

Consequences of phase transitions occurred during inflation

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第九十五期物理讲座

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2009.12381, 2201.05171 w/ Kun-Feng Lyu, Lian-Tao Wang and Siyi Zhou

2208.14857 w/ Xi Tong and Siyi Zhou

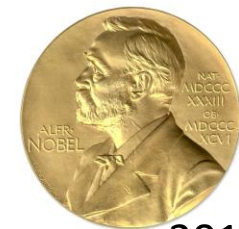
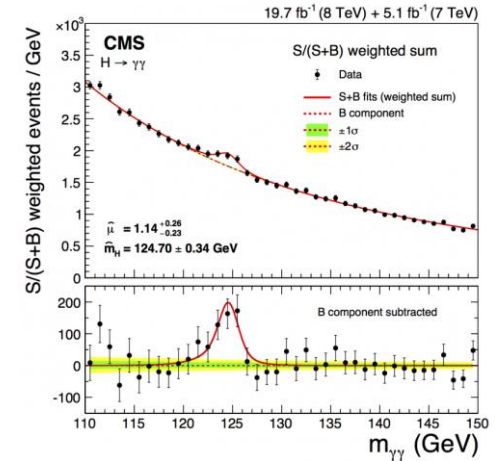
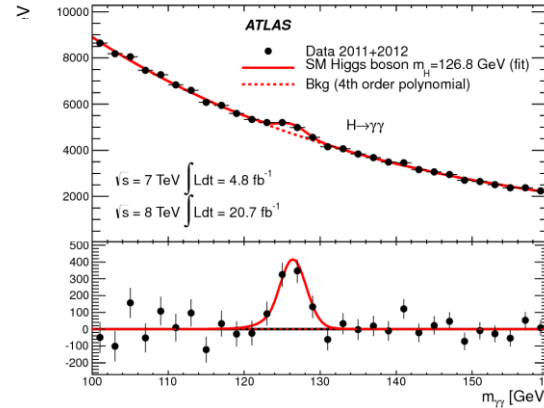
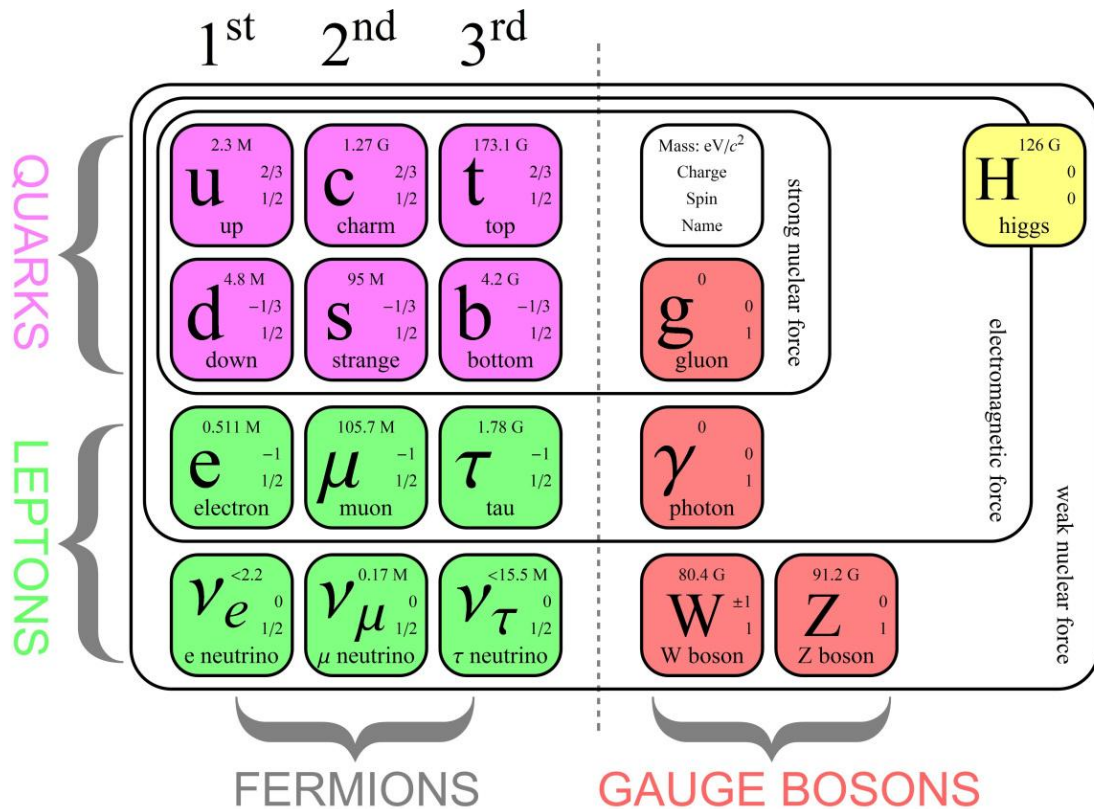
2304.02361 w/ Chen Yang

2308.00070 w/ Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang

2409.05833 w/ Qi Chen, Yuan Yin

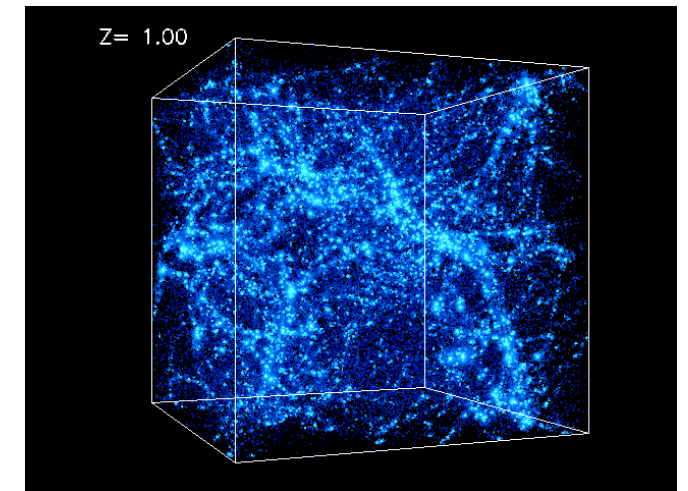
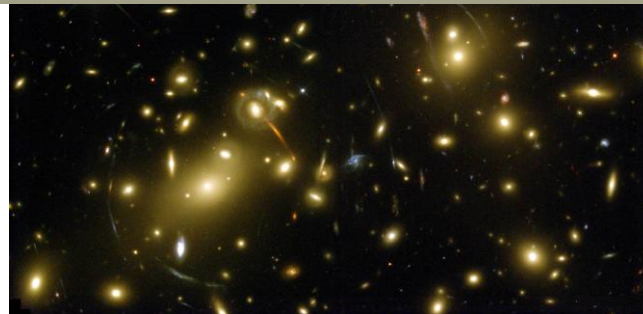
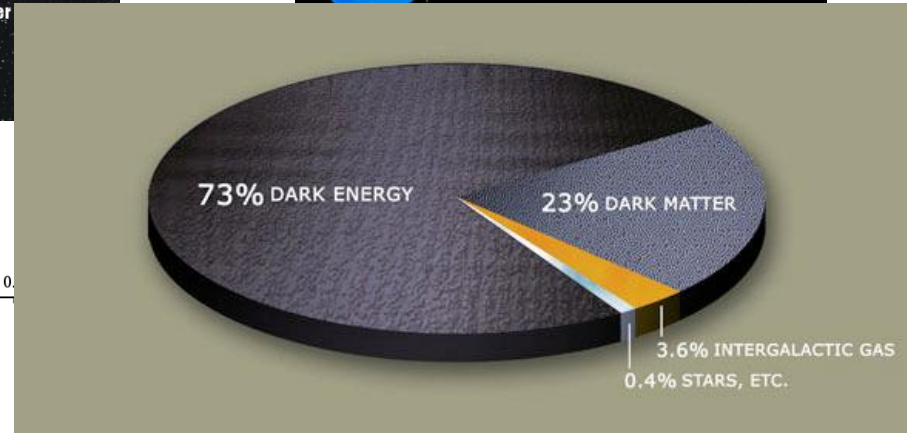
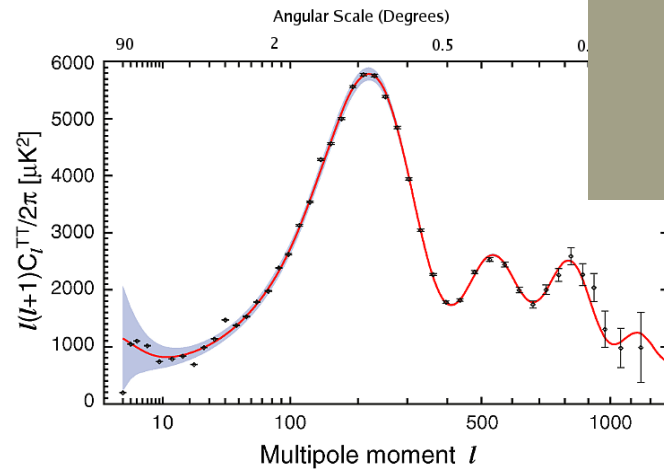
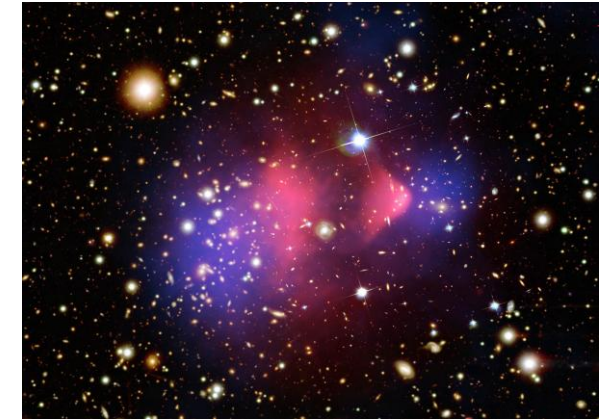
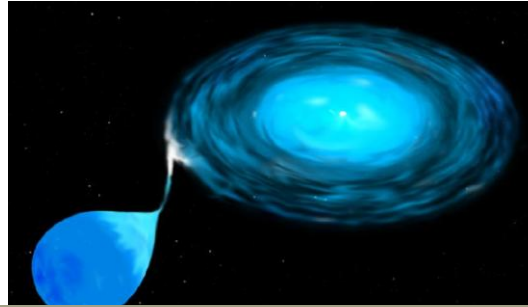
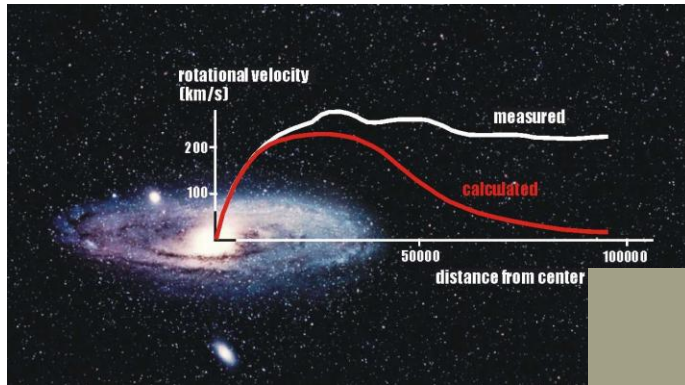
2411.12699 w/ Qi Chen, Yuhang Li, Yuan Yin

The Standard Model of Particle Physics

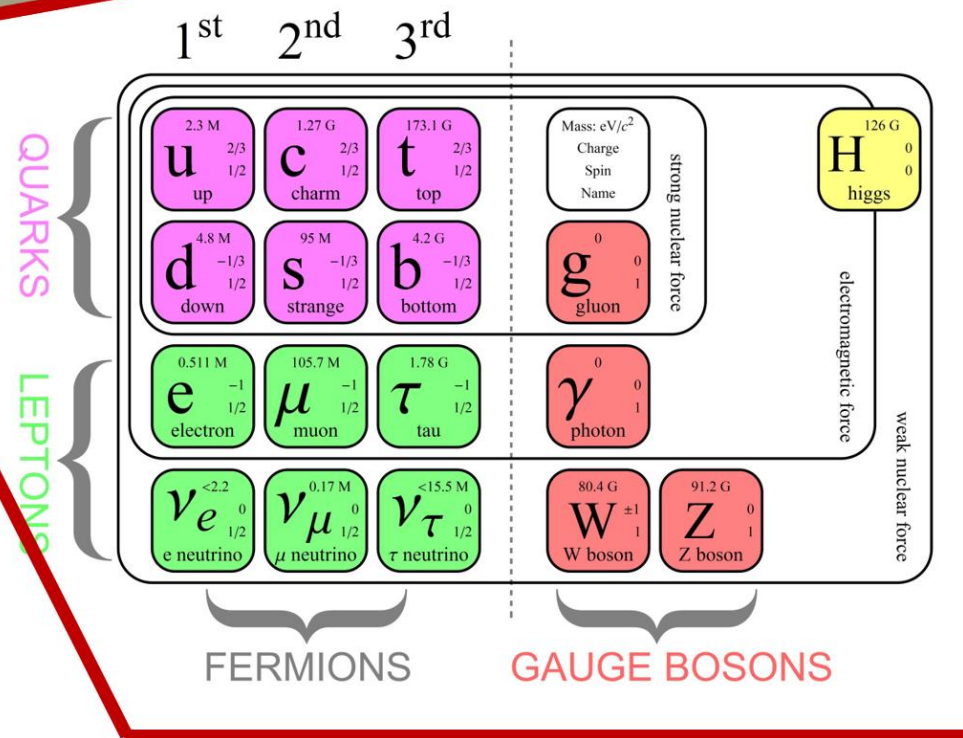
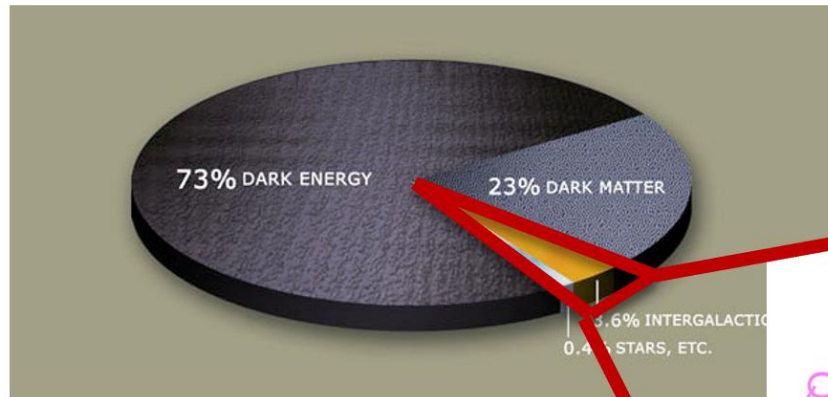


2013

But when we look up into the Sky

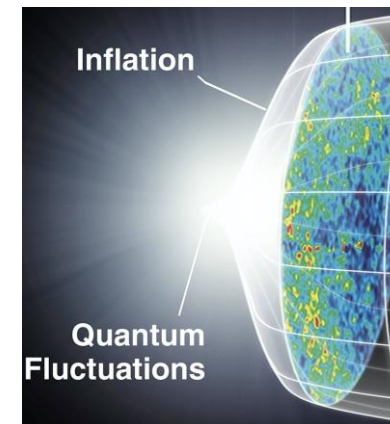
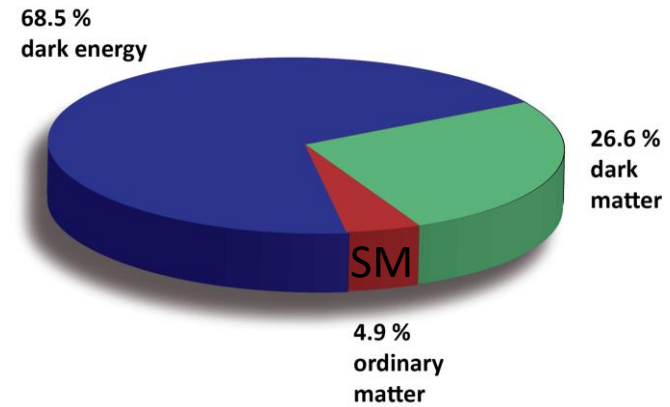
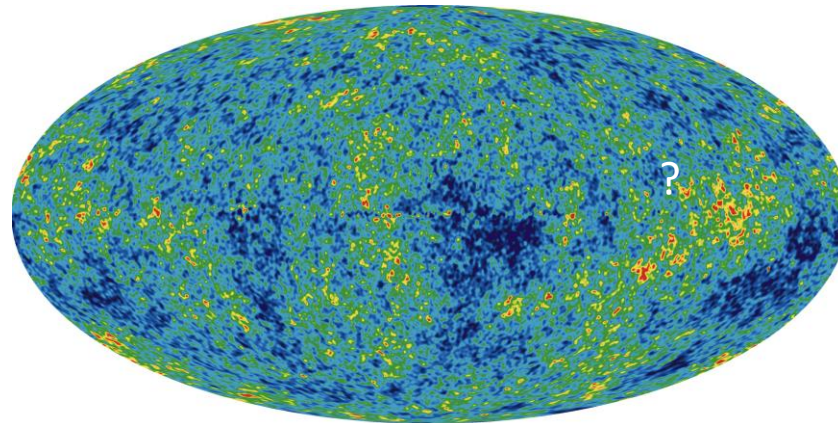
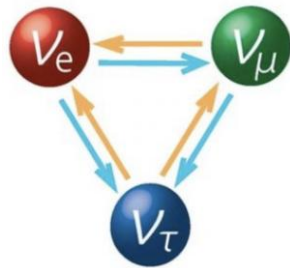


Big Problems in the Standard Model

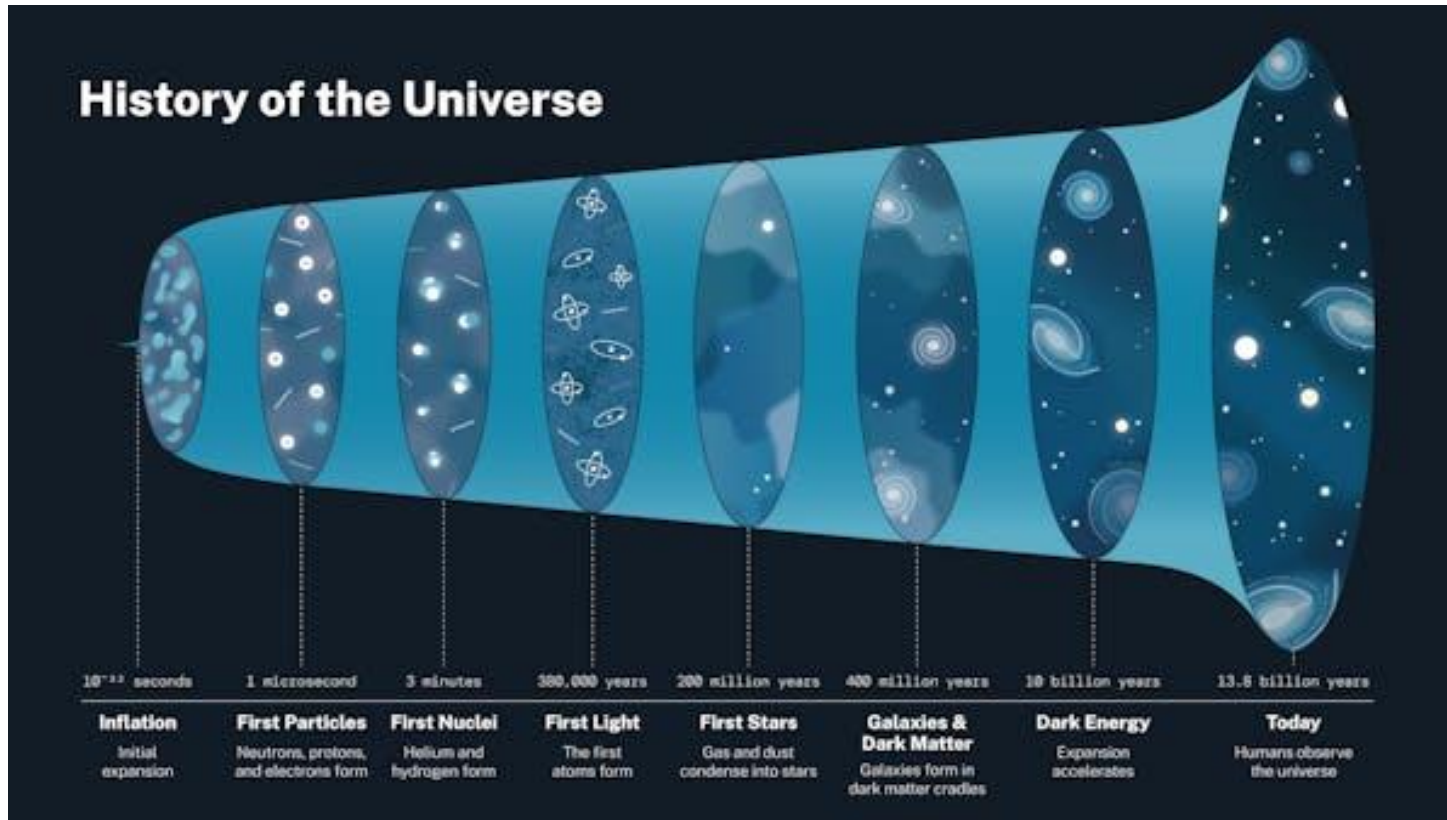


Big Problems in the Standard Model

$$\frac{n_B}{n_\gamma} = (6.047 \pm 0.074) \times 10^{-10}$$



The Big Bang Theory



Hubble



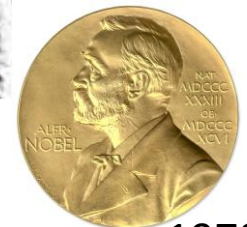
1953



Gamow



Penzias and Wilson



1978

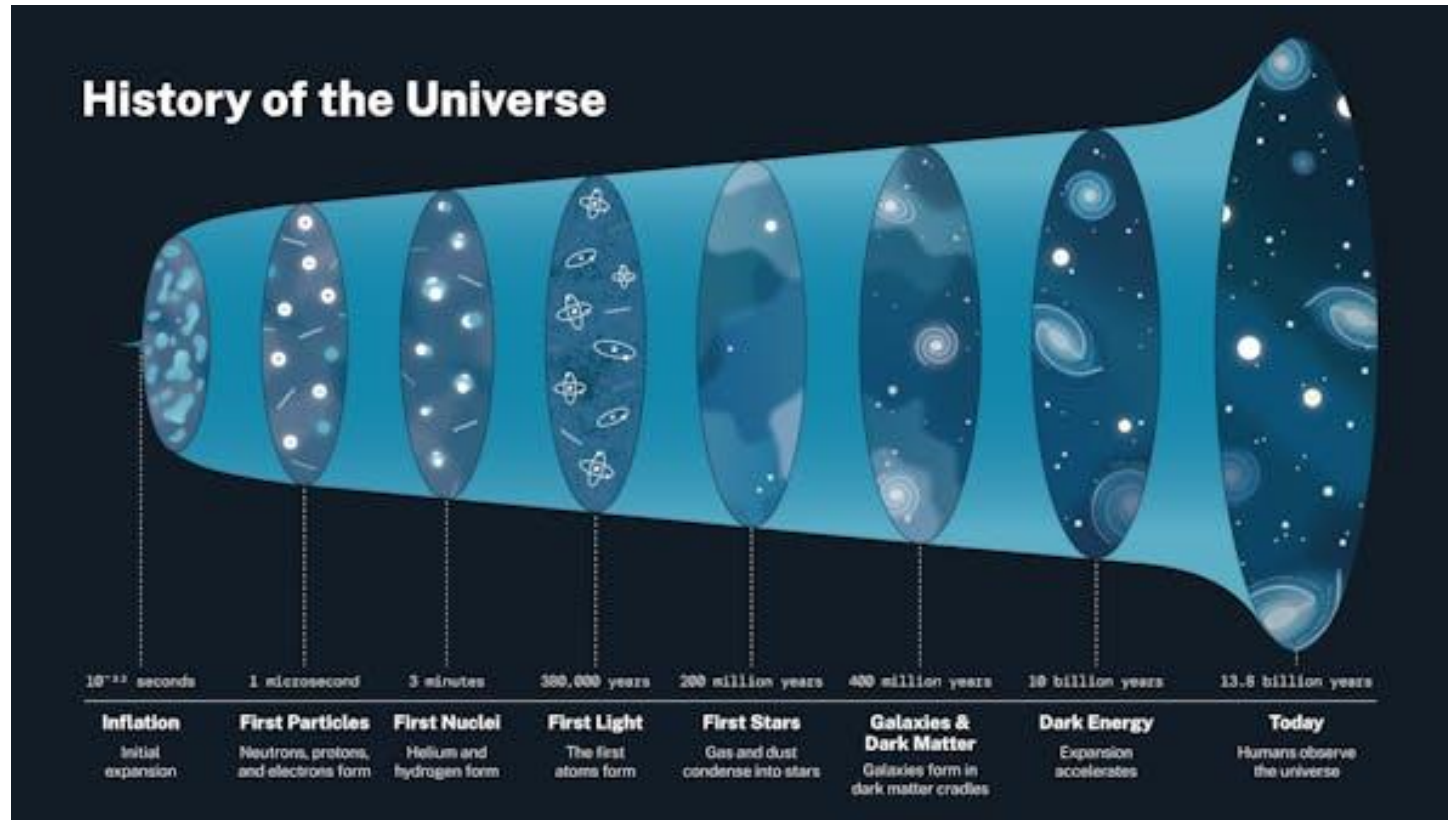
$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

$$H = \dot{a}/a$$

The Big Bang Theory

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

$$H = \dot{a}/a$$



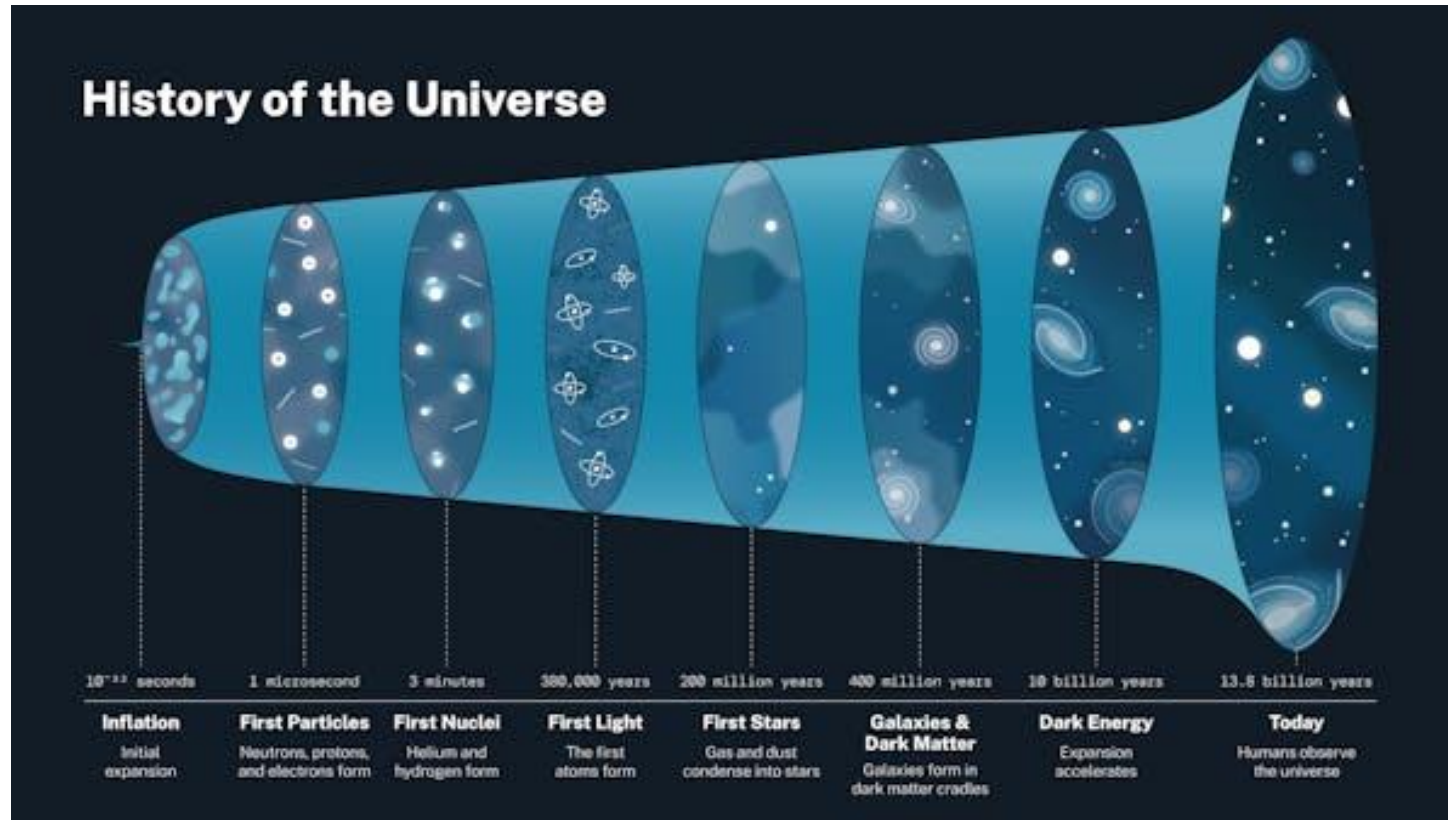
The expansion of the universe **decelerates** in the RD and MD regime.

Radiation
domination

Matter
domination

Dark energy
domination

Problems with the Big Bang Theory



Radiation
domination

Matter
domination

Dark energy
domination

1. Causality problem
2. Flatness problem
3. Magnetic monopole problem

The conformal time

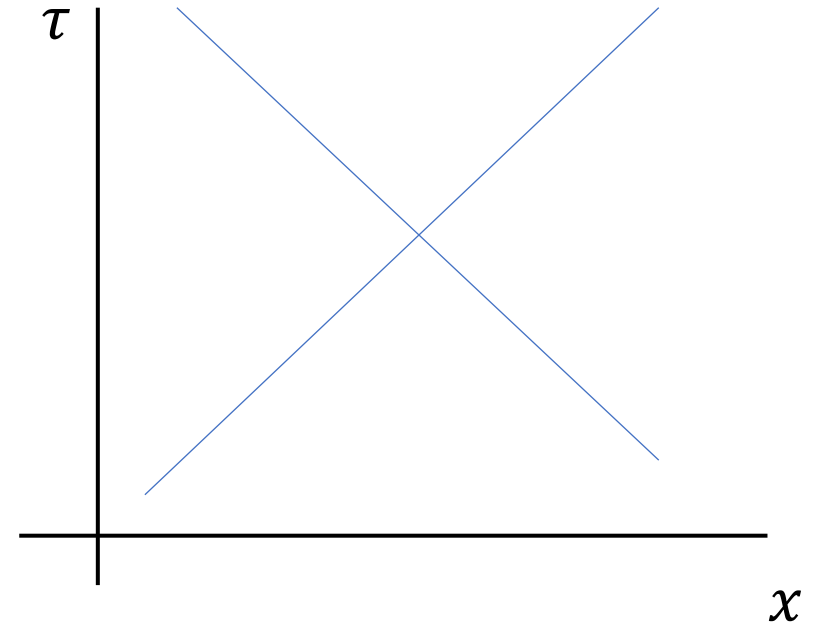
$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

$$ds^2 = a^2(\tau)(-d\tau^2 + dx^2 + dy^2 + dz^2)$$

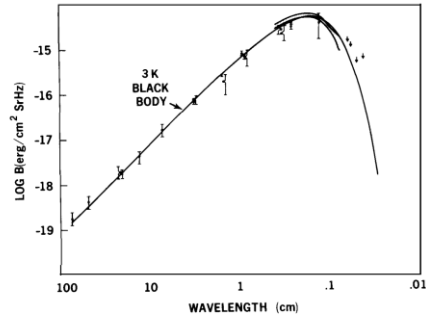
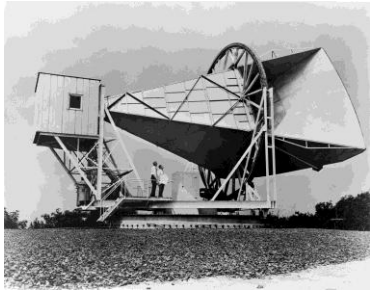


Conformal time

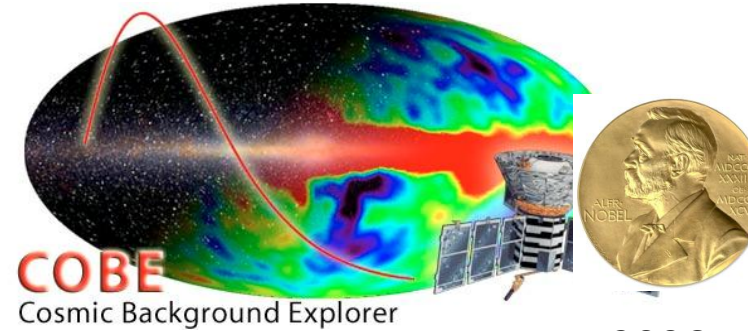
Light cone: $ds^2 = 0$



The causality problem

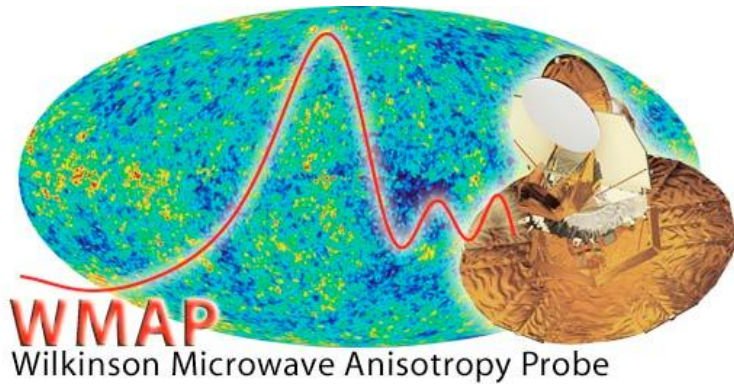


Bell Laboratory Penzias and Wilson 1964

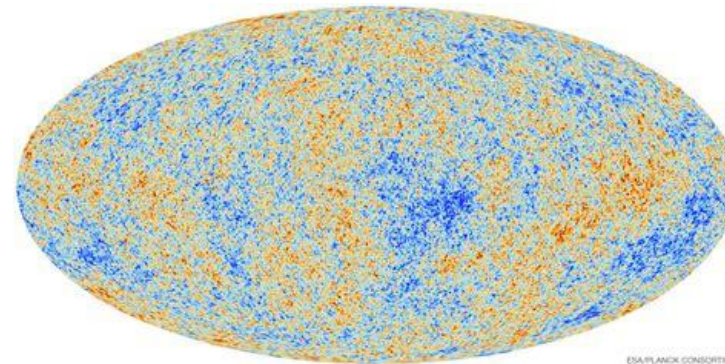


NASA 1989-1996

2006

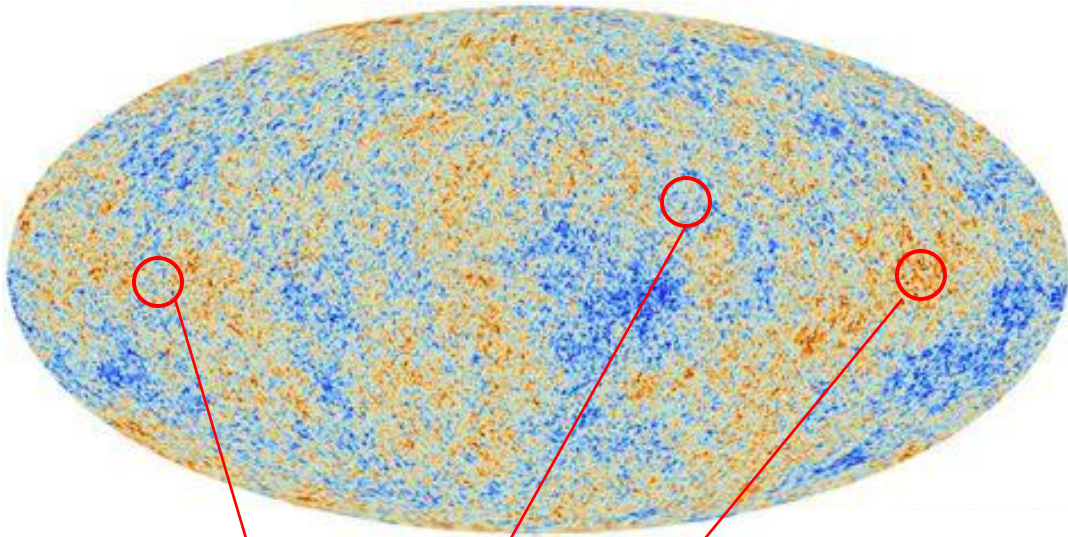


NASA 2001-2010

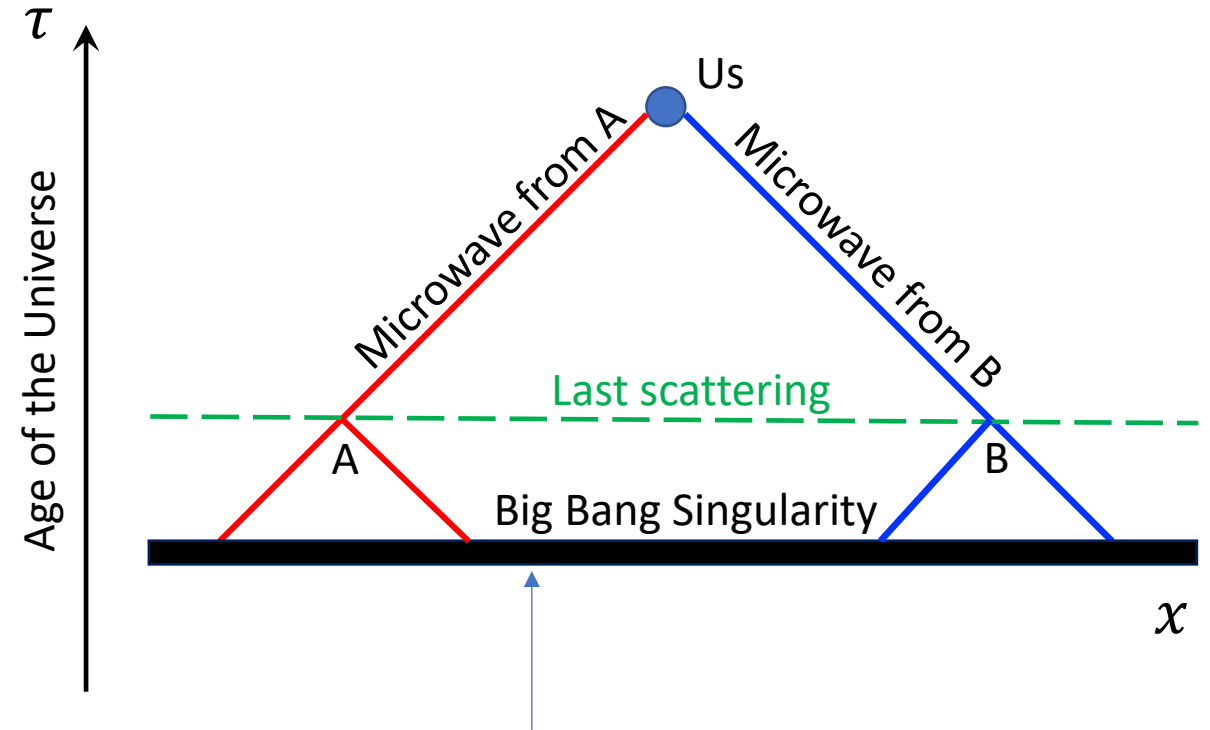


European space agency
Planck spacecraft

The causality problem



Same temperature and similar fluctuations.

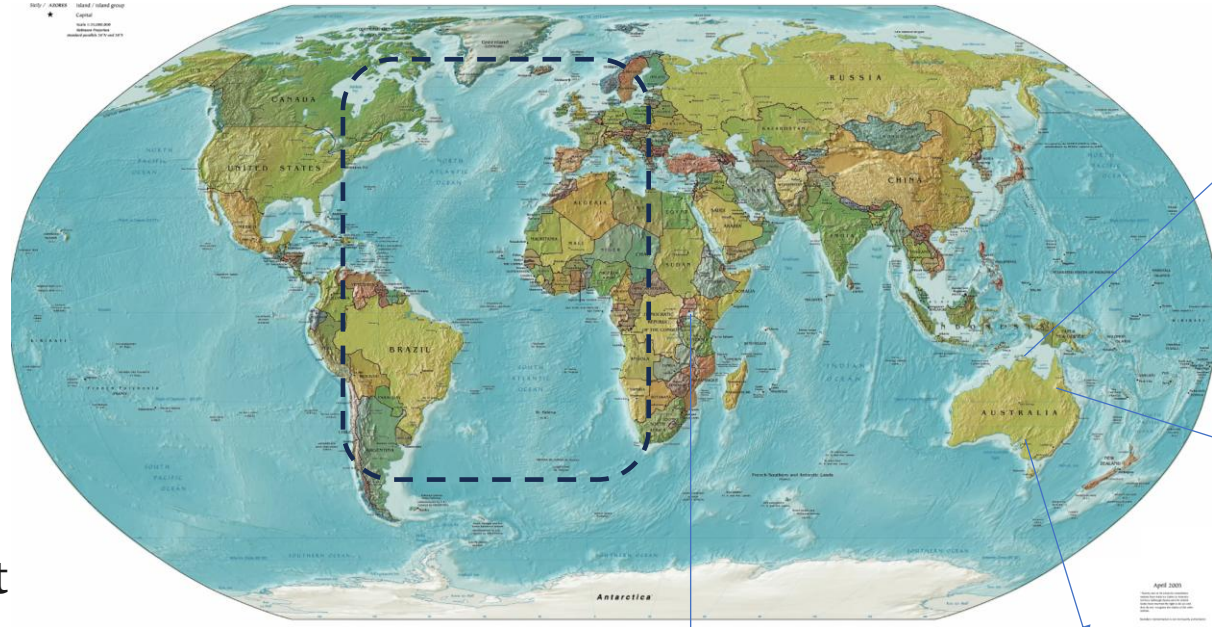


The expansion of the
Universe decelerates.

Causality problems usually indicate big discoveries!



**Alfred Wegener:
Continental drift
hypothesis**



Animals with
brood pouch
育儿袋



ostrich



emu

Causality problems usually indicate big discoveries!



