Neutrino oscillations in DM

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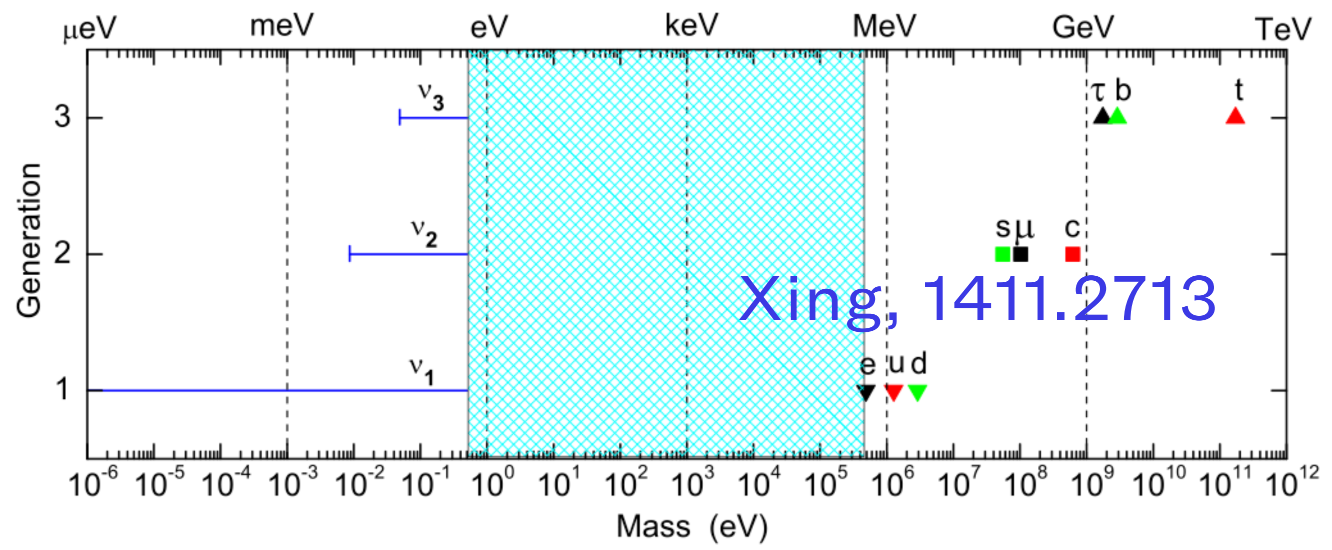
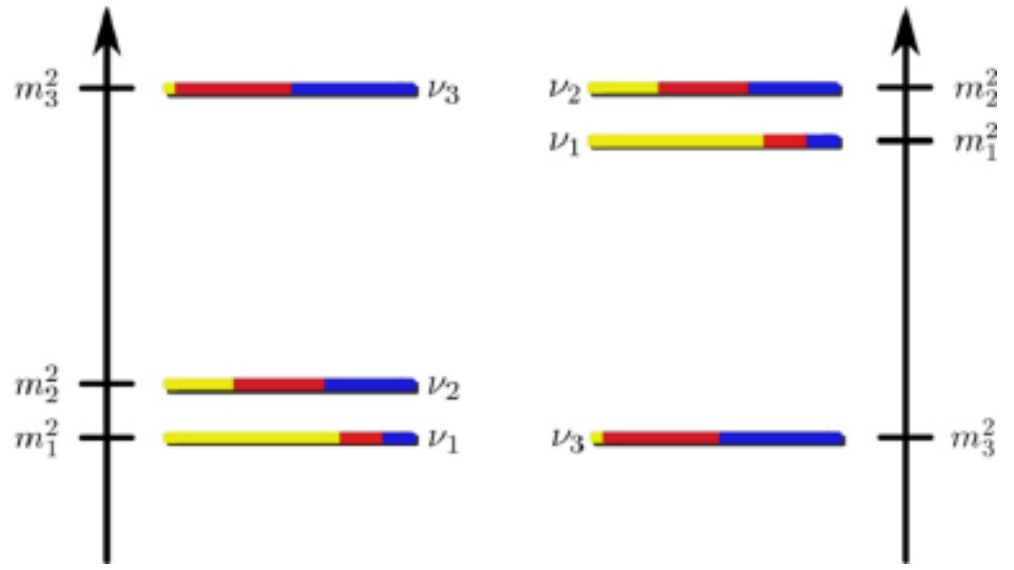
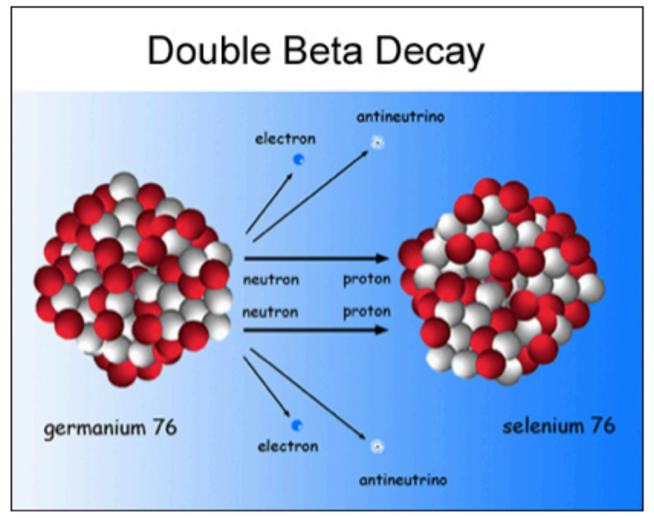
Outline

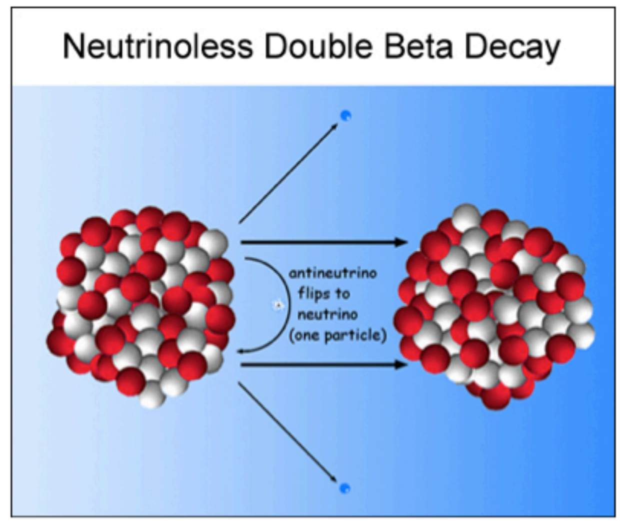
- Neutrino oscillation in Fermionic DM background.
- Neutrino oscillation in ultralight bosonic DM (QM approach).
- Quantum field theory approach to neutrino oscillations in scalar DM.
- Summary.

