$0\nu\beta\beta$ decay from RG-improved dim-7 SMEFT interactions + BNV nucleon decays

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第五届JUNO相关的理论和唯象研讨会

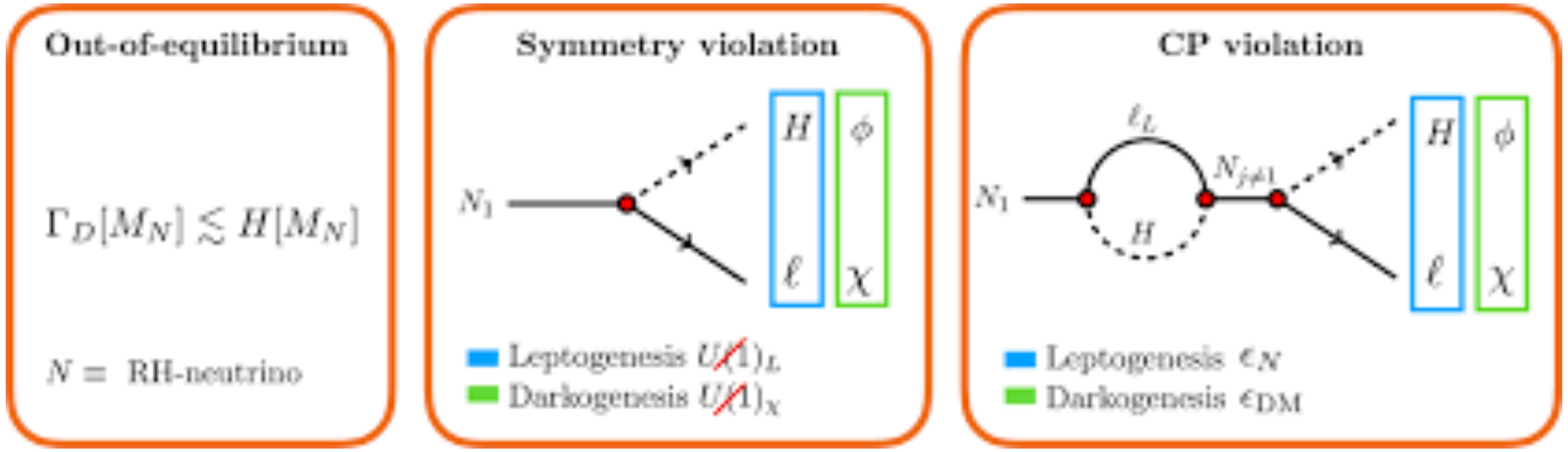
2026年1月27日-30日，杭州

Lepton number violation

- Neutrino oscillation ⇒ neutrino mass

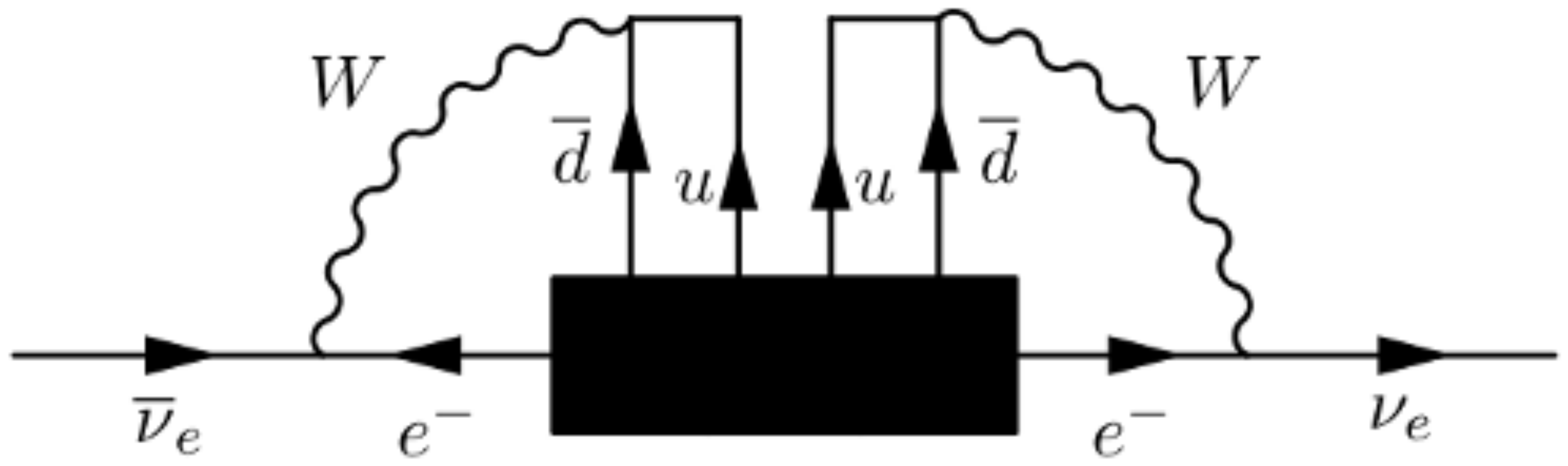
https://neutrino.syr.edu/research/neutrino-oscillations/

- Majorana nature of neutrinos is a well-motivated picture for massive neutrinos
- Baryogenesis via leptogenesis



https://x.com/higgsinocat/status/1375404211974893570

- Dark matter



Black box theorem

Schechter, Valle, Phys.Rev. D25 (1982) 774

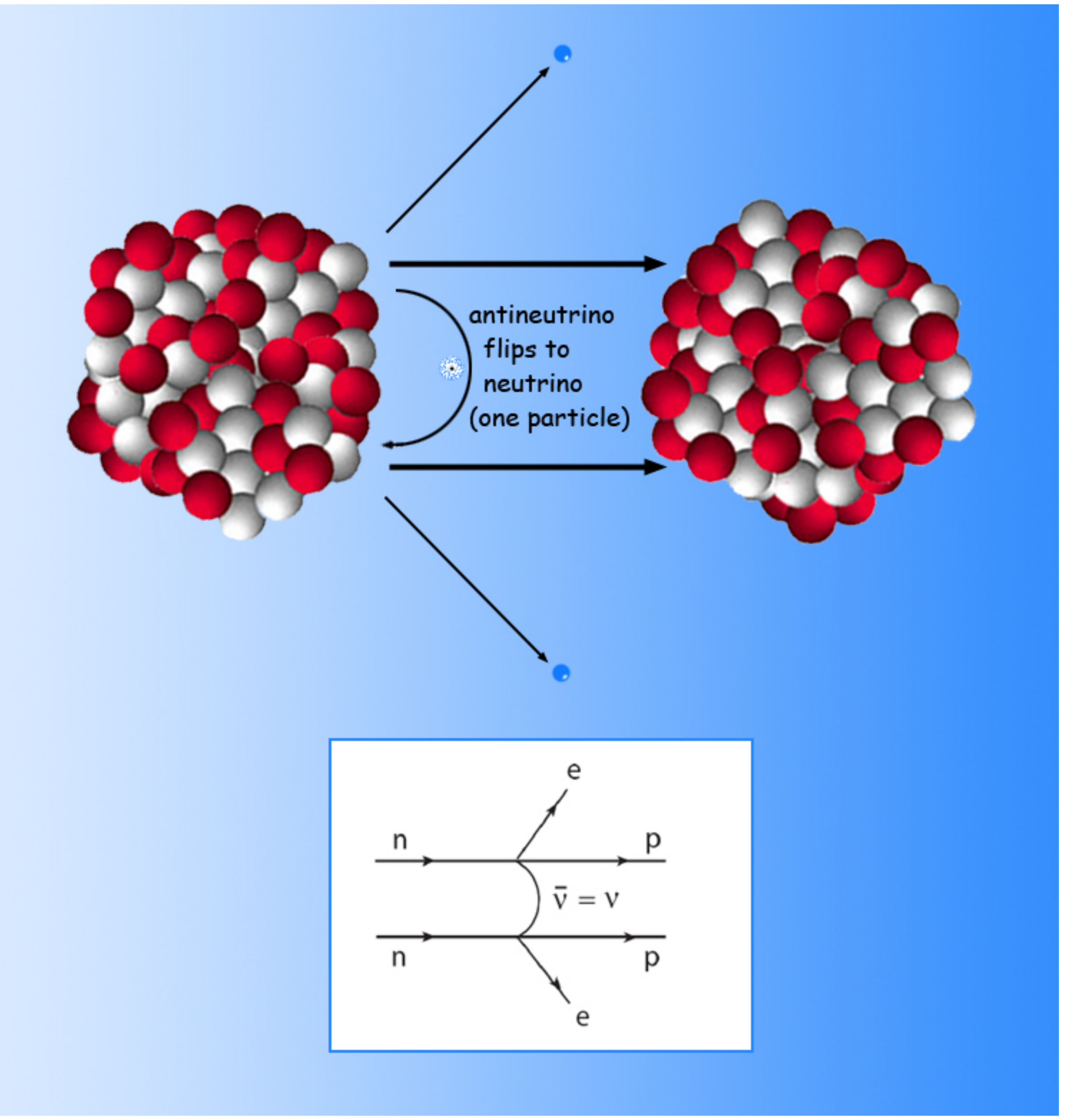
LNV signals as a test of Majorana nature of ν

Nuclear and collider test of LNV

- Neutrinoless double beta decay(0vββ)

$$^{A}_{Z}X \to ^{A}_{Z+2}X + 2e^-$$

Gerda, Majorana, LEGEND, CUPID, NEXT, CUORE, nEXO, KamLAND-ZenSNO, DARWIN, PandaX, etc



- Muon-positron conversion in nuclei

$$\mu^- + ^{A}_{Z}X \to ^{A}_{Z-2}X + e^+$$

Pontecorvo, 1967

COMET, Mu2e

- Neutrinoless Double-Electron Capture (0v2EC)

$$e_b^- + e_b^- + (A, Z) \to (A, Z-2)**$$

2007.14908

- Trimuon production from neutrino-neutron collision

$$\nu_\mu N \to \mu^- \mu^+ \mu^+ X$$

Flanz, Rodejohann, Zuber, 9907203

- $e - p$ collider:

$$e^\pm p \to \bar{\nu}_e(\nu_e)\ell^\pm \ell'^\pm X$$

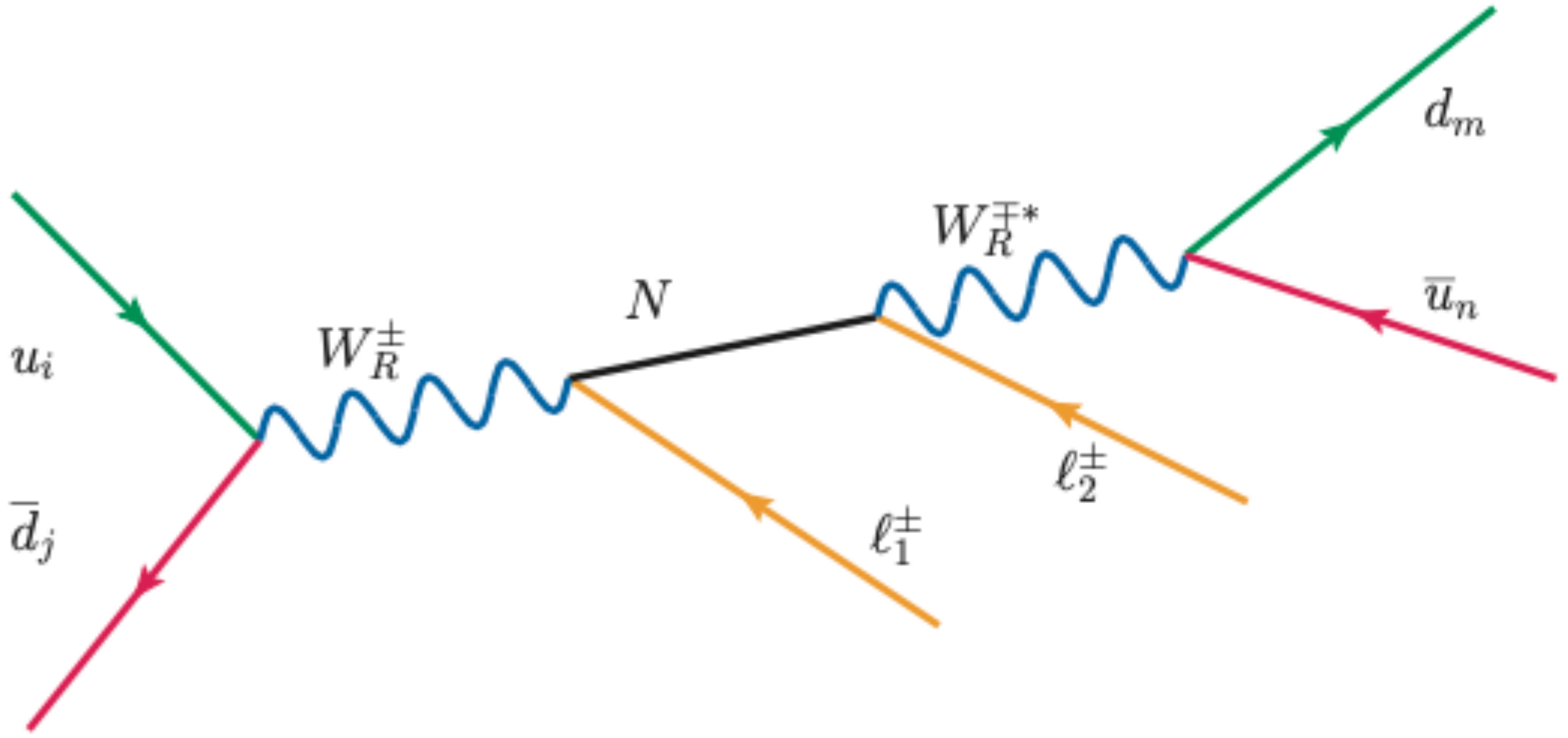
Flanz, Rodejohann, Zuber, 9911298

- LHC search:

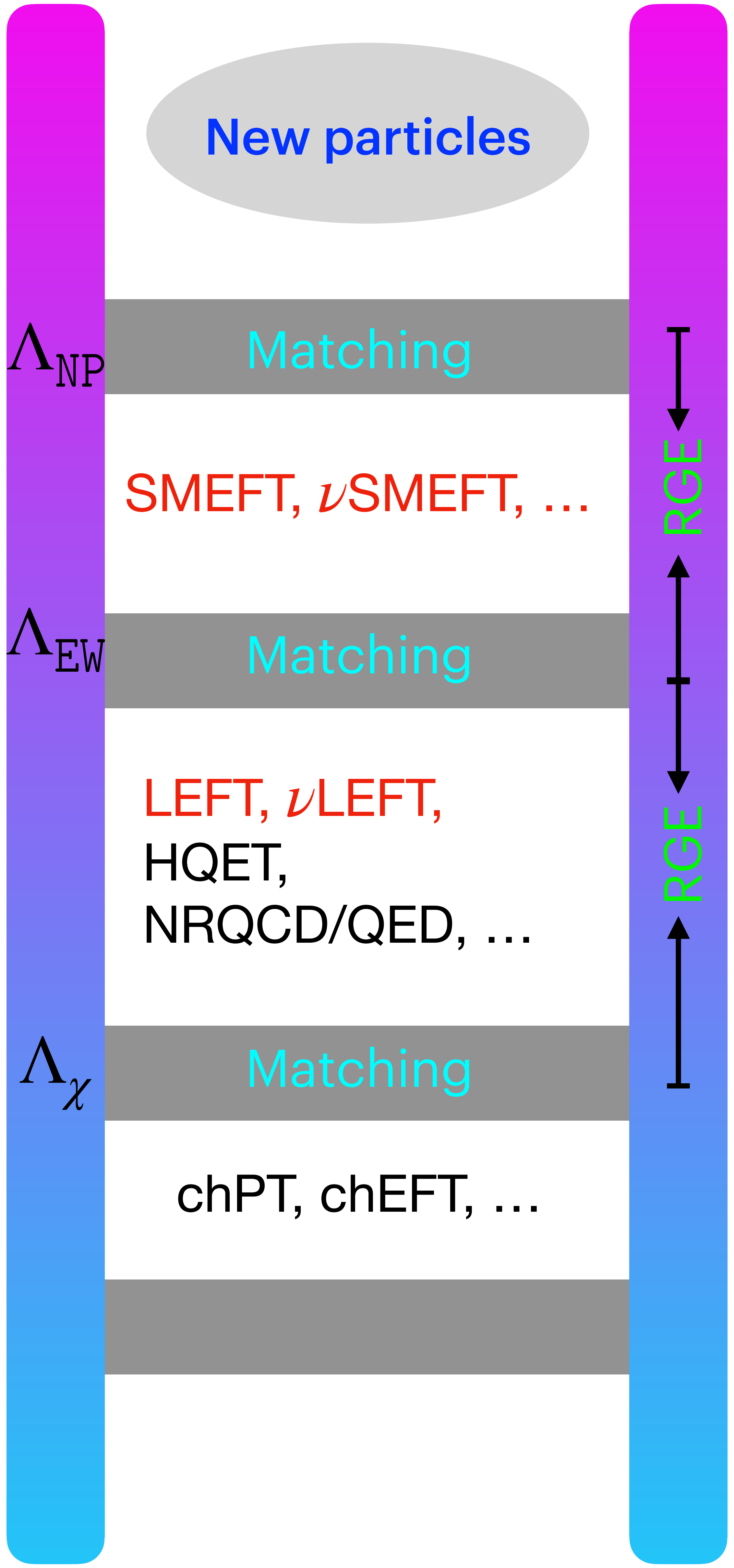
$$pp \to \ell^\pm \ell'^\pm + nj + X$$

Cai, Han, Li, Ruiz, 1711.02180

....



