi^-\pi^0} \left|F_{\pi\pi}^{(-)}(t)\right|^2 G_{\text{EM}}(t)$$

- Non-radiative two-pion tau decay

$$\frac{d\Gamma^{(0)}_{\tau_{\pi\pi}}}{dt} = \frac{G_F^2 \left|V_{ud}\right|^2 m_\tau^3 S_{\text{EW}}}{384\pi^3} \left(1 - \frac{t}{m_\tau^2}\right)^2 \left(1 + \frac{2t}{m_\tau^2}\right) \beta^3_{\pi^-\pi^0} \left|F_{\pi\pi}^{(-)}(t)\right|^2$$

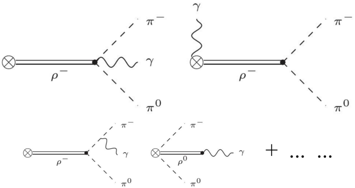
virtual photon (loops)

[Cirigliano, et al.,PLB’ 01]



real photon

$(\tau \to \pi\pi\nu_\tau\gamma)$



$$\frac{d\Gamma_{\tau_{\pi\pi[\gamma]}}}{dt} = \frac{d\Gamma^{(0)}_{\tau_{\pi\pi}}}{dt} G_{\text{EM}}(t) \Rightarrow G_{\text{EM}}(t) = 1 + G^{(v)}_{\text{EM}}(t) + G^{(r)}_{\text{EM}}(t)$$

The electromagnetic correction function $G_{\mathrm{EM}}(t)$

- Amplitude of $\tau \to \pi\pi\nu_\tau\gamma$

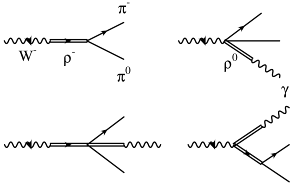
$$\mathcal{M} = eG_F V_{ud}^* \varepsilon^u(k)^* \left\{ F_\nu \bar{u}(q)\gamma^\nu(1-\gamma_5)(m_\tau + \not{P} - \not{k})\gamma_\mu u(P) + \right.$$
$$\left. + (V_{\mu\nu} - A_{\mu\nu})\bar{u}(q)\gamma^\nu(1-\gamma_5)u(P) \right\}$$

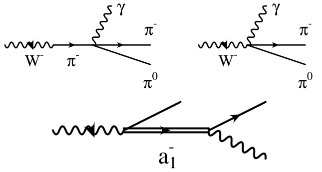
- The hadronic tensor amplitude

$$\begin{aligned}
V_{\mu\nu} =& F_{\pi\pi}^{(-)}(u)\frac{p_{1\mu}}{p_1\cdot k}(p_1+k-p_2)_\nu - F_{\pi\pi}^{(-)}(u)g_{\mu\nu} + \frac{F_{\pi\pi}^{(-)}(u)-F_{\pi\pi}^{(-)}(t)}{(p_1+p_2)\cdot k}(p_1+p_2)_\mu(p_2-p_1)_\nu \\
&+ v_1(g_{\mu\nu}p_1\cdot k - p_{1\mu}k_\nu) + v_2(g_{\mu\nu}p_2\cdot k - p_{2\mu}k_\nu) + v_3(p_{1\mu}p_2\cdot k - p_{2\mu}p_1\cdot k)p_{1\nu} + \\
&+ v_4(p_{1\mu}p_2\cdot k - p_{2\mu}p_1\cdot k)(p_1+p_2+k)_\nu \\
A_{\mu\nu} =& i\left(a_1\epsilon_{\mu\nu\rho\sigma}p_1^\rho k^\sigma + a_2\epsilon_{\mu\nu\rho\sigma}p_2^\rho k^\sigma + a_3 p_{1\nu}\epsilon_{\mu\rho\beta\sigma}k^\rho p_1^\beta p_2^\sigma + a_4 p_{2\nu}\epsilon_{\mu\rho\beta\sigma}k^\rho p_1^\beta p_2^\sigma\right) \\
F_\nu =& (p_2-p_1)_\nu F_{\pi\pi}^{(-)}(t)/(2P\cdot k)
\end{aligned}$$

