中國科學院高能物理研究所
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Probing Unitarity Violation of Lepton Flavor Mixing Matrix @ JUNO & TAO

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Based on: JH & Shun Zhou, 2511.15525
Phys. Lett. B 873 (2026) 140160

The 5th Workshop on JUNO-related Theory and Phenomenology
Specified Workshop on First Physical Achievement in JUNO, Hangzhou, 2026/01/27-30

Neutrino Masses and Mixing

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3}\\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3}\\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

flavor states **PMNS matrix** **mass states**

$$P\left(\nu_\alpha \to \nu_\beta\right) = \delta_{\alpha\beta} - 4\sum_{i<j}^{3} \mathrm{Re}\left(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i}\right)\sin^2\frac{\Delta m^2_{ji}L}{4E}$$

$$+2\sum_{i<j}^{3} \mathrm{Im}\left(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i}\right)\sin\frac{\Delta m^2_{ji}L}{2E}$$

oscillation probability

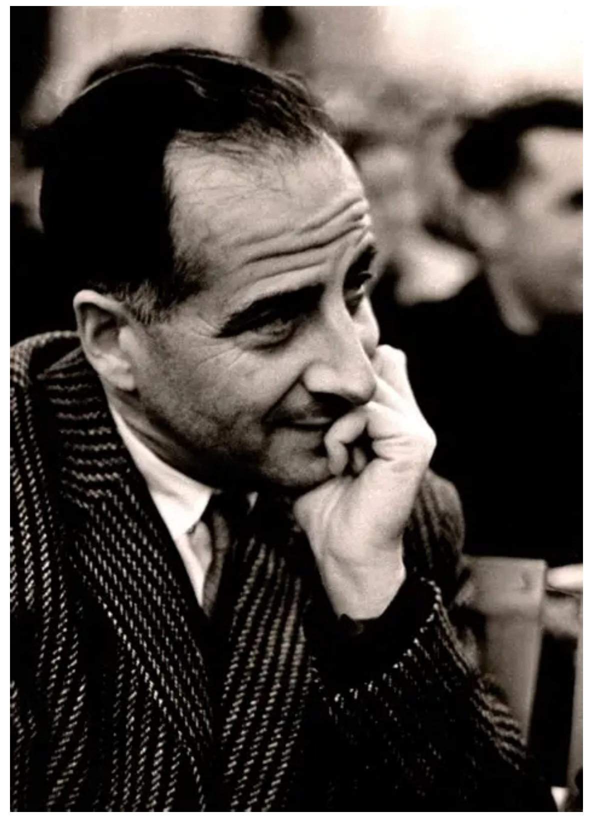
✓ Massive neutrinos

✓ Leptonic flavor mixing

⬇

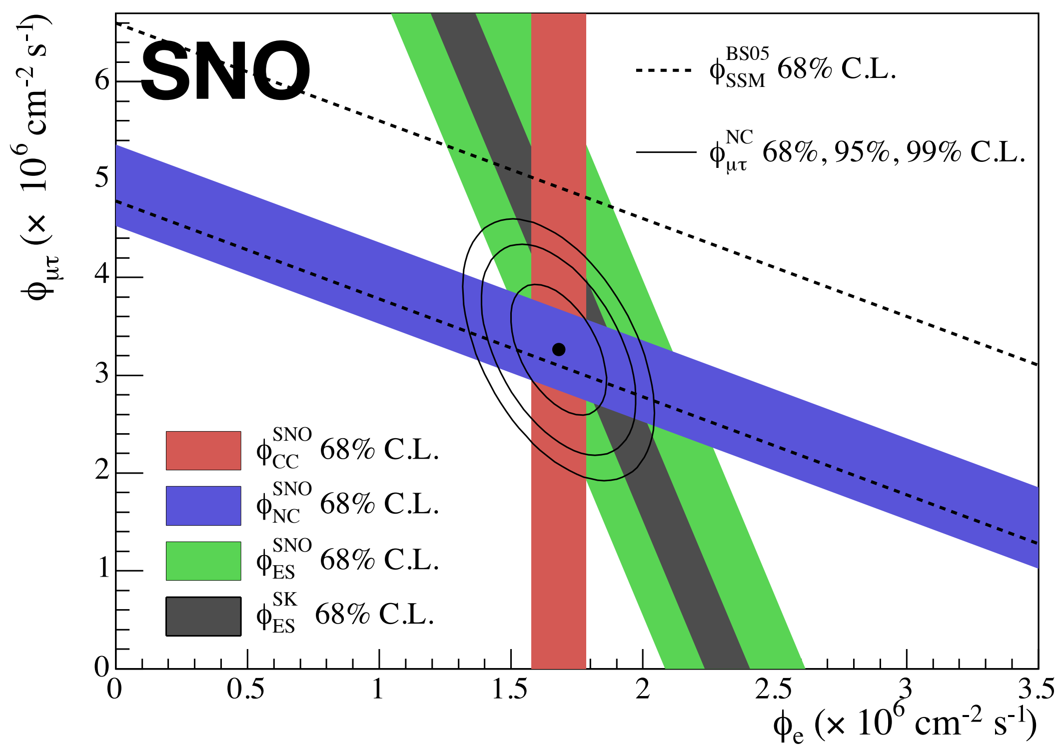
◉ *Neutrino mass models*

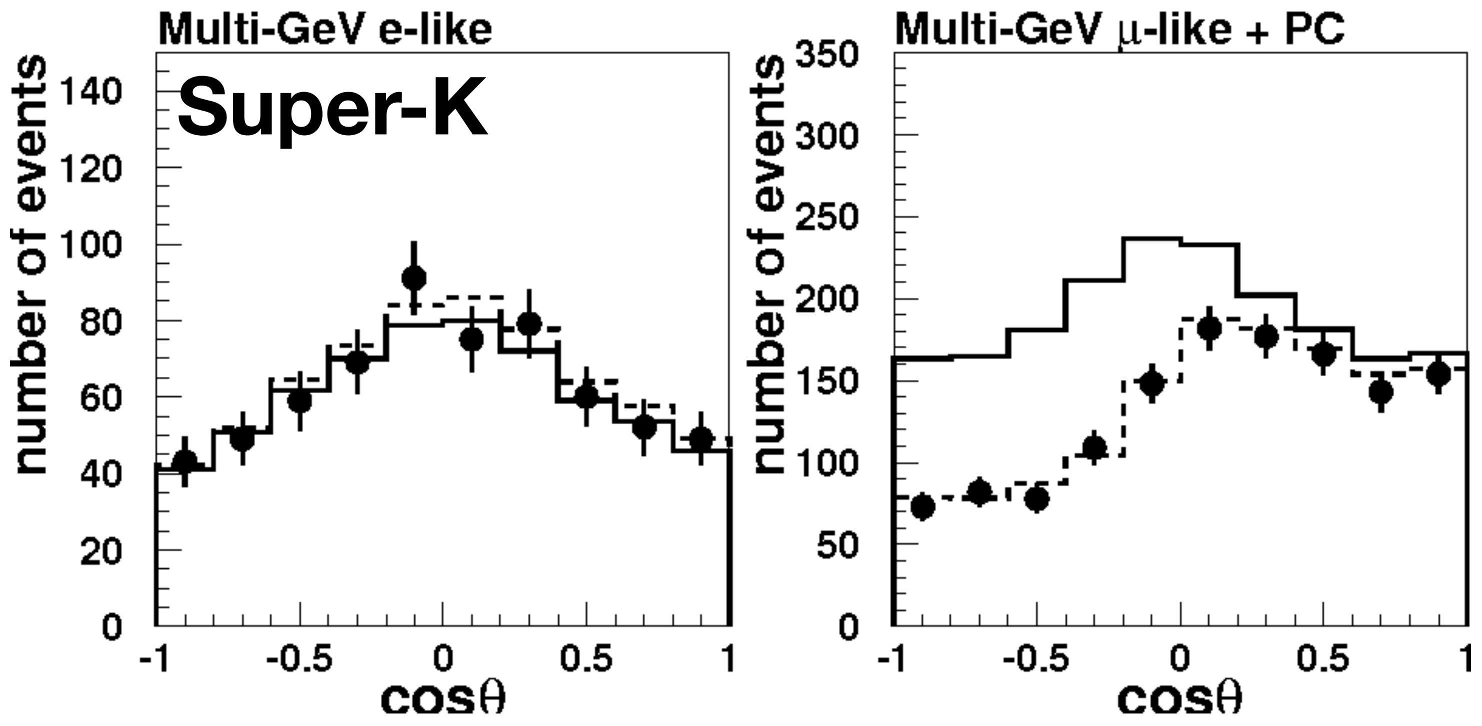
◉ Flavor mixing patterns

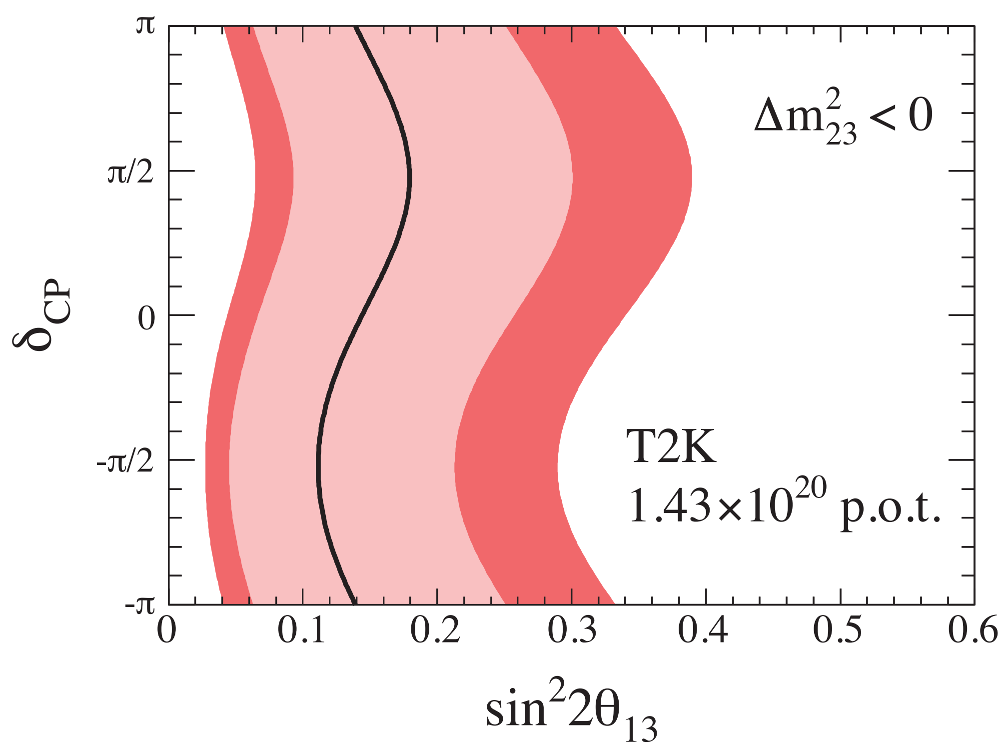


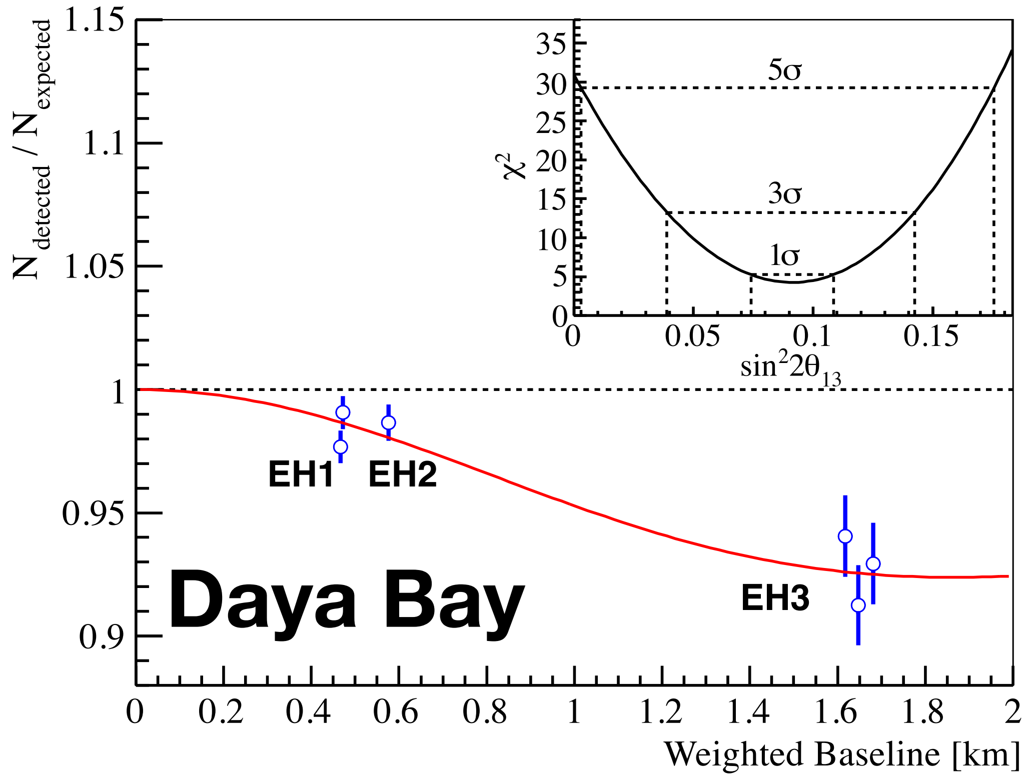
$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}}\\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23}\\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23}\end{pmatrix}\cdot P_\nu$$

- **Three mixing angles** & **One** (Dirac) **CP-violating phase**
- **Two** independent **mass-squared differences**