**Alfred Wegener:
Continental drift
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Animals with
brood pouch
育儿袋

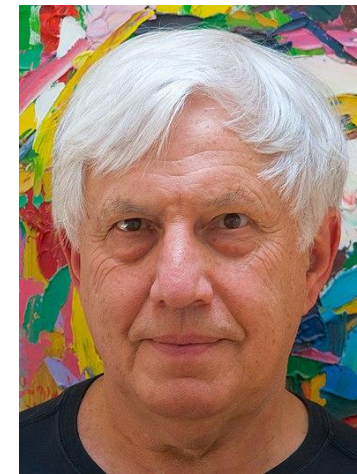
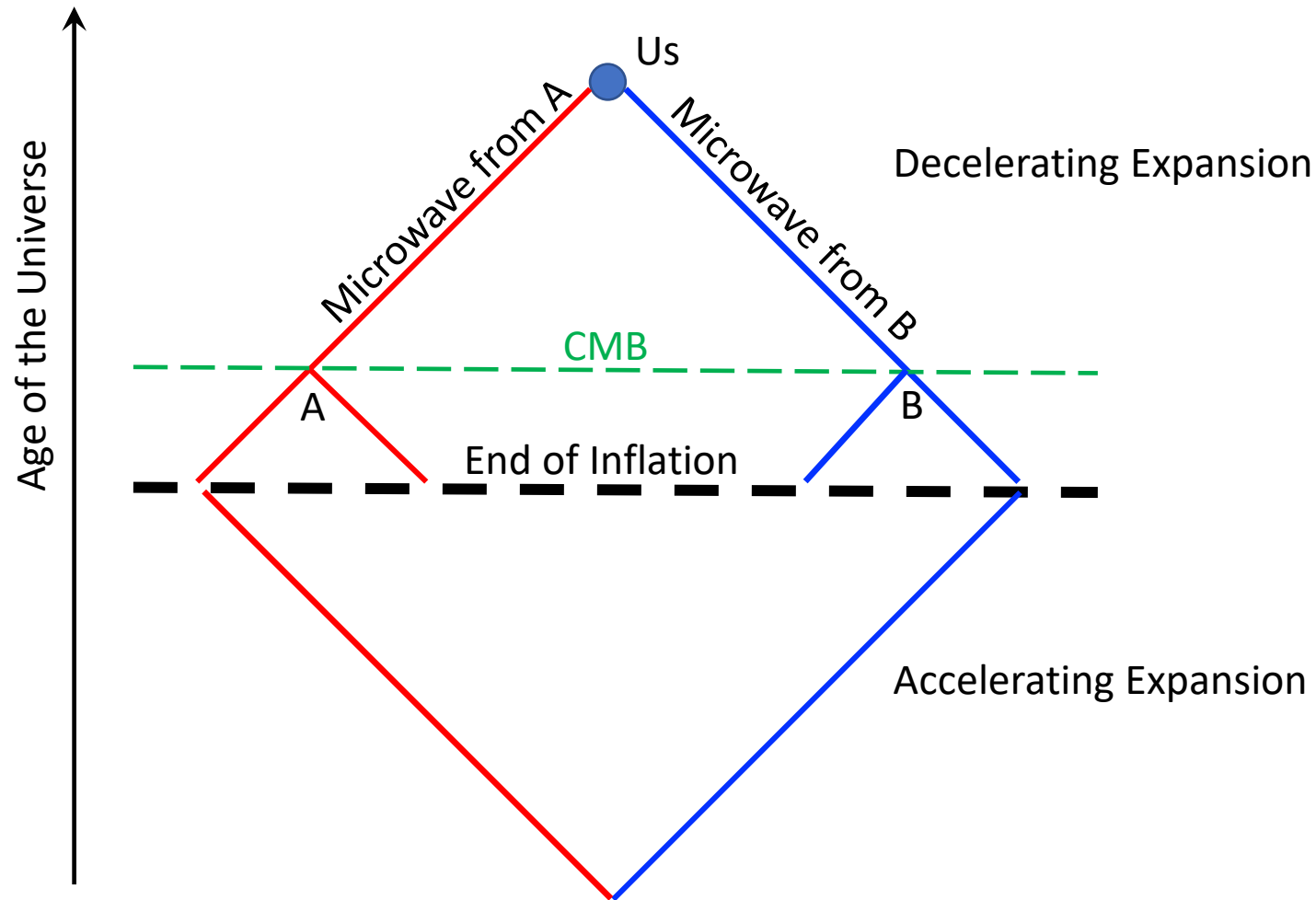


ostrich



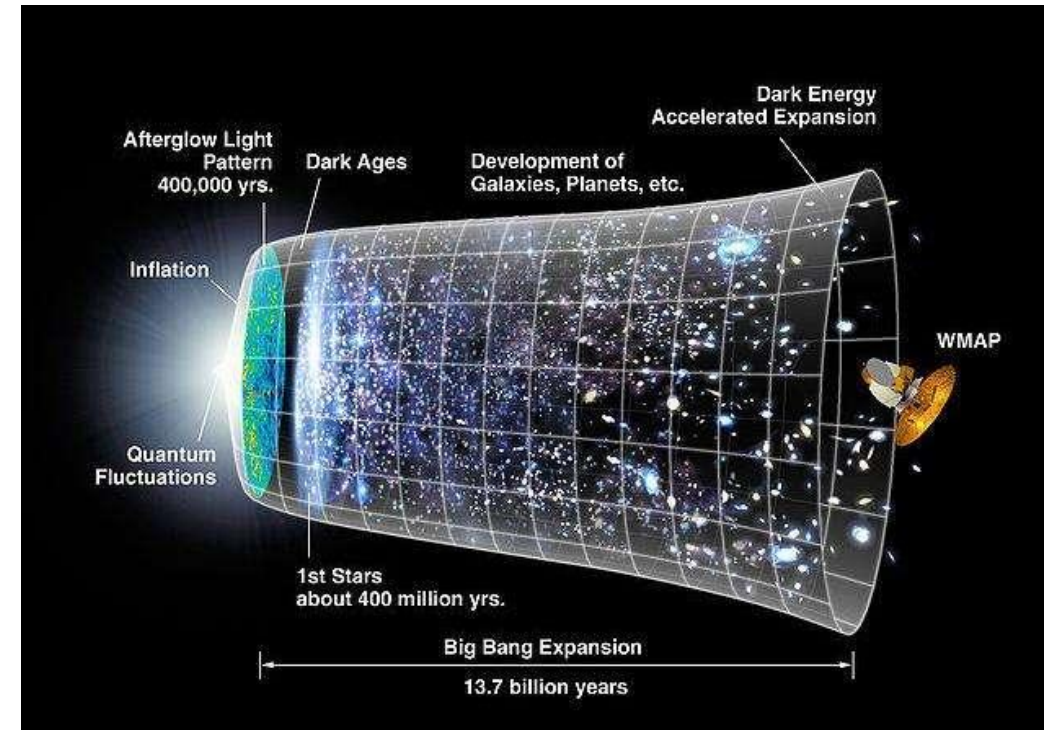
emu

Inflation theory

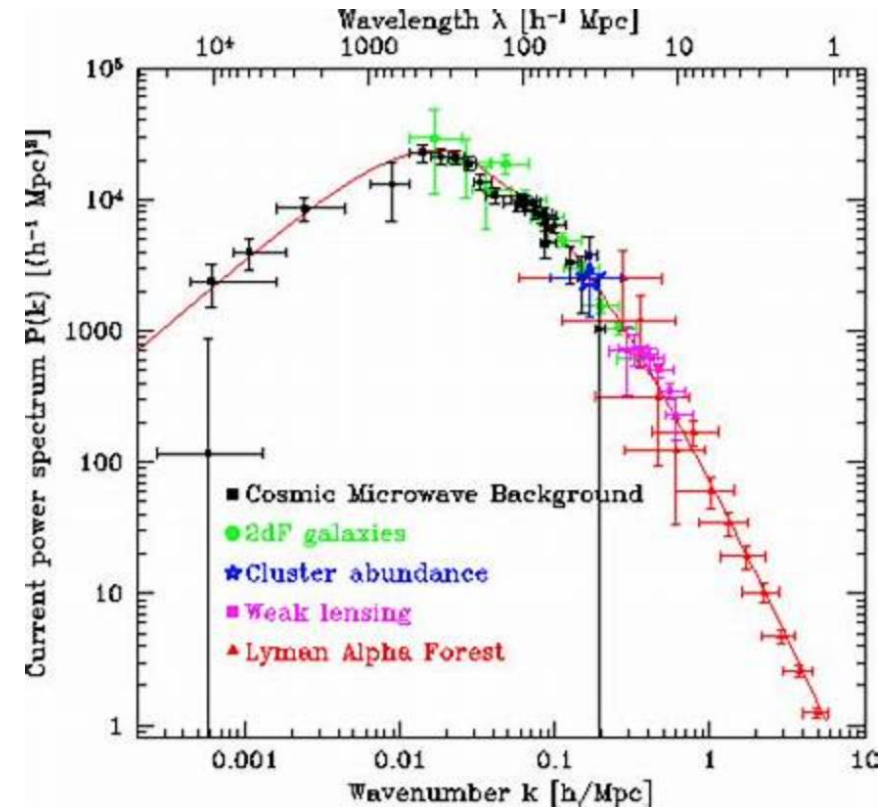
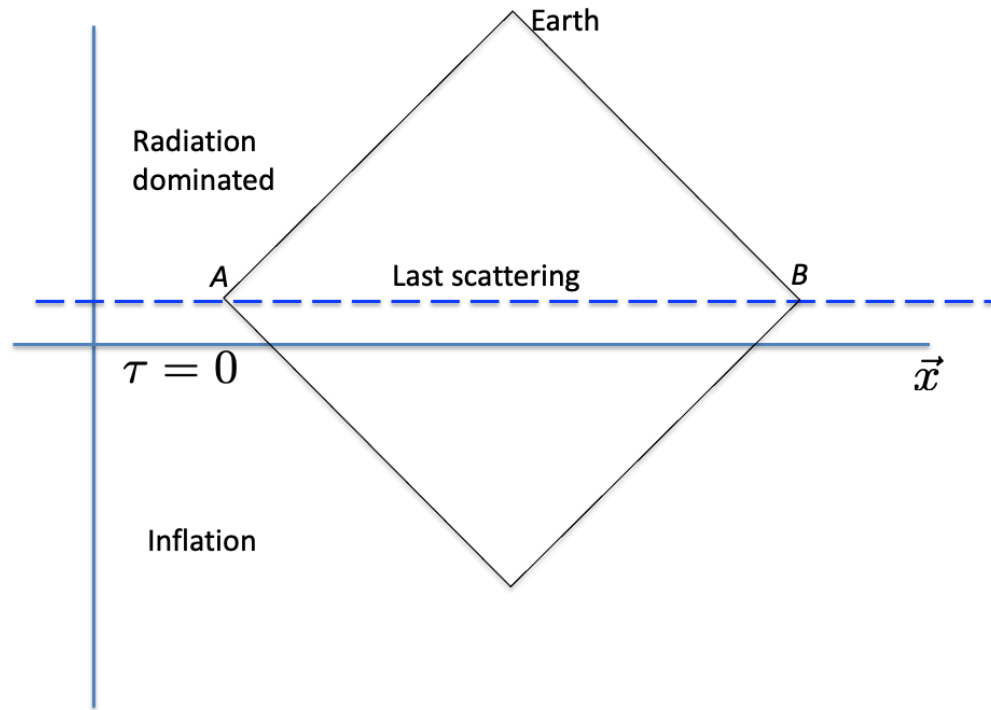


Very brief introduction of inflation

1. Solves the causality problem
2. Solves the flatness problem
3. Solves the magnetic monopole problem
4. Generates the seed of large scale structure

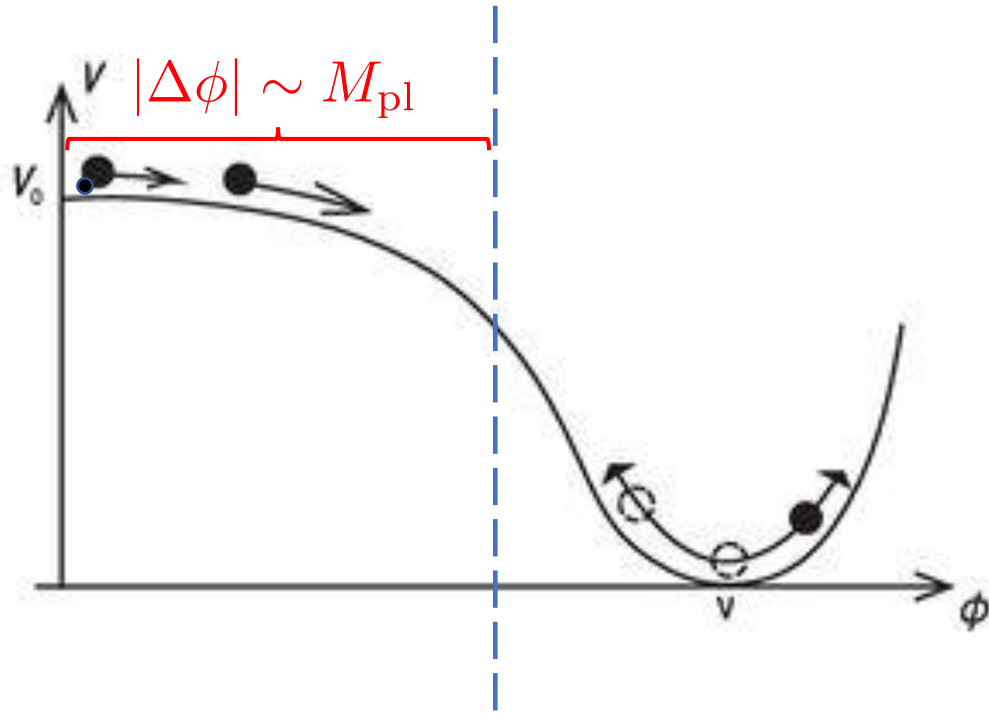


Very brief introduction of inflation



- To solve the problems, 40 to 60 e-folds is required, BUT we can only observe ten!

Slow roll inflation



- The large potential energy of the inflaton field drives the accelerating expansion of the Universe.
 - Hubble is almost a constant.
 $H = \dot{a}/a, a \sim e^{Ht}$
 - The inflaton field slowly rolling down the potential hill.
-
- To generate enough e-folds, the excursion of the inflaton field must be very large, comparable or even larger than the M_{pl} .

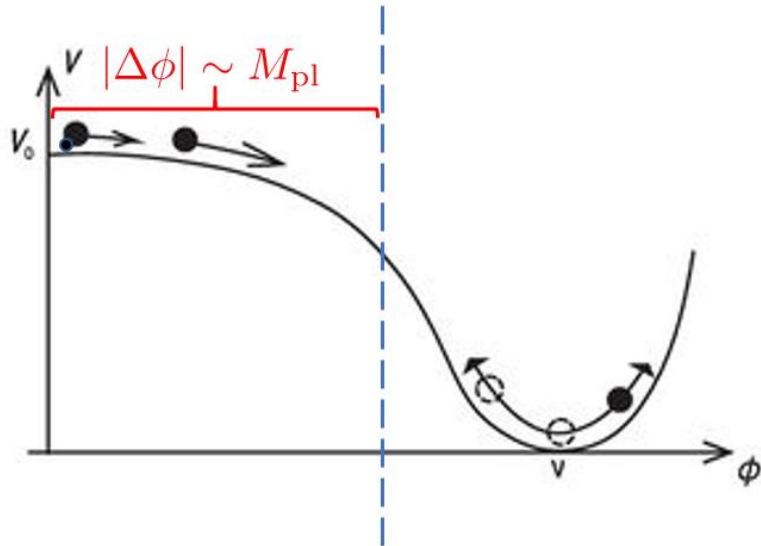
Phase transitions in spectator sector triggered by the evolution of the inflaton field

- ϕ : inflaton field

σ : spectator field

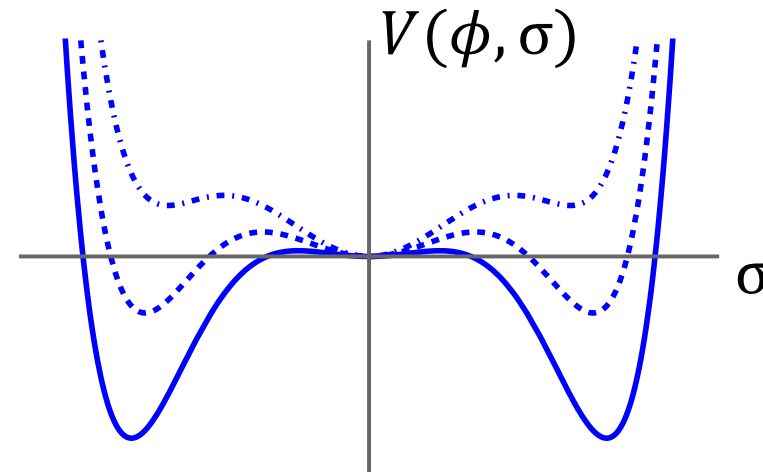
Example 1:

$$V_1(\phi, \sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$



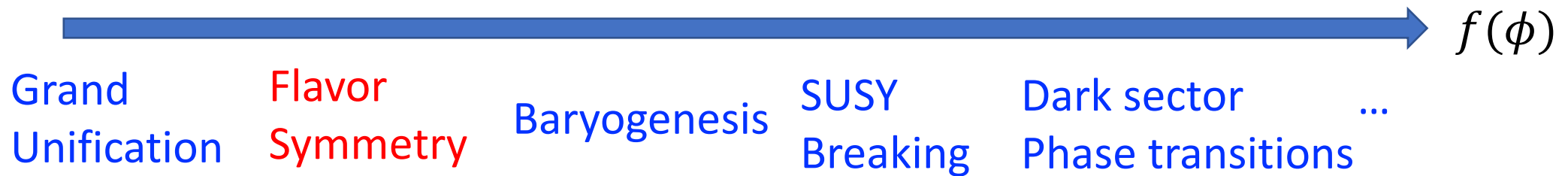
Example 2:

$$\mathcal{L}_\sigma = -\left(1 - \frac{c^2\phi^2}{\Lambda^2}\right)\frac{1}{4g^2}G_{\mu\nu}^a G^{a\mu\nu}$$



Evolution in the early universe

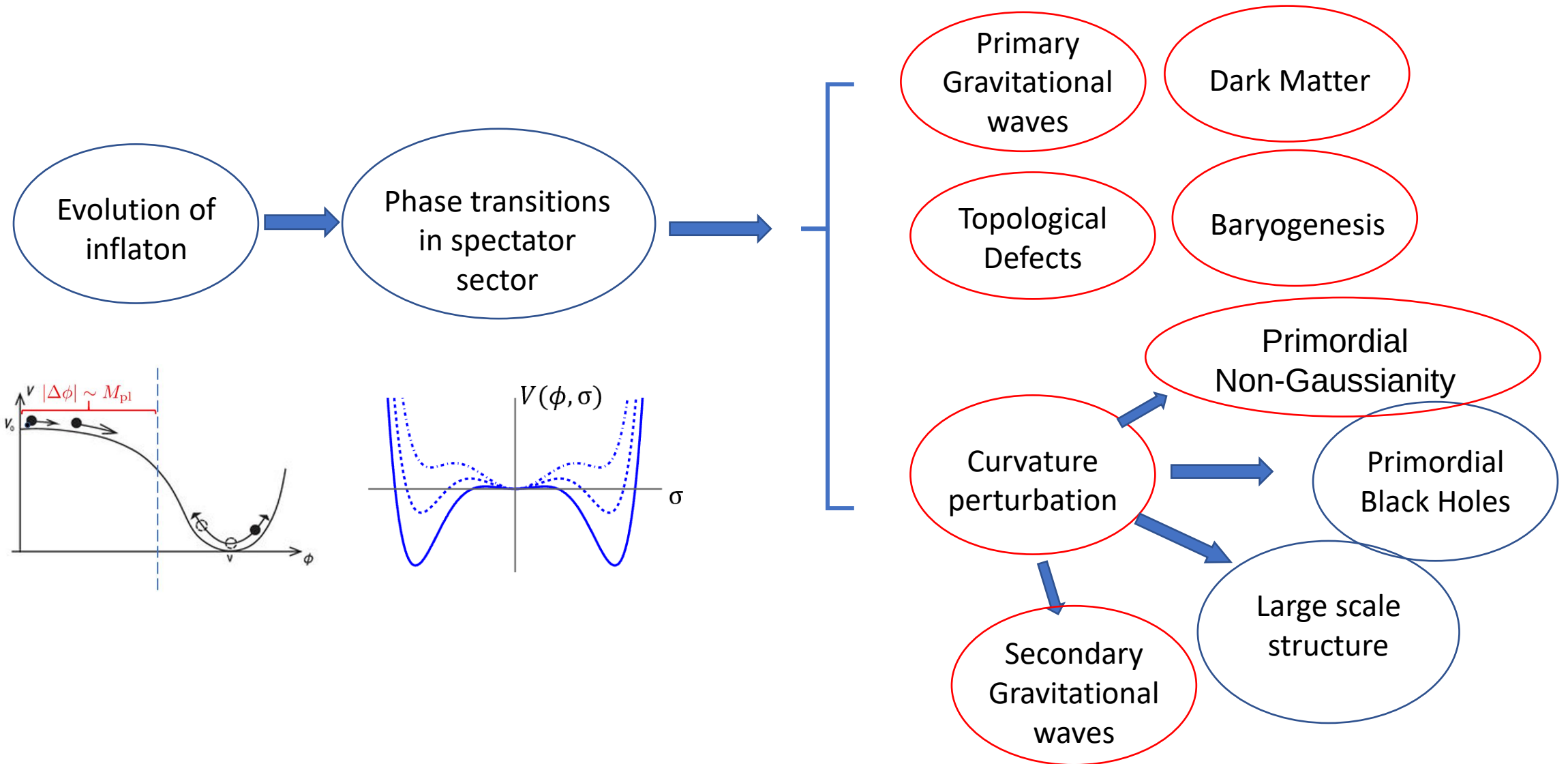
- Inflation: ϕ coupled to spectator sectors $f(\phi)g(\sigma)$



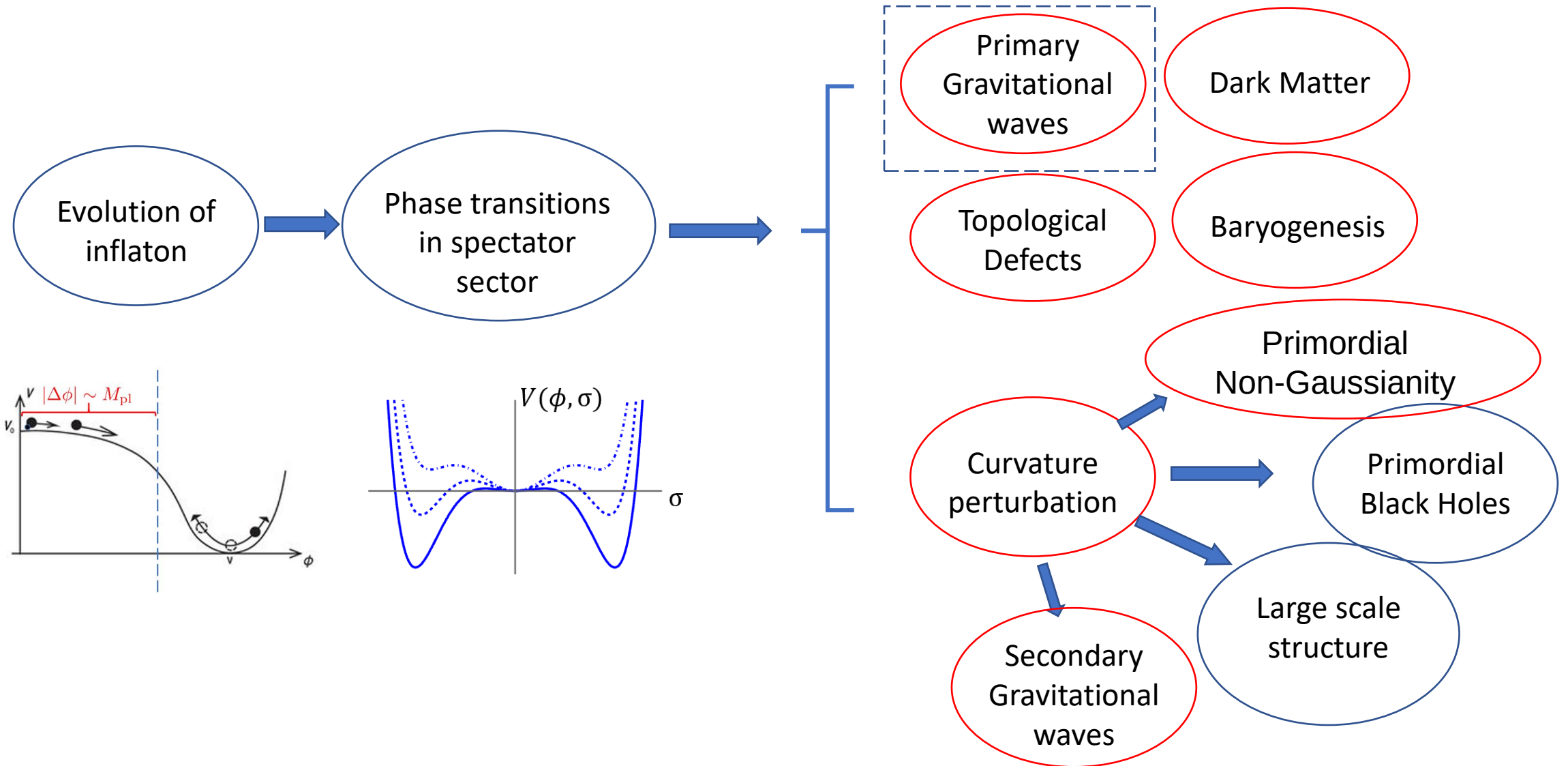
- Thermal expansion: temperature coupled to SM sector $T^2|H^2|$



Consequences of the phase transitions



Consequences of the phase transitions



GWs from first-order phase transitions during inflation

- Cosmology 101

Robertson-Walker Metric with conformal time and flat space

$$ds^2 = -a^2(d\tau^2 - dx^2 - dy^2 - dz^2)$$

Perturbation of the metric

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$$

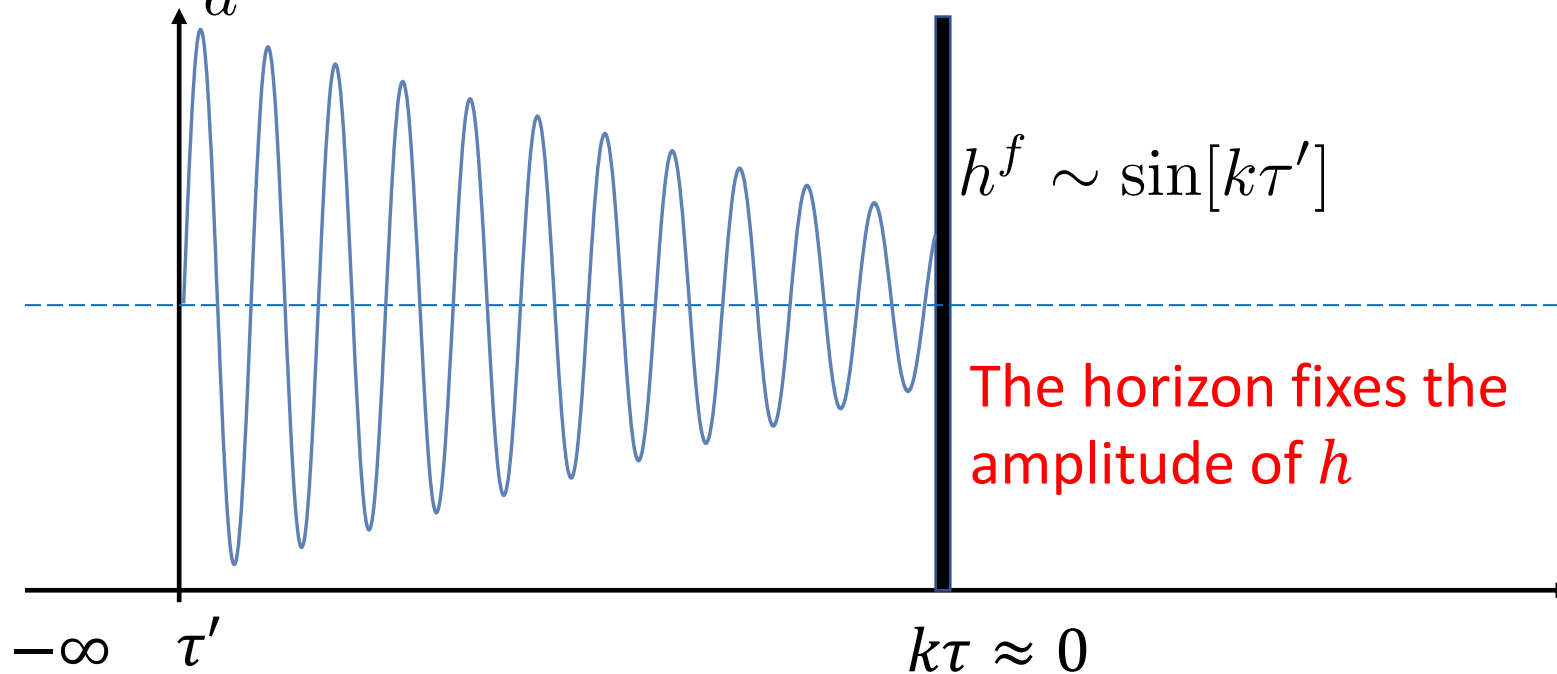
GWs are in the traceless and transverse part of h_{ij} .

GWs from first-order phase transitions during inflation

- How to calculate GWs?
- In E&M: $\partial_\mu F^{\mu\nu} = J^\nu$
 - We solve the Green's function first.
 - We convolve the Green's function with the source.
- In GR: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$
 - We linearize the Einstein equation: $g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$. GW is h_{ij}^{TT} .
 - We solve the Green's function first. (instantaneous and local source)
 - We convolve the Green's function with the source.

GWs from first-order phase transitions during inflation

- $$h''(\tau, \mathbf{k}) + \frac{2a'}{a}h'(\tau, \mathbf{k}) + k^2h(\tau, \mathbf{k}) = 16\pi G_N a^{-1}T\delta(\tau - \tau')$$



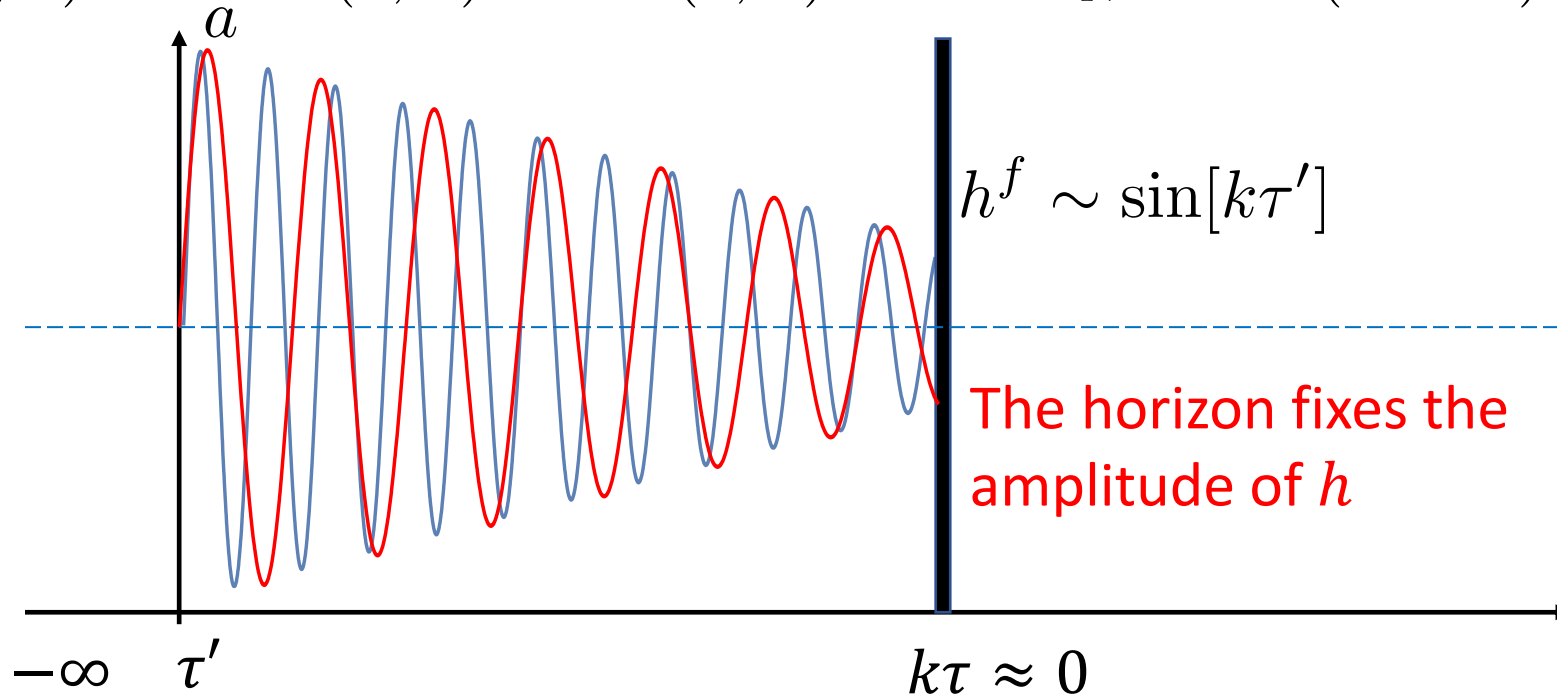
$$a = e^{Ht} = -\frac{1}{H\tau}$$

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$$

GW is h_{ij}^{TT} .

GWs from first-order phase transitions during inflation

- $$h''(\tau, \mathbf{k}) + \frac{2a'}{a}h'(\tau, \mathbf{k}) + k^2h(\tau, \mathbf{k}) = 16\pi G_N a^{-1}T\delta(\tau - \tau')$$



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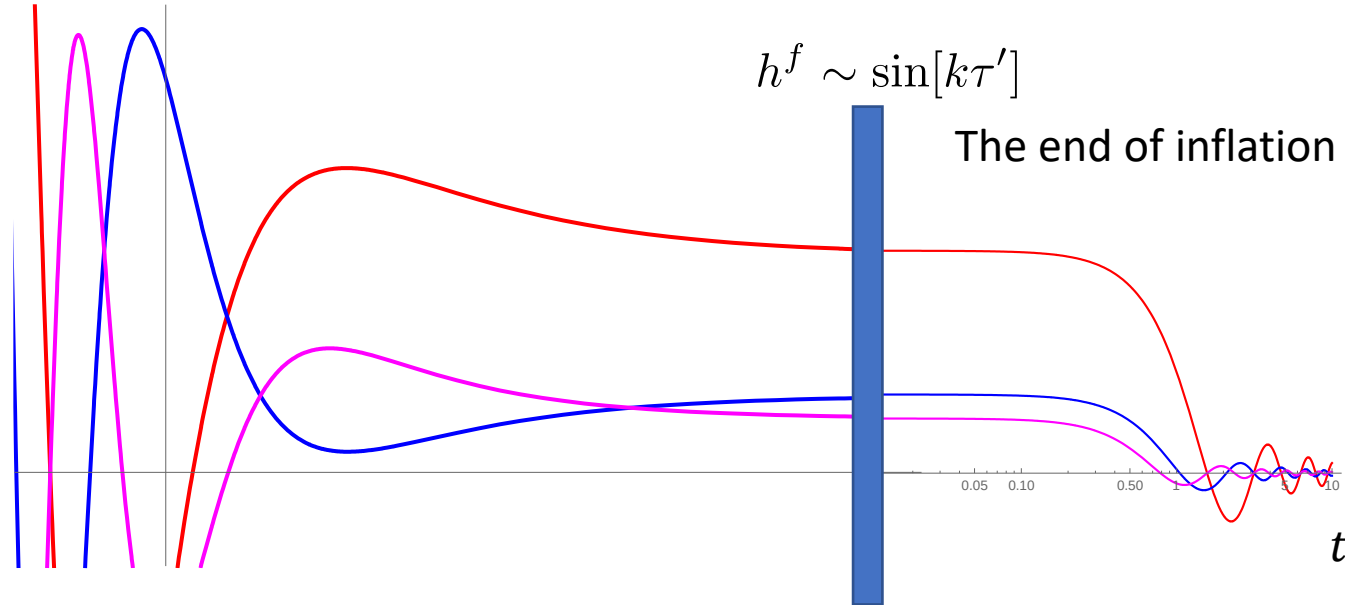
$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$$

GW is h_{ij}^{TT} .

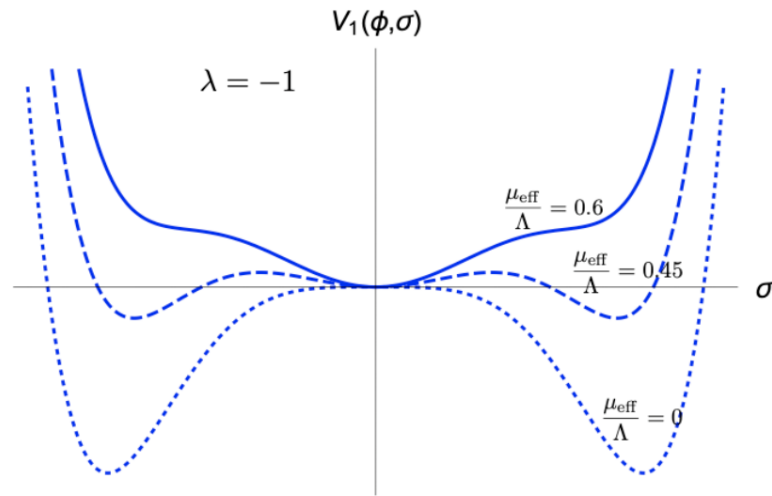
After inflation

- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$.

$$a = e^{Ht} = -\frac{1}{H\tau}$$

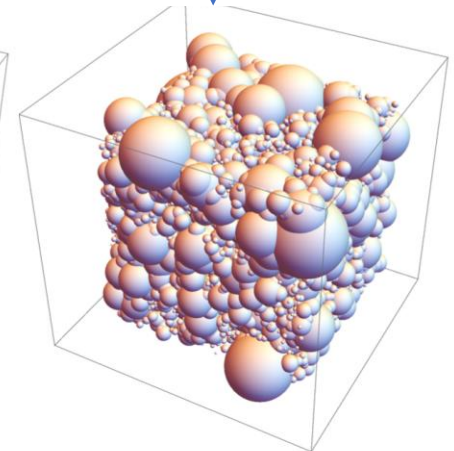
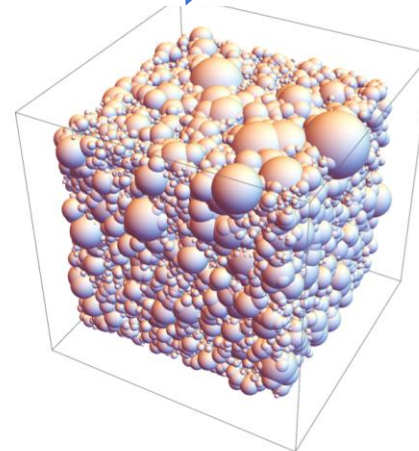
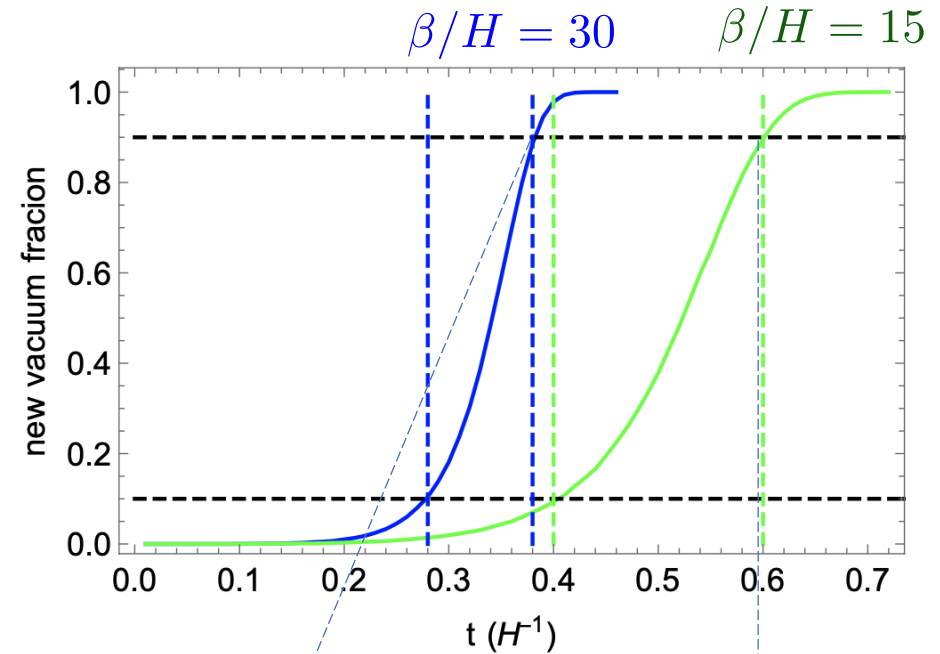


First-order phase transition during inflation

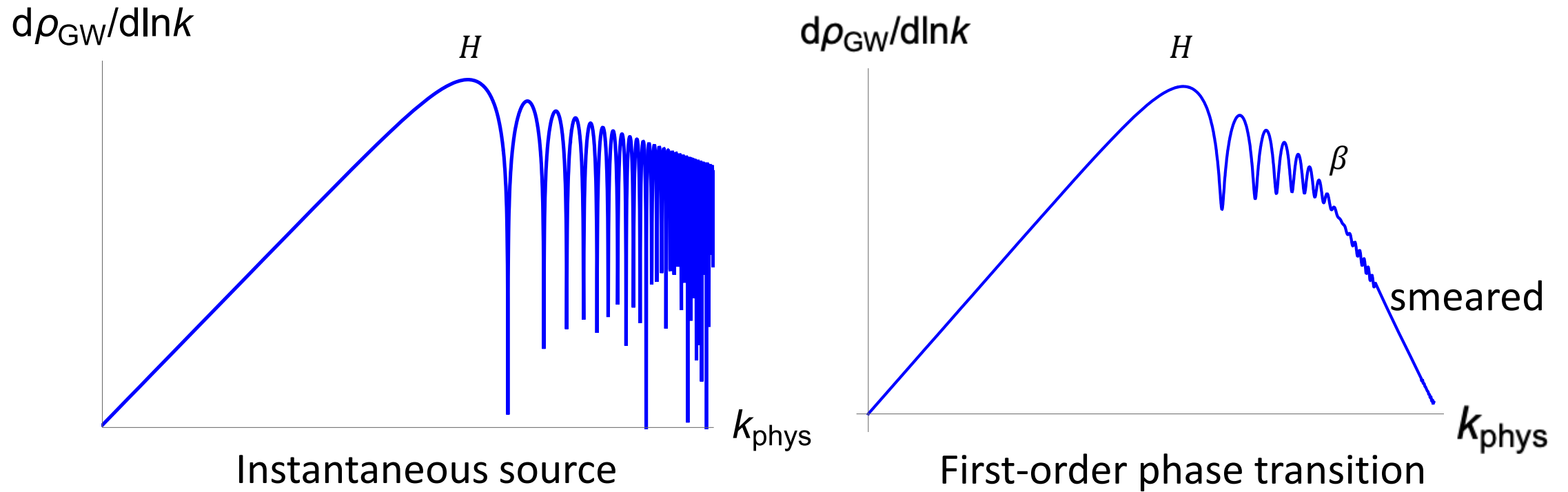


S_4 becomes smaller during

- $\beta = -\frac{dS_4}{dt}$, determines the rate of the phase transition.
- Phase transition completes if $\beta \gg H$.