- **Form Factor $F_{\pi\pi}^{(-)}(t)$ and $v_i(a_i)$**

  Minimal resonance chiral theory contributions (CEN) [V. Cirigliano, G. Ecker, and H. Neufeld,JHEP'02]



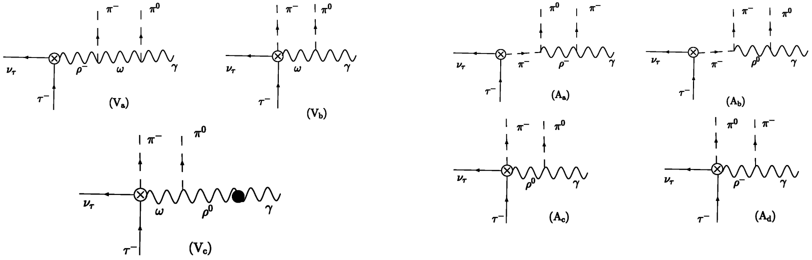


The electromagnetic correction function $G_{\text{EM}}(t)$

- **Contributions from VVP and VJP operators in resonance chiral theory**

[P. D. Ruiz-Femenia, et al.,JHEP'03] [C.Chen, C.-G.Duan and Z.-H.Guo, JHEP'22]

$$
\begin{aligned}
\mathcal{L}_{VVP} =& d_1 \epsilon_{\mu\nu\rho\sigma} \left\langle \{ V^{\mu\nu}, V^{\rho\alpha} \} \nabla_\alpha u^\sigma \right\rangle + i d_2 \epsilon_{\mu\nu\rho\sigma} \left\langle \{ V^{\mu\nu}, V^{\rho\sigma} \} \chi_- \right\rangle \\
&+ d_3 \epsilon_{\mu\nu\rho\sigma} \left\langle \{ \nabla_\alpha V^{\mu\nu}, V^{\rho\alpha} \} u^\sigma \right\rangle + d_4 \epsilon_{\mu\nu\rho\sigma} \left\langle \{ \nabla^\sigma V^{\mu\nu}, V^{\rho\alpha} \} u_\alpha \right\rangle , \\
\mathcal{L}_{VJP} =& \frac{c_1}{M_V} \epsilon_{\mu\nu\rho\sigma} \left\langle \left\{ V^{\mu\nu}, f_+^{\rho\alpha} \right\} \nabla_\alpha u^\sigma \right\rangle + \frac{c_2}{M_V} \epsilon_{\mu\nu\rho\sigma} \left\langle \left\{ V^{\mu\alpha}, f_+^{\rho\sigma} \right\} \nabla_\alpha u^\nu \right\rangle \\
&+ \frac{i c_3}{M_V} \epsilon_{\mu\nu\rho\sigma} \left\langle \left\{ V^{\mu\nu}, f_+^{\rho\sigma} \right\} \chi_- \right\rangle + \frac{i c_4}{M_V} \epsilon_{\mu\nu\rho\sigma} \left\langle V^{\mu\nu} \left[f_-^{\rho\sigma}, \chi_+ \right] \right\rangle \\
&+ \frac{c_5}{M_V} \epsilon_{\mu\nu\rho\sigma} \left\langle \left\{ \nabla_\alpha V^{\mu\nu}, f_+^{\rho\alpha} \right\} u^\sigma \right\rangle + \frac{c_6}{M_V} \epsilon_{\mu\nu\rho\sigma} \left\langle \left\{ \nabla_\alpha V^{\mu\alpha}, f_+^{\rho\sigma} \right\} u^\nu \right\rangle \\
&+ \frac{c_7}{M_V} \epsilon_{\mu\nu\rho\sigma} \left\langle \left\{ \nabla^\sigma V^{\mu\nu}, f_+^{\rho\alpha} \right\} u_\alpha \right\rangle .
\end{aligned}
$$