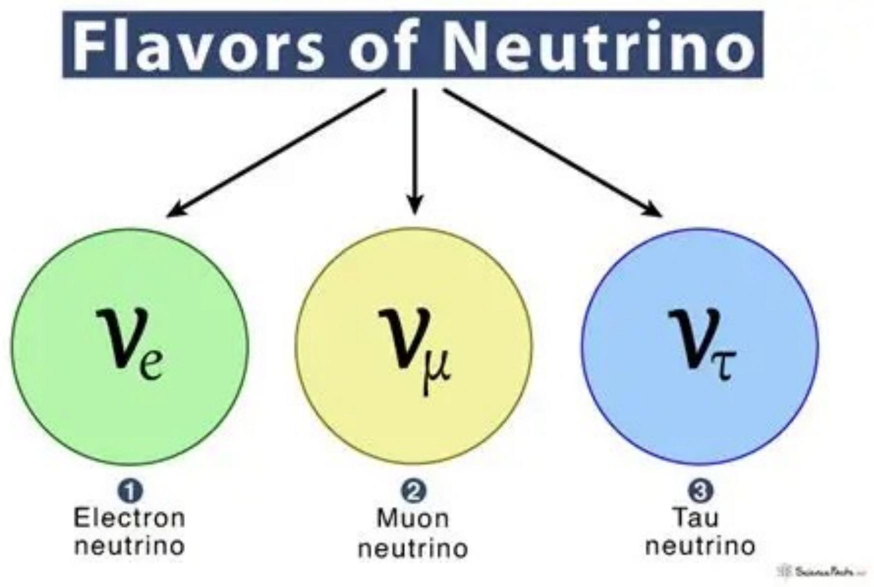
Topics relevant to neutrino physics

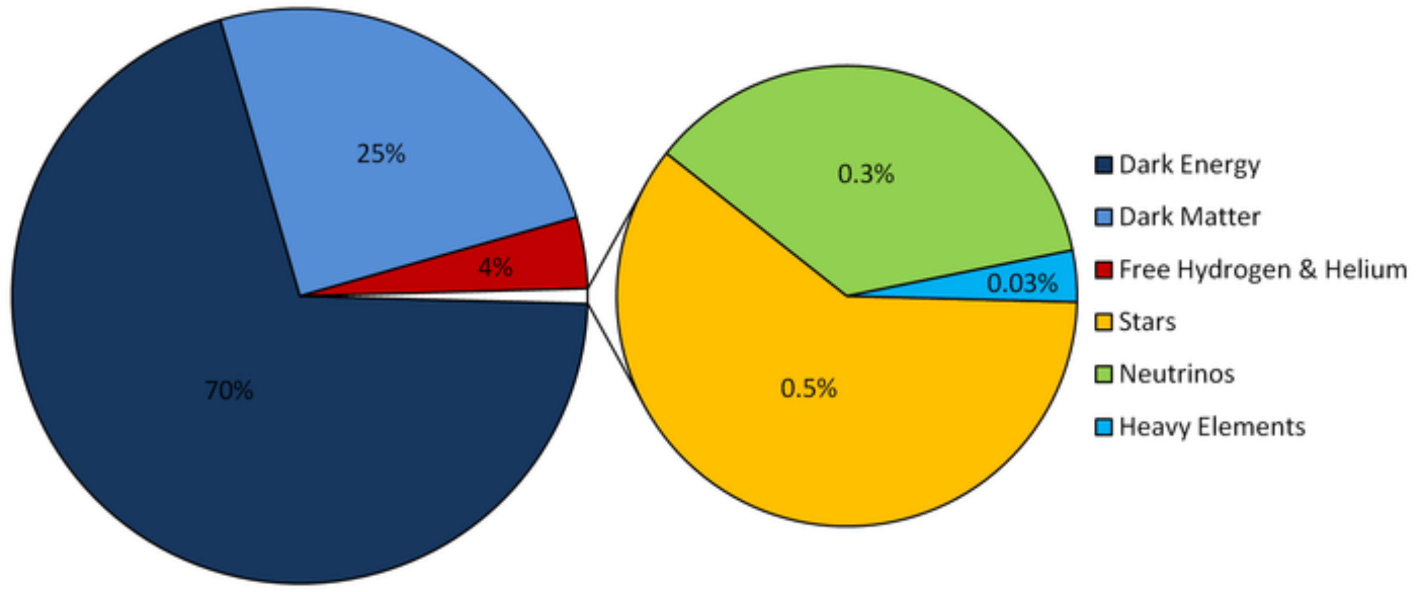


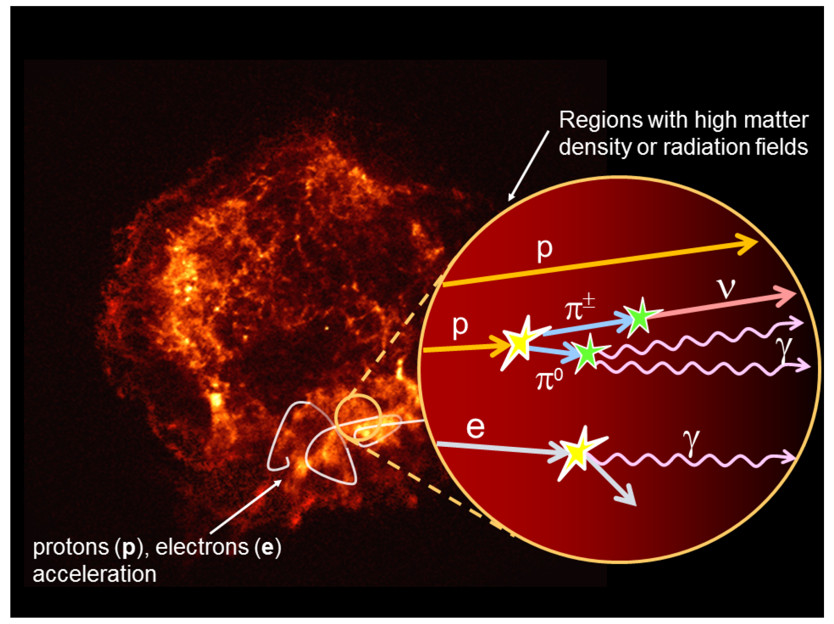




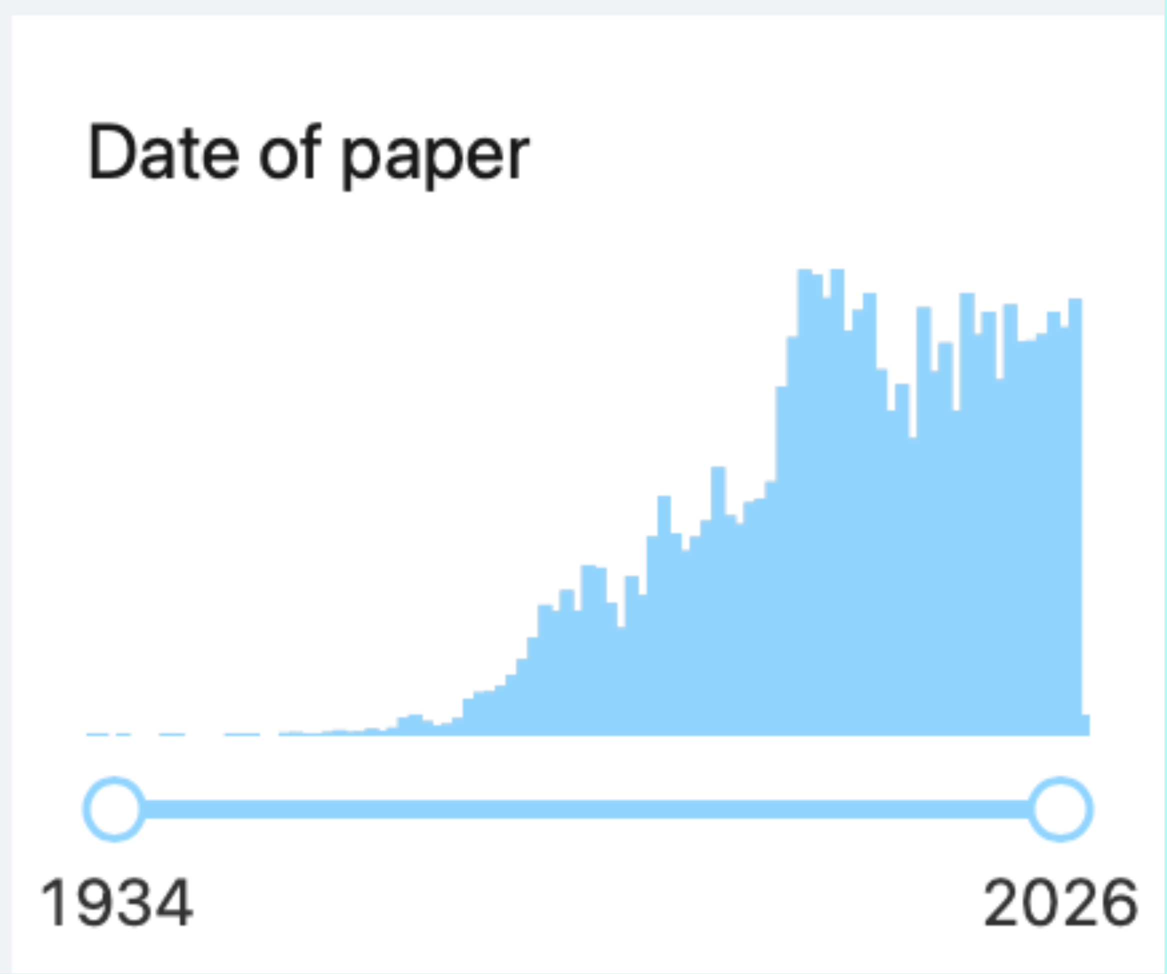








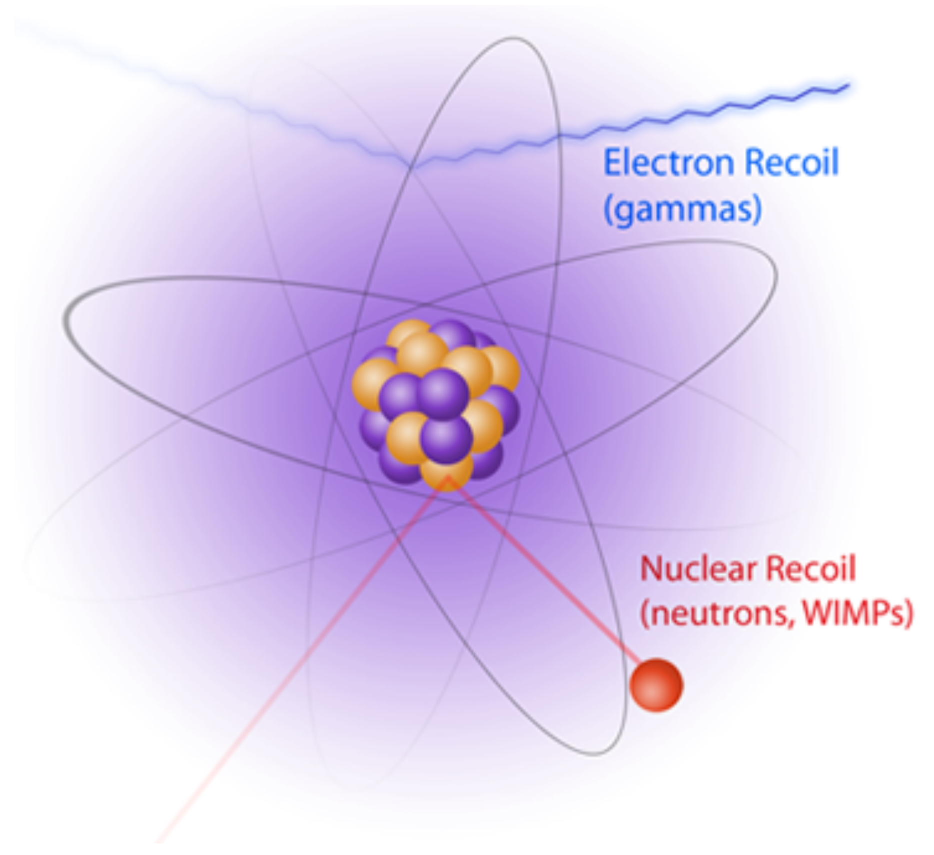
- ✦Origin of neutrino masses
- ✦Majorana VS Dirac
- ✦Absolute mass of neutrinos
- ✦Mixings and CP violation
- ✦Neutrino cosmology
- ✦Neutrino astrophysics
- ✦。。。

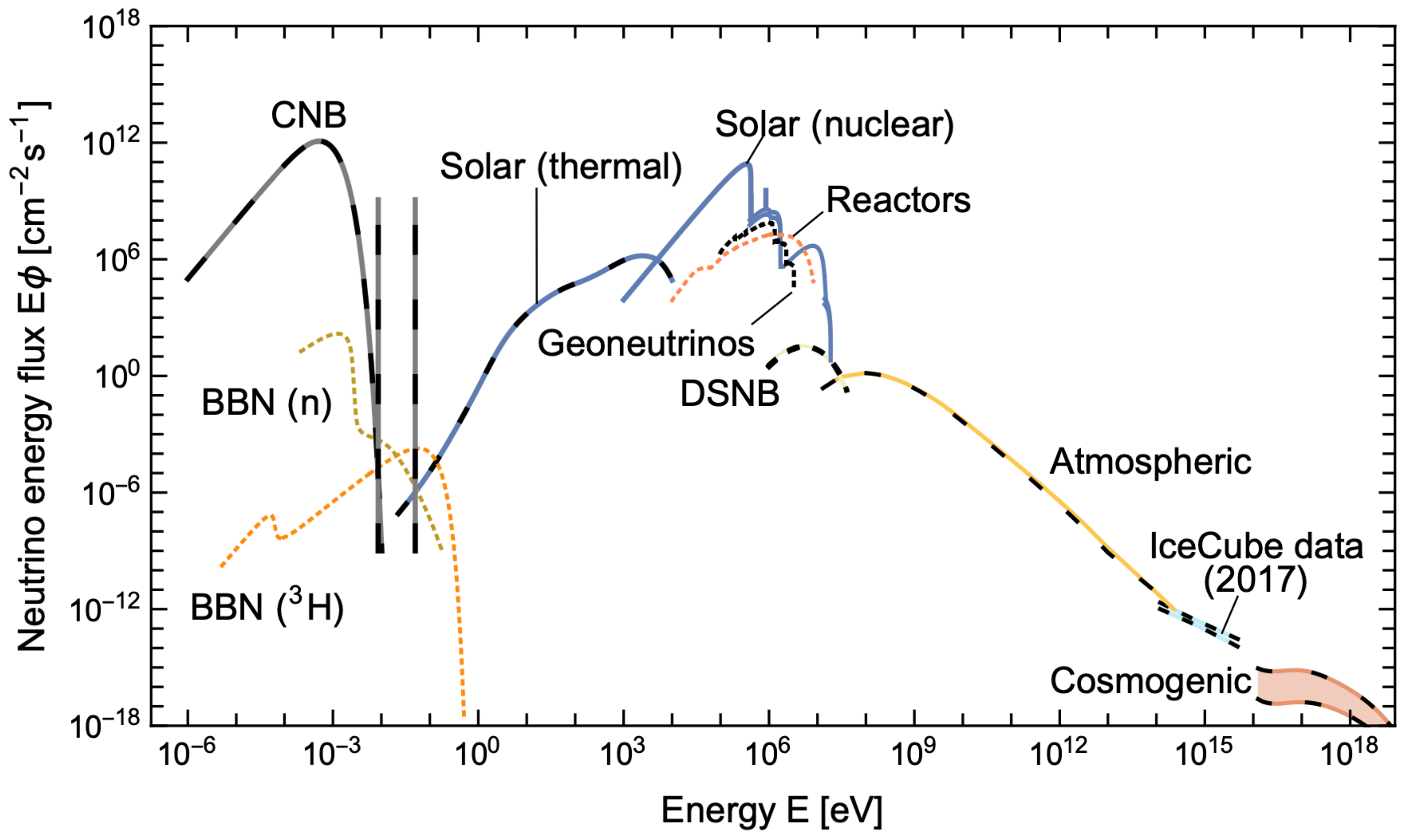


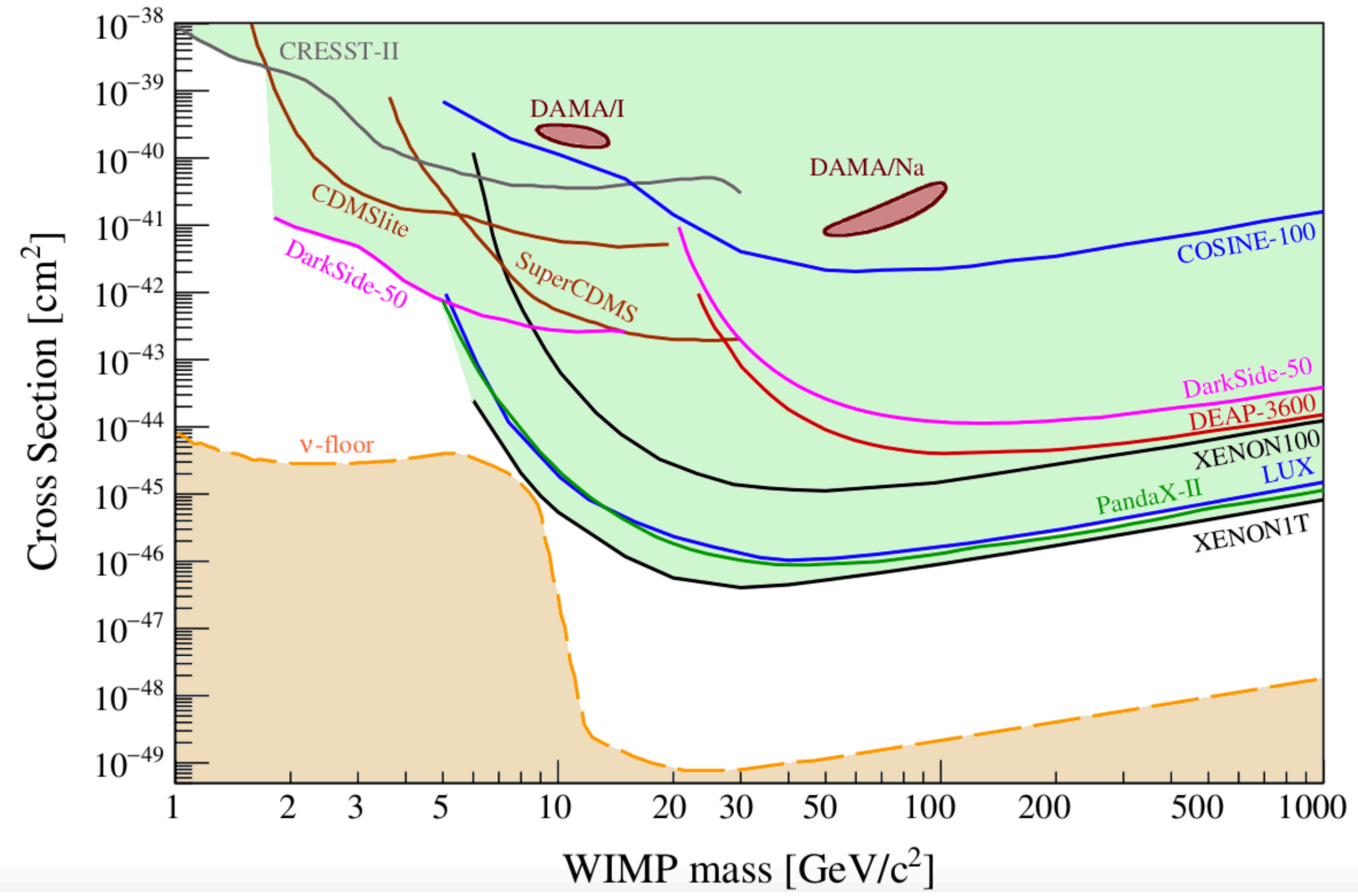
What we concern: ν-DM connection

$$\nu \leftrightarrow \text{DM}$$

- **Properties of neutrinos are similar to these of dark matter**
- **Neutrino is a hot dark matter candidate**
- **Sterile neutrino is typical warm/cold dark matter candidate**
- **The signal of neutrino in direct detection experiments is similar to that of DM**







Neutrino-DM connection

The WIMP event rate

$$\frac{dR}{dE_R} = MT \times \frac{\rho_{DM}\sigma_n^0 A^2}{2m_{DM}\mu_n^2} F^2(E_R) \int_{v_{min}} \frac{f(\vec{v})}{v} d^3v$$

Exposure — DM density — Nuclear Form Factor

Coherent neutrino-nucleus scattering in the SM

$$\frac{d\sigma_\nu}{dE_R} = \frac{G_F^2}{4\pi} Q_{\nu N}^2 m_N \left(1 - \frac{m_N E_R}{2E_\nu^2}\right) F^2(E_R)$$

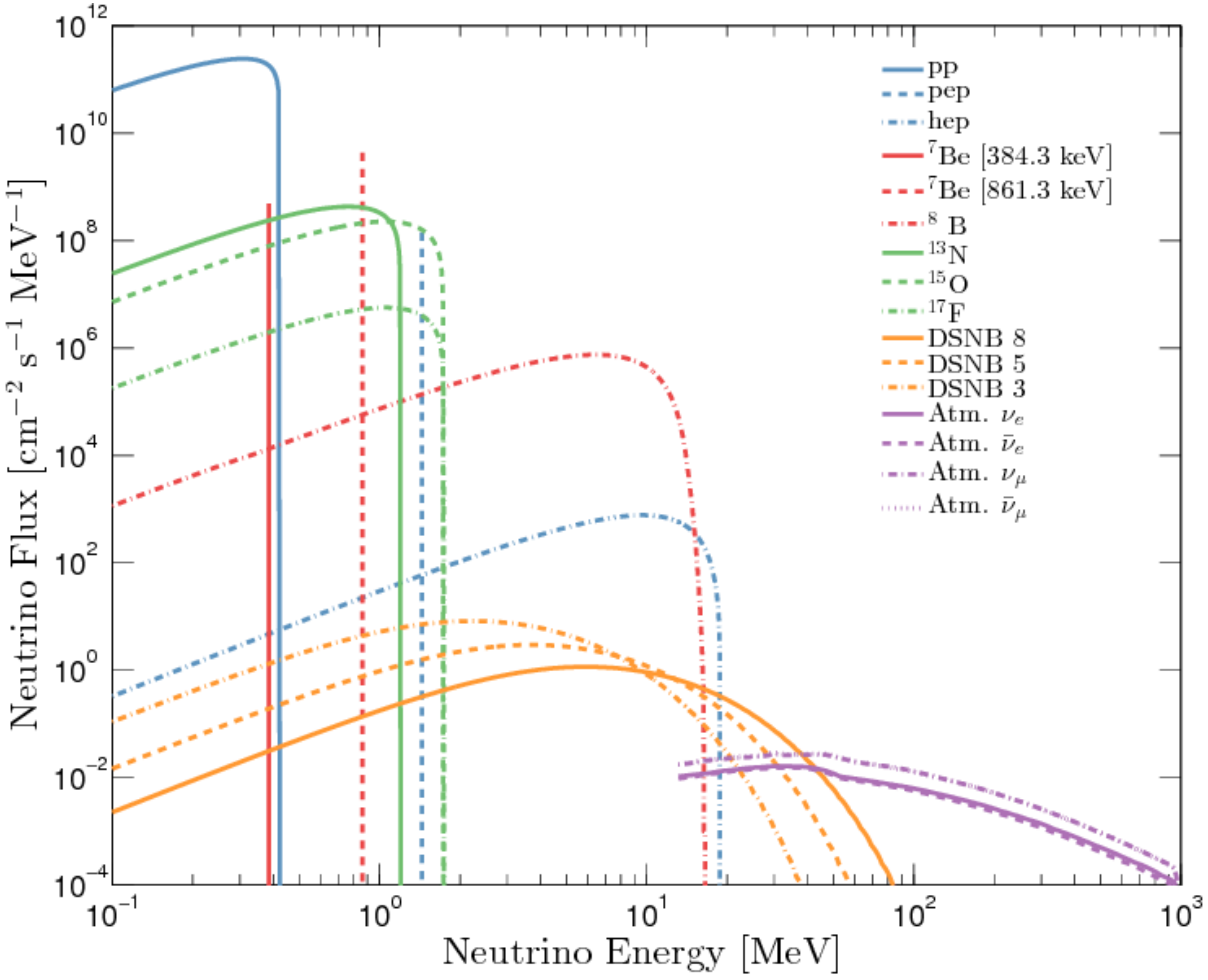
Weak hyper-charge of target nucleus — Nuclear form factor