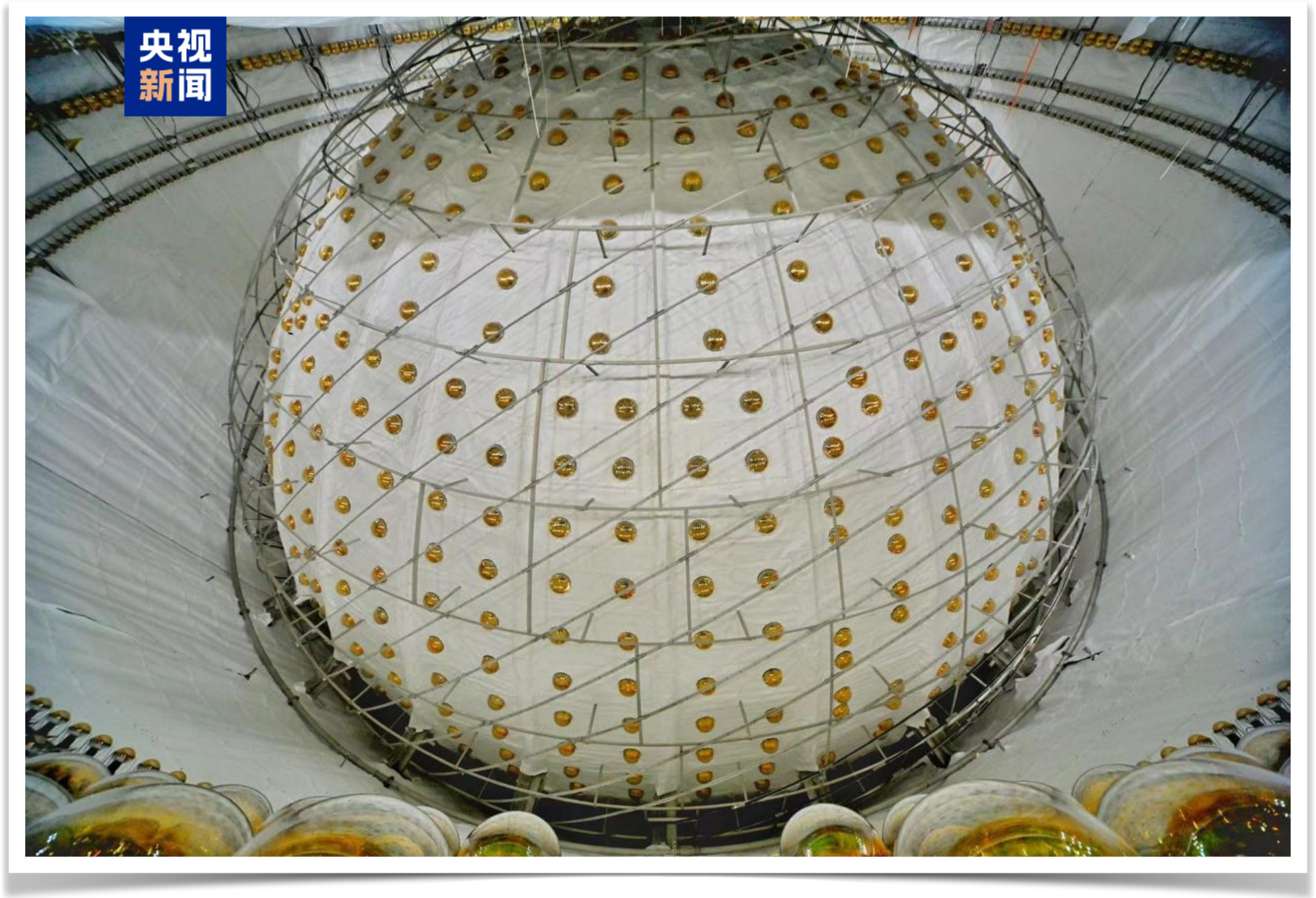


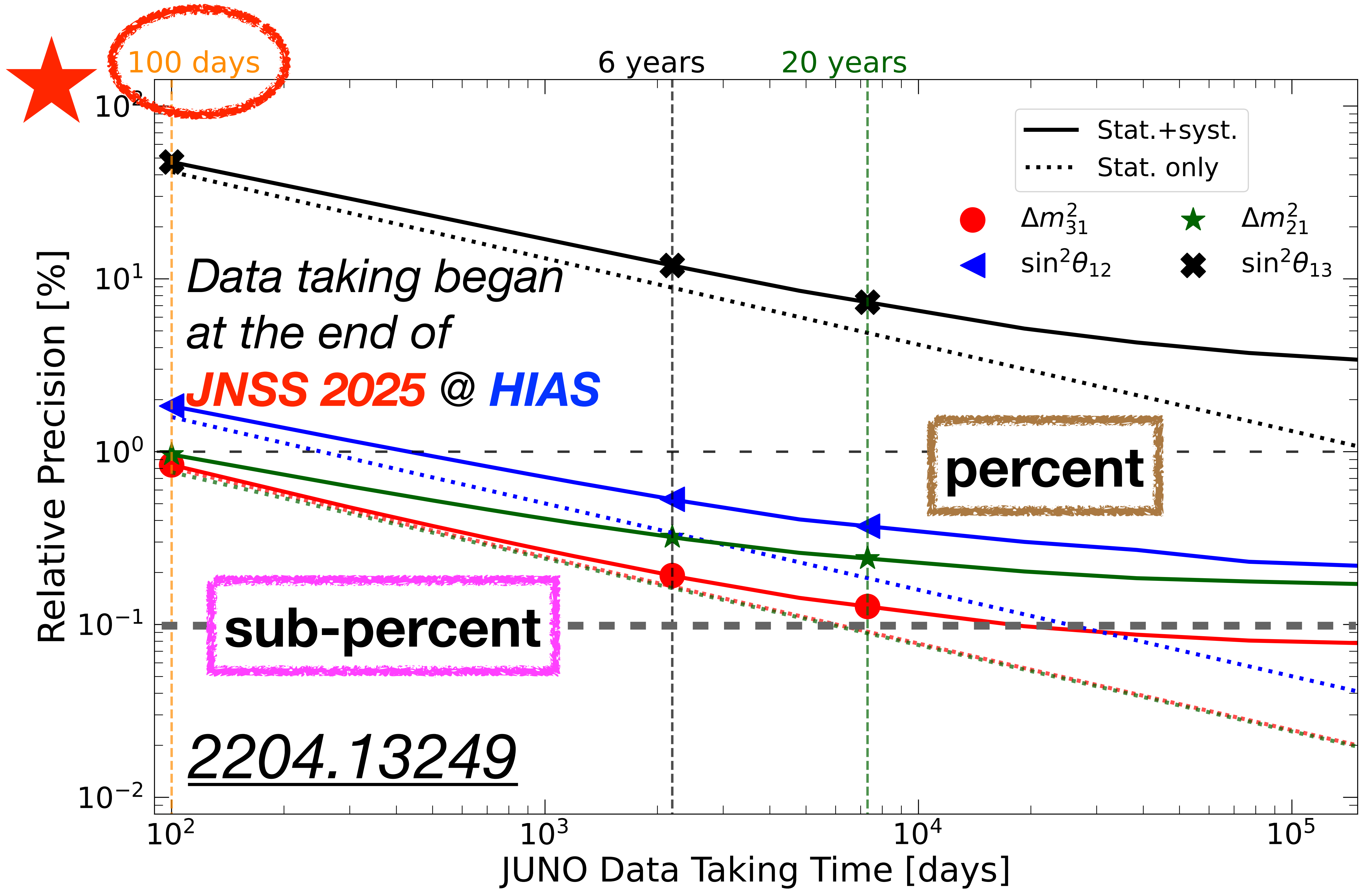


Precise Measurements @ JUNO

Global fit by Bari Group 2511.21650

Global analysis [Ref.]	Parameter	Best fit	1σ range	2σ range	3σ range	"1σ" (%)
All data 2024	$\delta m^2/10^{-5}$ eV^2	7.37	7.21 – 7.52	7.06 – 7.71	6.93 – 7.93	2.3
[3]	$\sin^2\theta_{12}/10^{-1}$	3.03	2.91 – 3.17	2.77 – 3.31	2.64 – 3.45	4.5
w/ SNO+ 2025	$\delta m^2/10^{-5}$ eV^2	7.44	7.30 – 7.61	7.17 – 7.80	7.04 – 7.99	2.1
[This work]	$\sin^2\theta_{12}/10^{-1}$	3.06	2.93 – 3.19	2.80 – 3.33	2.67 – 3.47	4.4
w/ SNO+ & JUNO 2025	$\delta m^2/10^{-5}$ eV^2	7.48	7.39 – 7.58	7.30 – 7.68	7.21 – 7.78	1.3
[This work]	$\sin^2\theta_{12}/10^{-1}$	3.085	3.010 – 3.156	2.939 – 3.230	2.866 – 3.303	2.4



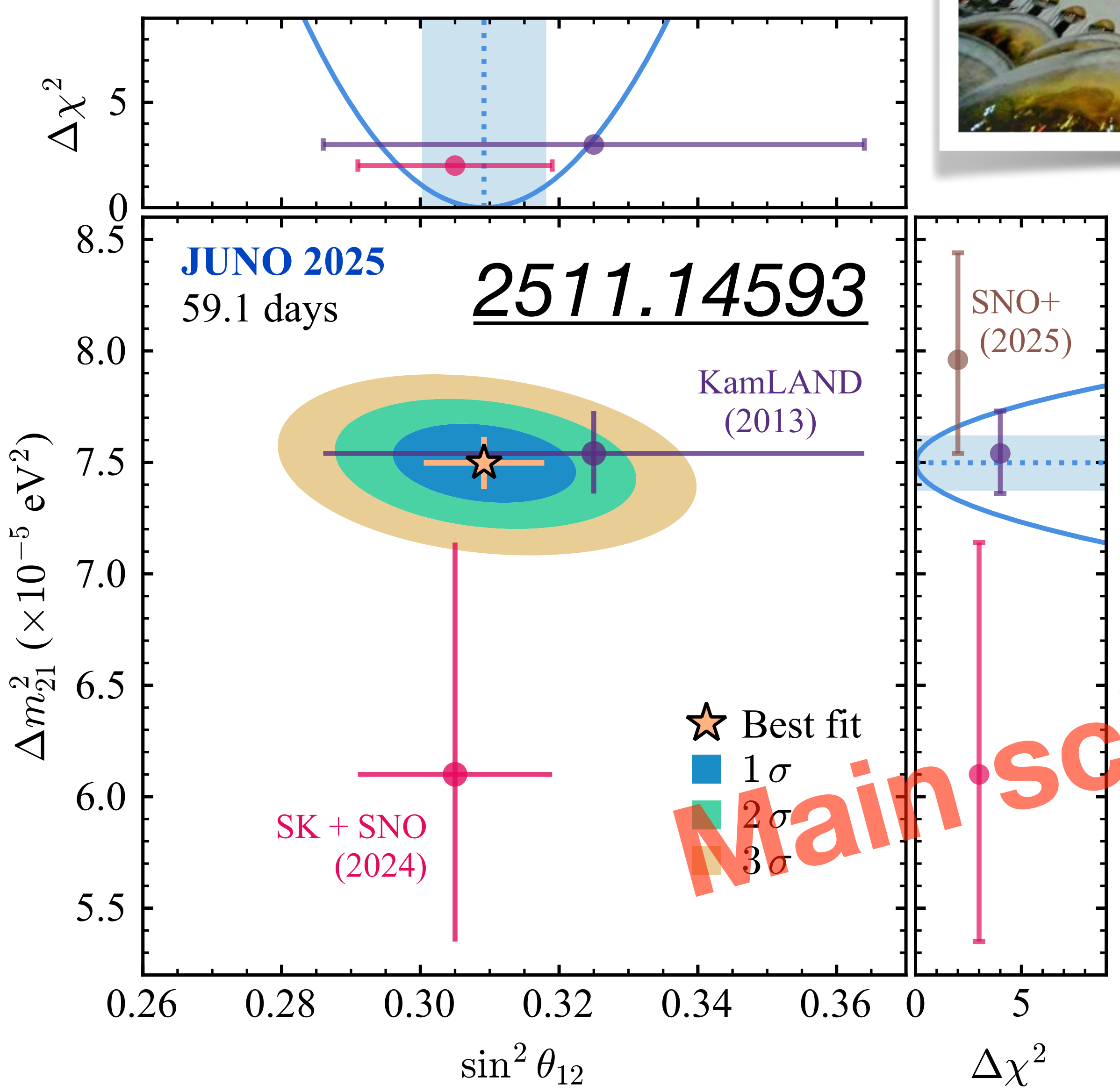


Data taking began at the end of ***JNSS 2025*** *@* ***HIAS***

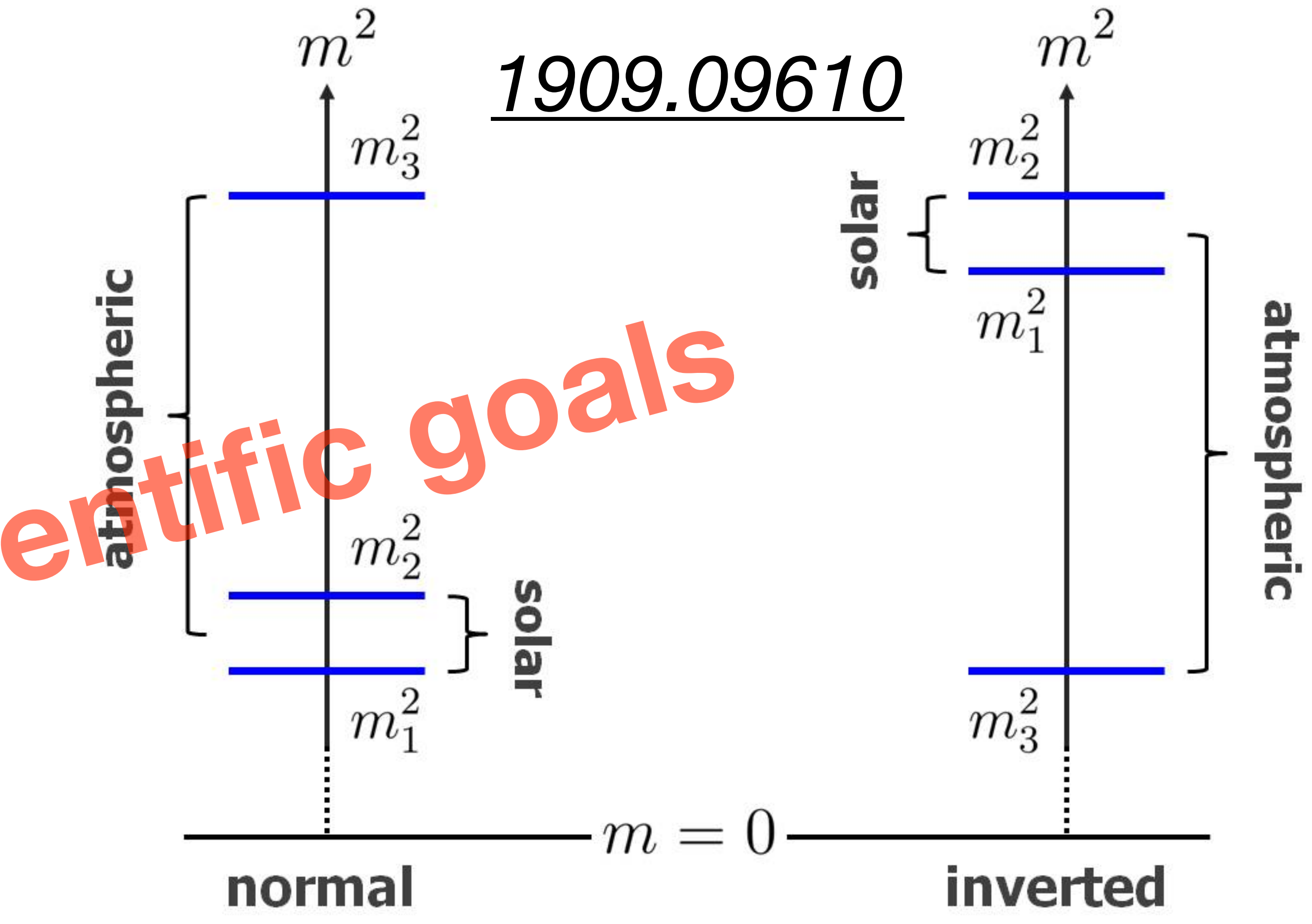
percent

sub-percent

2204.13249



2511.14593



1909.09610

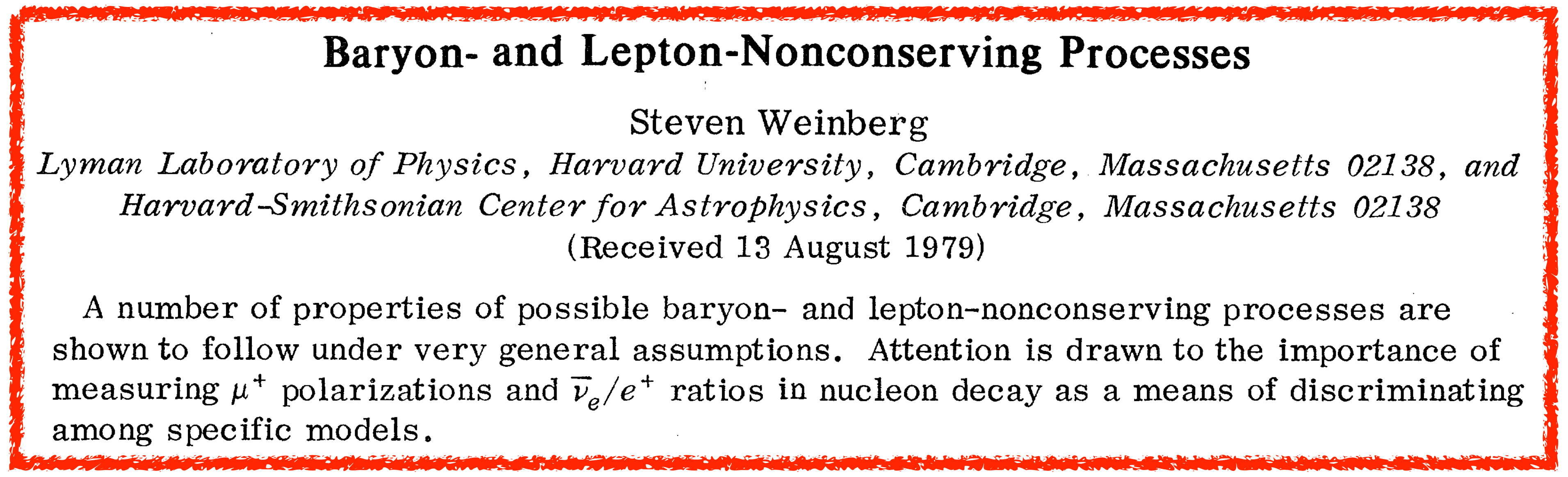
Main scientific goals

- **Test new physics with the precise measurements from JUNO: *non-unitary flavor mixing***

Neutrino mass & Non-unitarity

✴ Neutrino mass in SMEFT

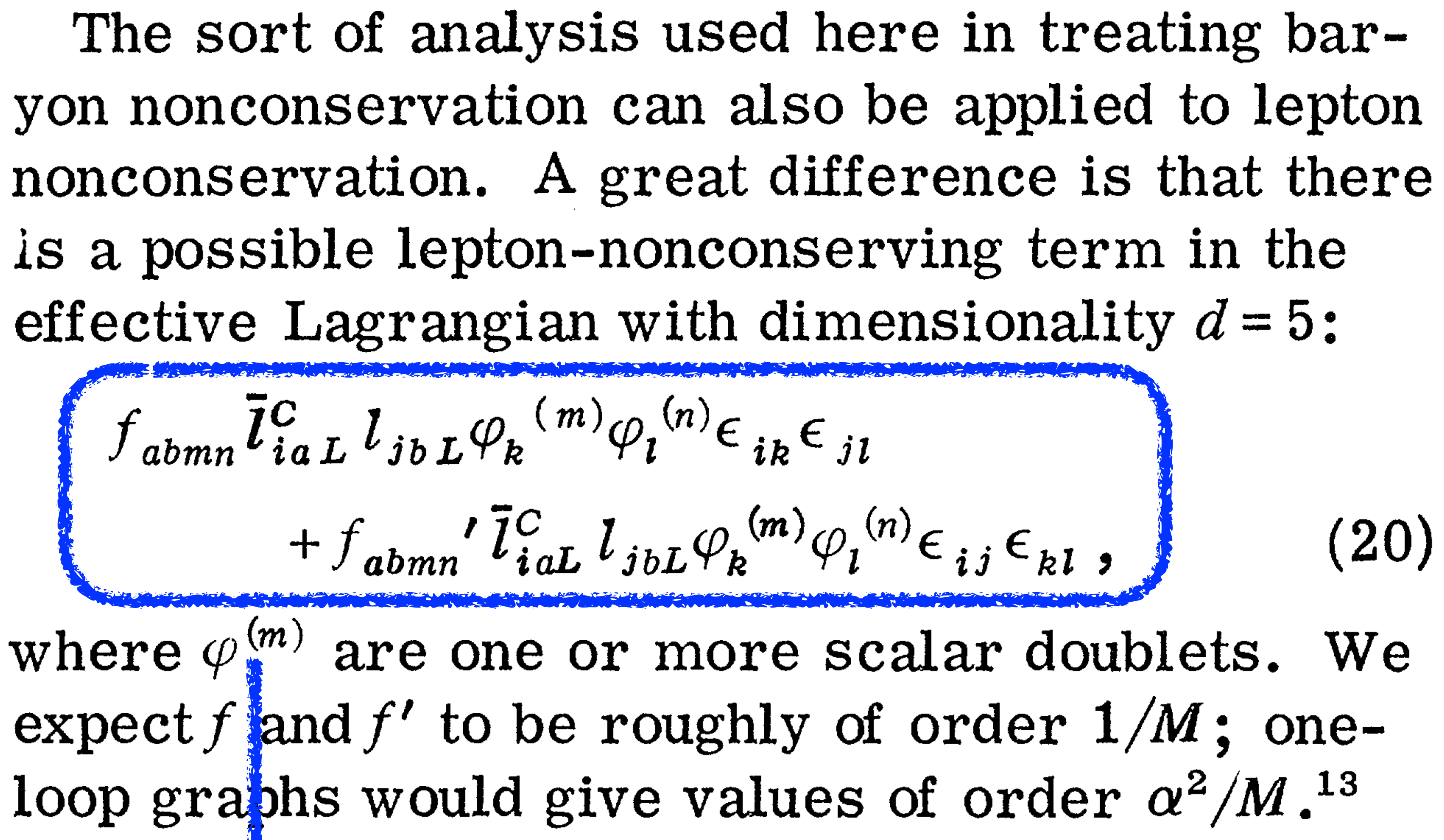
Weinberg, Phys. Rev. Lett. 43 (1979) 1566-1570



Baryon- and Lepton-Nonconserving Processes

Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, and
Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138
(Received 13 August 1979)

A number of properties of possible baryon- and lepton-nonconserving processes are shown to follow under very general assumptions. Attention is drawn to the importance of measuring μ^+ polarizations and $\bar{\nu}_e/e^+$ ratios in nucleon decay as a means of discriminating among specific models.