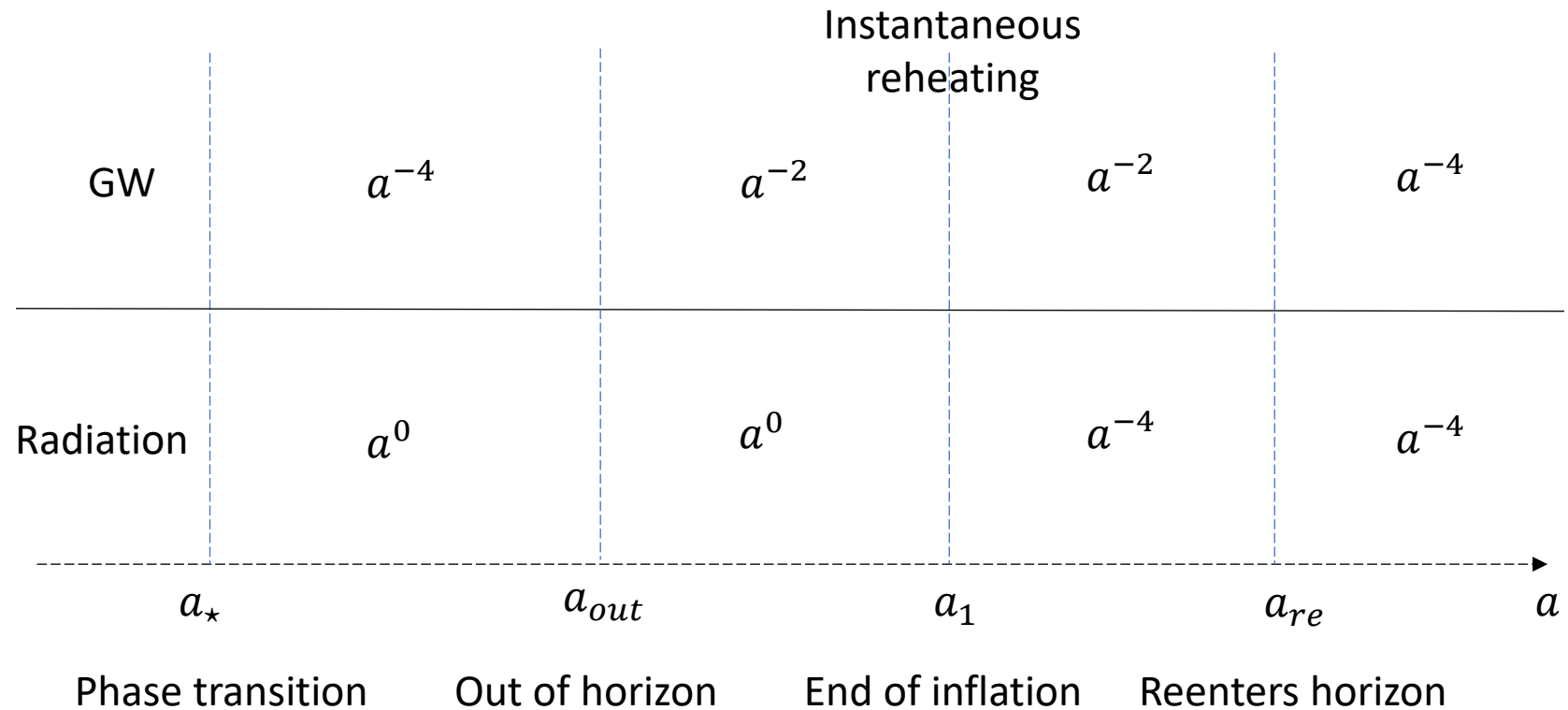


Spectrum of GW from a real source

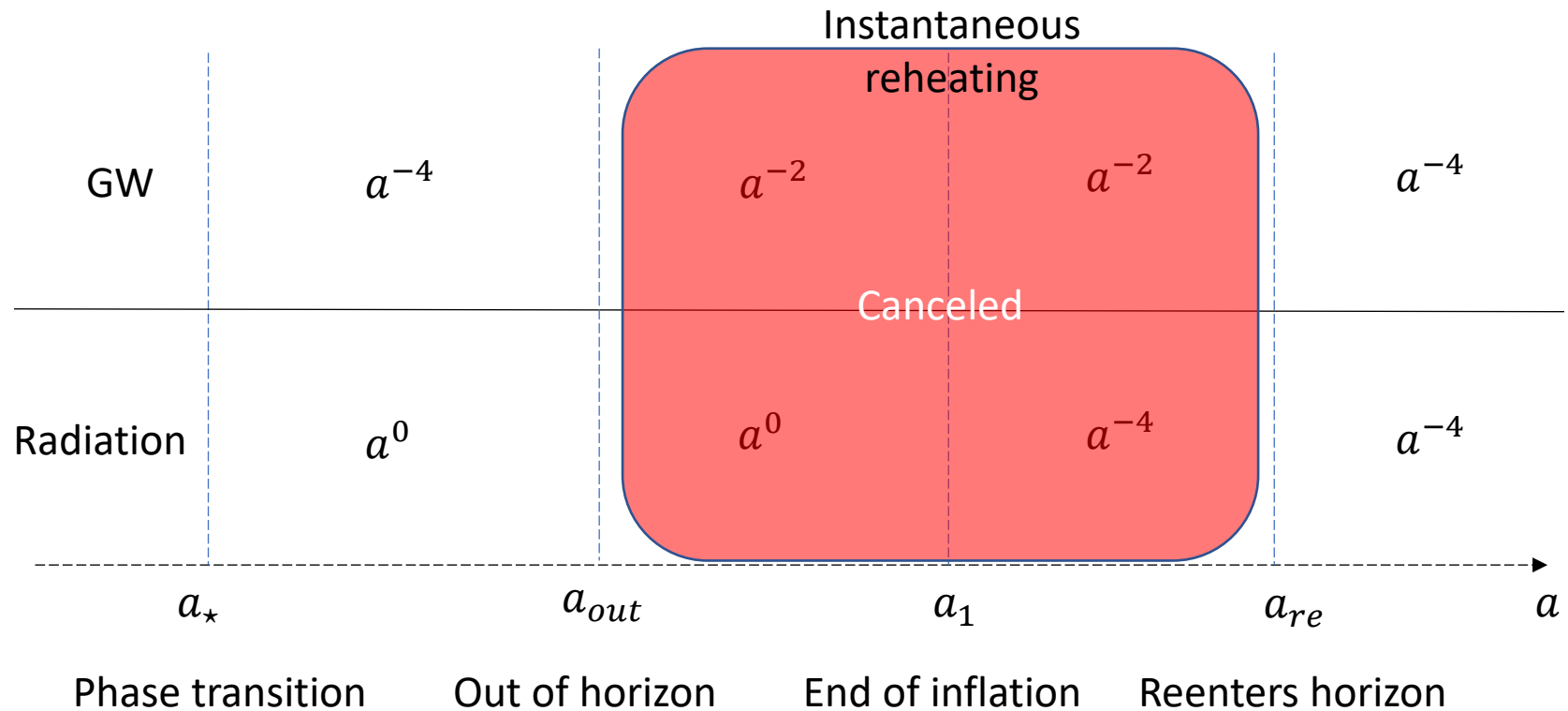


$$\beta = -\frac{dS_b}{dt} \gg H$$

Redshifts of the GW signal

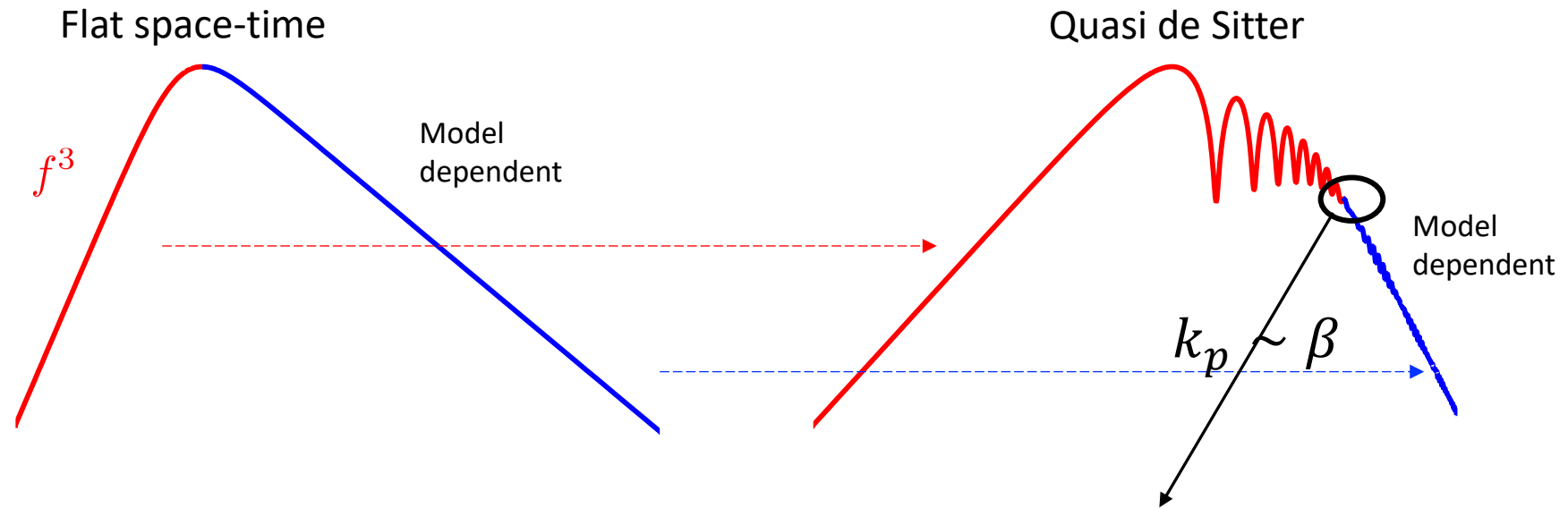


Redshifts of the GW signal



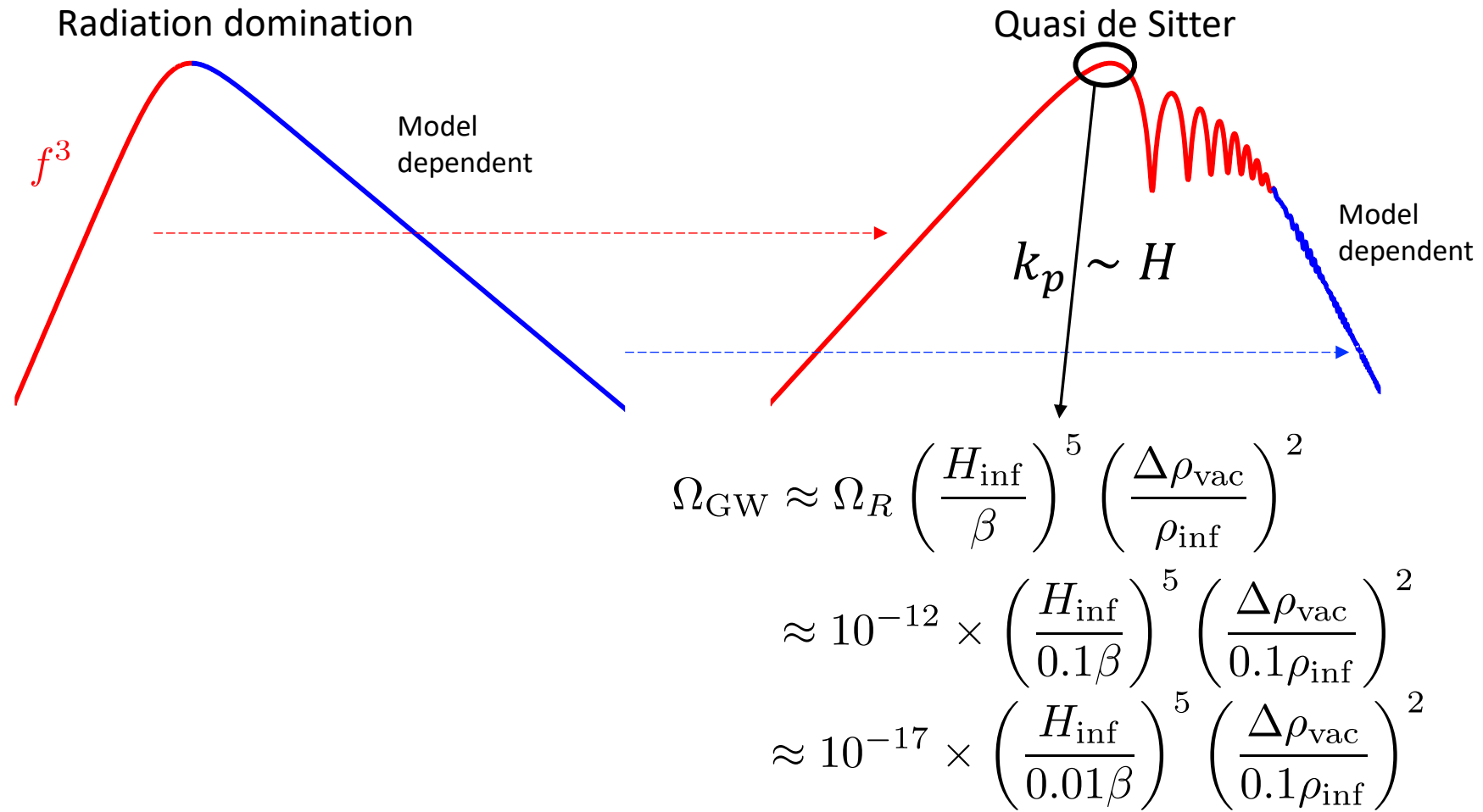
$$\frac{\Omega_{\text{GW}}}{\Omega_{\gamma}} \sim \left(\frac{a_{*}}{a_{\text{out}}} \right)^4 \sim \left(\frac{H}{\beta} \right)^4$$

Spectrum distortion by inflation



$$\Omega_{\text{GW}} \approx \Omega_R \left(\frac{H_{\text{inf}}}{\beta} \right)^6 \left(\frac{\Delta \rho_{\text{vac}}}{\rho_{\text{inf}}} \right)^2$$

Spectrum distortion by inflation

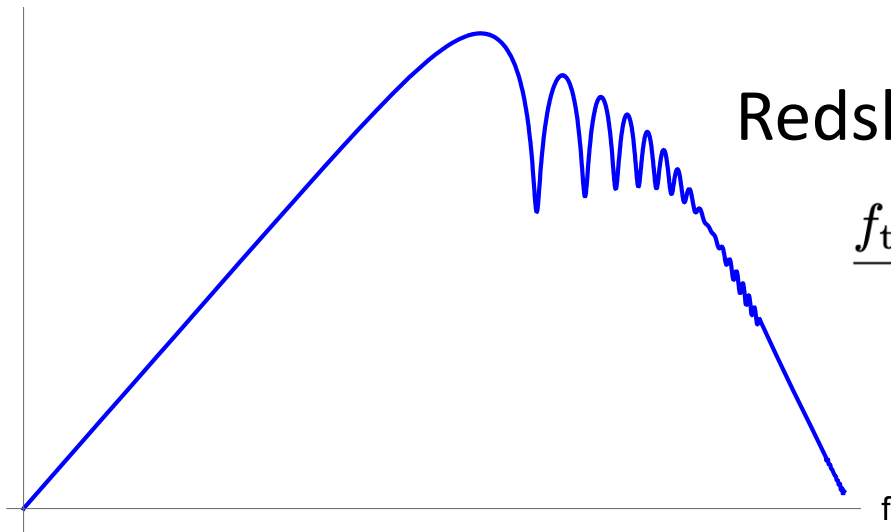


First-order phase transition during inflation

- Assume quasi-dS inflation, RD re-entering, and fast reheating

$$\Omega_{\text{GW}}(k_{\text{today}}) = \Omega_R \frac{H_{\text{inf}}^4}{k_p^4} \left[\frac{1}{2} + \mathcal{S}(k_p \beta^{-1}) \cos \left(\frac{2k_p}{H_{\text{inf}}} \right) \right] \left(\frac{\Delta \rho_{\text{vac}}}{\rho_{\text{inf}}} \right)^2 \frac{d\rho_{\text{GW}}^{\text{flat}}}{\Delta \rho_{\text{vac}} d \log k_p}$$

$\frac{d\rho}{d \log k}$



Dilution factor

Smearing

Suppressed by
the energy
faction

Redshift

$$\frac{f_{\text{today}}}{f_{\star}} = \frac{a(\tau_{\star})}{a_1} \left(\frac{g_{\star S}^{(0)}}{g_{\star S}^{(R)}} \right)^{1/3} \frac{T_{\text{CMB}}}{\left[\left(\frac{30}{g_{\star}^{(R)} \pi^2} \right) \left(\frac{3H_{\text{inf}}^2}{8\pi G_N} \right) \right]^{1/4}}$$

e^{-N_e}

N_e : e-folds before the end of inflation

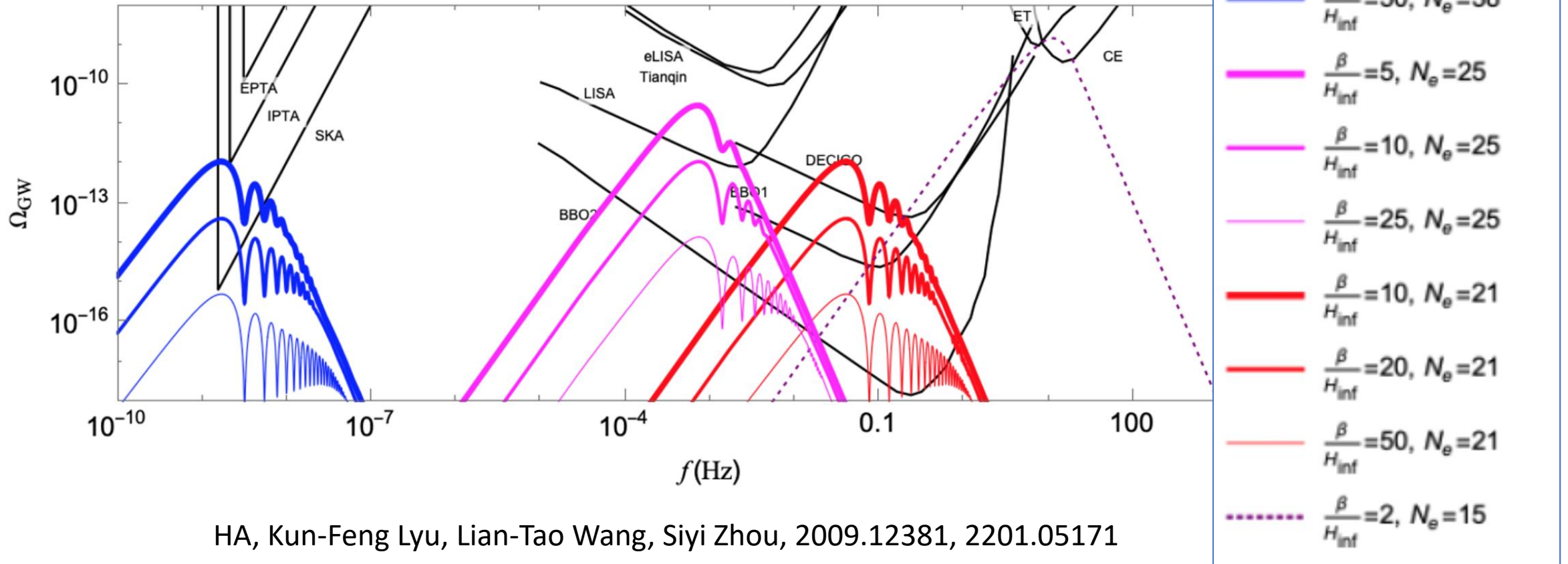
First-order phase transition during inflation

- Primordial stochastic GW signals

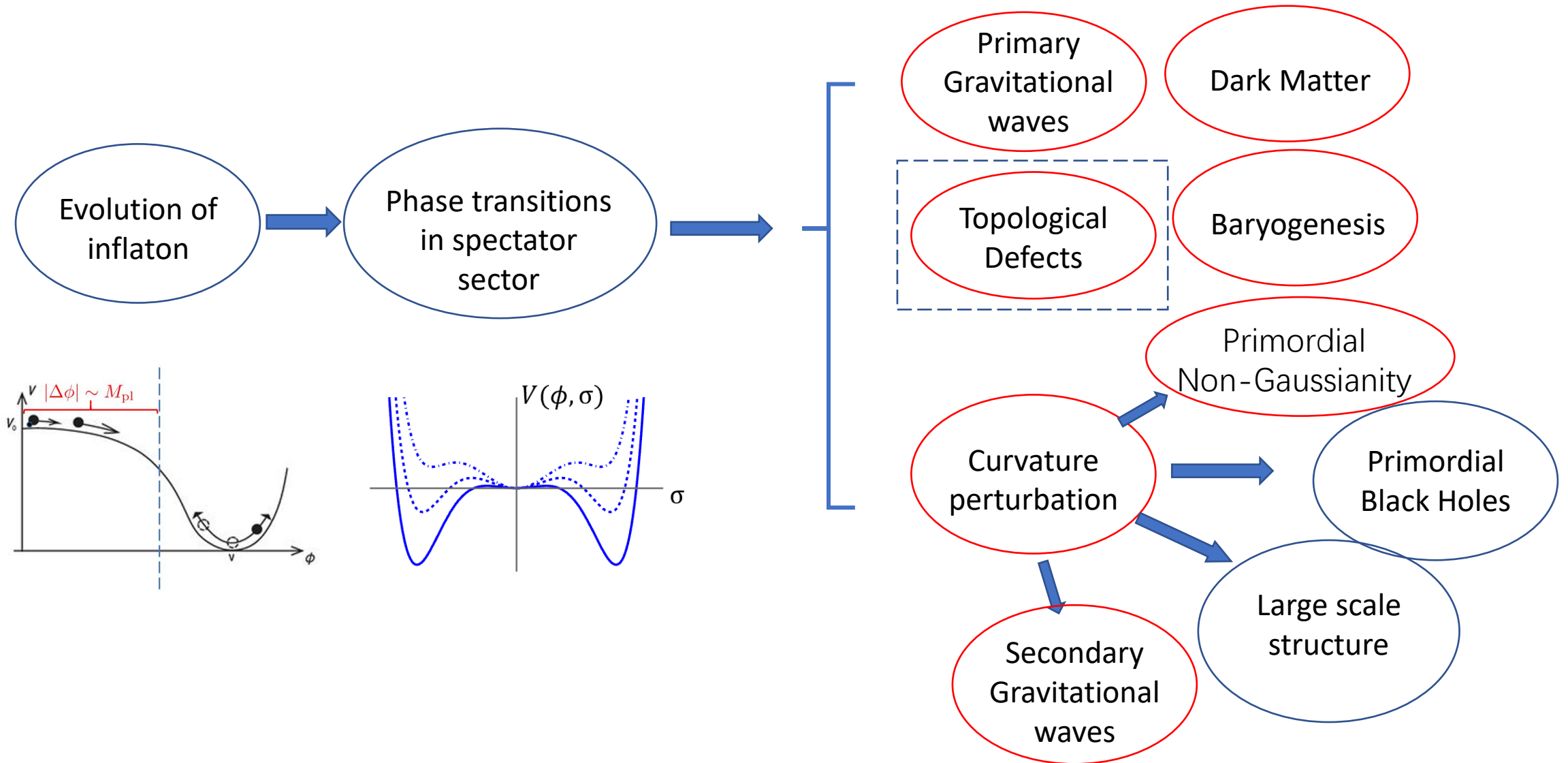
Instantaneous reheating

$$H_{\text{inf}} = 10^{12} \text{ GeV}$$

$$\Delta\rho_{\text{vac}}/\rho_{\text{inf}} = 0.3$$

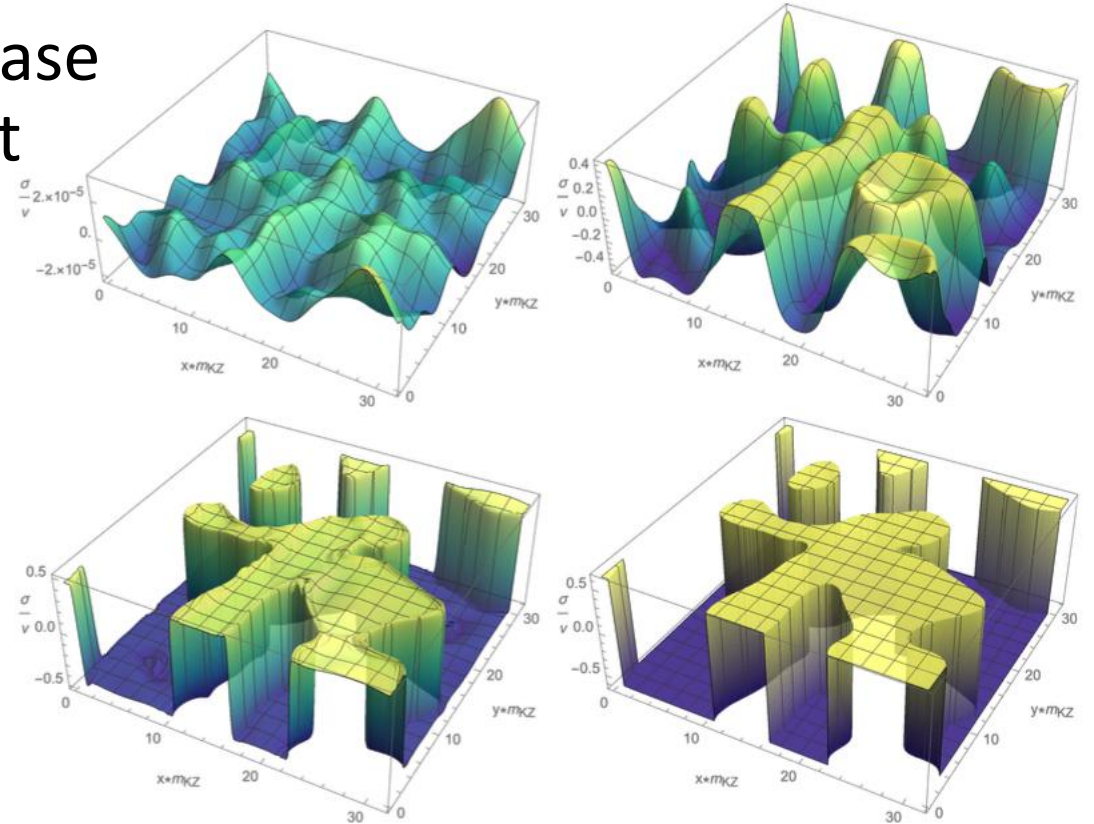


Consequences of the phase transitions



GWs from topological defects

- GWs directly from second-order phase transitions are small, usually cannot be detected.
- Phase transitions can produce topological defects:
 - Domain walls
 - Cosmic strings
 - Monopoles



Formation of domain walls

- Tachyonic growth

$$V_{\text{KZ}} = -\frac{1}{2}m_{\text{KZ}}^3 a_c^{-1}(\tau - \tau_c)\sigma^2 + \frac{\lambda}{4}\sigma^4$$

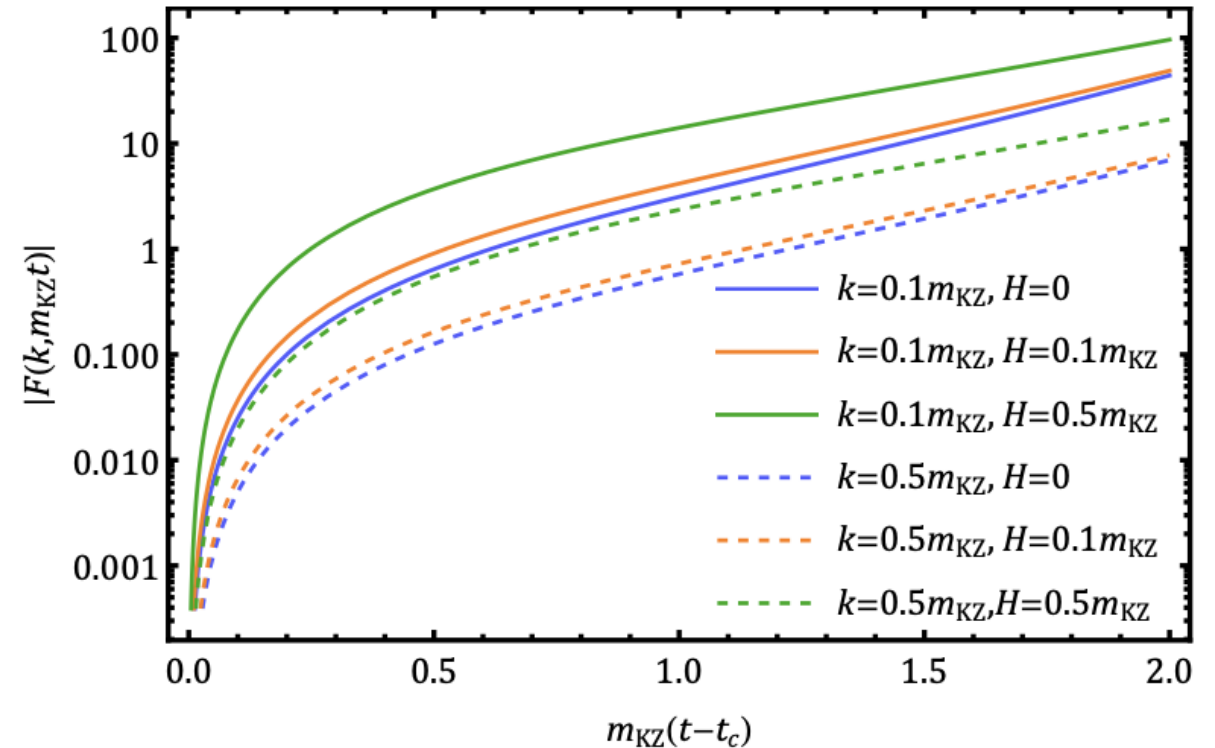


$$\sigma_{\mathbf{k}}'' + \frac{2a'}{a}\sigma_{\mathbf{k}}' + \omega_{\mathbf{k}}^2(\tau)\sigma_{\mathbf{k}} = 0$$



$$k^2 - a_c^2 m_{\text{KZ}}^3(\tau - \tau_c) + \frac{\lambda}{2}\langle\sigma^2(\tau, \mathbf{x})\rangle$$

$\omega_k^2 < 0$ for small k around τ_c .



$F(k, m_{\text{KZ}}t)$ can be seen as the occupation number in the k mode.

Formation of domain walls

- Matching to classical nonlinear evolution

Quantum
ensemble



Classical
ensemble

$$\begin{aligned}\tilde{\pi}(\mathbf{k}, \tau) &= a_{\mathbf{k}} a(\tau)^2 f'(k, \tau) + a_{-\mathbf{k}}^\dagger a(\tau)^2 f'^*(k, \tau), \\ \tilde{\sigma}(\mathbf{k}, \tau) &= a_{\mathbf{k}} f(k, \tau) + a_{-\mathbf{k}}^\dagger f^*(k, \tau).\end{aligned}$$

$$F(k, \tau) = a(\tau)^2 \text{Re} [f'(k, \tau) f^*(k, \tau)]$$

$$W(\sigma_{\mathbf{k}}, \pi_{\mathbf{k}}) = \frac{1}{\pi^2} \exp \left[-\frac{|\sigma_{\mathbf{k}}|^2}{|f(\mathbf{k}, \tau)|^2} - 4|f(\mathbf{k}, \tau)|^2 \left| \pi_{\mathbf{k}} - \frac{F(\mathbf{k}, \tau)}{|f(\mathbf{k}, \tau)|^2} \sigma_{\mathbf{k}} \right|^2 \right]$$

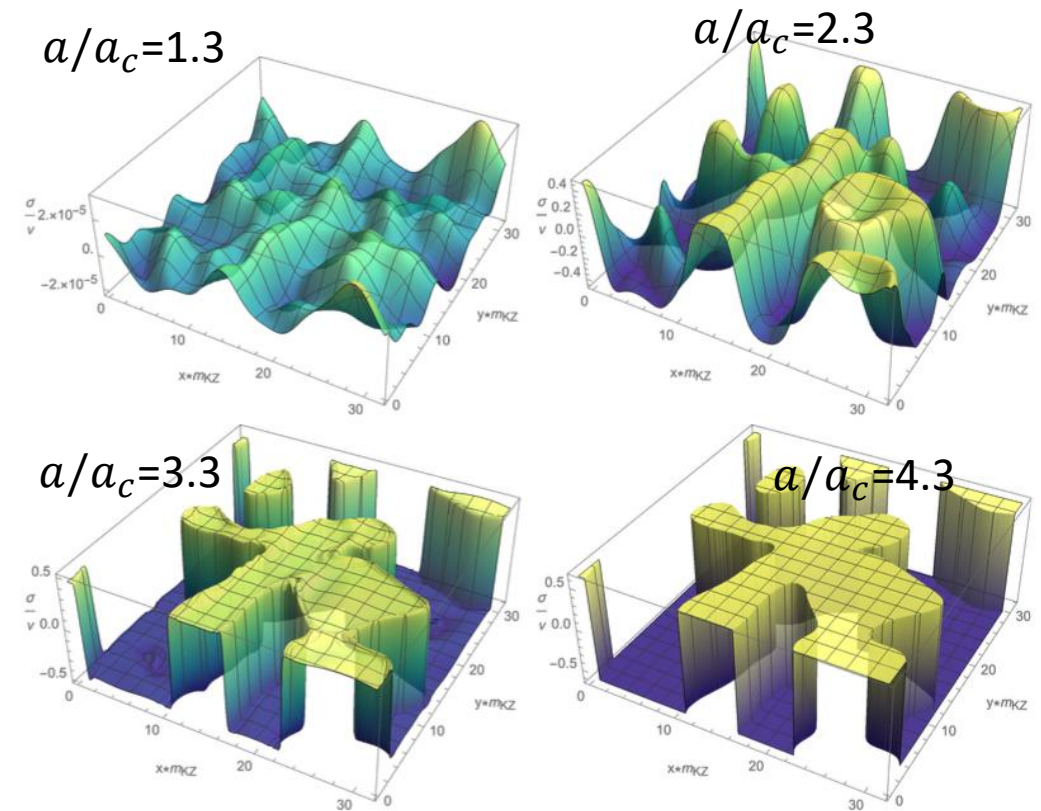
We randomly generate the σ_k and π_k according to W as the initial condition for classical lattice simulation.

Polarski and Starobinsky 1996,
Lesgourgues, Polarski and Starobinsky, gr-qc/9611019
Kiefer, Polarski and Starobinsky, gr-qc/9802003

...

Formation of domain walls

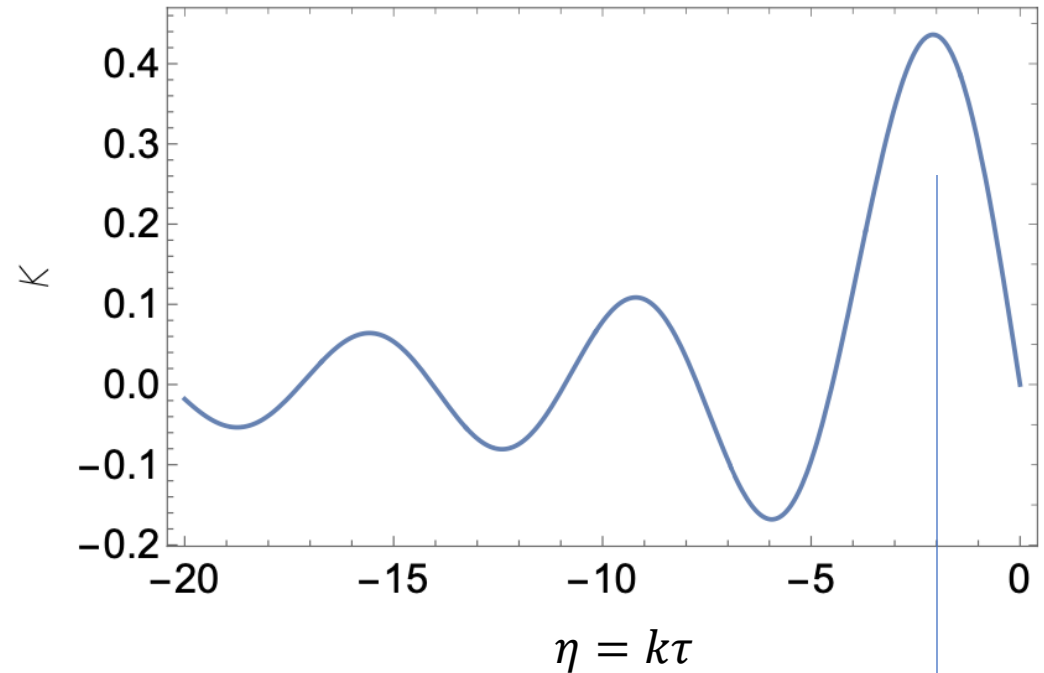
- Symmetry breaking via a **second order phase transition**.
- We numerically solve the nonlinear evolution of σ field with $1000 \times 1000 \times 1000$ lattice.
- At the beginning there are fluctuations, dying out after a few e-folds.
- The configuration becomes comovingly static after a few e-folds.