Number of expected events

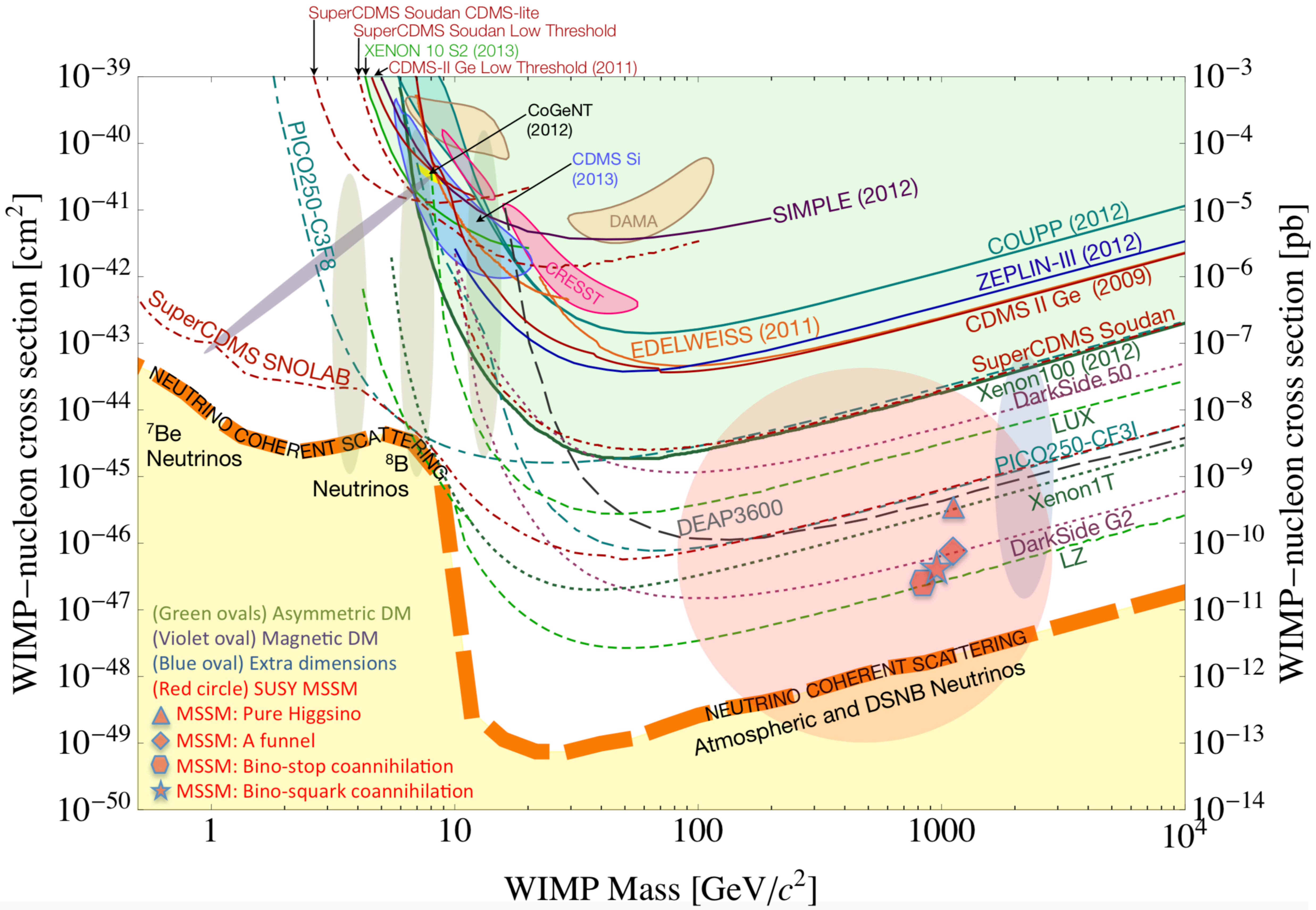
$$N = \frac{\varepsilon}{m_N} \int_{E_T}^{E_{max}} dE_R \int dE_\nu \frac{d\phi_\nu}{dE_\nu} \frac{d\sigma_\nu}{dE_R}$$

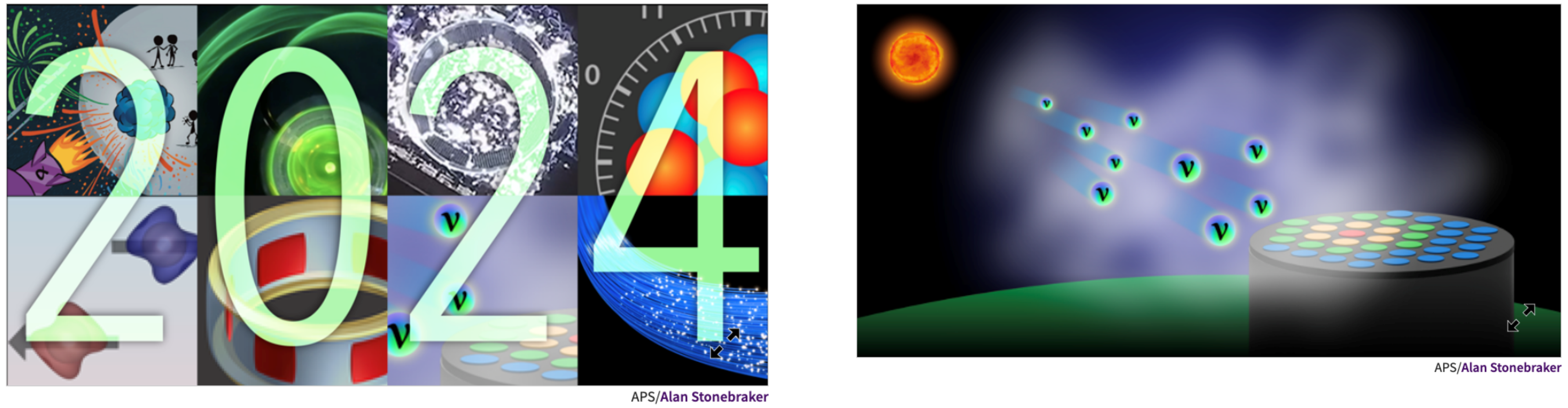
Neutrino floor

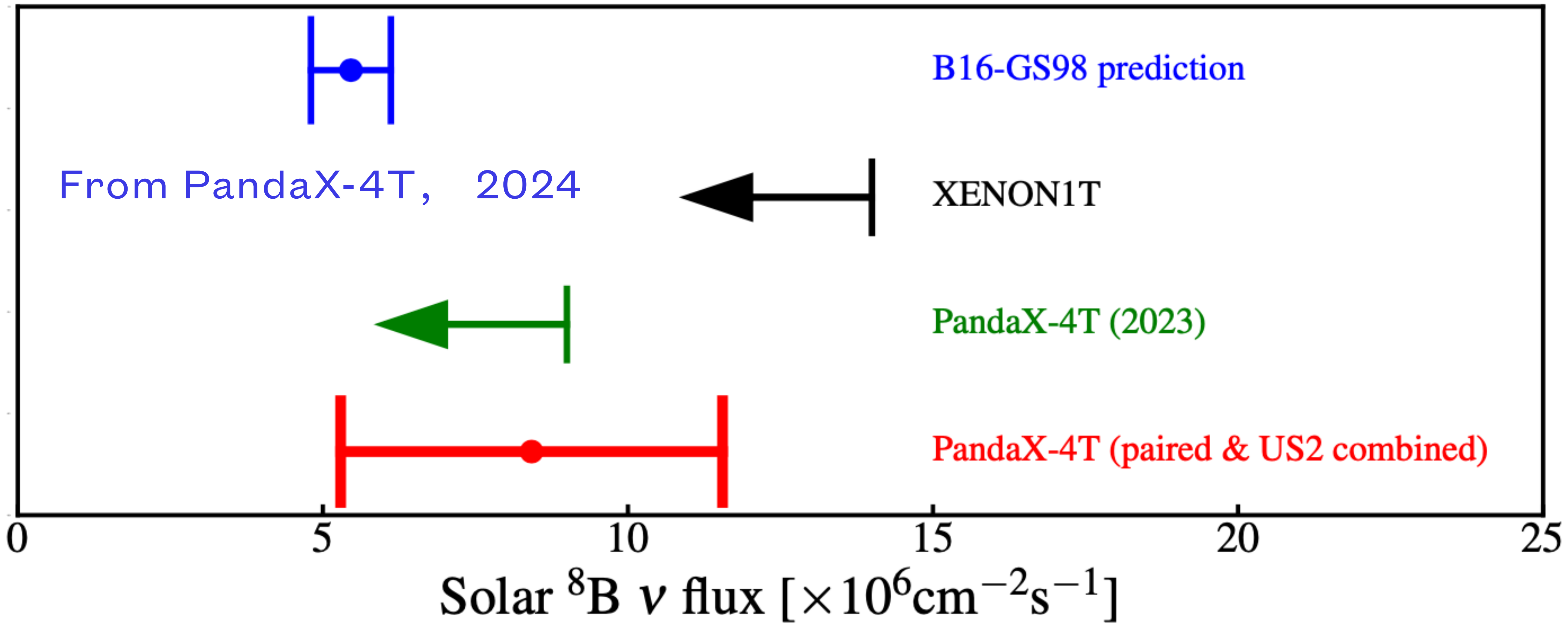
$$\sigma_n^0 = \frac{2.3}{m} \int_{E_R} \left(\frac{1}{m_N} \int_{E_\nu^{min}} \frac{d\phi_\nu}{dE_\nu} \frac{d\sigma_\nu}{dE_R} \right) \left(\frac{\rho_{DM} A^2}{2m_{DM}\mu_n^2} \int_{E_R}^{E_R^{max}} F^2(E_R) dE_R \int_{v_{min}} \frac{f(\vec{v})}{v} d^3v \right)^{-1}$$



Neutrino-DM connection







Neutrino-DM connection

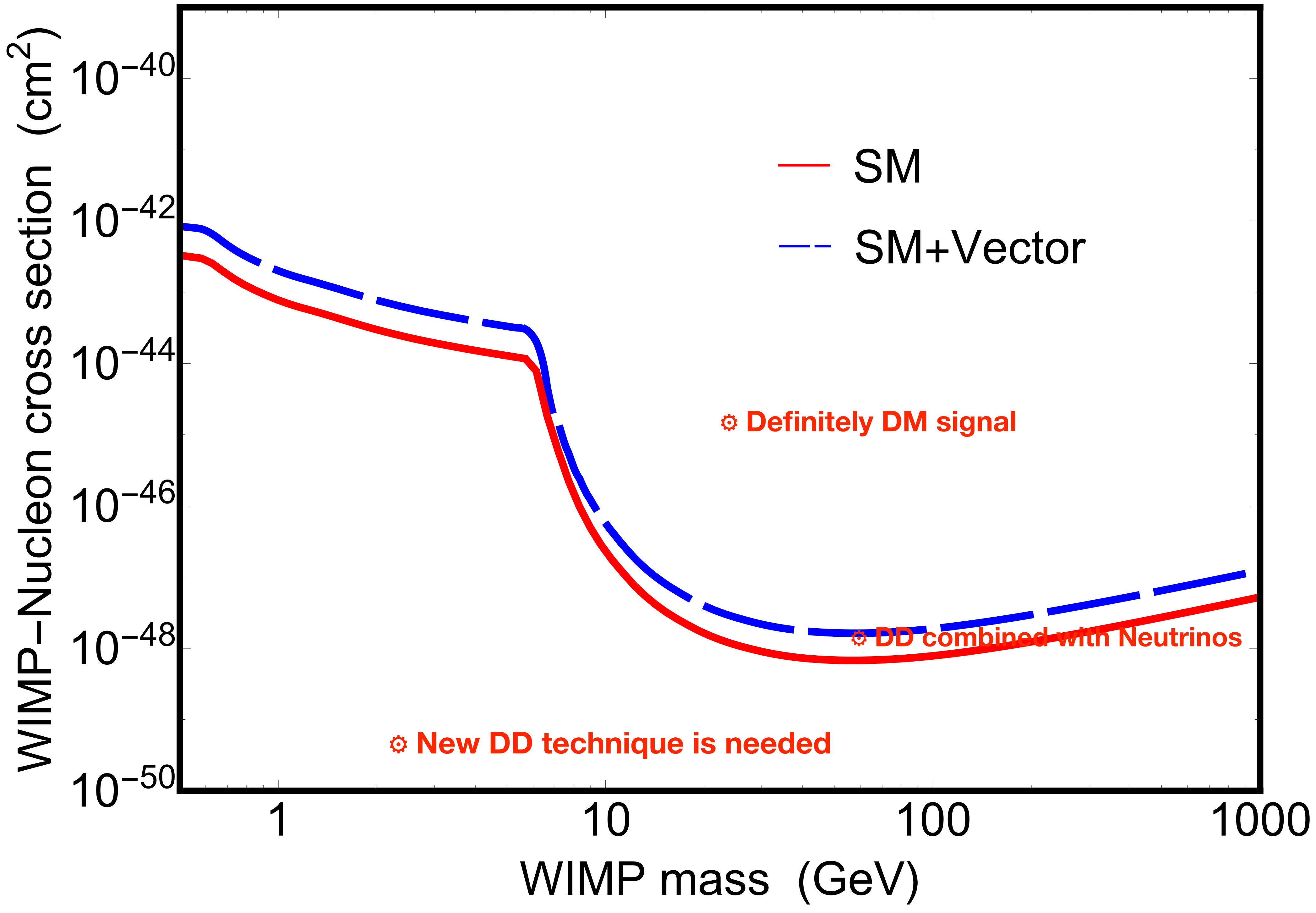
Impacts of non-standard neutrino interactions to the neutrino floor was studied

$$\frac{d\sigma_\nu}{dE_R} = \frac{2G_F^2 m_A}{(2J_A+1)E_\nu^2}\left\{\sum_{\alpha\beta=0,1}(4E_\nu^2 - 2m_A E_R)\zeta_V^\alpha \zeta_V^{\beta*} W_M^{\alpha\beta}(q^2) + \sum_{\alpha,\beta=0,1}\left(E_\nu^2 + \frac{1}{2}m_A E_R\right)\zeta_A^\alpha\zeta_A^{\beta*}W_{\Sigma'}^{\alpha\beta}(q^2) + \sum_{\alpha\beta=0,1}\frac{E_R}{4m_A}(2E_\nu^2 - m_A E_R)\zeta_A^\alpha\zeta_A^{\beta*}W_{\Sigma''}^{\alpha\beta}(q^2) + 8(2E_\nu^2 - m_A E_R)\zeta_T^2 W_{\Sigma'}^{00}(q^2) + 16E_\nu^2\zeta_T^2 W_{\Sigma''}^{00}(q^2) + 2m_A E_R \zeta_S^2 W_M^{00}(q^2) + \sum_{\alpha\beta=0,1}\frac{E_R^2 m_A^2}{m_N^2}\zeta_P^\alpha\zeta_P^{\beta*}W_{\Sigma''}^{\alpha\beta}(q^2)\right\} \quad (4$$

$$\zeta_\alpha^0 = \frac{1}{2}\left(\zeta_\alpha^p + \zeta_\alpha^n\right) \qquad \zeta_\alpha^1 = \frac{1}{2}\left(\zeta_\alpha^p - \zeta_\alpha^n\right)$$