The sort of analysis used here in treating baryon nonconservation can also be applied to lepton nonconservation. A great difference is that there is a possible lepton-nonconserving term in the effective Lagrangian with dimensionality $d=5$:

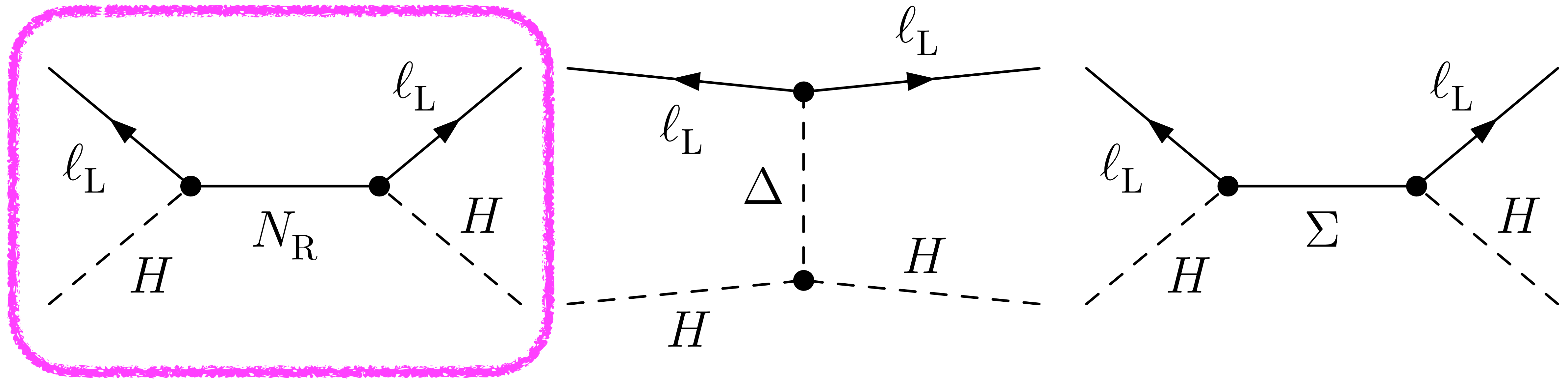
$$f_{abmn}\bar{l}^C_{iaL}\,l_{jbL}\varphi_k^{(m)}\varphi_l^{(n)}\epsilon_{ik}\epsilon_{jl} + f_{abmn}'\bar{l}^C_{iaL}\,l_{jbL}\varphi_k^{(m)}\varphi_l^{(n)}\epsilon_{ij}\epsilon_{kl}\,, \tag{20}$$

where $\varphi^{(m)}$ are one or more scalar doublets. We expect f and f' to be roughly of order $1/M$; one-loop graphs would give values of order α^2/M.[13]

tree-level realization

Steven Weinberg (Dec 1979) @ CERN
"*The rise and fall of baryon number*"

✴ New heavy degrees of freedom

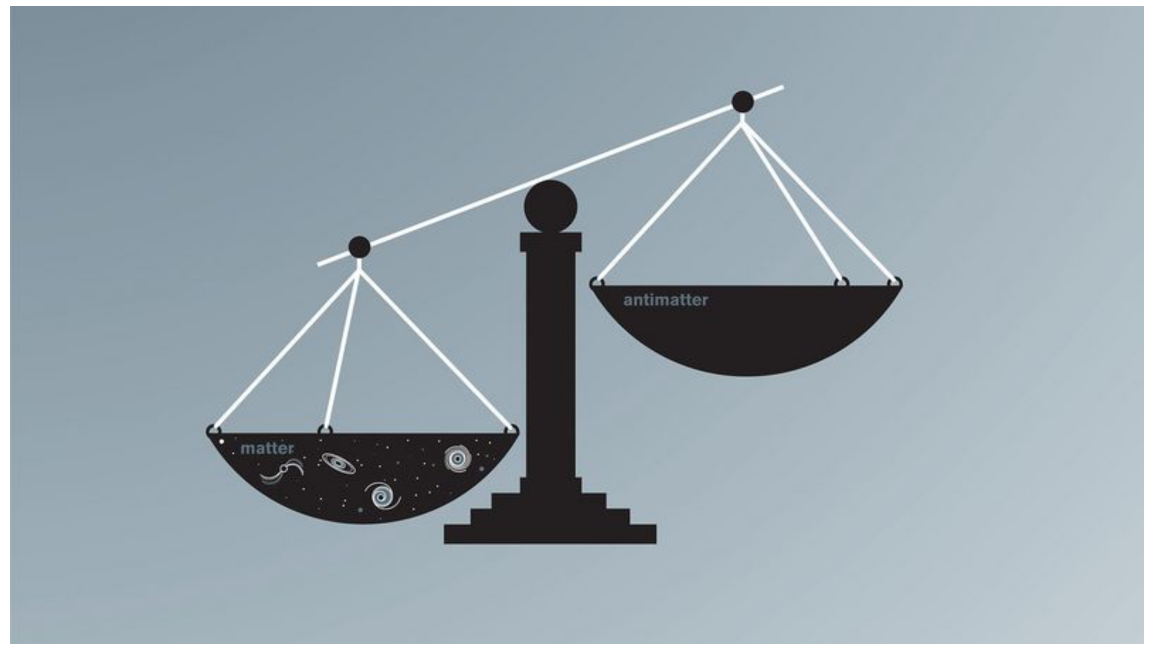


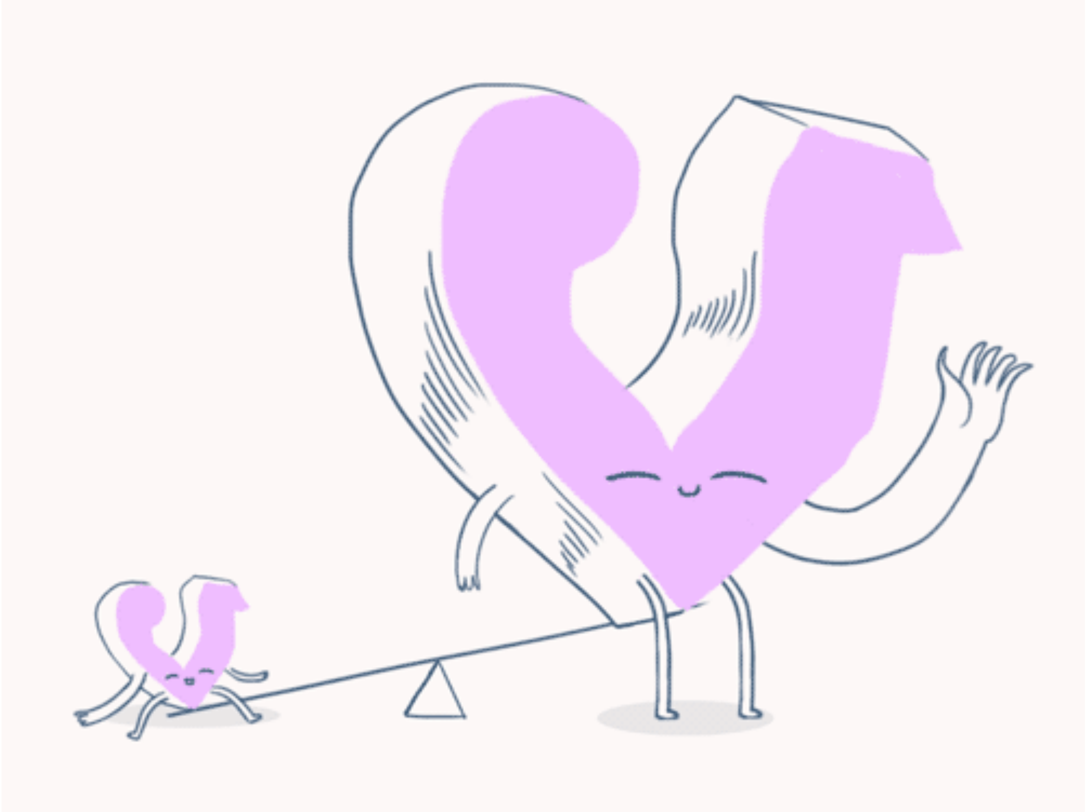
(a) (b) (c)

$$m_\nu^{\rm I} = -\frac{1}{2}y_\nu\frac{v^2}{m_{\rm R}}y_\nu^{\rm T} \qquad m_\nu^{\rm II} = \lambda_\Delta y_\Delta\frac{v^2}{M_\Delta} \qquad m_\nu^{\rm III} = -\frac{1}{2}y_\Sigma\frac{v^2}{m_\Sigma}y_\Sigma^{\rm T}$$

- ***Seesaw mechanism (+ leptogenesis)***:

*Solving **neutrino mass** & **BAU** at the same time!*





***Heavy fermions** will cause the derivation from the **unitary** leptonic flavor mixing.*

Interactions with Non-unitarity

❖ From type-I seesaw *EFT*

Broncano, Gavela & Jenkins,
hep-ph/0210271, hep-ph/0307058

$$\mathscr{L}^{\rm I}_{\rm EFT} = \mathscr{L}_{\rm SM} + \underline{\frac{1}{2}\left[\left(y_\nu m_{\rm R}^{-1} y_\nu^{\rm T}\right)_{\alpha\beta}\overline{\ell_{\alpha{\rm L}}}\tilde{H}\tilde{H}^{\rm T}\ell^{\rm c}_{\beta{\rm L}} + {\rm h.c.}\right]} + \underline{\left(y_\nu m_{\rm R}^{-2} y_\nu^{\dagger}\right)_{\alpha\beta}\left(\overline{\ell_{\alpha{\rm L}}}\tilde{H}\right){\rm i}\gamma^\mu\partial_\mu\left(\tilde{H}^\dagger\ell_{\beta{\rm L}}\right)}$$

Dim-5 (Weinberg operator): ***neutrino mass***

Dim-6: non-canonical neutrino kinetic terms

↓ *Warsaw basis*

$$\frac{1}{4}\underline{\left(y_\nu m_{\rm R}^{-2} y_\nu^{\dagger}\right)_{\alpha\beta}}\left[\left(\overline{\ell_{\alpha{\rm L}}}\gamma^\mu\ell_{\beta{\rm L}}\right)\underline{\left(H^\dagger{\rm i}\overleftrightarrow{D}_\mu H\right)} - \left(\overline{\ell_{\alpha{\rm L}}}\gamma^\mu\sigma^I\ell_{\beta{\rm L}}\right)\underline{\left(H^\dagger{\rm i}\overleftrightarrow{D}^I_\mu H\right)}\right]$$

←

Non-unitary mixing matrix:

$$N = (\mathbf{1} - \boxed{\eta})\cdot\boxed{V'}$$

Hermitian matrix *unitary matrix*

- A consequence of the ***polar decomposition theorem***: any matrix can be decomposed as the product of a Hermitian and a unitary matrix.
- Provide the connection with the ***dim-6 operator*** and the (Hermitian) ***Wilson coefficient***.

modify the NC & CC interactions ↓

$$\mathscr{L}_{\rm CC} = \frac{g}{\sqrt{2}}\overline{l_{\alpha{\rm L}}}\gamma^\mu P_{\rm L}\boxed{N_{\alpha i}}\nu_{i{\rm L}}W^-_\mu + {\rm h.c.}\,, \quad \mathscr{L}_{\rm NC} = \frac{g}{2\cos\theta_{\rm w}}\overline{\nu_{i{\rm L}}}\gamma^\mu P_{\rm L}\boxed{\left(N^\dagger N\right)}_{ij}\nu_{j{\rm L}}Z_\mu$$

non-unitary

$\neq \mathbf{1}$

Type-I Seesaw Model

❖ *From type-I seesaw model*

Peter Minkowski
(1941-2025)

Dirac + Majorana mass terms

$$\mathscr{L}_{\rm mass} = -\frac{1}{2}\overline{\left(\nu_{\rm L} \quad N_{\rm R}^{\rm c}\right)} \begin{pmatrix} \mathbf{0} & m_{\rm D} \\ m_{\rm D}^{\rm T} & m_{\rm R} \end{pmatrix} \begin{pmatrix} \nu_{\rm L}^{\rm c} \\ N_{\rm R} \end{pmatrix} + {\rm h.c.}$$

PMNS matrix

$$\xrightarrow{\text{diagonalization}} \begin{pmatrix} N & R \\ S & U \end{pmatrix}^{\dagger} \begin{pmatrix} \mathbf{0} & m_{\rm D} \\ m_{\rm D}^{\rm T} & m_{\rm R} \end{pmatrix} \begin{pmatrix} N & R \\ S & U \end{pmatrix}^{*} = \begin{pmatrix} \widehat{m} & \mathbf{0} \\ \mathbf{0} & \widehat{M} \end{pmatrix}$$

← *light neutrino masses*

← *heavy neutrino masses*

The 6×6 matrix is unitary, while the 3×3 matrix N is **not** !

- ***Euler parametrization of 6×6 matrix***

$$\mathscr{U} = \begin{pmatrix} N & R \\ S & U \end{pmatrix} = \underbrace{\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0 \end{pmatrix}}_{\text{heavy}} \underbrace{\begin{pmatrix} A & R \\ D & B \end{pmatrix}}_{\text{heavy-light}} \underbrace{\begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}}_{\text{light}}$$

$$= \left(\mathcal{O}_{56}\mathcal{O}_{46}\mathcal{O}_{45}\right) \times \left(\mathcal{O}_{36}\mathcal{O}_{26}\mathcal{O}_{16}\mathcal{O}_{35}\mathcal{O}_{25}\mathcal{O}_{15}\mathcal{O}_{34}\mathcal{O}_{24}\mathcal{O}_{14}\right) \times \left(\mathcal{O}_{23}\mathcal{O}_{13}\mathcal{O}_{12}\right)$$

$c_{ij} \equiv \cos\theta_{ij}$

$\hat{s}_{ij} \equiv {\rm e}^{{\rm i}\delta_{ij}} \sin\theta_{ij}$

$$\mathcal{O}_{24} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & \hat{s}_{24}^{*} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\hat{s}_{24} & 0 & c_{24} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

*Xing, **0709.2220; 1110.0083***

Euler Parametrization

❖ *From type-I seesaw model*

Xing, 0709.2220; 1110.0083

$$\mathcal{U} = \begin{pmatrix} N & R \\ S & U \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0 \end{pmatrix} \begin{pmatrix} \boxed{A} & R \\ D & B \end{pmatrix} \begin{pmatrix} \boxed{V_0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} = \left(\mathcal{O}_{56}\mathcal{O}_{46}\mathcal{O}_{45}\right) \times \left(\mathcal{O}_{36}\mathcal{O}_{26}\mathcal{O}_{16}\mathcal{O}_{35}\mathcal{O}_{25}\mathcal{O}_{15}\mathcal{O}_{34}\mathcal{O}_{24}\mathcal{O}_{14}\right) \times \left(\mathcal{O}_{23}\mathcal{O}_{13}\mathcal{O}_{12}\right)$$

unitary matrix

$$V_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}^*_{12}c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{12}c_{23} - \hat{s}^*_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{13}\hat{s}^*_{23} \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -\hat{s}^*_{12}\hat{s}_{13}c_{23} - c_{12}\hat{s}_{23} & c_{13}c_{23} \end{pmatrix}$$

$$A = \begin{pmatrix} c_{14}c_{15}c_{16} & 0 & 0 \\ \begin{array}{c} -c_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} - c_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} \\ -\hat{s}_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & c_{24}c_{25}c_{26} & 0 \\ \begin{array}{c} -c_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} + c_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + \hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} \\ +\hat{s}_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} - \hat{s}_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{c} -c_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} - c_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -\hat{s}_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & c_{34}c_{35}c_{36} \end{pmatrix}$$

lower-triangular matrix

A similar structure appears in the ***inverse seesaw model***: *Han & Xing, **2110.12705***—Structure is property.

Low-energy ***derivatives*** can be calculated from high-energy ***original*** parameters:

✓ *Xing, **2306.02362**; **2406.01142***

✓ *Xing & Zhu, **2412.17698***

✓ ***JH** & Zhou, **2507.21691***

Non-unitary mixing matrix:

$$N = \boxed{A} \cdot \boxed{V_0}$$

*More suitable for discussing the **non-unitarity effects***

VS

Non-unitary mixing matrix:

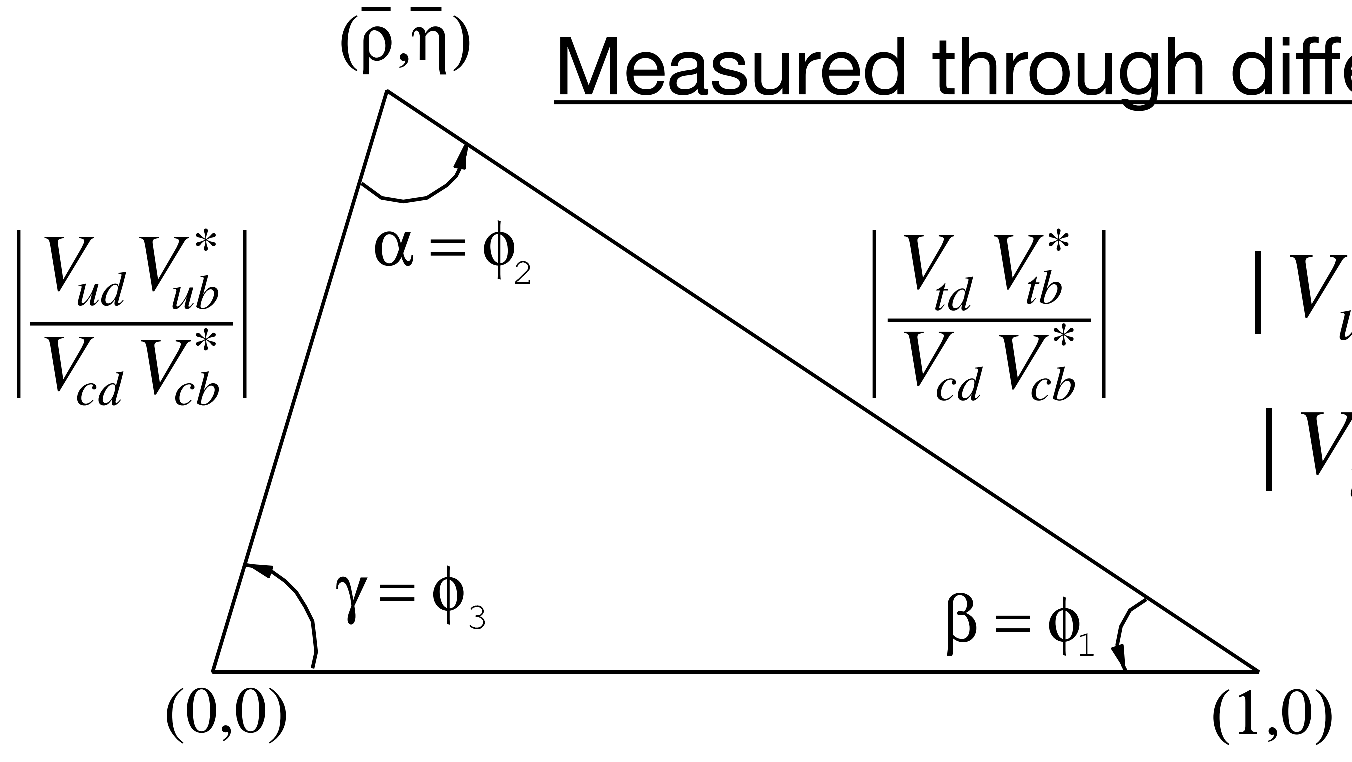
$$N = (\mathbf{1} - \boxed{\eta}) \cdot \boxed{V'}$$

- $V_0 \neq V' \neq V$ (unitary case), **matching relations**: *Blennow et al., **1609.08637***; *Wang & Zhou, **2109.13622***

How to Test the Non-unitarity?