EFT as a powerful tool for low-energy LNV processes



Assumption: scales are well separated with $\Lambda_{NP} \gg \Lambda_{EW}$

Parametrize the derivation of low energy observables w.r.t. the SM prediction by non-SM interactions based on SM particles and symmetries

SMEFT-like framework

Study NP effect in low energy observables indirectly

SMEFT framework

Weinberg, 1979 | Buchmuller & Wyler, 1986; Warsaw basis, 2010 | Lehman, 2014; **Liao & Ma, 2016** | Murphy, 2020; Li et al, 2020 | Li et al, 2020; **Liao & Ma, 2020** | Harlander et al, 2023

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \mathscr{L}_{\rm dim5} + \mathscr{L}_{\rm dim6} + \mathscr{L}_{\rm dim7} + \mathscr{L}_{\rm dim8} + \mathscr{L}_{\rm dim9} + \mathscr{L}_{\rm dim10-12} \cdots$$

$$\mathscr{L}_{\rm dim5} = \frac{\hat{C}_{LH}}{\Lambda} \epsilon_{im}\epsilon_{jn}(\overline{L_i^{\rm C}} L_j)\tilde{H}_n \tilde{H}_m + {\rm h.c.},$$

Hilbert series method: *Henning, Lu, Melia, Murayama 2015, 2017*

- Matter fields: $Q, L; u, d, e; H$
- Gauge symmetry: $\mathrm{SU(3)_c} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y}$
- $D \in$ even (odd) if $|B-L|/2$ is even (odd)
- $D = 6 : |B-L| = 0$ vs $D = 5, 7 : |B-L| = 2$
- $D \in$ odd: B/L is violated Kobach, 1604.05726

$\Delta L = 2$ processes such as $0\nu\beta\beta$ are described by odd-D operators

Operator basis @ dim 7

$\psi^2 H^4$ + h.c.		$\psi^2 H^3 D$ + h.c.	
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^i C L^m)H^j H^n (H^\dagger H)$	$\mathcal{O}_{LeHD}$	$\epsilon_{ij}\epsilon_{mn}(L^i C\gamma_\mu e)H^j H^m iD^\mu H^n$
$\psi^2 H^2 D^2$ + h.c.		$\psi^2 H^2 X$ + h.c.	
$\mathcal{O}_{LHD1}$ $\mathcal{O}_{LHD2}$	$\epsilon_{ij}\epsilon_{mn}(L^i C D^\mu L^j)H^m(D_\mu H^n)$ $\epsilon_{im}\epsilon_{jn}(L^i C D^\mu L^j)H^m(D_\mu H^n)$	$\mathcal{O}_{LHB}$ $\mathcal{O}_{LHW}$	$\epsilon_{ij}\epsilon_{mn}(L^i C\sigma_{\mu\nu}L^m)H^j H^n B^{\mu\nu}$ $\epsilon_{ij}(\epsilon\tau^I)_{mn}(L^i C\sigma_{\mu\nu}L^m)H^j H^n W^{I\mu\nu}$
$\psi^4 D$ + h.c.		$\psi^4 H$ + h.c.	
$\mathcal{O}_{\bar{d}uLLD}$ $\mathcal{O}_{\bar{L}QddD}$ $\mathcal{O}_{\bar{e}dddD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C iD^\mu L^j)$ $(\bar{L}\gamma_\mu Q)(dCiD^\mu d)$ $(\bar{e}\gamma_\mu d)(dCiD^\mu d)$	$\mathcal{O}_{\bar{e}LLLH}$ $\mathcal{O}_{\bar{d}LQLH1}$ $\mathcal{O}_{\bar{d}LQLH2}$ $\mathcal{O}_{\bar{d}LueH}$ $\mathcal{O}_{\bar{Q}uLLH}$ $\mathcal{O}_{\bar{L}dud\tilde{H}}$ $\mathcal{O}_{\bar{L}dddH}$ $\mathcal{O}_{\bar{e}Qdd\tilde{H}}$ $\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^j C L^m)H^n$ $\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^j C L^m)H^n$ $\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^j C L^m)H^n$ $\epsilon_{ij}(\bar{d}L^i)(uCe)H^j$ $\epsilon_{ij}(\bar{Q}u)(LCL^i)H^j$ $(\bar{L}d)(uCd)\tilde{H}$ $(\bar{L}d)(dCd)H$ $\epsilon_{ij}(\bar{e}Q^i)(dCd)\tilde{H}^j$ $\epsilon_{ij}(\bar{L}d)(QCQ^i)\tilde{H}^j$
$\mathcal{O}^{(2)}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C\sigma^{\mu\nu}D_\nu L^j)$	$\mathcal{O}_{\bar{L}dQdD}$	$(\bar{L}iD^\mu d)(QC\gamma_\mu d)$

Counting operators depend on flavors

Provide leading-order $\Delta L = 2$ interactions beyond the Weinberg operator

$13(B) + 7(\not{B})$ given by [L. Lehman:1410.4193]. Yi Liao & XDM 1607.07309

Dim-7 Majorana neutrino mass operator;

Baryon number violating operators with $\Delta B = -\Delta L = 1$;

Redundant operators.

Flavor-specific basis

Classes	Original operator basis in [17, 18]	Basis in [19, 20]	Relations of the two different notations	WC name in D7RGESolver
Dim-7 SMEFT operators: $(\Delta B, \Delta L) = (0, 2)$				
$\psi^2 H^4$	$\mathcal{O}_{LH}^{pr} = \epsilon_{ij}\epsilon_{mn}\left(\overline{L_p^{i\mathtt{C}}} L_r^m\right) H^j H^n (H^\dagger H)$	$\mathcal{O}_{LH}^{pr}$	same	LH_pr
$\psi^2 H^3 D$	$\mathcal{O}_{LeHD}^{pr} = \epsilon_{ij}\epsilon_{mn}\left(\overline{L_p^{i\mathtt{C}}}\gamma_\mu e_r\right) H^j H^m iD^\mu H^n$	$\mathcal{O}_{LeDH}^{pr}$	$= \mathcal{O}_{LeHD}^{pr}$	LeDH_pr
$\psi^2 H^2 D^2$	$\mathcal{O}_{LHD1}^{pr} = \epsilon_{ij}\epsilon_{mn}\left(\overline{L_p^{i\mathtt{C}}} D^\mu L_r^j\right) H^m D_\mu H^n$	$\mathcal{O}_{DLDH1}^{pr}$	$= \frac{1}{2}\left(\mathcal{O}_{LHD1}^{pr} + \mathcal{O}_{LHD1}^{rp}\right)$	DLDH1_pr
	$\mathcal{O}_{LHD2}^{pr} = \epsilon_{im}\epsilon_{jn}\left(\overline{L_p^{i\mathtt{C}}} D^\mu L_r^j\right) H^m D_\mu H^n$	$\mathcal{O}_{DLDH2}^{pr}$	$= \frac{1}{2}\left(\mathcal{O}_{LHD2}^{pr} + \mathcal{O}_{LHD2}^{rp}\right)$	DLDH2_pr
$\psi^2 H^2 X$	$\mathcal{O}_{LHB}^{pr} = \epsilon_{ij}\epsilon_{mn}\left(\overline{L_p^{i\mathtt{C}}}\sigma_{\mu\nu} L_r^m\right) H^j H^n B^{\mu\nu}$	$\mathcal{O}_{LHB}^{pr}$	same	LHB_pr
	$\mathcal{O}_{LHW}^{pr} = \epsilon_{ij}(\epsilon\tau^I)_{mn}\left(\overline{L_p^{i\mathtt{C}}}\sigma_{\mu\nu} L_r^m\right) H^j H^n W^{I\mu\nu}$	$\mathcal{O}_{LHW}^{pr}$	same	LHW_pr
$\psi^4 H$	$\mathcal{O}_{\bar{e}LLLH}^{prst} = \epsilon_{ij}\epsilon_{mn}\left(\overline{e_p} L_r^i\right)\left(\overline{L_s^{j\mathtt{C}}} L_t^m\right) H^n$	$\mathcal{O}_{\bar{e}LLLH}^{(\mathtt{S}),prst}$	$= \frac{1}{6}\left(\mathcal{O}_{\bar{e}LLLH}^{prst} + \mathcal{O}_{\bar{e}LLLH}^{pstr} + \mathcal{O}_{\bar{e}LLLH}^{ptrs}\right) + s \leftrightarrow t$	eLLLHS_prst
		$\mathcal{O}_{\bar{e}LLLH}^{(\mathtt{A}),prst}$	$= \frac{1}{6}\left(\mathcal{O}_{\bar{e}LLLH}^{prst} + \mathcal{O}_{\bar{e}LLLH}^{pstr} + \mathcal{O}_{\bar{e}LLLH}^{ptrs}\right) - s \leftrightarrow t$	eLLLHA_prst
		$\mathcal{O}_{\bar{e}LLLH}^{(\mathtt{M}),prst}$	$= \frac{1}{3}\left(\mathcal{O}_{\bar{e}LLLH}^{prst} + \mathcal{O}_{\bar{e}LLLH}^{psrt}\right) - t \leftrightarrow r$	eLLLHM_prst
	$\mathcal{O}_{\bar{d}LQLH1}^{prst} = \epsilon_{ij}\epsilon_{mn}\left(\overline{d_p} L_r^i\right)\left(\overline{Q_s^{j\mathtt{C}}} L_t^m\right) H^n$	$\mathcal{O}_{\bar{d}LQLH1}^{prst}$	same	dLQLH1_prst
	$\mathcal{O}_{\bar{d}LQLH2}^{prst} = \epsilon_{im}\epsilon_{jn}\left(\overline{d_p} L_r^i\right)\left(\overline{Q_s^{j\mathtt{C}}} L_t^m\right) H^n$	$\mathcal{O}_{\bar{d}LQLH2}^{prst}$	same	dLQLH2_prst
	$\mathcal{O}_{\bar{d}LueH}^{prst} = \epsilon_{ij}\left(\overline{d_p} L_r^i\right)\left(\overline{u_s^{\mathtt{C}}} e_t\right) H^j$	$\mathcal{O}_{\bar{d}LueH}^{prst}$	same	dLueH_prst
	$\mathcal{O}_{\bar{Q}uLLH}^{prst} = \epsilon_{ij}\left(\overline{Q_p} u_r\right)\left(\overline{L_s^{\mathtt{C}}} L_t^i\right) H^j$	$\mathcal{O}_{\bar{Q}uLLH}^{prst}$	same	QuLLH_prst
$\psi^4 D$	$\mathcal{O}_{\bar{d}uLLD}^{prst} = \epsilon_{ij}\left(\overline{d_p}\gamma_\mu u_r\right)\left(\overline{L_s^{i\mathtt{C}}} iD^\mu L_t^j\right)$	$\mathcal{O}_{\bar{d}uLDL}^{prst}$	$= \frac{1}{2}\left(\mathcal{O}_{\bar{d}uLLD}^{prst} + \mathcal{O}_{\bar{d}uLLD}^{prts}\right)$	duLDL_prst
Dim-7 SMEFT operators: $(\Delta B, \Delta L) = (1, -1)$				
$\psi^4 H$	$\mathcal{O}_{\bar{L}dud\tilde{H}}^{prst} = \epsilon_{\alpha\beta\gamma}\left(\overline{L_p} d_r^\alpha\right)\left(\overline{u_s^{\beta\mathtt{C}}} d_t^\gamma\right)\tilde{H}$	$\mathcal{O}_{\bar{L}dud\tilde{H}}^{prst}$	same	LdudH_prst
	$\mathcal{O}_{\bar{L}dddH}^{prst} = \epsilon_{\alpha\beta\gamma}\left(\overline{L_p} d_r^\alpha\right)\left(\overline{d_s^{\beta\mathtt{C}}} d_t^\gamma\right) H$	$\mathcal{O}_{\bar{L}dddH}^{(\mathtt{M}),prst}$	$= \frac{1}{3}\left(\mathcal{O}_{\bar{L}dddH}^{prst} + \mathcal{O}_{\bar{L}dddH}^{psrt}\right) - s \leftrightarrow t$	LdddHM_prst
	$\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{prst} = \epsilon_{ij}\epsilon_{\alpha\beta\gamma}\left(\overline{e_p} Q_r^{i\alpha}\right)\left(\overline{d_s^{\beta\mathtt{C}}} d_t^\gamma\right)\tilde{H}^j$	$\mathcal{O}_{\bar{e}Qdd\tilde{H}}^{prst}$	same	eQddH_prst
	$\mathcal{O}_{\bar{L}dQQ\tilde{H}}^{prst} = \epsilon_{ij}\epsilon_{\alpha\beta\gamma}\left(\overline{L_p} d_r^\alpha\right)\left(\overline{Q_s^{\beta\mathtt{C}}} Q_t^{i\gamma}\right)\tilde{H}^j$	$\mathcal{O}_{\bar{L}dQQ\tilde{H}}^{prst}$	same	LdQQH_prst
$\psi^4 D$	$\mathcal{O}_{\bar{e}dddD}^{prst} = \epsilon_{\alpha\beta\gamma}\left(\overline{e_p}\gamma_\mu d_r^\alpha\right)\left(\overline{d_s^{\beta\mathtt{C}}} iD^\mu d_t^\gamma\right)$	$\mathcal{O}_{\bar{e}ddDd}^{prst}$	$= \frac{1}{6}\left(\mathcal{O}_{\bar{e}dddD}^{prst} + \mathcal{O}_{\bar{e}dddD}^{ptrs} + \mathcal{O}_{\bar{e}dddD}^{pstr}\right) + s \leftrightarrow t$	eddDd_prst
	$\mathcal{O}_{\bar{L}QddD}^{prst} = \epsilon_{\alpha\beta\gamma}\left(\overline{L_p}\gamma_\mu Q_r^\alpha\right)\left(\overline{d_s^{\beta\mathtt{C}}} iD^\mu d_t^\gamma\right)$	$\mathcal{O}_{\bar{L}QdDd}^{prst}$	$= \frac{1}{2}\left(\mathcal{O}_{\bar{L}QddD}^{prst} + \mathcal{O}_{\bar{L}QddD}^{prts}\right)$	LQdDd_prst

- Organized in terms of operators with manifest flavor symmetries
- Easy to write down the RGEs in a non-redundant way
- Adopted notation in our code

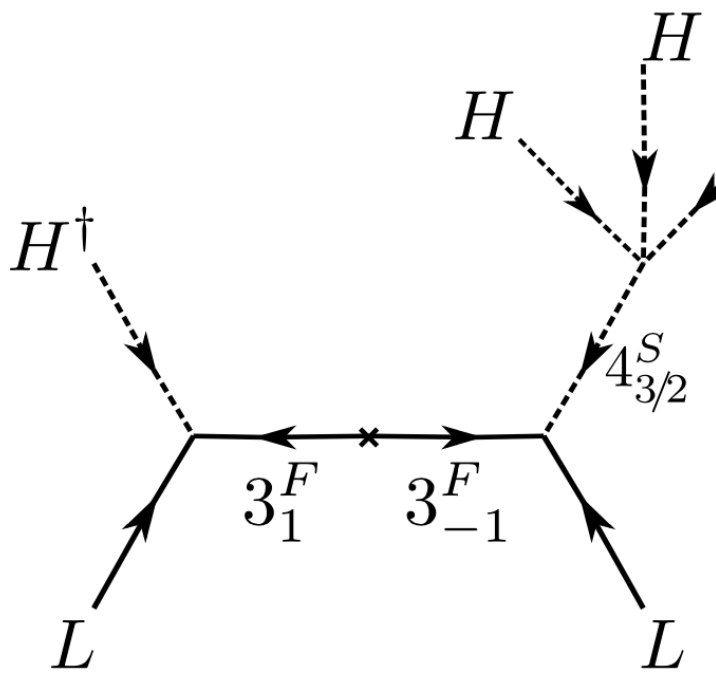
Yi Liao, XDM: 1901.10302
D. Zhang: 2310.11055

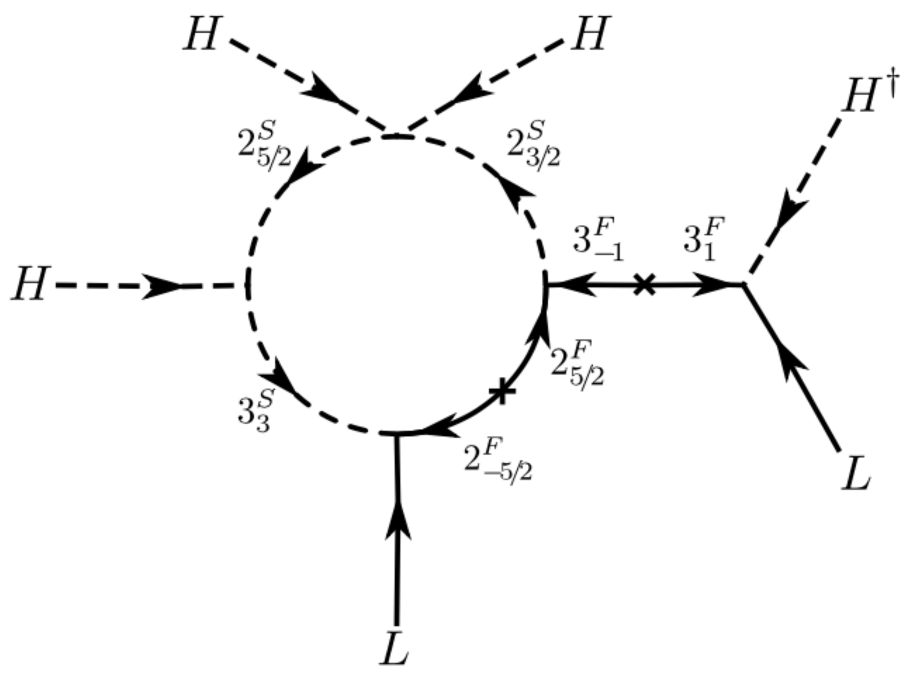
UV completion of dim-7 operators

EFT operators ⇔ UV models

Tree-level ⇒ 1-loop ⇒ 2-loop

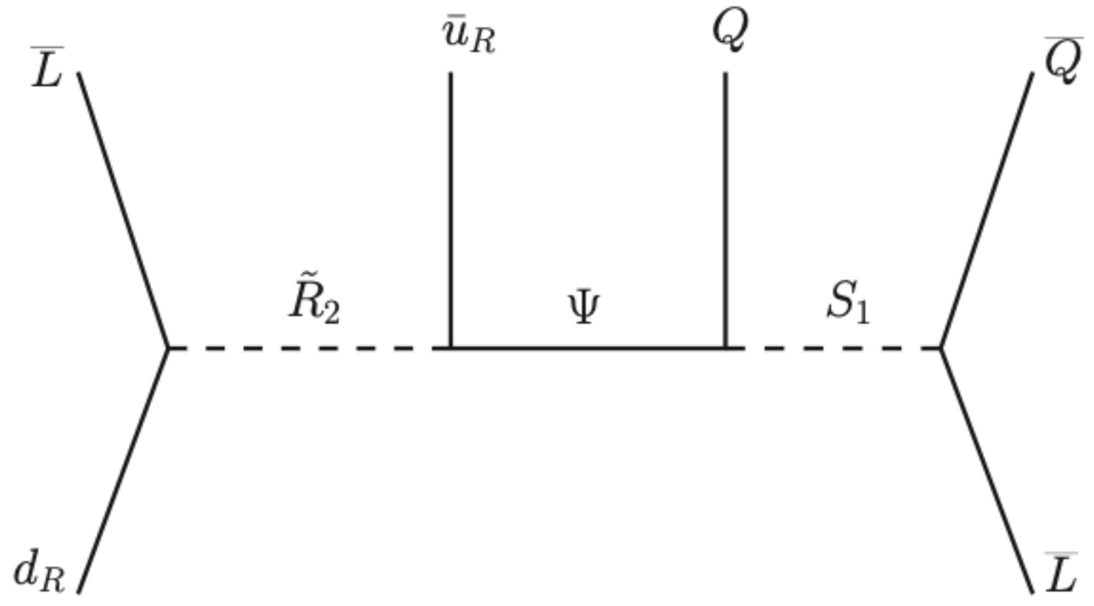
- ✱ Usually, assume the SM gauge symmetry intact
- ✱ Internal heavy fields: scalar, fermion, vector





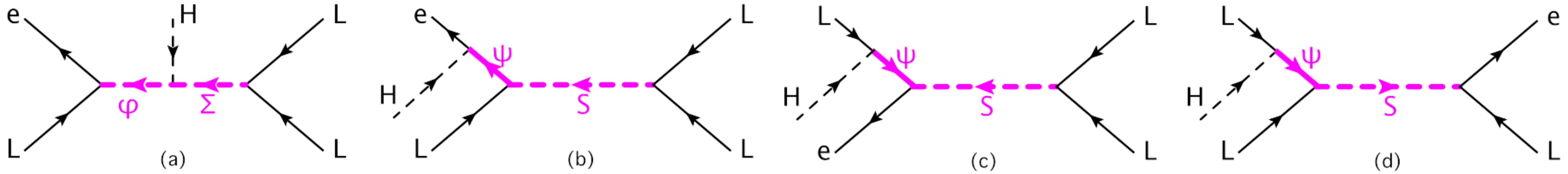
Anamiati et al: 1806.07264
Bonnet et al: 1204.5862
Cai et a : 1706.08524
Hirsch: 1411.7038, …

- Generate topologies and diagrams
- Assign external and internal fields
- Select genuine topologies



Chen, Ding, Yao: 2110.15347, 2301.02503

Li, Zhao, Yu: 2311.10079



Yi Liao, XDM, Quan-Yu Wnag: 2005.08013

RG running effects in phenomenological studies

- Improve perturbative calculation;
- Operator renormalization ⇒ **operator mixing effects**
- RGE effects are crucial for precise low energy analyses