Quark level	Nucleon level	Matching conditions
$\frac{G_F}{\sqrt{2}}\zeta_{q,S}\bar{\nu}_\alpha P_L \nu_\beta \bar{q}q$	$\frac{G_F}{\sqrt{2}}\zeta_{N,S}\bar{\nu}_\alpha P_L \nu_\beta \bar{N}N$	$\zeta_{N,S} = \sum_{q=u,d}\zeta_{q,S}\frac{m_N}{m_q}f_{T_q}^N$
$\frac{G_F}{\sqrt{2}}\zeta_{q,P}\bar{\nu}_\alpha P_L \nu_\beta \bar{q}i\gamma^5 q$	$\frac{G_F}{\sqrt{2}}\zeta_{N,P}\bar{\nu}_\alpha P_L \nu_\beta \bar{N}i\gamma^5 N$	$\zeta_{N,P} = \sum_{q=u,d}\zeta_{q,P}\frac{m_N}{m_q}\left(1 - \frac{\bar{m}}{m_q}\right)\Delta_q^N$
$\frac{G_F}{\sqrt{2}}\zeta_{q,V}\bar{\nu}_\alpha \gamma_\mu P_L \nu_\beta \bar{q}\gamma^\mu q$	$\frac{G_F}{\sqrt{2}}\zeta_{N,V}\bar{\nu}_\alpha \gamma_\mu P_L \nu_\beta \bar{N}\gamma^\mu N$	$\zeta_{p,V} = 2\zeta_{u,V} + \zeta_{d,V}; \quad \zeta_{n,V} = \zeta_{u,V} + 2\zeta_{d,V}$
$\frac{G_F}{\sqrt{2}}\zeta_{q,A}\bar{\nu}_\alpha \gamma_\mu P_L \nu_\beta \bar{q}\gamma^\mu\gamma^5 q$	$\frac{G_F}{\sqrt{2}}\zeta_{N,A}\bar{\nu}_\alpha \gamma_\mu P_L \nu_\beta \bar{N}\gamma^\mu\gamma^5 N$	$\zeta_{N,A} = \sum_q \zeta_{q,A}\Delta_q^N$
$\frac{G_F}{\sqrt{2}}\zeta_{q,T}\bar{\nu}_\alpha \sigma_{\mu\nu} P_L \nu_\beta \bar{q}\sigma^{\mu\nu} q$	$\frac{G_F}{\sqrt{2}}\zeta_{N,T}\bar{\nu}_\alpha \sigma_{\mu\nu} P_L \nu_\beta \bar{N}\sigma^{\mu\nu} N$	$\zeta_{N,T} = \sum_q \zeta_{q,T}\delta_q^N$

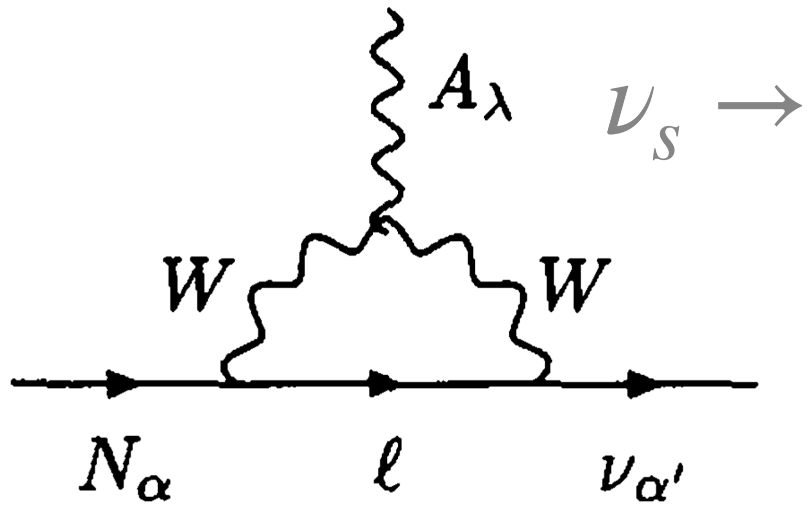
NSI	Enhancement
Vector	✓
Axial-vector	✗
Tensor	✗
Scalar	✓
P-Scalar	✓



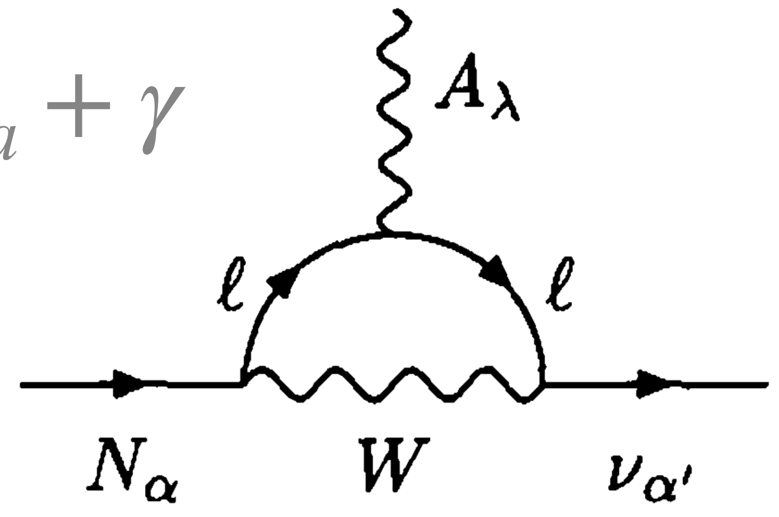
Wei Chao, J. Zhang, X. Wang and X. Zhang, JCAP, 19

Neutrino-DM connection

Neutrino portal DM： Sterile neutrino + ⋯

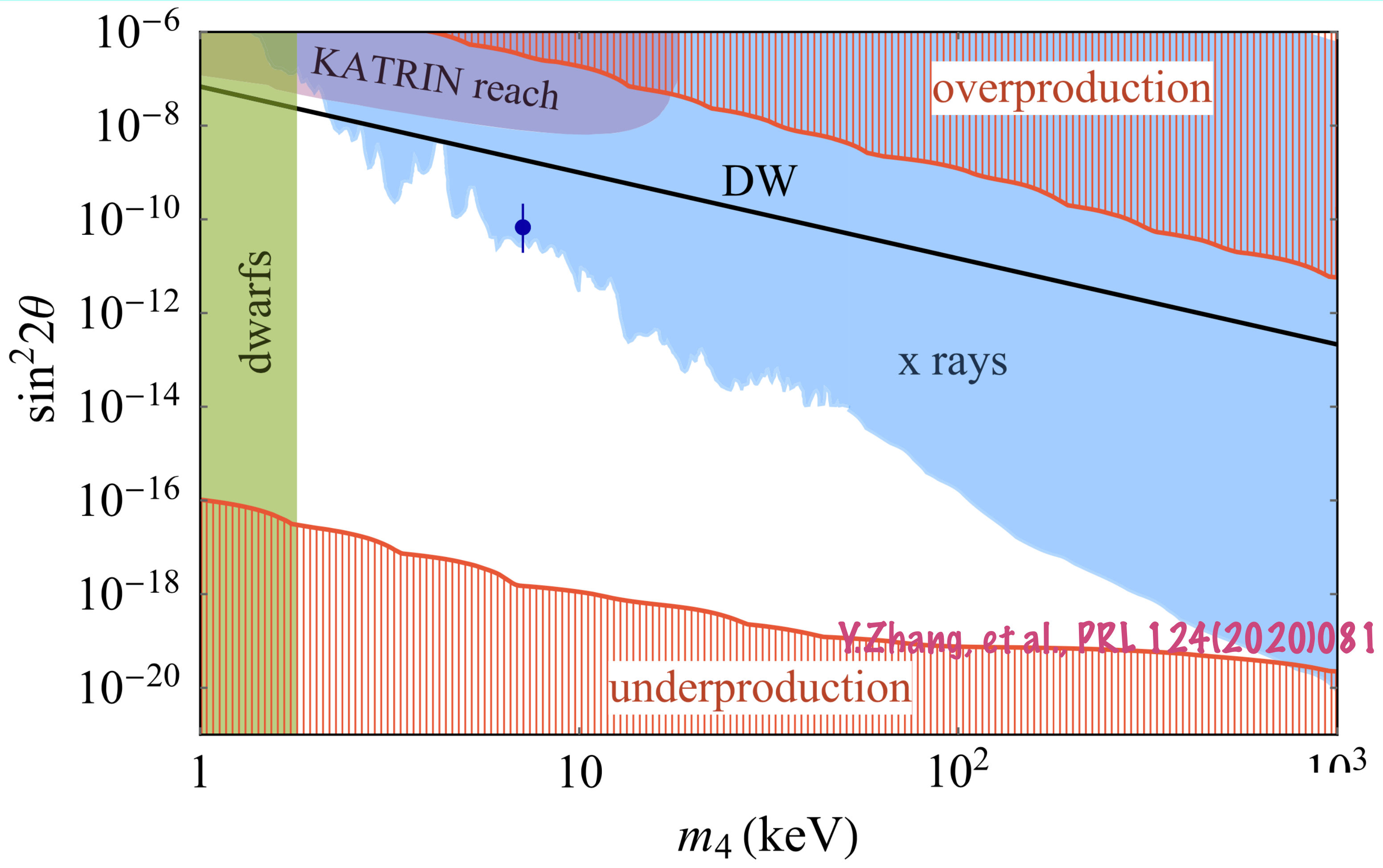


$\nu_s \to \nu_a + \gamma$

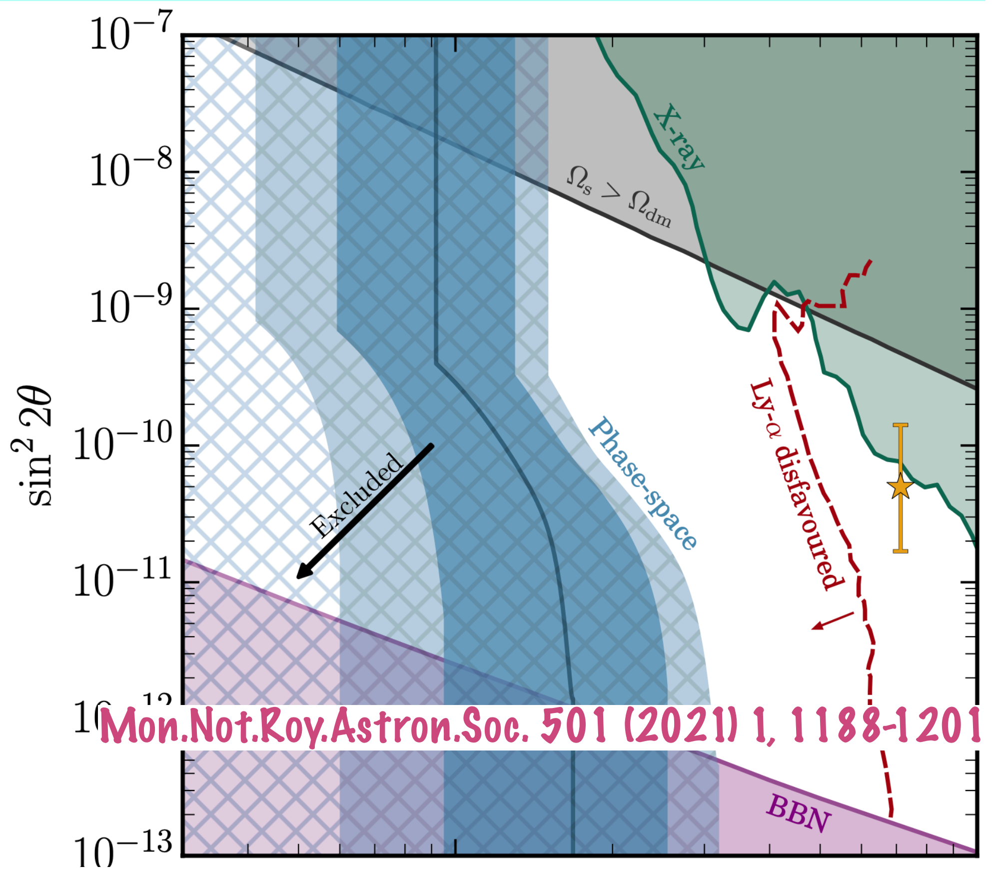


$$\Gamma_\gamma = 1.62 \times 10^{-28} s^{-1} \left(\frac{\sin^2 2\theta}{7 \times 10^{-11}} \right) \left(\frac{m_s}{7\ keV} \right)^5$$