- **High energy constraints** to the resonance couplings

$$c_1 + 4c_3 = 0 \qquad c_1 - c_2 + c_5 = 0 \qquad c_5 - c_6 = \frac{N_C M_V}{64\sqrt{2}\pi^2} F_V$$

$$d_1 + 8d_2 = -\frac{N_C M_V^2}{(8\pi F_V)^2} + \frac{F^2}{4F_V^2} \qquad d_3 = -\frac{N_C M_V^2}{(8\pi F_V)^2} + \frac{F^2}{8F_V^2}$$

- Determination of low-energy coupling constant d_4

[C.Chen, C.-G.Duan and Z.-H.Guo, JHEP'22]

$\omega \to \pi^0\pi^0\gamma$

Scalar

- Conditions for cancelling ultraviolet divergence:

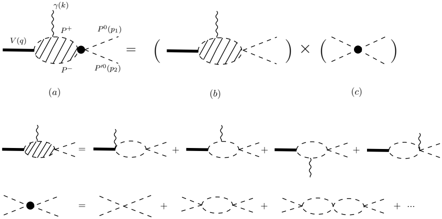
$$F_A = F_\pi\,, \quad F_V = \sqrt{2}F_\pi\,, \quad G_V = F_\pi/\sqrt{2}$$

- Determine d_4 from the radiative decay $\omega \to \pi^0\pi^0\gamma$

$$\Gamma^{\text{Exp}}_{\omega\to\pi^0\pi^0\gamma} = (5.8 \pm 1.0) \times 10^{-4}\ \text{MeV}$$

$d_4 = -0.42 \pm 0.07$ Sol-A

$d_4 = 1.01 \pm 0.07$ Sol-B



- We are left with a parameter free theoretical amplitude for the $\tau \to \pi\pi\nu_\tau\gamma$ process

The electromagnetic correction function $G_{\text{EM}}(t)$

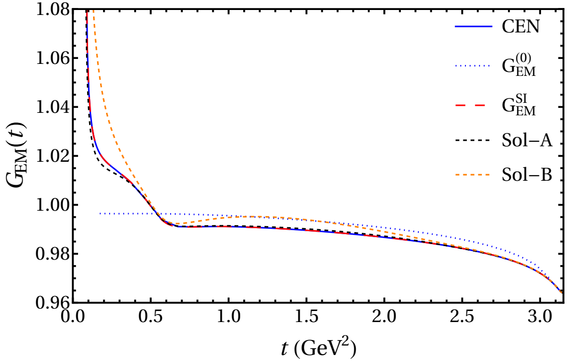
- Results for G_{EM}

$$G_{\text{EM}}(t) = 1 + G_{\text{EM}}^{(v)}(t) + G_{\text{EM}}^{(r)}(t)$$

- real-photon correction function

$$G_{\text{EM}}^{(r)}(t) = \frac{1}{\frac{d\Gamma^{(0)}_{\tau\pi\pi}}{dt}} \frac{\pi^2}{32(2\pi)^{12} m_\tau^2} \int_{S^-_{\pi\pi\nu}}^{S^+_{\pi\pi\nu}} dS_{\pi\pi\nu} \int_{S^-_{\nu\gamma}}^{S^+_{\nu\gamma}} dS_{\nu\gamma} \int_{S^-_{\pi\nu}}^{S^+_{\pi\nu}} dS_{\pi\nu} \int_{S^-_{\pi\nu\gamma}}^{S^+_{\pi\nu\gamma}} \frac{dS_{\pi\nu\gamma}}{\sqrt{-\Delta_4}} \overline{|\mathcal{M}|}^2_{\tau\to\pi\pi\nu_\tau\gamma}$$

- virtual correction $G_{\text{EM}}^{(v)}(t) = \dfrac{12 \int_{u_{\text{min}}}^{u_{\text{max}}} D(t,u)\, f_{\text{loop}}^{\text{elm}}(u, M_\gamma)\, du}{m_\tau^6 \left(1 - \frac{4m_\pi^2}{t}\right)^{3/2} \left(1 - \frac{t}{m_\tau^2}\right)^2 \left(1 + \frac{2t}{m_\tau^2}\right)}$



