GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

INFN
LNGS
Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali del Gran Sasso

PARTICLE ACCELERATION IN SNR AND THE SHAPING OF THEIR CR SPECTRUM

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The Symposium of Ultra-High-Energy Gamma Rays from Supernova Remnants and the Origin of Galactic Cosmic Rays
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THE SNR PARADIGM

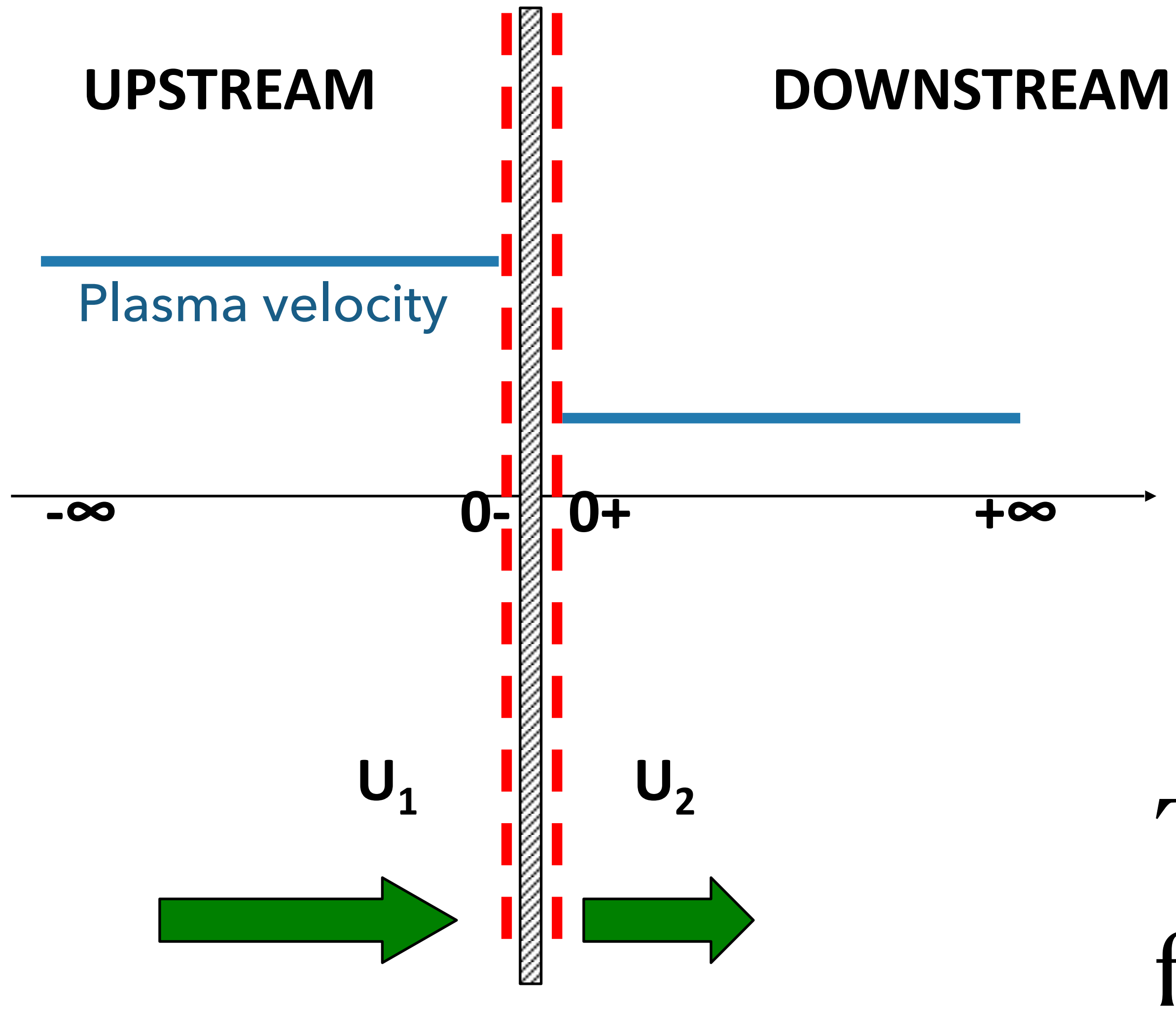
- From the energetic point of view, SNR are the optimal source of Galactic CR, requiring an efficiency of conversion of kinetic energy to CR energy of order 3-10%

- The mechanism of conversion is mostly Diffusive Shock Acceleration at the shock accompanying SN explosions - the injection is from the ISM material and the chemical enrichment is guaranteed by the preferential injection of heavy nuclei, due to the collisionless nature of the shocks

- The paradigm requires that protons are injected with a power law spectrum ~2.3 and that the Galactic transport be energy dependent with $D(E) \sim E^{0.5}$

- All these considerations apply to $E \leq 1$ TeV, while it is all but clear what is the SNR contribution at higher energies and what is the energy dependence of D(E) at higher energies (see Sarah's talk)

PARTICLE ACCELERATION AT SHOCKS



$$\boxed{\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - u\frac{\partial f}{\partial x} + \frac{1}{3}\frac{du}{dx}\,p\frac{\partial f}{\partial p} + Q(x,p,t)}$$

DIFFUSION **ADVECTION** **COMPRESSION** **INJECTION**

The transport equation describes the **isotropic part** of the distribution function of non-thermal particles

All gradients are to be intended on scales **larger than the thickness of the shock**, which in turn is set by collisionless processes

The distribution function is **continuous** across the shock surface

The physics of the injection term is determined by the **microphysics** of the shock formation

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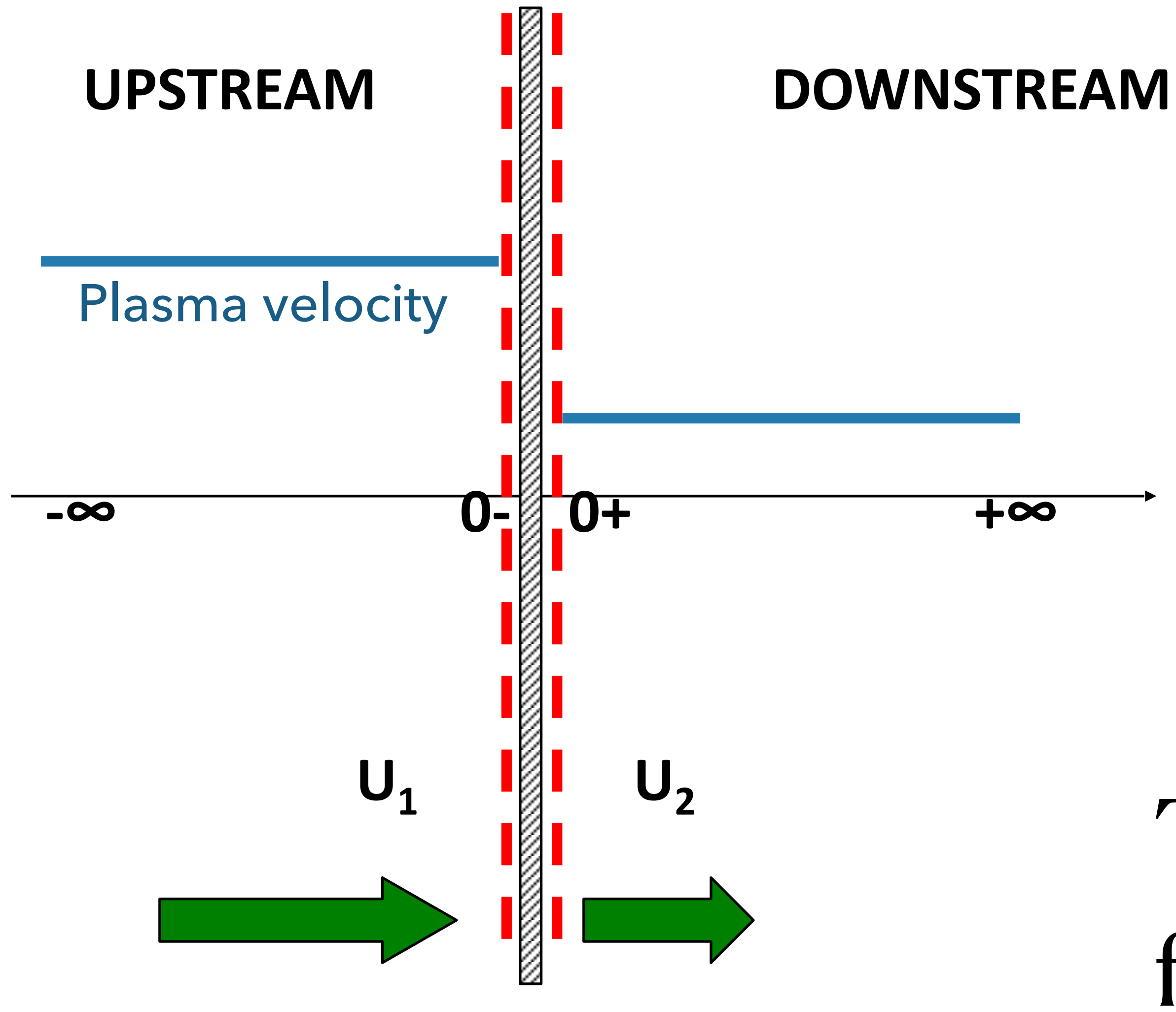

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UPSTREAM SOLUTION

LET US ASSUME STATIONARITY (LATER WE SHALL DISCUSS IMPLICATIONS)

IN THE UPSTREAM THE EQUATION READS

$$\frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - u\frac{\partial f}{\partial x} = 0 \quad \rightarrow \quad \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x} - uf\right] = 0$$

FLUX IS CONSERVED!

THE SOLUTION THAT HAS VANISHING f AND VANISHING DERIVATIVE AT UPSTREAM INFINITY IS

$$f(x,p) = f_0 \exp\left[\frac{u_1 x}{D}\right] \quad \rightarrow \quad \boxed{D\frac{\partial f}{\partial x}|_{x\to 0^-} = u_1 f_0(p)}$$

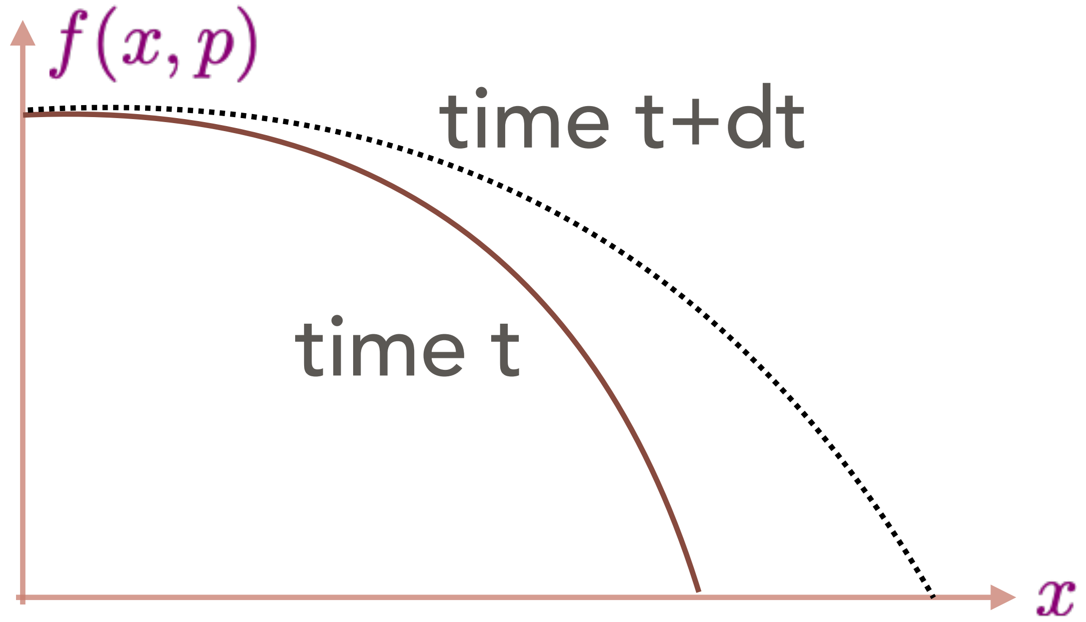
DOWNSTREAM SOLUTION

IN THE DOWNSTREAM THE EQUATION READS

$$\frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right]-u\frac{\partial f}{\partial x}=0 \quad \rightarrow \quad \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}-uf\right]=0$$

FLUX IS CONSERVED!