$$E_\gamma \approx \frac{m_s}{2}$$



Y.Zhang, et al. PRL 124(2020)081802



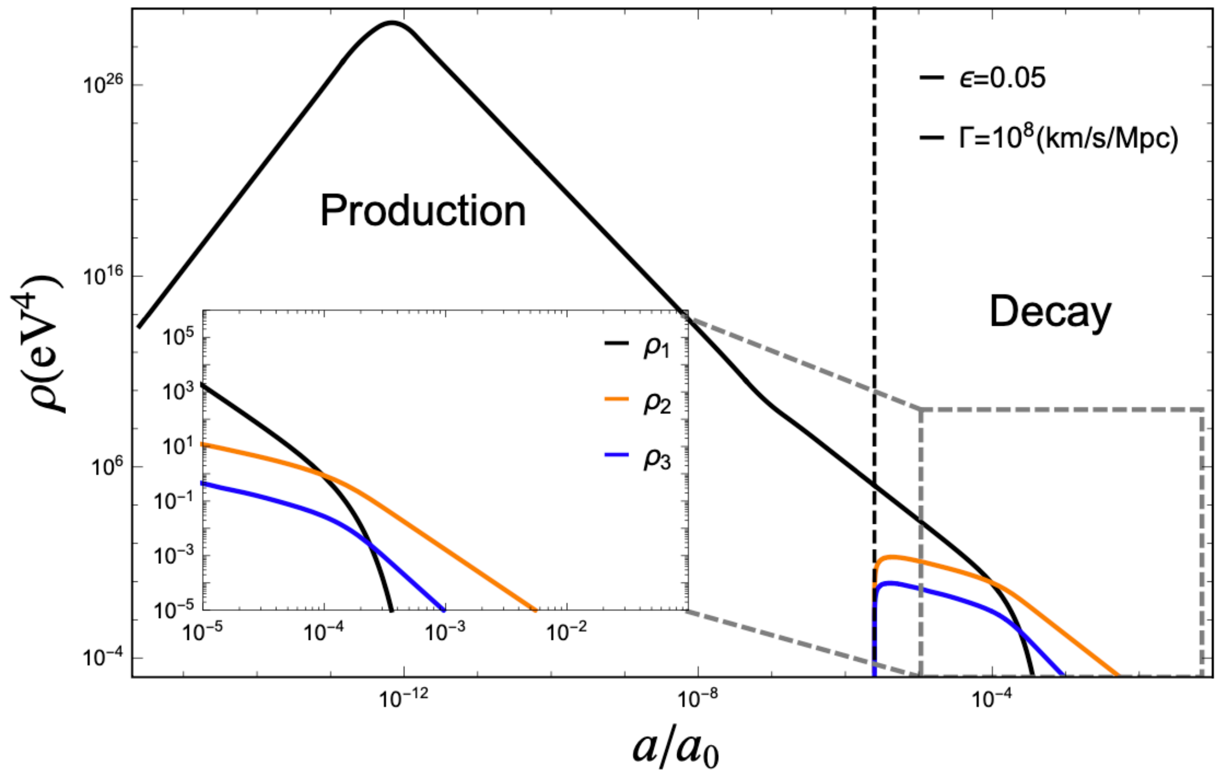
Mon.Not.Roy.Astron.Soc. 501 (2021) 1, 1188-1201

Good: The life-time of a keV scale sterile neutrino is longer than the age of the universe.

Not too good: Too much sterile neutrino dark matter in galaxies, so it can be constrained by results of standard X-ray astronomy.

Strategy: introducing a pseudo-Dirac sterile neutrino

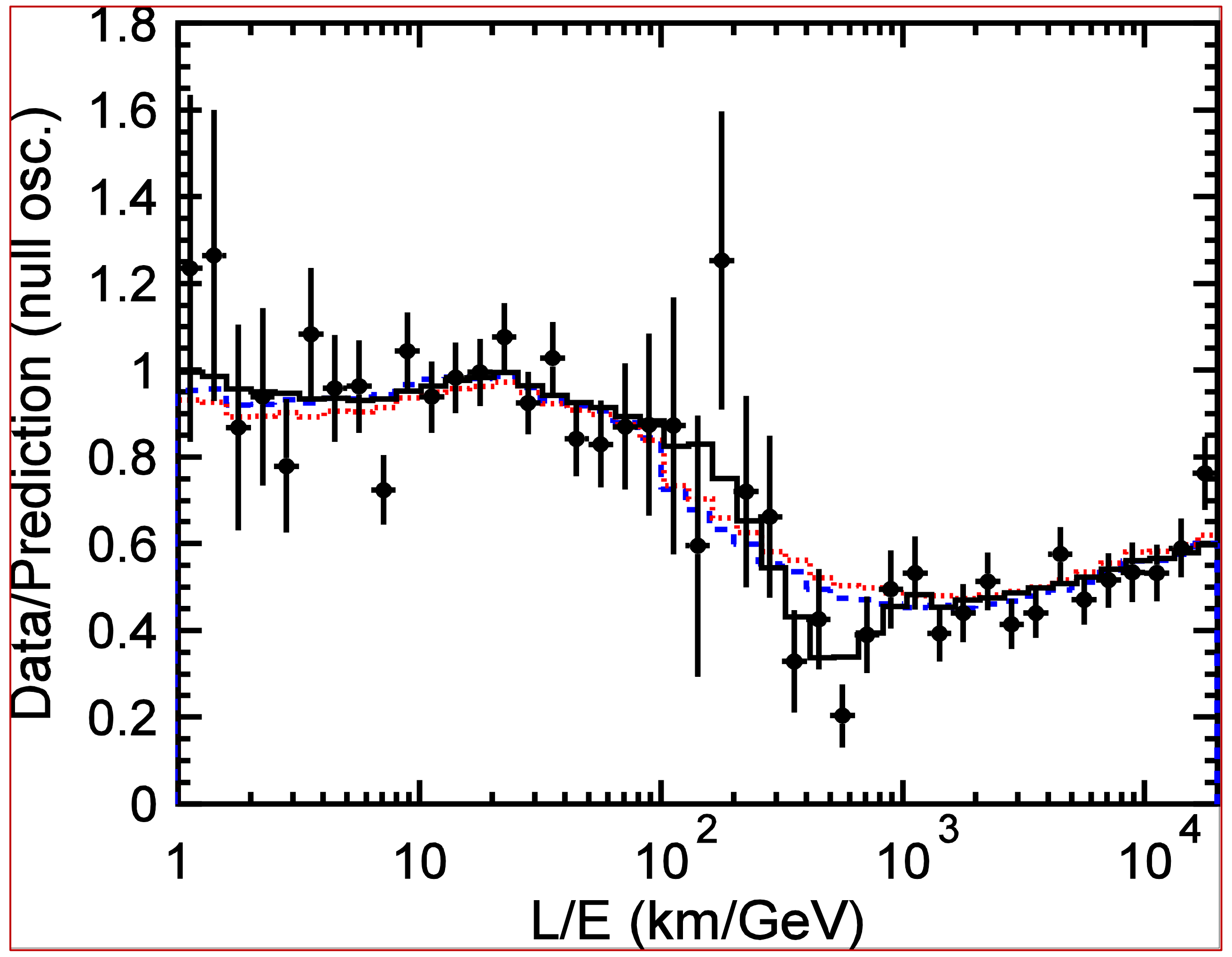
Chao *et al.*, EPJC 25

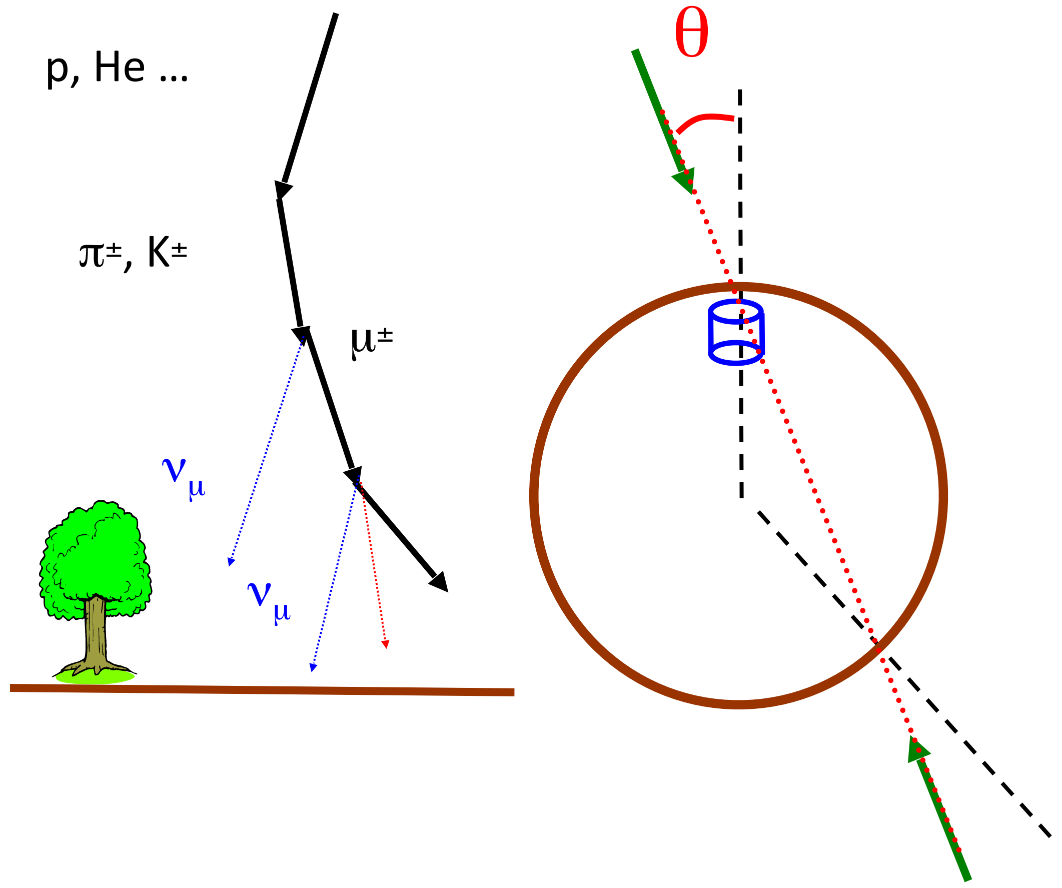


Neutrino oscillation

First discovery of neutrino oscillation

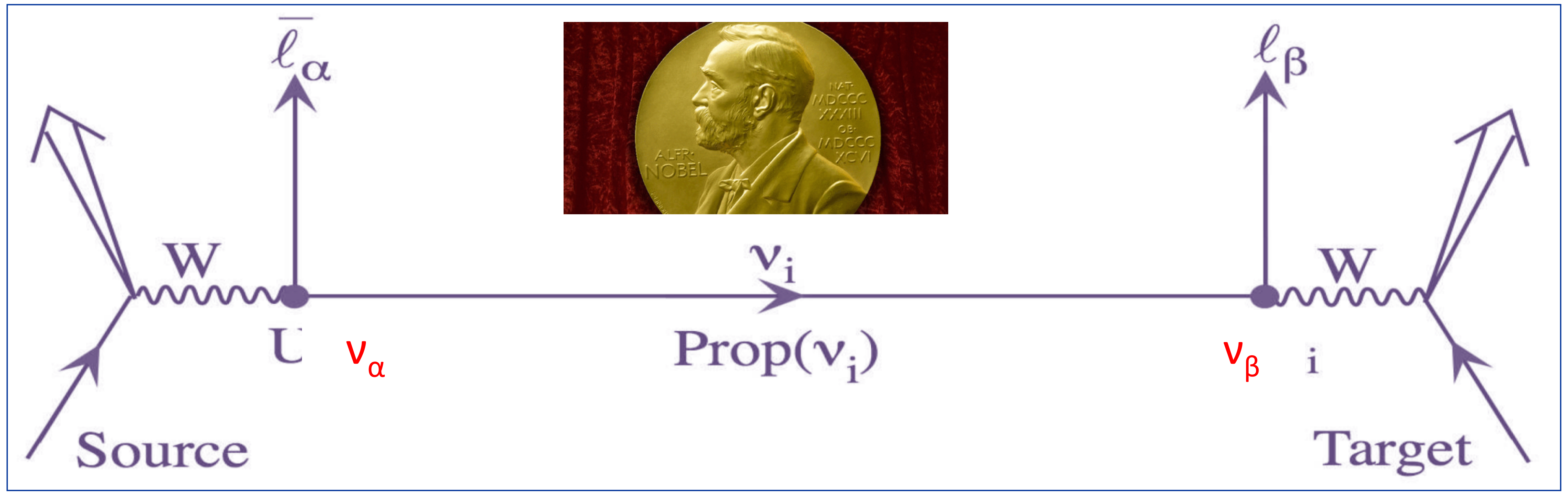
大气中微子振荡的实验证据（Ashie et al., PRL 04）





该结果被长基线中微子振荡实验K2K和MINOS证实

Picture



$$|\nu_\alpha(0)\rangle = \sum_{i=1}^{3} V_{\alpha i}^{*}|\nu_i\rangle$$

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^{3} V_{\alpha i}^{*} e^{-iE_i t}|\nu_i\rangle$$

+薛定谔方程

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j} \mathrm{Re}\left(V_{\alpha i}V_{\beta j}V_{\alpha j}^{*}V_{\beta i}^{*}\right)\sin^2\left(\frac{\Delta m_{ji}^2 L}{4E}\right) + 2\sum_{i<j} \mathrm{Im}\left(V_{\alpha i}V_{\beta j}V_{\alpha j}^{*}V_{\beta i}^{*}\right)\sin^2\left(\frac{\Delta m_{ji}^2 L}{2E}\right)$$

$$\mathscr{L}_{cc} = \frac{g}{\sqrt{2}}\overline{(e \quad \mu \quad \tau)}\gamma^\mu V_{PMNS}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} + \mathrm{h.c.}$$

NOBEL PRIZE IN 2015: TAKAAKI KAJITA, ARTHUR B. MCDONALD

Neutrino oscillation parameters

Parameters

F. Capozzi et al , 2107.00532

Parameter	Ordering	Best fit	1σ range	2σ range	3σ range	"1σ" (%)
$\delta m^2/10^{-5}$ eV^2	NO, IO	7.36	7.21 – 7.52	7.06 – 7.71	6.93 – 7.93	2.3
$\sin^2\theta_{12}/10^{-1}$	NO, IO	3.03	2.90 – 3.16	2.77 – 3.30	2.63 – 3.45	4.5
$\|\Delta m^2\|/10^{-3}$ eV^2	NO	2.485	2.454 – 2.508	2.427 – 2.537	2.401 – 2.565	1.1
	IO	2.455	2.430 – 2.485	2.403 – 2.513	2.376 – 2.541	1.1
$\sin^2\theta_{13}/10^{-2}$	NO	2.23	2.17 – 2.30	2.11 – 2.37	2.04 – 2.44	3.0
	IO	2.23	2.17 – 2.29	2.10 – 2.38	2.03 – 2.45	3.1
$\sin^2\theta_{23}/10^{-1}$	NO	4.55	4.40 – 4.73	4.27 – 5.81	4.16 – 5.99	6.7
	IO	5.69	5.48 – 5.82	4.30 – 5.94	4.17 – 6.06	5.5
δ/π	NO	1.24	1.11 – 1.42	0.94 – 1.74	0.77 – 1.97	16
	IO	1.52	1.37 – 1.66	1.22 – 1.78	1.07 – 1.90	9
$\Delta\chi^2_{\mathrm{IO-NO}}$	IO−NO	+6.5				