Unitarity triangle $\Delta_s : V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$

Measured through different channels



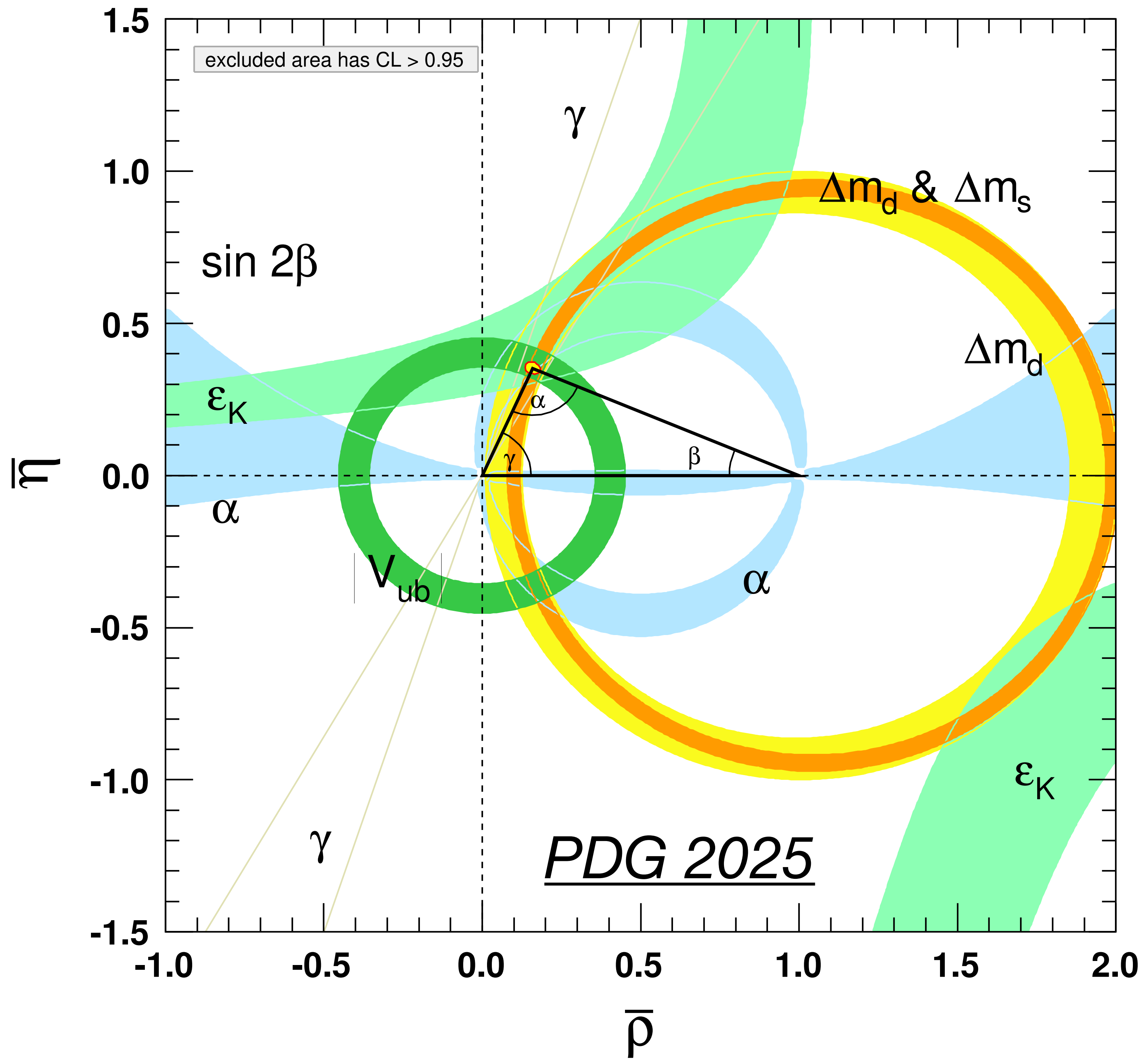
- ❖ Assume the **unitarity** of mixing matrix (V) and measure oscillation parameters **in different experiments** precisely.

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0007 \quad \text{(1st row)}$$

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.9971 \pm 0.0020 \quad \text{(1st column)}$$

$$\alpha + \beta + \gamma = (172 \pm 5)^\circ$$

- Jarlskog invariant can be calculated from the moduli of CKM matrix elements: ***Luo** & **Xing, 2309.07656***

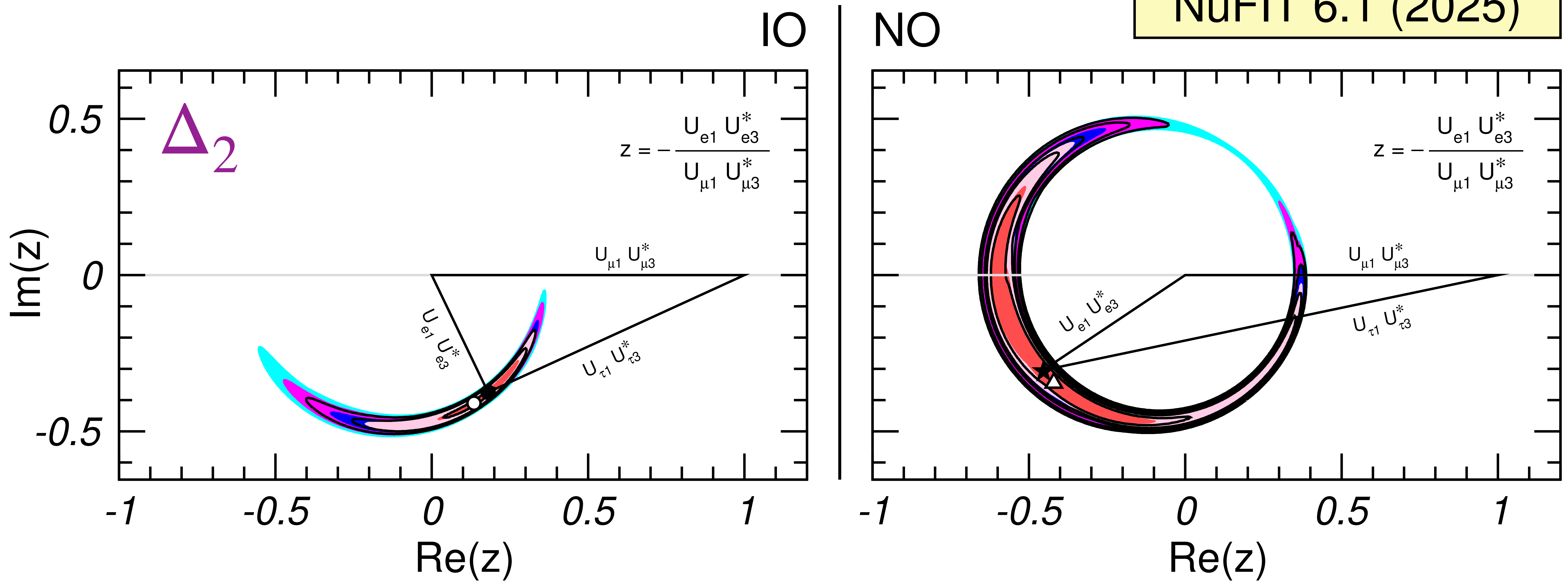


PDG 2025

θ_{12}: solar, long-baseline reactor; θ_{13}: short-baseline reactor, accelerator

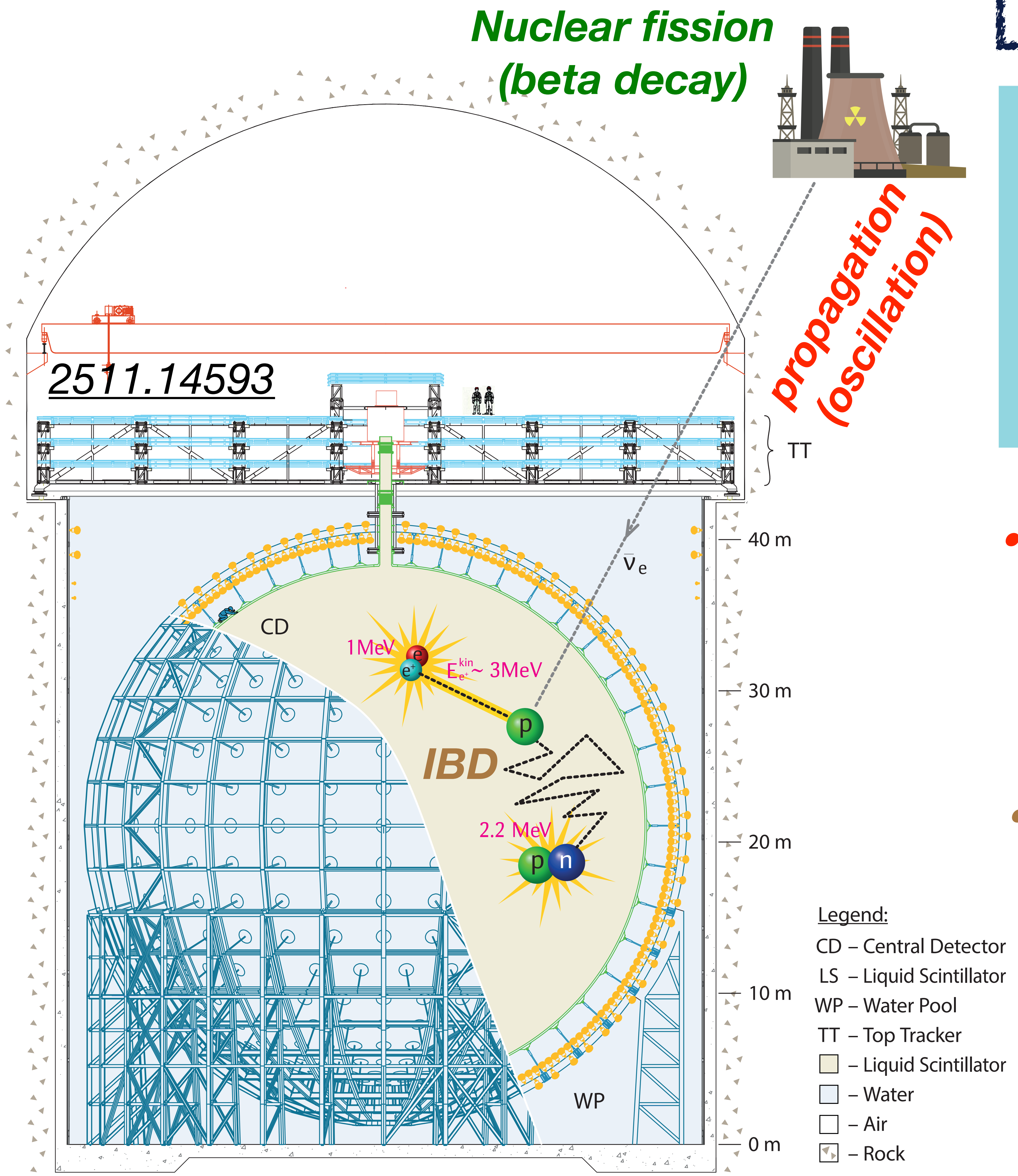
θ_{23}: atmospheric, accelerator; δ_{CP}: accelerator

NuFIT 6.1 (2025)



Procedure in the Unitary Case

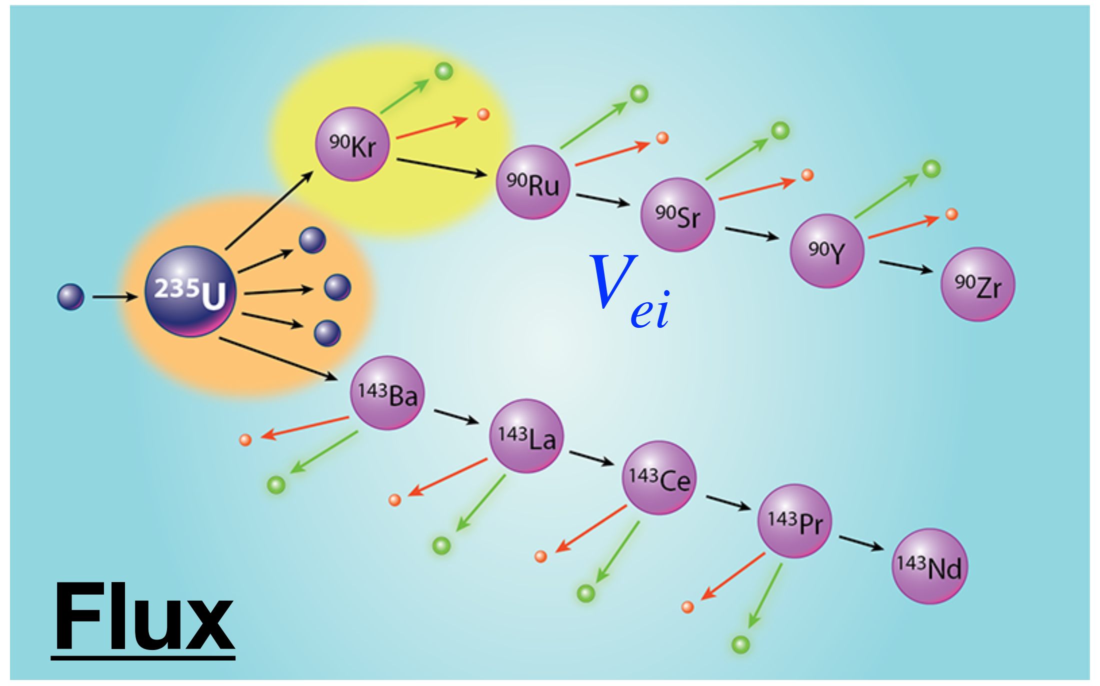
✦ *How to test the unitarity?*



Nuclear fission
(beta decay)

propagation
(oscillation)

✓ Adopt the complete theory with **non-unitary** flavor mixing and analyze the **production**, **propagation** and **detection** of neutrinos.



Flux

$$\mathscr{L}_{\rm CC} = \frac{g}{\sqrt{2}} \overline{l_{\alpha \rm L}} \gamma^\mu P_{\rm L} \boxed{V_{\alpha i}} \nu_{i\rm L} W^-_\mu + {\rm h.c.}$$

unitary

- ***Neutrino flavor states :*** $\left|\nu_\alpha\right\rangle = \sum_{i=1}^{3} V^*_{\alpha i} \left|\nu_i\right\rangle$

time evolution

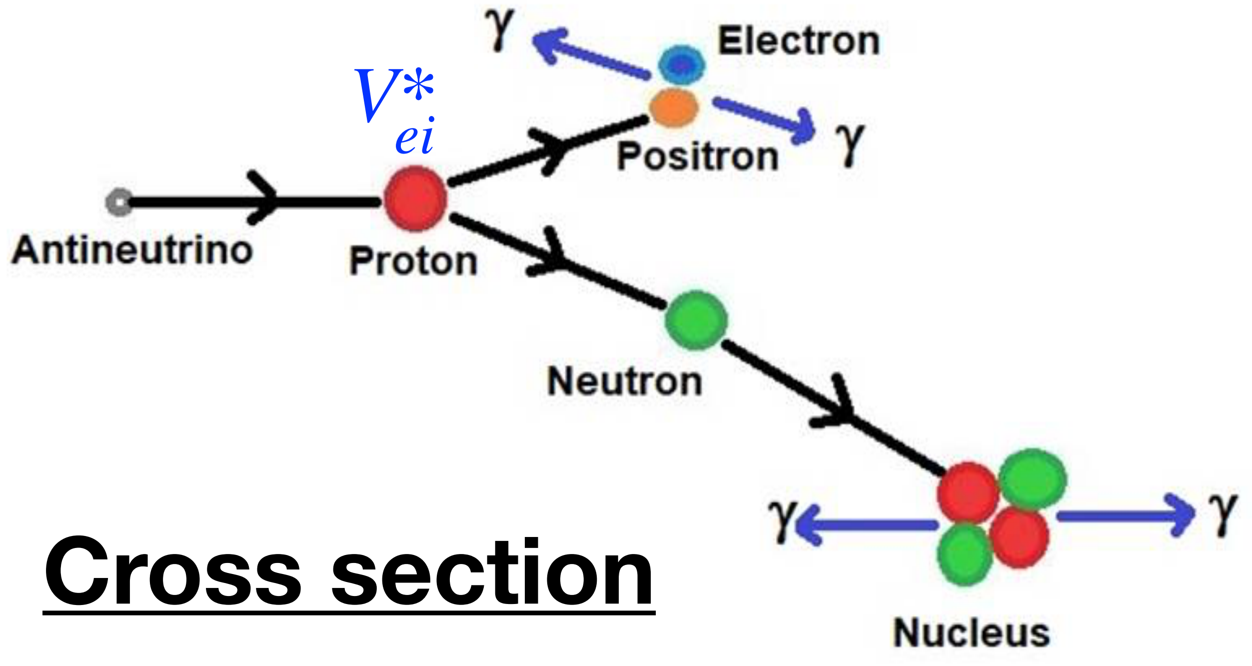
- ***Oscillation probabilities :***

$$P\left(\nu_\alpha \rightarrow \nu_\beta\right) \equiv \left|\left\langle \nu_\beta \middle| \nu_\alpha(L)\right\rangle\right|^2 \longleftarrow \left|\nu_\alpha(L)\right\rangle = \sum_{i=1}^{3} V^*_{\alpha i}\, {\rm e}^{-{\rm i}EL} \left|\nu_i(0)\right\rangle$$

- ***Event rates:***

$$\frac{{\rm d}N}{{\rm d}E_{\rm obs}} \propto N_{\rm target} t \int {\rm d}E_\nu \frac{{\rm d}\Phi_\alpha}{{\rm d}E_\nu} \times \frac{P_{\alpha\beta}(E, L)}{4\pi L^2} \times \sigma(E)$$

Q: What impact does non-unitarity have on those three parts?