NOTICE THAT WE HAVE REQUIRED STATIONARITY AND OBVIOUSLY THE ONLY SOLUTION THAT IS CONSISTENT WITH THAT ASSUMPTION IS



$$f(x,p) = \text{constant} = \mathrm{f}_0(\mathrm{p})$$

$$\boxed{D\frac{\partial f}{\partial x}|_{x\to 0^+}=0}$$

AROUND THE SHOCK

INTEGRATING THE TRANSPORT EQUATION IN A NARROW NEIGHBORHOOD OF THE SHOCK WE GET

$$D\frac{\partial f}{\partial x}|_{x\to 0^+} - D\frac{\partial f}{\partial x}|_{x\to 0^-} + \frac{1}{3}(u_2 - u_1)p\frac{df_0}{dp} + \frac{\eta n_1 u_1}{4\pi p_{inj}^2}\delta(p - p_{inj})$$

WHERE WE USED du/dx=$(u_2$-$u_1)\delta(x)$

REPLACING THE EXPRESSIONS FOR THE DERIVATIVES DERIVED BEFORE:

$$-u_1 f_0(p) + \frac{1}{3}(u_2 - u_1)p\frac{df_0}{dp} = 0 \qquad p > p_{inj}$$

WHICH HAS THE SOLUTION:

$$\boxed{f_0(p) = Kp^{-\alpha} \qquad \alpha = \frac{3u_1}{u_1 - u_2} = \frac{3r}{r - 1}}$$

THE SPECTRUM IS A POWER LAW IN MOMENTUM

INTEGRATION OF THIS SIMPLE EQUATION GIVES:

$$f_0(p)=\frac{3u_1}{u_1-u_2}\frac{N_{inj}}{4\pi p_{inj}^2}\left(\frac{p}{p_{inj}}\right)^{\frac{-3u_1}{u_1-u_2}}$$

DEFINE THE COMPRESSION FACTOR

$r=u_1/u_2$ —> 4 (strong shock)

THE SLOPE OF THE SPECTRUM IS

$$\frac{3u_1}{u_1-u_2}=\frac{3}{1-1/r}\to 4 \quad \text{if r} \to 4$$

THE SPECTRUM OF THE PARTICLES ACCELERATED AT A STRONG SHOCK IS UNIVERSAL AND IS ALWAYS PROPORTIONAL TO p^{-4}

IT IS **NOT** A POWER LAW IN ENERGY!!! UNLESS YOU ARE EITHER...

ULTRA-RELATIVISTIC $N(E)dE=4\pi p^2 f(p)dp \to N(E)\propto E^{-2}$

NON-RELATIVISTIC $N(E)dE=4\pi p^2 f(p)dp \to N(E)\propto E^{-3/2}$

THE SPECTRUM IS A POWER LAW IN MOMENTUM

- THE SPECTRUM OF ACCELERATED PARTICLES IS A POWER LAW IN MOMENTUM
- THE POWER LAW EXTENDS TO INFINITE MOMENTA!!!
- THE SLOPE DEPENDS UNIQUELY ON THE COMPRESSION FACTOR AND IS INDEPENDENT OF THE DIFFUSION PROPERTIES
- NO DEPENDENCE UPON DIFFUSION (MICRO-PHYSICS)

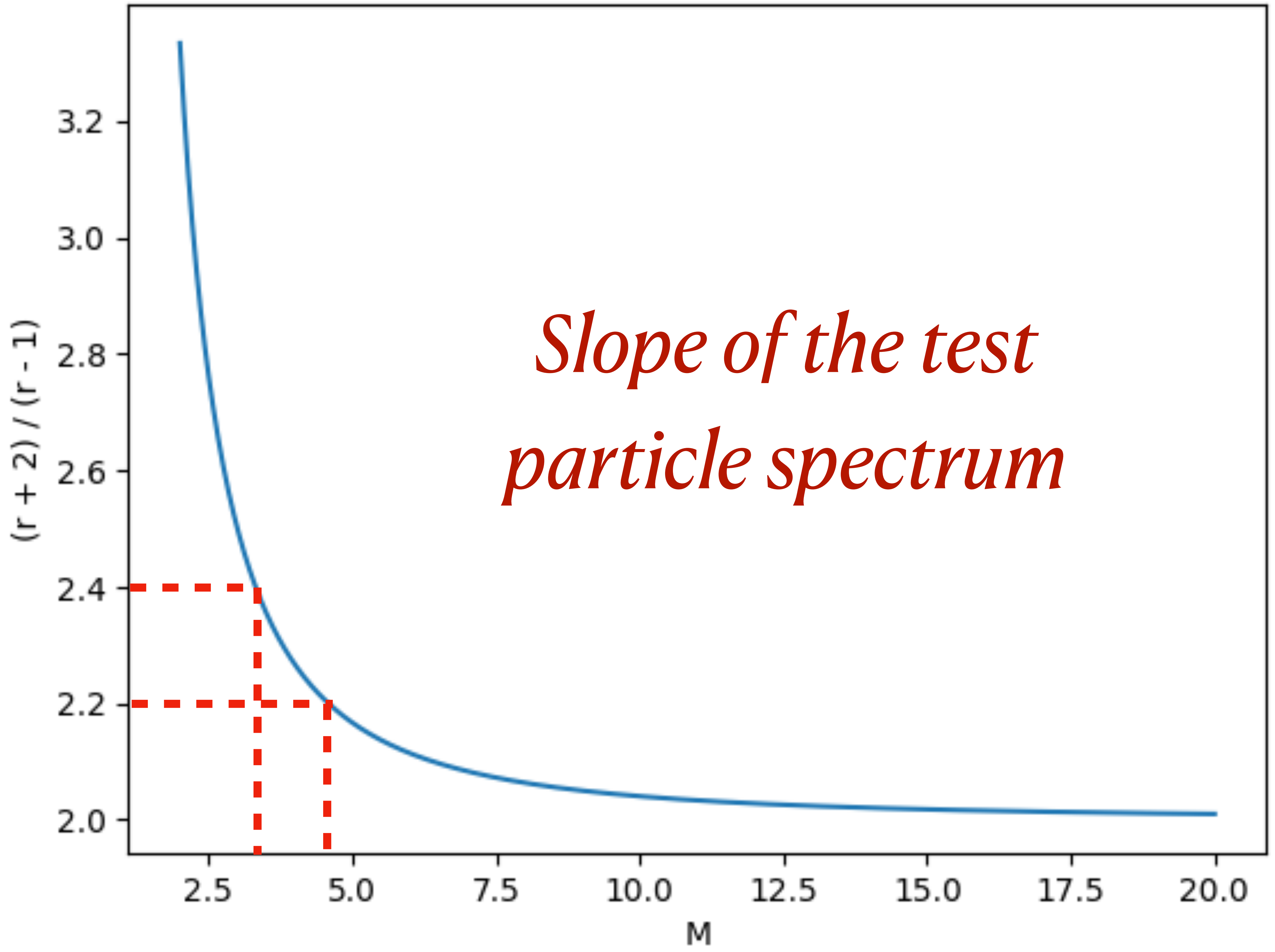
AND HERE IS WHEN YOU START GETTING CONCERNED...

☑ *ASSUMPTION OF STATIONARITY —> THERE IS NO MAXIMUM ENERGY! (if there were one, at a time t+dt it would be higher, violating stationarity)*

☑ *...BUT THE TOTAL ENERGY CARRIED BY PARTICLES IS* $E_{tot} \gtrsim \int_m^{E_{max}} dE E^{-2} E \to \ln(\infty)$ *(contradicting the assumption of test particles)*

☑ *EVEN IF THE TOTAL ENERGY WERE NOT INFINITE, THERE IS NO CHECK THAT IT IS NOT LARGER THAT ρu^2, THE TOTAL ENERGY WE CAN TAP FROM*

TEST PARTICLE SPECTRUM



Slope of the test particle spectrum

FOR ALL RELEVANT VALUES OF THE MACH NUMBER THE SPECTRAL SLOPE IS 2 - THE REQUIRED VALUES OF 2.2-2.4 ARE ONLY ACHIEVED FOR M<5. **Such a slope is NOT compatible with the vanilla version of DSA, and as we will see, other aspects as well are not compatible with such a version of DSA.**

FAILURE OF THE BASIC THEORY OF DSA: E_{MAX}

In the simple case of a SN exploding in the standard ISM, the Sedov phase starts at time:

$$t_{ST} \approx 430 \text{ yrs} \left(\frac{\mathrm{M_{ej}}}{\mathrm{M_\odot}}\right)^{5/6} \left(\frac{\mathrm{E_{SN}}}{10^{51}\mathrm{erg}}\right)^{-1/2} \left(\frac{\mathrm{n_H}}{0.1\mathrm{cm}^{-3}}\right)^{-1/3}$$

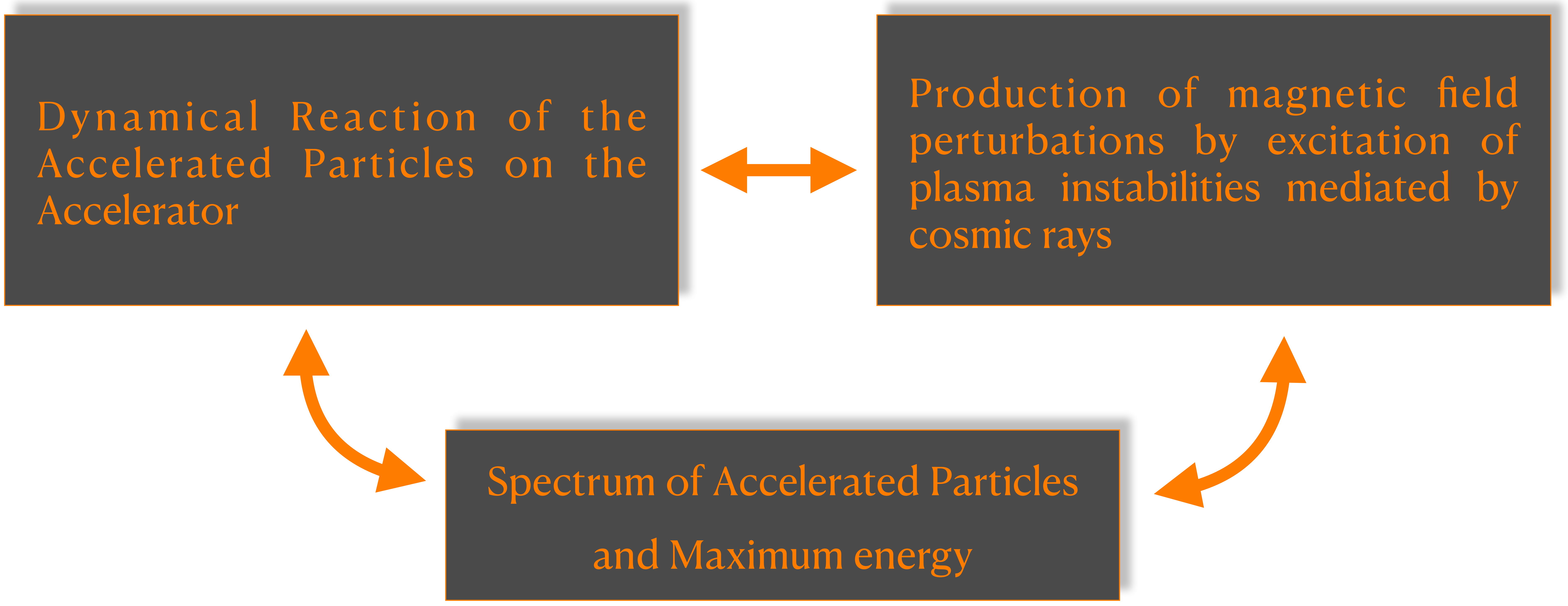
Requiring that the acceleration time, assuming Galactic $D(E)=3\times10^{28}\ E(\mathrm{GeV})^{1/2}$, equals the Sedov time:

$$\frac{D(E)}{v^2} = t_{ST} \rightarrow E_{max} \approx 0.2 \text{ GeV} \left(\frac{\mathrm{M_{ej}}}{\mathrm{M_\odot}}\right)^{-1/3} \left(\frac{\mathrm{E_{SN}}}{10^{51}\mathrm{erg}}\right) \left(\frac{\mathrm{n_H}}{0.1\mathrm{cm}^{-3}}\right)^{-2/3}$$

In the absence of any action making the magnetic field **UPSTREAM** of the shock larger **and more disordered** on the scale of the Larmor radius, SNR can accelerate at uselessly low energies

Modern Theory of DSA in SNRs

These theories aim at a description of the interplay between accelerated particles and the accelerator itself – the theory becomes non-linear and often untreatable analytically, but Physics is clear