- Dim-6 RGEs: 1308.2627, 1310.4838, 1312.2014
- Automatic RGE solver: DsixTools (1704.04504), wilson (1804.05033)
- Extensive phenomenological analyses incorporating RGEs

- Dim-5 RGEs: hep-ph/9309223, hep-ph/0108005
- Dim-7 RGEs: 1607.07309 , 1901.10302, 2306.03008, 2310.11055
- Automatic RGE solver: D7RGESolver (2505.06499)

Process relevant to dim-5/7 SMEFT
•Neutrino masses
•Neutrinoless double beta decay
•LNV signals at collider
•LNV meson and charged lepton decay
•Majorana neutrino magnetic moment
……

The accurate solutions of dim-5&7 RGEs are necessary for physical analysis

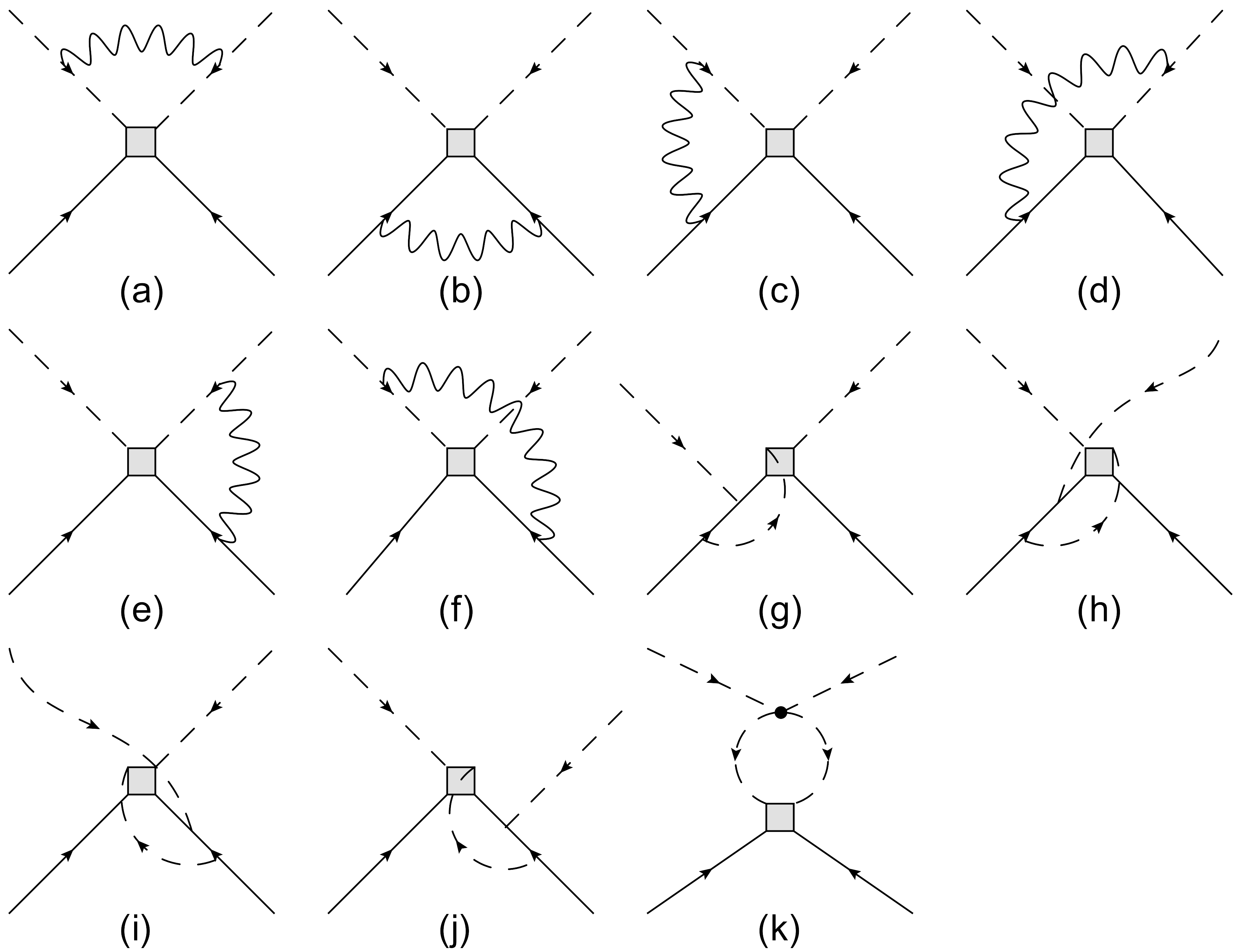
RGEs of dim-5 operators

$\mathcal{O}_5 = \epsilon_{ij}\epsilon_{mn}(L^iCL^m)H^jH^n$

$$16\pi^2\mu\frac{d}{d\mu}C_5^{pr} = (4\lambda + 2Y - 3g_2^2)C_5^{pr} - \frac{3}{2}\left[(Y_eY_e^\dagger)_{vp}C_5^{vr} + p \to r\right]$$

$$Y \equiv \mathrm{Tr}\left[3(Y_u^\dagger Y_u) + 3(Y_d^\dagger Y_d) + (Y_e^\dagger Y_e)\right]$$

K. S. Babu et al: hep-ph/9309223
S. Antusch et al: hep-ph/0108005



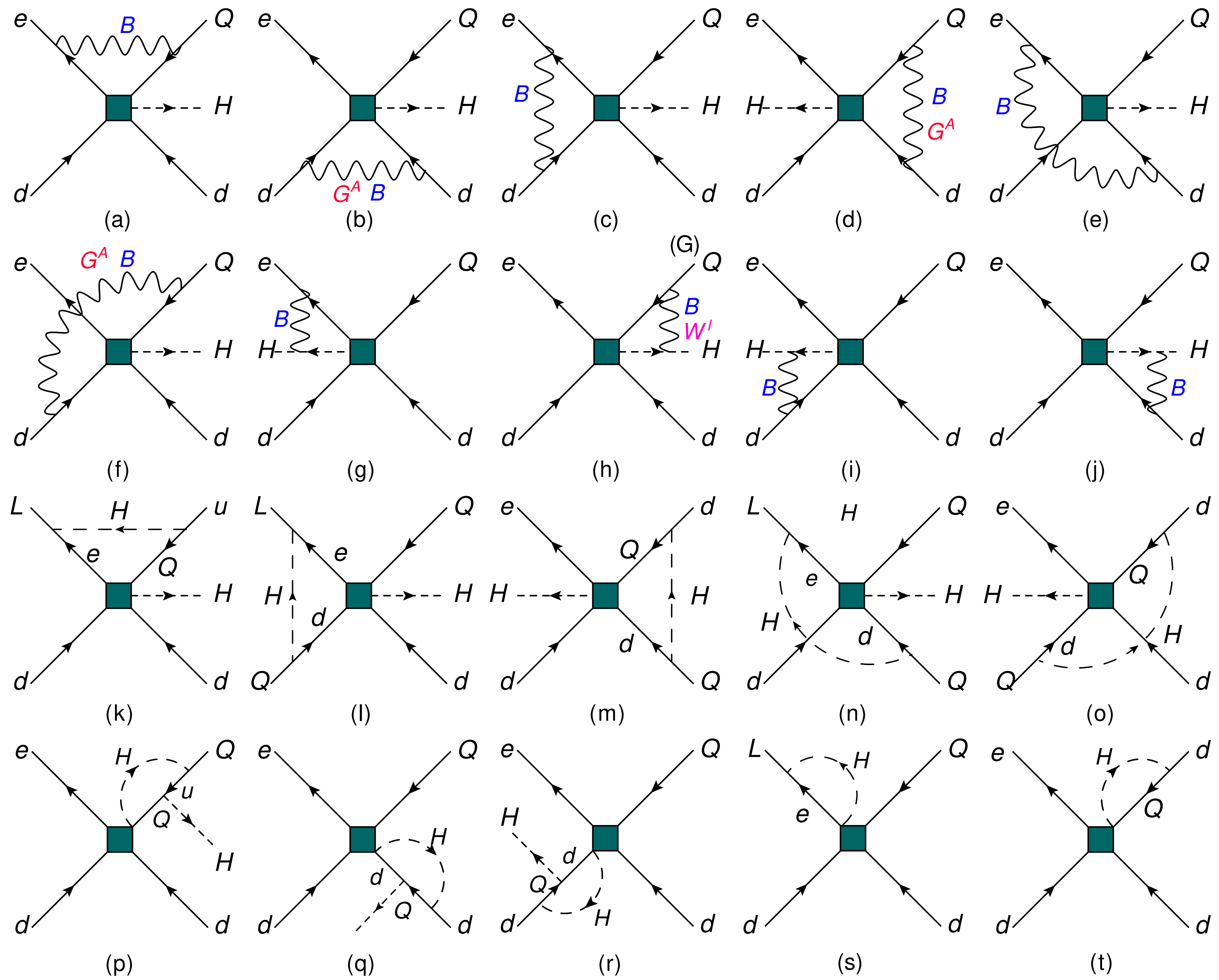
No g_1^2 terms—cancelled

$$16\pi^2\mu\frac{d}{d\mu}C_{LH}^{d\,pr} = \left[(3d^2-18d+19)\lambda - \frac{3}{4}(d-5)g_1^2 - \frac{3}{4}(3d-11)g_2^2 + (d-3)W_H\right]C_{LH}^{d\,pr} - \frac{3}{2}\left[(Y_eY_e^\dagger)_{vp}C_{LH}^{d\,vr} + (Y_eY_e^\dagger)_{vr}C_{LH}^{d\,pv}\right],$$

Yi Liao, XDM: 1701.08019

$$\mathcal{O}^d_{\rm n.m.} = [(L^{\rm T}\epsilon H)C(L^{\rm T}\epsilon H)](H^\dagger H)^{(d-5)/2}$$

RGEs of dim-7 $\Delta L = 2$ operators



$$
\begin{aligned}
\langle (C\mathcal{O})_{\bar{d}LueH}\rangle^{(k)}\delta &= -\frac{1}{2}(Y_d)_{pv}(Y_e^\dagger)_{ws}C^{vtrw}_{\bar{d}LueH}\mathcal{O}^{prst}_{\bar{d}LueH},\\
\langle (C\mathcal{O})_{\bar{d}LueH}\rangle^{(l)}\delta &= (Y_u^\dagger)_{vs}(Y_e^\dagger)_{wt}C^{prvw}_{\bar{d}LueH}\left(\mathcal{O}^{prst}_{\bar{d}LQLH1}-\mathcal{O}^{prst}_{\bar{d}LQLH2}\right),\\
\langle (C\mathcal{O})_{\bar{d}LueH}\rangle^{(m)}\delta &= -\frac{1}{2}(Y_e)_{vt}(Y_e^\dagger)_{wr}C^{pvsw}_{\bar{d}LueH}\mathcal{O}^{prst}_{\bar{d}LueH},\\
\langle (C\mathcal{O})_{\bar{d}LueH}\rangle^{(n)}\delta &= -(Y_d^\dagger Y_d)_{pv}C^{vrst}_{\bar{d}LueH}\mathcal{O}^{prst}_{\bar{d}LueH},\\
\langle (C\mathcal{O})_{\bar{d}LueH}\rangle^{(o)}\delta &= -(Y_u^\dagger Y_u)_{vs}C^{prvt}_{\bar{d}LueH}\mathcal{O}^{prst}_{\bar{d}LueH},\\
\langle (C\mathcal{O})_{\bar{d}LueH}\rangle^{(p)}\delta &= (Y_e Y_e^\dagger)_{vr}C^{pvst}_{\bar{d}LueH}\mathcal{O}^{prst}_{\bar{d}LueH},\\
\langle (C\mathcal{O})_{\bar{d}LueH}\rangle^{(q)}\delta &= -\frac{1}{2}(Y_e^\dagger Y_e)_{vt}C^{prsv}_{\bar{d}LueH}\mathcal{O}^{prst}_{\bar{d}LueH},\\
\langle (C\mathcal{O})_{\bar{d}LueH}\rangle^{(r)}\delta &= 3(Y_u^\dagger Y_d)_{vw}C^{wpvr}_{\bar{d}LueH}\mathcal{O}^{pr}_{LeHD},
\end{aligned}
$$

	$(\omega,\bar\omega)\|\chi$	$(5,3)\|3$	$(5,3)\|3$	$(5,3)\|3$	$(5,5)\|2$	$(5,5)\|2$	$(5,5)\|2$	$(7,3)\|2$	$(7,3)\|2$	$(7,3)\|2$	$(7,3)\|3$	$(7,3)\|3$	$(7,5)\|1$
$(\omega,\bar\omega)\|\chi$	γ_{ij}	$\mathcal{O}_{LHD1}$	$\mathcal{O}_{LHD2}$	$\mathcal{O}_{\bar{d}uLLD}$	$\mathcal{O}_{LeHD}$	$\mathcal{O}_{\bar{d}LueH}$	$\mathcal{O}_{\bar{Q}uLLH}$	$\mathcal{O}_{\bar{e}LLLH}$	$\mathcal{O}_{\bar{d}LQLH1}$	$\mathcal{O}_{\bar{d}LQLH2}$	$\mathcal{O}_{LHB}$	$\mathcal{O}_{LHW}$	$\mathcal{O}_{LH}$
$(5,3)\|3$	$\mathcal{O}_{LHD1}$	g^2	g^2	g^2	0	0	0	0	0	0	0	0	0
$(5,3)\|3$	$\mathcal{O}_{LHD2}$	g^2	g^2	0	0	0	0	0	0	0	0	0	0
$(5,3)\|3$	$\mathcal{O}_{\bar{d}uLLD}$	g^2	g^2	g^2	0	0	0	0	0	0	0	0	0
$(5,5)\|2$	$\mathcal{O}_{LeHD}$	g^3	g^3	0	g^2	g^2	0	0	0	0	$\Sigma\to 0$	$\Sigma\to 0$	0
$(5,5)\|2$	$\mathcal{O}_{\bar{d}LueH}$	g^3	g^3	g^3	g^2	g^2	g^2	0	Y_uY_e	Y_uY_e	0	0	0
$(5,5)\|2$	$\mathcal{O}_{\bar{Q}uLLH}$	g^3	g^3	g^3	0	g^2	g^2	Y_uY_e	Y_uY_d	Y_uY_d	0	0	0
$(7,3)\|2$	$\mathcal{O}_{\bar{e}LLLH}$	g^3	g^3	0	0	0	$Y_u^\dagger Y_e^\dagger$	g^2	g^2	g^2	g^3	g^3	0
$(7,3)\|2$	$\mathcal{O}_{\bar{d}LQLH1}$	g^3	g^3	g^3	0	$Y_u^\dagger Y_e^\dagger$	$Y_u^\dagger Y_d^\dagger$	g^2	g^2	g^2	g^3	g^3	0
$(7,3)\|2$	$\mathcal{O}_{\bar{d}LQLH2}$	g^3	g^3	g^3	0	$Y_u^\dagger Y_e^\dagger$	$Y_u^\dagger Y_d^\dagger$	g^2	g^2	g^2	g^3	g^3	0
$(7,3)\|3$	$\mathcal{O}_{LHB}$	g^2	g^2	0	$\Sigma\to 0$	0	0	g^1	g^1	0	g^2	g^2	0
$(7,3)\|3$	$\mathcal{O}_{LHW}$	g^2	g^2	0	$\Sigma\to 0$	0	0	g^1	g^1	g^1	g^2	g^2	0
$(7,5)\|1$	$\mathcal{O}_{LH}$	g^4	g^4	0	g^3	0	g^3	g^3	g^3	0	0	g^4	g^2

Rich operator mixing effects

RG mixing between dim-5 and dim-7 operators

$$
\begin{aligned}
\dot{C}^{pr}_{LH5} = {}& \frac{1}{2}(-3g^2+4\lambda+2W_H)C^{pr}_{LH5} - \frac{3}{2}[C_{LH5}Y_lY_l^\dagger]^{pr} + \mu_h^2\{8C^{pr}_{LH} + 2[C_{LeDH}Y_l^\dagger]^{pr}\\
&+\frac{3}{2}g^2\left(2C^{pr}_{DLDH1}+C^{pr}_{DLDH2}\right) + [C_{DLDH1}Y_lY_l^\dagger]^{pr} - \frac{1}{2}[C_{DLDH2}Y_lY_l^\dagger]^{pr}\\
&-[Y_l]_{ts}\left(3C^{(\mathtt{S}),stpr}_{\bar{e}LLLH}+2C^{(\mathtt{M}),stpr}_{\bar{e}LLLH}\right) - 3[Y_d]_{ts}C^{sptr}_{\bar{d}LQLH1} + 6[Y_u^\dagger]_{st}C^{tspr}_{\bar{Q}uLLH}\} + p\leftrightarrow r\,.
\end{aligned}
$$

Yi Liao, XDM: 1901.10302

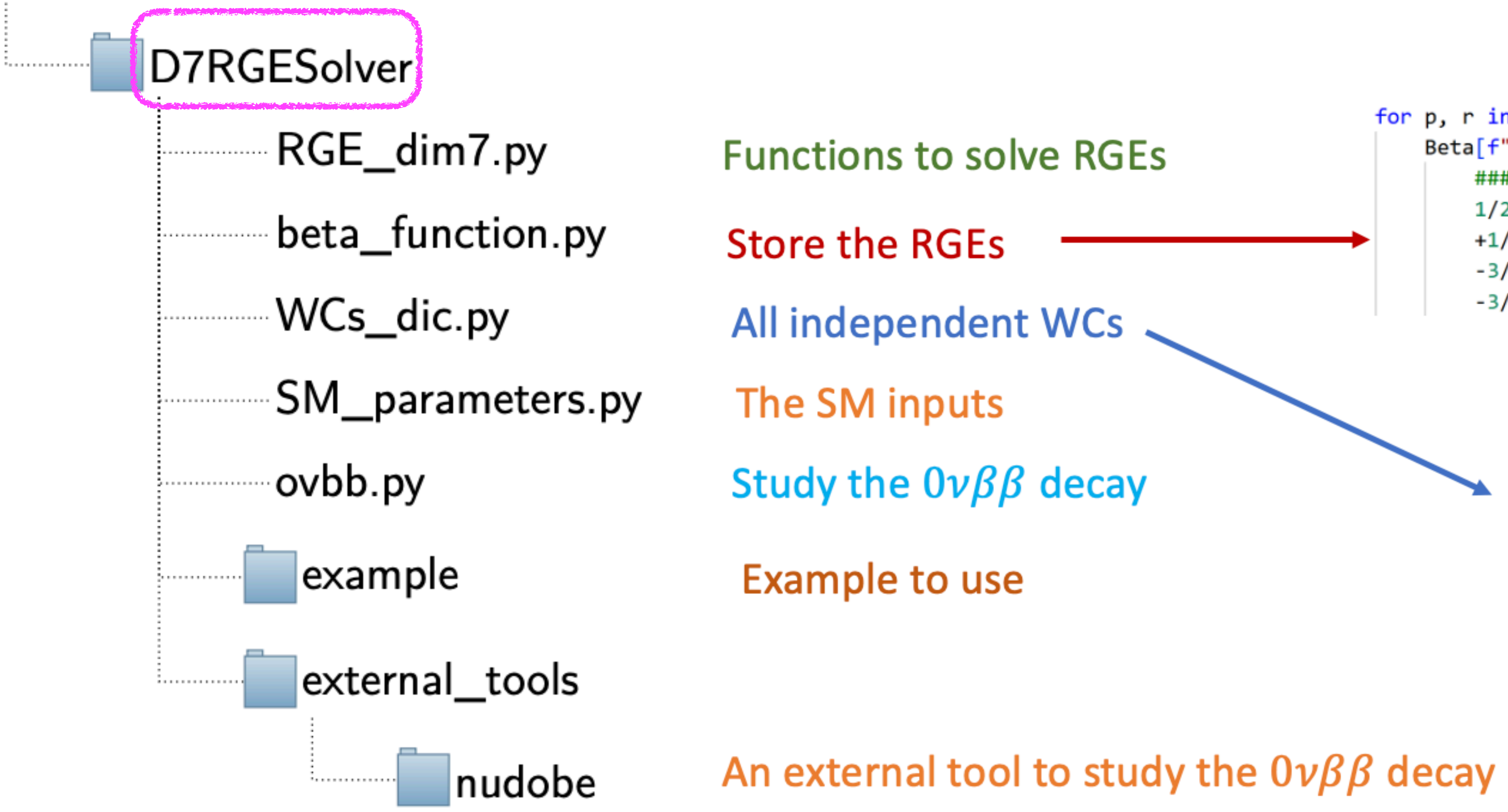
D. Zhang: 2310.11055; 2306.03008

D7RGESolver — a python package for solving dim5&7 RGEs

https://github.com/ZhaoXiang210/D7RGESolver

Yi Liao, XDM, Hao-Lin Wang, Xiang Zhao, 2505.06499

Solve the complete RGEs of dim-5&7 operators




```
for p, r in indices:
    Beta[f"LH5_{p}{r}"] = (
        ###### dim-5 contribution to LH5
        1/2*(-3*g**2 + 4*Lambda + 2*WH)*C_LH5[p-1,r-1]
        +1/2*(-3*g**2 + 4*Lambda + 2*WH)*C_LH5[r-1,p-1]
        -3/2*(C_LH5 @ Yl @ Yl.conj().T)[p-1,r-1]
        -3/2*(C_LH5 @ Yl @ Yl.conj().T)[r-1,p-1]
```

```
"LH5_11": 0,
"LH5_12": 0,
"LH5_13": 0,
"LH5_22": 0,
"LH5_23": 0,
"LH5_33": 0,
```

Oliver Scholer et al., 2304.05415

The structure of the D7RGESolver package.