Cross section

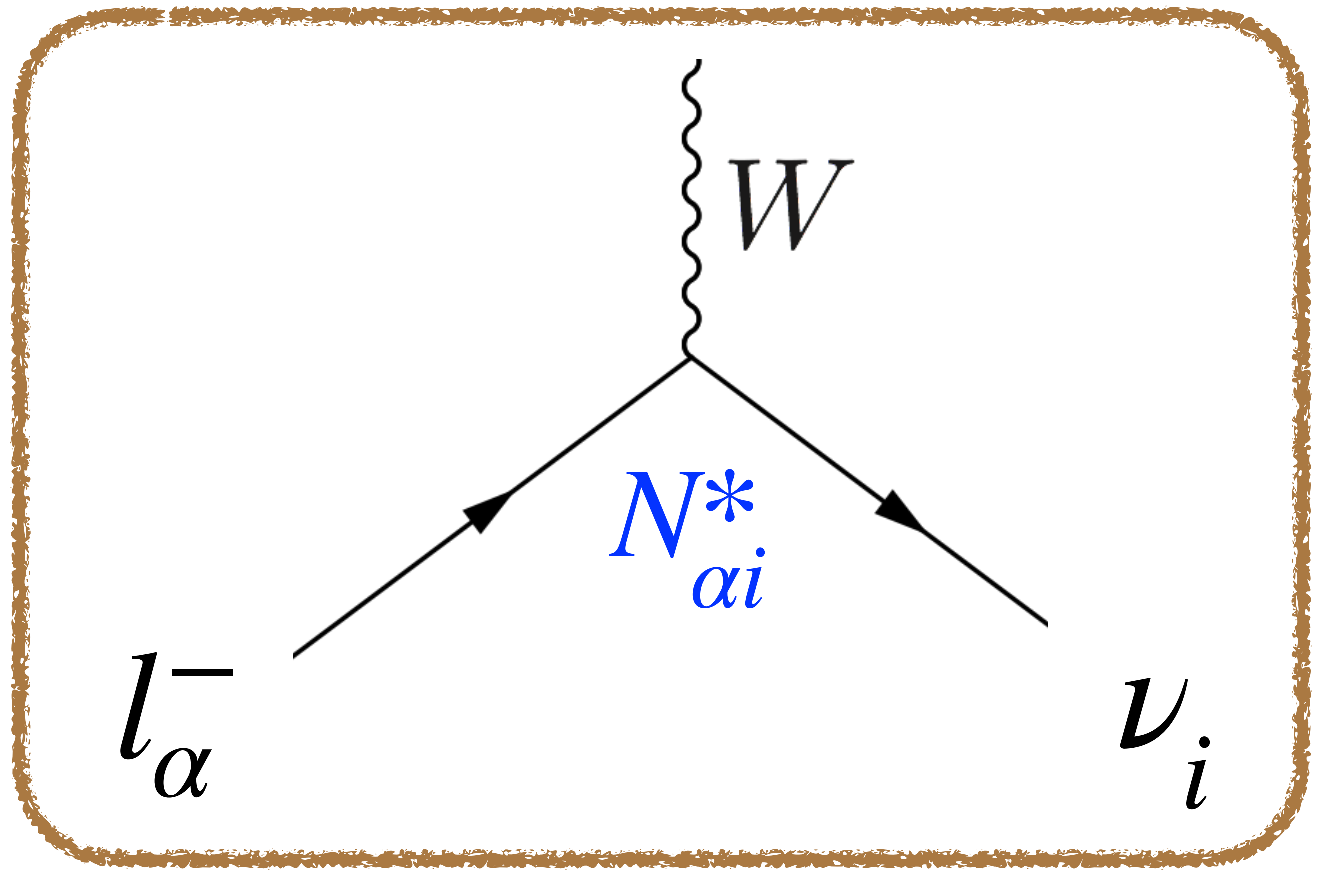
Flavor States with Non-unitarity

- ***Charged-current interaction:*** $$\mathscr{L}_{\rm CC} = \frac{g}{\sqrt{2}} \overline{l_{\alpha \rm L}} \gamma^\mu P_{\rm L} \boxed{N_{\alpha i}} \nu_{i \rm L} W^-_\mu + {\rm h\,.\,c\,.}$$

non-unitary

↓

- ***Neutrino flavor states (non-unitary case):*** $$\left|\nu_\alpha\right\rangle \equiv \sum_{i=1}^{3} \frac{\boxed{N^*_{\alpha i}}}{\sqrt{\left(NN^\dagger\right)_{\alpha\alpha}}} \left|\nu_i\right\rangle$$



✦ Oscillation probabilities :

$$P\left(\nu_\alpha \to \nu_\beta\right) \equiv \left|\langle \nu_\beta | \nu_\alpha(L) \rangle\right|^2 = \frac{1}{\left(NN^\dagger\right)_{\alpha\alpha}\left(NN^\dagger\right)_{\beta\beta}} \left[\left|\left(NN^\dagger\right)_{\alpha\beta}\right|^2 - 4\sum_{i<j} {\rm Re}\, \mathscr{N}^{ij}_{\alpha\beta} \sin^2 F_{ji} + 2\sum_{i<j} {\rm Im}\, \mathscr{N}^{ij}_{\alpha\beta} \sin 2F_{ji}\right]$$

✓ Starting from the ***definition*** of *flavor states*, independent of the ***production*** and ***detection*** processes.

$$\mathscr{N}^{ij}_{\alpha\beta} \equiv N_{\alpha i} N_{\beta j} N^*_{\alpha j} N^*_{\beta i}\,, \quad F_{ji} \equiv \Delta m^2_{ji} L/(4E)$$

✓ The mixing matrix $N = T \cdot V$

$$T \equiv \mathbf{1} - \zeta = \mathbf{1} - \begin{pmatrix} \zeta_{ee} & 0 & 0 \\ \zeta_{\mu e} & \zeta_{\mu\mu} & 0 \\ \zeta_{\tau e} & \zeta_{\tau\mu} & \zeta_{\tau\tau} \end{pmatrix}$$

real & positive

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}{\rm e}^{-{\rm i}\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}{\rm e}^{{\rm i}\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}{\rm e}^{{\rm i}\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}{\rm e}^{{\rm i}\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}{\rm e}^{{\rm i}\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix}$$

Oscillation Probabilities

- ***Case I: $\overline{\nu}_e \to \overline{\nu}_e$ Disappearance channels***

$$\overline{P}_{ee} = P_{ee} = \frac{1}{\left(1-\zeta_{ee}\right)^4}\left[\left(1-\zeta_{ee}\right)^4 - 4\left(1-\zeta_{ee}\right)^4 \sum_{i<j} \left|V_{ei}\right|^2 \left|V_{ej}\right|^2 \sin^2 F_{ji}\right]$$

$$= 1 - 4\sum_{i<j} \underline{\left|V_{ei}\right|^2 \left|V_{ej}\right|^2} \sin^2 F_{ji}$$

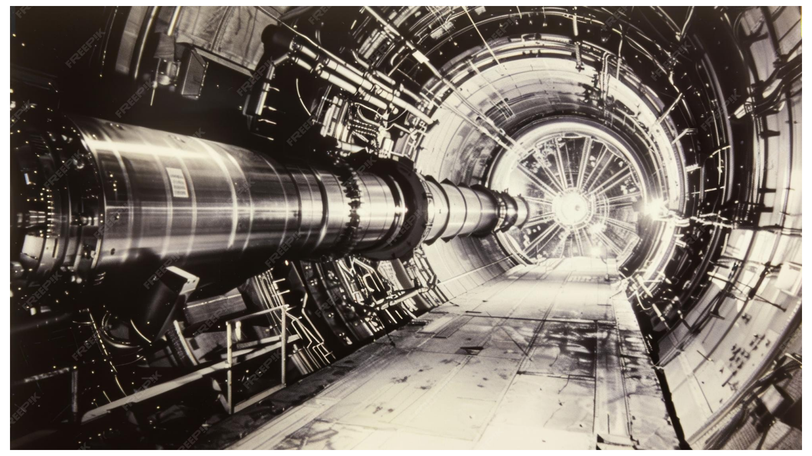
Insensitive to the non-unitarity effects!

  - The mixing parameters directly correspond to those in the **unitary** case.
  - It is necessary to consider the **production** and **detection** processes.

- ***Case II: $\overline{\nu}_e \to \overline{\nu}_\mu$ Appearance channels***

$$\overline{P}_{e\mu} \approx -4\sum_{i<j} \mathrm{Re}\left[V_{ei}V_{\mu j}V^*_{ej}V^*_{\mu i} - \left(\underline{\zeta_{\mu e}}V_{ei}V^*_{\mu i}\left|V_{ej}\right|^2 + \underline{\zeta^*_{\mu e}}V^*_{ej}V_{\mu j}\left|V_{ei}\right|^2\right)\right]\sin^2 F_{ji}$$

$$-2\sum_{i<j} \mathrm{Im}\left[V_{ei}V_{\mu j}V^*_{ej}V^*_{\mu i} - \left(\underline{\zeta_{\mu e}}V_{ei}V^*_{\mu i}\left|V_{ej}\right|^2 + \underline{\zeta^*_{\mu e}}V^*_{ej}V_{\mu j}\left|V_{ei}\right|^2\right)\right]\sin 2F_{ji}\,.$$



Zero-distance Effect

$$N = T \cdot V \qquad T \equiv \mathbf{1} - \zeta = \mathbf{1} - \begin{pmatrix} \zeta_{ee} & 0 & 0 \\ \zeta_{\mu e} & \zeta_{\mu\mu} & 0 \\ \zeta_{\tau e} & \zeta_{\tau\mu} & \zeta_{\tau\tau} \end{pmatrix}$$

$$P_{\alpha\beta} = \frac{1}{\left(NN^\dagger\right)_{\alpha\alpha}\left(NN^\dagger\right)_{\beta\beta}} \left[\left| \left(NN^\dagger\right)_{\alpha\beta} \right|^2 - 4 \sum_{i<j} \mathrm{Re} \left(N_{\alpha i} N_{\beta j} N^*_{\alpha j} N^*_{\beta i} \right) \sin^2 \frac{\Delta m^2_{ji} L}{4E} + 2 \sum_{i<j} \mathrm{Im} \left(N_{\alpha i} N_{\beta j} N^*_{\alpha j} N^*_{\beta i} \right) \sin \frac{\Delta m^2_{ji} L}{2E} \right]$$

- ***Disappearance channel*** $P_{\alpha\alpha}(L=0) = \frac{1}{\left(NN^\dagger\right)_{\alpha\alpha}\left(NN^\dagger\right)_{\alpha\alpha}} \left[\left| \left(NN^\dagger\right)_{\alpha\alpha} \right|^2 \right] = 1$

Zero-distance effect !

- ***Appearance channel*** $P_{e\mu}(L=0) = \frac{1}{\left(NN^\dagger\right)_{ee}\left(NN^\dagger\right)_{\mu\mu}} \left[\left| \left(NN^\dagger\right)_{e\mu} \right|^2 \right] \approx |\zeta_{\mu e}|^2 \neq 0$

✓ **Near detectors are useful for constraining ζ**

Oscillations without Unitarity

$$N = T \cdot V \qquad T \equiv \mathbf{1} - \zeta = \mathbf{1} - \begin{pmatrix} \zeta_{ee} & 0 & 0 \\ \zeta_{\mu e} & \zeta_{\mu\mu} & 0 \\ \zeta_{\tau e} & \zeta_{\tau\mu} & \zeta_{\tau\tau} \end{pmatrix}$$

$$P_{e\mu}(L=0) = \frac{1}{\left(NN^\dagger\right)_{ee}\left(NN^\dagger\right)_{\mu\mu}}\left[\left|\left(NN^\dagger\right)_{e\mu}\right|^2\right] \approx |\zeta_{\mu e}|^2 \neq 0$$

$$P_{\alpha\beta} = \frac{1}{\left(NN^\dagger\right)_{\alpha\alpha}\left(NN^\dagger\right)_{\beta\beta}}\left[\left|\left(NN^\dagger\right)_{\alpha\beta}\right|^2 - 4\sum_{i<j}\mathrm{Re}\left(N_{\alpha i}N_{\beta j}N^*_{\alpha j}N^*_{\beta i}\right)\sin^2\frac{\Delta m^2_{ji}L}{4E} + 2\sum_{i<j}\mathrm{Im}\left(N_{\alpha i}N_{\beta j}N^*_{\alpha j}N^*_{\beta i}\right)\sin\frac{\Delta m^2_{ji}L}{2E}\right]$$

✦ All the oscillation probabilities are reduced to those in the unitary case by replacing $N \to V$ with $VV^\dagger = \mathbf{1}$.

✤ In the full type-I seesaw model, it is ***FORBIDDEN*** to take $T \to \mathbf{1}$ or $\zeta \to \mathbf{0}$

Don't switch off the Yukawa couplings between N_R and ℓ_L !

$$A \simeq \mathbf{1} - \frac{1}{2}\begin{pmatrix} s^2_{14} + s^2_{15} + s^2_{16} & 0 & 0 \\ 2\hat{s}_{14}\hat{s}^*_{24} + 2\hat{s}_{15}\hat{s}^*_{25} + 2\hat{s}_{16}\hat{s}^*_{26} & s^2_{24} + s^2_{25} + s^2_{26} & 0 \\ 2\hat{s}_{14}\hat{s}^*_{34} + 2\hat{s}_{15}\hat{s}^*_{35} + 2\hat{s}_{16}\hat{s}^*_{36} & 2\hat{s}_{24}\hat{s}^*_{34} + 2\hat{s}_{25}\hat{s}^*_{35} + 2\hat{s}_{26}\hat{s}^*_{36} & s^2_{34} + s^2_{35} + s^2_{36} \end{pmatrix} \simeq \mathbf{1} - \mathcal{O}\left(m^2_{\rm D}/m^2_{\rm R}\right)$$

Xing, 0709.2220; 1110.0083

Muon Decay with Non-unitarity

$$\mathscr{L}_{\rm CC} = \frac{g}{\sqrt{2}} \overline{l_{\alpha \rm L}} \gamma^{\mu} P_{\rm L} N_{\alpha i} \nu_{i \rm L} W^{-}_{\mu} + \rm h.c.$$

$$G_{\mu}^{\rm SM} = \left[g^2/(4\sqrt{2} m_W^2) \right] (1 + \Delta r)$$

$$G_{\mu}^{\rm exp} \approx 1.166 \times 10^{-5}\ \text{GeV}^{-2}$$

- ***<u>Decay amplitude</u>***

$$\mathscr{M}\left(\mu^- \to e^- + \nu_\mu + \overline{\nu}_e\right) \simeq -\frac{g^2}{2m_W^2} \left\langle e^- \underline{\nu_\mu \overline{\nu}_e} \right| \left(\overline{\nu}_i \gamma_\lambda P_{\rm L} N^*_{\mu i} \mu\right) \left(\overline{e} \gamma^\lambda P_{\rm L} N_{ej} \nu_j\right) \left| \mu^- \right\rangle$$