Calculation of GWs

- In Minkowski spacetime, static source cannot radiate due to energy-momentum conservation.
- During inflation, energy conservation is badly broken, so the even static source can produce GWs.

$$\tilde{h}_{ij}^f(\mathbf{k}) = \frac{16\pi G_N}{k} \int_{-\infty}^0 d\tau' \mathcal{K}(k\tau') \tilde{T}_{ij}^{TT}(\tau', \mathbf{k})$$



Numerical results for GWs

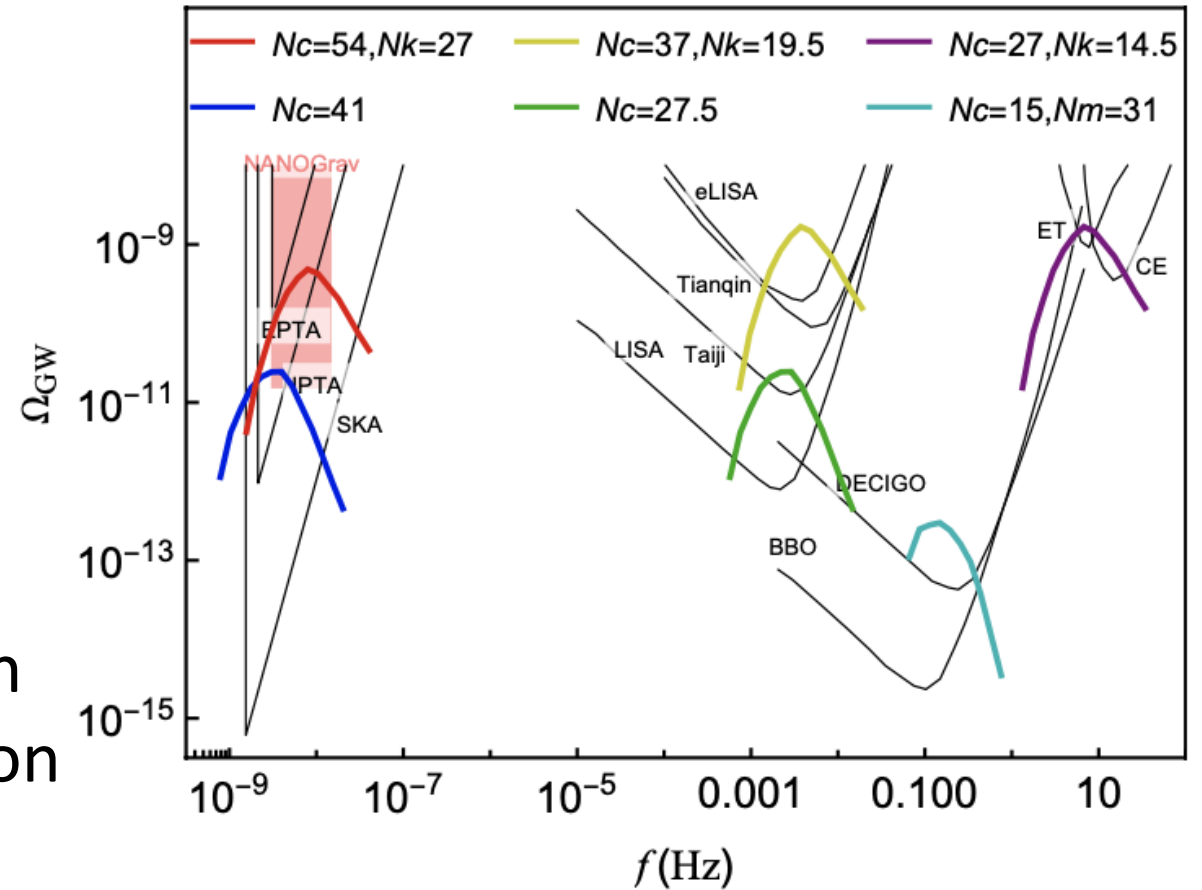
HA, Chen Yang, 2304.02361

$$\Omega_{\text{GW}}(f) = \Omega_R \times \rho_R^{-1} \left. \frac{d\rho_{\text{GW}}}{d \ln f} \right|_{\text{today}}$$

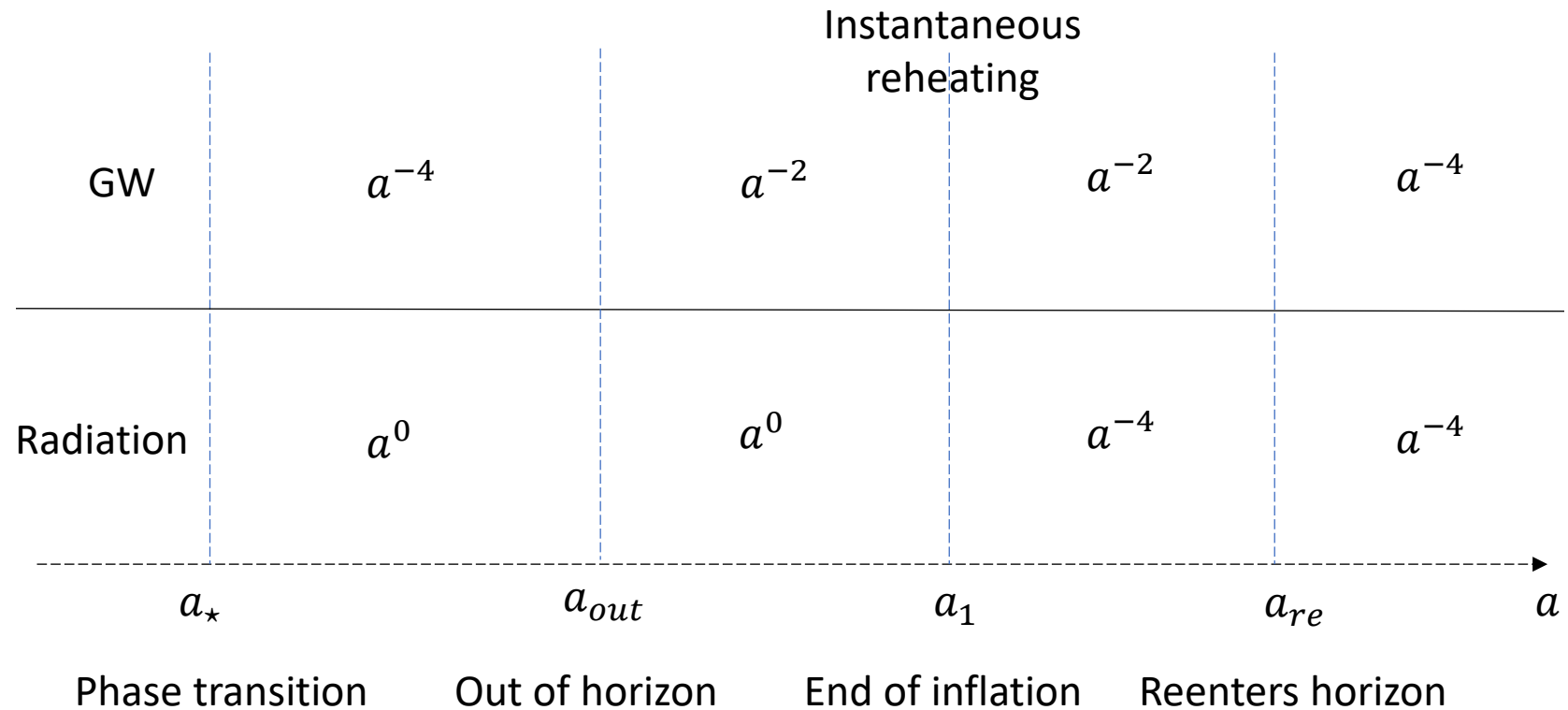
$$\frac{f_{\text{today}}}{f_{\star}} = \frac{a(\tau_{\star})}{a_1} \left(\frac{g_{\star S}^{(0)}}{g_{\star S}^{(R)}} \right)^{1/3} \frac{T_{\text{CMB}}}{\left[\left(\frac{30}{g_{\star}^{(R)}} \right) \left(\frac{3H_{\text{inf}}^2}{8\pi G_N} \right) \right]^{1/4}}$$

Intermediate stages matter:

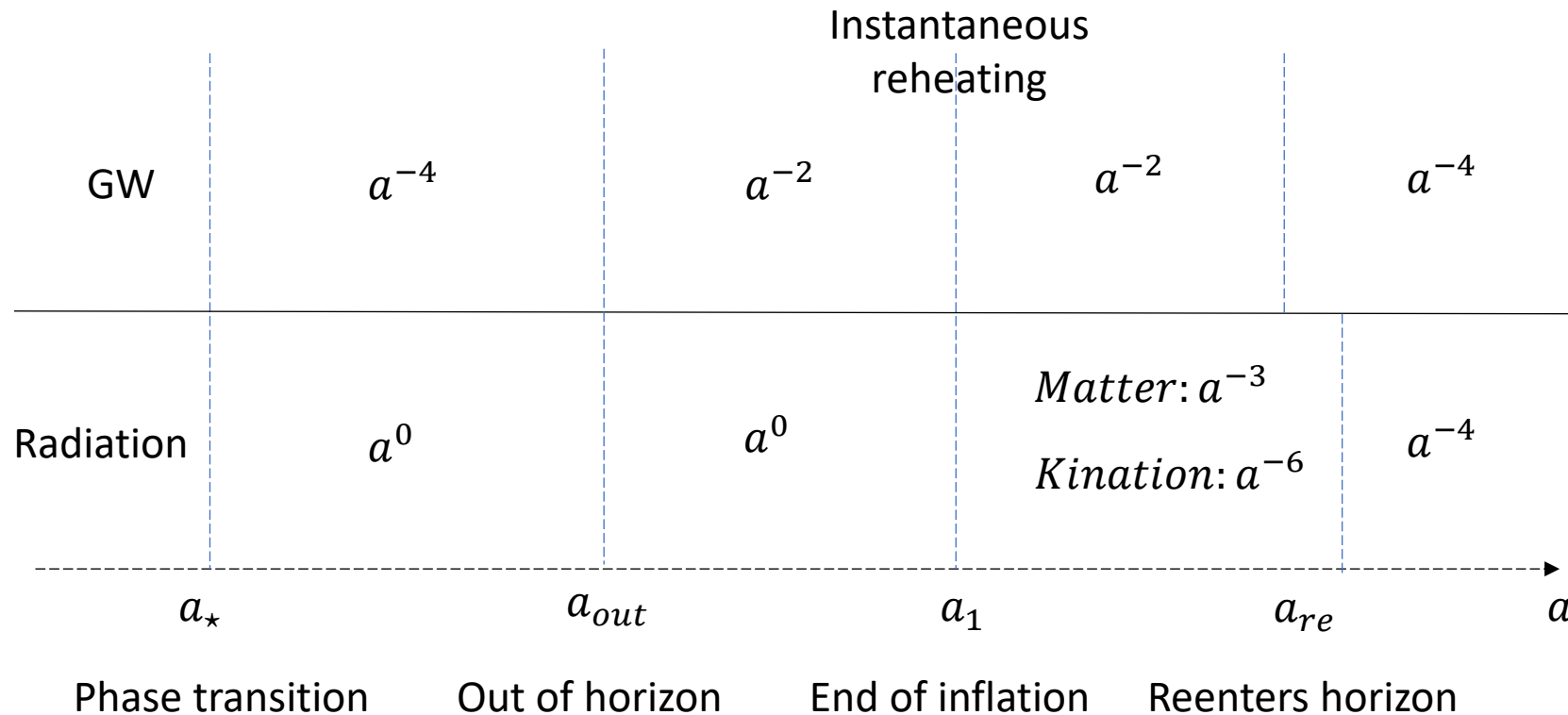
- Instantaneous reheating
- Intermediate matter domination
- Intermediate kination domination



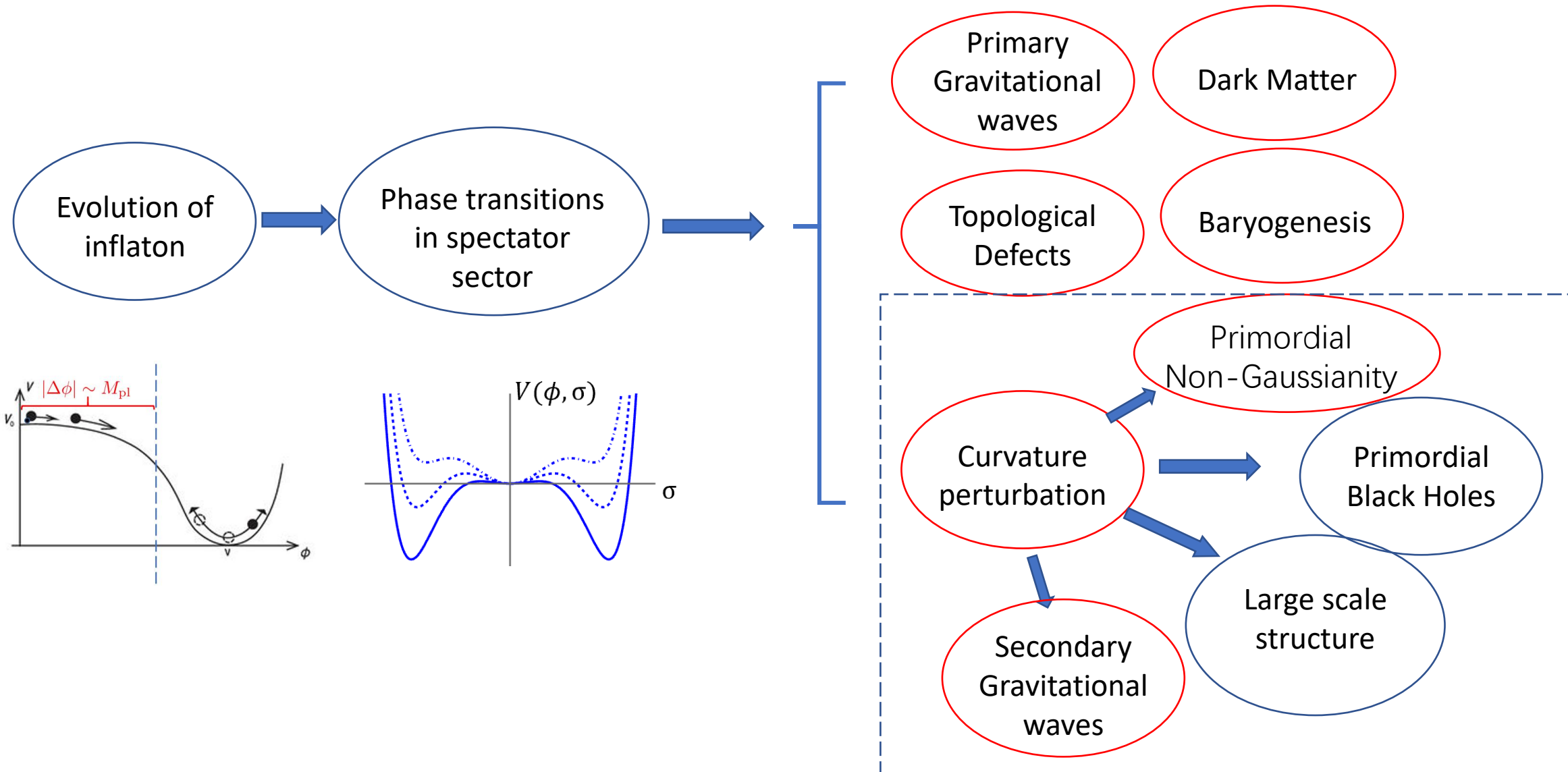
Redshifts of the GW signal



Intermediate stages between inflation and reheating



Consequences of the phase transitions



Induced classical scalar perturbation $\delta\phi$

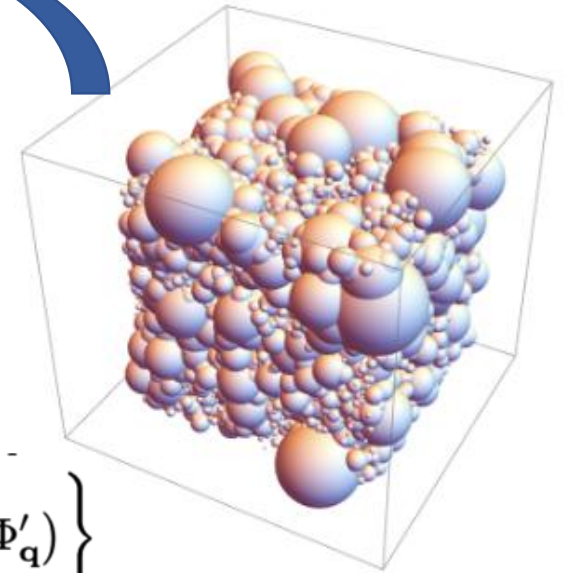
- Interactions

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\phi, \sigma)$$

$$V(\phi, \sigma) = V_0(\phi) + V_1(\phi, \sigma) \quad \xrightarrow{\phi = \phi_0 + \delta\phi} \quad \frac{\partial V_1}{\partial \phi_0} \delta\phi$$

$$\delta\tilde{\phi}''_{\mathbf{q}} - \frac{2}{\tau}\delta\tilde{\phi}'_{\mathbf{q}} + \left(q^2 + \frac{1}{H^2\tau^2} \frac{\partial^2 V_0}{\partial \phi_0^2} \right) \delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}} ,$$

$$\mathcal{S}_{\mathbf{q}} = -\frac{1}{H^2\tau^2} \left[\frac{\partial V_1}{\partial \phi} \right]_{\mathbf{q}} - \underbrace{\left\{ \frac{2\Phi_{\mathbf{q}}}{H^2\tau^2} \left(\frac{\partial V_0}{\partial \phi_0} + \left[\frac{\partial V_1}{\partial \phi} \right]_0 \right) + \frac{\dot{\phi}_0}{H\tau} (3\Psi'_{\mathbf{q}} + \Phi'_{\mathbf{q}}) \right\}}_{\text{Pure gravitational, subdominant}}$$



Pure gravitational, subdominant

Induced classical scalar perturbation $\delta\phi$

- Interactions

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\phi, \sigma)$$

$$V(\phi, \sigma) = V_0(\phi) + V_1(\phi, \sigma) \quad \xrightarrow{\phi = \phi_0 + \delta\phi} \quad \frac{\partial V_1}{\partial \phi_0} \delta\phi$$

Source term for $\delta\phi$

- The source is different from T_{ij}^{TT}
- No one has done the simulation before

$$\delta\tilde{\phi}_{\mathbf{q}}'' - \frac{2}{\tau}\delta\tilde{\phi}_{\mathbf{q}}' + \left(q^2 + \frac{1}{H^2\tau^2} \frac{\partial^2 V_0}{\partial \phi_0^2}\right) \delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}} ,$$

$$\mathcal{S}_{\mathbf{q}} = -\frac{1}{H^2\tau^2} \left[\frac{\partial V_1}{\partial \phi} \right]_{\mathbf{q}} - \underbrace{\left\{ \frac{2\Phi_{\mathbf{q}}}{H^2\tau^2} \left(\frac{\partial V_0}{\partial \phi_0} + \left[\frac{\partial V_1}{\partial \phi} \right]_0 \right) + \frac{\dot{\phi}_0}{H\tau} (3\Psi'_{\mathbf{q}} + \Phi'_{\mathbf{q}}) \right\}}_{\text{Pure gravitational, subdominant}}$$

Pure gravitational, subdominant

Induced curvature perturbation ζ

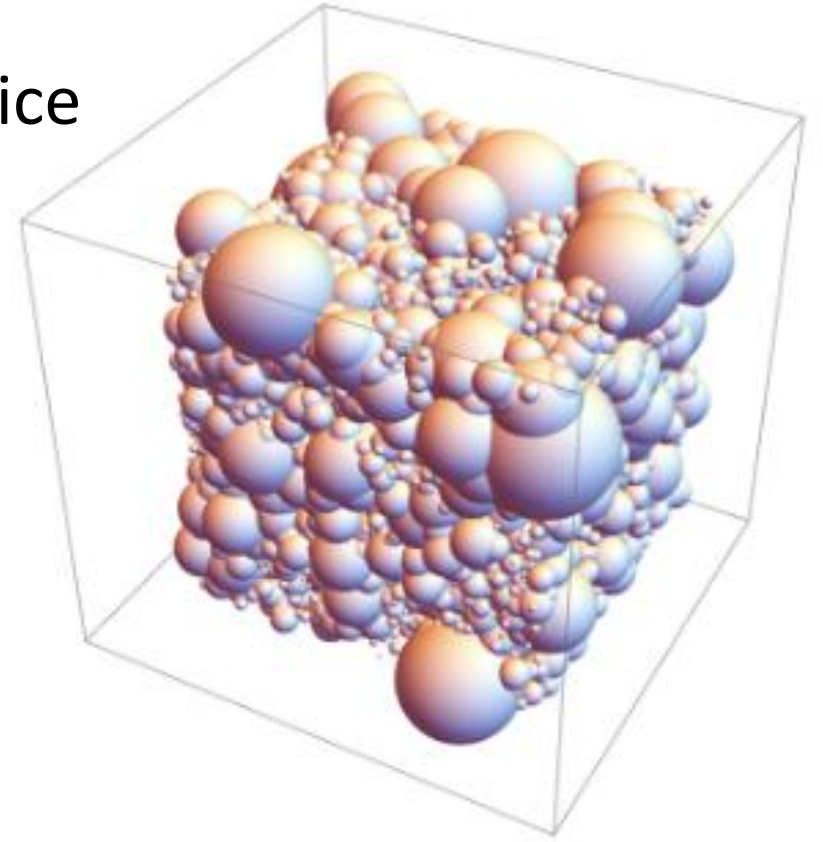
- We solve the following equations of motion numerically with a $1000 \times 1000 \times 1000$ lattice

$$\delta\tilde{\phi}_{\mathbf{q}}'' - \frac{2}{\tau}\delta\tilde{\phi}_{\mathbf{q}}' + \left(q^2 + \frac{1}{H^2\tau^2} \frac{\partial^2 V_0}{\partial\phi_0^2}\right) \delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}} ,$$

$$\tilde{\Psi}_{\mathbf{q}}' - \frac{\tilde{\Phi}_{\mathbf{q}}}{\tau} = -4\pi G_N \left(\frac{\dot{\phi}_0 \delta\tilde{\phi}_{\mathbf{q}}}{H_{\text{inf}}\tau} + \left[\frac{\partial_i}{\partial^2} (\sigma' \partial_i \sigma) \right]_{\mathbf{q}} \right)$$

$$\tilde{\pi}_{\mathbf{q}}^S = -\frac{3}{2} H_{\text{inf}}^2 \tau^2 q_i q_j q^{-4} [(\partial_i \sigma \partial_j \sigma)^{\text{TL}}]_{\mathbf{q}}$$

$$\zeta_{\mathbf{q}} = -\tilde{\Psi}_{\mathbf{q}} - \frac{H_{\text{inf}} \delta\tilde{\phi}_{\mathbf{q}}}{\dot{\phi}_0}$$



Power spectrum of ζ

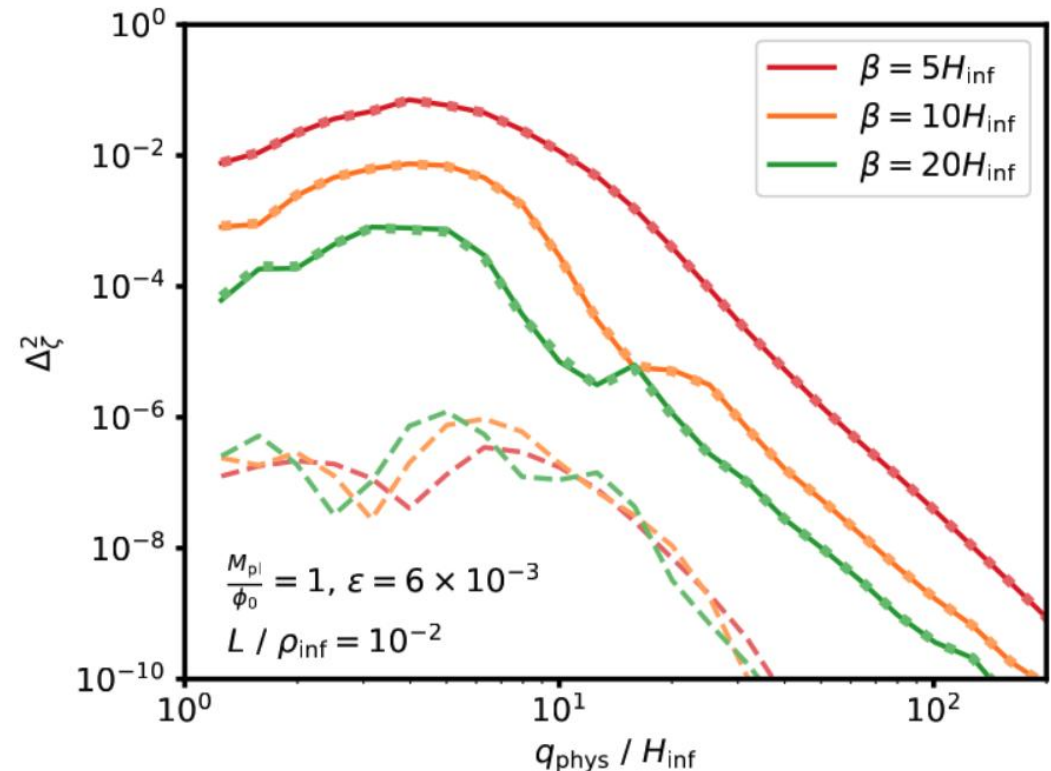
- After the collision of the bubbles, σ field oscillates and keeps producing ζ .
- The production of ζ lasts about H^{-1} , longer than β^{-1} .

$$\Delta_{\zeta}^{2(\text{emp})}(q) = A_{\text{ref}} \mathcal{F} \left(\frac{q_{\text{phys}}}{H_{\text{inf}}} \right)$$

$$\mathcal{F}(x) = \frac{x^3}{1 + (\alpha_1 x)^4 + (\alpha_2 x)^9}$$

$$A_{\text{ref}} = \frac{\mathcal{A}}{\epsilon} \left(\frac{M_{\text{pl}}}{\phi_0} \right)^2 \left(\frac{H_{\text{inf}}}{\beta} \right)^3 \left(\frac{\Delta\rho}{\rho_{\text{inf}}} \right)^2$$

$$\mathcal{A} \approx 24 \quad \alpha_1 \approx 0.31, \alpha_2 \approx 0.17$$



Secondary GWs

- After inflation $\zeta \rightarrow \Phi, \Psi$ $g_{\mu\nu} = a^2[(1 + 2\Phi)d\tau^2 - ((1 + 2\Psi)\delta_{ij} + h_{ij})dx^i dx^j]$
- Expand the Einstein equation to second order:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\hat{\mathcal{T}}_{ij}{}^{lm}\mathcal{S}_{lm},$$

$$\begin{aligned}\mathcal{S}_{ij} \equiv & 2\Phi\partial^i\partial_j\Phi - 2\Psi\partial^i\partial_j\Phi + 4\Psi\partial^i\partial_j\Psi + \partial^i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi \\ & - \frac{4}{3(1+w)\mathcal{H}^2}\partial_i(\Psi' + \mathcal{H}\Phi)\partial_j(\Psi' + \mathcal{H}\Phi) - \frac{2c_s^2}{3w\mathcal{H}^2}[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi]\partial_i\partial_j(\Phi - \Psi).\end{aligned}$$

Scalar induced GWs

Matarrese, Mollerach, and Bruni, astro-hp/9707278

Mollerach, Harari, and Matarrese, astro-hp/0310711

Ananda, Clarkson, and Wands, gr-qc/0612013

Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290

...

Secondary GWs

$$\Omega_{\text{GW}}^{(2)}(f) = \Omega_R A_{\text{ref}}^2 \mathcal{F}_2 \left(\frac{q_{\text{phys}}}{H_{\text{inf}}} \right)$$

$$f = \frac{q}{2\pi a_0} = f_{\text{ref}} \times \frac{q_{\text{phys}}}{H_{\text{inf}}}$$

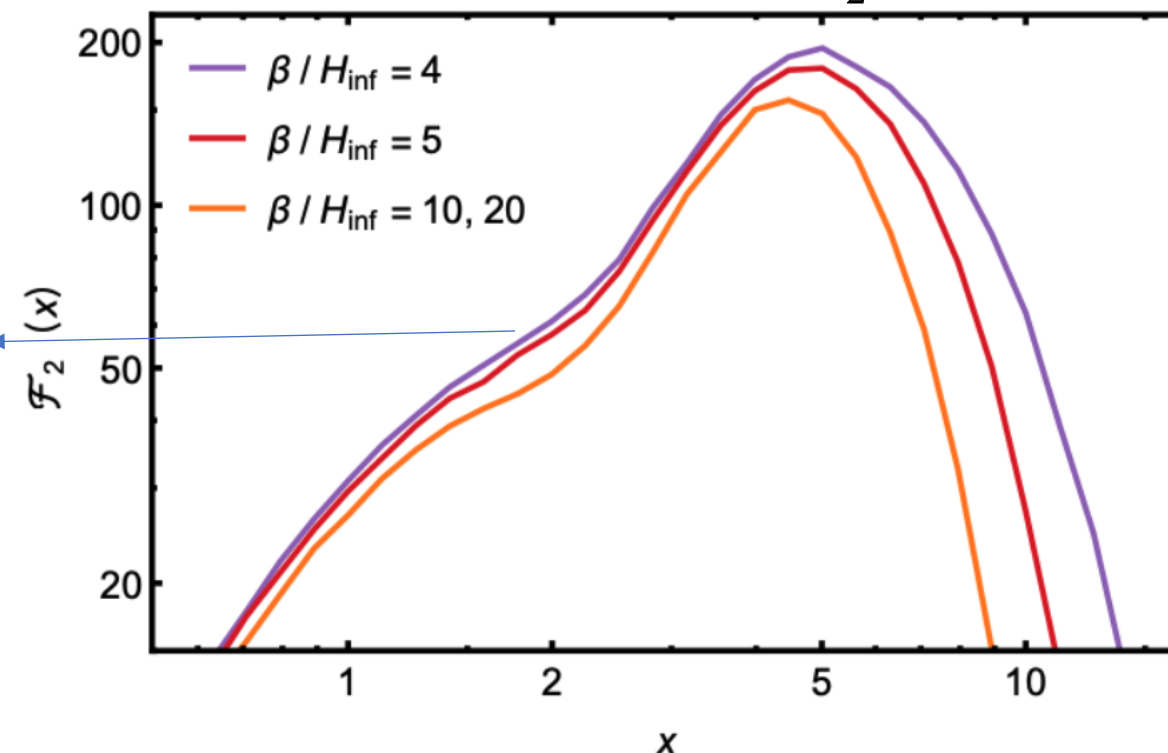
$$f_{\text{ref}} = 10^{-9} \text{ Hz} \times e^{40-N_e} \left(\frac{H_{\text{inf}}}{10^{14} \text{ GeV}} \right)^{1/2}$$

$$\mathcal{F}_2^{\text{IR}}(x) \approx x^3 \left(\frac{6}{5} \log^2 x + \frac{16}{25} \log x + \frac{28}{125} \right)$$

$\mathcal{F}_2$ collects information of the transfer functions.

$$A_{\text{ref}} = \frac{\mathcal{A}}{\epsilon} \left(\frac{M_{\text{pl}}}{\phi_0} \right)^2 \left(\frac{H_{\text{inf}}}{\beta} \right)^3 \left(\frac{\Delta\rho}{\rho_{\text{inf}}} \right)^2$$

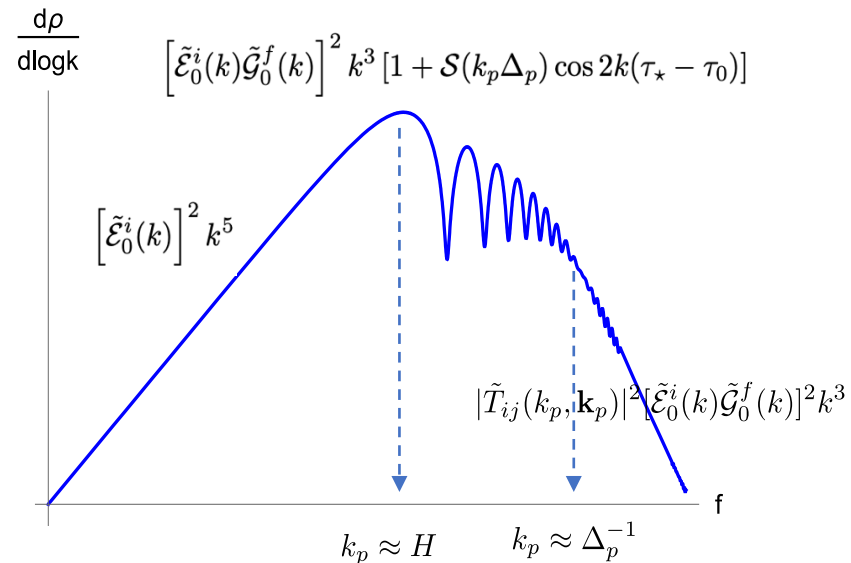
$$\mathcal{F}_2^{\text{max}} \approx 200$$



Comparison between primary GW and secondary GW

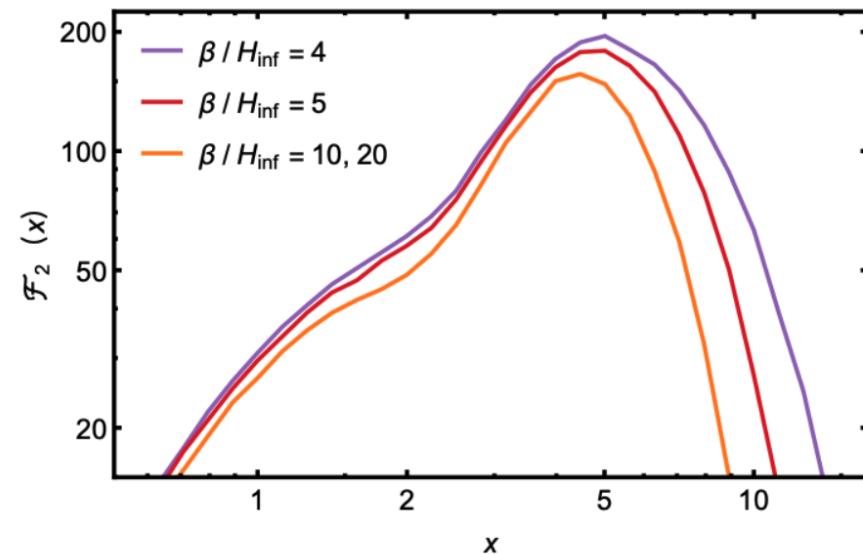
- Primary

$$\Omega_{\text{GW}} \approx \Omega_R \left(\frac{H_{\text{inf}}}{\beta} \right)^5 \left(\frac{\Delta\rho_{\text{vac}}}{\rho_{\text{inf}}} \right)^2$$

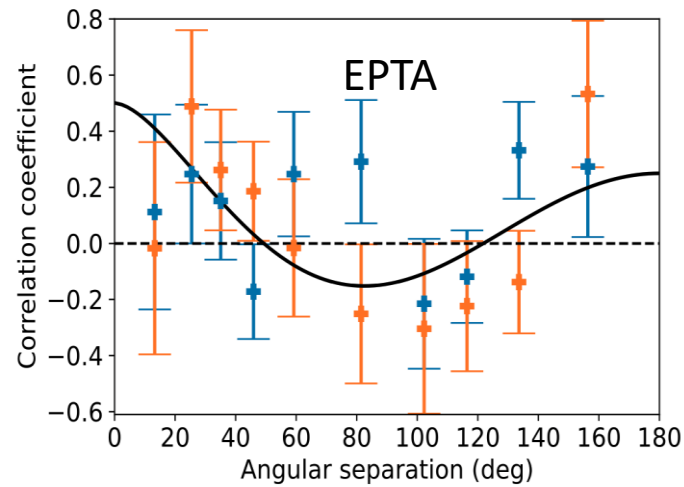
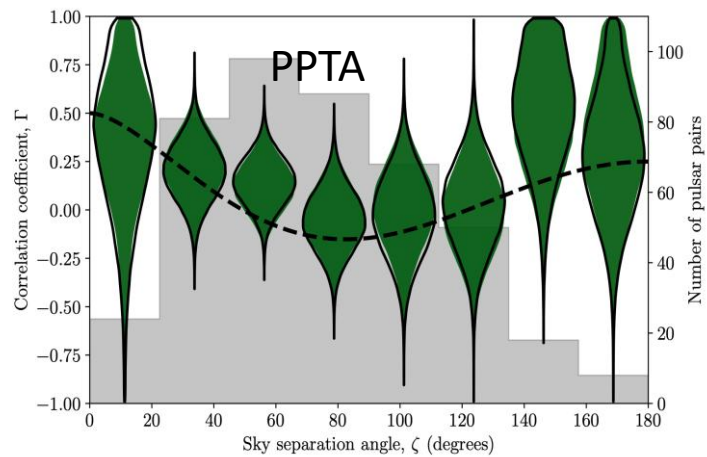
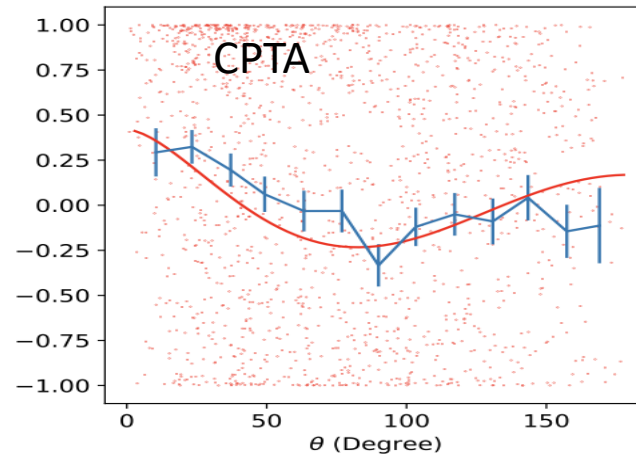
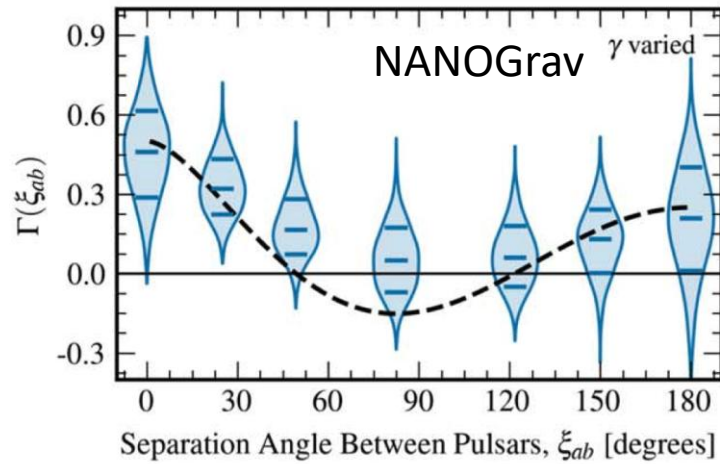


- Secondary

$$\Omega_{\text{GW}} \approx \Omega_R \left(\frac{\mathcal{A}}{\epsilon} \right)^2 \left(\frac{M_{\text{pl}}}{\phi_0} \right)^4 \left(\frac{H_{\text{inf}}}{\beta} \right)^6 \left(\frac{\Delta\rho_{\text{vac}}}{\rho_{\text{inf}}} \right)^4$$



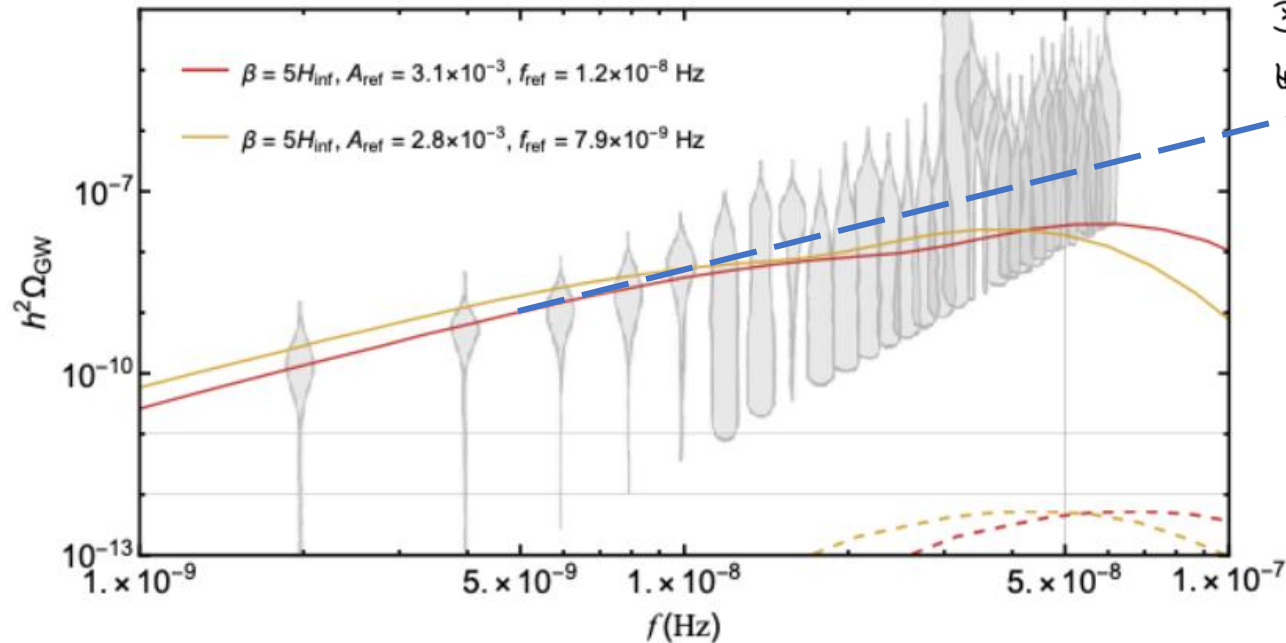
Observation from PTAs



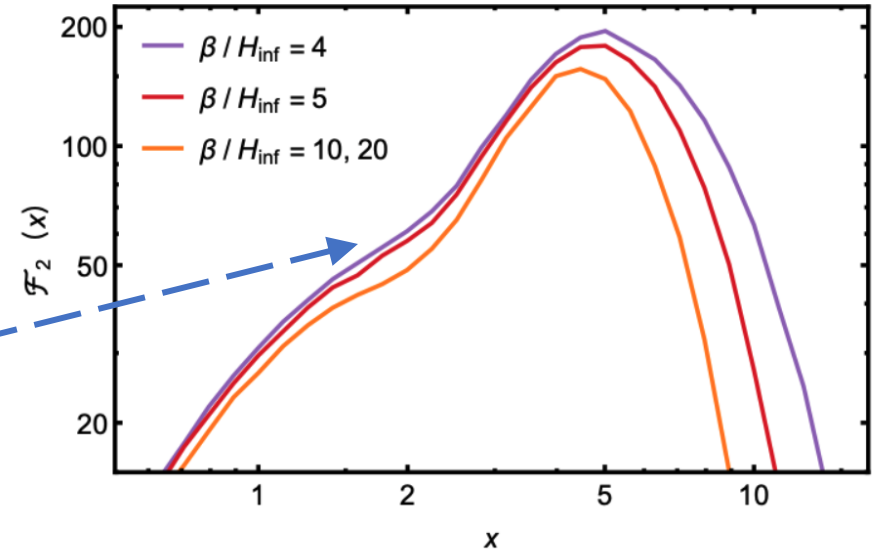
Observation from PTAs

HA, Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang, 2308.00070

- The slope is around 2 in the IR region



$$\mathcal{F}_2^{\text{IR}}(x) \approx x^3 \left(\frac{6}{5} \log^2 x + \frac{16}{25} \log x + \frac{28}{125} \right)$$



$$\Omega_{\text{GW}}^{(2)}(f) = \Omega_R A_{\text{ref}}^2 \mathcal{F}_2 \left(\frac{q_{\text{phys}}}{H_{\text{inf}}} \right)$$

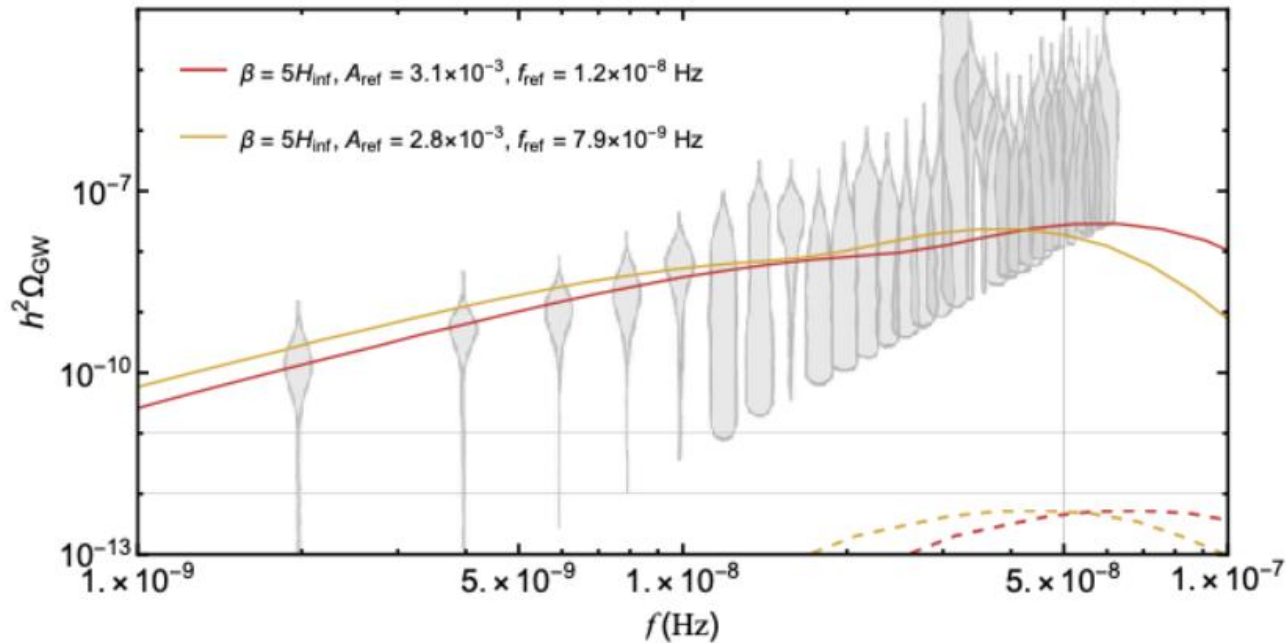
$$f = \frac{q}{2\pi a_0} = f_{\text{ref}} \times \frac{q_{\text{phys}}}{H_{\text{inf}}}$$

$$f_{\text{ref}} = 10^{-9} \text{ Hz} \times e^{40-N_e} \left(\frac{H_{\text{inf}}}{10^{14} \text{ GeV}} \right)^{1/2}$$

Observation from PTAs

HA, Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang, 2308.00070

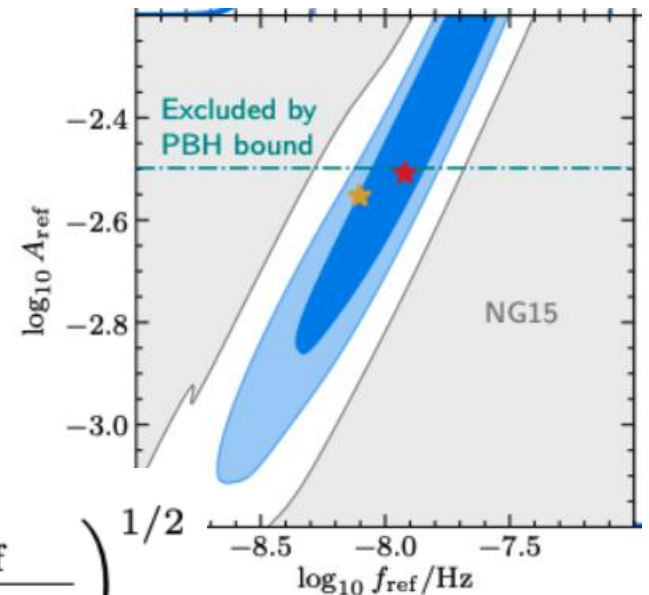
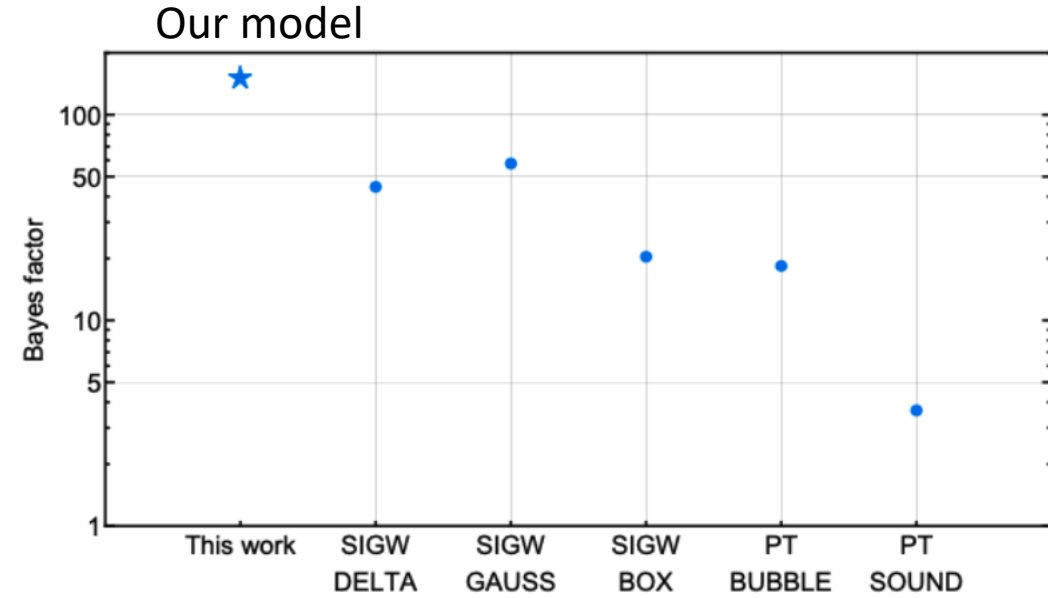
- Bayes factor against SMBHB