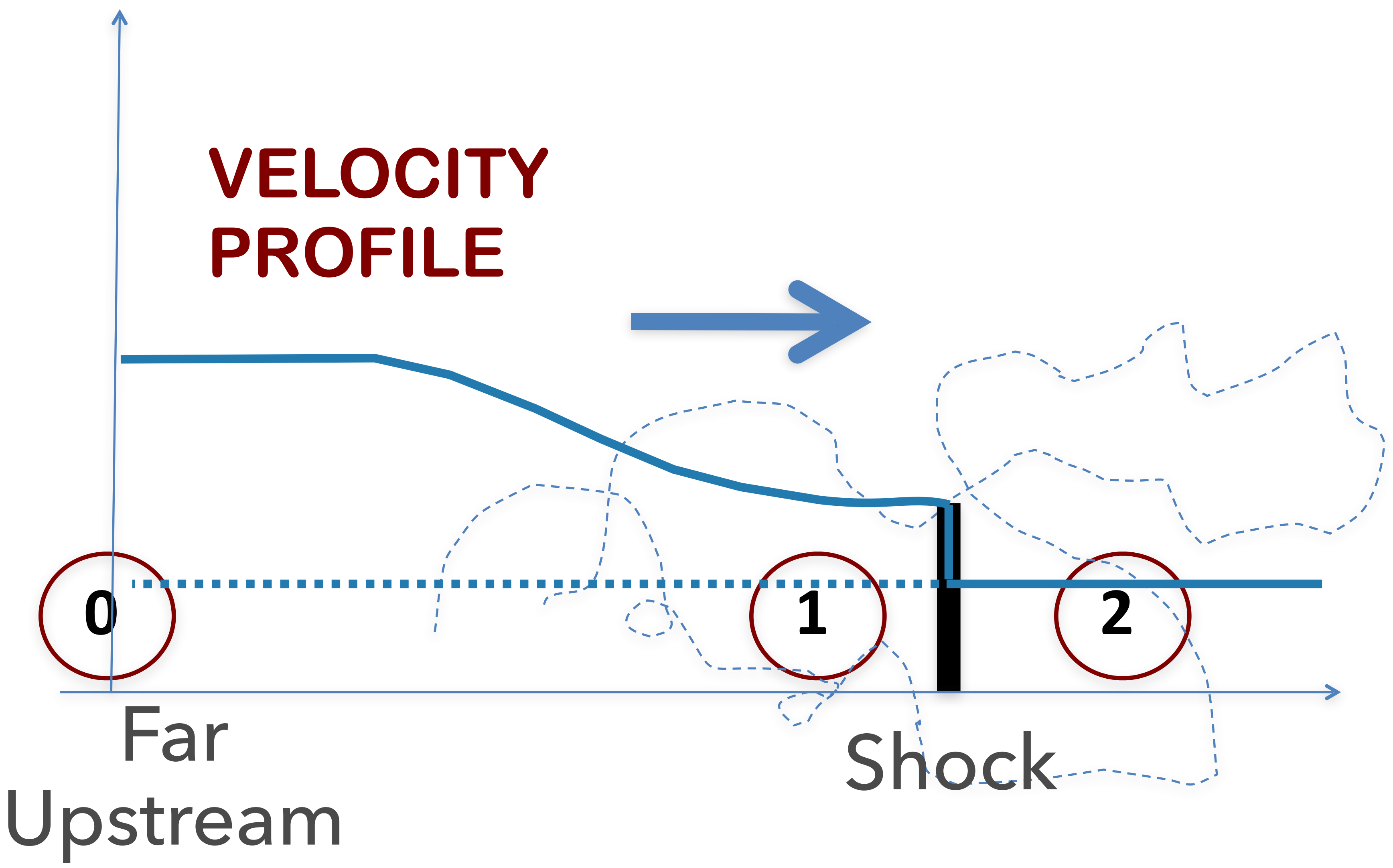


Dynamical Reaction of the Accelerated Particles on the Accelerator

Production of magnetic field perturbations by excitation of plasma instabilities mediated by cosmic rays

Spectrum of Accelerated Particles and Maximum energy

DYNAMICAL REACTION OF COSMIC RAYS



$$P_g(x) + \rho u^2 + P_{CR} = P_{g,0} + \rho_0 u_0^2$$

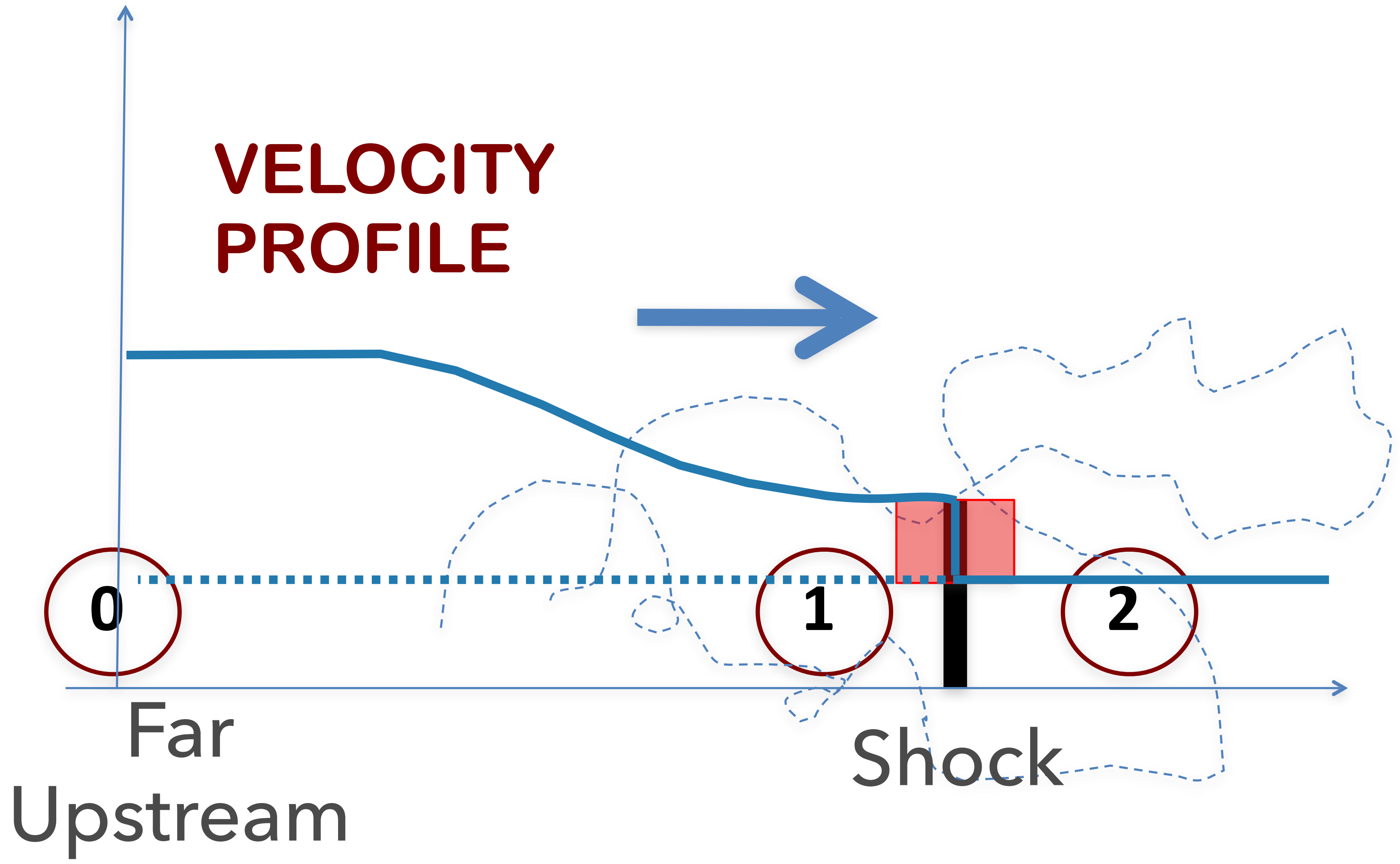
$$\frac{P_g}{\rho_0 u_0^2} + \frac{u}{u_0} + \frac{P_{CR}}{\rho_0 u_0^2} = \frac{P_{g,0}}{\rho_0 u_0^2} + 1$$

$$\frac{u}{u_0} \approx 1 - \xi_{CR}(x)$$

$$\xi_{CR}(x) = \frac{P_{CR}(x)}{\rho_0 u_0^2}$$

- Compression factor becomes a function of energy
- Spectra are not perfect power laws (concavity)
- Gas behind the shock is cooler because part of the energy has been used to energise CR

DYNAMICAL REACTION OF COSMIC RAYS



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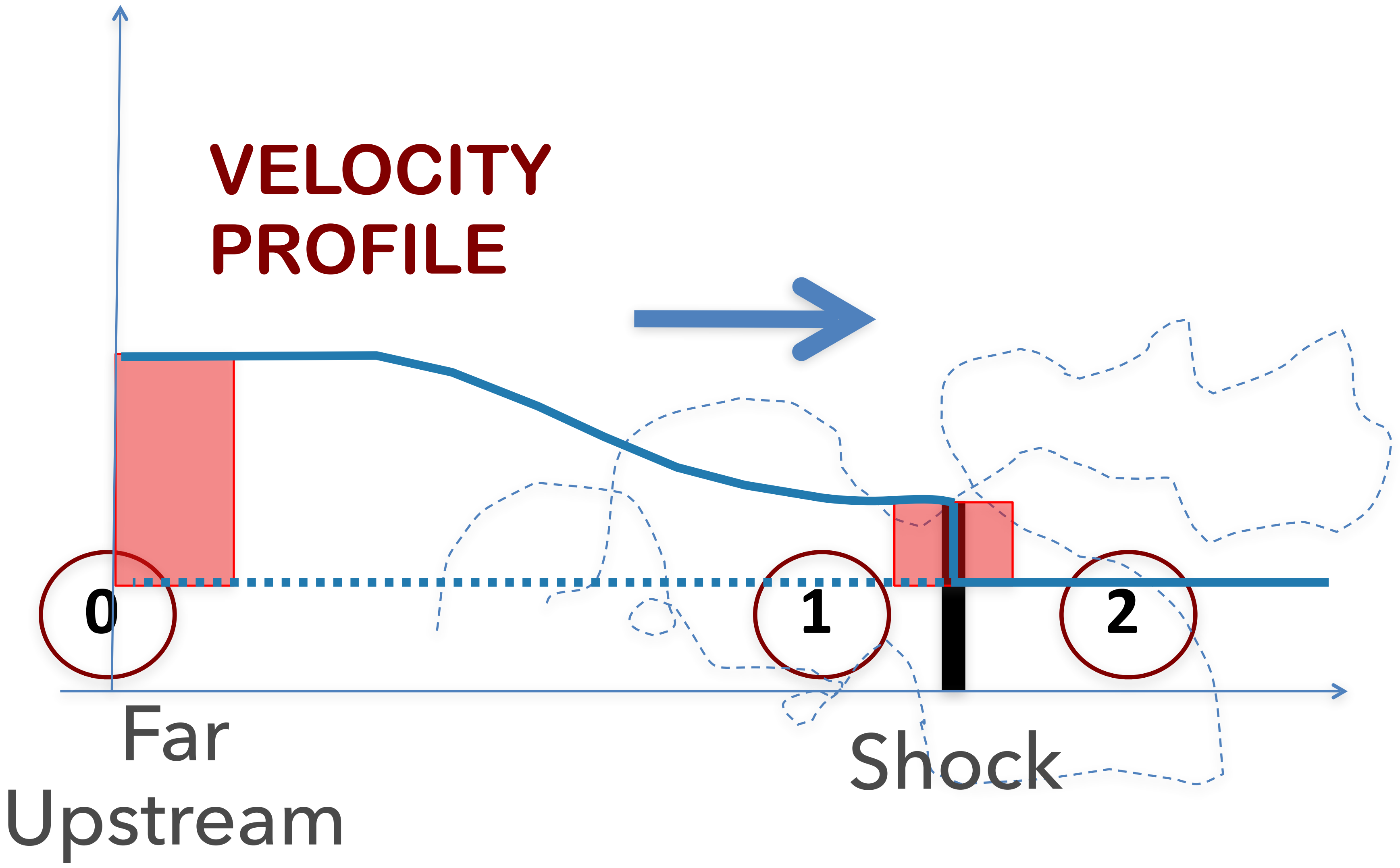