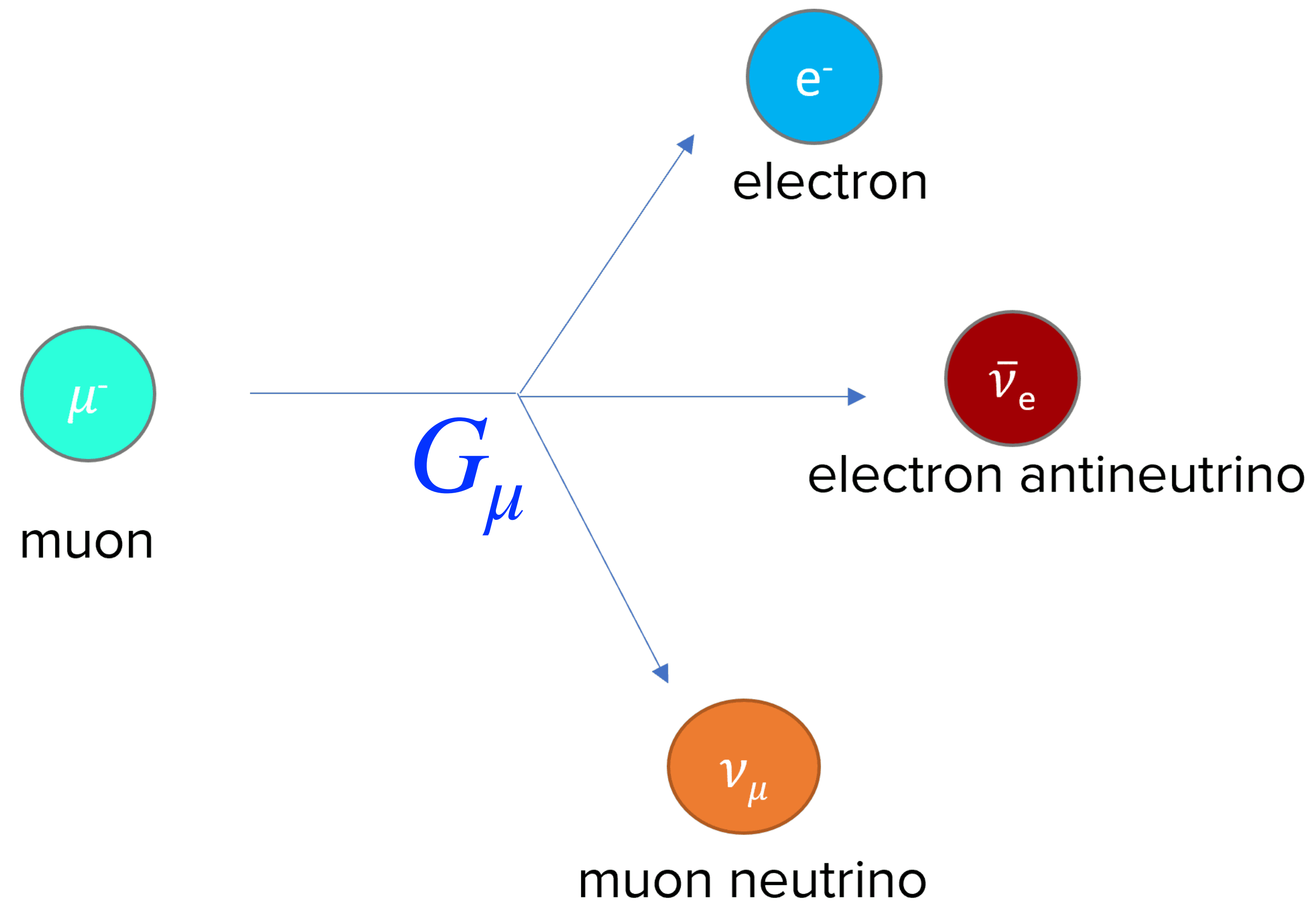
$$\langle \nu_j | \nu_i \rangle = \delta_{ij}$$

$$\left\langle \nu_\mu \right| = \sum_m \left\langle \nu_m \right| \frac{N_{\mu m}}{\sqrt{\left(NN^\dagger\right)_{\mu\mu}}}, \quad \left\langle \overline{\nu}_e \right| = \sum_n \left\langle \nu_n \right| \frac{N^*_{en}}{\sqrt{\left(NN^\dagger\right)_{ee}}}$$

$$\mathscr{M}\left(\mu^- \to e^- + \nu_\mu + \overline{\nu}_e\right) = \mathscr{M}_{\rm SM}\left(\mu^- \to e^- + \nu_\mu + \overline{\nu}_e\right) \sqrt{\left(NN^\dagger\right)_{ee} \left(NN^\dagger\right)_{\mu\mu}}$$

*<u>extracted from **muon lifetime** measurement</u>*

<u>Antusch et al., hep-ph/0607020</u>



- ***<u>Modified Fermi coupling constant</u>***

$$\boxed{G_\mu^{\rm exp} = G_\mu^{\rm SM} \sqrt{\left(NN^\dagger\right)_{ee} \left(NN^\dagger\right)_{\mu\mu}}}$$

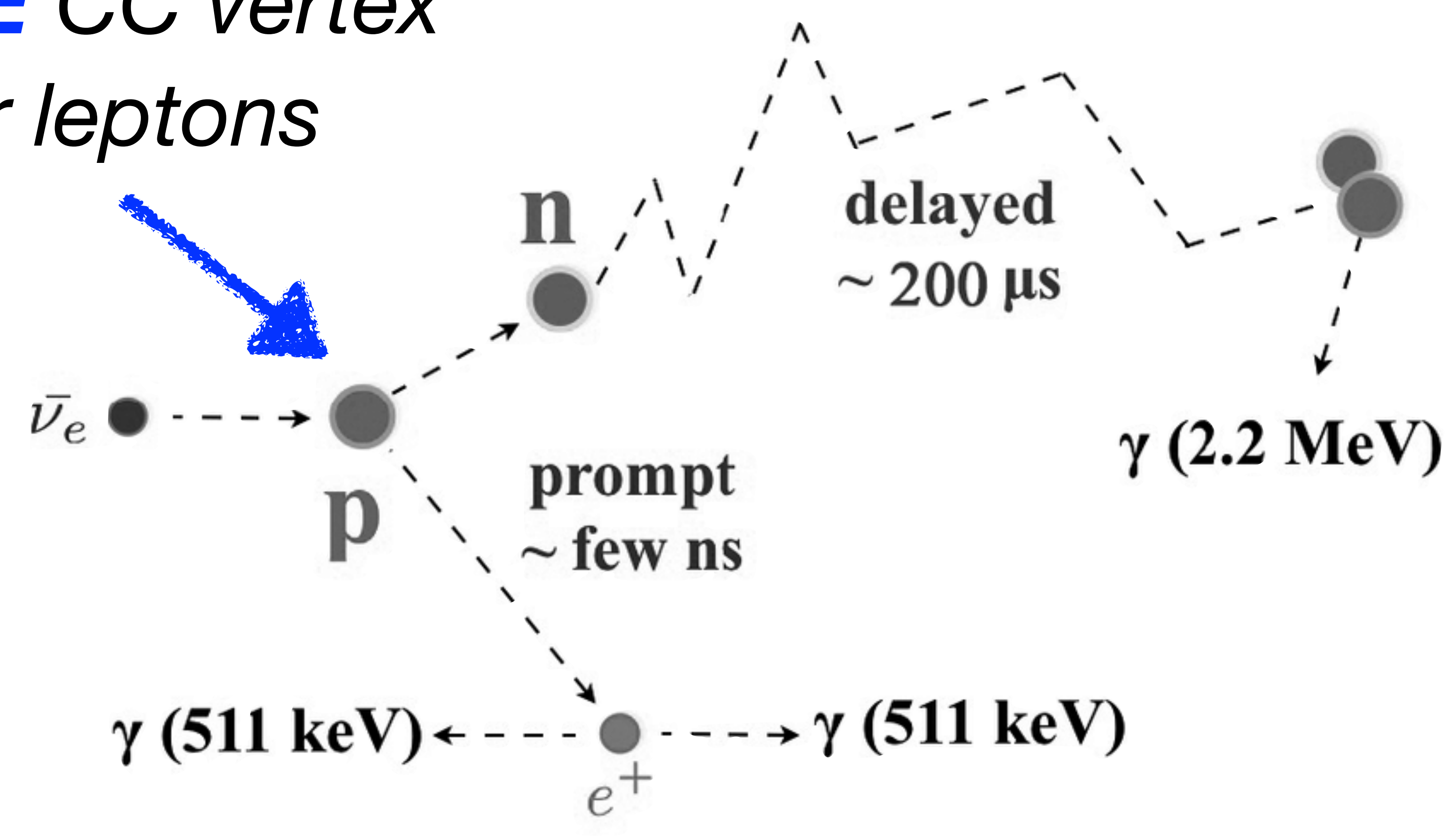
Cross Section: IBD

$$\mathscr{L}_{\rm CC} = \frac{g}{\sqrt{2}} \overline{l_{\alpha \rm L}} \gamma^\mu P_{\rm L} N_{\alpha i} \nu_{i \rm L} W^-_\mu + {\rm h.c.}$$

$$\boxed{\sigma_{\rm IBD} = \sigma^{\rm SM}_{\rm IBD}\left[G^{\rm SM}_\mu\right] \cdot \left(NN^\dagger\right)_{ee} = \sigma^{\rm SM}_{\rm IBD}\left[G^{\rm exp}_\mu\right] / \left(NN^\dagger\right)_{\mu\mu}}$$

$$NN^\dagger = \begin{pmatrix} \left(1-\zeta_{ee}\right)^2 & -\left(1-\zeta_{ee}\right)\zeta^*_{\mu e} & -\left(1-\zeta_{ee}\right)\zeta^*_{\tau e} \\ -\left(1-\zeta_{ee}\right)\zeta_{\mu e} & \boxed{\left(1-\zeta_{\mu\mu}\right)^2 + \left|\zeta_{\mu e}\right|^2} & -\left(1-\zeta_{\mu\mu}\right)\zeta^*_{\tau\mu} + \zeta_{\mu e}\zeta^*_{\tau e} \\ -\left(1-\zeta_{ee}\right)\zeta_{\tau e} & -\left(1-\zeta_{\mu\mu}\right)\zeta_{\tau\mu} + \zeta^*_{\mu e}\zeta_{\tau e} & \left(1-\zeta_{\tau\tau}\right)^2 + \left|\zeta_{\tau e}\right|^2 + \left|\zeta_{\tau\mu}\right|^2 \end{pmatrix}$$

Only ***ONE*** *CC vertex*
N_{ei} for leptons



$$G^{\rm exp}_\mu = G^{\rm SM}_\mu \sqrt{\left(NN^\dagger\right)_{ee} \left(NN^\dagger\right)_{\mu\mu}}$$

✦ An empirical formula for the **IBD cross section** in the SM

Strumia & Vissani, astro-ph/0302055

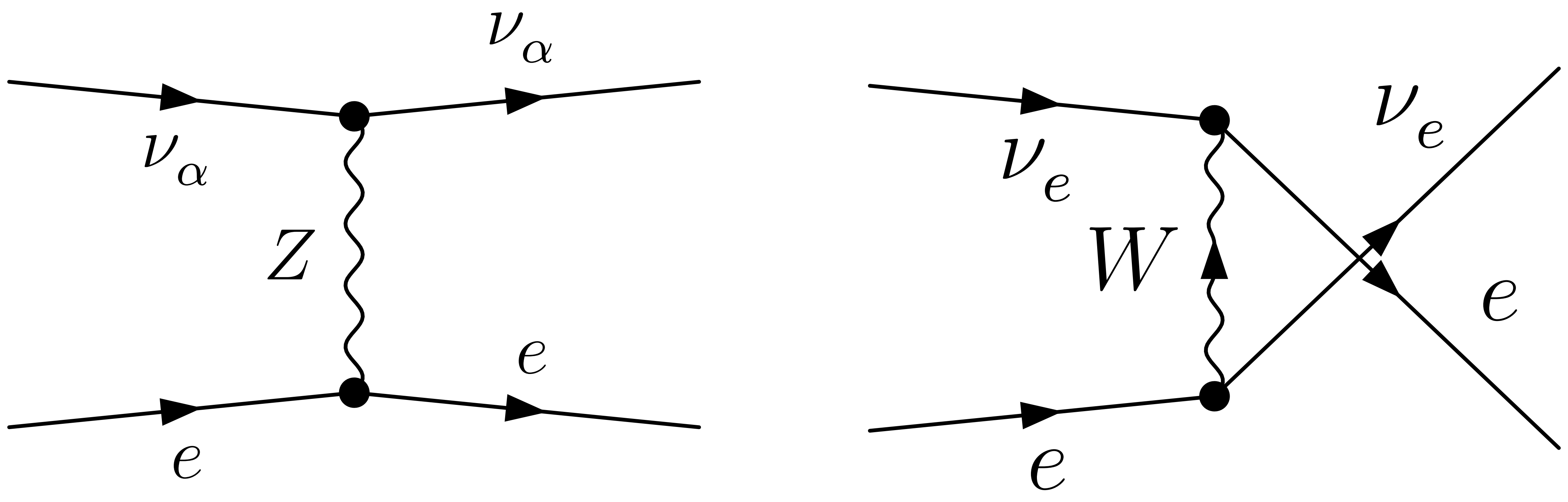
$$\sigma^{\rm SM}_{\rm IBD}(E) \approx 10^{-43}\ {\rm cm}^2 \left|\mathbf{p}_{e^+}\right| E_{e^+} E^{-0.07056+0.02018\ln E - 0.001953\ln^3 E}$$

✓ Discussions on **radiative corrections**, see, *e.g.*, **Tomalak**'s talk and *2512.07956, 2512.07957*

Cross Section: $\overline{\nu}$-e Scattering (SM)

- In the **SM** with the unitary mixing matrix

't Hooft, PLB 1971



NC

CC

$$\frac{\mathrm{d}\sigma_e^{\mathrm{SM}}}{\mathrm{d}T_e} = \frac{\left(G_\mu^{\exp}\right)^2 m_e}{2\pi}\left\{\frac{m_e T_e}{E^2}\left[\left(c_{\mathrm{A}}+1\right)^2 - \left(c_{\mathrm{V}}+1\right)^2\right] + \left(1-\frac{T_e}{E}\right)^2\left(c_{\mathrm{A}}+c_{\mathrm{V}}+2\right)^2 + \left(c_{\mathrm{A}}-c_{\mathrm{V}}\right)^2\right\}$$

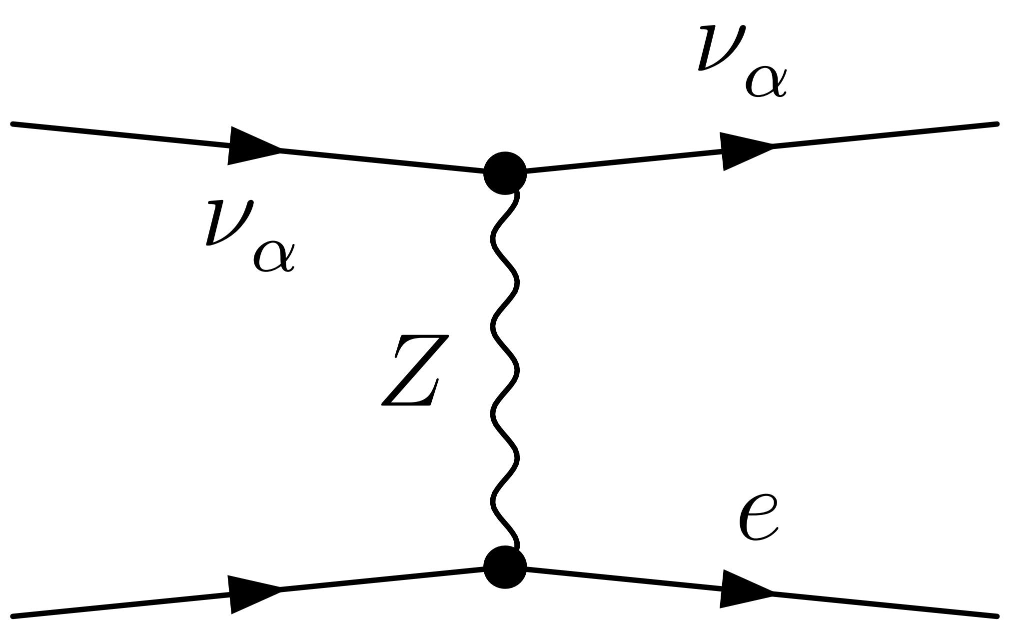
$$\frac{\mathrm{d}\sigma_{\mu,\tau}^{\mathrm{SM}}}{\mathrm{d}T_e} = \frac{\left(G_\mu^{\exp}\right)^2 m_e}{2\pi}\left\{\frac{m_e T_e}{E^2}\left(c_{\mathrm{A}}^2 - c_{\mathrm{V}}^2\right) + \left(1-\frac{T_e}{E}\right)^2\left(c_{\mathrm{A}}+c_{\mathrm{V}}\right)^2 + \left(c_{\mathrm{A}}-c_{\mathrm{V}}\right)^2\right\}$$

with $c_{\mathrm{V}} = -1/2 + 2\sin^2\theta_{\mathrm{w}}$ and $c_{\mathrm{A}} = -1/2$

✓One-loop corrections:
Sarantakos, Sirlin & Marciano, NPB 1983
Marciano & Parsa, hep-ph/0403168
*Tomalak & Hill, 1907.03379; **JH** & Zhou, 2412.17047*

$\overline{\nu}$-e Scattering with Non-unitarity

$$\mathscr{L}_{\rm CC} = \frac{g}{\sqrt{2}} \overline{l_{\alpha {\rm L}}} \gamma^\mu P_{\rm L} N_{\alpha i} \nu_{i{\rm L}} W^-_\mu + {\rm h.c.}, \qquad \mathscr{L}_{\rm NC} = \frac{g}{2\cos\theta_{\rm w}} \overline{\nu_{i{\rm L}}} \gamma^\mu P_{\rm L} \boxed{\left(N^\dagger N\right)}_{ij} \nu_{j{\rm L}} Z_\mu$$



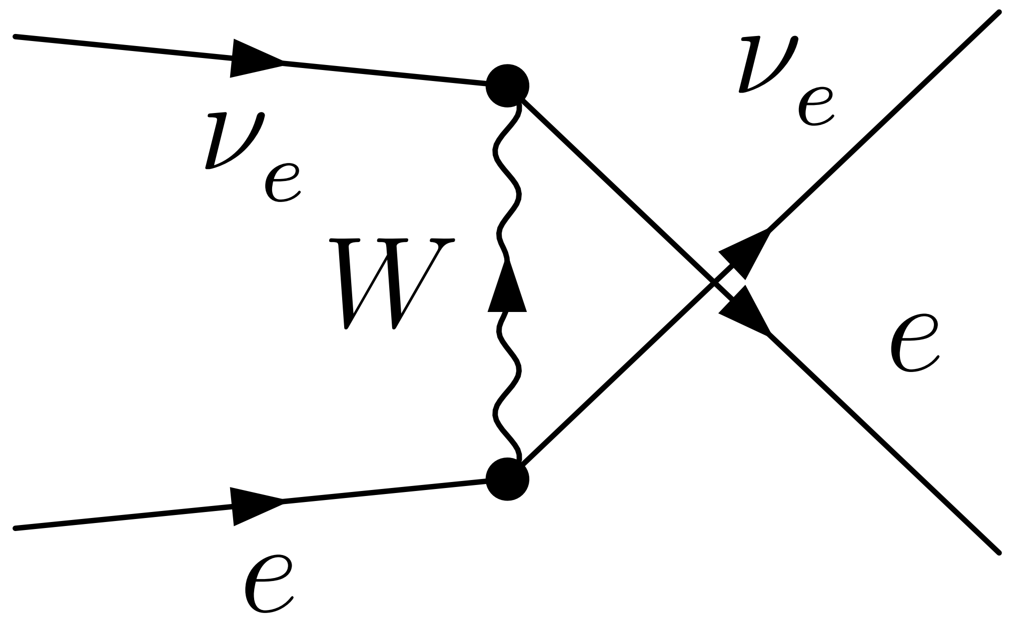
NC

$$\mathscr{M}_{\rm NC}\left(\overline{\nu}_\mu + e^- \to \overline{\nu}_\mu + e^-\right) = \boxed{\frac{\left(NN^\dagger NN^\dagger\right)_{\mu\mu}}{\left(NN^\dagger\right)_{\mu\mu}}} \mathscr{M}^{\rm SM}_{\rm NC}\left(\overline{\nu} + e^- \to \overline{\nu} + e^-\right)$$

Leading order contribution !

$$\mathscr{M}_{\rm CC}\left(\overline{\nu}_\mu + e^- \to \overline{\nu}_\mu + e^-\right) = \frac{\left|\left(NN^\dagger\right)_{\mu e}\right|^2}{\left(NN^\dagger\right)_{\mu\mu}} \mathscr{M}^{\rm SM}_{\rm CC}\left(\overline{\nu} + e^- \to \overline{\nu} + e^-\right)$$

Next-to-leading order contribution !