D7RGESolver — a python package for solving dim5&7 RGEs

- SM inputs at electroweak scale
- The RGEs of the SM parameters:

  one loop + two loop

- Dim-5&7 operators: only focus on independent flavors
- Two choices of quark flavor basis:

Up-quark flavor basis: $Y_u = \frac{\sqrt{2}}{v} M_u, \quad Y_d = \frac{\sqrt{2}}{v} V_{\rm CKM} M_d.$

Down-quark flavor basis: $Y_u = \frac{\sqrt{2}}{v} V^\dagger_{\rm CKM} M_u, \quad Y_d = \frac{\sqrt{2}}{v} M_d.$

$$Y_l = \frac{\sqrt{2}}{v} M_e$$

Input notation in the code

Class	2F	2FS	2FA	4F3S	4F3A	4F2S
#	9	6	3	30	3	54
WC in the code	LeDH_pr LHW_pr	LH5_pr LH_pr DLDH1_pr DLDH2_pr	LHB_pr	eLLLHS_prst eddDd_prst	eLLLHA_prst	duLDL_prst LQdDd_prst
sym.	\	pr = rp	pr = −rp	prst = prts = ptsr	prst = −prts = −ptsr	prst = prts
Adopted flavors	{p, r}	{1, 1} {1, 2} {1, 3} {2, 2} {2, 3} {3, 3}	{1, 2} {1, 3} {2, 3}	{p, 1, 1, 1} {p, 1, 1, 2} {p, 1, 1, 3} {p, 1, 2, 2} {p, 1, 2, 3} {p, 1, 3, 3} {p, 2, 2, 2} {p, 2, 2, 3} {p, 2, 3, 3} {p, 3, 3, 3}	{p, 1, 2, 3}	{p, r, 1, 1} {p, r, 1, 2} {p, r, 1, 3} {p, r, 2, 2} {p, r, 2, 3} {p, r, 3, 3}

Diagonal at the electroweak scale ⇒ become non-diagonal at other scales

D7RGESolver — Example for its basic usage

Run $\mathcal{O}^{11}_{LH5}$ and $\mathcal{O}^{11}_{DLDH1}$ from 10 TeV to 80 GeV

```
from RGE_dim7 import solve_rge, print_WCs
C_in = {"LH5_11": 1e-15+0j,"DLDH1_11": 1e-15+0j}          # Input the WCs
scale_in = 1e4                          # Input energy scale in units of GeV
scale_out = 80                          # Output energy scale in units of GeV
```

```
C_out = solve_rge(scale_in, scale_out, C_in, basis="down", method="integrate")
```

Main function

method="leadinglog" for fast calculation

```
print_WCs(C_out)
```

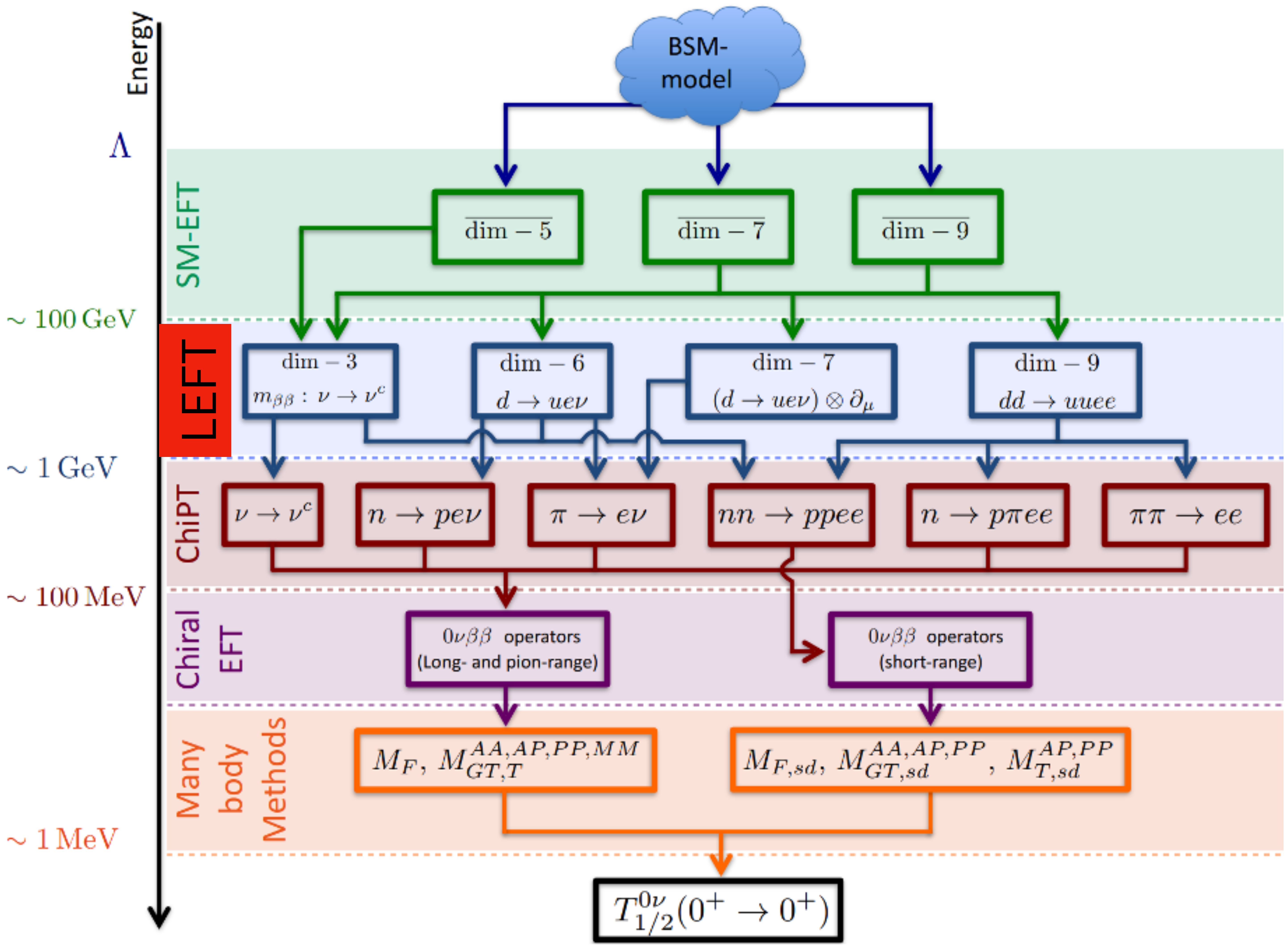
```
## Wilson coefficients

**EFT:** ‘SMEFT‘
| WC name | Value |
|-------------------|-----------------------------------------------|
| ‘LH5_11‘ | (5.51729127984111e-13-1.1697182471147425e-28j) |
| ‘LH_11‘ | (-2.7271477267102585e-18+1.9639321856872016e-33j) |
| ‘DLDH1_11‘ | (8.183376576714679e-16+1.539722194653356e-39j) |
| ‘DLDH2_11‘ | (8.253541710427263e-17-3.157387110573996e-39j) |
| ‘QuLLH_3311‘ | (-2.976790814412032e-17+6.616906663619594e-33j) |
| ‘LHW_11‘ | (-3.399190512252408e-18-1.1024467172929897e-33j) |
| ... | ... |
```

Enhanced due to mixing from $\mathcal{O}^{11}_{DLDH1}$

- Dim-5 and -7 Weinberg operators on top
- Other dim-7 ones are in descending order of absolute values

Application to $0\nu\beta\beta$



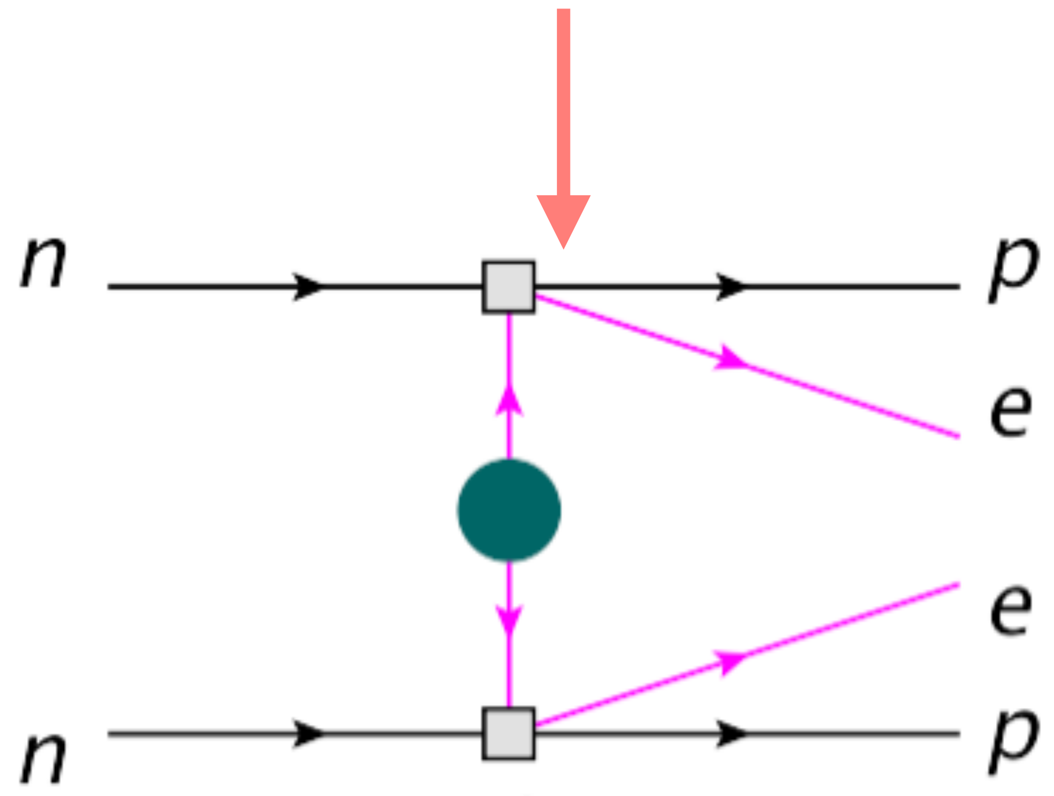
V. Cirigliano et al.,1806.02780

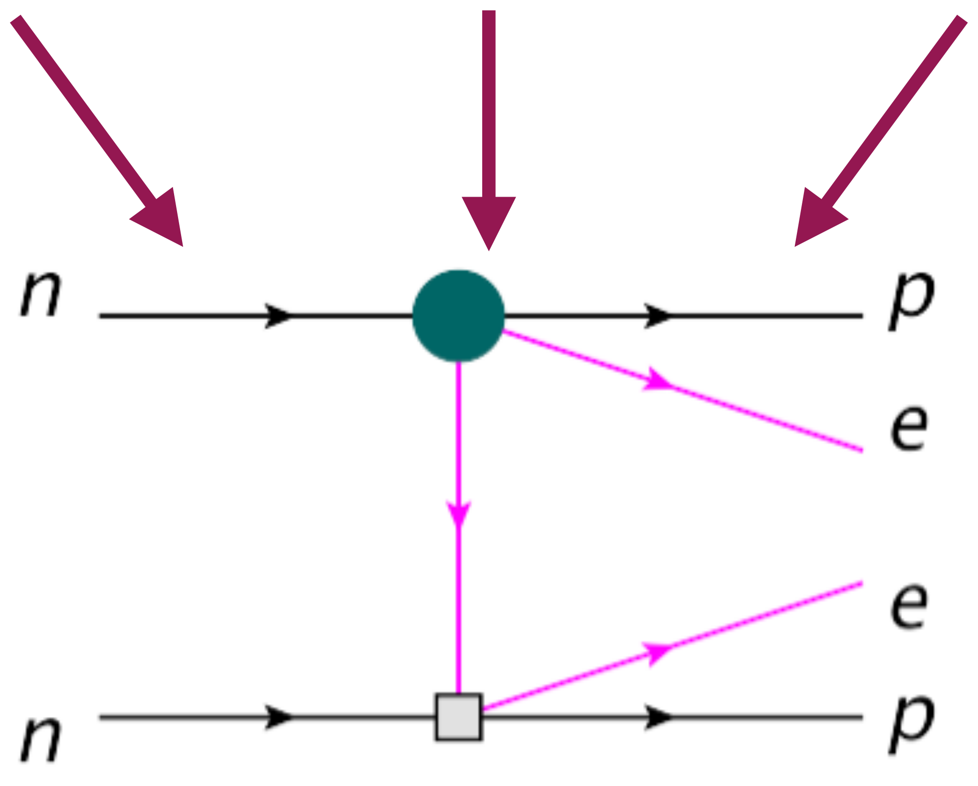
Low-energy EFT (LEFT) below electroweak scale

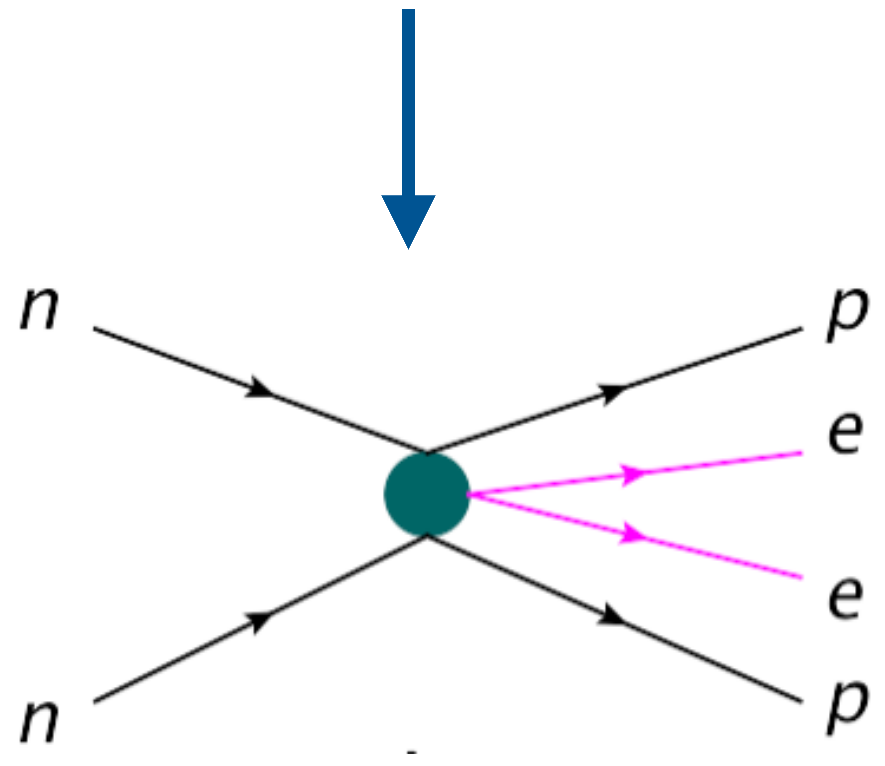
$$u,\ d, s,\ c,\ b;\ \ e,\ \mu,\ \tau;\ \ \nu_e,\ \nu_\mu,\ \nu_\tau$$

$$SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{em}$$

$$\mathcal{L}^{0\nu\beta\beta}_{\mathrm{LEFT}} = \mathcal{L}^{(3)}_{\Delta L=2} + \mathcal{L}^{(6)}_{\Delta L=2} + \mathcal{L}^{(7)}_{\Delta L=2} + \mathcal{L}^{(8)}_{\Delta L=2} + \mathcal{L}^{(9)}_{\Delta L=2} + \cdots ,$$







Mass mechanism | Long-distance | Short-distance

V. Cirigliano et al.,1806.02780

L.Graf, F. Deppisch, F. Iachello, J. Kotila:1806.06058

LEFT description of $0\nu\beta\beta$ process due to dim-7 operators

V. Cirigliano et al.,1806.02780

$$\mathcal{L}^{(3)}_{\Delta L=2} = -\frac{1}{2} m_{\beta\beta}\, \overline{\nu^{\mathtt{C}}_{\mathrm{L},e}} \nu_{\mathrm{L},e} + \mathrm{h.c.}\,, \quad 1$$

$$\mathcal{L}^{(6)}_{\Delta L=2} = \sqrt{2} G_F \Big[C^{(6)}_{\mathrm{VL}} (\overline{u}_{\mathrm{L}} \gamma^\mu d_{\mathrm{L}})(\overline{e}_{\mathrm{R}} \gamma_\mu \nu^{\mathtt{C}}_{\mathrm{L},e}) + C^{(6)}_{\mathrm{VR}} (\overline{u}_{\mathrm{R}} \gamma^\mu d_{\mathrm{R}})(\overline{e}_{\mathrm{R}} \gamma_\mu \nu^{\mathtt{C}}_{\mathrm{L},e}) \quad 6$$
$$+ C^{(6)}_{\mathrm{SR}} (\overline{u}_{\mathrm{L}} d_{\mathrm{R}})(\overline{e}_{\mathrm{L}} \nu^{\mathtt{C}}_{\mathrm{L},e}) + C^{(6)}_{\mathrm{SL}} (\overline{u}_{\mathrm{R}} d_{\mathrm{L}})(\overline{e}_{\mathrm{L}} \nu^{\mathtt{C}}_{\mathrm{L},e}) + C^{(6)}_{\mathrm{T}} (\overline{u}_{\mathrm{L}} \sigma^{\mu\nu} d_{\mathrm{R}})(\overline{e}_{\mathrm{L}} \sigma_{\mu\nu} \nu^{\mathtt{C}}_{\mathrm{L},e}) \Big] + \mathrm{h.c.}\,,$$

$$\mathcal{L}^{(7)}_{\Delta L=2} = \frac{\sqrt{2} G_F}{v} \Big[C^{(7)}_{\mathrm{VL}} (\overline{u}_{\mathrm{L}} \gamma^\mu d_{\mathrm{L}})(\overline{e}_{\mathrm{L}} i \overleftrightarrow{\partial}_\mu \nu^{\mathtt{C}}_{\mathrm{L},e}) + C^{(7)}_{\mathrm{VR}} (\overline{u}_{\mathrm{R}} \gamma^\mu d_{\mathrm{R}})(\overline{e}_{\mathrm{L}} i \overleftrightarrow{\partial}_\mu \nu^{\mathtt{C}}_{\mathrm{L},e}) \Big] + \mathrm{h.c.}\,, \quad 2$$