Parameter	TT+lowE	TT, TE, EE+lowE	TT, TE, EE+lowE+lensing	TT, TE, EE+lowE+lensing+BAO
Ω_K	$-0.056^{+0.044}_{-0.050}$	$-0.044^{+0.033}_{-0.034}$	$-0.011^{+0.013}_{-0.012}$	$0.0007^{+0.0037}_{-0.0037}$
Σm_ν [eV]	< 0.537	< 0.257	< 0.241	< 0.120
N_{eff}	$3.00^{+0.57}_{-0.53}$	$2.92^{+0.36}_{-0.37}$	$2.89^{+0.36}_{-0.38}$	$2.99^{+0.34}_{-0.33}$
Y_P	$0.246^{+0.039}_{-0.041}$	$0.240^{+0.024}_{-0.025}$	$0.239^{+0.024}_{-0.025}$	$0.242^{+0.023}_{-0.024}$
$dn_s/d\ln k$	$-0.004^{+0.015}_{-0.015}$	$-0.006^{+0.013}_{-0.013}$	$-0.005^{+0.013}_{-0.013}$	$-0.004^{+0.013}_{-0.013}$
$r_{0.002}$	< 0.102	< 0.107	< 0.101	< 0.106
w_0	$-1.56^{+0.60}_{-0.48}$	$-1.58^{+0.52}_{-0.41}$	$-1.57^{+0.50}_{-0.40}$	$-1.04^{+0.10}_{-0.10}$

Aghanim et al. [Planck] 2018

- ★ Neutrino oscillaitions in dense matter, S.Luo, PRD 2020, ZZ Xing and J.Y. Zhu, NPB 2019,……
- ★ Matter effects with exotic BSM neutrino interactions, A.Smirnov and X.j. Xu JHEP 2019, S.F. Ge and S.Parke, PRL 2019, GJ.Ding and F. Feruglio, JHEP 2020, ……
- ★ Interpretation the discrepancy of measured CP phase by T2k and Nova: Denton, Gehrlein and Pestes, PRL 2021; Chatterjee and Palazzo PRL 2021.
- ★ Neutrino oscillation in dark matter, J. Liao et al., 2018 J. Liu et al., PRD 2018, S.F. Ge and Muryama, 2019, W. Chao et al., 2020, Dev et al., 2020,……

ν oscillate in a specific lepton portal Fermionic DM

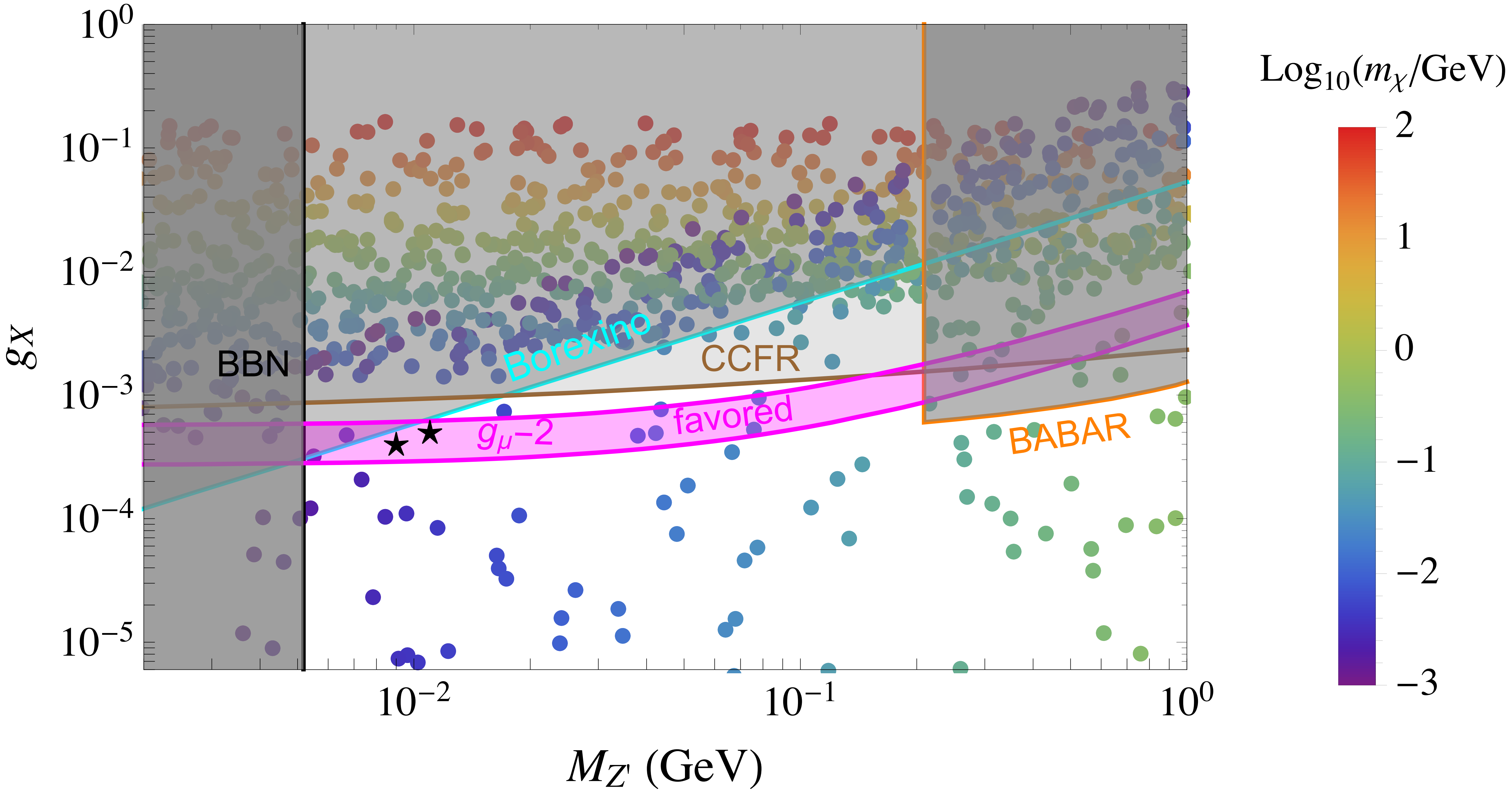
Gauge portal to fermionic DM via the $U(1)_{L_\mu - L_\tau}$ gauge symmetry

Particles	ℓ_e	ℓ_μ	ℓ_τ	E_R	μ_R	τ_R	$\chi_{L,R}$	Φ
Charges	0	1	-1	0	1	-1	1	1

$$\mathscr{L} \sim \bar{\chi} i \gamma^\mu D_\mu \chi + (D_\mu \Phi)^\dagger (D^\mu \Phi) - m\bar{\chi}\chi + \mu^2 \Phi^\dagger \Phi - \lambda(\Phi^\dagger \Phi)^2$$

$$\mathscr{L}_{\nu-mass} \sim \frac{1}{4}\kappa_{fg} \overline{\ell^C_{Lc}}^f \varepsilon_{cd} H_d \ell^g_{Lb} \varepsilon_{ba} H_a + \mathrm{h.c.}$$

$$\mathscr{L}'_{\nu-mass} \sim \frac{1}{4}\kappa'_{e\mu} \Phi^\dagger \overline{\ell^C_{Lc}}^e \varepsilon_{cd} H_d \ell^\mu_{Lb} \varepsilon_{ba} H_a + \frac{1}{4}\kappa'_{e\tau} \Phi \overline{\ell^C_{Lc}}^e \varepsilon_{cd} H_d \ell^\tau_{Lb} \varepsilon_{ba} H_a + \mathrm{h.c.}$$



DM Relic density

Neutrino oscillations in Fermionic DM

Effective potential arising from DM: $V_\chi = \pm \frac{g_\chi^2}{m_{Z'}^2} n_\chi$

$$\tilde{\mathscr{H}} = \mathscr{H} + \mathscr{H}' = \frac{1}{2E}\left[V \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} V^\dagger + \begin{pmatrix} A_{CC} & & \\ & A_\chi & \\ & & -A_\chi \end{pmatrix} \right]$$

Ignoring electron induced matter effect for simplicity!

$$\tilde{\mathscr{H}} = \frac{1}{2E}\left[V \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} V^\dagger + \begin{pmatrix} 0 & & \\ & A_\chi & \\ & & -A_\chi \end{pmatrix} \right] = \frac{1}{2E} \tilde{V} \begin{pmatrix} \tilde{m}_1^2 & & \\ & \tilde{m}_2^2 & \\ & & \tilde{m}_3^2 \end{pmatrix} \tilde{V}^\dagger ,$$

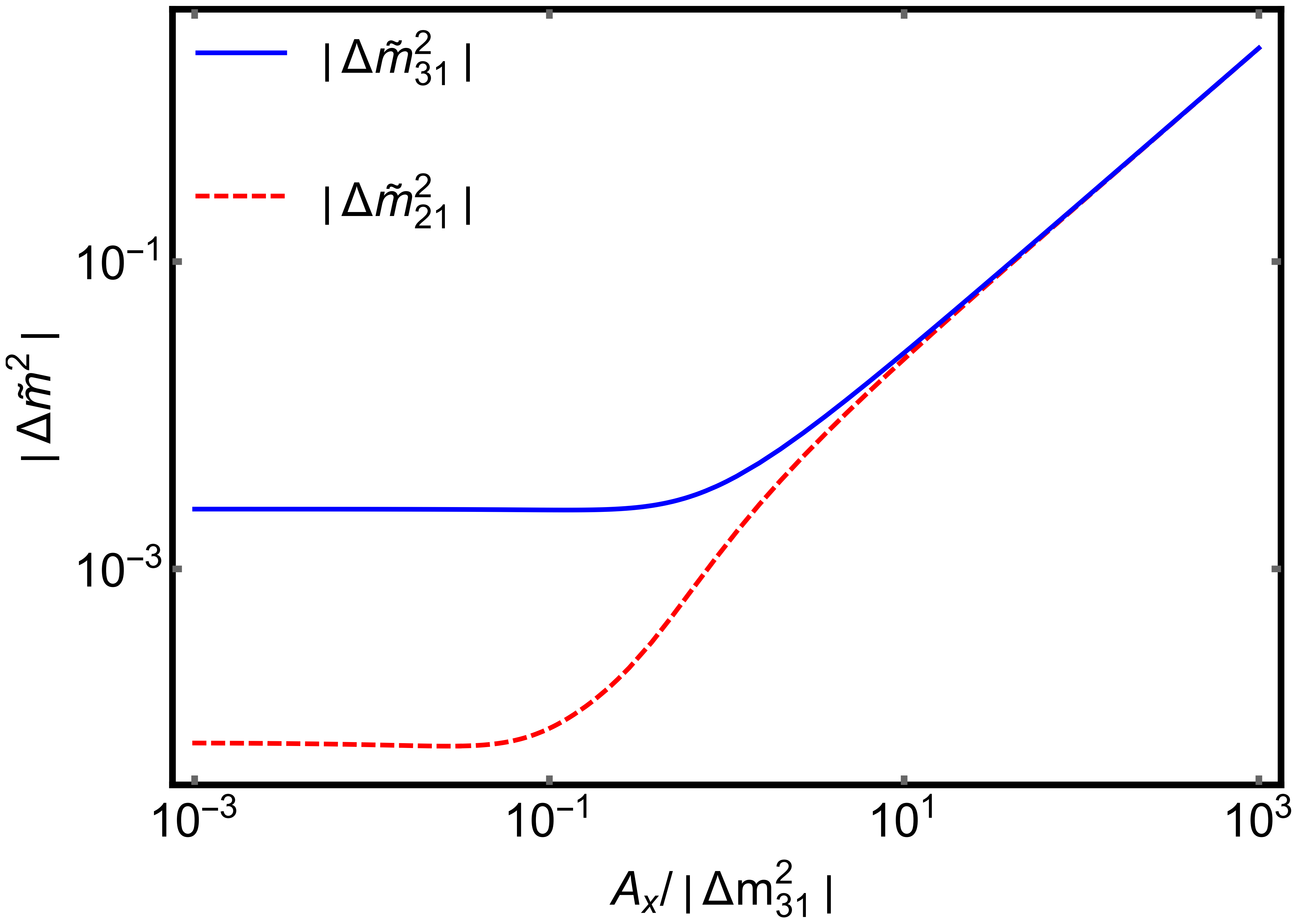
$$|\tilde{V}_{\alpha i}|^2 = \frac{(\lambda_i - \xi_\alpha)(\lambda_i - \zeta_\alpha)}{(\lambda_i - \lambda_j)(\lambda_i - \lambda_k)} = \frac{\lambda_i^2 - \lambda_i(\xi_\alpha + \zeta_\alpha) + \xi_\alpha \zeta_\alpha}{(\lambda_i - \lambda_j)(\lambda_i - \lambda_k)} ,$$

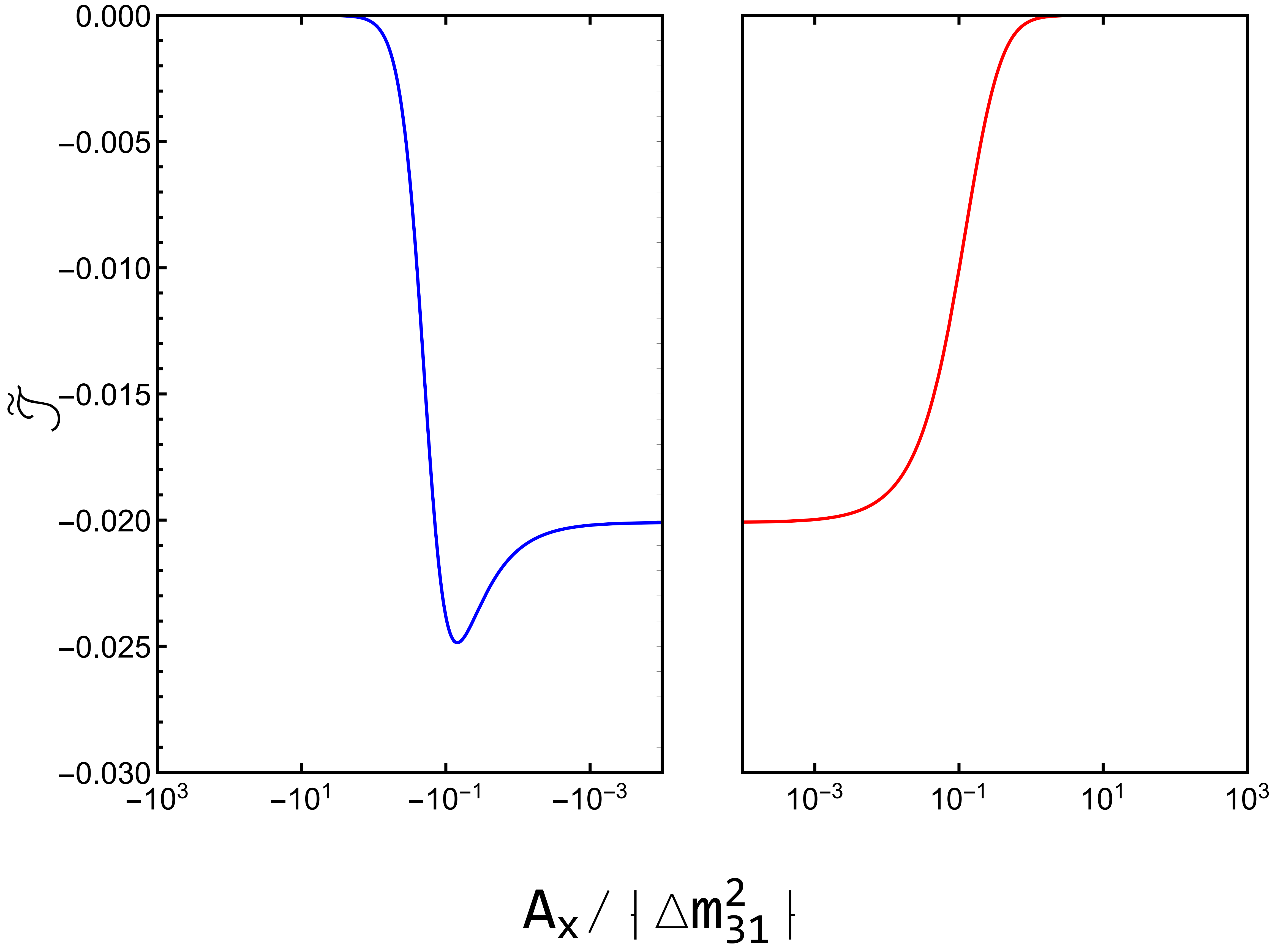
$\frac{\lambda_i}{2E}$: **Eigenvalue of** $\tilde{\mathscr{H}}$