Updated evaluation of a_μ from τ data

The **relative shifts** of a_μ caused by the IB corrections in the $\pi\pi$ channel

$$\Delta a_\mu^{\text{HVP,LO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{t_{max}} dt\, K(t) \left[\frac{K_\sigma(t)}{K_\Gamma(t)} \frac{d\Gamma_{\tau_{\pi\pi[\gamma]}}}{dt}\right] \times \left(\frac{R_{\text{IB}}(t)}{S_{\text{EW}}} - 1\right).$$

$\Delta a_\mu^{\text{HVP,LO}}[\pi\pi]$ from $G_{\text{EM}}(t)$ in units of 10^{-11}						
$[t_{min}, t_{max}]$	Sol-A	Sol-B	CEN	MR[$\mathcal{O}(p^4)$]		MR[$\mathcal{O}(p^6)$]
$[4m_\pi^2, 1\ \text{GeV}^2]$	-7.1	-44.9	-10.6	-10.4	-15.9	-63.2 ± 16.5
$[4m_\pi^2, 2\ \text{GeV}^2]$	-6.4	-44.5	-9.8	-9.6	-15.2	-58.1 ± 12.2
$[4m_\pi^2, 3\ \text{GeV}^2]$	-6.3	-44.4	-9.7	-9.5	-15.1	-67.8 ± 17.5
$[4m_\pi^2, m_\tau^2]$	-6.3	-44.4	-9.7	-9.5	-15.1	-64.9 ± 13.4

- The dominant contributions arise from the **low-energy** regions.

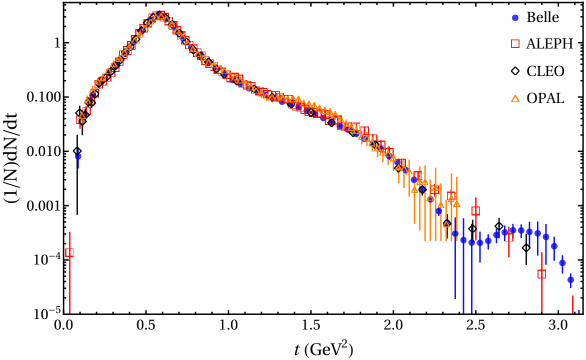
$\Delta a_\mu^{\text{HVP,LO}}[\pi\pi]$ from various IB sources in units of 10^{-11}					
Parameters	t_{max}/GeV^2	S_{EW}	$\beta^3_{\pi^+\pi^-}/\beta^3_{\pi^-\pi^0}$	$\lvert F^{(0)}_{\pi\pi}/F^{(-)}_{\pi\pi}\rvert^2$	FSR
Sol-A	1.0	-112.7	-74.1	76.8	42.7
	2.0	-114.6	-74.3	76.7	43.3
	3.0	-114.7	-74.3	76.7	43.3
	m_τ^2	-114.7	-74.3	76.7	43.3
Sol-B	1.0	-113.6	-76.9	77.1	43.2
	2.0	-115.4	-77.1	77.0	43.8
	3.0	-115.5	-77.1	77.0	43.8
	m_τ^2	-115.5	-77.1	77.0	43.8

Updated evaluation of a_μ from τ data

- Evaluation of $e^+e- \to \pi^+\pi^-$ cross section by using tau data with IB corrections

$$\sigma^0_{e^+e^-\to\pi^+\pi^-}(t) = \frac{2\pi\alpha^2 m_\tau^2}{3|V_{ud}|^2 t} \frac{1}{\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+\frac{2t}{m_\tau^2}\right)} \frac{\mathcal{B}_{\tau_{\pi\pi}}}{\mathcal{B}_{\tau_e}} \frac{1}{N_{\tau_{\pi\pi}}} \frac{dN_{\tau_{\pi\pi}}}{dt} \frac{R_{\mathrm{IB}}(t)}{S_{\mathrm{EW}}}$$

- Input two-pion tau decay data



Updated evaluation of a_μ from τ data

- LO HVP

$$a_\mu^{\text{HVP,LO}|_{\pi\pi,\tau\text{data}}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} dt\, K(t)\, \sigma^0_{e^+e^-\to\pi^+\pi^-}(t)$$