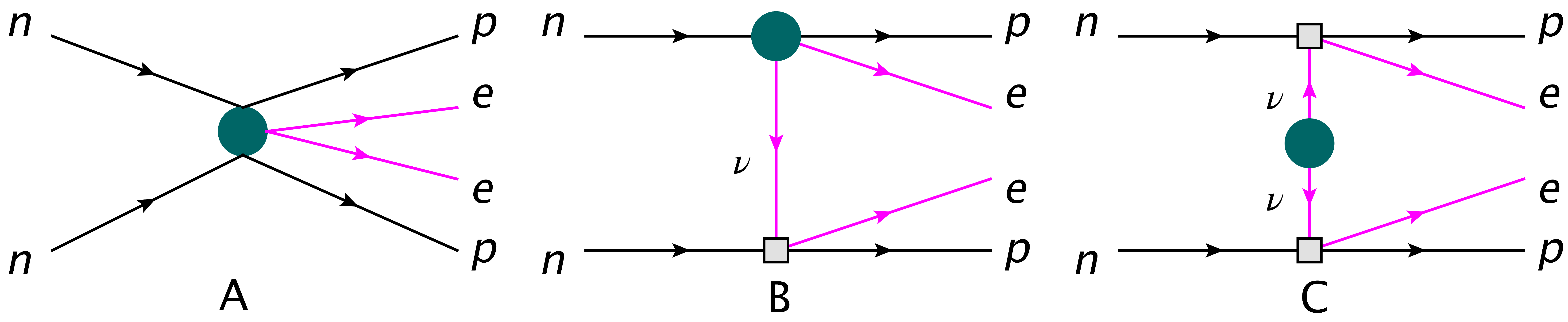
$$\mathcal{L}^{(9)}_{\Delta L=2} = \frac{1}{v^5} \Big[C^{(9)}_{1\mathrm{L}} (\overline{u}_{\mathrm{L}} \gamma_\mu d_{\mathrm{L}})(\overline{u}_{\mathrm{L}} \gamma^\mu d_{\mathrm{L}})(\overline{e_{\mathrm{L}}} e^{\mathtt{C}}_{\mathrm{L}}) + C^{(9)}_{4\mathrm{L}} (\overline{u}_{\mathrm{L}} \gamma_\mu d_{\mathrm{L}})(\overline{u}_{\mathrm{R}} \gamma^\mu d_{\mathrm{R}})(\overline{e_{\mathrm{L}}} e^{\mathtt{C}}_{\mathrm{L}}) \Big] + \mathrm{h.c.}\,, \quad 2$$

Yi Liao, XDM, Quan-Yu Wang, 2020

SMEFT @dim 7

↓

A limited number of LEFT operators are required.



Short-distance

Long-distance

Mass mechanism

Matching onto the LEFT

Tree-level matching: 1 dim-5 and 11 dim-7 operators $\Rightarrow 0\nu\beta\beta$

Basis	LEFT operators	Matching results at electroweak scale $\Lambda_{\rm EW}$
Up flavor basis	$\mathcal{O}^{(6)}_{\rm SL} = \sqrt{2}G_F(\overline{u_{\rm R}}d_{\rm L})(\overline{e_{\rm L}}\nu^{\rm c}_{{\rm L},e})$	$C^{(6)}_{\rm SL} = v^3\left(\frac{1}{\sqrt{2}}V_{w1}C^{w111*}_{\bar{Q}uLLH} + \frac{1}{2v}m_u V_{ud}C^{11*}_{DLDH2}\right)$
	$\mathcal{O}^{(6)}_{\rm SR} = \sqrt{2}G_F(\overline{u_{\rm L}}d_{\rm R})(\overline{e_{\rm L}}\nu^{\rm c}_{{\rm L},e})$	$C^{(6)}_{\rm SR} = v^3\left(\frac{1}{2\sqrt{2}}C^{1111*}_{\bar{d}LQLH1} - \frac{1}{2v}m_d V_{ud}C^{11*}_{DLDH2}\right)$
	$\mathcal{O}^{(6)}_{\rm T} = \sqrt{2}G_F(\overline{u_{\rm L}}\sigma^{\mu\nu}d_{\rm R})(\overline{e_{\rm L}}\sigma_{\mu\nu}\nu^{\rm c}_{{\rm L},e})$	$C^{(6)}_{\rm T} = v^3\left(\frac{1}{8\sqrt{2}}C^{1111*}_{\bar{d}LQLH1} + \frac{1}{4\sqrt{2}}C^{1111*}_{\bar{d}LQLH2}\right)$
	$\mathcal{O}_{m\beta\beta} = -\frac{1}{2}m_{\beta\beta}\overline{\nu^{\rm c}_{{\rm L},e}}\nu_{L,e}$	$m_{\beta\beta} = -v^2C^{11}_{LH5} - \frac{1}{2}v^4C^{11}_{LH}$
	$\mathcal{O}^{(6)}_{\rm VL} = \sqrt{2}G_F(\overline{u_{\rm L}}\gamma^\mu d_{\rm L})(\overline{e_{\rm R}}\gamma_\mu\nu^{\rm c}_{{\rm L},e})$	$C^{(6)}_{\rm VL} = v^3V_{ud}\left(-\frac{1}{\sqrt{2}}C^{11*}_{LeDH} + 4\frac{m_e}{v}g^{-1}C^{11*}_{LHW}\right)$
	$\mathcal{O}^{(6)}_{\rm VR} = \sqrt{2}G_F(\overline{u_{\rm R}}\gamma^\mu d_{\rm R})(\overline{e_{\rm R}}\gamma_\mu\nu^{\rm c}_{{\rm L},e})$	$C^{(6)}_{\rm VR} = \frac{v^3}{2\sqrt{2}}C^{1111*}_{\bar{d}LueH}$
	$\mathcal{O}^{(7)}_{\rm VL} = \frac{\sqrt{2}}{v}G_F(\overline{u_{\rm L}}\gamma^\mu d_{\rm L})(\overline{e_{\rm L}}\overleftrightarrow{\partial}_\mu\nu^{\rm c}_{{\rm L},e})$	$C^{(7)}_{\rm VL} = -v^3V_{ud}\left(C^{11*}_{DLDH1} + \frac{1}{2}C^{11*}_{DLDH2} + 4g^{-1}C^{11*}_{LHW}\right)$
	$\mathcal{O}^{(7)}_{\rm VR} = \frac{\sqrt{2}}{v}G_F(\overline{u_{\rm R}}\gamma^\mu d_{\rm R})(\overline{e_{\rm L}}\overleftrightarrow{\partial}_\mu\nu^{\rm c}_{{\rm L},e})$	$C^{(7)}_{\rm VR} = -v^3C^{1111*}_{\bar{d}uLDL}$
	$\mathcal{O}^{(9)}_{\rm 1L} = \frac{1}{v^5}(\overline{u_{\rm L}}\gamma_\mu d_{\rm L})(\overline{u_{\rm L}}\gamma^\mu d_{\rm L})(\overline{e_{\rm L}}e^{\rm c}_{\rm L})$	$C^{(9)}_{\rm 1L} = -v^3V^2_{ud}\left(2C^{11*}_{DLDH1} + 8g^{-1}C^{11*}_{LHW}\right)$
	$\mathcal{O}^{(9)}_{\rm 4L} = \frac{1}{v^5}(\overline{u_{\rm L}}\gamma_\mu d_{\rm L})(\overline{u_{\rm R}}\gamma^\mu d_{\rm R})(\overline{e_{\rm L}}e^{\rm c}_{\rm L})$	$C^{(9)}_{\rm 4L} = -2v^3V_{ud}C^{1111*}_{\bar{d}uLDL}$
Down flavor basis	$\mathcal{O}^{(6)}_{\rm SL} = \sqrt{2}G_F(\overline{u_{\rm R}}d_{\rm L})(\overline{e_{\rm L}}\nu^{\rm c}_{{\rm L},e})$	$C^{(6)}_{\rm SL} = v^3\left(\frac{1}{\sqrt{2}}C^{1111*}_{\bar{Q}uLLH} + \frac{1}{2v}m_u V_{ud}C^{11*}_{DLDH2}\right)$
	$\mathcal{O}^{(6)}_{\rm SR} = \sqrt{2}G_F(\overline{u_{\rm L}}d_{\rm R})(\overline{e_{\rm L}}\nu^{\rm c}_{{\rm L},e})$	$C^{(6)}_{\rm SR} = v^3\left(\frac{1}{2\sqrt{2}}V_{1w}C^{11w1*}_{\bar{d}LQLH1} - \frac{1}{2v}m_d V_{ud}C^{11*}_{DLDH2}\right)$
	$\mathcal{O}^{(6)}_{\rm T} = \sqrt{2}G_F(\overline{u_{\rm L}}\sigma^{\mu\nu}d_{\rm R})(\overline{e_{\rm L}}\sigma_{\mu\nu}\nu^{\rm c}_{{\rm L},e})$	$C^{(6)}_{\rm T} = v^3V_{1w}\left(\frac{1}{8\sqrt{2}}C^{11w1*}_{\bar{d}LQLH1} + \frac{1}{4\sqrt{2}}C^{11w1*}_{\bar{d}LQLH2}\right)$

Depending on the convention of the relationship between mass vs flavor eigenstates

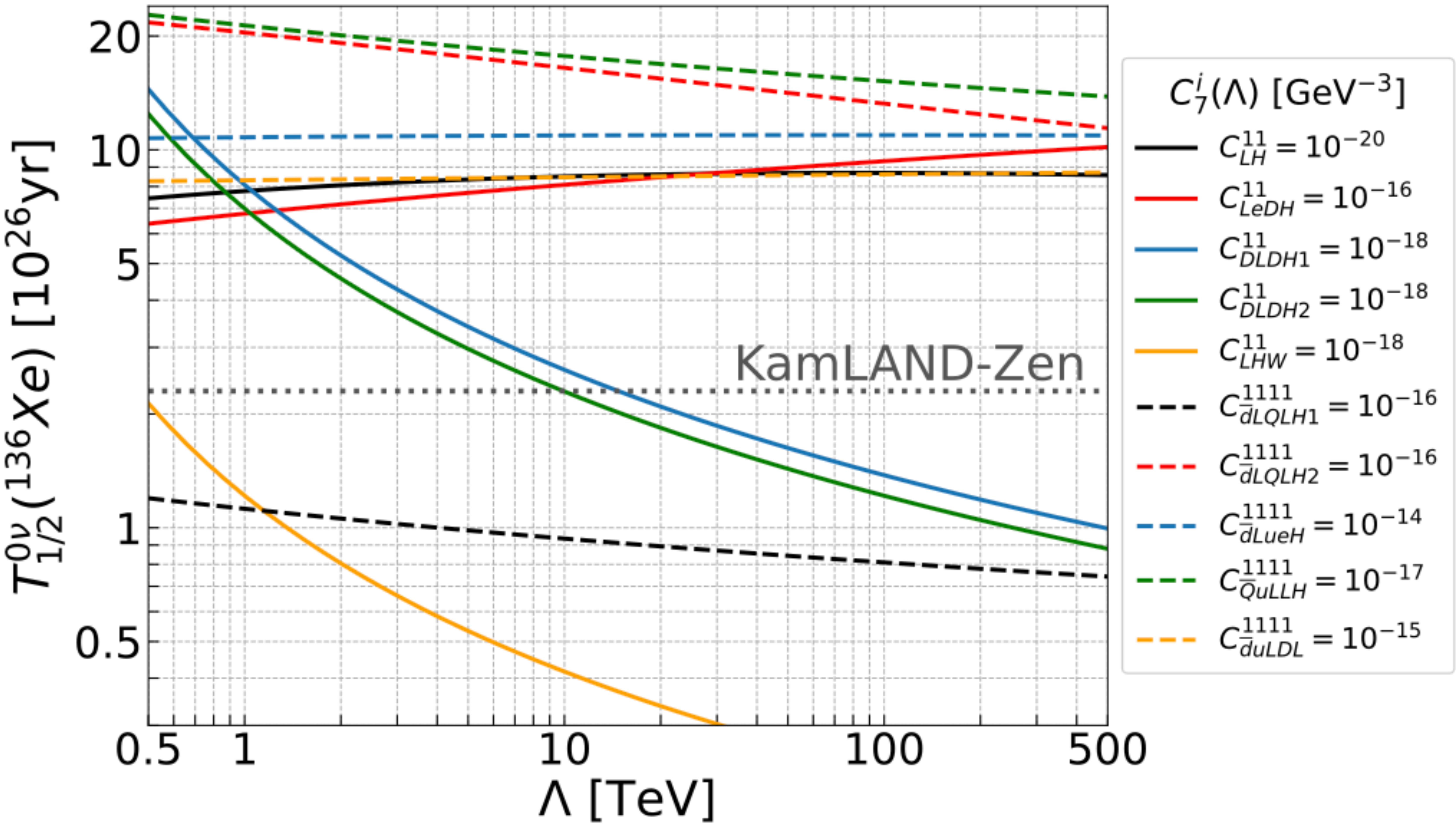
Up-quark flavor basis

$$Y_u = \frac{\sqrt{2}}{v}M_u, \quad Y_d = \frac{\sqrt{2}}{v}V_{\rm CKM}M_d\,.$$

Down-quark flavor basis

$$Y_u = \frac{\sqrt{2}}{v}V^\dagger_{\rm CKM}M_u, \quad Y_d = \frac{\sqrt{2}}{v}M_d\,.$$

RG effects on the half lifetime of $^{136}\mathrm{Xe}$ $0\nu\beta\beta$ decay



Yi Liao, XDM, Hao-Lin Wang, Xiang Zhao, 2505.06499

Large impact from

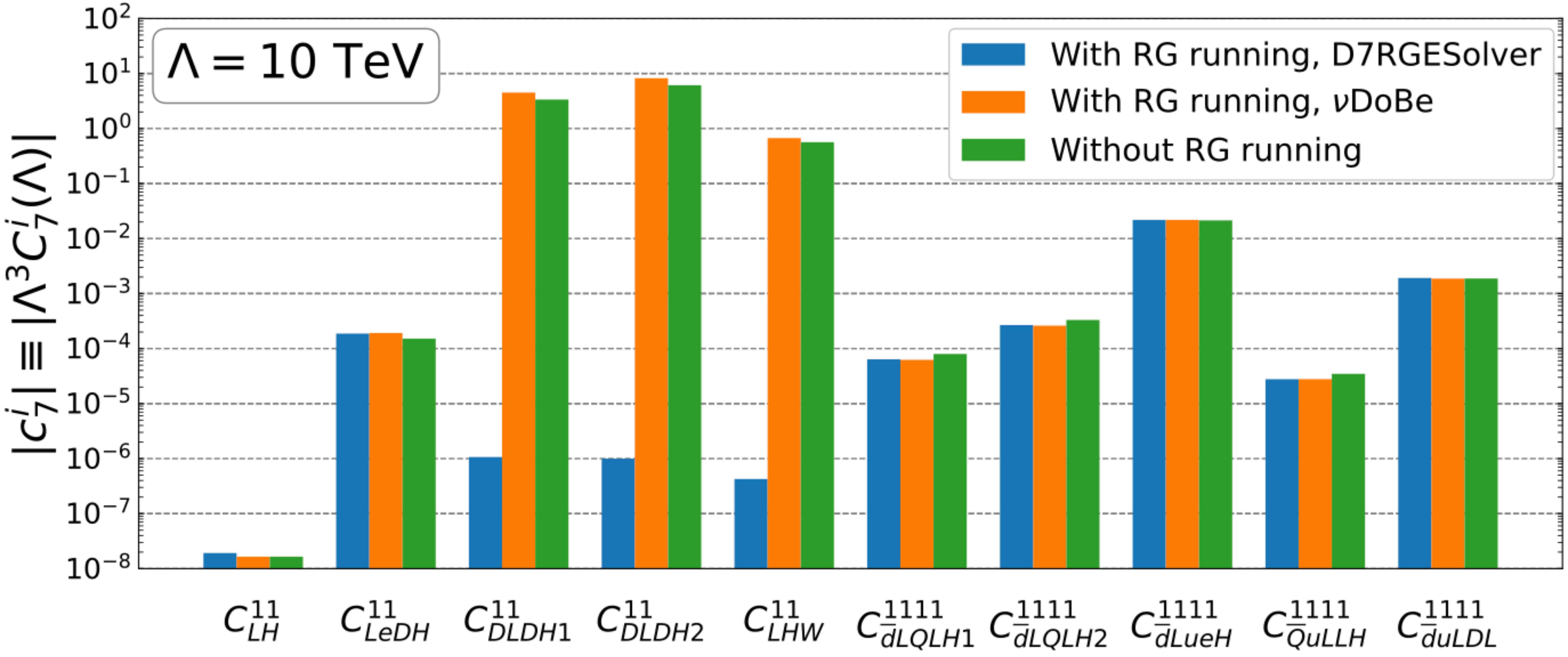
$\mathcal{O}^{11}_{DLDH1,2}$, $\mathcal{O}^{11}_{LHW}$

Mixing into $\mathcal{O}_{LH(5)}$ operator

RGE effect plays a significant role in predicting the $0\nu\beta\beta$ decay half lifetime

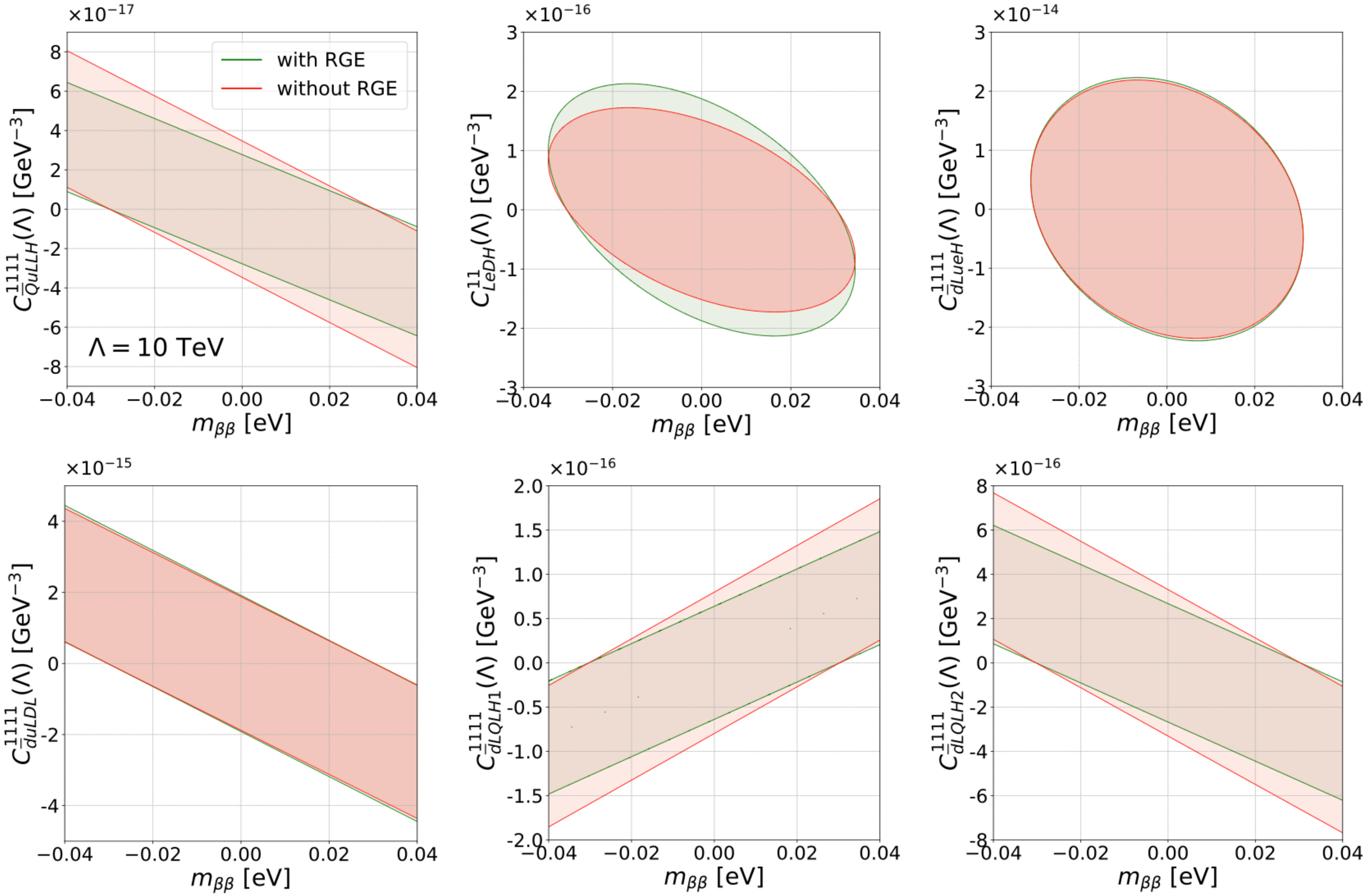
O. Scholer, J. de Vries and L. Gráf, νDoBe, 2306.08709
Di Zhang, 2310.11055
L. Gráf, C. Hati, A. Martín-Galán and O. Scholer, 2504.00081