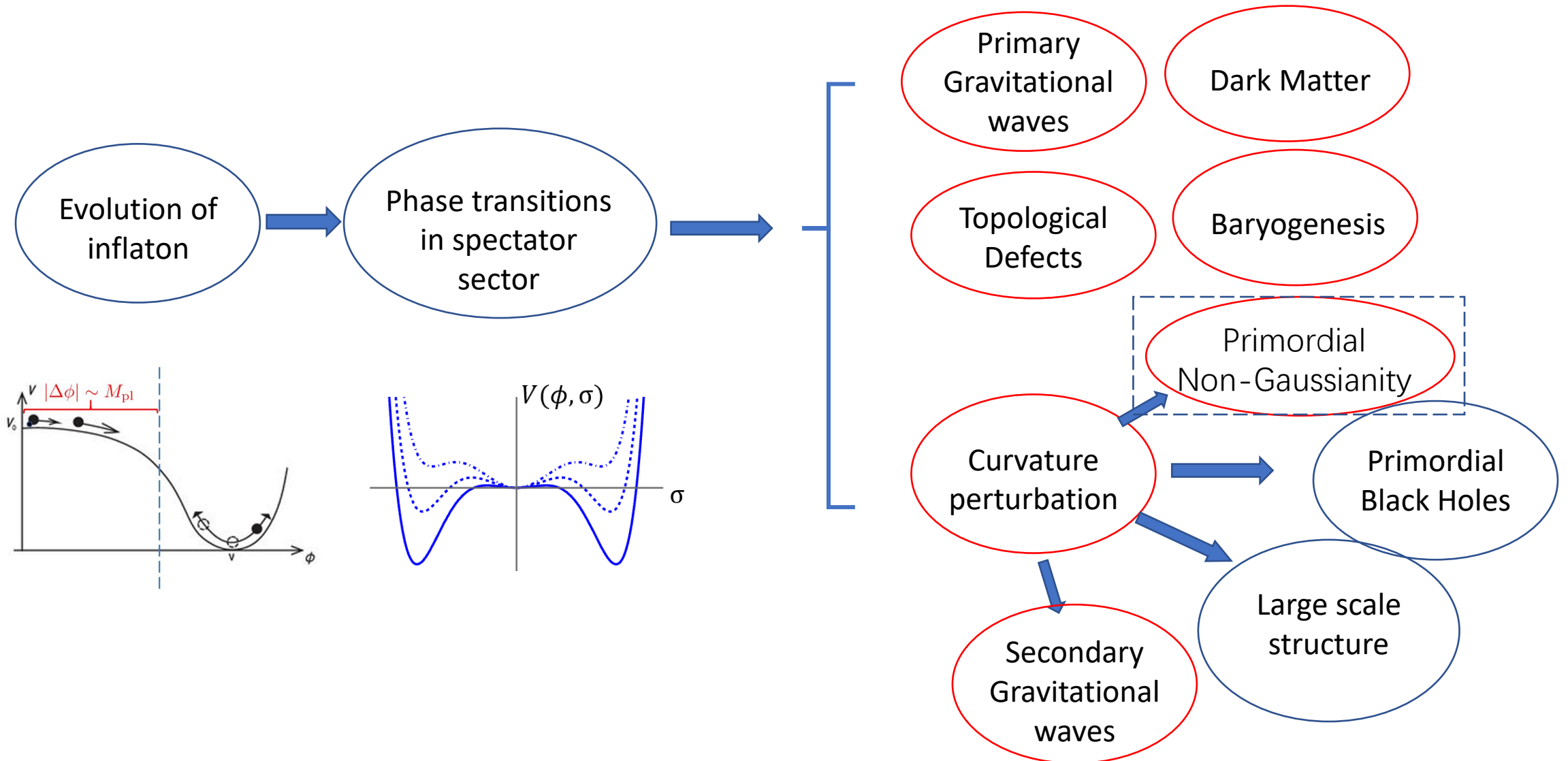
$$\Omega_{\text{GW}}^{(2)}(f) = \Omega_R \underline{A_{\text{ref}}^2} \mathcal{F}_2 \left(\frac{q_{\text{phys}}}{\underline{H_{\text{inf}}}} \right)$$

$$f = \frac{q}{2\pi a_0} = f_{\text{ref}} \times \frac{q_{\text{phys}}}{H_{\text{inf}}}$$

$$f_{\text{ref}} = 10^{-9} \text{ Hz} \times e^{40-N_e} \left(\frac{H_{\text{inf}}}{10^{14} \text{ GeV}} \right)^{1/2}$$



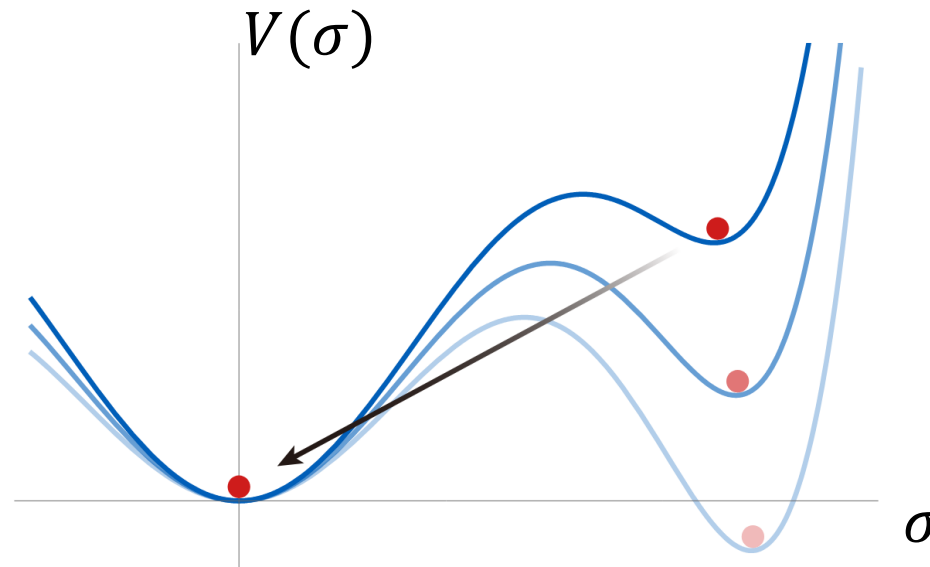
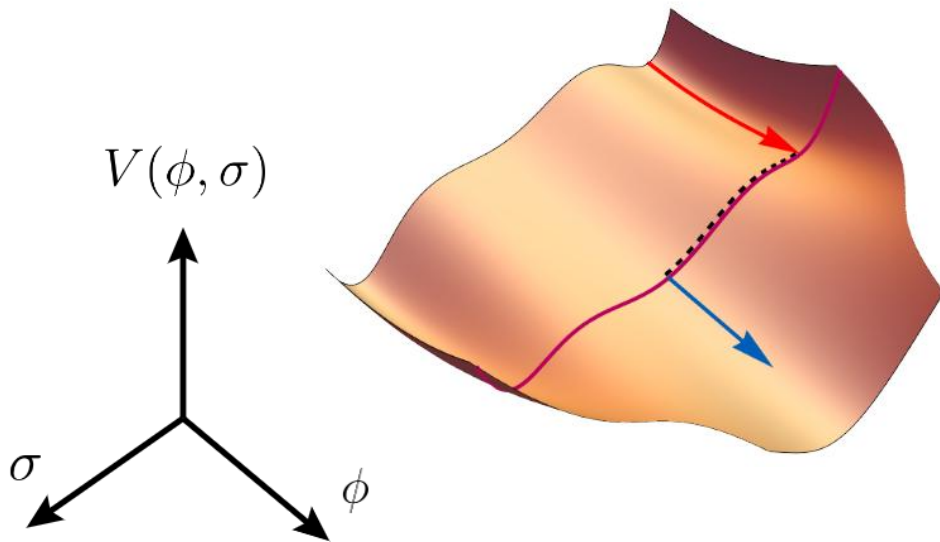
Consequences of the phase transitions



Primordial non-Gaussianity (quantum fluctuation)

- The evolution of ϕ_0 induces the evolution of σ_0 .

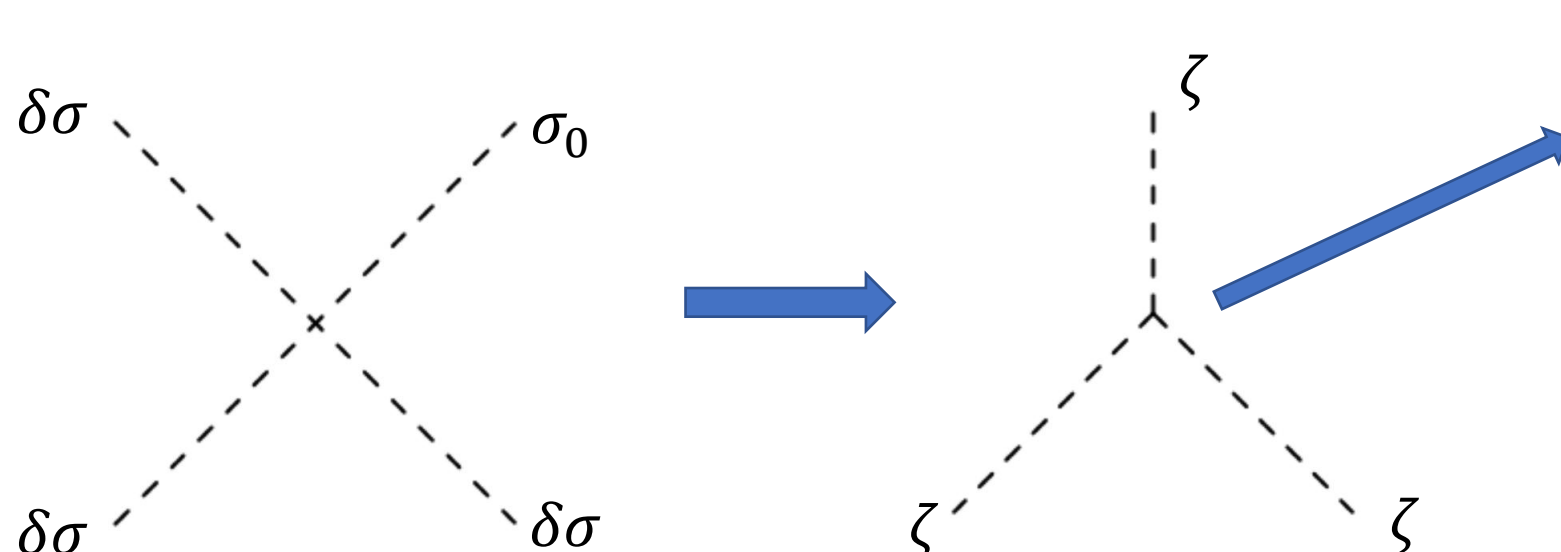
$$\zeta = -H \left[\frac{\dot{\phi}_0 \delta\phi + \dot{\sigma}_0 \delta\sigma}{\dot{\phi}_0^2 + \dot{\sigma}_0^2} \right]$$



- $\delta\sigma$ also contributes to the curvature perturbation, and the interaction in the σ sector is strong.

Primordial non-Gaussianity (quantum fluctuation)

- 3pt function in the symmetry breaking phase



The diagram illustrates the transition from a 4-point function to a 3-point function in the symmetry breaking phase. On the left, a 4-point function is shown with four external legs labeled $\delta\sigma$, σ_0 , $\delta\sigma$, and $\delta\sigma$, meeting at a central vertex marked with an 'x'. A blue arrow points to the right, where a 3-point function is shown with three external legs labeled ζ , ζ , and ζ , meeting at a central vertex. A blue arrow points from the 3-point function to the right, towards the equations.

$$\mathcal{H}_I \sim \left[\frac{1}{6} \frac{\partial^3 V_\sigma}{\partial \sigma^3} \dot{\sigma}_0^3 \right] \zeta^3$$
$$\dot{\sigma}_0 \propto \dot{\phi}_0$$
$$\mathcal{H}_I \sim \kappa^3 L \epsilon^{3/2}$$

Enhancement: $\kappa^3 \frac{L}{\rho_{\text{inf}}} \epsilon^{-1/2}$

Compared to single field inflation.

Primordial non-Gaussianity

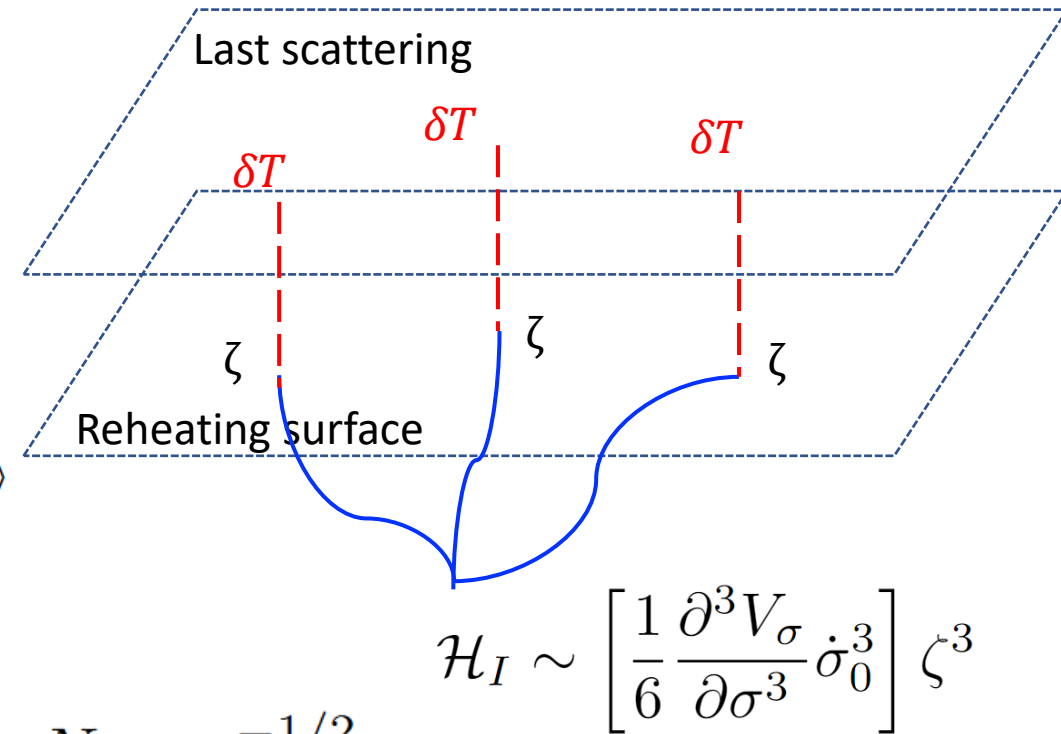
- Calculate the three-point function using the in-in formalism.

$$\begin{aligned} \langle Q(t) \rangle &= \sum_{N=0}^{\infty} i^N \int_{-\infty}^t dt_N \int_{-\infty}^{t_N} dt_{N-1} \cdots \int_{-\infty}^{t_2} dt_1 \\ &\times \left\langle \left[H_I(t_1), \left[H_I(t_2), \cdots \left[H_I(t_N), Q^I(t) \right] \cdots \right] \right] \right\rangle \end{aligned}$$

S. Weinberg, hep-th/0506236

- Relevant operator, IR dominant. $\int d\tau \sim N_e \sim \epsilon^{-1/2}$

- $f_{NL} \sim \mathcal{O}(1)$



Primordial non-Gaussianity

- Calculate the three-point function using the in-in formalism.

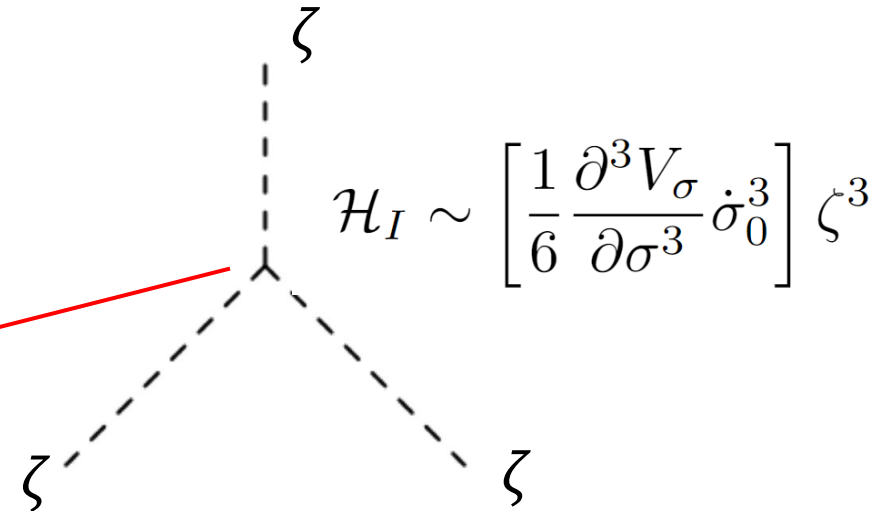
$$\begin{aligned} \langle Q(t) \rangle &= \sum_{N=0}^{\infty} i^N \int_{-\infty}^t dt_N \int_{-\infty}^{t_N} dt_{N-1} \cdots \int_{-\infty}^{t_2} dt_1 \\ &\times \left\langle \left[H_I(t_1), \left[H_I(t_2), \cdots \left[H_I(t_N), Q^I(t) \right] \cdots \right] \right] \right\rangle \end{aligned}$$

S. Weinberg, hep-th/0506236

$$\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle' = \frac{3}{4} \int_{-\infty}^0 \frac{d\tau}{\tau} \frac{H^8}{\dot{\phi}_0^6} \frac{\lambda(\tau)}{k_1^2 k_2^3 k_3^2} f(k_1, k_2, k_3)$$

Dominated in the region
 $|k_1 \tau| \ll 1, |k_2 \tau| \ll 1, |k_3 \tau| \ll 1$

$$\text{Re} \left[\left(1 + \frac{i}{k_1 \tau} \right) \left(1 + \frac{i}{k_2 \tau} \right) \left(1 + \frac{i}{k_3 \tau} \right) e^{i(k_1 + k_2 + k_3) \tau} \right]$$

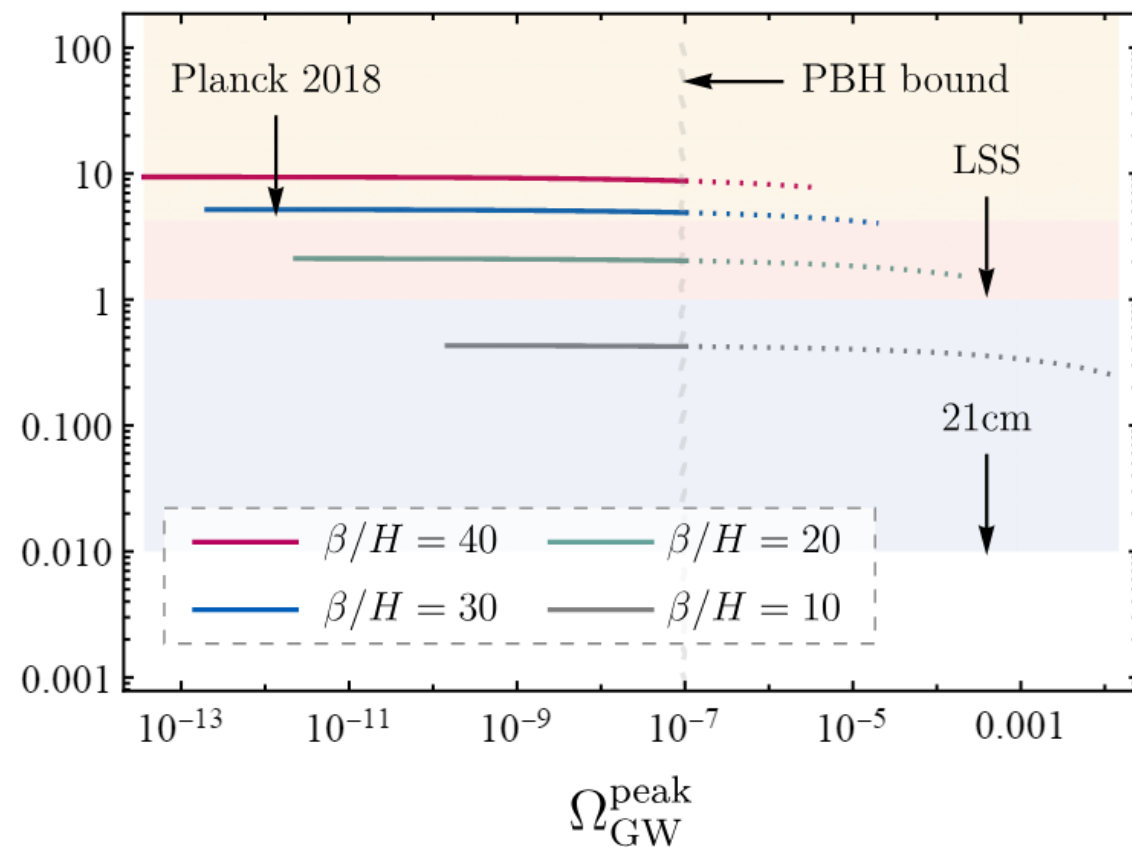
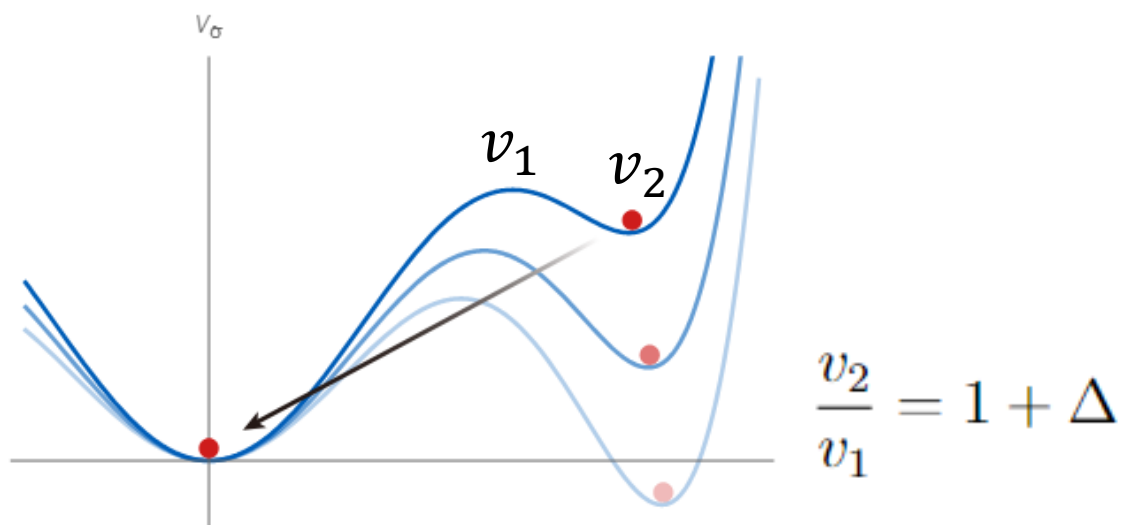


Primordial non-Gaussianity

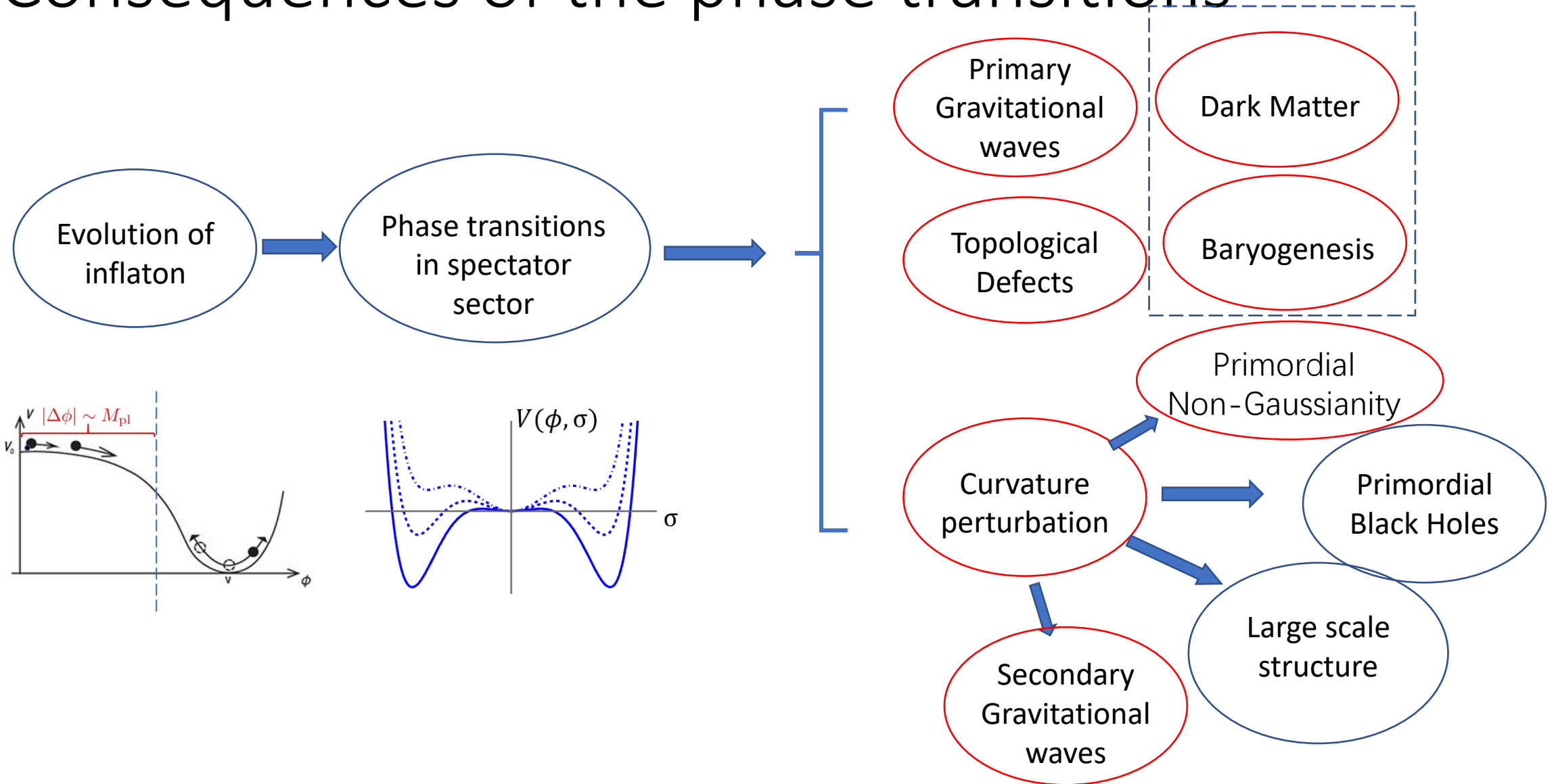
$$f_{\text{NL}} = \left(\frac{\beta_*}{H}\right)^3 \left(\frac{L}{\rho_{\text{inf}}}\right) \left(\frac{\Delta_*}{\mathcal{S}_{E_*}}\right)^3 \mathcal{F}(\Delta_*)$$

$$\Omega_{\text{GW}} \sim \Omega_R \left(\frac{\mathcal{A}}{\epsilon}\right)^2 \left(\frac{M_{\text{pl}}}{\phi_0}\right)^4 \left(\frac{H_{\text{inf}}}{\beta}\right)^6 \left(\frac{\Delta\rho}{\rho_{\text{inf}}}\right)^4$$

$|f_{\text{NL}}|$

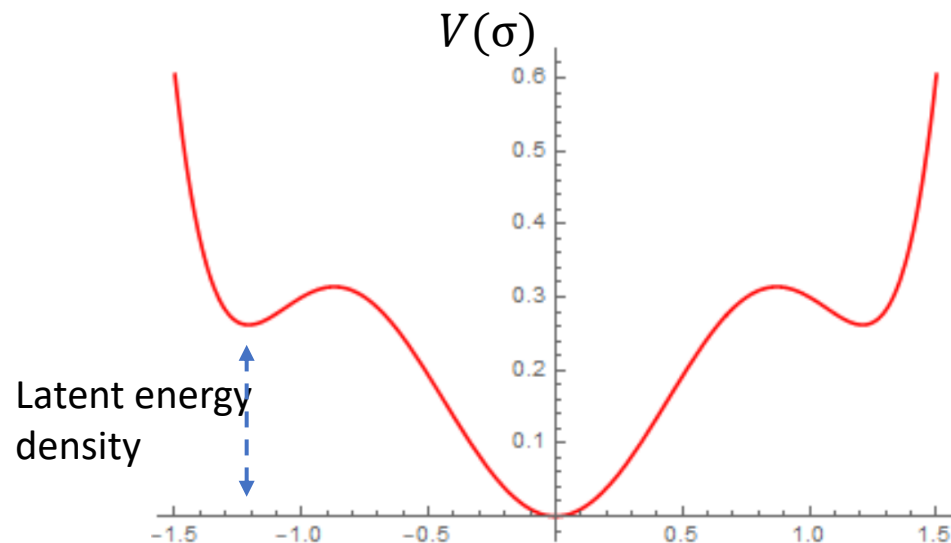


Consequences of the phase transitions



Producing superheavy DM

- Where does the latent energy go?
- σ particles produced during bubble collision and thermalization.
- If the phase transition is ***symmetry-restoration***, σ particles can be DM.



Producing superheavy DM

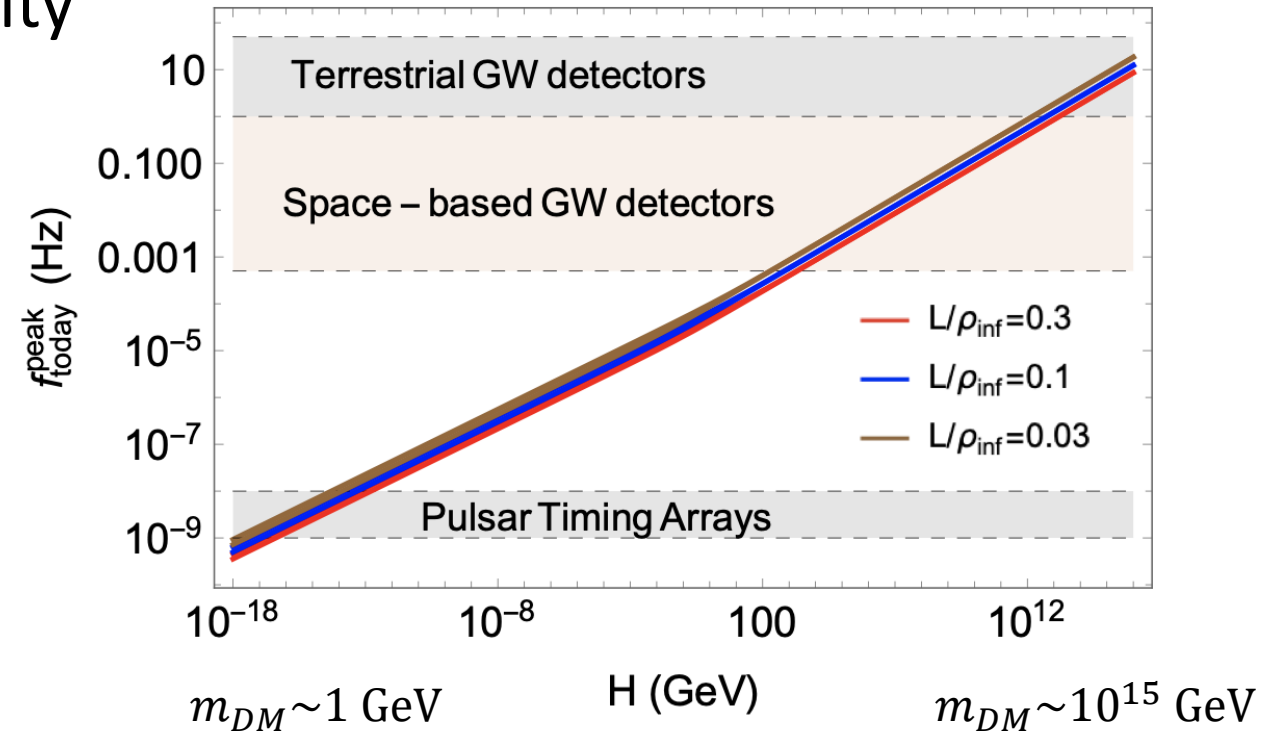
- Today's dark matter energy density

$$\rho_{DM}^{(0)} \approx \Delta\rho_{\text{vac}} e^{-3(N_{\star} - N_{\text{after}})}$$

$$\Omega_{DM} \sim \frac{\Delta\rho_{\text{vac}}}{\rho_{\text{inf}}} \times \eta^{-1} \times e^{-3N_{\star}}$$

$$\eta = \frac{n_B^{(0)}}{n_{\gamma}} \approx 10^{-9}$$

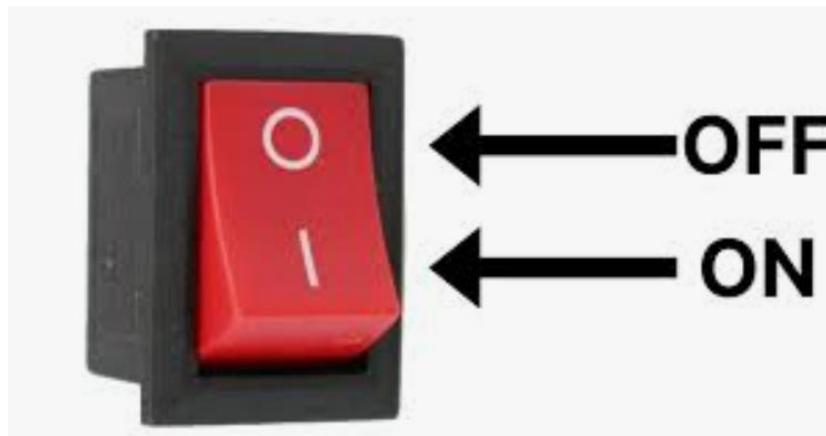
$$f_{\text{today}}^{\text{peak}} \sim \frac{1}{2\pi} \left(\frac{\Delta\rho_{\text{vac}}}{\rho_{\text{inf}}} \right)^{-1/3} (H_{\text{inf}} H_0^2)^{1/3}$$



HA, Xi Tong, Siyi Zhou, 2208.14857

Baryon number as an accidental symmetry

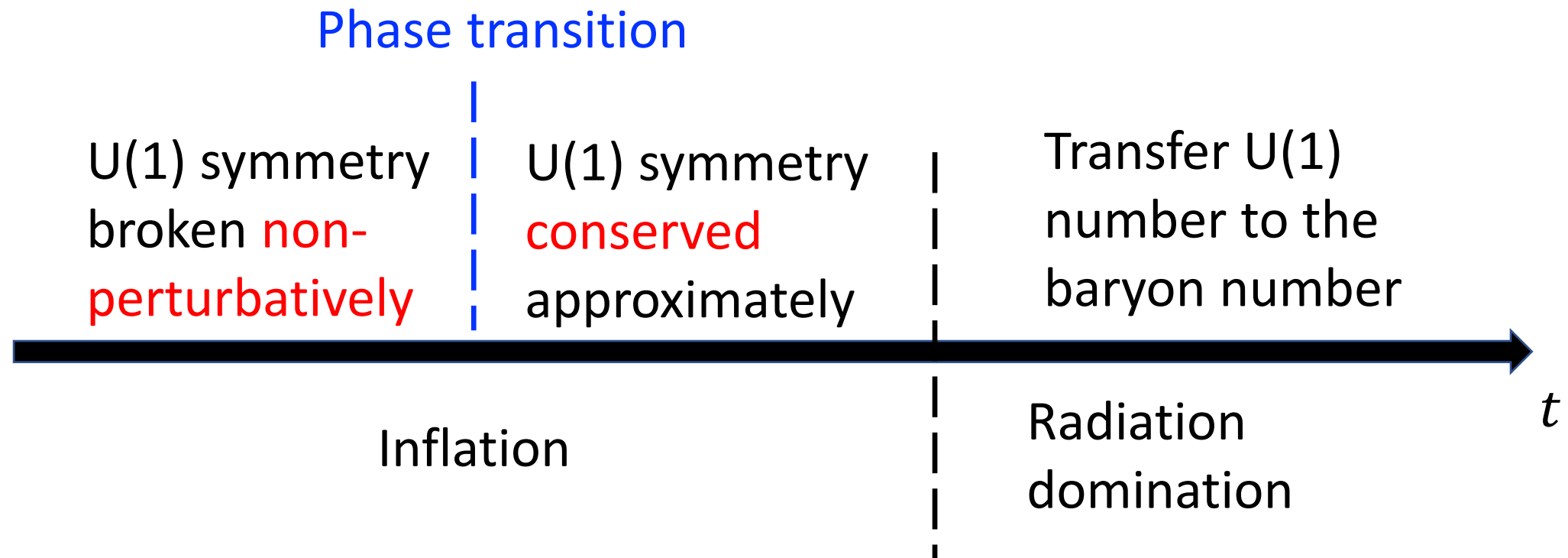
- The baryon number symmetry must be broken in the early universe.
- Today the baryon number is approximately conserved.
- There must be a “switch” of baryon number violation that was active in the early universe but is inactive today.



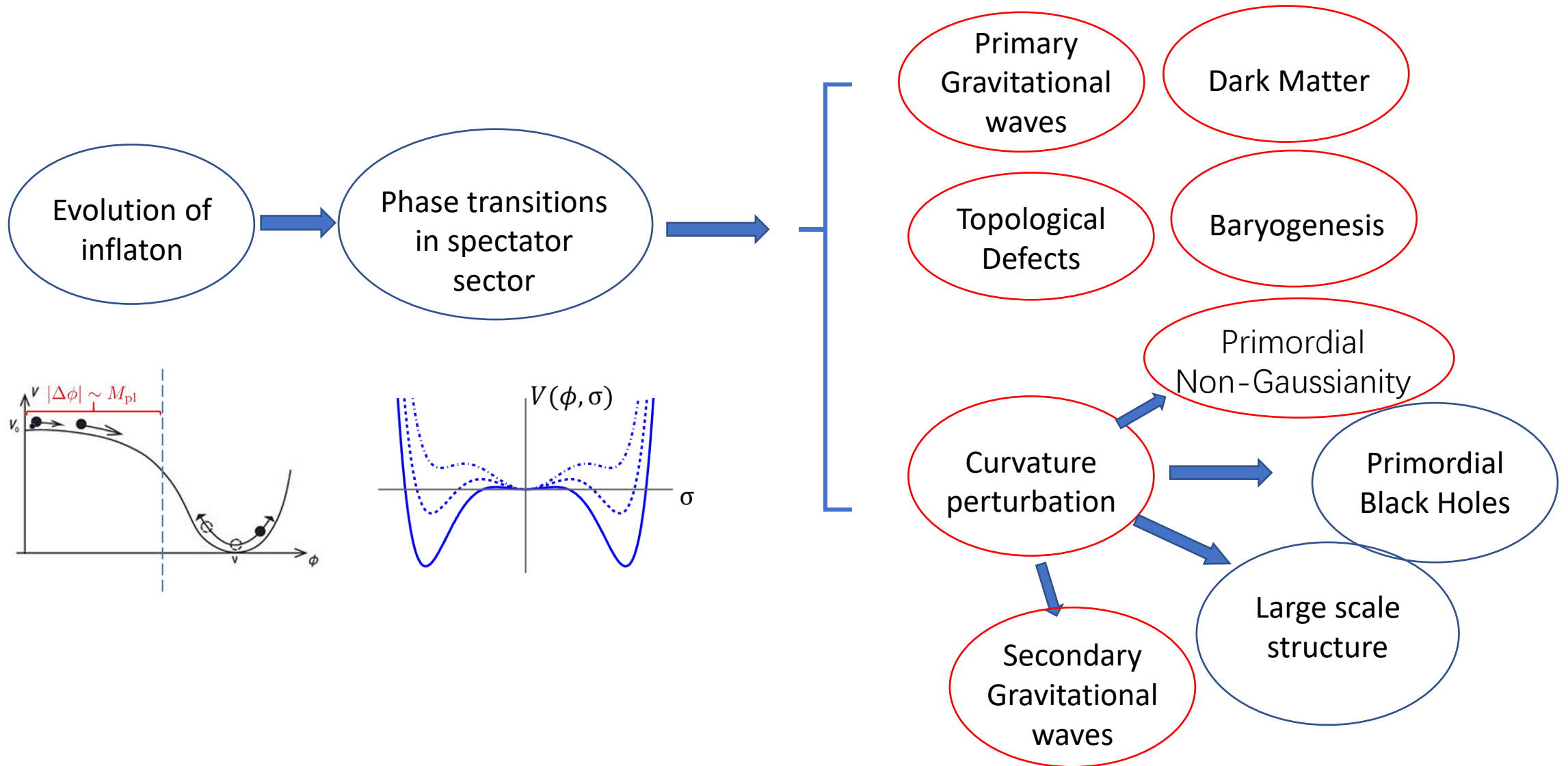
Baryogenesis during inflation

HA, Qi Chen and Yuan Yin, 2409.05833

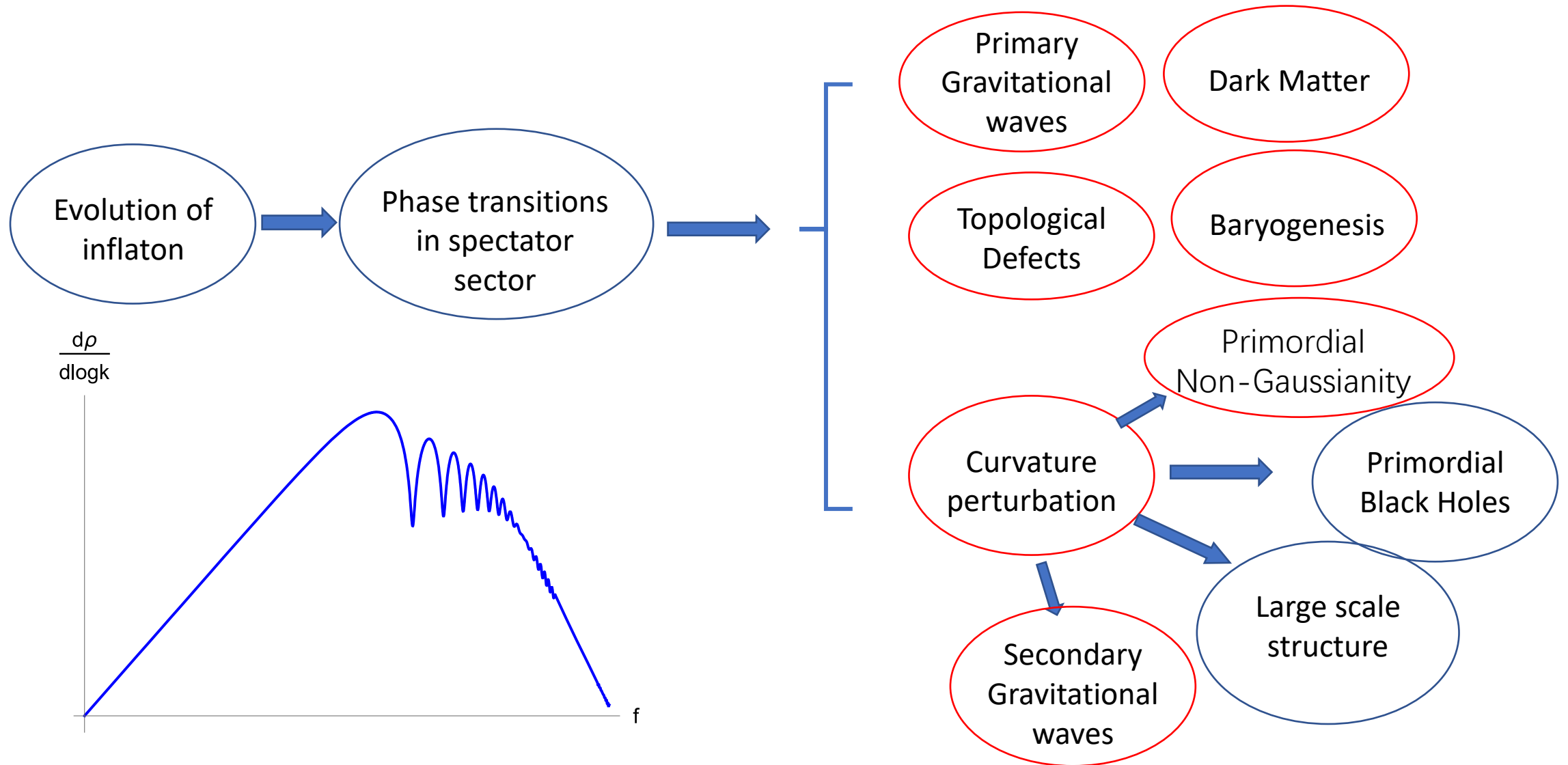
- We first generate a $U(1)$ number during inflation.
- We transfer the $U(1)$ number to baryon number.