CC

$$\mathscr{M}_{\rm NC}\left(\overline{\nu}_e + e^- \to \overline{\nu}_e + e^-\right) = \frac{\left(NN^\dagger NN^\dagger\right)_{ee}}{\left(NN^\dagger\right)_{ee}} \mathscr{M}^{\rm SM}_{\rm NC}\left(\overline{\nu} + e^- \to \overline{\nu} + e^-\right)$$

$$\mathscr{M}_{\rm CC}\left(\overline{\nu}_e + e^- \to \overline{\nu}_e + e^-\right) = \left(NN^\dagger\right)_{ee} \mathscr{M}^{\rm SM}_{\rm CC}\left(\overline{\nu} + e^- \to \overline{\nu} + e^-\right)$$

Both are leading order contribution !

$\overline{\nu}$-e Scattering Cross Sections

$$\frac{\mathrm{d}\sigma_\beta}{\mathrm{d}T_e} = \frac{\left(G_\mu^{\mathrm{exp}}\right)^2 m_e}{2\pi \left(NN^\dagger\right)_{ee} \left(NN^\dagger\right)_{\mu\mu}} \left\{ \frac{m_e T_e}{E^2} \left[\left(\tilde{c}_{\mathrm{A}}^\beta + \tilde{c}_{\mathrm{C}}^\beta\right)^2 - \left(\tilde{c}_{\mathrm{V}}^\beta + \tilde{c}_{\mathrm{C}}^\beta\right)^2 \right] + \left(1 - \frac{T_e}{E}\right)^2 \left(\tilde{c}_{\mathrm{A}}^\beta + \tilde{c}_{\mathrm{V}}^\beta + 2\tilde{c}_{\mathrm{C}}^\beta\right)^2 + \left(\tilde{c}_{\mathrm{A}}^\beta - \tilde{c}_{\mathrm{V}}^\beta\right)^2 \right\}$$

$$\frac{\mathrm{d}\sigma_\beta}{\mathrm{d}T_e} \approx \frac{\mathrm{d}\sigma_\beta^{\mathrm{SM}}}{\mathrm{d}T_e} \times \begin{cases} 1 - 2\left(\zeta_{ee} - \zeta_{\mu\mu}\right) & \beta = e \\ 1 + 2\left(\zeta_{ee} - \zeta_{\mu\mu}\right) & \beta = \mu \\ 1 + 2\left(\zeta_{ee} + \zeta_{\mu\mu} - 2\zeta_{\tau\tau}\right) & \beta = \tau \end{cases} ,$$

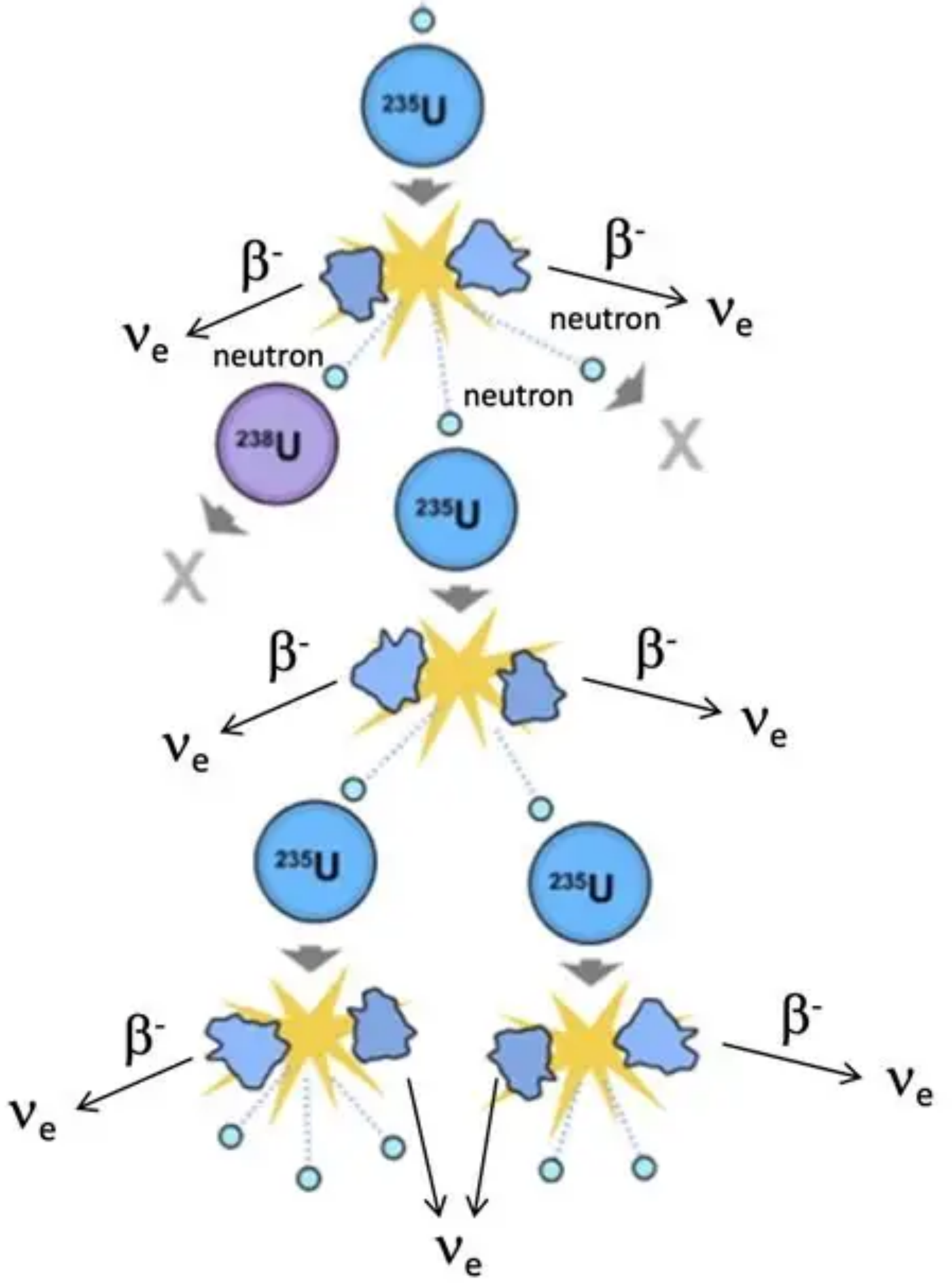
Only depends on ***diagonal*** *elements !*

Reactor Antineutrino Flux

✓ **Six** antineutrinos produced in each fission with **200 MeV** energy

Thermal power (measurement) → **Antineutrino flux**

Four main isotopes:
^{235}U, ^{238}U, ^{239}Pu and ^{241}Pu

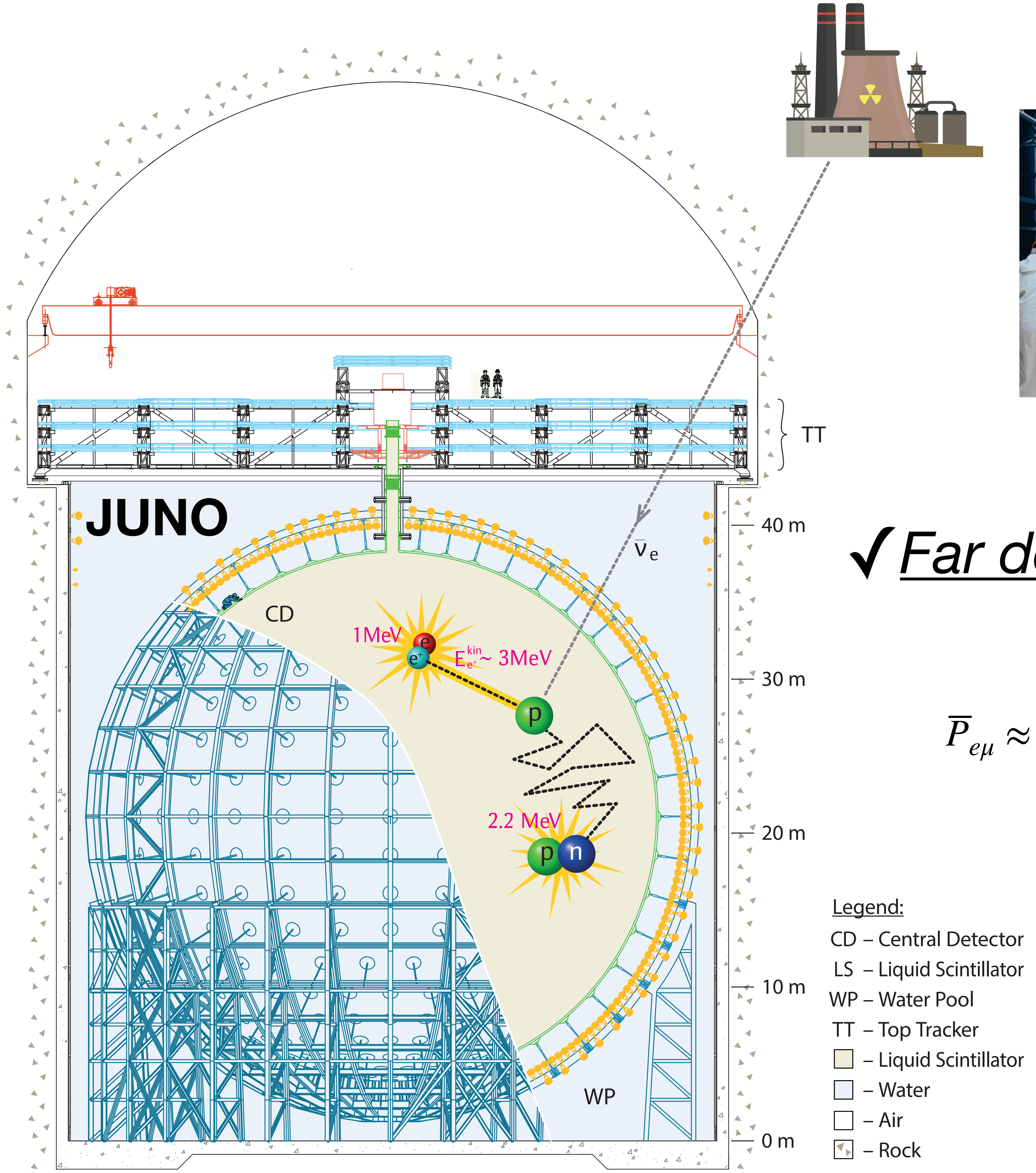


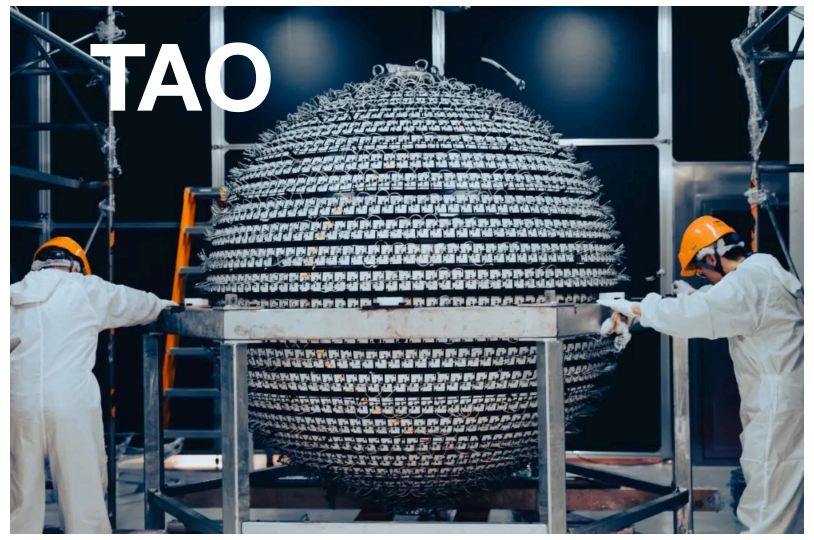
The antineutrino flux does not need any correction !

Non-unitarity Effects

Q: What impact does non-unitarity have on those three parts?

- *The antineutrino flux does not need any correction !*





✓ *Near detector*

$$P_{\alpha\alpha}(L=0) = \frac{1}{\left(NN^\dagger\right)_{\alpha\alpha}\left(NN^\dagger\right)_{\alpha\alpha}}\left[\left|\left(NN^\dagger\right)_{\alpha\alpha}\right|^2\right] = 1$$

$$\sigma_{\mathrm{IBD}} = \sigma_{\mathrm{IBD}}^{\mathrm{SM}}\left[G_\mu^{\exp}\right] / \left(NN^\dagger\right)_{\mu\mu}$$

✓ *Far detector*

$$\overline{P}_{ee} == 1 - 4\sum_{i<j}\left|V_{ei}\right|^2\left|V_{ej}\right|^2 \sin^2 F_{ji}$$

$$\overline{P}_{e\mu} \approx -4\sum_{i<j}\mathrm{Re}\left[V_{ei}V_{\mu j}V_{ej}^*V_{\mu i}^* - \left(\zeta_{\mu e}V_{ei}V_{\mu i}^*\left|V_{ej}\right|^2 + \zeta_{\mu e}^*V_{ej}^*V_{\mu j}\left|V_{ei}\right|^2\right)\right]\sin^2 F_{ji}$$

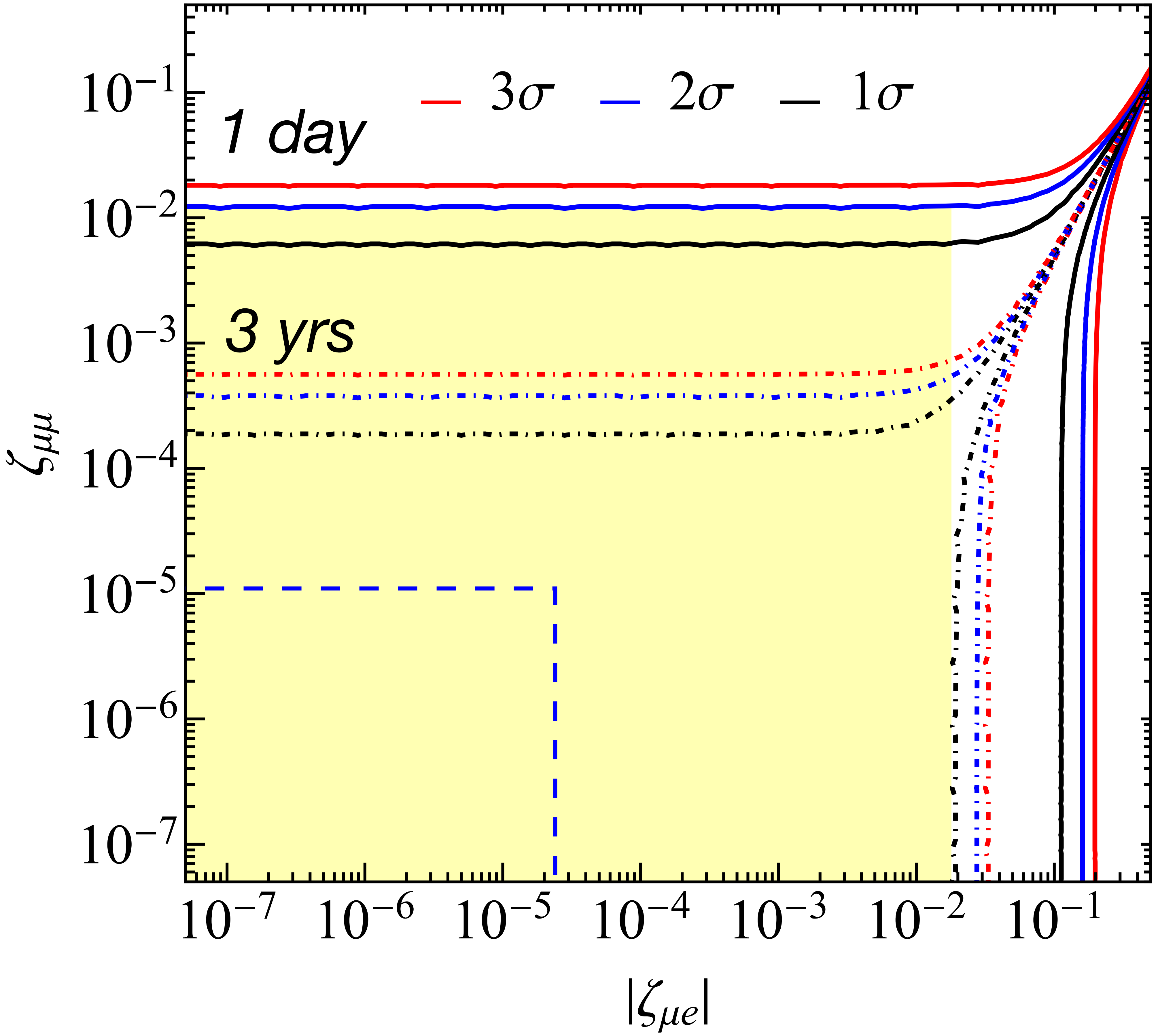
$$-2\sum_{i<j}\mathrm{Im}\left[V_{ei}V_{\mu j}V_{ej}^*V_{\mu i}^* - \left(\zeta_{\mu e}V_{ei}V_{\mu i}^*\left|V_{ej}\right|^2 + \zeta_{\mu e}^*V_{ej}^*V_{\mu j}\left|V_{ei}\right|^2\right)\right]\sin 2F_{ji}\,.$$

$$\frac{\mathrm{d}\sigma_\beta}{\mathrm{d}T_e} \approx \frac{\mathrm{d}\sigma_\beta^{\mathrm{SM}}}{\mathrm{d}T_e} \times \frac{\left[\left(NN^\dagger\right)_{\beta\beta}\right]^2}{\left(NN^\dagger\right)_{ee}\left(NN^\dagger\right)_{\mu\mu}}\,.$$