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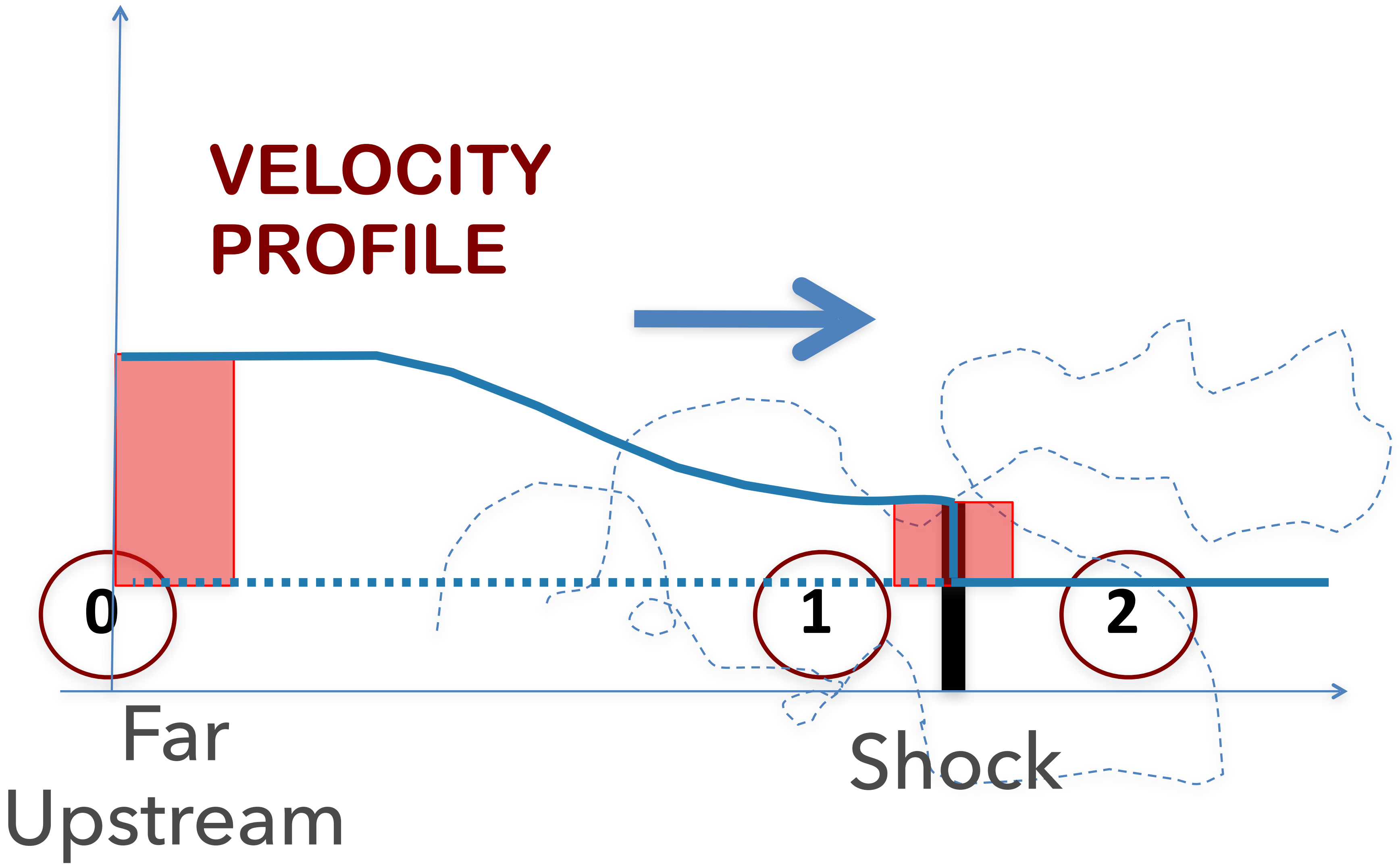

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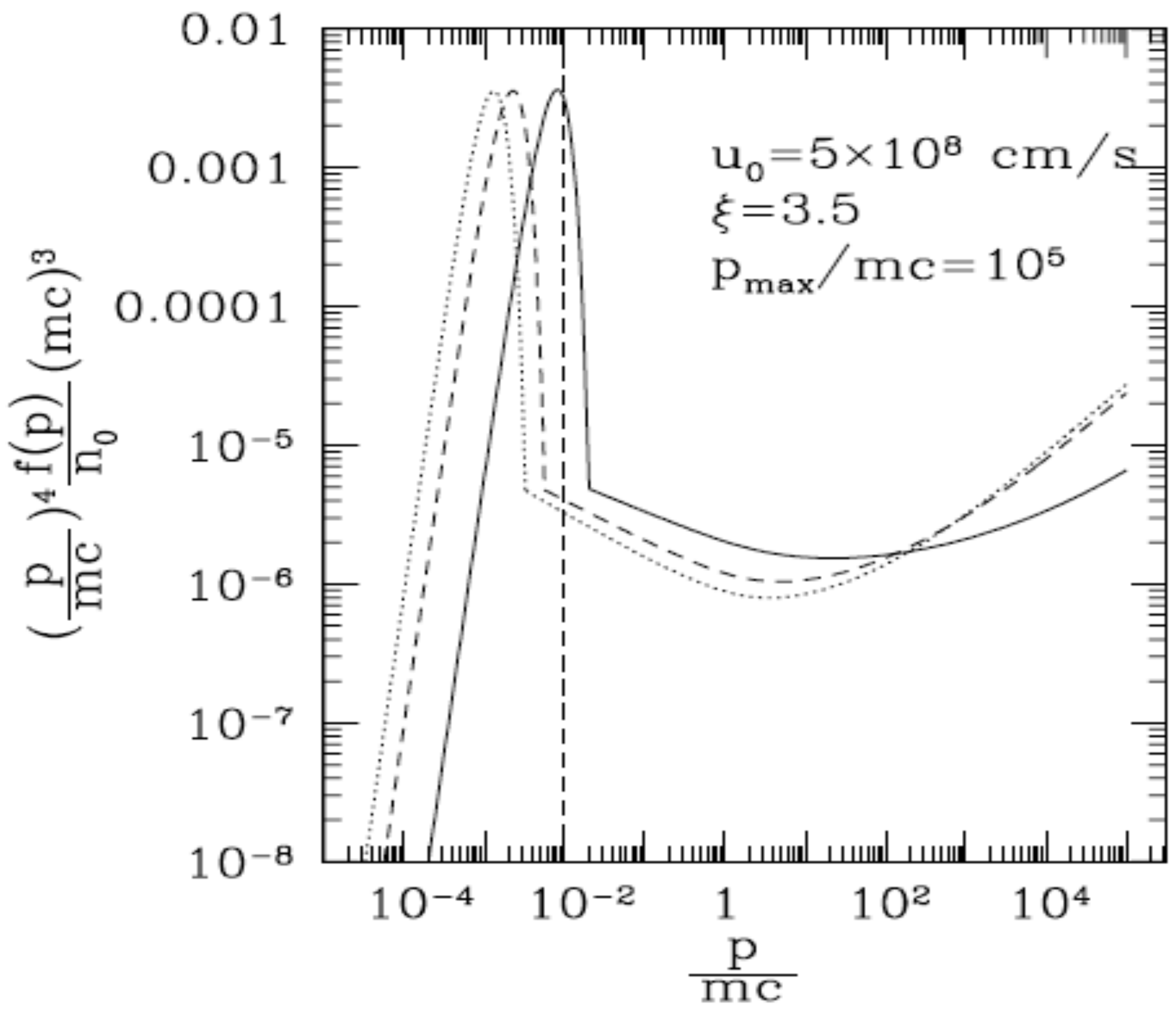
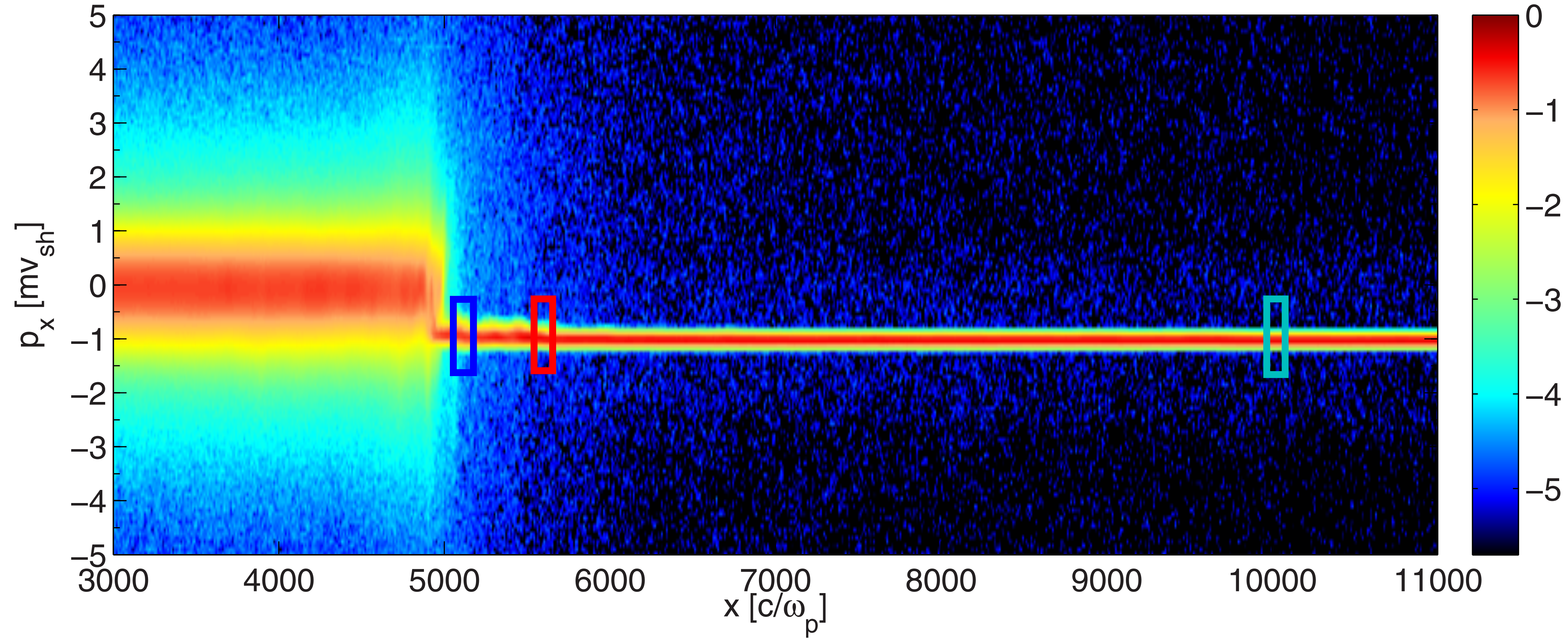
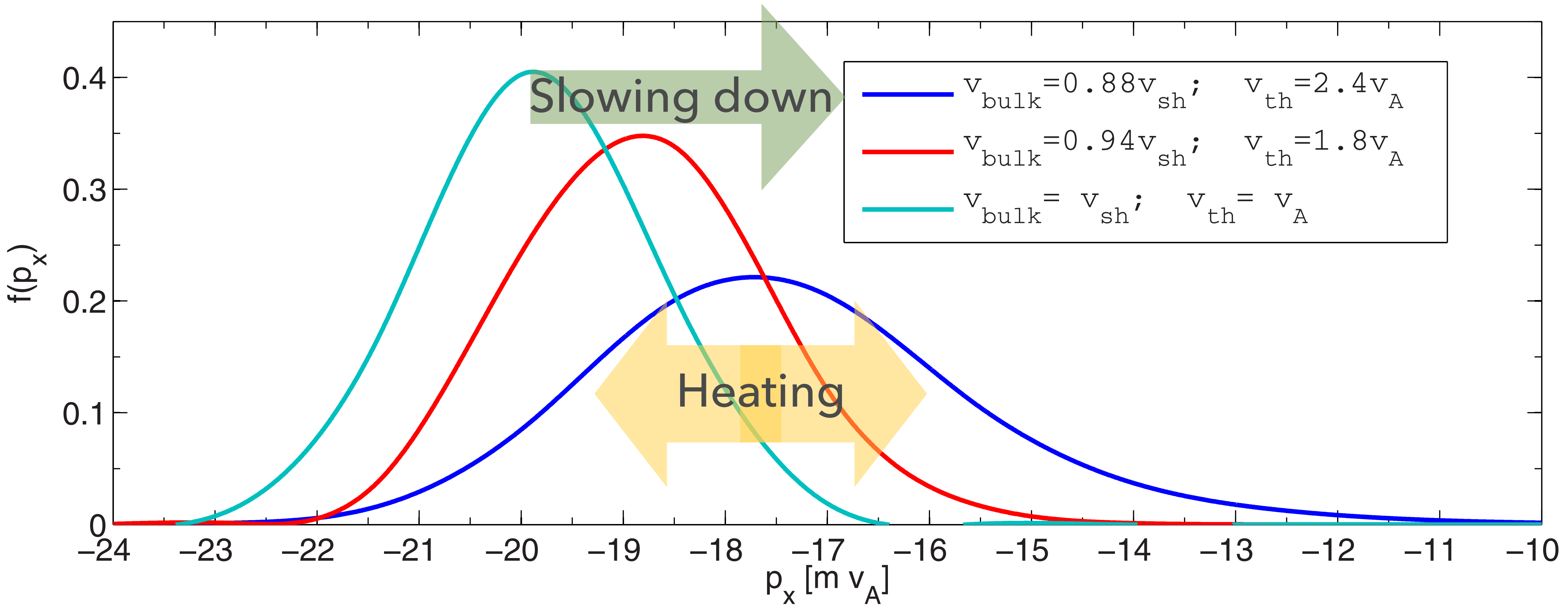


Malkov 2001, PB 2002, PB, Gabici & Vannoni 2005,...