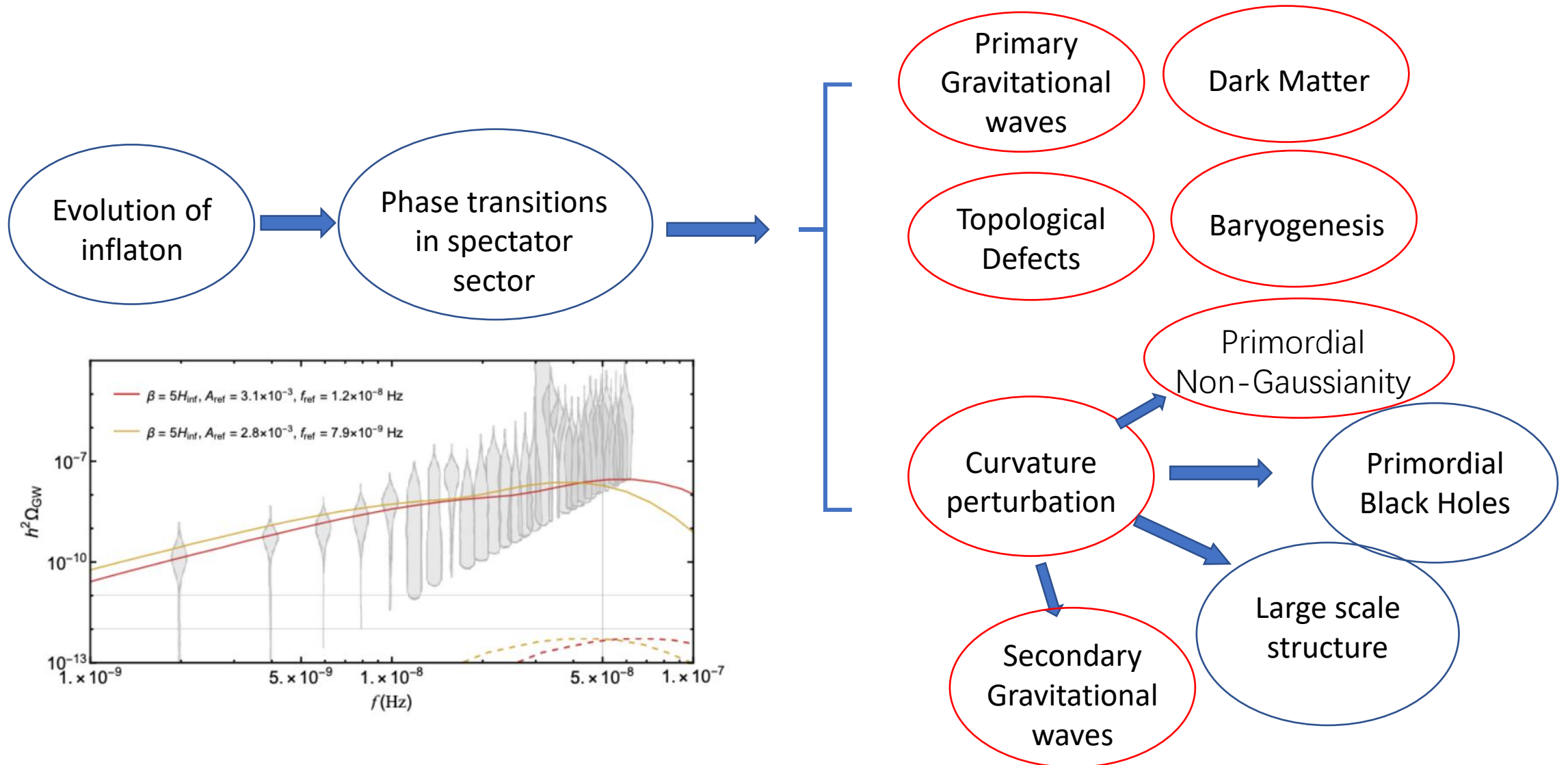
Summary



Summary

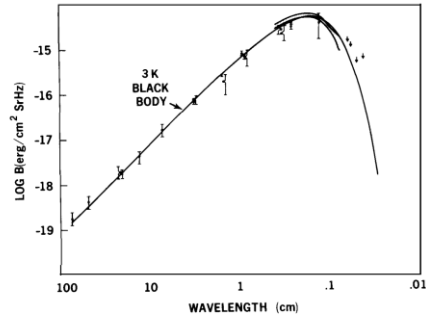
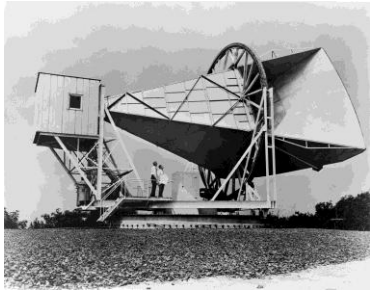


Summary

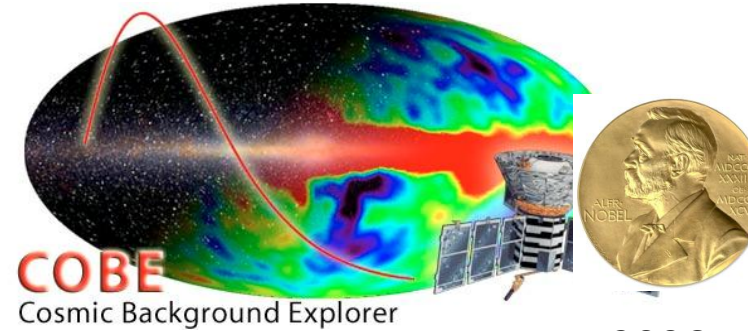


Summary

Why do we need inflation?

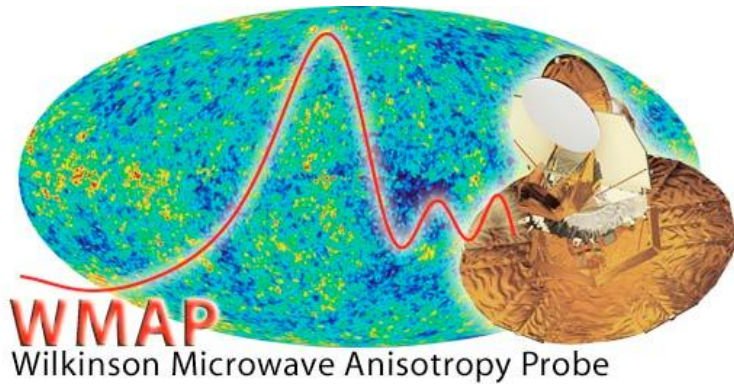


Bell Laboratory Penzias and Wilson 1964

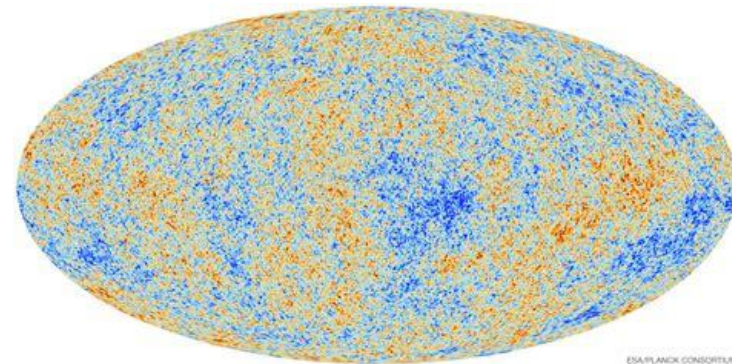


NASA 1989-1996

2006

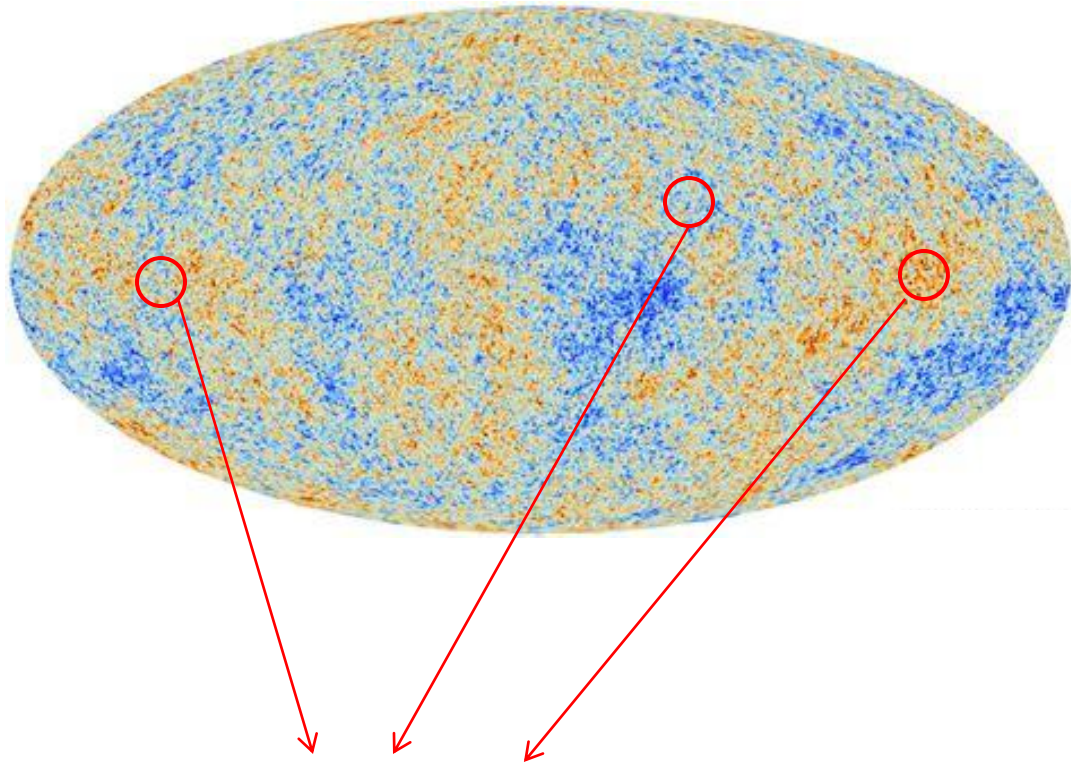


NASA 2001-2010

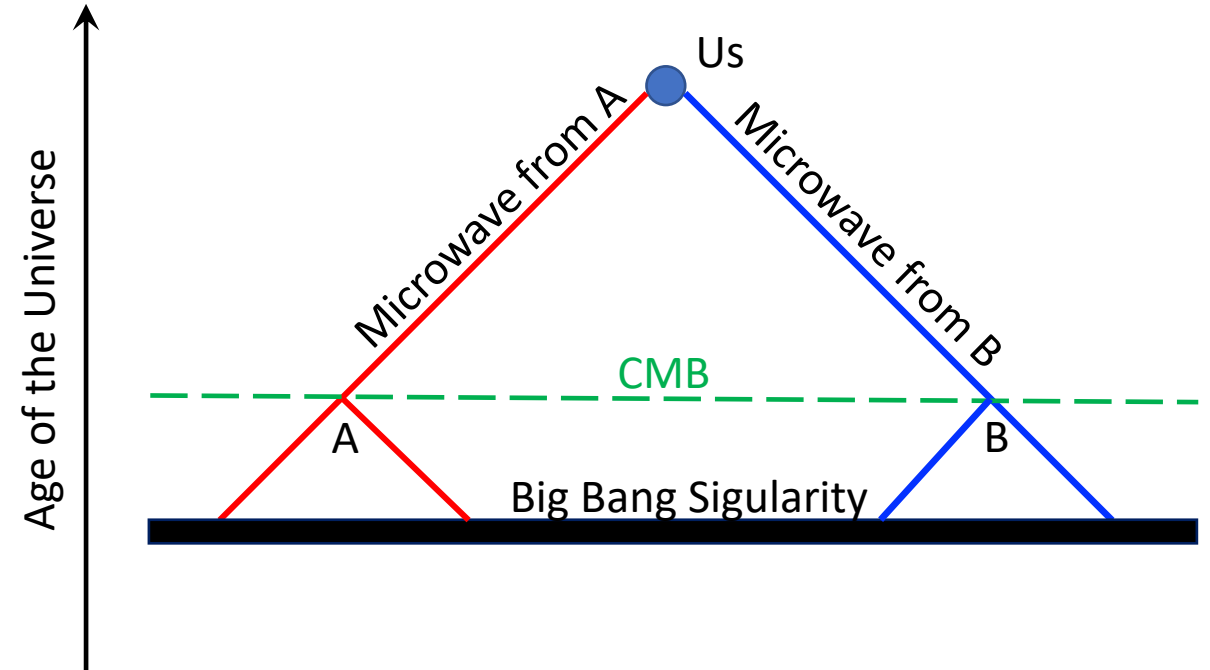


European space agency
Planck spacecraft

The causality problem



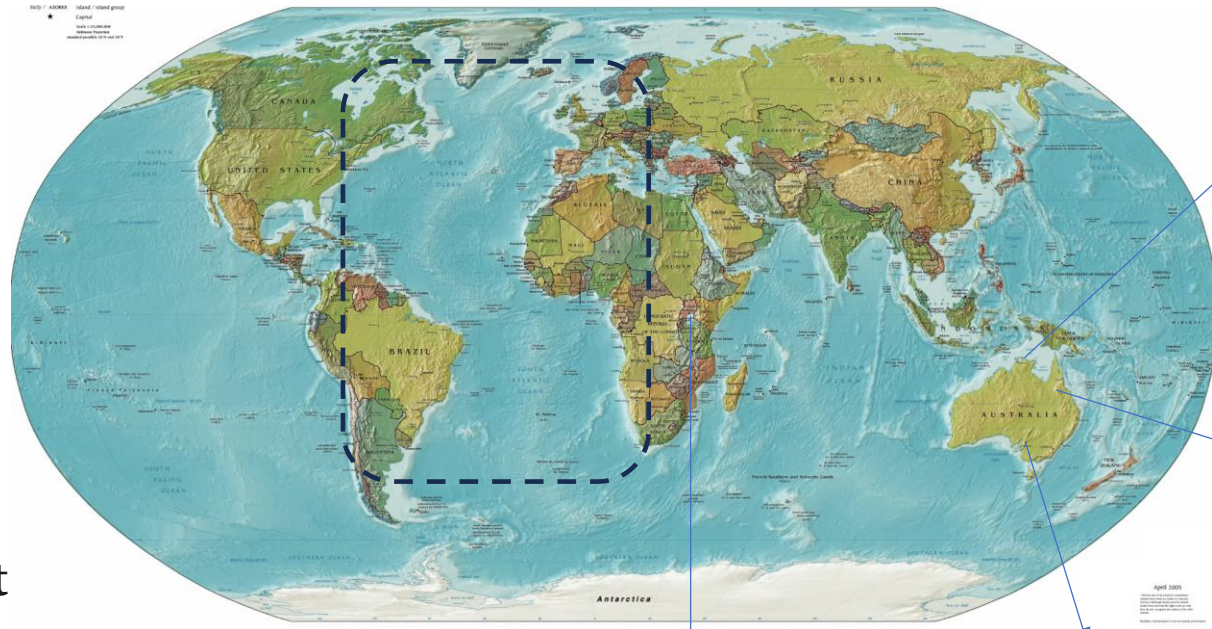
Same temperature and similar fluctuations.



Causality problems usually indicate big discoveries!



**Alfred Wegener:
Continental drift
hypothesis**



Animals with
brood pouch
育儿袋



ostrich



emu

Causality problems usually indicate big discoveries!



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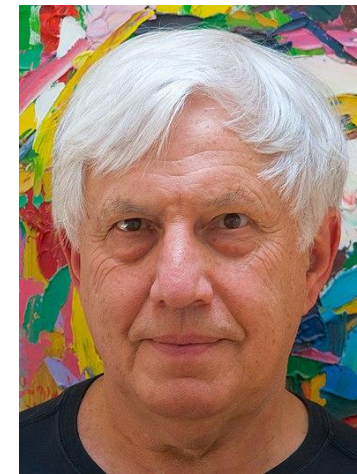
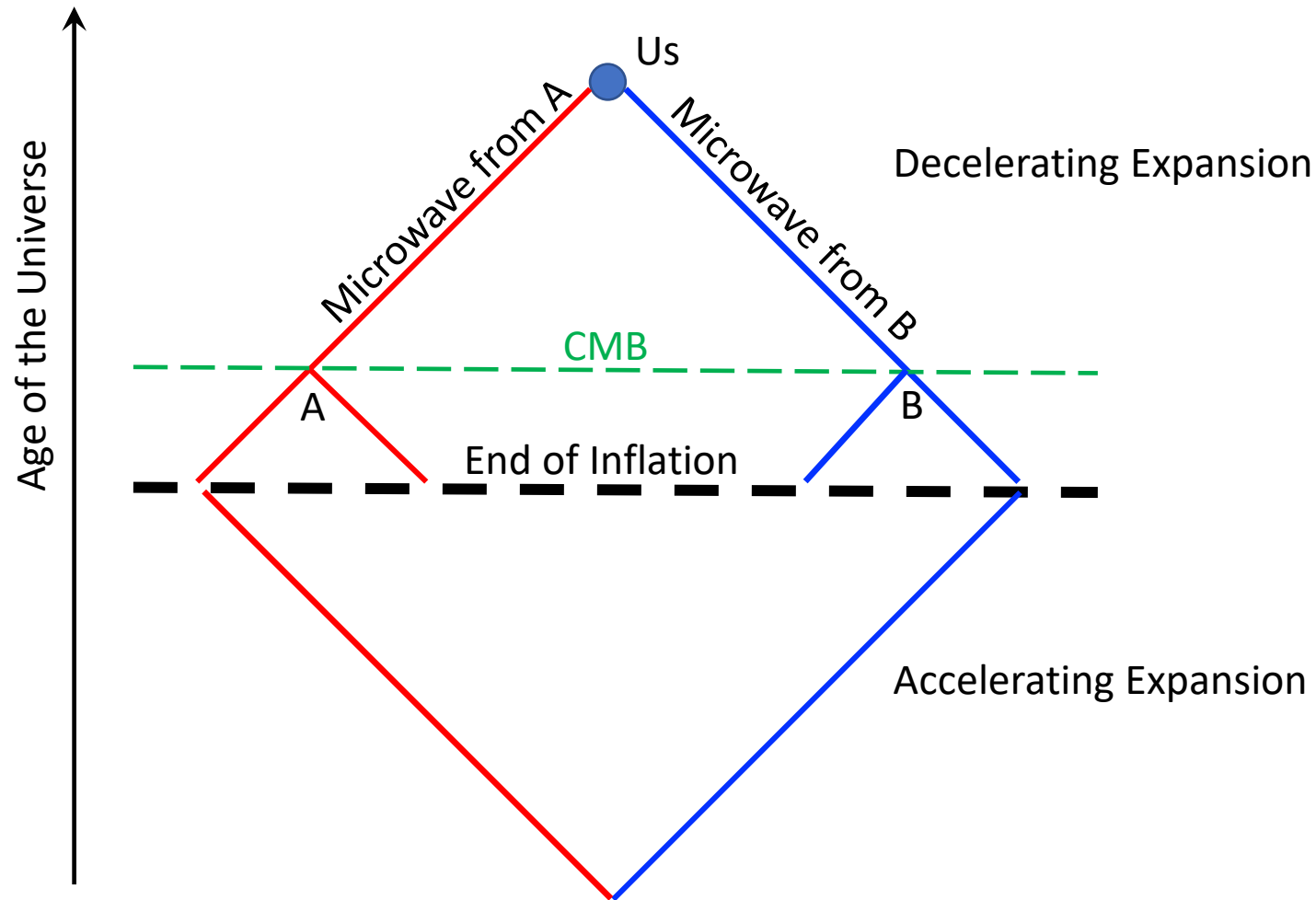


ostrich



emu

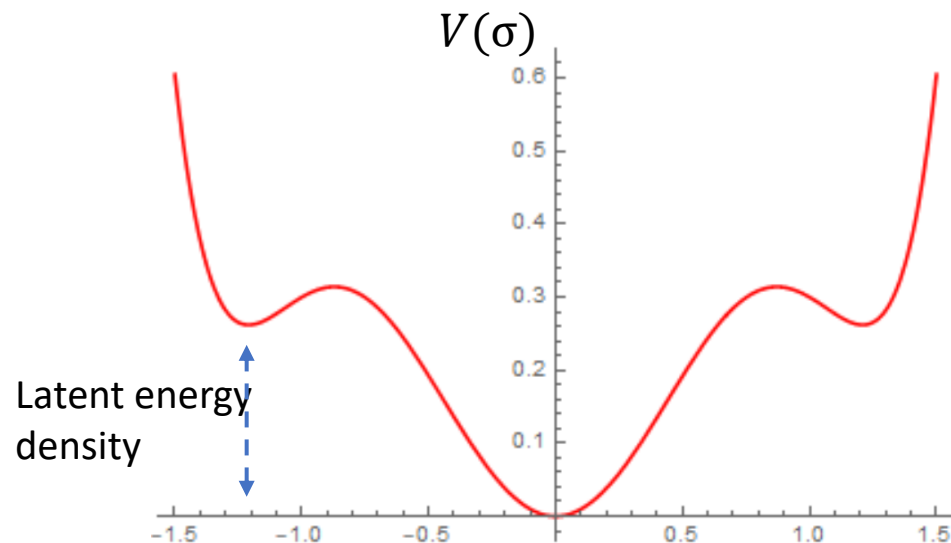
Inflation theory



Backups

Producing superheavy DM

- Where does the latent energy go?
- σ particles produced during bubble collision and thermalization.
- If the phase transition is ***symmetry-restoration***, σ particles can be DM.



Producing superheavy DM

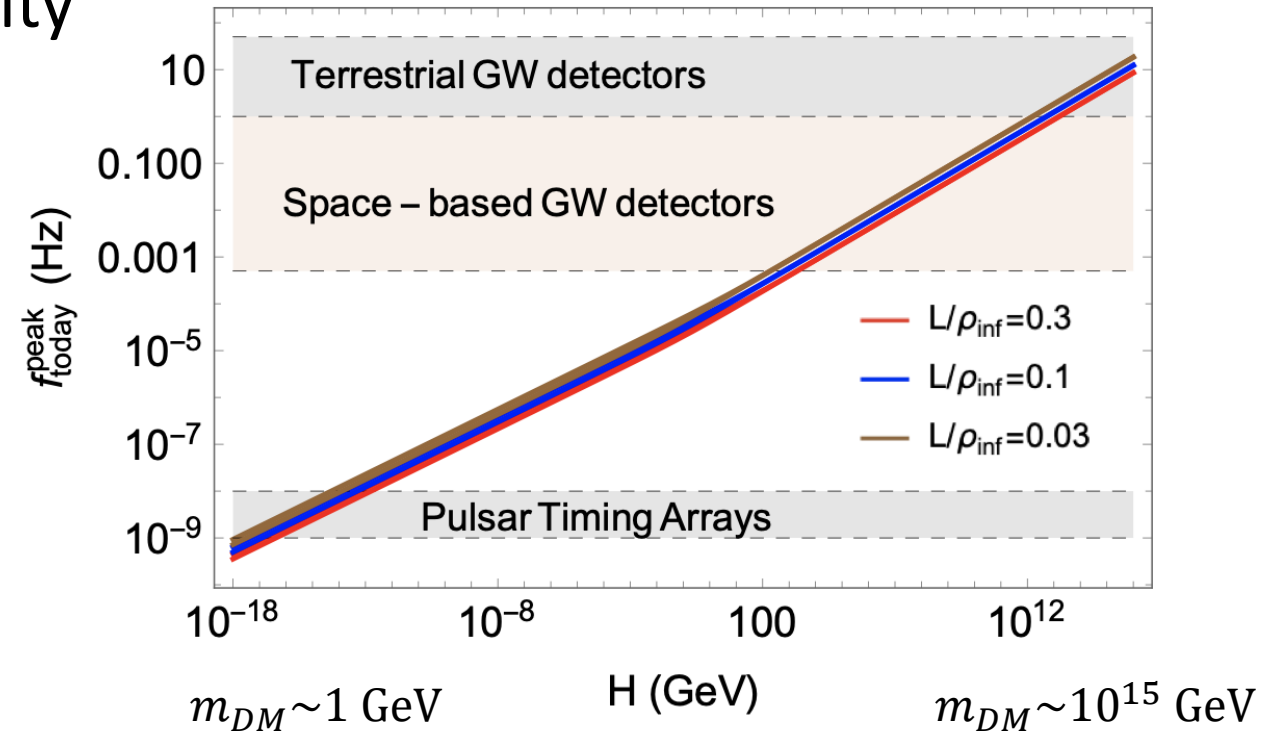
- Today's dark matter energy density

$$\rho_{DM}^{(0)} \approx \Delta\rho_{\text{vac}} e^{-3(N_{\star} - N_{\text{after}})}$$

$$\Omega_{DM} \sim \frac{\Delta\rho_{\text{vac}}}{\rho_{\text{inf}}} \times \eta^{-1} \times e^{-3N_{\star}}$$

$$\eta = \frac{n_B^{(0)}}{n_\gamma} \approx 10^{-9}$$

$$f_{\text{today}}^{\text{peak}} \sim \frac{1}{2\pi} \left(\frac{\Delta\rho_{\text{vac}}}{\rho_{\text{inf}}} \right)^{-1/3} (H_{\text{inf}} H_0^2)^{1/3}$$



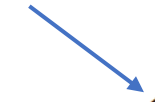
HA, Xi Tong, Siyi Zhou, 2208.14857

Baryogenesis during inflation

- Conserved numbers must be diluted as a^{-3} , even for spontaneously broken symmetries.
- For a U(1) number to survive inflation, it must be broken explicitly.
- In our model:

	ϕ	χ	σ
$U(1)$	0	1	0
$\mathbb{Z}_2$	1	1	-1

$$\mathcal{L}_{\text{dim-5}} = -\frac{i}{\Lambda} \partial_\mu \phi (\chi \partial^\mu \chi^* - \chi^* \partial^\mu \chi)$$



$$-i \frac{\dot{\phi}_0}{\Lambda} (\chi \dot{\chi}^* - \chi^* \dot{\chi})$$

- Explicit U(1) breaking term: $A\sigma^2\chi + h.c.$

Trivial if no
explicit broken.

Field content of our model

Field contains the U(1) number

Inflaton

The switch field

The number we want to generate

	ϕ	χ	σ
$U(1)$	0	1	0
$\mathbb{Z}_2$	1	1	-1

σ

Baryogenesis during inflation

- We use the phase transition of σ as a switch:
- In the Z_2 breaking phase: $A\sigma^2\chi \rightarrow A\sigma_0^2\chi$, a tadpole for χ .

$$\sqrt{-g}\mathcal{L} = a^3|\dot{\chi}|^2 - a|\partial_i\chi|^2 + ia^3\mu(\chi\dot{\chi}^* - \chi^*\dot{\chi}) - a^3(m_\chi^2|\chi|^2 - Av_\sigma^2(\chi + \chi^*))$$



Chemical potential

Initial U(1) number density:

$$n_\chi^{(\text{ini})} = -2\mu v_\chi^2 = -\frac{2\mu A^2 v_\sigma^4}{m_\chi^4 + 9H^2\mu^2}$$

does not dilute with inflation!

Baryogenesis during inflation

- We use the phase transition of σ as a switch:
- In the Z_2 restored phase:
 - No tadpole for χ , the U(1) breaking interactions become perturbative.
 - We need to consider the washout effects from the explicit breaking term.
 - We need to further engineer the model to transfer this U(1) number to the baryon number.
- The phase transition happened in a very short period ($\beta \gg H$), the change of U(1) number during the phase transition can be neglected.

Baryogenesis during inflation

- Today's baryon number

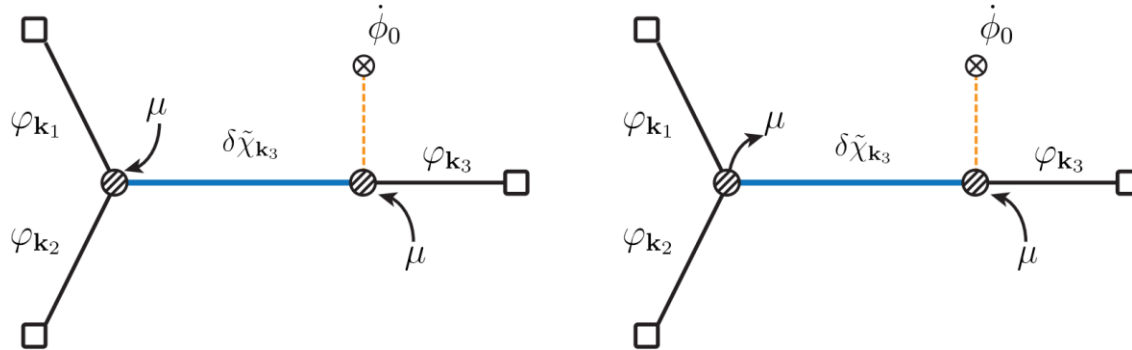
$$n_B^{(0)} = \frac{2\mu A^2 v_\sigma^4}{m_\chi^4 + 9H^2 \mu^2} z_{\text{ph}}^{-3}$$

$$\eta = \frac{n_B^{(0)}}{n_\gamma} \approx 10^{-9} \times \left(\frac{H}{10^{14} \text{ GeV}} \right)^{-1/2} \times \frac{c_A^2 c_\mu \theta}{9c_\mu^2 + c_{m_\chi}^4} \times e^{-(3N_e - 29)}$$

$$c_A = \frac{A}{H}, \quad c_\mu = \frac{\mu}{H}, \quad c_{m_\chi} = \frac{m_\chi}{H}, \quad \theta = \frac{v_\sigma^4}{\rho_{\text{inf}}}$$

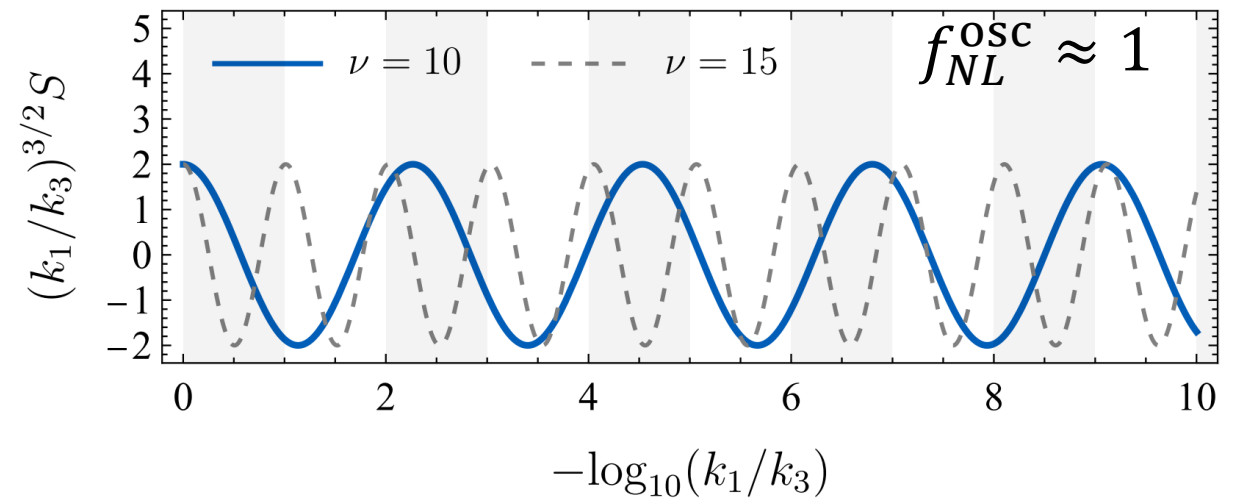
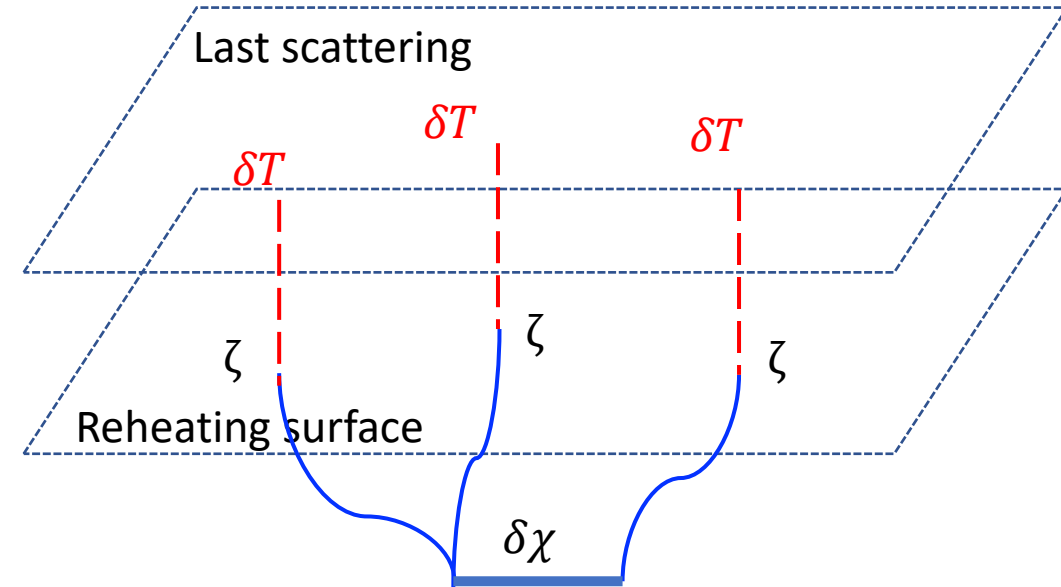
Baryogenesis during inflation

- Cosmological collider signal
Bodas, Kumar, Sundrum, 2010.04727

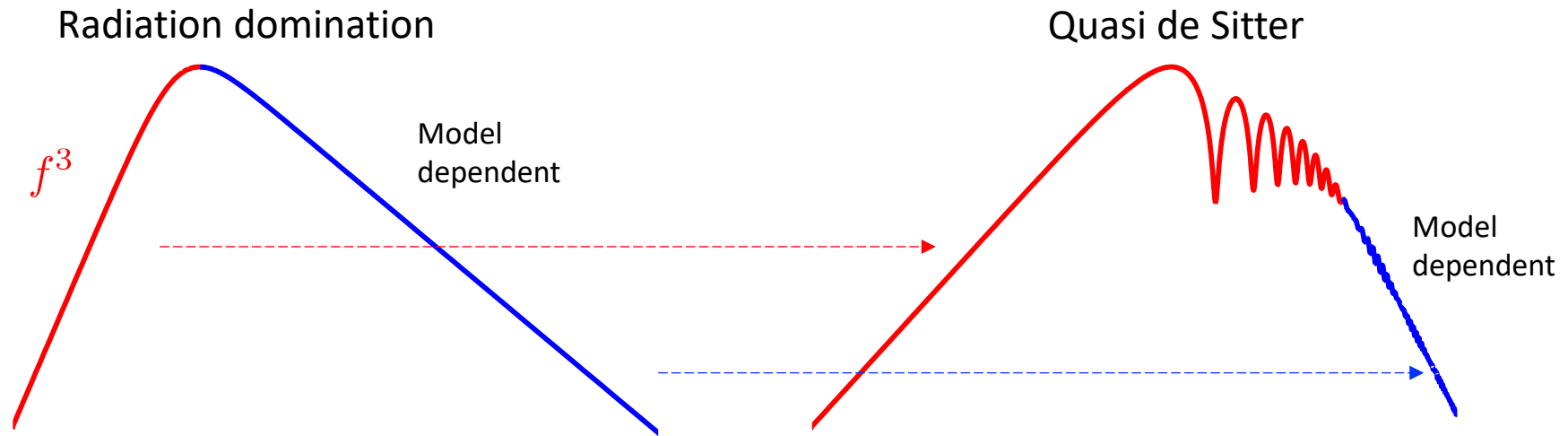


$$\nu \equiv \sqrt{m_{\text{eff}}^2/H^2 - 9/4}$$

$$m_{\text{eff}}^2 \equiv m_\chi^2 + \mu^2$$



Spectrum distortion by inflation



GW from instantaneous and local sources (qualitative study)

- E.O.M. of GW

$$h''_{ij} + \frac{2a'}{a}h'_{ij} - \nabla^2 h_{ij} = 16\pi^2 G_N a^2 \sigma_{ij}$$

$$ds^2 = a^2(\tau) [-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$



Traceless and transverse

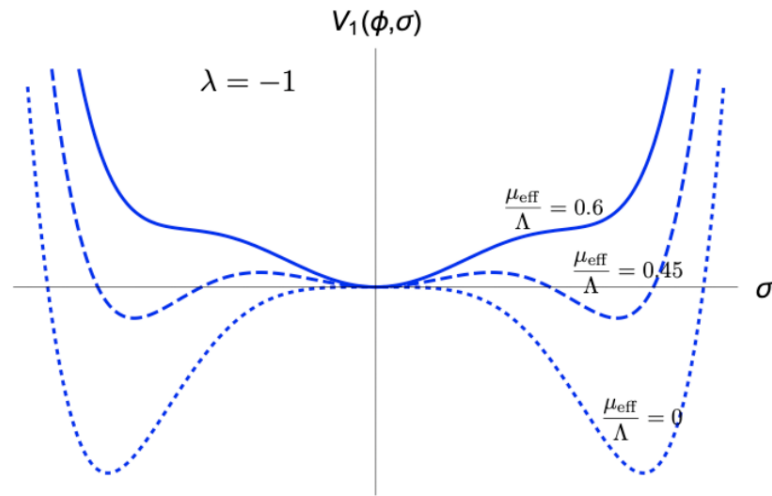
- For an instantaneous and local source,

$$\sigma_{ij} \sim \delta(\mathbf{x})\delta(\tau - \tau')$$

- E.O.M. in Fourier space

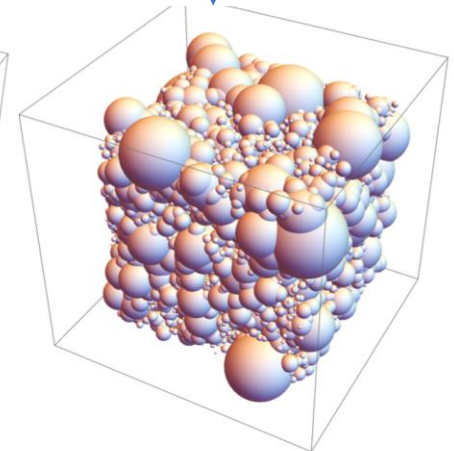
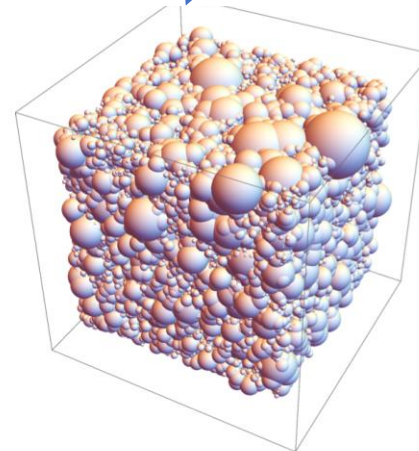
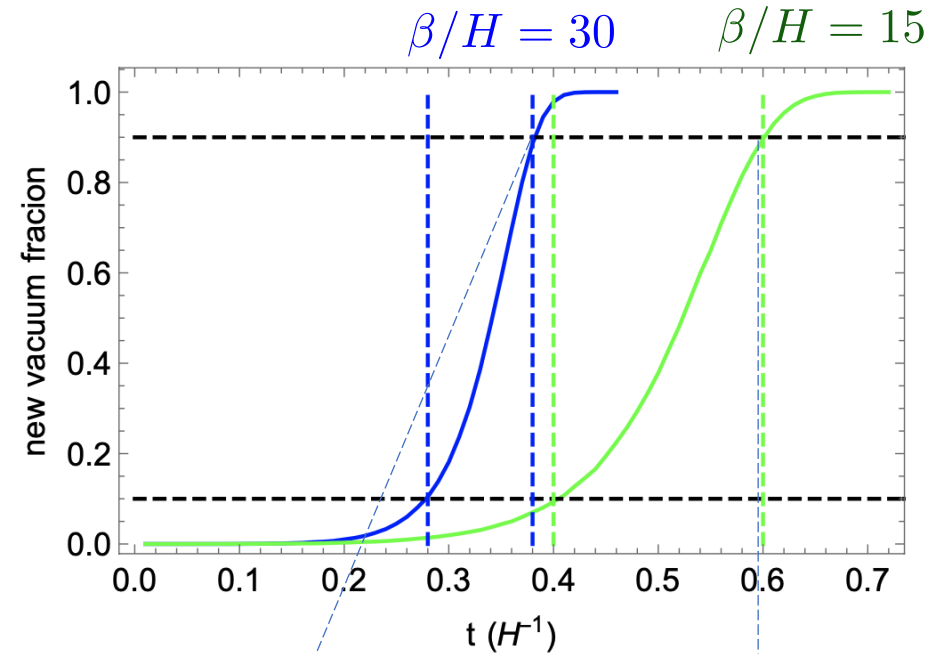
$$h''(\tau, \mathbf{k}) + \frac{2a'}{a}h'(\tau, \mathbf{k}) + k^2 h(\tau, \mathbf{k}) = 16\pi G_N a^{-1} T \delta(\tau - \tau')$$

First-order phase transition during inflation

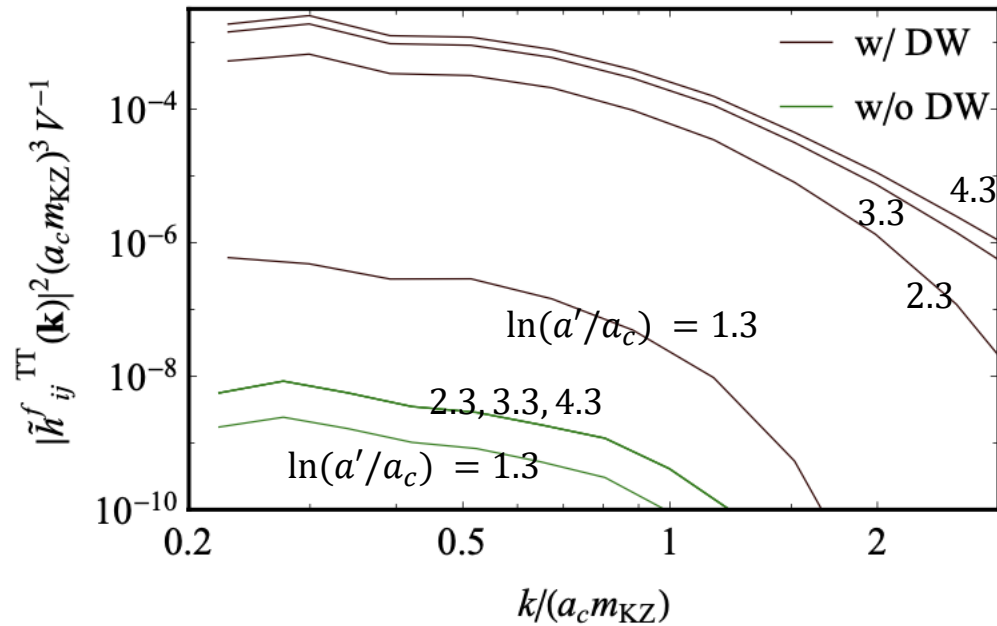


S_4 becomes smaller during

- $\beta = -\frac{dS_4}{dt}$, determines the rate of the phase transition.
- Phase transition completes if $\beta \gg H$.