Constraints on 10 1st generation operators



- KamLAND-Zen experimental limit $[T_{1/2}^{0\nu}]_{\texttt{SMEFT}}(c_i) \gtrsim [T_{1/2}^{0\nu}]_{\texttt{KamLAND}}^{90\,\%} = 2.3 \times 10^{26}\,\text{yr.}$
- The full and precise RGE evolution of these operators has a significant impact on their numerical constraints
- Mainly from their contributions to the $\mathcal{O}_{LH(5)}$ through RGE mixing

2D constraints in the plane $m_{\beta\beta} - C_7^i$



- KamLAND-Zen experimental limit

$$[T^{0\nu}_{1/2}]_{\texttt{SMEFT}}(c_i) \gtrsim [T^{0\nu}_{1/2}]^{90\,\%}_{\texttt{KamLAND}} = 2.3 \times 10^{26}\,\mathrm{yr.}$$

- RG effect is negligible—$\mathcal{O}^{11}_{\bar{d}LueH,\bar{d}uLDL}$

- Ellipse: $\Gamma^{-1} \propto m^2_{\beta\beta} + c\,|C^i_7|^2$

- Band: $\Gamma^{-1} \propto |m_{\beta\beta} + cC^i_7|^2$

The mixing of these operators and the neutrino mass operators from RGE evolution is negligible

RGE improved constraints

RGE-improved constraints on the dimensionless WCs c_7^i at $\Lambda = 10$ TeV				
Operator	$\lvert c_7^i\rvert \equiv \lvert\Lambda^3 C_7^i(\Lambda)\rvert$ Same in both bases	Operator	$\lvert c_7^i\rvert \equiv \lvert\Lambda^3 C_7^i(\Lambda)\rvert$ Up-quark flavor basis	Down-quark flavor basis
$\mathcal{O}_{LH}^{11}$	1.92×10^{-8}	$\mathcal{O}_{\bar{Q}uLLH}^{1111}$	2.85×10^{-5}	2.78×10^{-5}
$\mathcal{O}_{LeDH}^{11}$	1.88×10^{-4}	$\mathcal{O}_{\bar{Q}uLLH}^{1211}$	8.58×10^{5}	4.48×10^{-4}
$\mathcal{O}_{DLDH1}^{11}$	1.07×10^{-6}	$\mathcal{O}_{\bar{Q}uLLH}^{1311}$	3.51	6.50×10^{-6}
$\mathcal{O}_{DLDH2}^{11}$	9.99×10^{-7}	$\mathcal{O}_{\bar{Q}uLLH}^{2111}$	1.24×10^{-4}	2.65×10^{-1}
$\mathcal{O}_{LHW}^{11}$	4.26×10^{-7}	$\mathcal{O}_{\bar{Q}uLLH}^{2211}$	1.00×10^{-4}	1.03×10^{-4}
$\mathcal{O}_{\bar{e}LLLH}^{(M),2112}$	1.76×10^{-3}	$\mathcal{O}_{\bar{Q}uLLH}^{2311}$	3.01×10^{-1}	1.43×10^{-6}
$\mathcal{O}_{\bar{e}LLLH}^{(M),3113}$	1.05×10^{-4}	$\mathcal{O}_{\bar{Q}uLLH}^{3111}$	3.24×10^{-3}	5.38×10^{-2}
$\mathcal{O}_{\bar{e}LLLH}^{(S),1111}$	2.59×10^{-1}	$\mathcal{O}_{\bar{Q}uLLH}^{3211}$	5.65×10^{1}	2.38×10^{-3}
$\mathcal{O}_{\bar{e}LLLH}^{(S),2112}$	1.25×10^{-3}	$\mathcal{O}_{\bar{Q}uLLH}^{3311}$	5.90×10^{-8}	5.90×10^{-8}
$\mathcal{O}_{\bar{e}LLLH}^{(S),3113}$	7.44×10^{-5}			
$\mathcal{O}_{\bar{d}uLDL}^{1111}$	1.92×10^{-3}	$\mathcal{O}_{\bar{d}LQLH1}^{1111}$	6.22×10^{-5}	6.38×10^{-5}
$\mathcal{O}_{\bar{d}uLDL}^{1211}$	3.58×10^{2}	$\mathcal{O}_{\bar{d}LQLH1}^{1121}$	2.17×10^{-1}	2.77×10^{-4}
$\mathcal{O}_{\bar{d}uLDL}^{1311}$	1.54×10^{1}	$\mathcal{O}_{\bar{d}LQLH1}^{1131}$	4.69	1.72×10^{-2}
$\mathcal{O}_{\bar{d}uLDL}^{2111}$	9.21×10^{3}	$\mathcal{O}_{\bar{d}LQLH1}^{2111}$	1.06×10^{-2}	4.50×10^{1}
$\mathcal{O}_{\bar{d}uLDL}^{2211}$	4.04	$\mathcal{O}_{\bar{d}LQLH1}^{2121}$	2.45×10^{-3}	2.39×10^{-3}
$\mathcal{O}_{\bar{d}uLDL}^{2311}$	1.66×10^{-1}	$\mathcal{O}_{\bar{d}LQLH1}^{2131}$	5.05×10^{-2}	4.09×10^{-1}
$\mathcal{O}_{\bar{d}uLDL}^{3111}$	1.34×10^{2}	$\mathcal{O}_{\bar{d}LQLH1}^{3111}$	1.27×10^{-2}	3.61×10^{-2}
$\mathcal{O}_{\bar{d}uLDL}^{3211}$	1.80	$\mathcal{O}_{\bar{d}LQLH1}^{3121}$	1.09×10^{-3}	7.91×10^{-3}
$\mathcal{O}_{\bar{d}uLDL}^{3311}$	1.33×10^{-4}	$\mathcal{O}_{\bar{d}LQLH1}^{3131}$	4.04×10^{-5}	4.04×10^{-5}
$\mathcal{O}_{\bar{d}LueH}^{1111}$	2.18×10^{-2}	$\mathcal{O}_{\bar{d}LQLH2}^{1111}$	2.61×10^{-4}	2.68×10^{-4}
$\mathcal{O}_{\bar{d}LueH}^{1121}$	9.27×10^{4}	$\mathcal{O}_{\bar{d}LQLH2}^{1121}$	3.57	1.16×10^{-3}
$\mathcal{O}_{\bar{d}LueH}^{1131}$	8.92×10^{3}	$\mathcal{O}_{\bar{d}LQLH2}^{1131}$	8.56×10^{1}	7.20×10^{-2}
$\mathcal{O}_{\bar{d}LueH}^{2111}$	2.27×10^{6}	$\mathcal{O}_{\bar{d}LQLH2}^{2111}$	1.75×10^{-1}	2.94×10^{3}
$\mathcal{O}_{\bar{d}LueH}^{2121}$	1.05×10^{3}	$\mathcal{O}_{\bar{d}LQLH2}^{2121}$	4.03×10^{-2}	3.93×10^{-2}
$\mathcal{O}_{\bar{d}LueH}^{2131}$	9.58×10^{1}	$\mathcal{O}_{\bar{d}LQLH2}^{2131}$	9.16×10^{-1}	2.66×10^{1}
$\mathcal{O}_{\bar{d}LueH}^{3111}$	2.72×10^{6}	$\mathcal{O}_{\bar{d}LQLH2}^{3111}$	2.09×10^{-1}	2.35
$\mathcal{O}_{\bar{d}LueH}^{3121}$	4.67×10^{2}	$\mathcal{O}_{\bar{d}LQLH2}^{3121}$	1.80×10^{-2}	5.14×10^{-1}
$\mathcal{O}_{\bar{d}LueH}^{3131}$	7.67×10^{-2}	$\mathcal{O}_{\bar{d}LQLH2}^{3131}$	7.33×10^{-4}	7.32×10^{-4}

RG mixing effects are important to constrain operators that cannot directly contribute to $0\nu\beta\beta$, especially the operators involving 3rd or 2nd generation quarks

55 dim-7 SMEFT operators are constrained vs 10 operators through tree-level contributions

Yi Liao, XDM, Hao-Lin Wang, Xiang Zhao, 2505.06499

BNV nucleon decays

JUNO sensitivity to nucleon or dinucleon decays

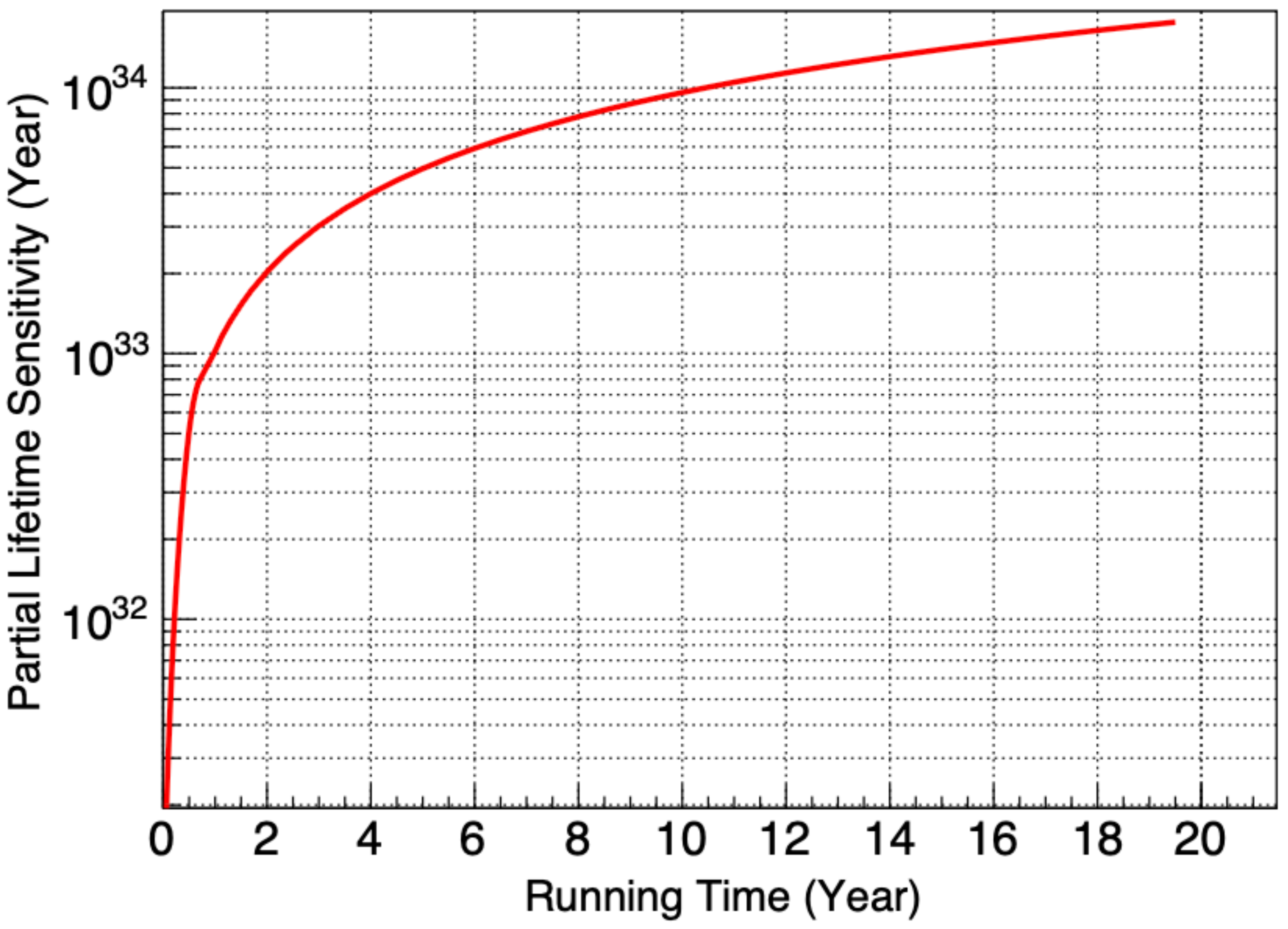
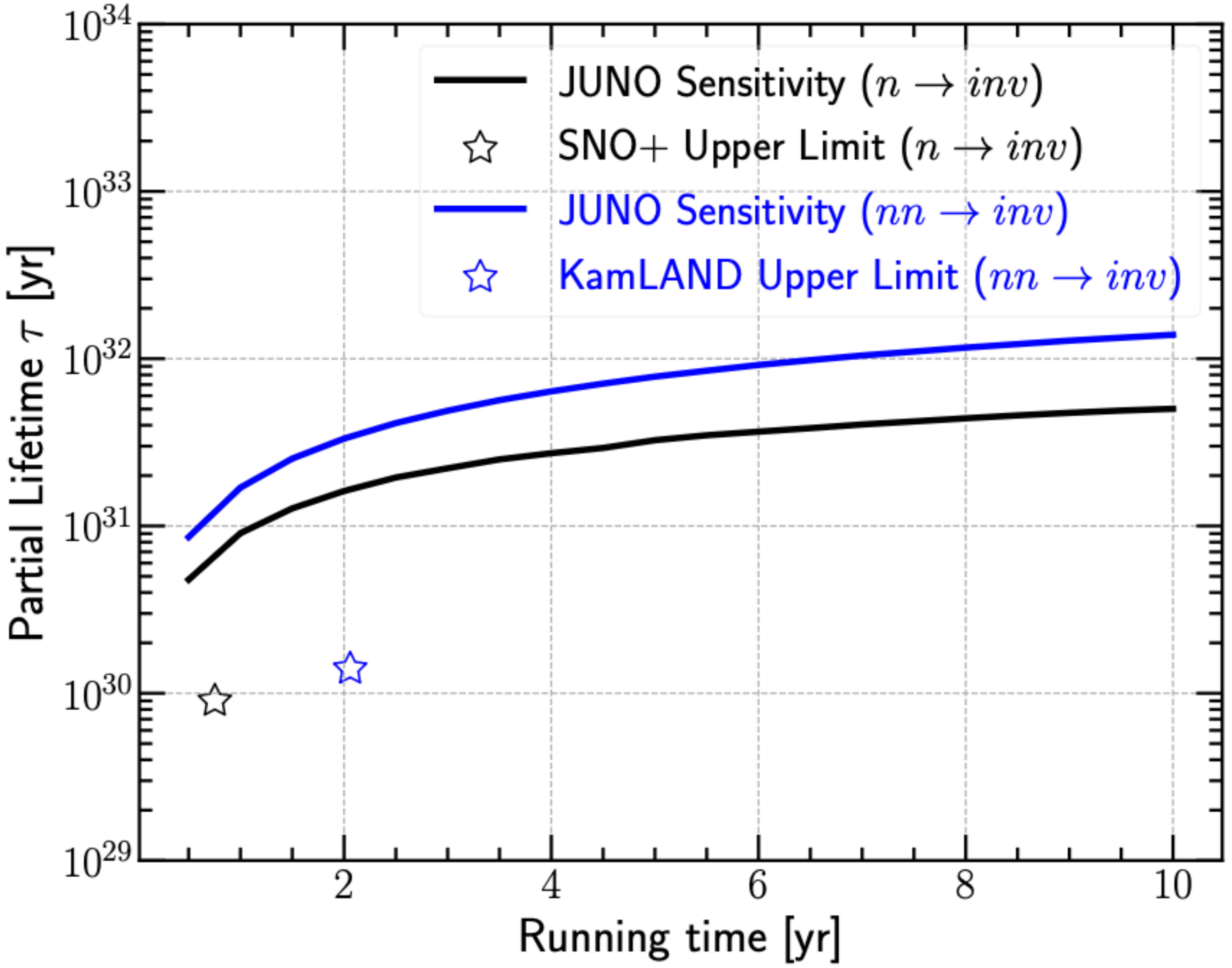


FIG. 12. JUNO sensitivity for $p \to \bar{\nu} K^+$ as a function of running time.



arXiv:2212.08502

arXiv:2405.17792

Expanding searches for exotic decay modes

DUNE, THEIA, Super-K, Hyper-K, ESSnuNB

Exotic nucleon decay modes

- Triple leptons: $p \to e^+e^-\mu^+, p \to e^+e^-\mu^+K^0\cdots$
- Sterile neutrino (N): $p \to N\pi^+,\ n \to N\pi^0, \cdots$

  $(\overline{N_R}d_L^\alpha)(\overline{u_L^{\beta C}}d_L^\gamma)\epsilon_{\alpha\beta\gamma},\ \cdots$

- ALP (a): $p \to e^+a,\ n \to e^+\pi^-a, \cdots$

  $(\partial_\mu a)(\overline{e_L^C}u_L^\alpha)(\overline{u_L^{\beta C}}\gamma^\mu d_R^\gamma)\epsilon_{\alpha\beta\gamma},\ \cdots$