DYNAMICAL REACTION OF COSMIC RAYS



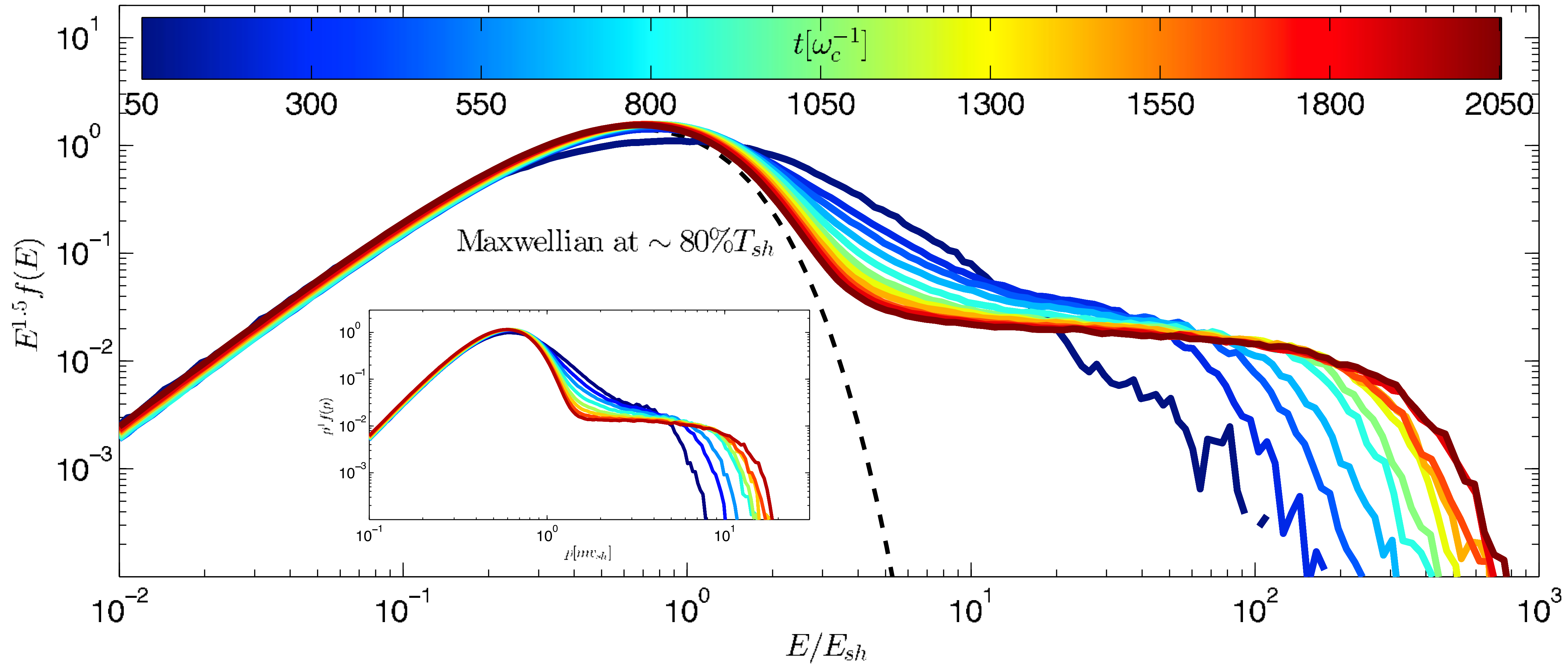


Caprioli & Spitkovsky 2014

Hybrid simulations now confirm that the shock is modified by the accelerated particles…

They also confirm that some level of heating occurs also upstream, resulting in lower Mach number and a reduced curvature

As a result: spectra close to power laws and efficiency of order 10%



MAGNETIC FIELD AMPLIFICATION (MFA)

The single most important non linear effect that makes DSA interesting if the turbulent amplification of magnetic fields induced by the accelerated particles

The necessary condition for the process to be important for acceleration is that enough power is created in magnetic fields on the scale of the gyration radius of the particles you want to accelerate

The main channels that have been investigated, both analytically and numerically, are:

RESONANT STREAMING INSTABILITY

Kulsrud & Pearce 1969, Bell 1978,
Lagage & Cesarsky 1982

NON RESONANT STREAMING INSTABILITY

Bell 2004, Amato & PB 2009, Zweibel & Everett 2010

ACOUSTIC INSTABILITY AND TURBULENT AMPLIFICATION

Drury & Falle 1986, Berezniak, Jones & Lazarian 2012

OBVIOUS AND LESS OBVIOUS FACTS

- Particles accelerated at the shock are mainly advected downstream, hence an escape process is needed for them to become Cosmic Rays

- When the shock speed decreases in time, $E_{max}(t)$ typically also drops in time: particles at $E_{max}(t)$ can leave the upstream region in the form of very peaked spectra at any given time

- The integral in time of these very peaked spectra leads to smoother (power law) escape spectra.

- Particles advected toward downstream escape later in a rather complex and model dependent manner

- The sum of these two contributions represents the real "source spectra", to be used in calculations of CR transport

- At the same time $E_{max}(t)$ is shaped by the escape process, at least for Bell modes

RESONANT STREAMING INSTABILITY

$$\boxed{n_{CR} m v_D \rightarrow n_{CR} m V_A \Rightarrow \frac{dP_{CR}}{dt} = \frac{n_{CR} m (v_D - V_A)}{\tau}}$$

RATE OF MOMENTUM LOST BY CR BEAM

$$\boxed{\frac{dP_w}{dt} = \gamma_W \frac{\delta B^2}{8\pi} \frac{1}{V_A}}$$

RATE OF MOMENTUM GAINED BY WAVES

Kulsrud Approach

Time for deflection by 90°

BY REQUIRING A SORT OF BALANCE BETWEEN THE TWO RATES ONE CAN ESTIMATE THE RATE OF GROWTH OF THE WAVES:

$$\gamma_W = \sqrt{2} \frac{n_{CR}}{n_{gas}} \frac{v_D - V_A}{V_A} \Omega_{cyc}$$

WITHIN A FACTOR OF ORDER UNITY THIS IS THE CORRECT GROWTH RATE

$$\gamma_W(k) = \frac{n_{CR}(>p)}{n_{gas}} \frac{v_D - v_A}{v_A} \Omega_c \approx \frac{4\pi p^3 f(p)}{n_{gas}} \frac{v_D - v_A}{v_A} \Omega_c \qquad k = k_{res}(p)$$

RESONANT STREAMING INSTABILITY

Since the spectrum of particles accelerated at a shock is a power law in momentum, we can normalise it so that CR take a fraction ξ_{CR} of the ram pressure:

$$f(p) = \frac{3\xi_{CR}\rho v_s^2}{4\pi p_0^4 c\Lambda}\left(\frac{p}{p_0}\right)^{-4} \qquad \Lambda = \ln\left(\frac{p_{max}}{p_0}\right)$$

Hence, replacing in the previous expression and rearranging things:

$$\gamma_W(k) \approx \frac{3\xi_{CR}\rho v_s^2}{c\Lambda p_0}\left(\frac{p}{p_0}\right)^{-1}\frac{1}{n_{gas}}\frac{v_s}{v_A}\frac{qB_0}{m_p c} = \frac{3\xi_{CR}\rho v_s^2}{\Lambda U_B}\frac{v_s}{c}\frac{v_A}{r_L(p)} \qquad U_B = \frac{B_0^2}{4\pi}$$

Upon introducing the Alfven Mach number $M_A = v_s/v_A$

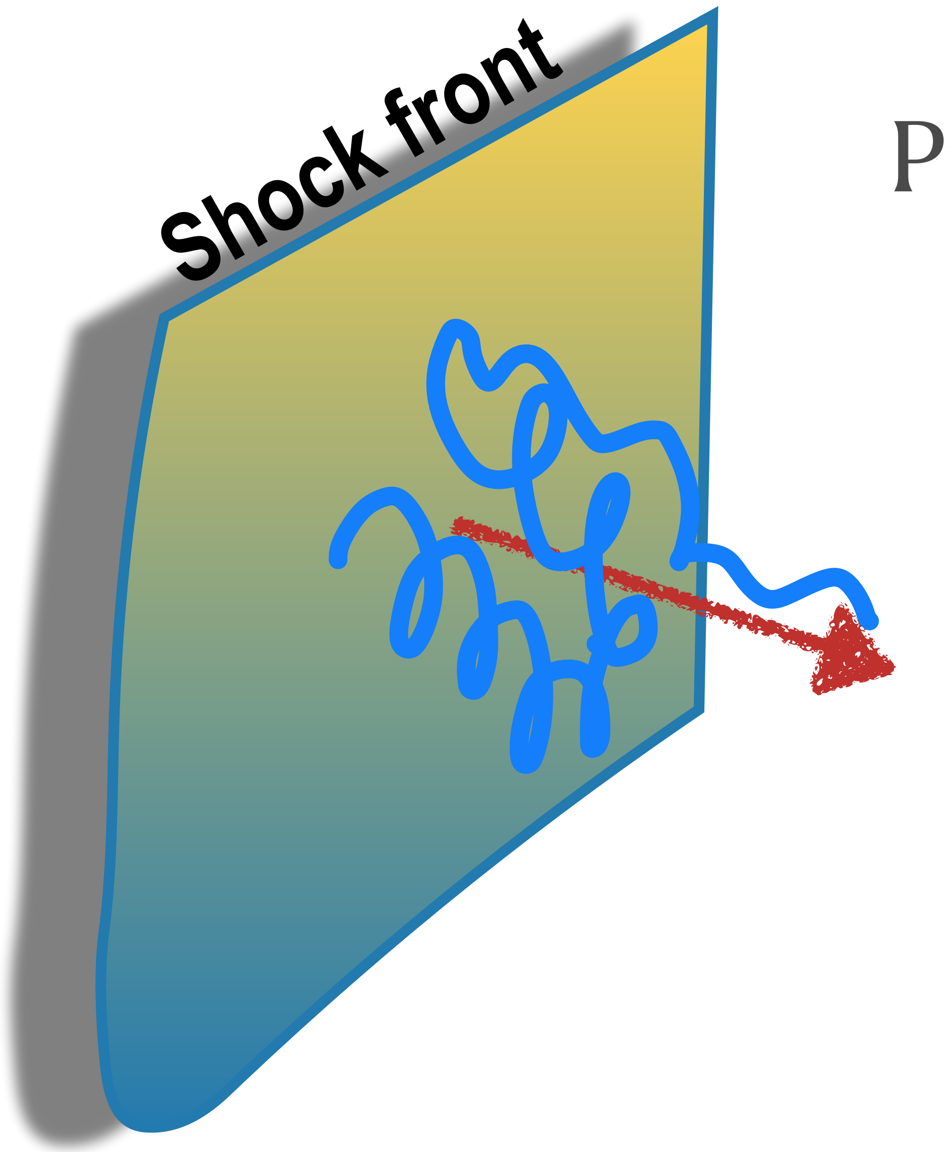
$$\gamma_W(k) = 3\frac{\xi_{CR}}{\Lambda}M_A^2\frac{v_s}{c}\frac{v_A}{r_L(p)}$$

→ FOR TYPICAL PARAMETERS

$$\gamma_W^{-1}(1GeV) \simeq 3\times 10^5 s$$

$$\gamma_W^{-1}(1PeV) \simeq 10^4 yr$$

The Bell (2004) Instability
[a.k.a. Non Resonant Instability]