Parameters	Experiments	$a_\mu^{\text{HVP,LO}\|_{\pi\pi,\tau\text{data}}}$
Sol-A	Belle	$516.7 \pm 2.1 \pm 7.9 \pm 2.2$
	ALEPH	$513.3 \pm 4.3 \pm 2.8 \pm 2.1$
	CLEO	$516.9 \pm 3.2 \pm 8.8 \pm 2.2$
	OPAL	$527.2 \pm 9.8 \pm 6.8 \pm 2.1$
Sol-B	Belle	$513.4 \pm 2.0 \pm 7.9 \pm 2.2$
	ALEPH	$510.2 \pm 4.2 \pm 2.8 \pm 2.1$
	CLEO	$513.7 \pm 3.2 \pm 8.8 \pm 2.2$
	OPAL	$523.5 \pm 9.5 \pm 6.8 \pm 2.1$

- The first error is from the uncertainties of the experimental invariant-mass spectra.
- The second error arises from the branching ratios of $\mathcal{B}_{\tau_{\pi\pi}}$ and $\mathcal{B}_{\tau_e}$.
- The third error comes from the IB correction function R_{IB}.

Updated evaluation of a_μ from τ data

- **We take Sol-A as the baseline solution**

$$a_\mu^{\text{HVP,LO}|_{\pi\pi,\tau\text{data, Belle}}} = 516.7 \pm 2.1_{\text{Spec}} \pm 7.9_{\text{BR}} \pm 2.2_{\text{IB}} \pm 3.3_{\text{Sys}}$$

$$a_\mu^{\text{HVP,LO}|_{\pi\pi,\tau\text{data, ALEPH}}} = 513.3 \pm 4.3_{\text{Spec}} \pm 2.8_{\text{BR}} \pm 2.1_{\text{IB}} \pm 3.2_{\text{Sys}}$$

$$a_\mu^{\text{HVP,LO}|_{\pi\pi,\tau\text{data, CLEO}}} = 516.9 \pm 3.2_{\text{Spec}} \pm 8.9_{\text{BR}} \pm 2.2_{\text{IB}} \pm 3.3_{\text{Sys}}$$

$$a_\mu^{\text{HVP,LO}|_{\pi\pi,\tau\text{data, OPAL}}} = 527.2 \pm 9.8_{\text{Spec}} \pm 6.8_{\text{BR}} \pm 2.1_{\text{IB}} \pm 3.7_{\text{Sys}}$$

- **The average of the central values weighted**

$$10^{10} \cdot a_\mu^{\text{HVP,LO}|_{\pi\pi,\tau\text{data}}} = 516.0 \pm 2.9_{\text{Spec+BR}} \pm 4.0_{\text{IB+Sys}}$$

- **Full LO HVP contribution** (including $\pi^+\pi^-\pi^0$, $2\pi^+2\pi^-$, ...)

$$10^{10} \cdot a_\mu^{\text{HVP,LO}|_{\tau\text{data}}} = 702.1 \pm 2.9_{\text{Spec+BR}} \pm 4.0_{\text{IB+Sys}} \pm 2.1_{\text{Others}}$$

- **The deviation between SM and BNL+FNAL experimental result**

$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (14.9 \pm 5.6) \times 10^{-10} \Rightarrow 2.7\sigma$$

Summary

- The real-photon electromagnetic correction to the $\tau \to \pi\pi\nu_\tau$ process has been calculated.
- By taking both vector and scalar contributions into account in $\omega \to \pi^0\pi^0\gamma$, we updated the determination of the key coupling d_4 .
- Using the experimental $\tau \to \pi\pi\nu_\tau$ data from Belle, ALEPH, CLEO and OPAL to calculate $a_\mu^{\text{HVP,LO}|_{\pi\pi,\tau\text{data}}}$.
- The resulting deviation of a_μ between the our estimation and the updated world average lies at the level of 2.7 σ.

Thanks!