- Dark photon (X): $p \to e^+X,\ n \to e^+\pi^-X, \cdots$

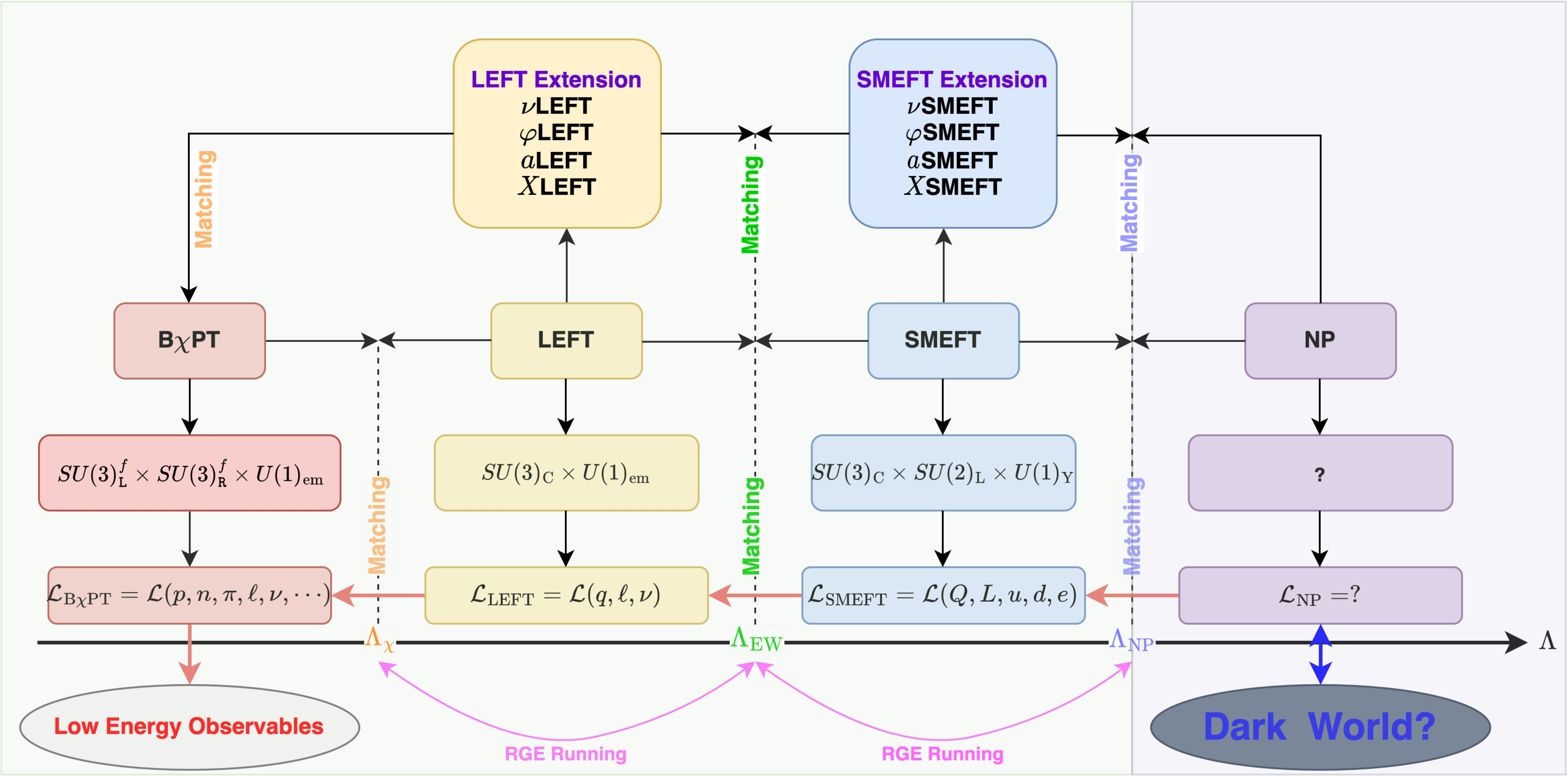
  $X_{\mu\nu}(\overline{\ell}_R d_L^\alpha)(\overline{d_L^{\beta C}}\sigma^{\mu\nu}d_L^\gamma)\epsilon_{\alpha\beta\sigma},\ \cdots$

- Scalar (φ): $p \to e^+\varphi,\ n \to e^+\pi^-\varphi, \cdots$

  $\varphi(\overline{\nu_L^C}d_L^\alpha)(\overline{u_L^{\beta C}}d_L^\gamma)\epsilon_{\alpha\beta\gamma},\ \cdots$

Sandbox Studio, Chicago

Nucleon decays in the EFT landscape



More on nucleon decays can be found in our recent works

* Xiao-Gang He, **XDM**, arXiv:2102.02562 (JHEP) Dinucleon decay

* Wei-Qi Fan, Yi Liao, **XDM**, Hao-Lin Wang, arXiv: 2412.20774 (PLB) BNV Hydrogen

* Yi Liao, **XDM**, Hao-Lin Wang, arXiv: 2504.14855 (PRL) General nucleon decay structure

* Yi Liao, **XDM**, Hao-Lin Wang, arXiv: 2506.05052 (PRD L) N decay w/ an vector meson

* Wei-Qi Fan, Yi Liao, **XDM**, Hao-Lin Wang, arXiv:2507.11844 (CPC-a) N decay w/ an ALP

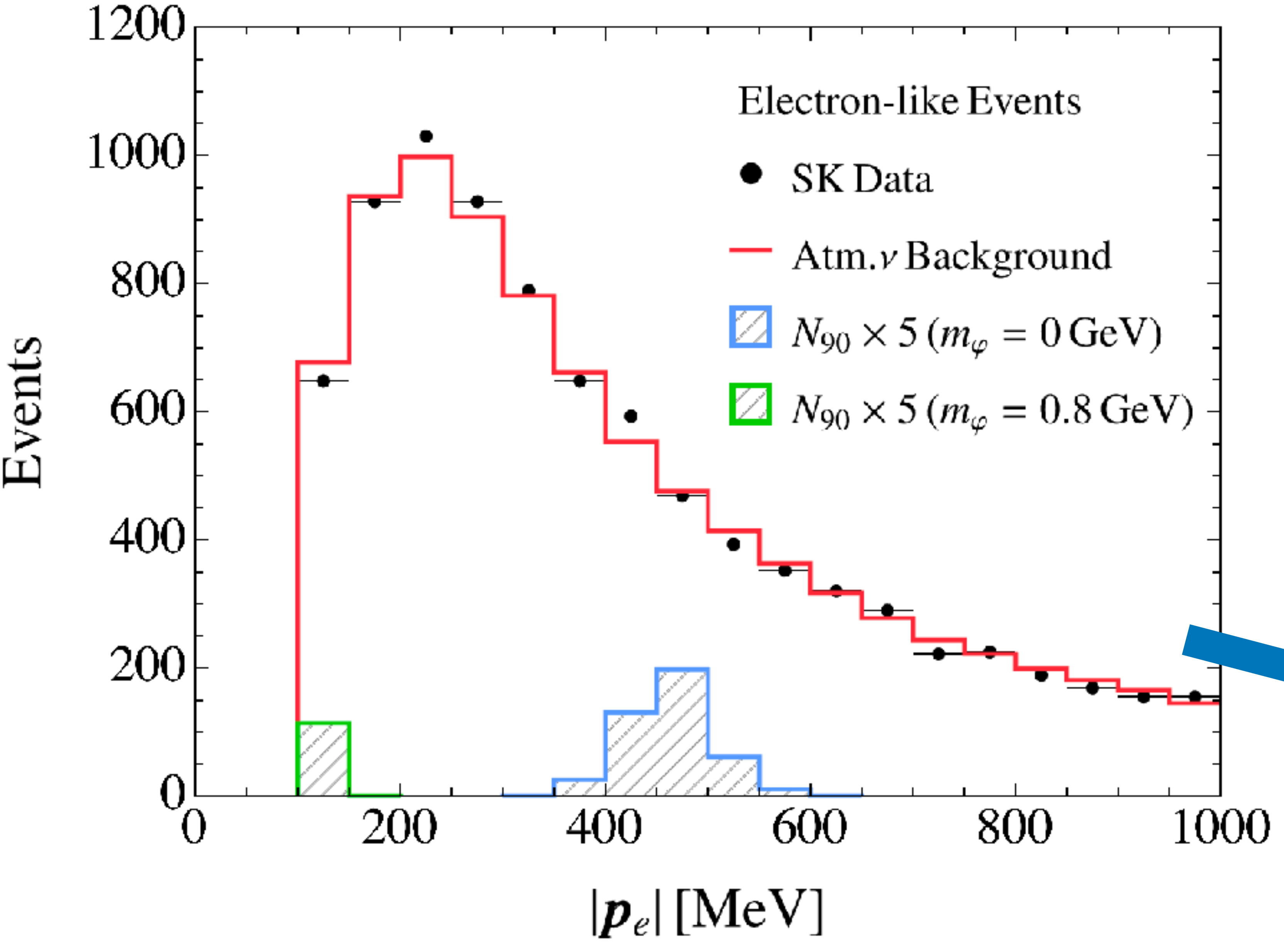
* **XDM**, Michael Schmidt, Wei-Hang Zhang, arXiv:2511.02169 (JHEP-a) N decay w/ a scalar

* Jing Chen, Yi Liao, **XDM** and Hao-Lin Wang, arXiv:2512.02692 N decay w/ triple leptons

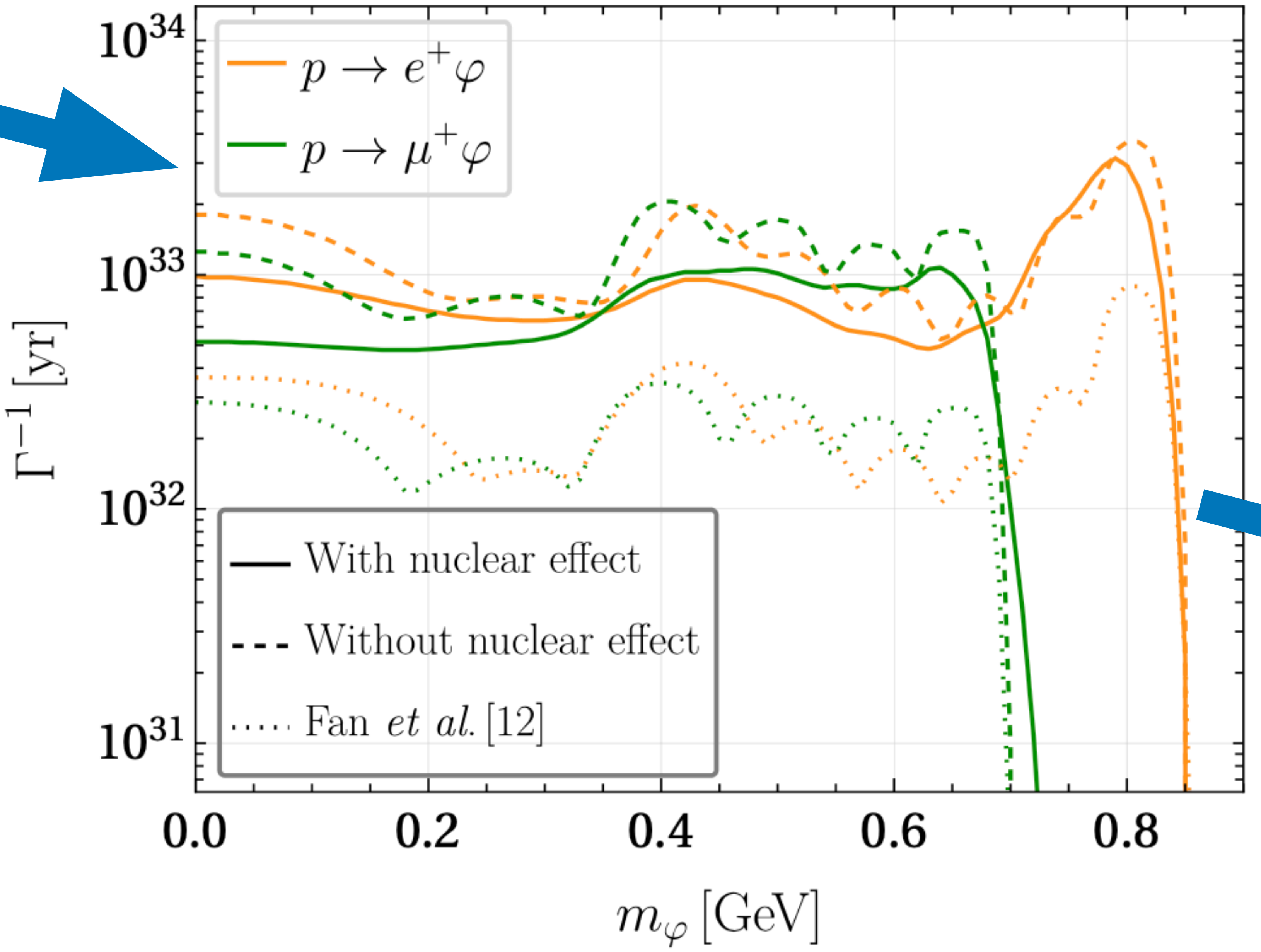
* Yi Liao, **XDM** and Xiang Zhao, arXiv:2512.09287   N decay w/ triple leptons

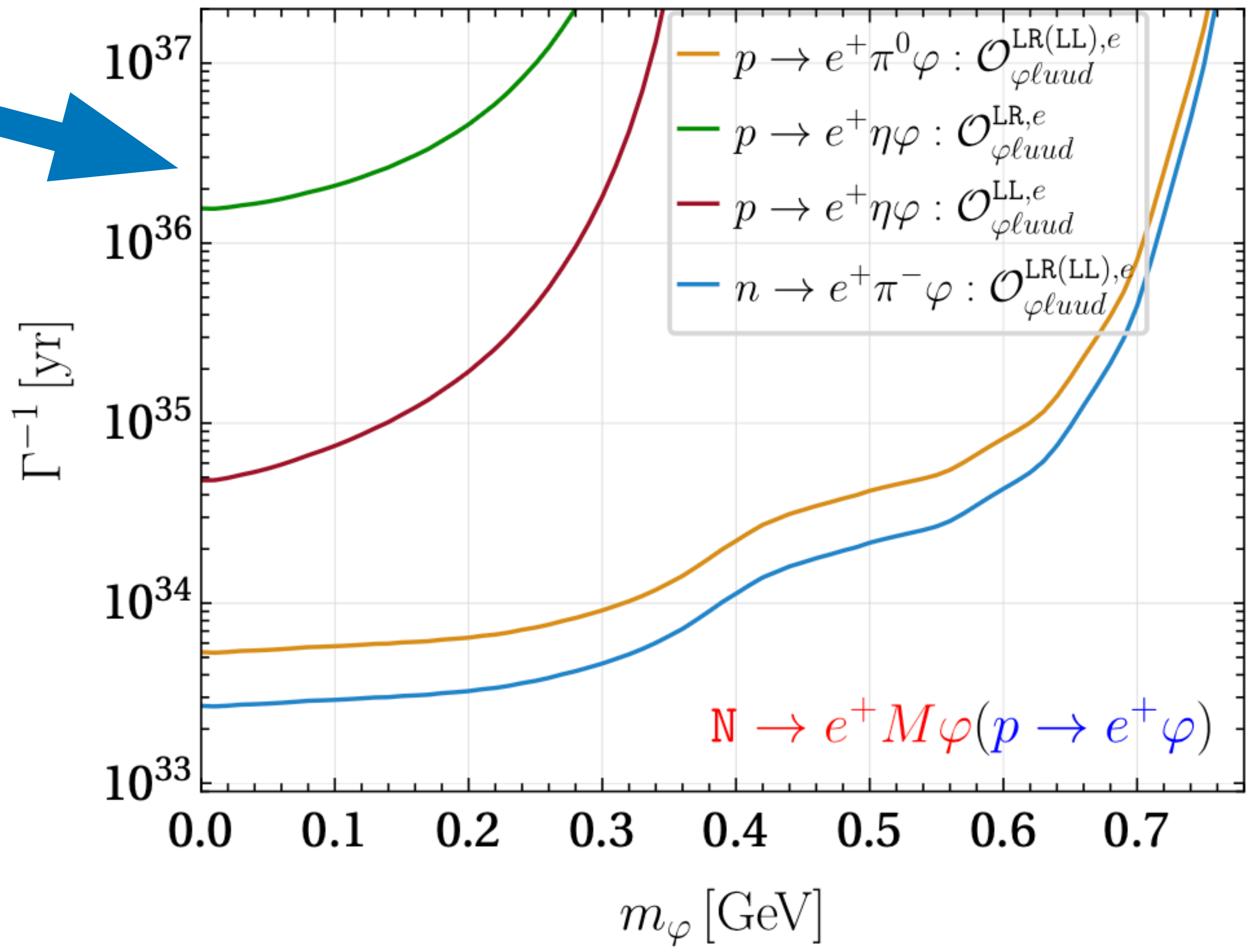
+ ongoing works

Nucleon decay with a light scalar: $\mathbb{N} \to l + \varphi + \ldots$



Reinterpret the existing Super-K data



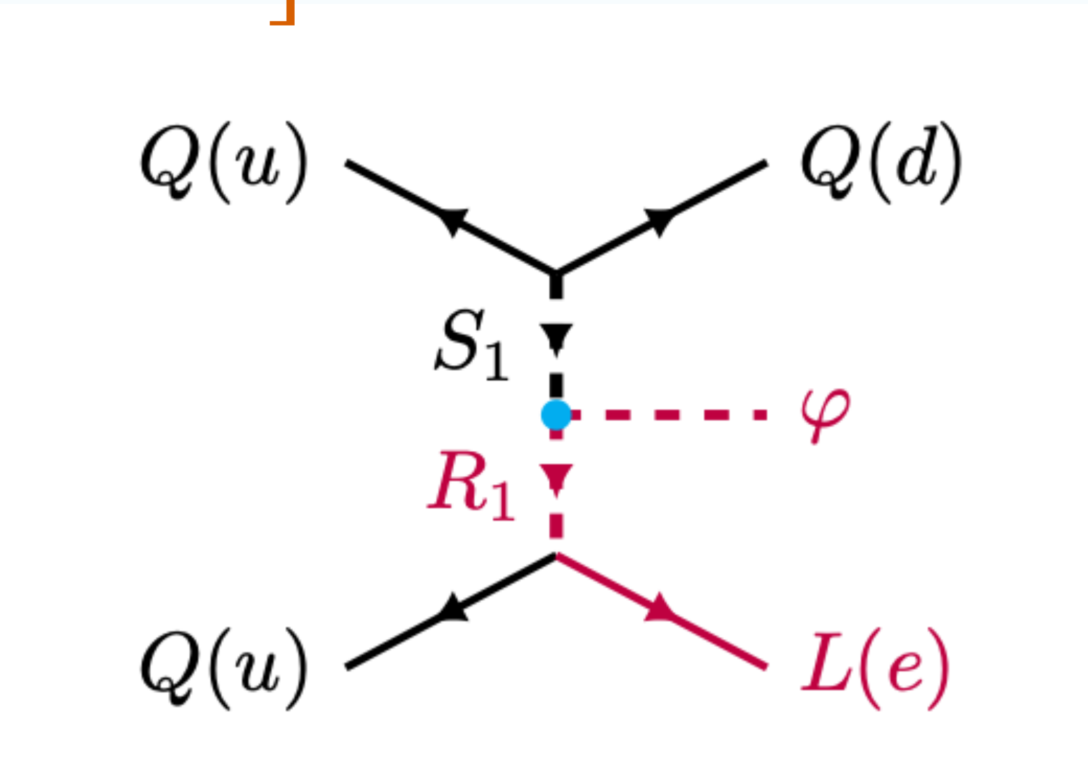


Bound on some modes are quite weak and worth of experimental search.