Protons of given energy upstream of the shock represent a current $J_{CR}=n_{CR}(>E)\,e\,v_{shock}$

The background plasma cancels the CR positive current with a return current created by a slight relative motion between thermal electrons and protons, thereby creating a two stream instability that grows the fastest on scales ***l***~1/k_{max} where

$$k_{max}B_0 = \frac{4\pi}{c}J_{CR}$$

The growth occurs at a rate that can be approximated as: $\Gamma_{max} = k_{max}v_A$

The condition for this instability to develop is that $k_{max}>1/r_L(E)$, which is equivalent to requiring that:

$$n_{CR}(>E)E\frac{v_{shock}}{c} > \frac{B_0^2}{4\pi} \Rightarrow M_A > \left(\frac{\Lambda}{\xi_{CR}}\frac{c}{v_{shock}}\right)^{1/2} \approx 500\left(\frac{\xi_{CR}}{0.1}\right)^{-1/2}\left(\frac{v_{shock}}{10^9\ \mathrm{cm/s}}\right)^{-1/2}$$

for E^{-2} spectrum

Only works in very young SNR with large Alfvenic Mach number

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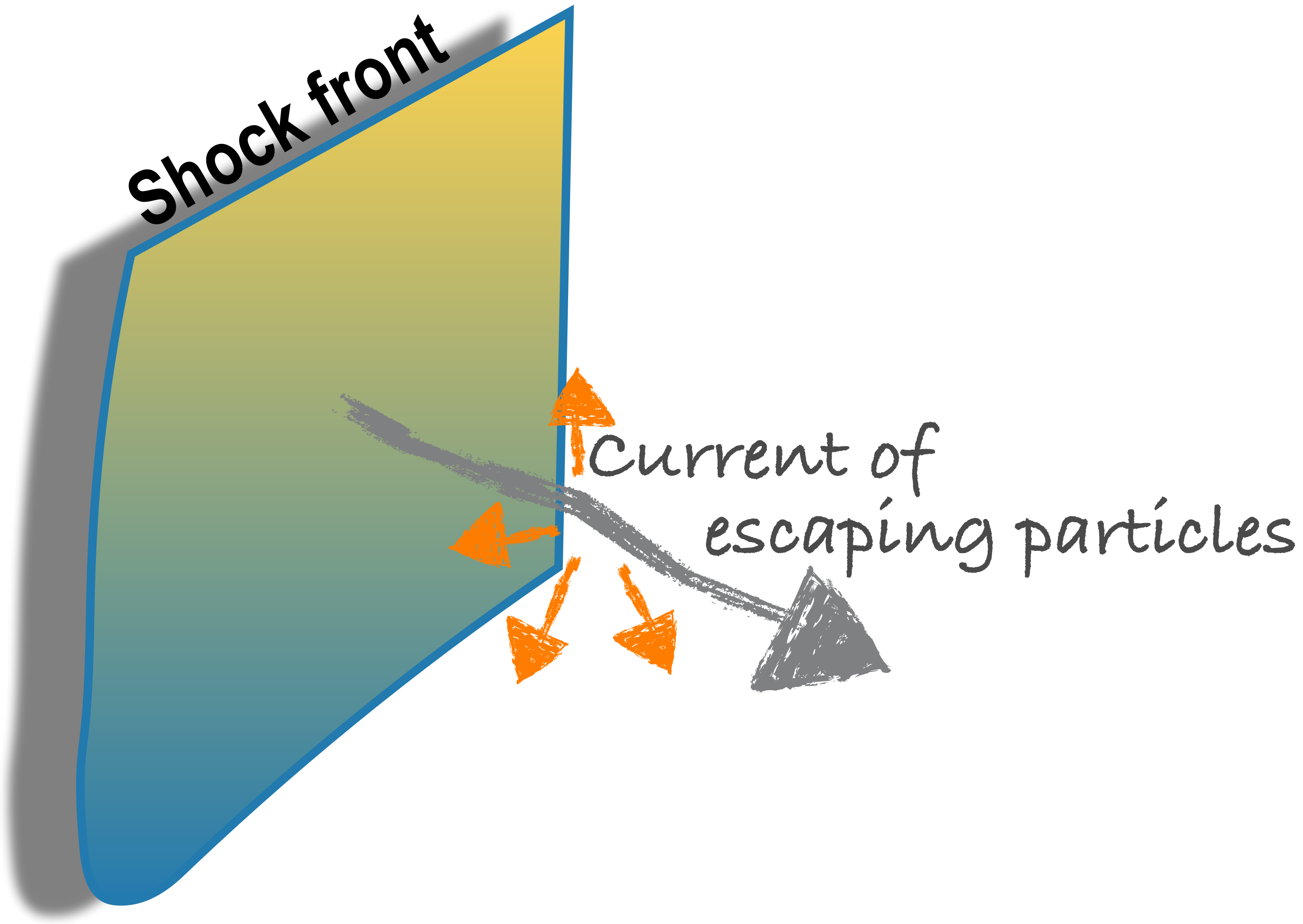


The current of escaping particles acts as a force on the background plasma in the direction perpendicular to both the current and the amplified field:

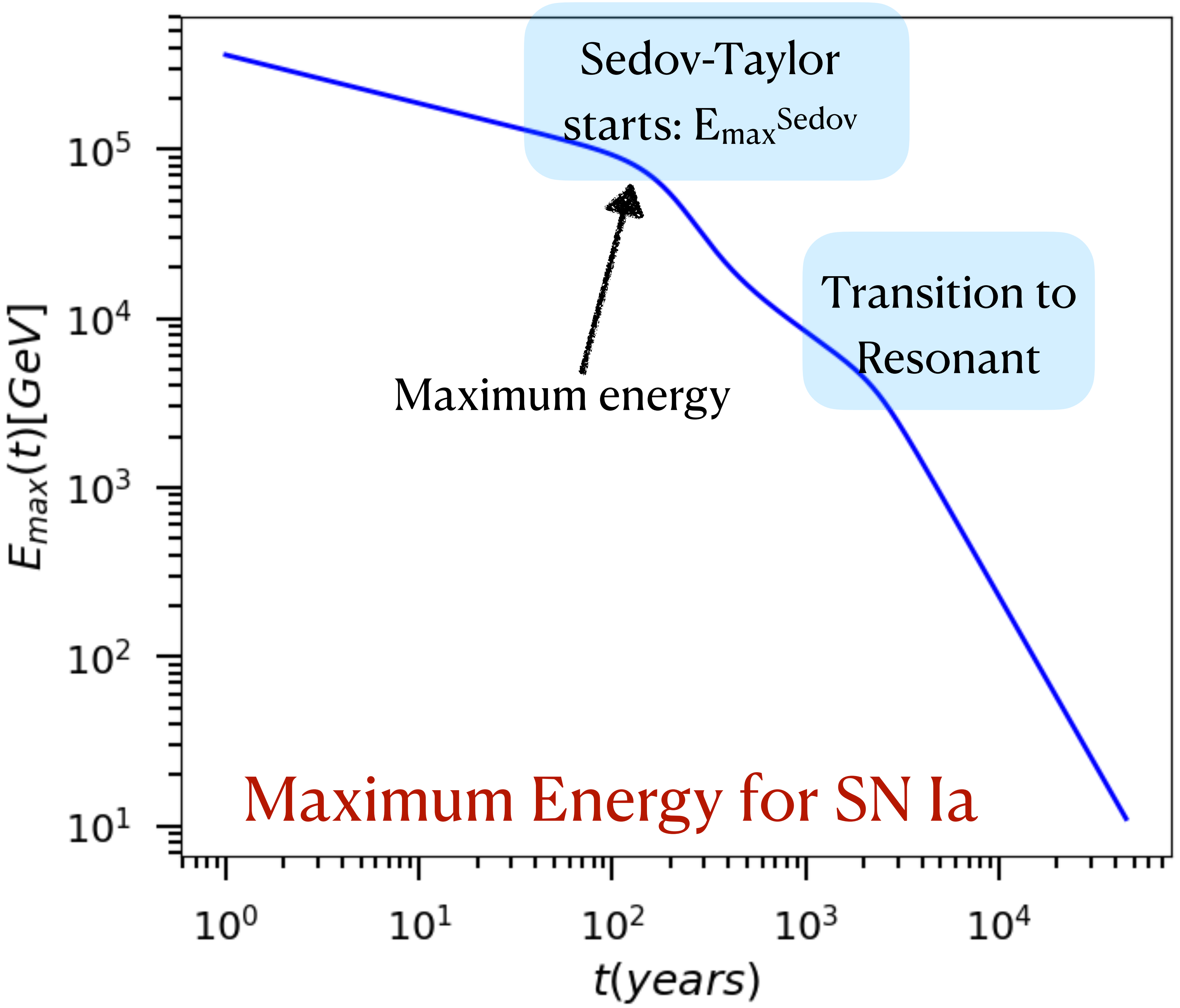
$$\rho \frac{dv}{dt} \sim \frac{1}{c} J_{CR} \delta B \longrightarrow \Delta x \sim \frac{J_{CR}}{c\rho} \frac{\delta B(0)}{\gamma_{max}^2} exp(\gamma_{max} t)$$

The current is weakly disturbed until the transverse displacement becomes of order the Larmor radius in the amplified field...this condition leads to the following saturation condition:

$$\frac{\delta B^2}{4\pi} = n_{CR}(> E) E \frac{v_{shock}}{c} \approx \frac{\xi_{CR}}{\Lambda} \rho v_{shock}^2 \frac{v_{shock}}{c}$$

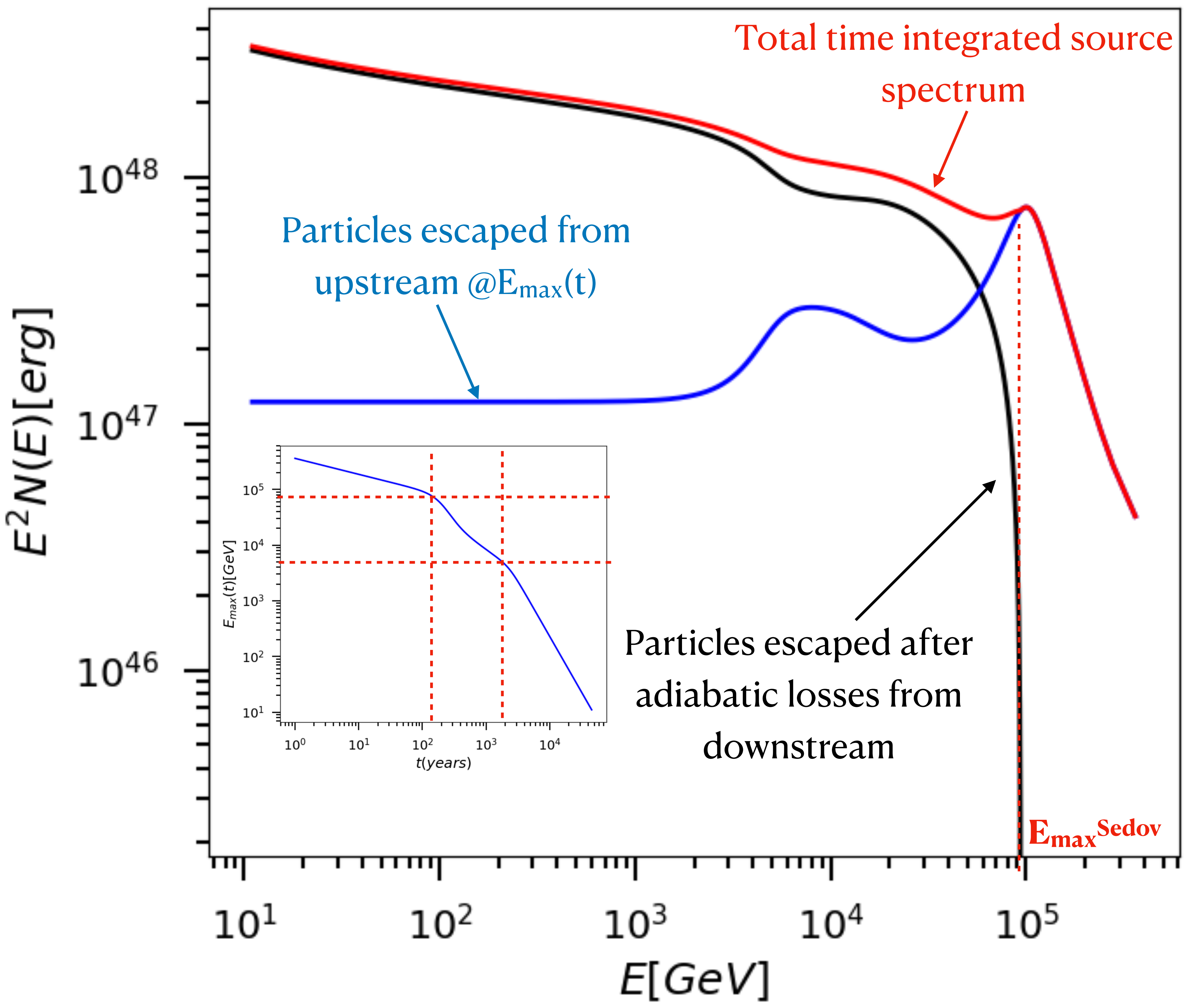
TRADE OFF BETWEEN RESONANT AND NON RESONANT MODES

For shocks slower than $v_{sh} \approx 1000 \text{ km/s} \left(\frac{\text{v}_\text{A}}{10\text{km/s}}\right)^{2/3} \left(\frac{\xi_{\text{CR}}}{0.1}\right)^{-1/3}$ the magnetic field that the Bell instability produces is smaller than B_0, hence there is a transition to **resonant streaming instability**, saturated by non-linear Landau damping (unless there are lots of neutral atoms around: ion-neutral damping)