Calculation of GWs



$$\tilde{h}_{ij}^f(\mathbf{k}) = \frac{16\pi G_N}{k} \int_{-\infty}^0 d\tau' \mathcal{K}(k\tau') \tilde{T}_{ij}^{TT}(\tau', \mathbf{k})$$

With domains, the dominant contribution to $\tilde{h}^f$ happens around $\ln(a'/a_c) \sim 2$ to 3.

Without domains ($\delta\sigma \rightarrow |\delta\sigma|$), the dominant contribution to $\tilde{h}^f$ stops around $\ln(a'/a_c) \sim 2$, and the magnitude is much smaller.

The dominant contribution to GWs is from domain walls.

Formation of domain walls

- Landau-Ginzburg type

$$V = -\frac{1}{2}m_{\text{eff}}^2\sigma^2 + \frac{\lambda}{4}\sigma^4$$

$$m_{\text{eff}}^2 = y\phi^2 - m^2$$

↓
Inflaton field

- Kibble-Zurek mechanism

c for critical

$$V_{\text{KZ}} = -\frac{1}{2}m_{\text{KZ}}^3 a_c^{-1}(\tau - \tau_c)\sigma^2 + \frac{\lambda}{4}\sigma^4$$

- m_{KZ} determines the average distances between the domain walls.

Kibble 1976, Zurek 1985

$$m_{\text{KZ}(B)}^3 = -y a_c \frac{d\phi_0^2}{d\tau} = \frac{2^{3/2}\epsilon^{1/2}m^2 H M_{\text{pl}}}{\phi_0(\tau_c)}$$

Murayama & Shu, 0905.1720

$$H^2 \ll m_{\text{KZ}}^2 \ll m^2$$



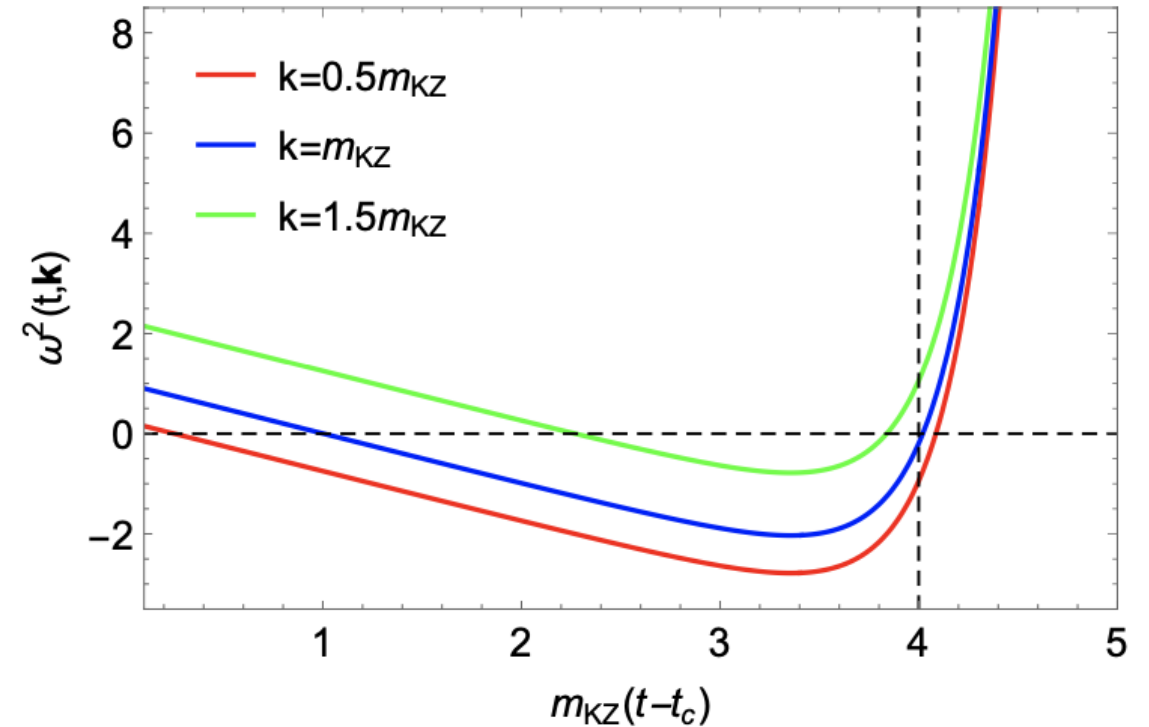
Formation of domain walls

- Stop of the tachyonic growth

$$k^2 - a_c^2 m_{KZ}^3 (\tau - \tau_c) + \frac{\lambda}{2} \langle \sigma^2(\tau, \mathbf{x}) \rangle$$

↓
Growth exponentially

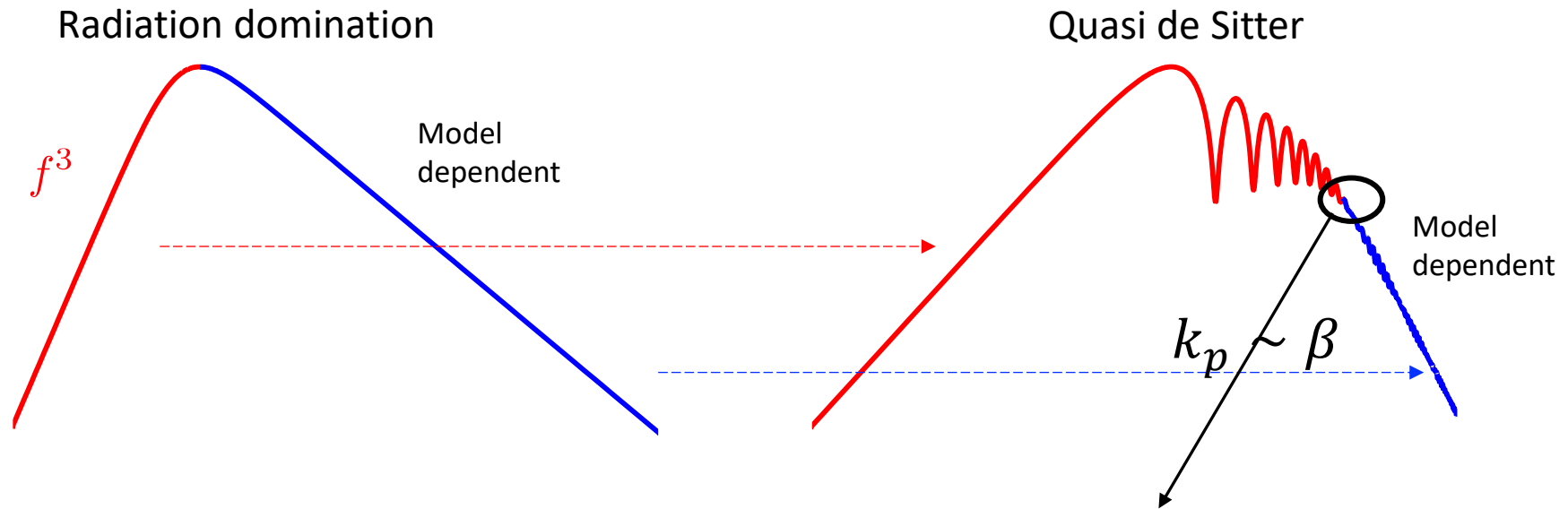
- Only modes with k smaller than about m_{KZ} can have a chance to grow exponentially.



Outlook

- The fate of the domain walls.
- Other topological defects.
- Application to high scale particle physics models.
- Baryogenesis (work in progress)

Spectrum distortion by inflation

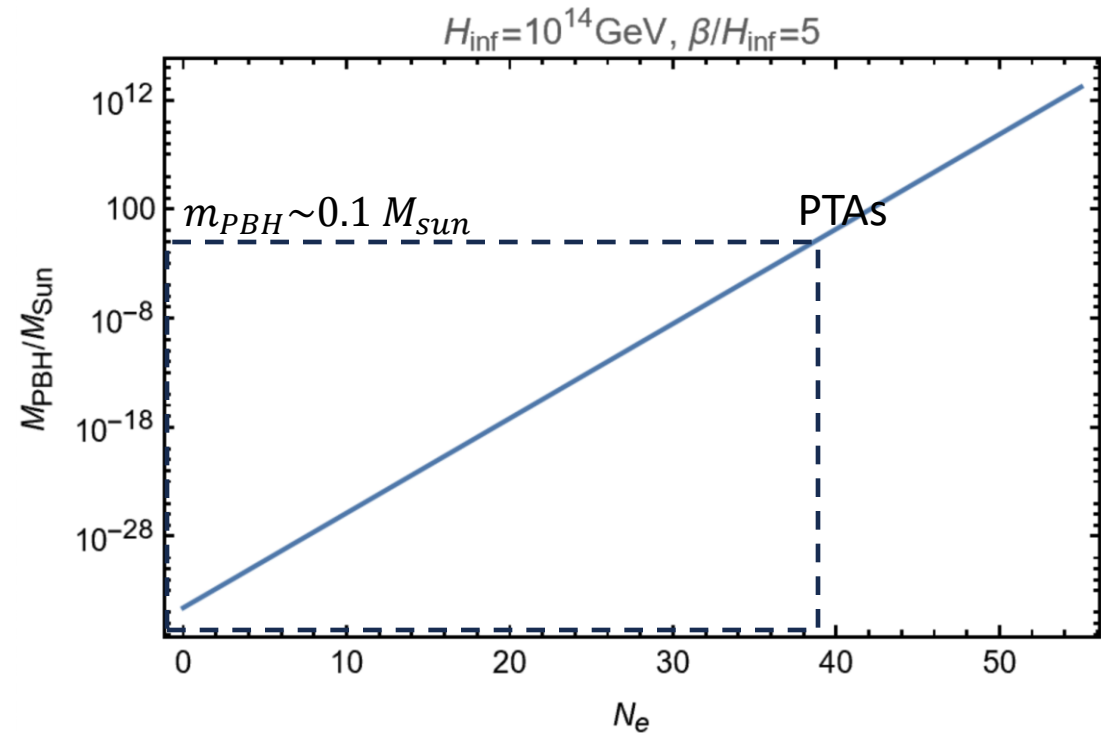
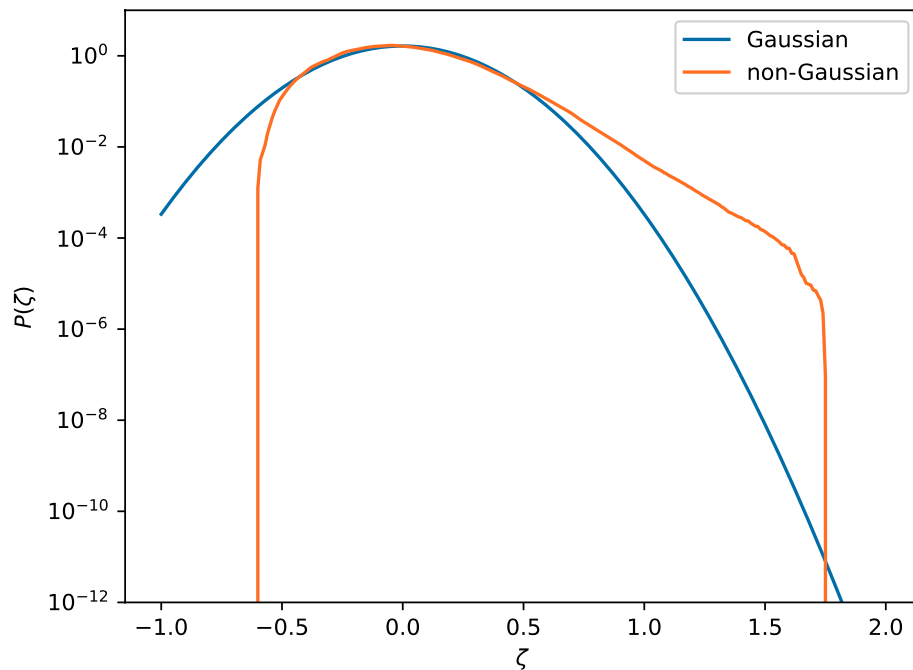


$$\Omega_{\text{GW}} \approx \Omega_R \left(\frac{H_{\text{inf}}}{\beta} \right)^6 \left(\frac{\Delta \rho_{\text{vac}}}{\rho_{\text{inf}}} \right)^2$$

Primordial Black Holes

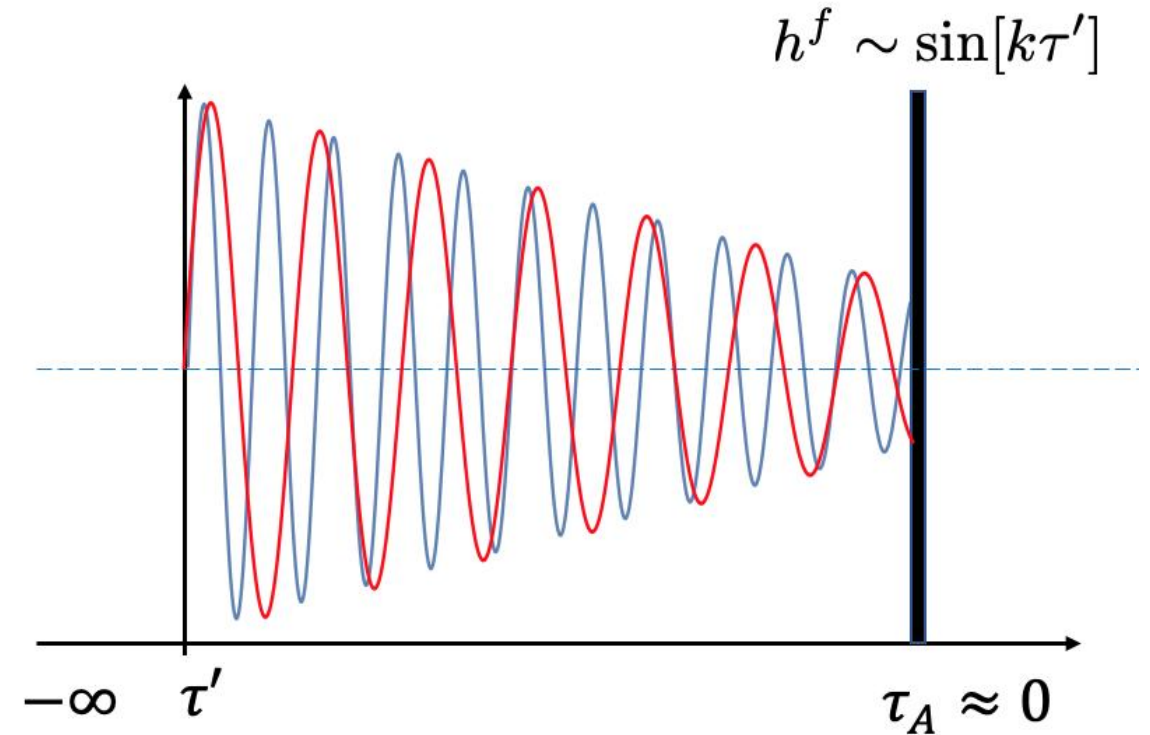
HA, Boye Su, Lian-Tao Wang, Chen Yang, work in progress

- PBHs will form if $\Delta_{\zeta}^2 \sim 0.01$
- The power spectrum is highly non-Gaussian



GW from instantaneous and local sources (qualitative study)

- The conformal time between the source and the horizon is fixed.
- The phase of h at the source is fixed.
- The value of h^f at the horizon **oscillates** with k .
- h^f is the **initial condition** for later evolution.



$$k\tau_A \approx 0$$

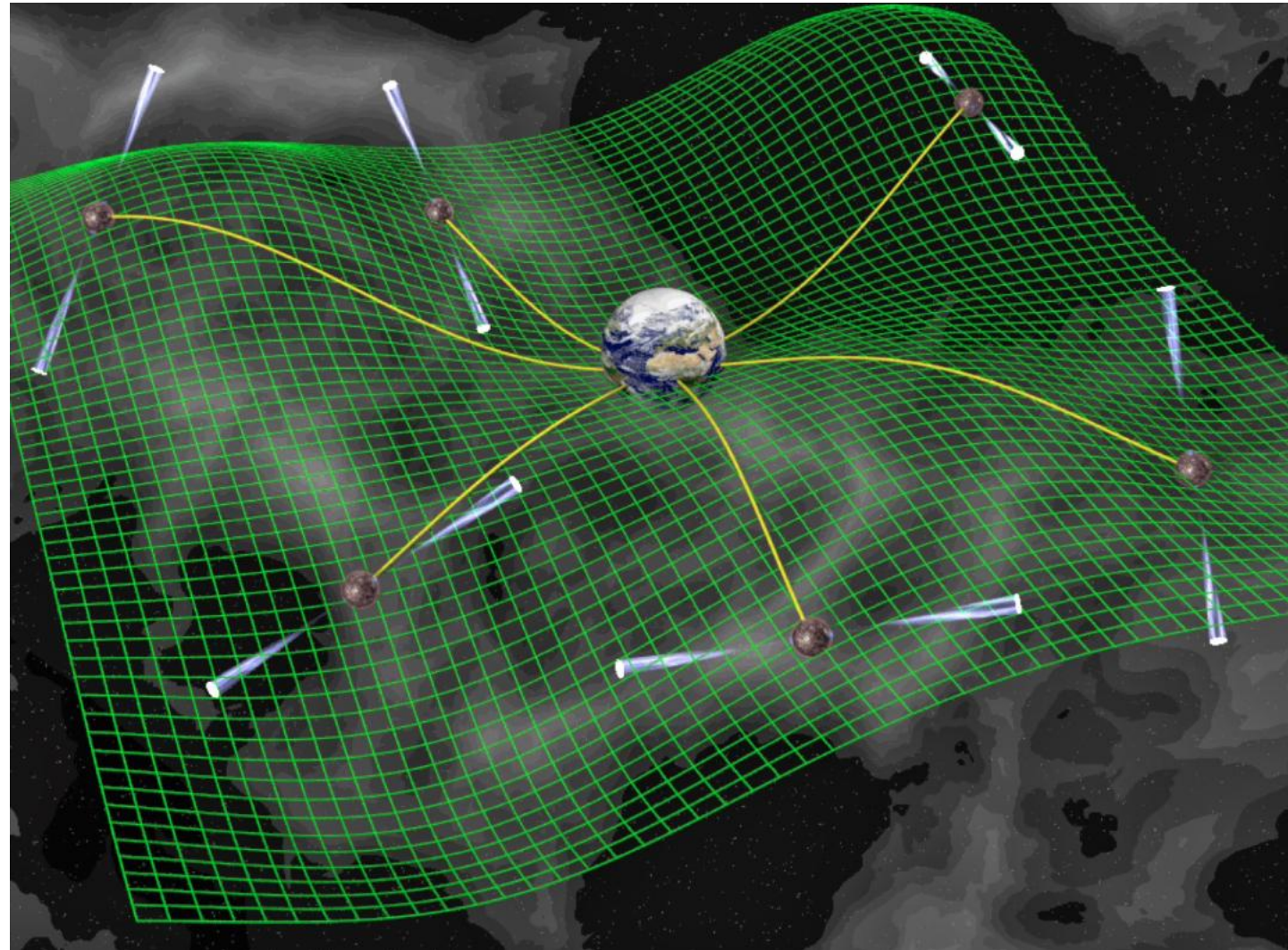
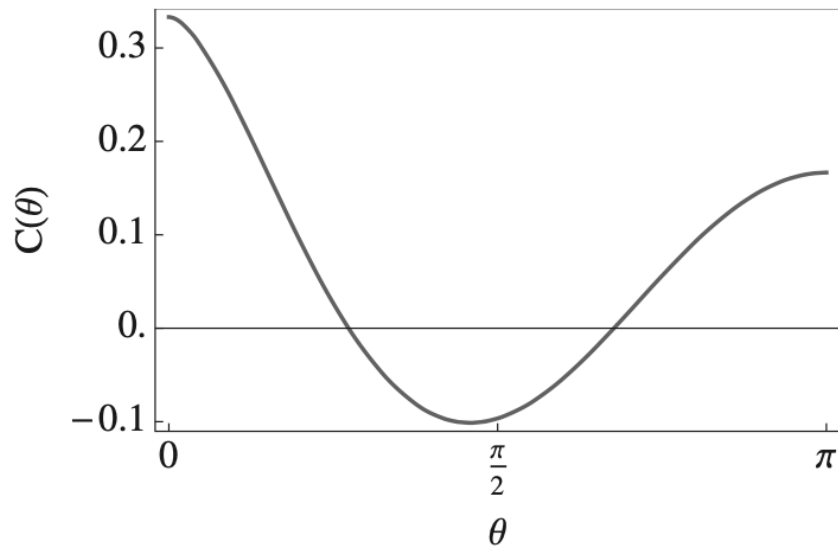
Observation from PTAs

- Hellings-Downs curve

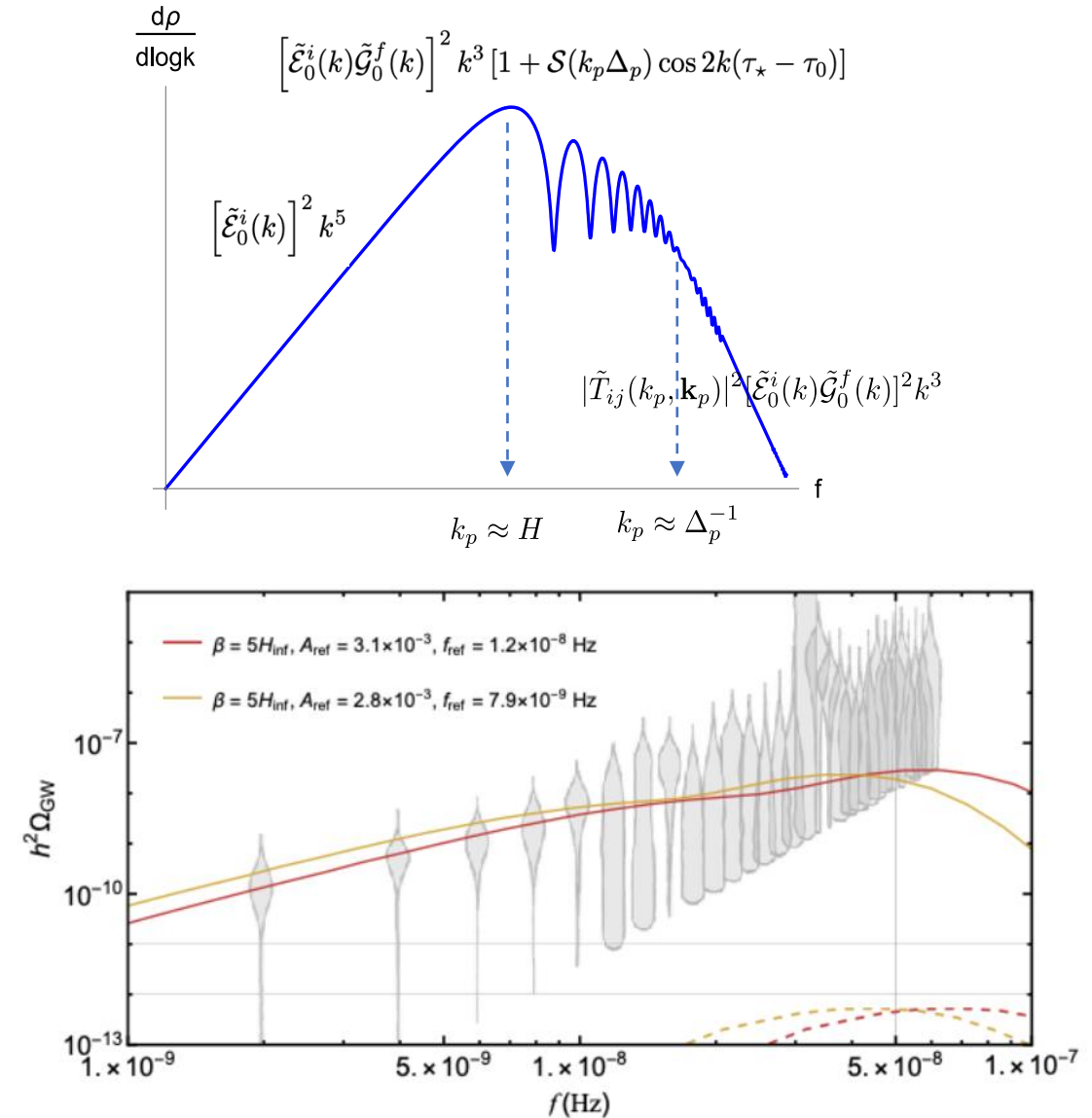
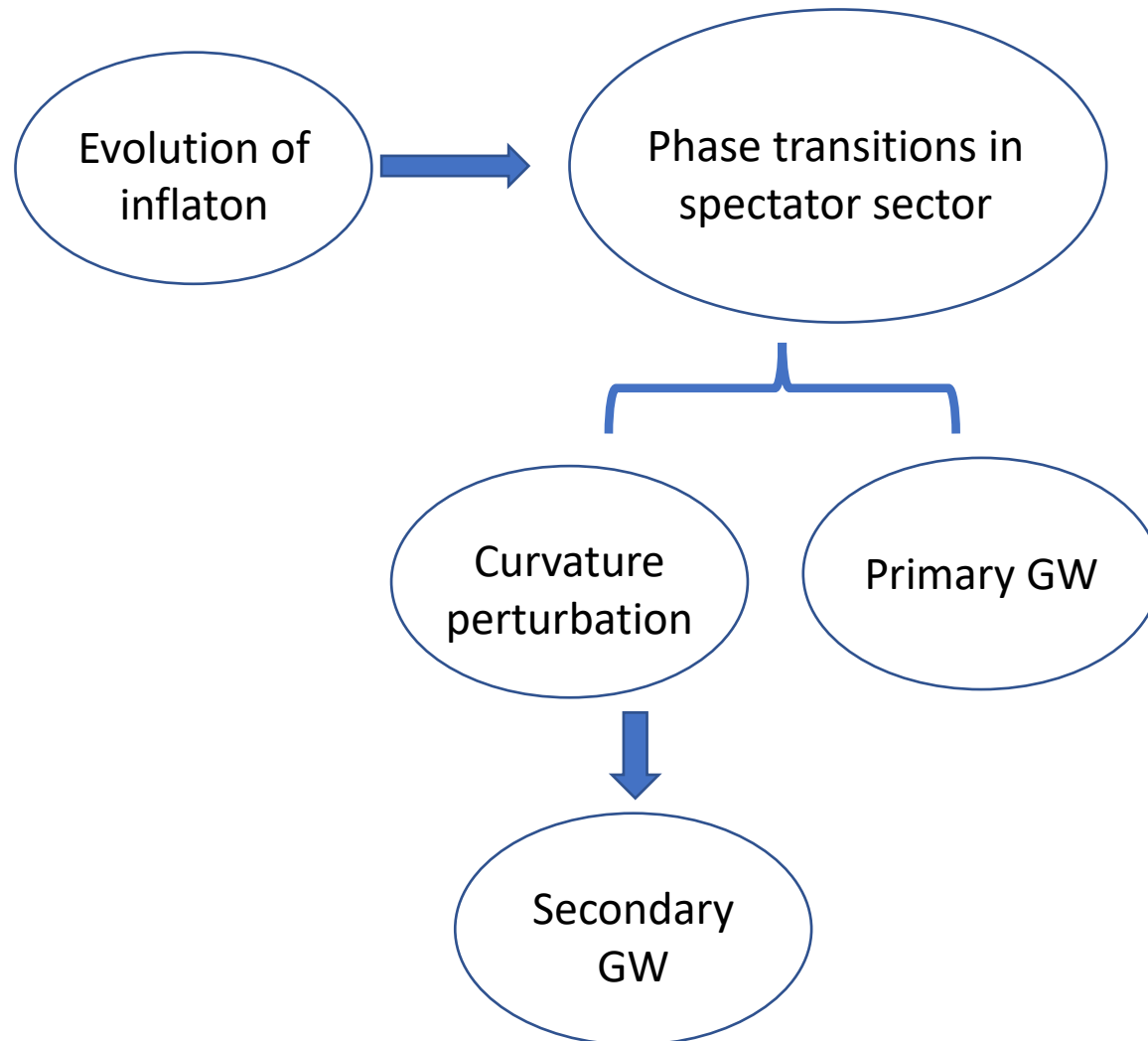
$$\langle z_a(t) z_b(t) \rangle = C(\theta_{ab}) \int_0^\infty df S_h(f)$$

Angular correlation

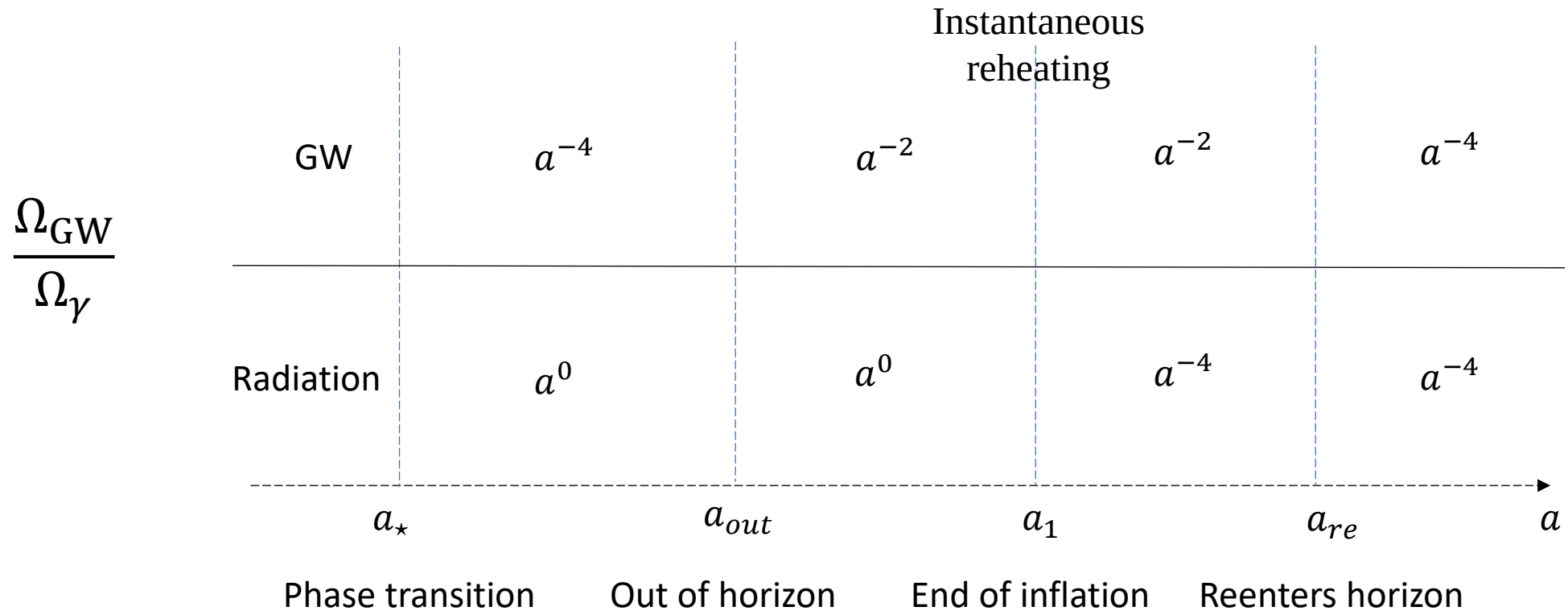
$$z_a(t) = -(\Delta\nu_a/\nu_a)(t) = \Delta T_a/T_a$$



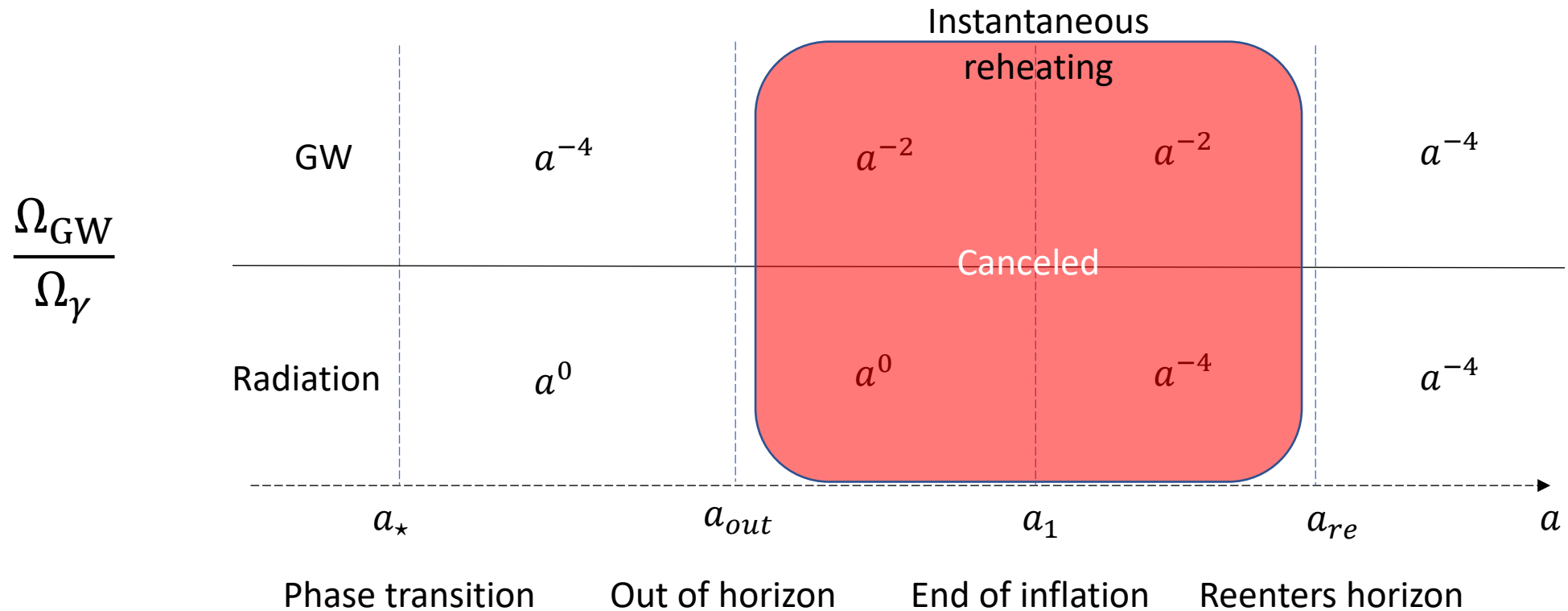
Summary for FOPT



Redshifts of the GW signal

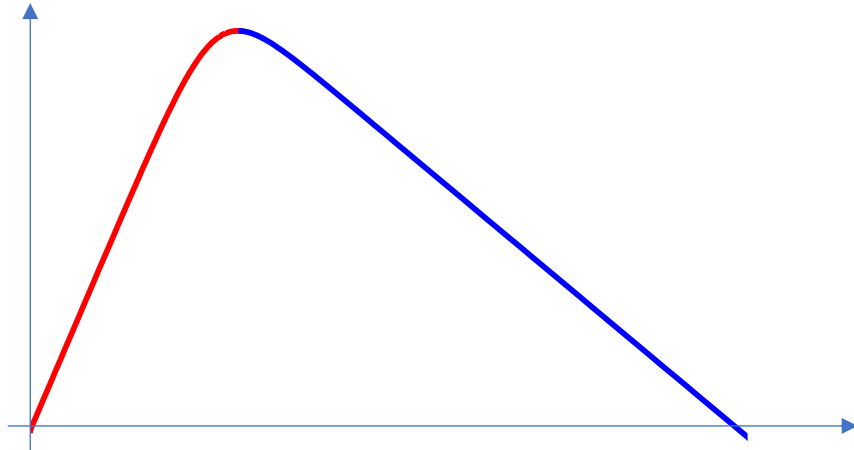


Redshifts of the GW signal



$$\frac{\Omega_{\text{GW}}}{\Omega_\gamma} \sim \left(\frac{a_\star}{a_{\text{out}}} \right)^4 \sim \left(\frac{H}{\beta} \right)^4$$

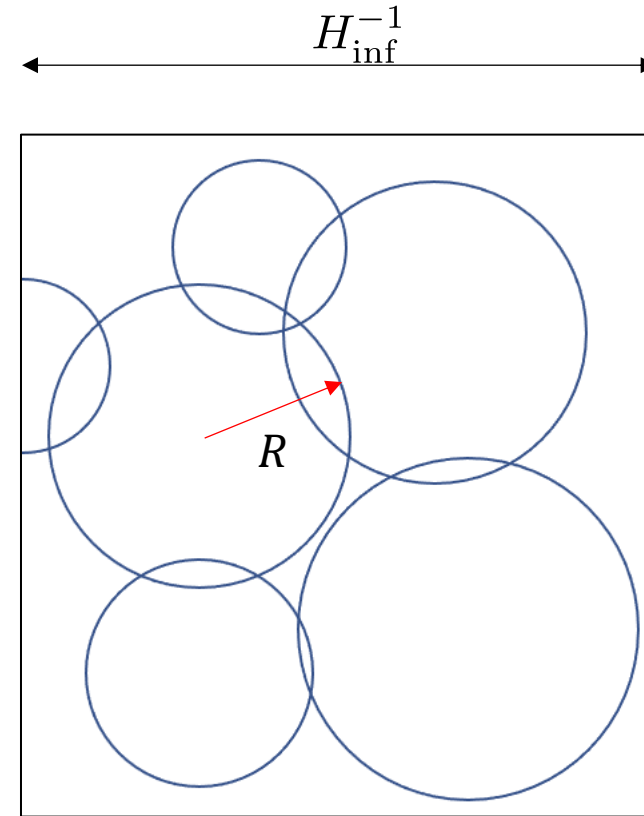
GWs produced in flat space-time



$$\frac{d\rho_{\text{GW}}^{\text{flat}}}{\Delta\rho_{\text{vac}} d\log k_p} \approx \left(\frac{H_{\text{inf}}}{\beta} \right)^2 \times \frac{\beta k_p^{2.8}}{\beta^{3.8} + 2.8 k_p^{3.8}}$$

Huber and Konstandin, 0806.1828

$$\Omega_{\text{GW}}^{(0)} \approx \Omega_R \left(\frac{H_{\text{inf}}}{\beta} \right)^2 \frac{\beta k_p^{2.8}}{\beta^{3.8} + 2.8 k_p^{3.8}}$$



First order phase transition during inflation

- $$\beta = \left| \frac{dS_4}{dt} \right| = \frac{dS_4}{d \log \mu_{\text{eff}}^2} \times \left| \frac{2\dot{\phi}}{\phi \left(1 - \frac{\mu^2}{c^2 \phi^2} \right)} \right|$$

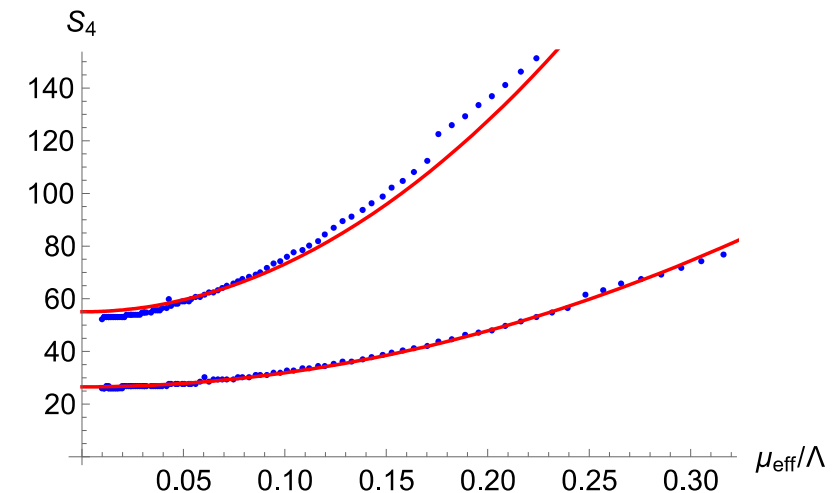
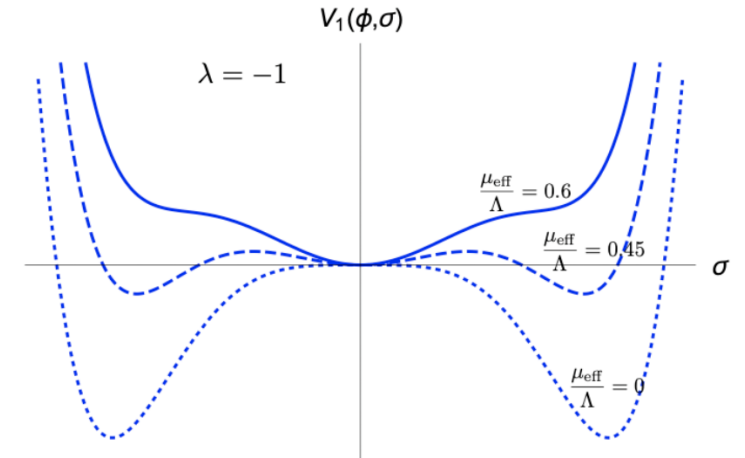
$$\Rightarrow \frac{\beta}{H} = \left| \frac{dS_4}{d \log \mu_{\text{eff}}^2} \right| (2\epsilon)^{1/2} \times \frac{M_{\text{pl}}}{\phi \left(1 - \frac{\mu^2}{c^2 \phi^2} \right)}$$

$$\int_{\phi_{\text{end}}}^{\phi_{\text{PT}}} \frac{d\phi}{\sqrt{2\epsilon} M_{\text{pl}}} = N_e$$

$$\sim \mu_{\text{eff}}^2 / \Lambda^2$$

$$\frac{\beta}{H} \sim \left| \frac{dS_4}{d \log \mu_{\text{eff}}^2} \right| \times \frac{\Lambda^2}{\mu_{\text{eff}}^2} \times \frac{1}{N_e}$$

It is natural to have $\beta/H \sim O(10)$.