Common in UV-complete models

$$\mathscr{L} \supset \left[R_1^\dagger(y_{Lpr}\overline{Q_p^{i\complement}}\epsilon_{ij}L_r^j + y_{Rpr}\overline{u_p^\complement}e_r) + S_1^\alpha(z_{Lpr}\overline{Q_p^{i\beta\complement}}\epsilon_{ij}Q_r^{j\gamma} + z_{Rpr}\overline{u_p^{\beta\complement}}d_r^\gamma)\epsilon_{\alpha\beta\gamma} - \kappa R_1^\dagger S_1\varphi + \mathrm{h.c.}\right],$$

Leptoquark model



$\mathbb{Z}_2$ symmetry

No dim-6 SMEFT

Dim-7 in φSMEFT

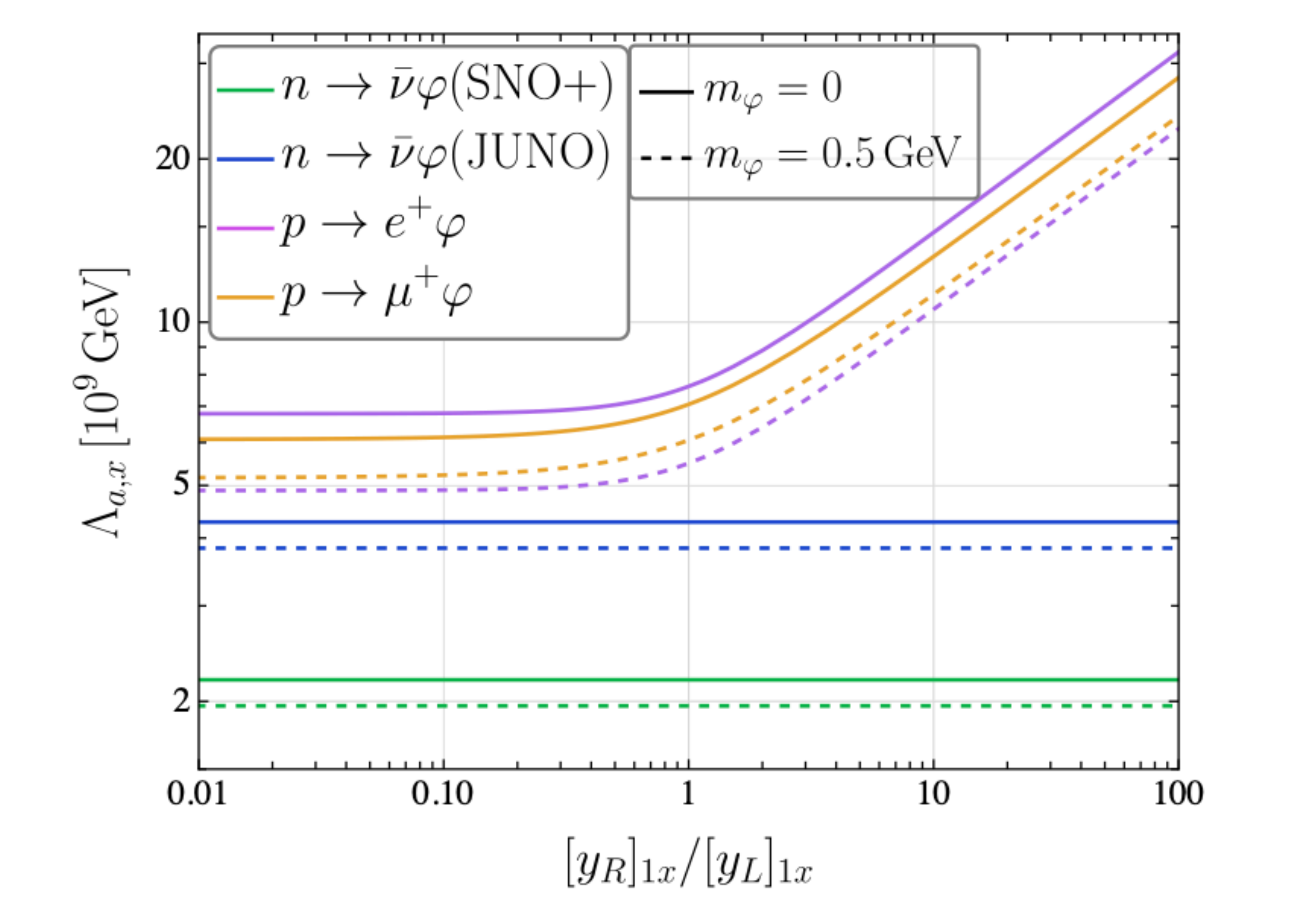
$$C^{prst}_{LQdu\varphi} = -\frac{\kappa^*[y_L]_{rp}[z_R]_{ts}}{m_S^2 m_R^2}, \qquad C^{prst}_{euQQ\varphi} = -\frac{\kappa^*[y_R]_{rp}[z_L]_{st}}{m_S^2 m_R^2},$$

$$C^{prst}_{LQQQ\varphi} = \frac{2\kappa^*[y_L]_{rp}[z_L]_{st}}{m_S^2 m_R^2}, \qquad C^{prst}_{eudu\varphi} = \frac{\kappa^*[y_R]_{rp}[z_R]_{ts}}{m_S^2 m_R^2}.$$

Dim-7 in φLEFT

$$C^{\mathrm{LR},x}_{\varphi\nu dud} = 1.32\frac{\kappa^*[y_L]_{1x}[z_R]_{11}}{m_S^2 m_R^2}, \; C^{\mathrm{LR},x}_{\varphi\ell uud} = -C^{\mathrm{LR},x}_{\varphi\nu dud}, \; C^{\mathrm{RL},x}_{\varphi\ell uud} = -1.32\frac{2\kappa^*[y_R]_{1x}[z_L]_{11}}{m_S^2 m_R^2},$$

$$C^{\mathrm{LL},x}_{\varphi\nu dud} = 1.32\frac{2\kappa^*[y_L]_{1x}[z_L]_{11}}{m_S^2 m_R^2}, \; C^{\mathrm{LL},x}_{\varphi\ell uud} = -C^{\mathrm{LL},x}_{\varphi\nu dud}, \; C^{\mathrm{RR},x}_{\varphi\ell uud} = -1.32\frac{\kappa^*[y_R]_{1x}[z_R]_{11}}{m_S^2 m_R^2},$$



$\Lambda_{a,x} \equiv (m_S^2 m_R^2/|\kappa^*[y_L]_{1x}([z_R]_{11} - 2[z_L]_{11})|)^{1/3}$

Spurion fields

$\mathcal{N}^{\mathrm{LL}}_{yzw}$ and $\mathcal{N}^{\mathrm{RL}}_{yzw}$ + L <—>R

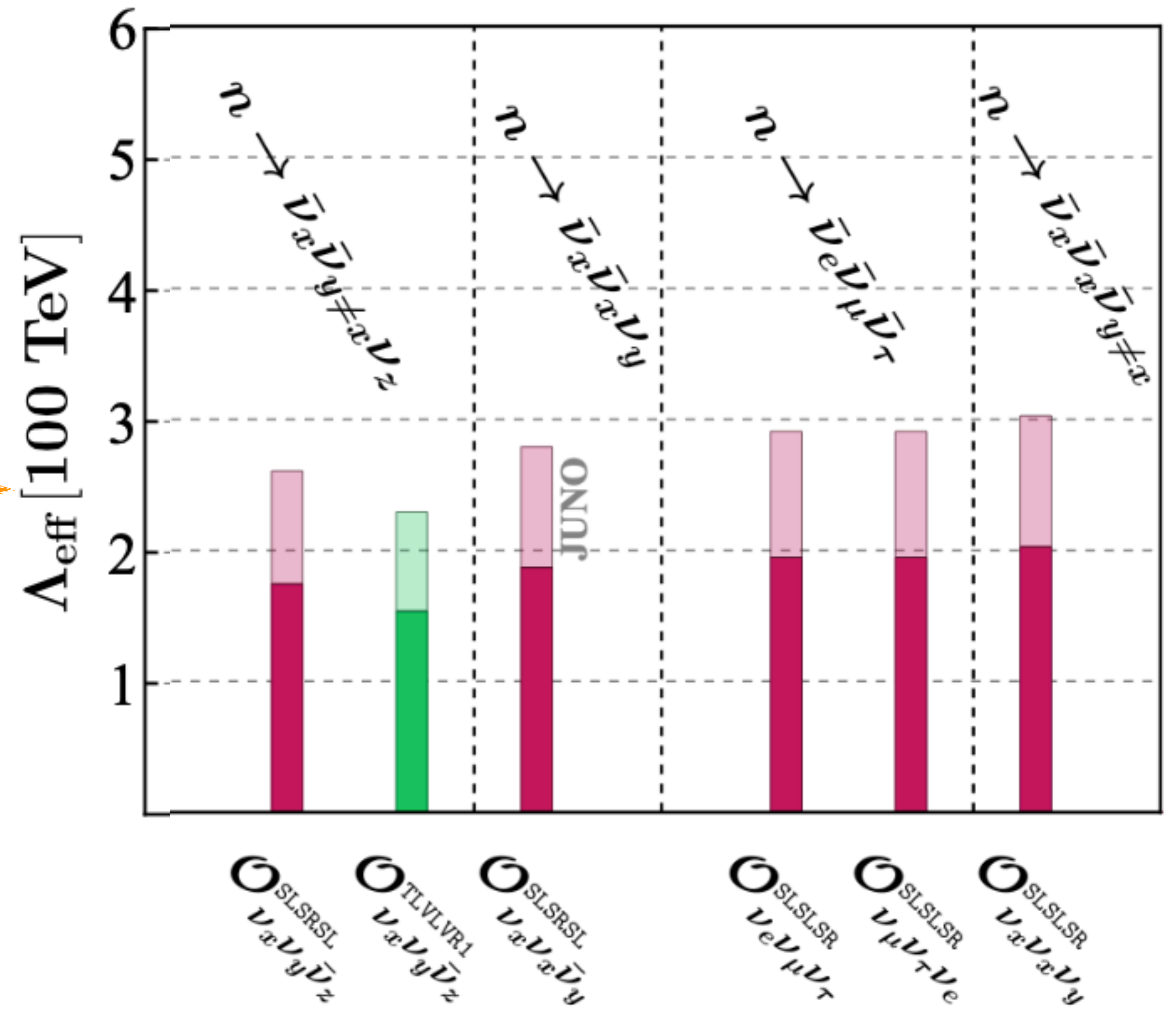
Nucleon decay involving three leptons

Yi Liao, **XDM**, Xiang Zhao, 2512.09287

Process	Exp. bound Γ^{-1}[yr]	Process	Exp. bound Γ^{-1}[yr]
$n \to e^- e^+ \hat{\nu}$	2.57×10^{32} [15]	$p \to e^+ \hat{\nu}\hat{\nu}$	1.7×10^{32} [16]
$n \to e^- \mu^+ \hat{\nu}$	8.3×10^{31} [15]	$p \to \mu^+ \hat{\nu}\hat{\nu}$	2.2×10^{32} [16]
$n \to \mu^- e^+ \hat{\nu}$	0.6×10^{30} [15]	$n \to \hat{\nu}\hat{\nu}\hat{\nu}$	0.9×10^{30} [17]
$n \to \mu^- \mu^+ \hat{\nu}$	7.9×10^{31} [15]		$(5 \times 10^{31}$ [18])
			JUNO: 2405.17792

$\Delta L = -1$	$\Delta L = +1$	$\Delta L = -3$	$\Delta L = +3$
$p \to \ell_x^+ \ell_y^+ \ell_z^- \; (z = x, y)$	$n \to \ell_x^- \ell_y^+ \nu_z \; (y = x, z)$	$p \to \bar{\nu}_x \bar{\nu}_y \ell_z^+$	$n \to \nu_x \nu_y \nu_z$
$n \to \ell_x^- \ell_y^+ \bar{\nu}_z \; (x = y, z)$	$p \to \nu_x \nu_y \ell_z^+ \; (z = y, z)$	$n \to \bar{\nu}_x \bar{\nu}_y \bar{\nu}_z$	
$p \to \nu_x \bar{\nu}_y \ell_z^+ \; (x = y, z)$	$n \to \nu_x \nu_y \bar{\nu}_z \; (z = x, y)$		
$n \to \bar{\nu}_x \bar{\nu}_y \nu_z \; (z = x, y)$			

	Notation	Operator	Chiral Irrep.	# of operators	Process
$\Delta L = -3$	$\mathcal{O}^{\rm SLSRSR}_{\nu\nu\ell,xyz}$	$(\overline{\nu^{\rm C}_{{\rm L}x}}\nu_{{\rm L}y})(\overline{\ell^{\rm C}_{{\rm R}z}}\mathcal{N}^{\rm RR}_{uud})$	$\mathbf{1}_{\rm L} \otimes \mathbf{8}_{\rm R}$	$\frac{1}{2}n_\ell n_\nu(n_\nu+1)$ [18]	$p \to \bar{\nu}_x \bar{\nu}_y \ell_z^+$
	$\mathcal{O}^{\rm SLSLSL}_{\nu\nu\ell,xyz}$	$(\overline{\nu^{\rm C}_{{\rm L}x}}\nu_{{\rm L}y})(\overline{\ell^{\rm C}_{{\rm L}z}}\mathcal{N}^{\rm LL}_{uud})$	$\mathbf{8}_{\rm L} \otimes \mathbf{1}_{\rm R}$	$\frac{1}{2}n_\ell n_\nu(n_\nu+1)$ [18]	
	$\mathcal{O}^{\rm TLTLSL}_{\nu\nu\ell,xyz}$	$(\overline{\nu^{\rm C}_{{\rm L}x}}\sigma_{\mu\nu}\nu_{{\rm L}y})(\overline{\ell^{\rm C}_{{\rm L}z}}\sigma^{\mu\nu}\mathcal{N}^{\rm LL}_{uud})$	$\mathbf{8}_{\rm L} \otimes \mathbf{1}_{\rm R}$	$\frac{1}{2}n_\ell n_\nu(n_\nu-1)$ [9]	
	$\mathcal{O}^{\rm TLSLTL}_{\nu\nu\ell,xyz}$	$(\overline{\nu^{\rm C}_{{\rm L}x}}\sigma_{\mu\nu}\nu_{{\rm L}y})(\overline{\ell^{\rm C}_{{\rm L}z}}\mathcal{N}^{{\rm LL},\mu\nu}_{uud})$	$\mathbf{10}_{\rm L} \otimes \mathbf{1}_{\rm R}$	$\frac{1}{2}n_\ell n_\nu(n_\nu-1)$ [9]	
	$\mathcal{O}^{\rm SLSRSL}_{\nu\nu\ell,xyz}$	$(\overline{\nu^{\rm C}_{{\rm L}x}}\nu_{{\rm L}y})(\overline{\ell^{\rm C}_{{\rm R}z}}\mathcal{N}^{\rm RL}_{uud})$	$\bar{\mathbf{3}}_{\rm L} \otimes \mathbf{3}_{\rm R}$	$\frac{1}{2}n_\ell n_\nu(n_\nu+1)$ [18]	
	$\mathcal{O}^{\rm TLVLVR1}_{\nu\nu\ell,xyz}$	$(\overline{\nu^{\rm C}_{{\rm L}x}}\sigma_{\mu\nu}\nu_{{\rm L}y})(\overline{\ell^{\rm C}_{{\rm R}z}}\gamma^\mu\mathcal{N}^{{\rm LR},\nu}_{udu})$	$\mathbf{6}_{\rm L} \otimes \mathbf{3}_{\rm R}$	$\frac{1}{2}n_\ell n_\nu(n_\nu-1)$ [9]	
	$\mathcal{O}^{\rm TLVLVR2}_{\nu\nu\ell,xyz}$	$(\overline{\nu^{\rm C}_{{\rm L}x}}\sigma_{\mu\nu}\nu_{{\rm L}y})(\overline{\ell^{\rm C}_{{\rm R}z}}\gamma^\mu\mathcal{N}^{{\rm LR},\nu}_{uud})$	$\mathbf{6}_{\rm L} \otimes \mathbf{3}_{\rm R}$	$\frac{1}{2}n_\ell n_\nu(n_\nu-1)$ [9]	
	$\mathcal{O}^{\rm SLSLSR}_{\nu\nu\ell,xyz}$	$(\overline{\nu^{\rm C}_{{\rm L}x}}\nu_{{\rm L}y})(\overline{\ell^{\rm C}_{{\rm L}z}}\mathcal{N}^{\rm LR}_{uud})$	$\mathbf{3}_{\rm L} \otimes \bar{\mathbf{3}}_{\rm R}$	$\frac{1}{2}n_\ell n_\nu(n_\nu+1)$ [18]	
	$\mathcal{O}^{\rm TLTLSR}_{\nu\nu\ell,xyz}$	$(\overline{\nu^{\rm C}_{{\rm L}x}}\sigma_{\mu\nu}\nu_{{\rm L}y})(\overline{\ell^{\rm C}_{{\rm L}z}}\sigma^{\mu\nu}\mathcal{N}^{\rm LR}_{uud})$	$\mathbf{3}_{\rm L} \otimes \bar{\mathbf{3}}_{\rm R}$	$\frac{1}{2}n_\ell n_\nu(n_\nu-1)$ [9]	
	$\mathcal{O}^{\rm SLSLSL}_{\nu\nu\nu,xyz}$	$(\overline{\nu^{\rm C}_{{\rm L}x}}\nu_{{\rm L}[y})(\overline{\nu^{\rm C}_{{\rm L}z]}}\mathcal{N}^{\rm LL}_{dud})$	$\mathbf{8}_{\rm L} \otimes \mathbf{1}_{\rm R}$	$\frac{1}{3}n_\nu(n_\nu^2-1)$ [8]	$n \to \bar{\nu}\bar{\nu}\bar{\nu}$
	$\mathcal{O}^{\rm TLSLTL}_{\nu\nu\nu,xyz}$	$(\overline{\nu^{\rm C}_{{\rm L}[x}}\sigma_{\mu\nu}\nu_{{\rm L}y})(\overline{\nu^{\rm C}_{{\rm L}z]}}\mathcal{N}^{{\rm LL},\mu\nu}_{udd})$	$\mathbf{10}_{\rm L} \otimes \mathbf{1}_{\rm R}$	$\frac{1}{6}n_\nu(n_\nu-2)(n_\nu-1)$ [1]	
	$\mathcal{O}^{\rm SLSLSR}_{\nu\nu\nu,xyz}$	$(\overline{\nu^{\rm C}_{{\rm L}x}}\nu_{{\rm L}[y})(\overline{\nu^{\rm C}_{{\rm L}z]}}\mathcal{N}^{\rm LR}_{dud})$	$\mathbf{3}_{\rm L} \otimes \bar{\mathbf{3}}_{\rm R}$	$\frac{1}{3}n_\nu(n_\nu^2-1)$ [8]	
$\Delta L = +3$	$\mathcal{O}^{\rm SRSRSR}_{\bar{\nu}\bar{\nu}\bar{\nu},xyz}$	$(\overline{\nu_{{\rm L}x}}\nu^{\rm C}_{{\rm L}[y})(\overline{\nu_{{\rm L}z]}}\mathcal{N}^{\rm RR}_{dud})$	$\mathbf{1}_{\rm L} \otimes \mathbf{8}_{\rm R}$	$\frac{1}{3}n_\nu(n_\nu^2-1)$ [8]	$n \to \nu\nu\nu$
	$\mathcal{O}^{\rm TRSRTR}_{\bar{\nu}\bar{\nu}\bar{\nu},xyz}$	$(\overline{\nu_{{\rm L}[x}}\sigma_{\mu\nu}\nu^{\rm C}_{{\rm L}y})(\overline{\nu_{{\rm L}z]}}\mathcal{N}^{{\rm RR},\mu\nu}_{udd})$	$\mathbf{1}_{\rm L} \otimes \mathbf{10}_{\rm R}$	$\frac{1}{6}n_\nu(n_\nu-2)(n_\nu-1)$ [1]	
	$\mathcal{O}^{\rm SRSRSL}_{\bar{\nu}\bar{\nu}\bar{\nu},xyz}$	$(\overline{\nu_{{\rm L}x}}\nu^{\rm C}_{{\rm L}[y})(\overline{\nu_{{\rm L}z]}}\mathcal{N}^{\rm RL}_{dud})$	$\bar{\mathbf{3}}_{\rm L} \otimes \mathbf{3}_{\rm R}$	$\frac{1}{3}n_\nu(n_\nu^2-1)$ [8]	



Summary

- We provide an automatic tool D7RGESolver to solve the full one-loop RGEs of dim-5&7 SMEFT operators;
- The running effects of dim-5&7 interactions can significantly affect the $0\nu\beta\beta$ rate;
- Current $0\nu\beta\beta$ half lifetime bound can contrain 55 dim-7 operators due to RGEs;
- The mixing through dim-5&7 Weinberg operators has the leading contribution to $0\nu\beta\beta$ process, and the mixing via Yukawa couplings involving 3rd generation quarks is also significant.
- Exotic nucleon decay modes are new avenues for test of BNV physics, worthy of experiemental searches.

Thank you for your time!