- For a typical Ia SN the maximum energy at the beginning of the Sedov phase is <100 TeV
- E_{max}^{Sedov} is what we call the **MAXIMUM ENERGY OF THE ACCELERATOR**
- Higher values can be achieved early on but they do not show in the time-integrated spectrum (see later)
- After ~1000 yr Bell instability stops being excited and only resonant Alfven waves can be excited

TIME INTEGRATED ESCAPE FLUX



At each given time particles with energy $E_{max}(t)$ are able to leave the shock region

These are the particles accelerated at that time + the ones accelerated at earlier times, that lose energy adiabatically (expansion) and are liberated when their energy drops below $E_{max}(t)$

The effective E_{max} of the source is ~E_{max}^{Sedov}

At higher energies the suppression is not exponential but rather a steeper power law

Even though the instantaneous spectrum is very close to E^{-2}, in the relativistic regime, the time integrated spectrum of the source is typically steeper

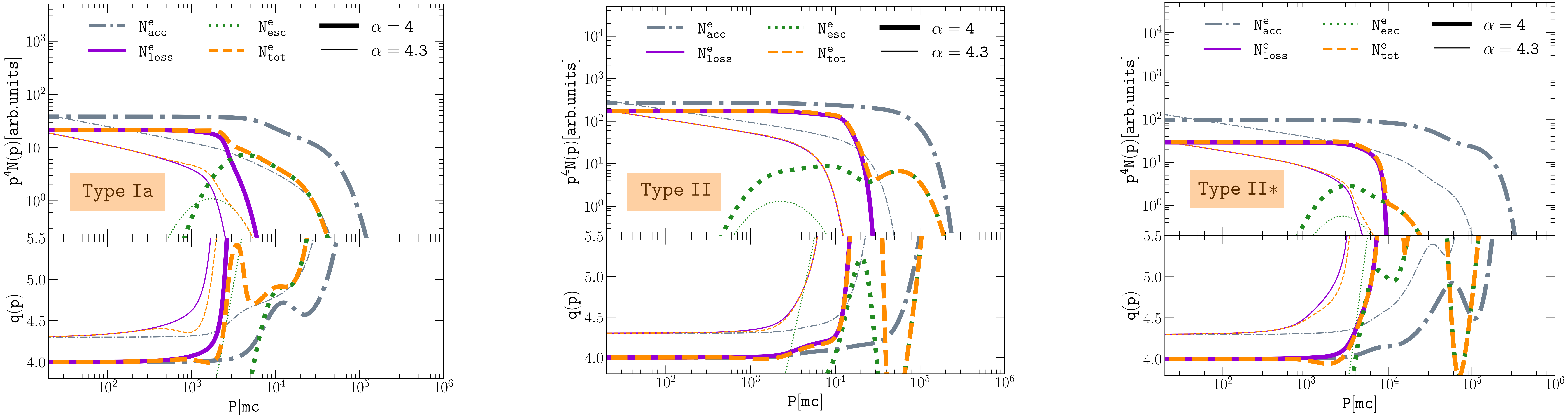
There are features in the source spectrum, not a pure power law: this should be the input for transport calculations

Situation similar but more complex in the case of core collapse SNR

THE SOURCE SPECTRUM OF ELECTRONS

- Based on transport models of CR on Galactic scales (see for instance Evoli+2020) the source spectra required for protons and nuclei are somewhat steeper than that for primary electrons, apparently contradicting the rigidity dependent nature of the acceleration process
- The difference in spectrum (~0.1 in slope) is at the same level as the difference between protons and He and clearly both require explanation
- The main difference between electrons and protons in SNR appears to be energy losses in the B-fields generated by protons: the effect depends rather sensibly upon the possible role of particle trapping and magnetic field levels during the radiative phase, both unknown
- Notice that at times in which $E_{max}^{(e)} < E_{max}^{(p)}$ due to energy losses, no escape occurs for electrons

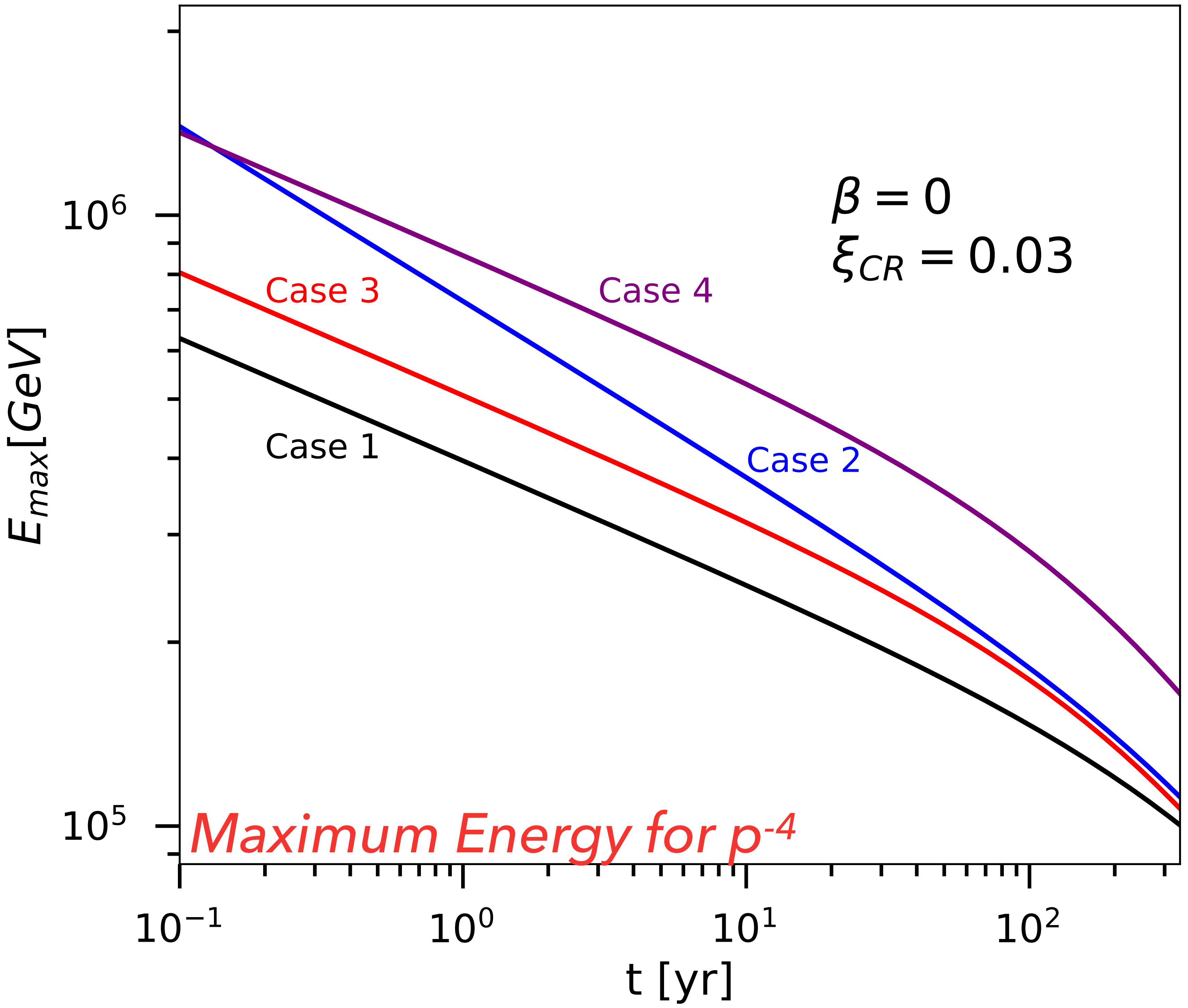
Cristofari, PB & Caprioli 2022

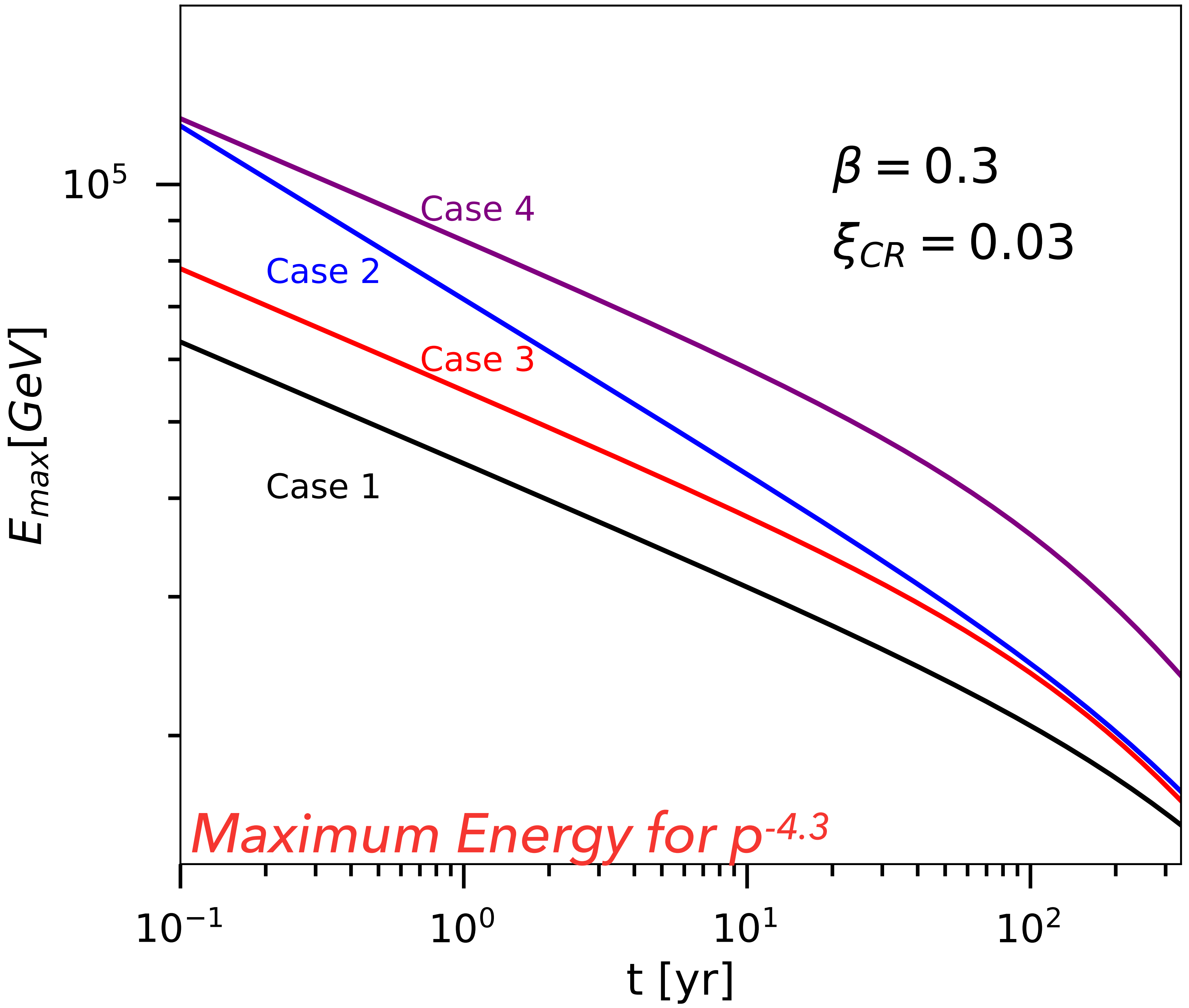


THE CASE OF CAS-A

This is the remnant of a core collapse SN explosion occurred 340 years ago. The SN exploded in the wind of its red giant progenitor.