Appearance channel (ν-e scattering)

IBD Event Rates @ TAO

$$\frac{\mathrm{d}N_{\mathrm{TAO}}}{\mathrm{d}E_{\mathrm{obs}}} = N_p t \sum_i \int_{E_{\mathrm{thr}}}^{\infty} \mathrm{d}E \, \frac{\mathrm{d}\Phi^i_{\overline{\nu}_e}}{\mathrm{d}E} \frac{\overline{P}_{ee}(E,L)}{4\pi L_i^2} \sigma_{\mathrm{IBD}}(E) \underbrace{\mathscr{G}\left(E_{\mathrm{obs}}; E_{\mathrm{vis}}, \delta_{\mathrm{E}}^{\mathrm{TAO}}\right)}_{\text{response function}}$$



- **Construct χ^2 function**

$$\chi^2_{\mathrm{TAO}} = \sum_i \frac{\left(N^{i,\mathrm{th}}_{\mathrm{TAO}} - N^{i,\mathrm{exp}}_{\mathrm{TAO}}\right)^2}{N^{i,\mathrm{exp}}_{\mathrm{TAO}}}$$

$$\Big\downarrow$$

6300 events per day

$$\boxed{\left|\zeta_{\mu e}\right|^2 + \left(1 - \zeta_{\mu\mu}\right)^2 = \frac{1}{1 \pm \sqrt{\chi^2_{\mathrm{TAO}}/(N_{\mathrm{TAO}}t)}} \simeq 1 \pm 0.0126\sqrt{\chi^2_{\mathrm{TAO}}/t}}$$

Blennow et al., 2306.01040; 2502.19480

$\overline{\nu}$-e Scattering Rates @ JUNO

$$\frac{\mathrm{d}N^{\beta}_{\rm JUNO}}{\mathrm{d}E_{\rm obs}} = \frac{N_e t}{4\pi L^2_{\rm JUNO}} \sum_i \int_0^\infty \mathrm{d}T_e\, \mathscr{G}\left(E_{\rm obs}; T_e, \delta_{\rm E}^{\rm JUNO}\right) \int_{E_{\rm min}}^\infty \mathrm{d}E\, \frac{\mathrm{d}\Phi^i_{\overline{\nu}_e}}{\mathrm{d}E} \overline{P}_{e\beta}\left(E, L_{\rm JUNO}\right) \frac{\mathrm{d}\sigma_\beta}{\mathrm{d}T_e}$$

✓ *60 IBD events and 10 $\overline{\nu}_e$-e scattering events per day*

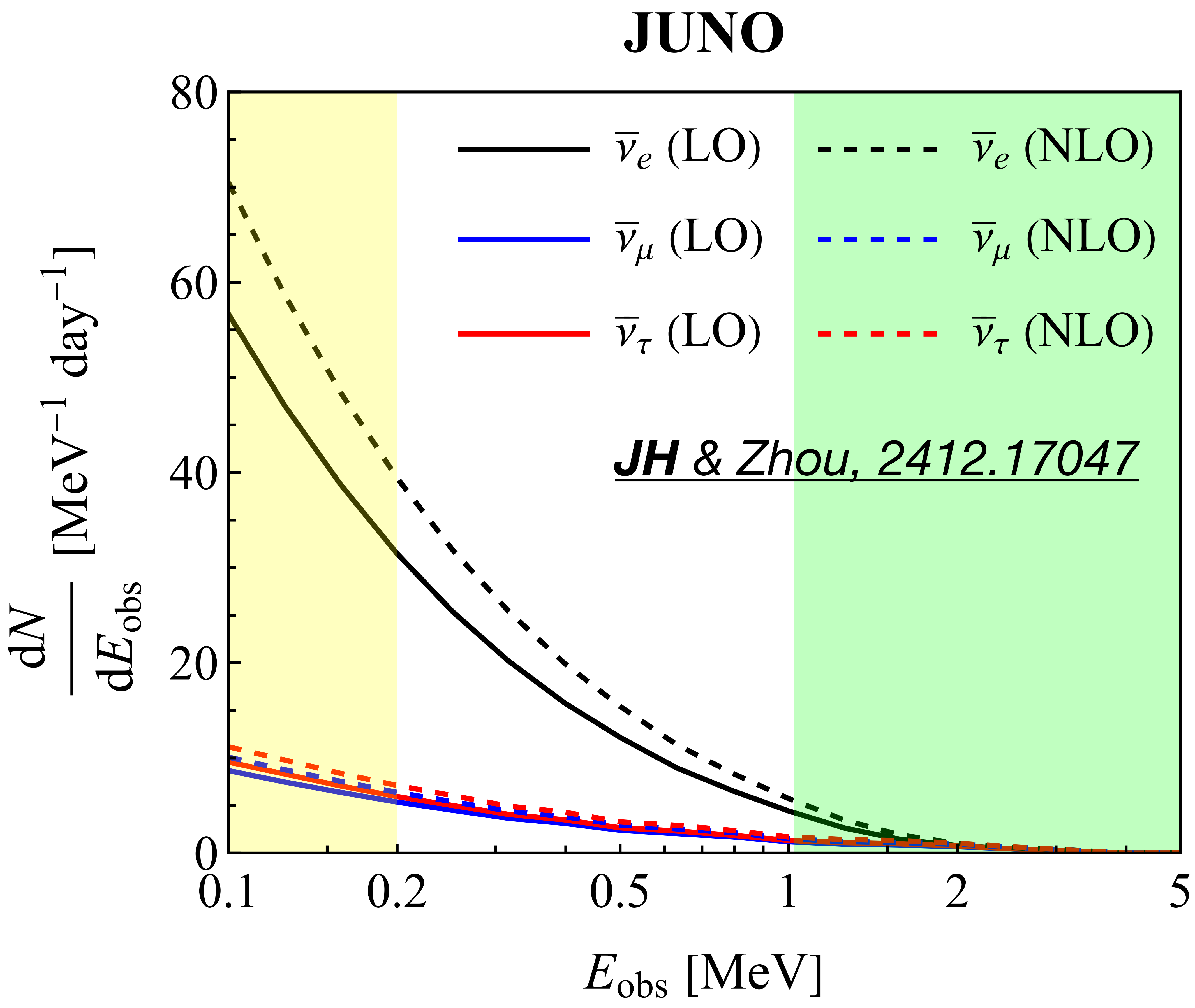
✓ *2 $\overline{\nu}_{\mu,\tau}$-$e$ scattering events per day*

❖ ***Retain all diagonal elements*** $\{\zeta_{ee}, \zeta_{\mu\mu}, \zeta_{\tau\tau}\}$

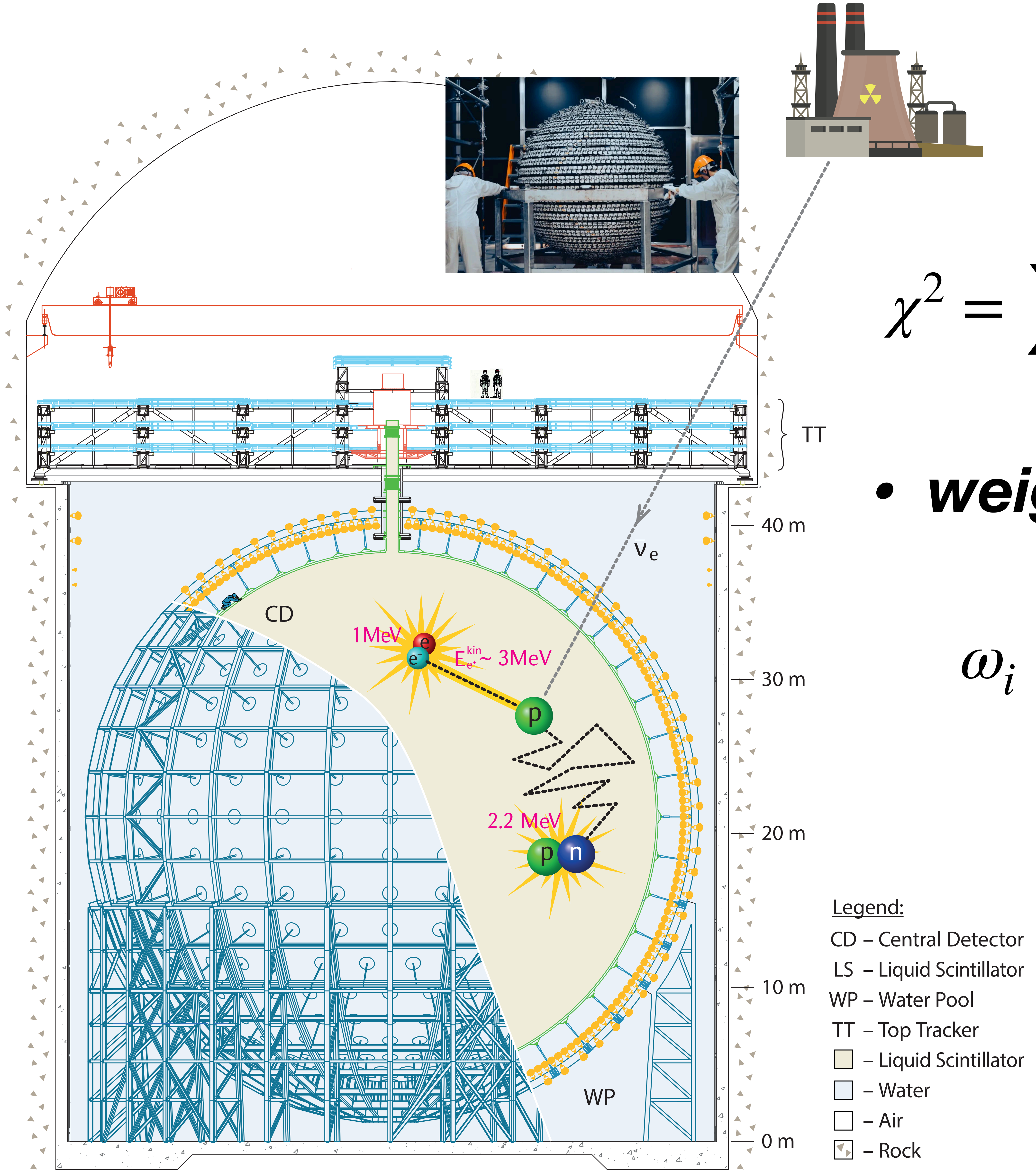
$$\frac{\mathrm{d}N^{\mu}_{\rm JUNO}}{\mathrm{d}E_{\rm obs}} \simeq \left[1 + 2\left(\zeta_{ee} - \zeta_{\mu\mu}\right)\right] \frac{\mathrm{d}N^{\mu,\rm SM}_{\rm JUNO}}{\mathrm{d}E_{\rm obs}}$$

$$\frac{\mathrm{d}N^{\tau}_{\rm JUNO}}{\mathrm{d}E_{\rm obs}} \simeq \left[1 + 2\left(\zeta_{ee} + \zeta_{\mu\mu} - 2\zeta_{\tau\tau}\right)\right] \frac{\mathrm{d}N^{\tau,\rm SM}_{\rm JUNO}}{\mathrm{d}E_{\rm obs}}$$

$$\boxed{\left|\zeta_{ee} - \zeta_{\tau\tau}\right| \lesssim 2.3 \times 10^{-2}\ (3\sigma,\ 3\text{yrs})}$$



Precise Measurements & Constraints



With both ***near*** and ***far*** detector:

$$\chi^2 = \sum_i \frac{\left(N_i^{\rm FD} - \omega_i N_i^{\rm ND}\right)^2}{N_i^{\rm FD}}$$

- ***weight factor :***

$$\omega_i \equiv \frac{\overline{P}^{i,{\rm FD}}_{ee}(E_i, L_{\rm FD})}{\overline{P}^{i,{\rm ND}}_{ee}(E_i, L_{\rm ND})}$$

✦ ***Independent*** *of the non-unitarity effects,*

✓ ***Precise measurement*** *of the mixing angles*

✤ *Analyze event rates at the far (near) detector* ***individually*** *to constrain non-unitarity effects*

Summary

- We investigate the non-unitarity effects of lepton flavor mixing matrix in JUNO and TAO. We separate the **production**, the **oscillation** and the **detection** into three independent parts (assuming neutrino masses are much smaller than the characteristic energy scales).
  - ✓ Derive the oscillation probabilities in the **lower-triangular** parametrization.
  - ✓ There is **NO** need to modify the reactor antineutrino flux from the theoretical aspect.
  - ✓ We calculate the cross sections for the IBD and the ν-e scattering with **NC** interactions.
- TAO and JUNO both have capabilities of **individually** limiting the non-unitarity parameters!
- One can combine the unitarity tests in the **neutrino oscillation** experiments, **precision EW and flavor observables** together, to derive much stronger constraints. Theoretical calculations of observables beyond the LO are necessary (in the **complete neutrino mass model**).

Thanks for your attention!

Backup Slides

$\overline{\nu}$-e Scattering Cross Sections

$$\frac{\mathrm{d}\sigma_\beta}{\mathrm{d}T_e} = \frac{\left(G_\mu^{\mathrm{exp}}\right)^2 m_e}{2\pi \left(NN^\dagger\right)_{ee} \left(NN^\dagger\right)_{\mu\mu}} \left\{ \frac{m_e T_e}{E^2} \left[\left(\tilde{c}^\beta_{\mathrm{A}} + \tilde{c}^\beta_{\mathrm{C}}\right)^2 - \left(\tilde{c}^\beta_{\mathrm{V}} + \tilde{c}^\beta_{\mathrm{C}}\right)^2 \right] + \left(1 - \frac{T_e}{E}\right)^2 \left(\tilde{c}^\beta_{\mathrm{A}} + \tilde{c}^\beta_{\mathrm{V}} + 2\tilde{c}^\beta_{\mathrm{C}}\right)^2 + \left(\tilde{c}^\beta_{\mathrm{A}} - \tilde{c}^\beta_{\mathrm{V}}\right)^2 \right\}$$

$$\tilde{c}^\beta_{\mathrm{V,A}} \equiv \frac{\left(NN^\dagger NN^\dagger\right)_{\beta\beta}}{\left(NN^\dagger\right)_{\beta\beta}} \times c_{\mathrm{V,A}} \,, \quad \tilde{c}^\beta_{\mathrm{C}} \equiv \frac{\left|\left(NN^\dagger\right)_{\beta e}\right|^2}{\left(NN^\dagger\right)_{\beta\beta}} \xrightarrow{\textit{unitary case}} \tilde{c}^\beta_{\mathrm{V,A}} = c_{\mathrm{V,A}} \,, \quad \tilde{c}^\beta_{\mathrm{C}} = \delta_{\beta e}$$

$$\frac{\mathrm{d}\sigma_\beta}{\mathrm{d}T_e} \approx \frac{\mathrm{d}\sigma_\beta^{\mathrm{SM}}}{\mathrm{d}T_e} \times \frac{\left[\left(NN^\dagger\right)_{\beta\beta}\right]^2}{\left(NN^\dagger\right)_{ee} \left(NN^\dagger\right)_{\mu\mu}} \,.$$

$$NN^\dagger = \begin{pmatrix} \left(1-\zeta_{ee}\right)^2 & -\left(1-\zeta_{ee}\right)\zeta^*_{\mu e} & -\left(1-\zeta_{ee}\right)\zeta^*_{\tau e} \\ -\left(1-\zeta_{ee}\right)\zeta_{\mu e} & \left(1-\zeta_{\mu\mu}\right)^2 + \left|\zeta_{\mu e}\right|^2 & -\left(1-\zeta_{\mu\mu}\right)\zeta^*_{\tau\mu} + \zeta_{\mu e}\zeta^*_{\tau e} \\ -\left(1-\zeta_{ee}\right)\zeta_{\tau e} & -\left(1-\zeta_{\mu\mu}\right)\zeta_{\tau\mu} + \zeta^*_{\mu e}\zeta_{\tau e} & \left(1-\zeta_{\tau\tau}\right)^2 + \left|\zeta_{\tau e}\right|^2 + \left|\zeta_{\tau\mu}\right|^2 \end{pmatrix}$$

$$\frac{\mathrm{d}\sigma_\beta}{\mathrm{d}T_e} \approx \frac{\mathrm{d}\sigma_\beta^{\mathrm{SM}}}{\mathrm{d}T_e} \times \begin{cases} 1 - 2\left(\zeta_{ee} - \zeta_{\mu\mu}\right) & \beta = e \\ 1 + 2\left(\zeta_{ee} - \zeta_{\mu\mu}\right) & \beta = \mu \\ 1 + 2\left(\zeta_{ee} + \zeta_{\mu\mu} - 2\zeta_{\tau\tau}\right) & \beta = \tau \end{cases} ,$$

Only depends on ***diagonal*** *elements !*

Different Strategies on NC

- *The NC interaction could not **distinguish** among three flavors of neutrinos.*

NC cross section defined in the mass eigenstate $\sigma_i^{\mathrm{NC}} = \sigma_{\mathrm{NC}}^{\mathrm{SM}} \sum_j \left| \left(N^\dagger N\right)_{ij} \right|^2$

✓ *Oscillation probabilities*

✳<u>*Antusch et al., hep-ph/0607020*</u> $\hat{P}_{\nu_\alpha \nu_i}(E, L) \equiv \left| N_{\alpha i} \right|^2 \longrightarrow \hat{P}_{\nu_\alpha \mathrm{NC}}(E, L) \equiv \sum_i \hat{P}_{\nu_\alpha \nu_i}(E, L)$

Neutrinos propagate as mass eigenstates, are already <u>*decoherent*</u> when they reach the detector <u>*without oscillations*</u>, and individually participate in the NC process.

✳<u>*Dutta & Roy, 1901.11298; Blennow et al., 2502.19480*</u> $\hat{P}_{\nu_\alpha \to \nu_{\mathrm{NC}}}(E, L) = \sum_j \left| \sum_i N_{\alpha i}^* \mathrm{e}^{-\mathrm{i} E_i L} \left(N^\dagger N\right)_{ij} \right|^2$

Neutrinos propagate as mass eigenstates and undergo NC interactions in a <u>*coherent*</u> manner, and the total probability is obtained through the <u>*summation over*</u> the final mass eigenstates (at the cross section level).