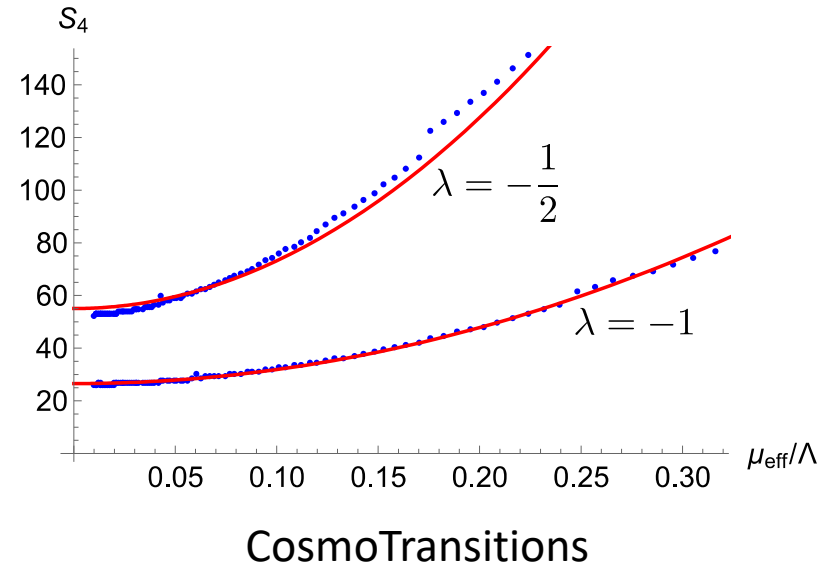
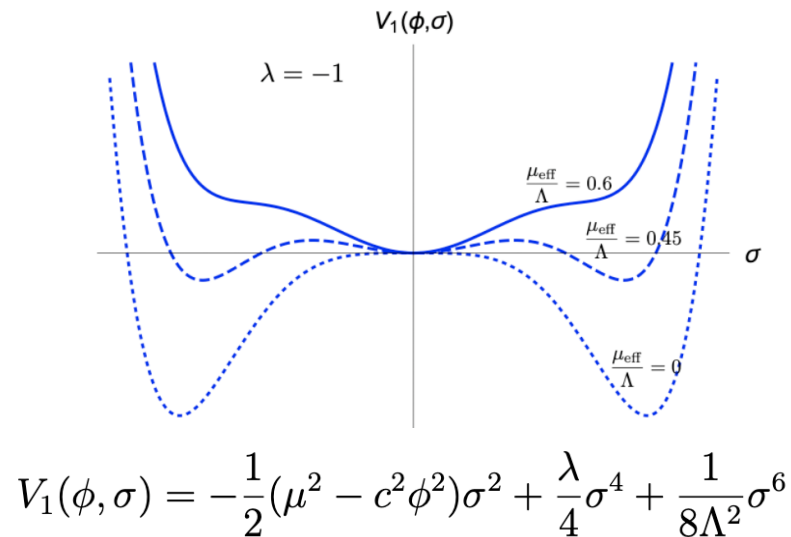


First order phase transition during inflation

- $$\beta = \left| \frac{dS_4}{dt} \right| = \frac{dS_4}{d \log \mu_{\text{eff}}^2} \times \left| \frac{2\dot{\phi}}{\phi \left(1 - \frac{\mu^2}{c^2 \phi^2} \right)} \right| \quad \mu_{\text{eff}}^2 = -(\mu^2 - c^2 \phi^2)$$

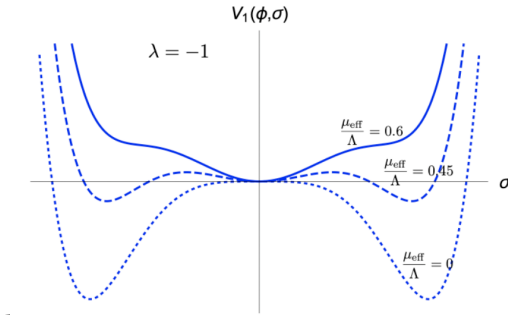


$$\frac{\beta}{H} = \left| \frac{dS_4}{d \log \mu_{\text{eff}}^2} \right| (2\epsilon)^{1/2} \times \frac{M_{\text{pl}}}{\left| \phi \left(1 - \frac{\mu^2}{c^2 \phi^2} \right) \right|}$$



First order phase transition during inflation

- $$\beta = \left| \frac{dS_4}{dt} \right| = \frac{dS_4}{d \log \mu_{\text{eff}}^2} \times \left| \frac{2\dot{\phi}}{\phi \left(1 - \frac{\mu^2}{c^2 \phi^2} \right)} \right|$$



$$\Rightarrow \frac{\beta}{H} = \left| \frac{dS_4}{d \log \mu_{\text{eff}}^2} \right| (2\epsilon)^{1/2} \times \frac{M_{\text{pl}}}{\left| \phi \left(1 - \frac{\mu^2}{c^2 \phi^2} \right) \right|}$$

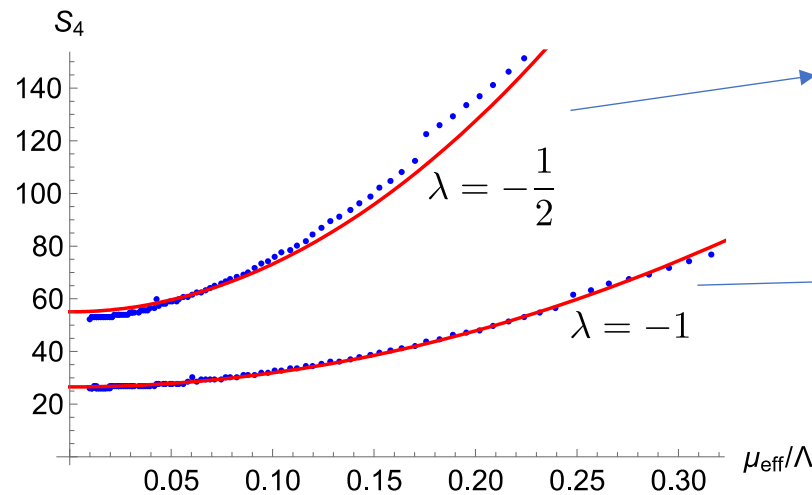
$$\int_{\phi_{\text{end}}}^{\phi_{\text{PT}}} \frac{d\phi}{\sqrt{2\epsilon} M_{\text{pl}}} = N_e$$

$$\sim \mu_{\text{eff}}^2 / \Lambda^2$$

$$\frac{\beta}{H} \sim \left| \frac{dS_4}{d \log \mu_{\text{eff}}^2} \right| \times \frac{\Lambda^2}{\mu_{\text{eff}}^2} \times \frac{1}{N_e}$$

First order phase transition during inflation

- $$\frac{\beta}{H} \sim \left| \frac{dS_4}{d \log \mu_{\text{eff}}^2} \right| \times \frac{\Lambda^2}{\mu_{\text{eff}}^2} \times \frac{1}{N_e}$$



$$\frac{\beta}{H} \sim \frac{3800}{N_e}$$

$$\frac{\beta}{H} \sim \frac{500}{N_e}$$

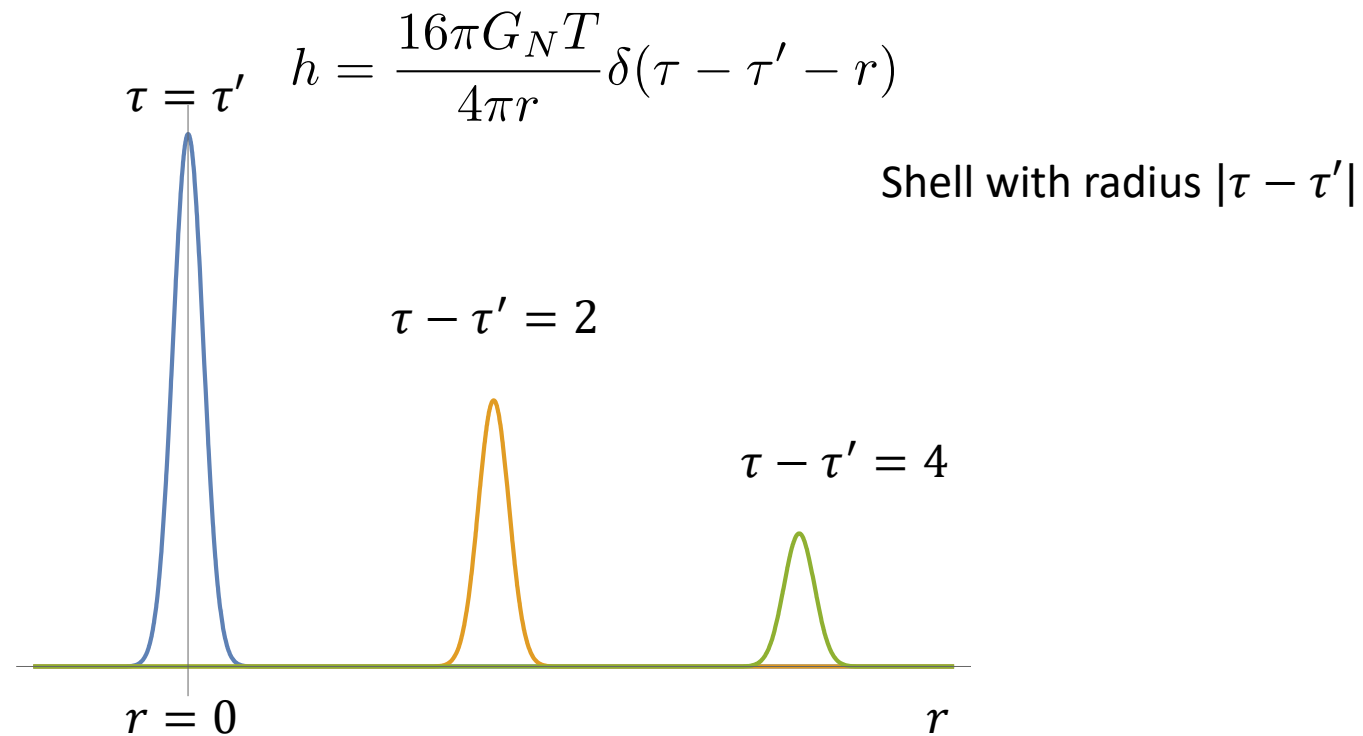
N_e : e-folds before the end of inflation

$$V_1(\phi, \sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

$$\frac{\beta}{H} \sim \mathcal{O}(10) - \mathcal{O}(100)$$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij} ?
- In Minkovski space



de Sitter inflation as an example

- What is the spatial configuration of h_{ij} ?
- In de Sitter space

$$h_{ij}(\tau, \mathbf{k}) = -16\pi G_N H T_{ij} \tau \Theta(\tau - \tau') \left[\frac{\sin k(\tau - \tau')}{k} + \left(\frac{1}{k^2 \tau} - \frac{1}{k^2 \tau'} \right) \cos k(\tau - \tau') + \frac{1}{k^3 \tau \tau'} \sin k(\tau - \tau') \right]$$

de Sitter inflation as an example

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$$h_{ij}(\tau, \mathbf{k}) = -16\pi G_N H T_{ij} \tau \Theta(\tau - \tau') \left[\frac{\sin k(\tau - \tau')}{k} + \left(\frac{1}{k^2 \tau} - \frac{1}{k^2 \tau'} \right) \cos k(\tau - \tau') + \frac{1}{k^3 \tau \tau'} \sin k(\tau - \tau') \right]$$

$\frac{1}{4\pi} \Theta(\tau - \tau' - |\mathbf{x}|)$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij} ?
- In de Sitter space

$$h(\tau, \mathbf{x}) \sim \underbrace{\frac{\tau}{4\pi x} \delta(\tau - \tau' - x)}_{\text{Similar to Minkovski}} + \underbrace{\frac{1}{4\pi} \Theta(\tau - \tau' - x)}_{\text{Intrinsic in de Sitter}}$$

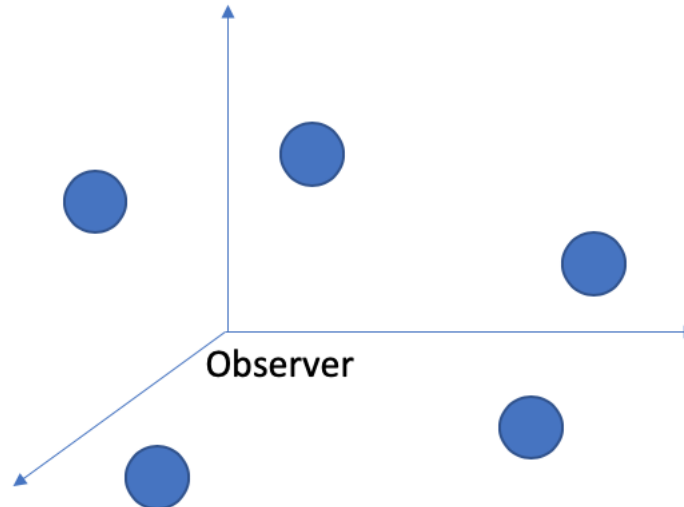
Decreases with both x and τ

constant

Vanishes out of horizon

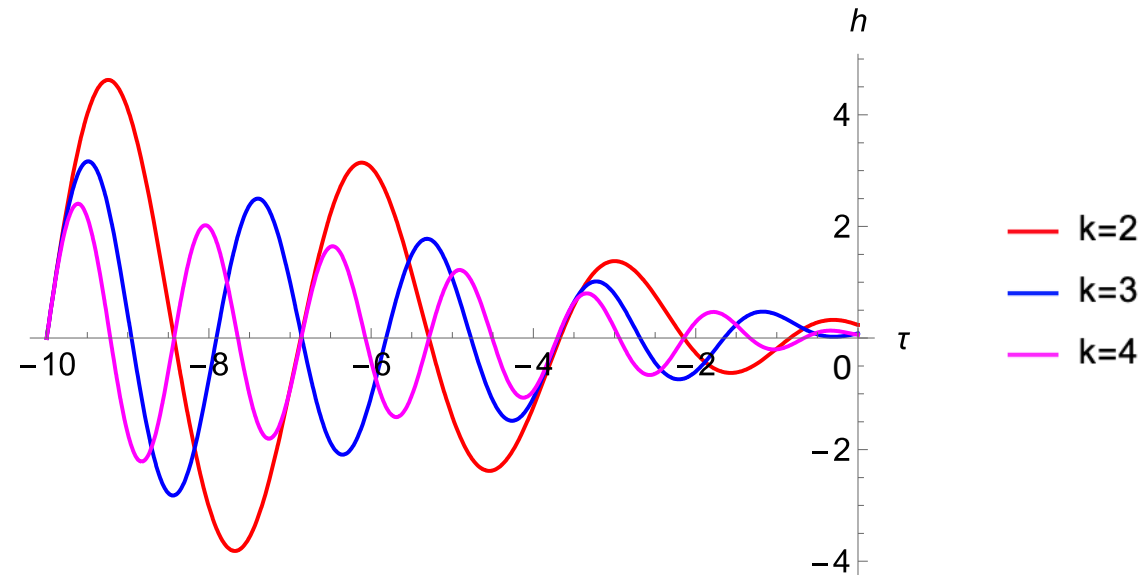
de Sitter inflation as an example

- At $\tau \rightarrow 0$ $h(\tau, \mathbf{x}) \sim \frac{1}{4\pi} \Theta(|\tau'| - x)$
- A ball of GW, with radius $|\tau'|$
- h uniformly distributed inside the GW balls.
- All the balls have the same radius.



Quasi-de Sitter inflation as an example

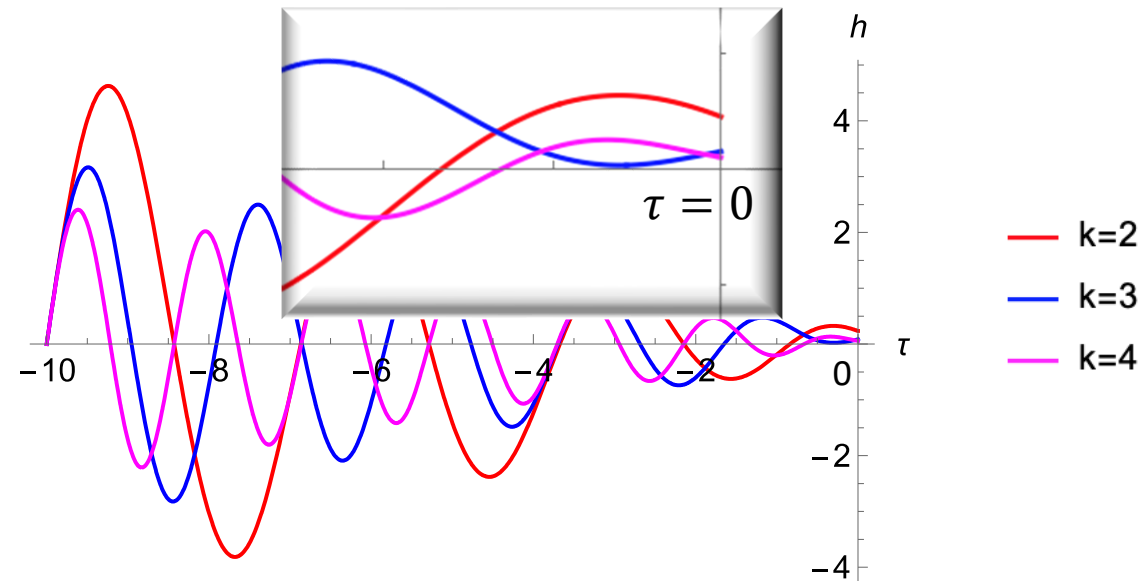
- $a = -\frac{1}{H\tau}$
- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij} \tau}{k} \left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'} \right) \cos k(\tau - \tau') + \left(1 + \frac{1}{k^2 \tau \tau'} \right) \sin k(\tau - \tau') \right]$$



De Sitter inflation as an example

- $$a = -\frac{1}{H\tau}$$

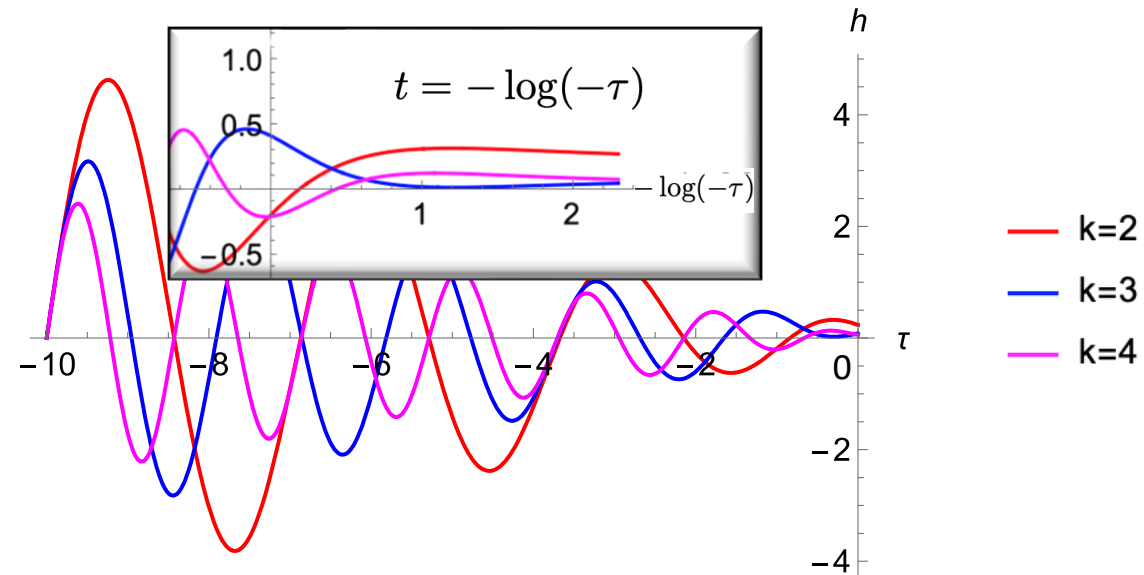
- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij} \tau}{k} \left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'} \right) \cos k(\tau - \tau') + \left(1 + \frac{1}{k^2 \tau \tau'} \right) \sin k(\tau - \tau') \right]$$



De Sitter inflation as an example

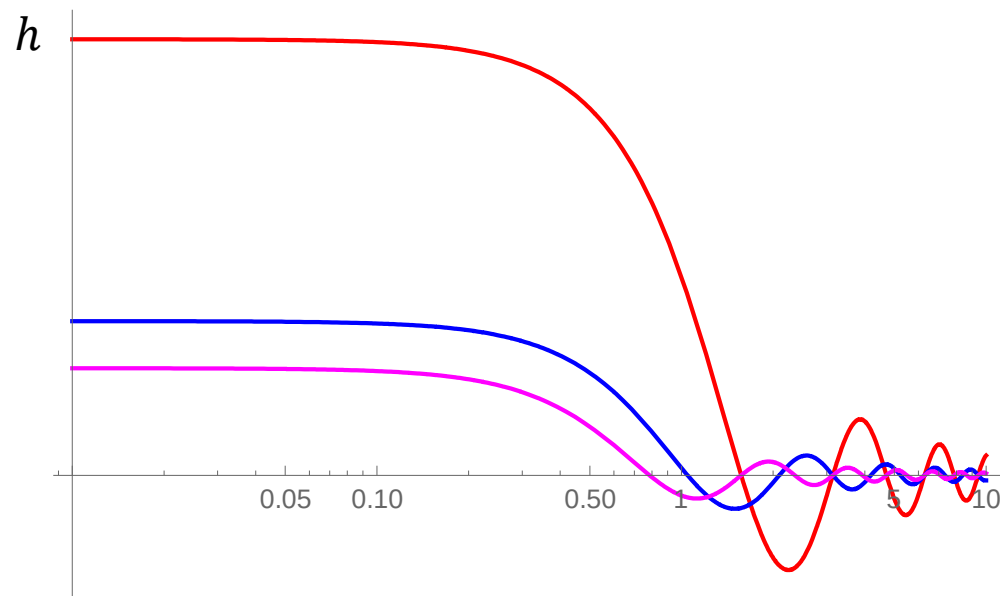
- $$a = -\frac{1}{H\tau}$$

- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij} \tau}{k} \left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'} \right) \cos k(\tau - \tau') + \left(1 + \frac{1}{k^2 \tau \tau'} \right) \sin k(\tau - \tau') \right]$$



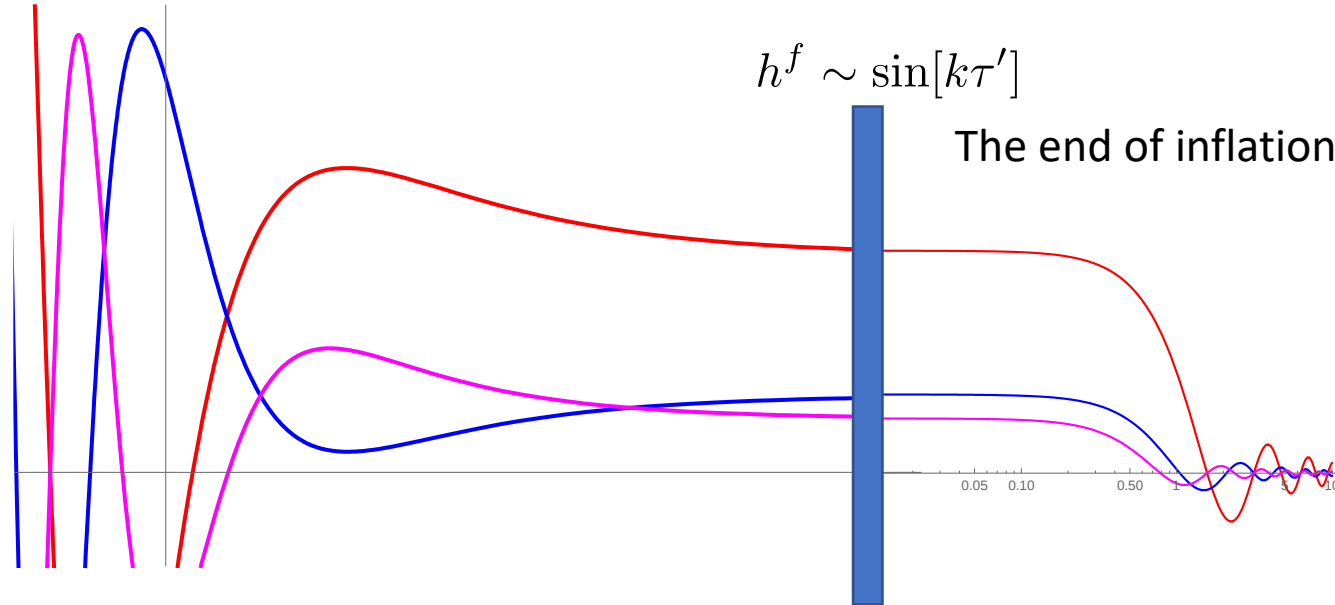
After inflation

- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$



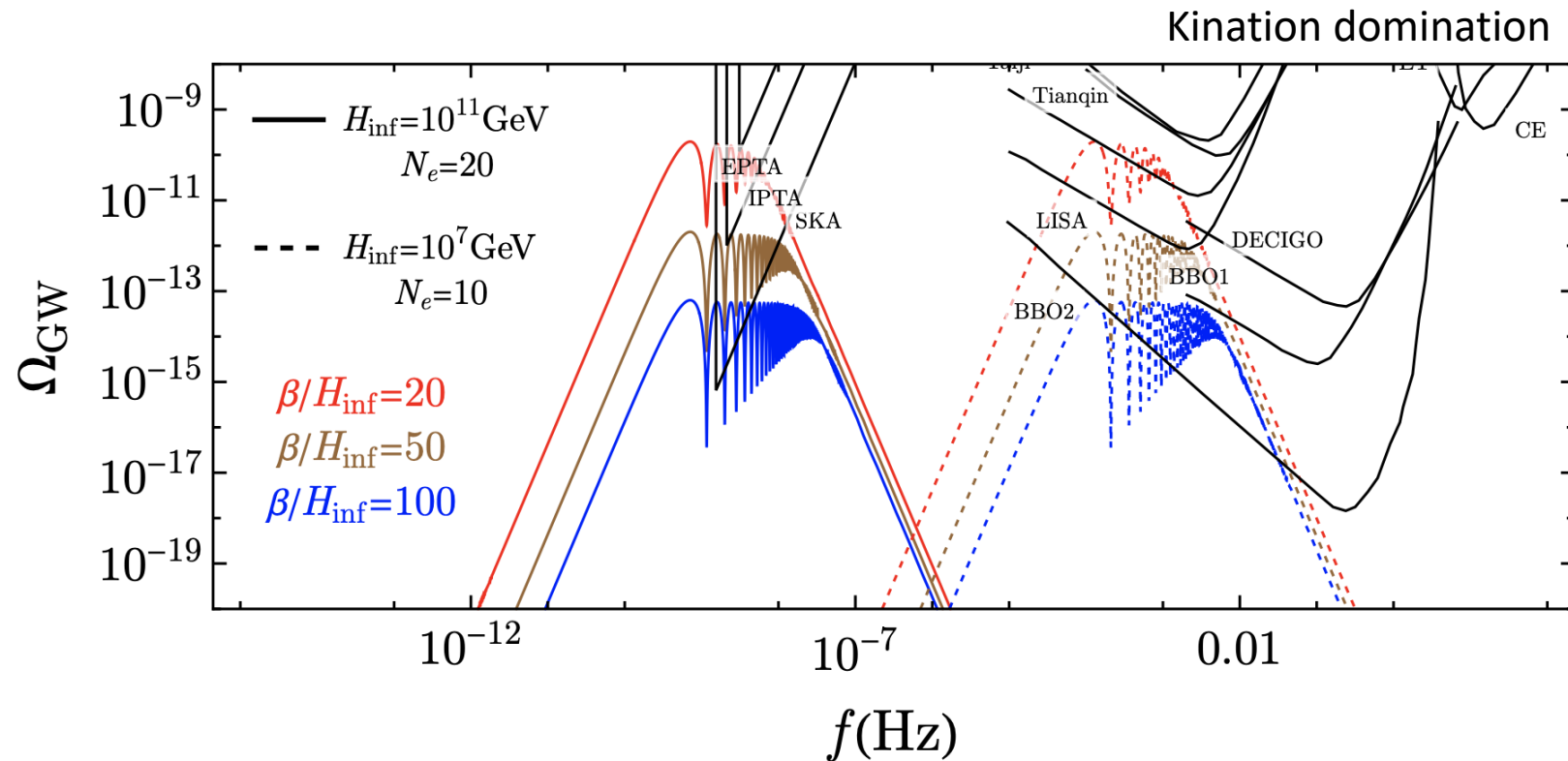
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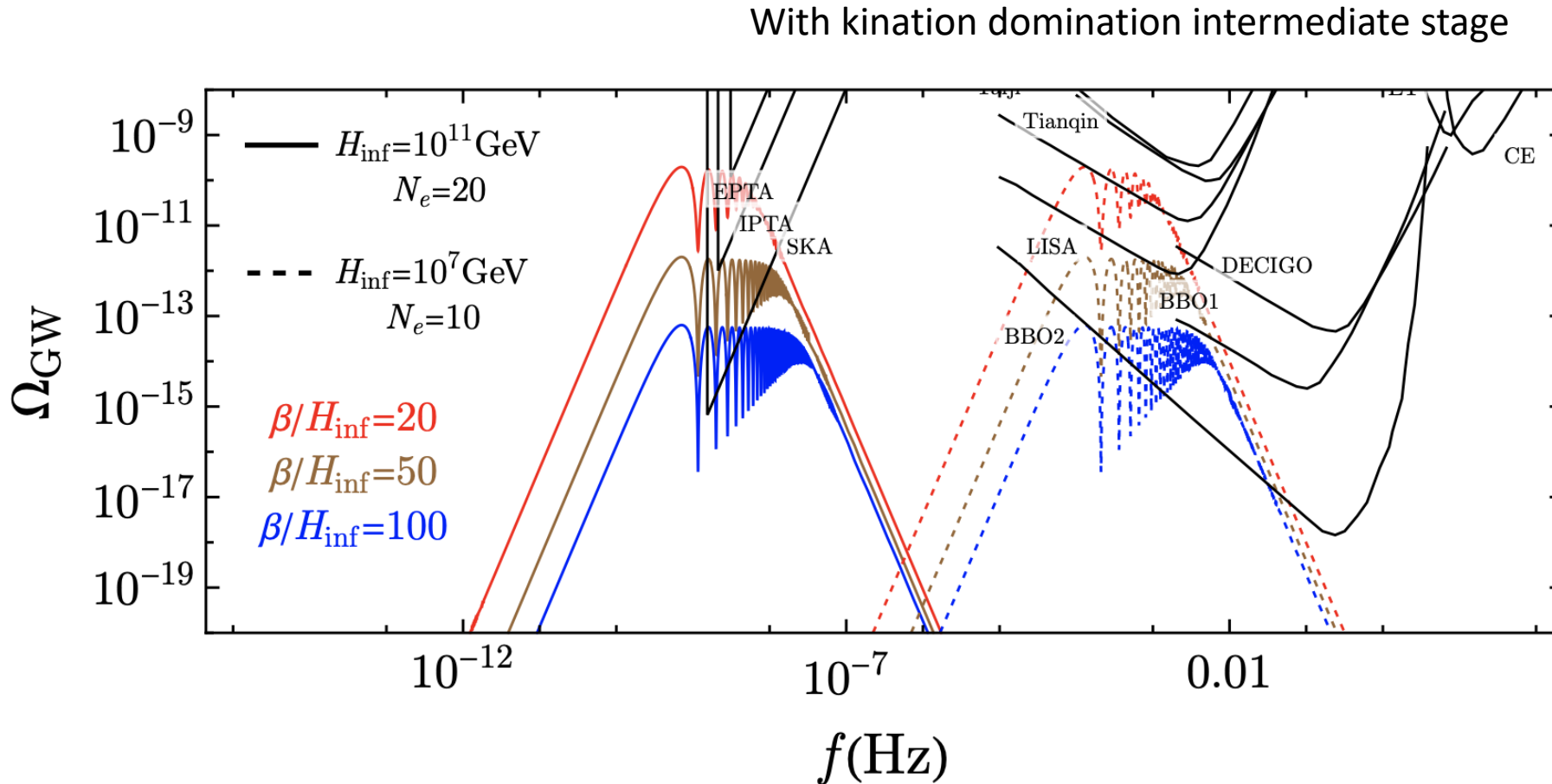


First order phase transition during inflation

- Signal strength is also sensitive to intermediate stages



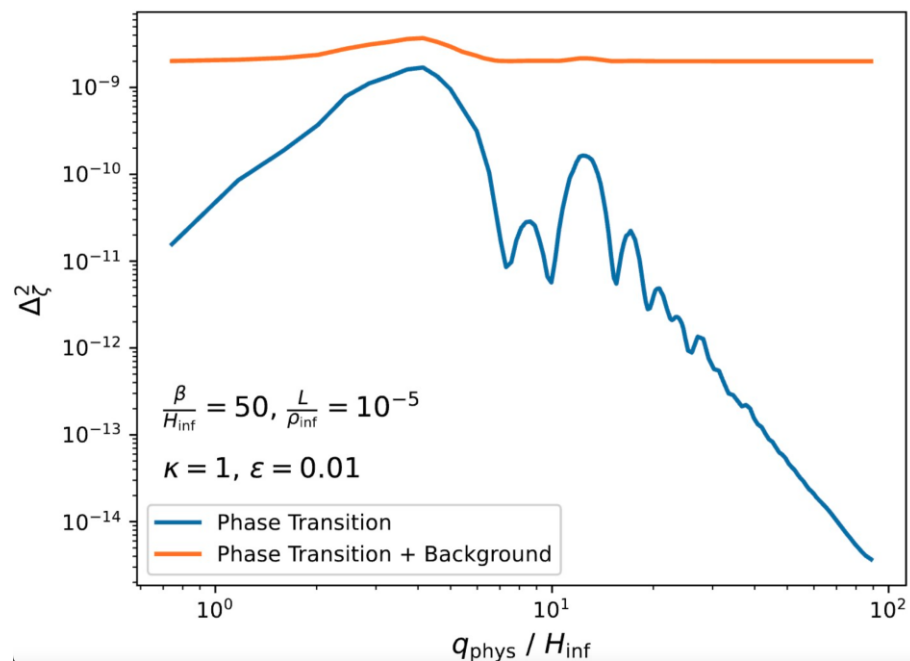
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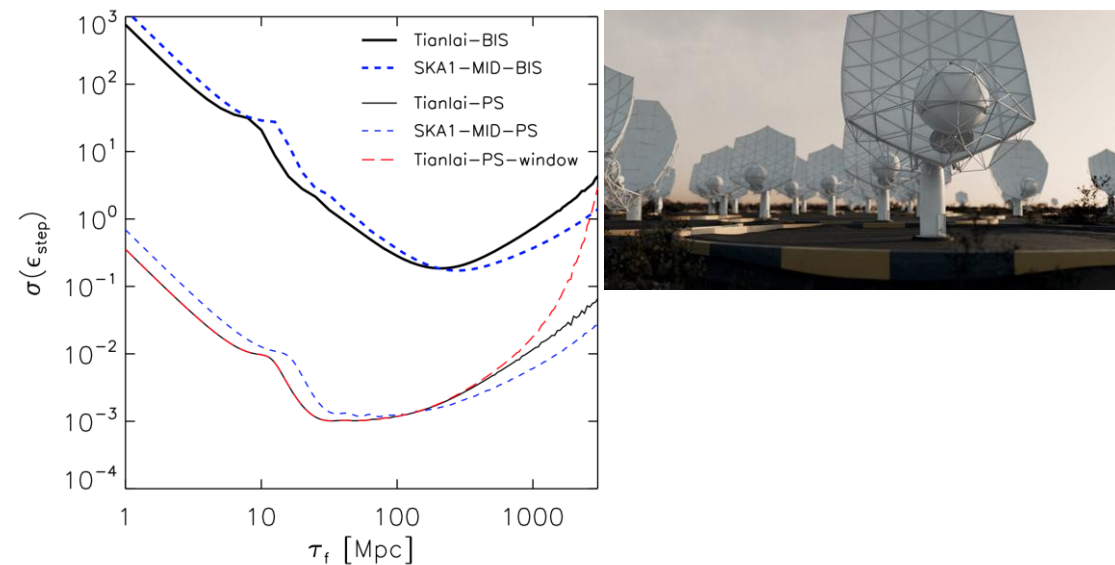
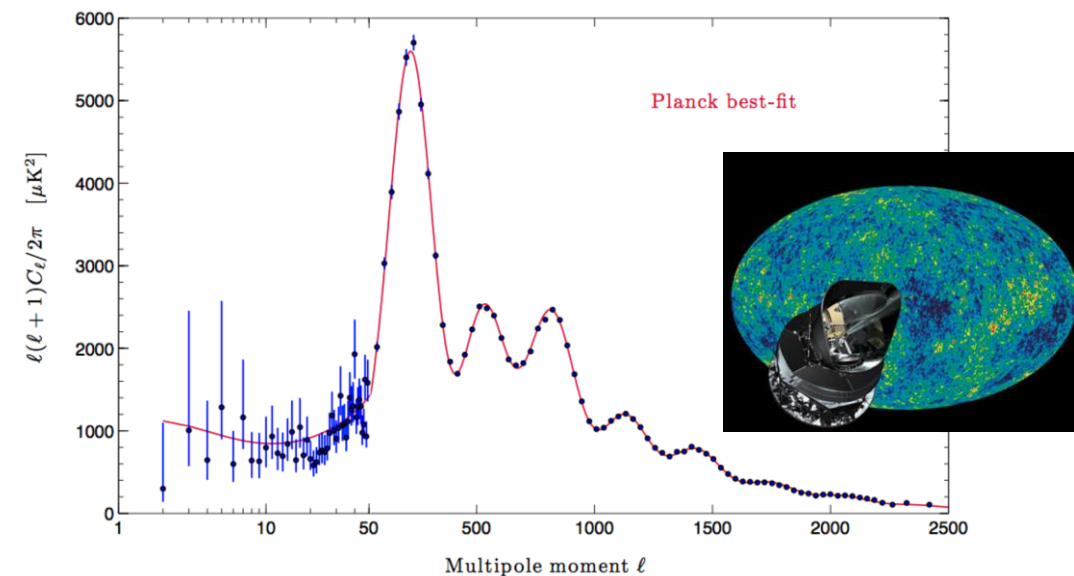
Power spectrum of ζ

$$k_{\text{today}} = (2000 \text{ Mpc})^{-1} \times e^{60-N_e} \times \left(\frac{H_{\text{inf}}}{10^{14} \text{ GeV}} \right)$$

$$A_{\text{ref}} = \frac{\mathcal{A}}{\epsilon} \left(\frac{M_{\text{pl}}}{\phi_0} \right)^2 \left(\frac{H_{\text{inf}}}{\beta} \right)^3 \left(\frac{L}{\rho_{\text{inf}}} \right)^2$$



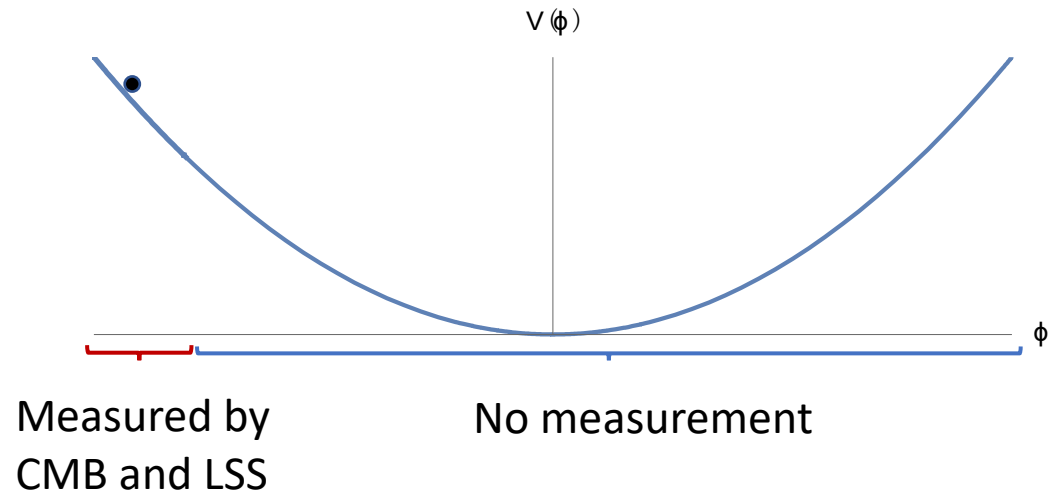
HA, Boye Su, Yidong Xu, Chen Yang, work in progress.



Xu, Hamann, Chen, 1607.00817

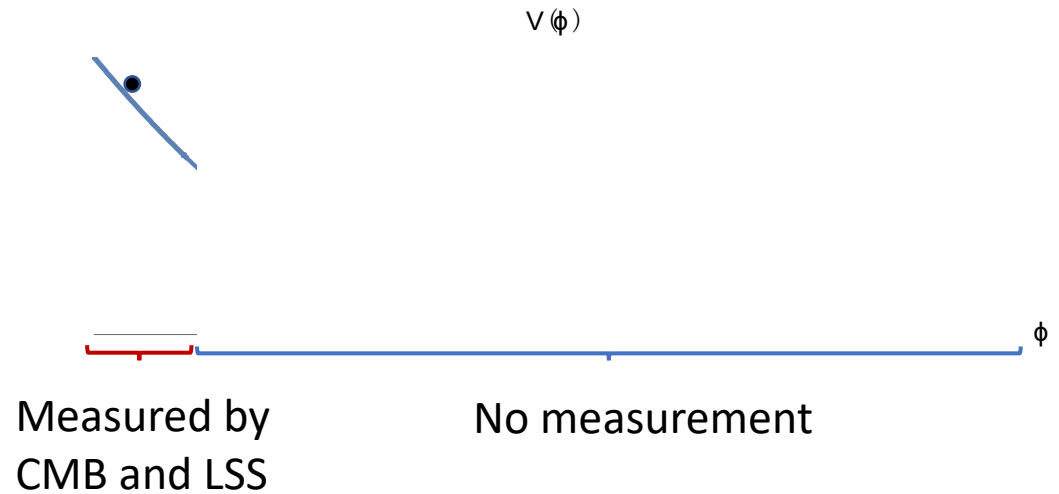
Slow roll models

- We usually assume a potential.
- Use it to calculate $n_s, r \dots$



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