ζ_α : **Eigenvalue of** $\tilde{\mathscr{H}}_\alpha = \begin{pmatrix} \tilde{\mathscr{H}}_{\beta\beta} & \tilde{\mathscr{H}}_{\beta\gamma} \\ \tilde{\mathscr{H}}_{\gamma\beta} & \tilde{\mathscr{H}}_{\gamma\gamma} \end{pmatrix}$

Neutrino oscillations in Fermionic DM

Squared mass differences and Jarlskog





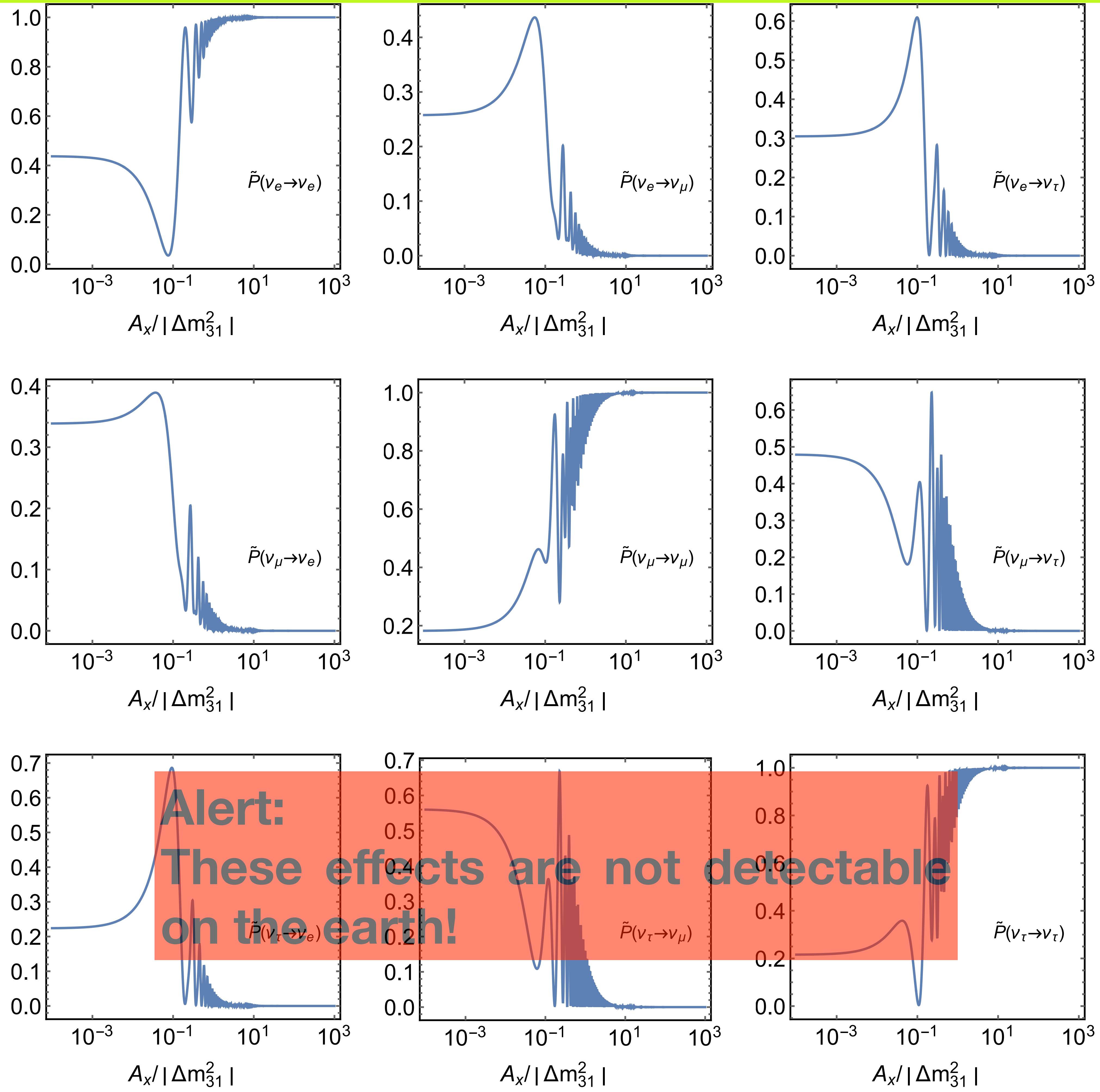
Neutrino oscillations in Fermionic DM

Oscillation probability

$$\tilde{P}(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta} - 4\sum_{j>i} \mathrm{Re}\left[\tilde{V}_{\alpha i}\tilde{V}^*_{\beta i}\tilde{V}^*_{\alpha j}\tilde{V}_{\beta j}\right]\sin^2\tilde{\Delta}_{ji}$$

$$\pm 2\sum_{j>i} \mathrm{Im}\left[\tilde{V}_{\alpha i}\tilde{V}^*_{\beta i}\tilde{V}^*_{\alpha j}\tilde{V}_{\beta j}\right]\sin 2\tilde{\Delta}_{ji}$$

$$\frac{L}{E} \sim 10^4 \text{ km/GeV} \longrightarrow$$



$\rho_\chi \sim 0.3 \sim 0.6$ GeV/cm^3 **and** $\mathcal{O}(g) \sim 10^{-3}$**, matter effect is unlikely to be observable, but this effect do exist in somewhere of the universe.**

Neutrino oscillations in Bosoinic DM

Ultralight bosonic DM

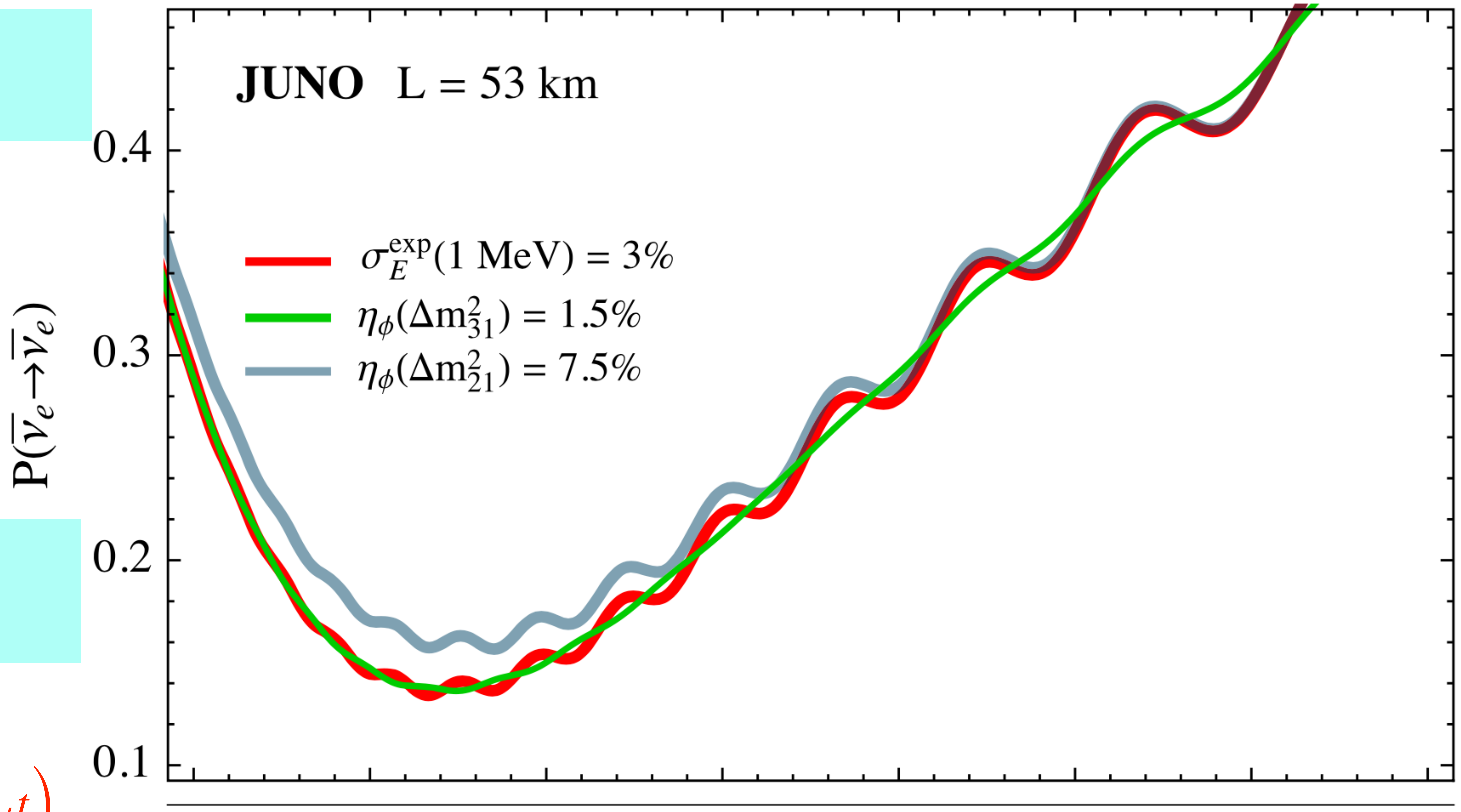
$$\mathcal{L}_s \supset -(m_\nu + g\phi)\nu\nu + h.c.$$

$$\mathcal{L}_V \supset gQ^{\alpha\beta}\overline{\nu_L^\alpha}\gamma^\mu A_\mu \nu_L^\beta$$

Effective potential derived from desperation relation

$$V_{eff}^1 = \frac{1}{2E_\nu}[\phi(ym_\nu + m_\nu y) + \phi^2 y^2] \qquad \phi = \frac{\sqrt{2\rho_\phi}}{m_\phi}\cos\left(m_\phi t\right)$$

$$V_{eff}^2 = -\frac{1}{2E_\nu}[2p\cdot AgQ + g^2Q^2A^2] \qquad A_\mu = \frac{\sqrt{2\rho_A}}{m_A}\xi_\mu \cos\left(m_A t\right)$$

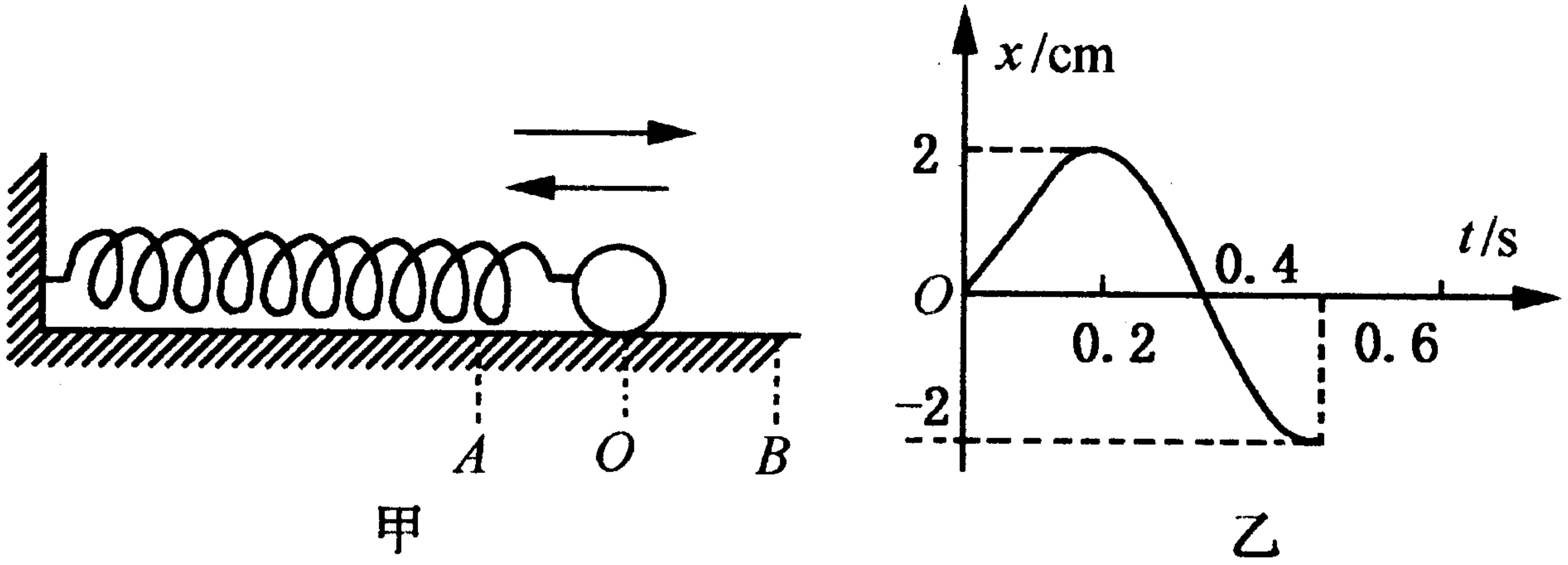


Krnjaic, et al., PHYS. REV. D 97, 075017 (2018)

Brdar, et al., PHYS. REV. D 97, 043001 (2018)

Neutrino oscillations in Bosoinic DM

Matching neutrino oscillation into classical oscillator



$$\ddot{x} + \frac{k}{m}x = 0 \quad \Rightarrow \quad x = A\cos(\omega t + \varphi)$$

$$\ddot{x} + (\delta + \varepsilon \cos \omega t)x = 0$$

Mathieu equation

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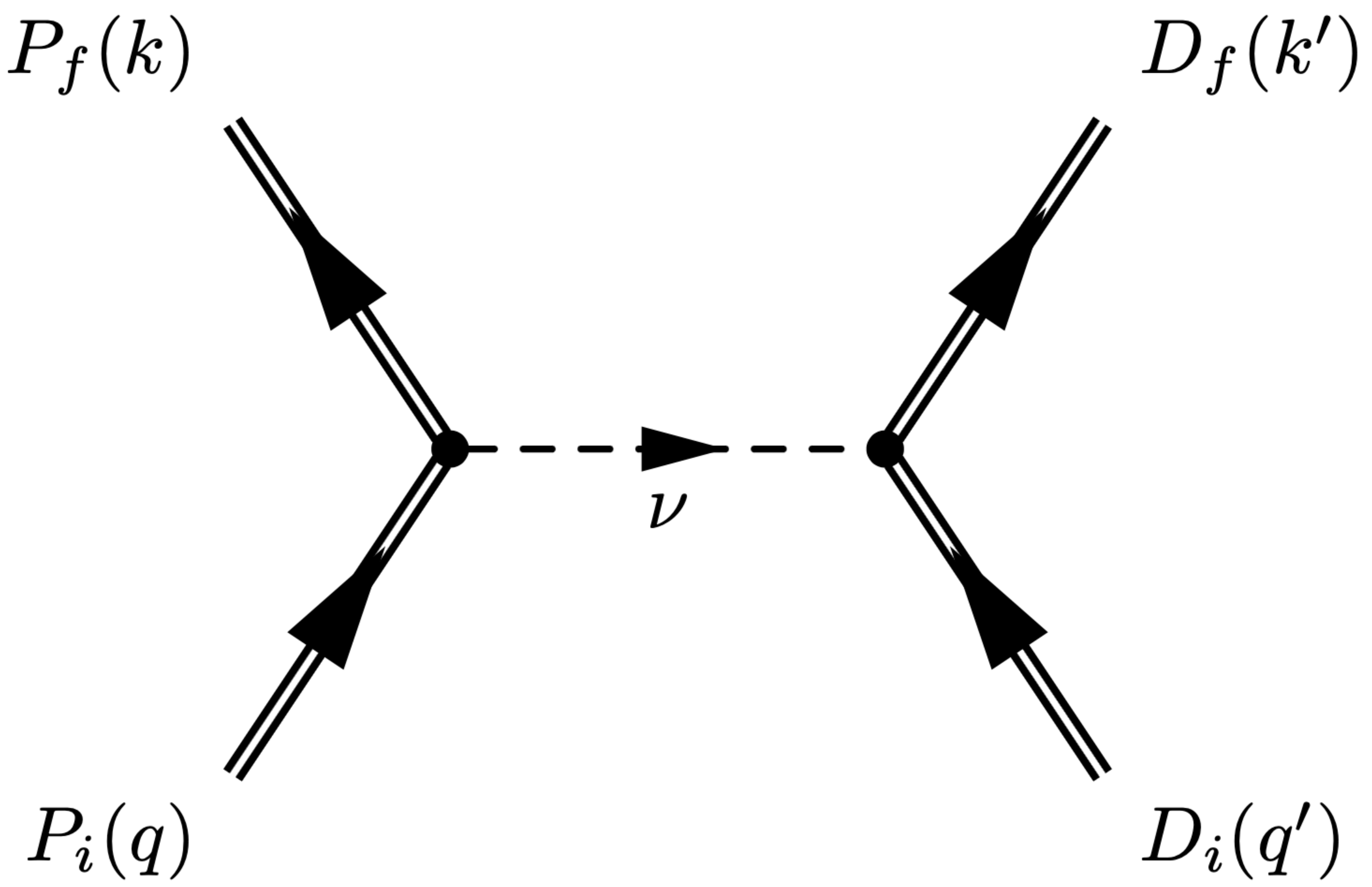
Quantum field theory approach



Setup

$$\mathcal{L} = \overline{\nu_L^\alpha} i\gamma^\mu \partial_\mu \nu_L^\alpha - \frac{1}{2}\mathcal{G}_{\alpha\beta}\varphi\overline{\nu_{L\alpha}^C}\nu_{L\beta} - \frac{1}{2}\overline{\nu_{L\alpha}^C} m_0^{\alpha\beta}\nu_{L\beta} + \mathrm{h.c.}$$

$$= \frac{1}{2}\overline{\nu^\alpha}\left(i\delta_{\alpha\beta}\gamma^\mu\partial_\mu - m_{0,\alpha\beta} - \mathcal{G}_{\alpha\beta}\varphi\right)\nu^\beta$$

EOM (taking DM as classical external field)

$$\left(i\gamma_\mu\partial^\mu - m_i\right)\hat{\nu}_i = \sum_j \bar{\mathcal{G}}_{ij}\varphi\hat{\nu}_j$$

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Free propagator:

$$S_F^i(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} \frac{i}{2}\left(1 - \frac{\vec{\sigma}\cdot\vec{p}}{E_i}\right)\frac{1}{p_0 - E_i + i\varepsilon}$$

Modified propagator

$$S_A^i(x_f, x_i) = S_F^i(x_f, x_i) + \iint d^4x_1 d^4x_2 S_F^i(x_f - x_1) g_{ij}\varphi(x_1) S_F^j(x_1, x_2) g_{ji}\varphi(x_2) S_F^i(x_2, x_i)$$

$2 \leftrightarrow 2$ *oscillation case*

Propagator in momentum space

$$\begin{cases} S_A^{1\to1}(p) \approx \dfrac{ip\cdot\sigma}{2p_0}\dfrac{p_0 - E_2}{(p_0 - E_1 + i\varepsilon)(p_0 - E_2 + i\varepsilon) - \kappa^2} \equiv \dfrac{ip\cdot\sigma}{2p_0}\Sigma^{11} \\ S_A^{2\to2}(p) = \dfrac{(p_0 - E_1)}{(p_0 - E_1 + i\varepsilon)(p_0 - E_2 + i\varepsilon) - \kappa^2} \equiv \dfrac{ip\cdot\sigma}{2p_0}\Sigma^{22} \\ S_A^{1\to2}(p) = \dfrac{ip\cdot\sigma}{2p_0}\dfrac{\kappa}{(p_0 - E_1 + i\varepsilon)(p_0 - E_2 + i\varepsilon) - \kappa^2} \equiv \dfrac{ip\cdot\sigma}{2p_0}\Sigma^{12} \end{cases}$$

$$\kappa = 2\pi\bar{\mathscr{G}}\frac{\sqrt{2\rho_\varphi}}{m_\varphi}$$

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Initial&final states

$$|\psi_i(x)\rangle = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_i(p)}} e^{-ip\cdot x} f_{\Psi_i}(p,P)\,|p\rangle$$

Amplitude

$$iT_{\alpha\to\beta} = \sum_{ij} \mathscr{U}^*_{\alpha i}\mathscr{U}_{\beta j}\int\frac{d^4p}{(2\pi)^4}\Phi_i^*(p)\Phi_j(p)\Sigma^{ij}(p)\exp\left(-ip_0T+i\vec{p}\cdot L\right)\ .$$

Oscillation probability

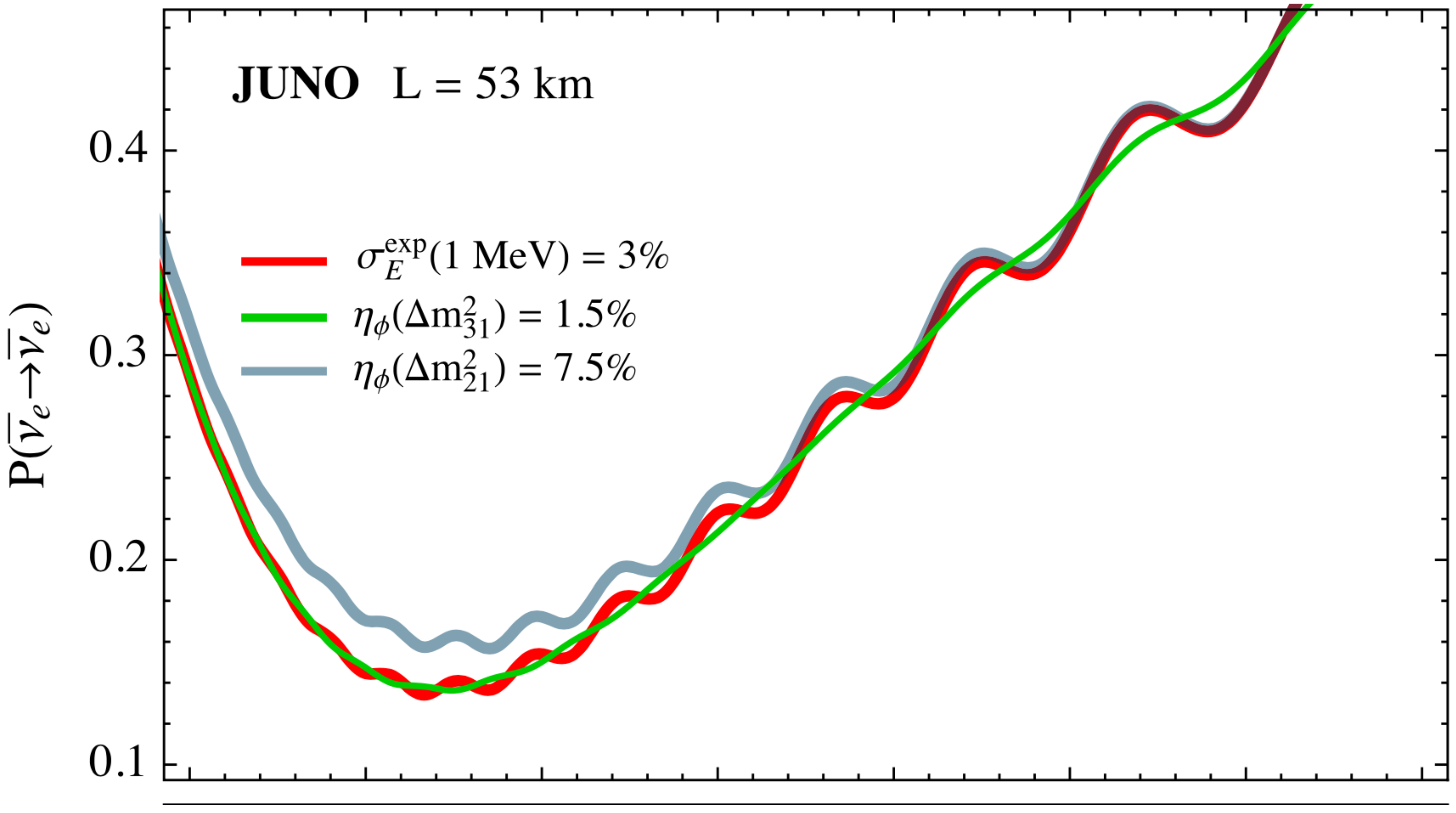
$$P_{e\to\mu} = \frac{[(\Delta\bar{m}^2/4E)s_{2\theta}+\kappa c_{2\theta}]^2}{\left(\Delta\bar{m}^2/4E\right)^2+\kappa^2}\sin^2\left[\sqrt{\left(\frac{\Delta\bar{m}^2}{4E}\right)^2+\kappa^2}\,L\right]$$

QM result:

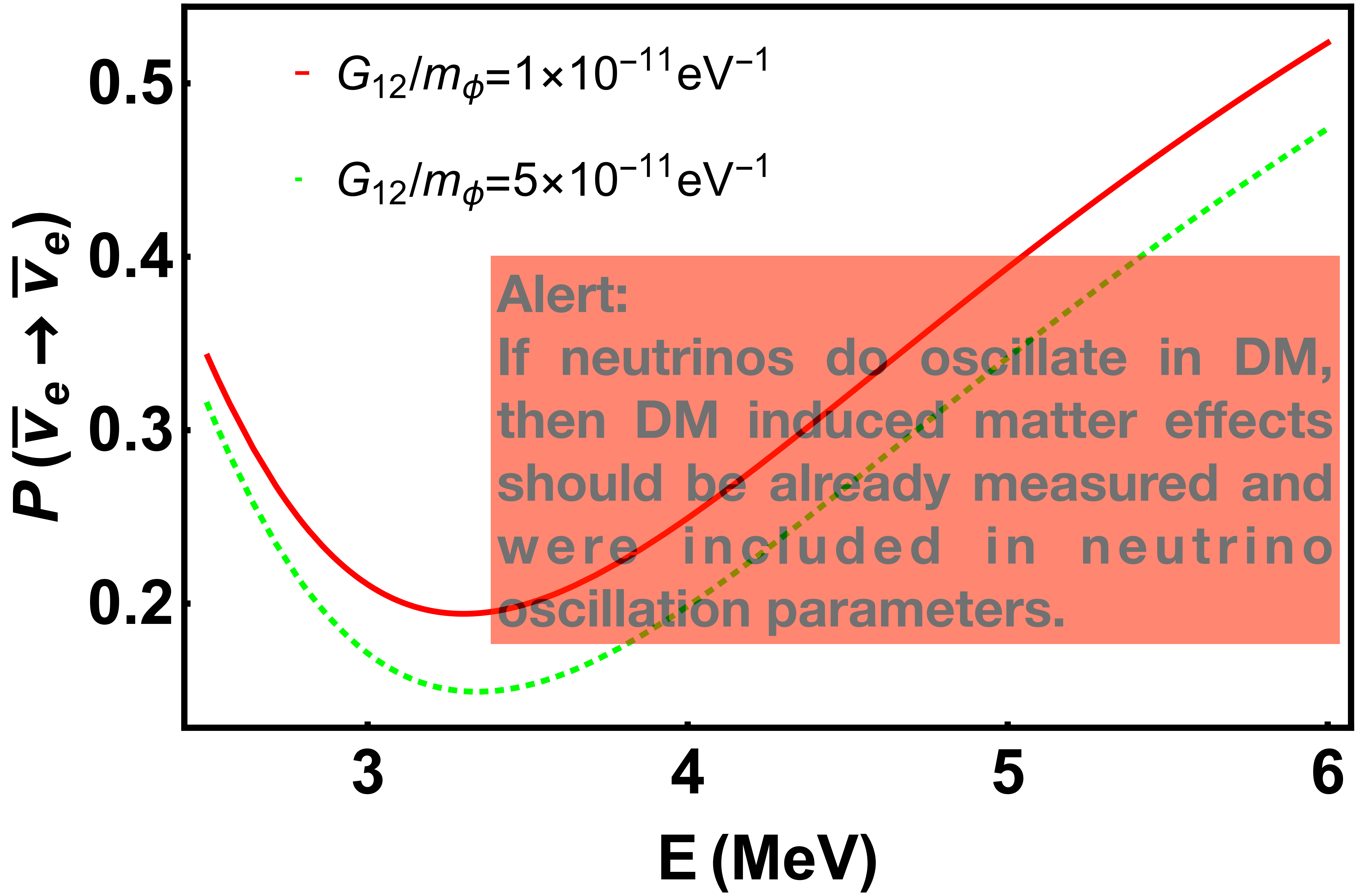
$$P_{e\to\mu} = \sin^2 2\theta_M \sin^2\left(\frac{\Delta m_M^2 L}{4E}\right)$$

Time dependence is removed !

Summary



From Krnjaic, et al., PHYS. REV. D 97, 075017 (2018)



THANK YOU FOR YOUR ATTENTION