It is now around the beginning of its Sedov phase...

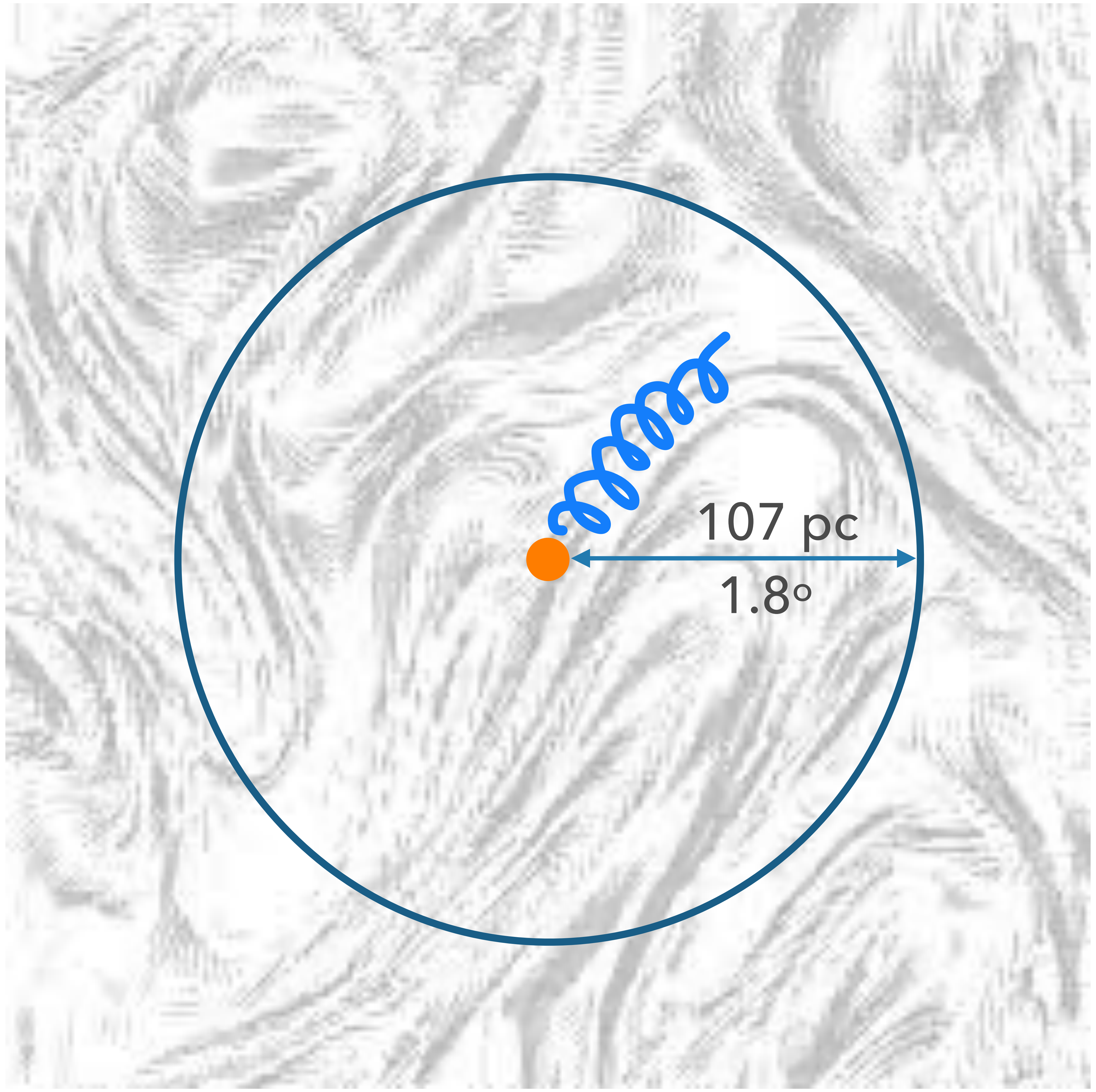




Model	E_{51}	$\frac{M_{ej}}{M_\odot}$	v_6	$\dot{M}_{-5}$	k	ζ	s
Case 1	1.1	3	2	1.5	12	1.14	3.81
Case 2	1.1	2	2	2	9	0.97	5.16
Case 3	1.0	2	1	1	12	1.14	3.81
Case 4	2.4	4	2	5	12	1.14	3.81

PB 2025

THE CASE OF *CAS-A*



If D(E) in the region around CasA is the Galactic one, the path length for diffusion is:

$$\lambda(E) \approx 1kpc \left(\frac{E}{PeV}\right)^{1/2} >> L_c$$

coherence scale of the Galactic B-field

$$r_L(E) \approx 1pc \left(\frac{E}{PeV}\right) << L_c$$

Hence on a scale of ~100 pc, 0.1-1PeV particles free stream parallel to B

The highest energy particles escaped CasA ~200 yrs ago, reaching ~60 pc at the present time, still inside 1.8 degrees from the source

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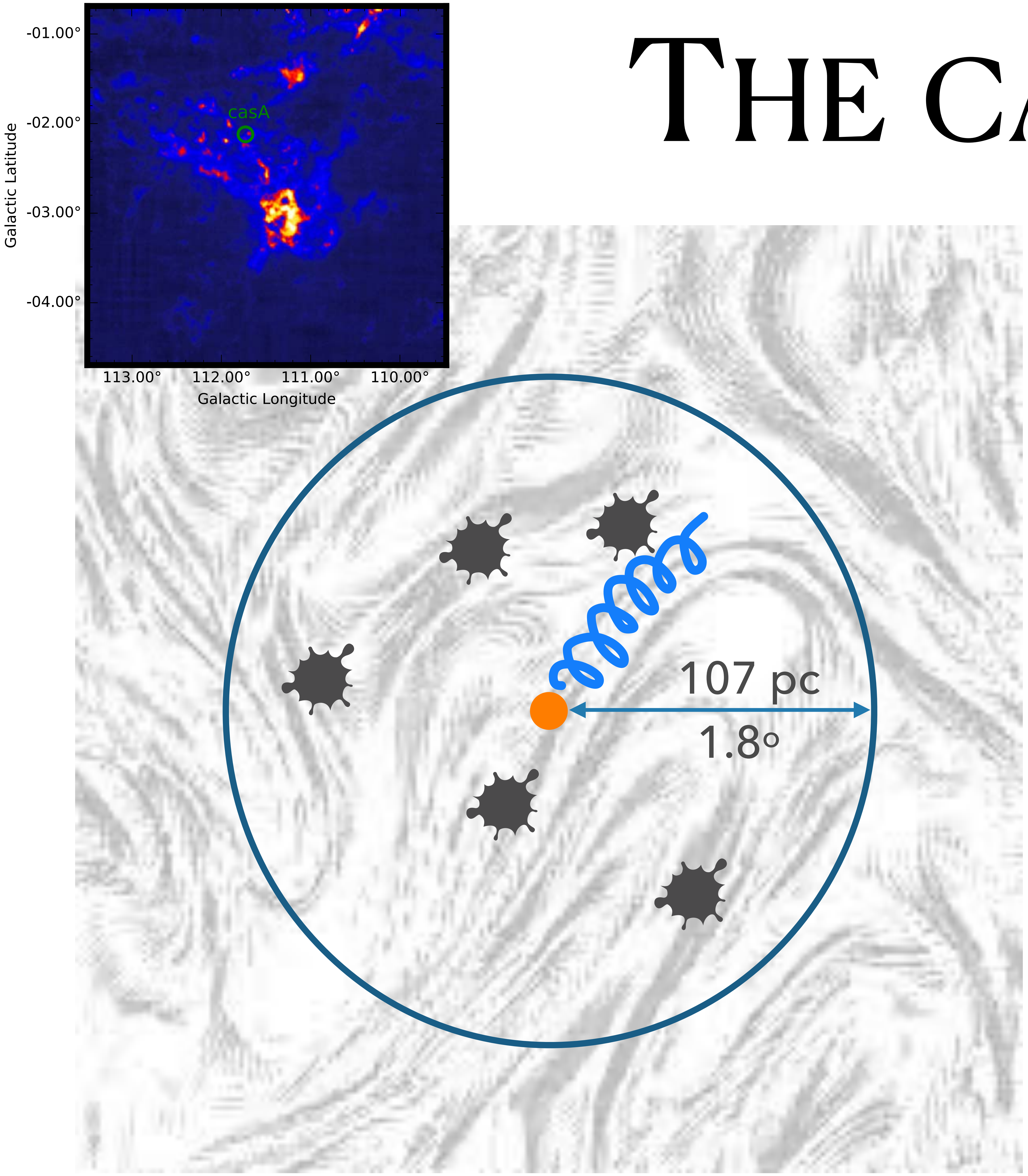

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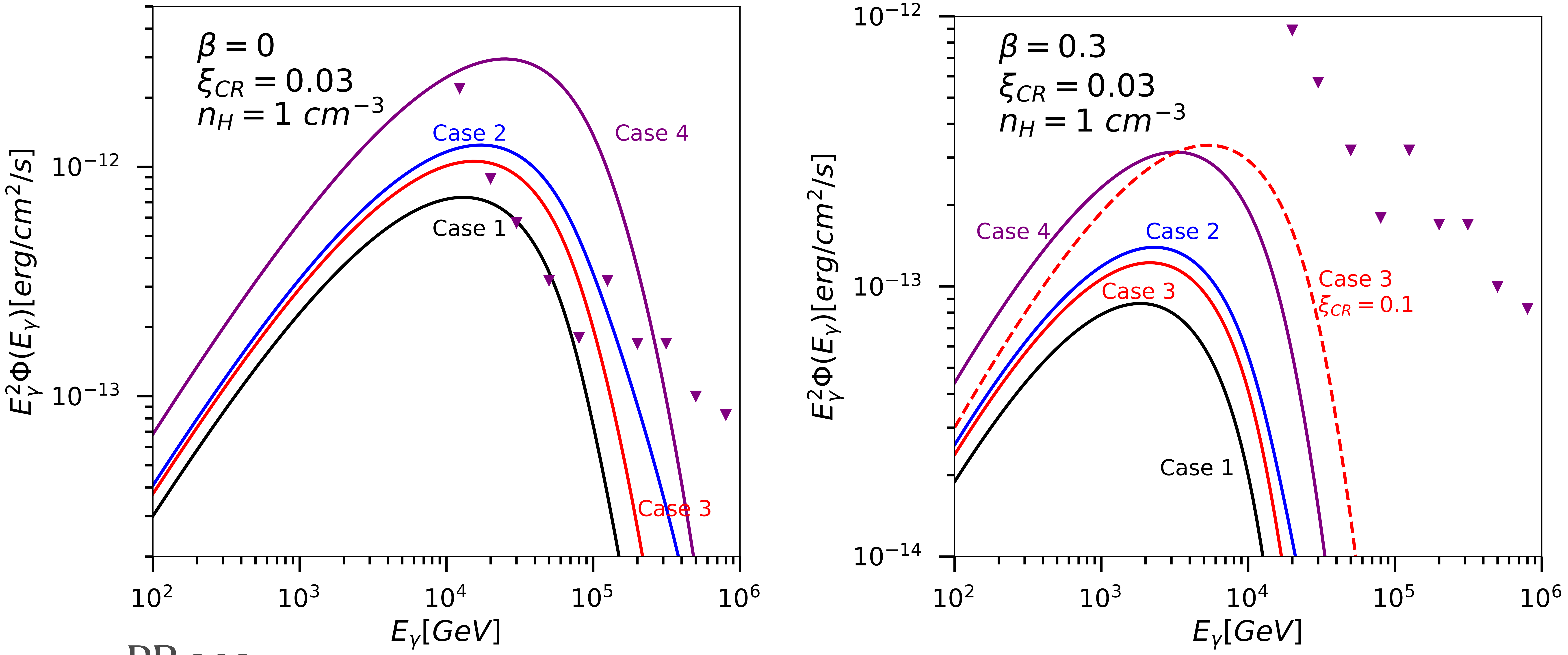
The highest energy particles escaped CasA ~200 yrs ago, reaching ~60 pc at the present time, still inside 1.8 degrees from the source

The gamma ray emission from the region reflects

1) the number of CR particles released as a function of energy
2) the structure of the local magnetic field
3) the gas distribution in the region

THE CASE OF *CAS-A*

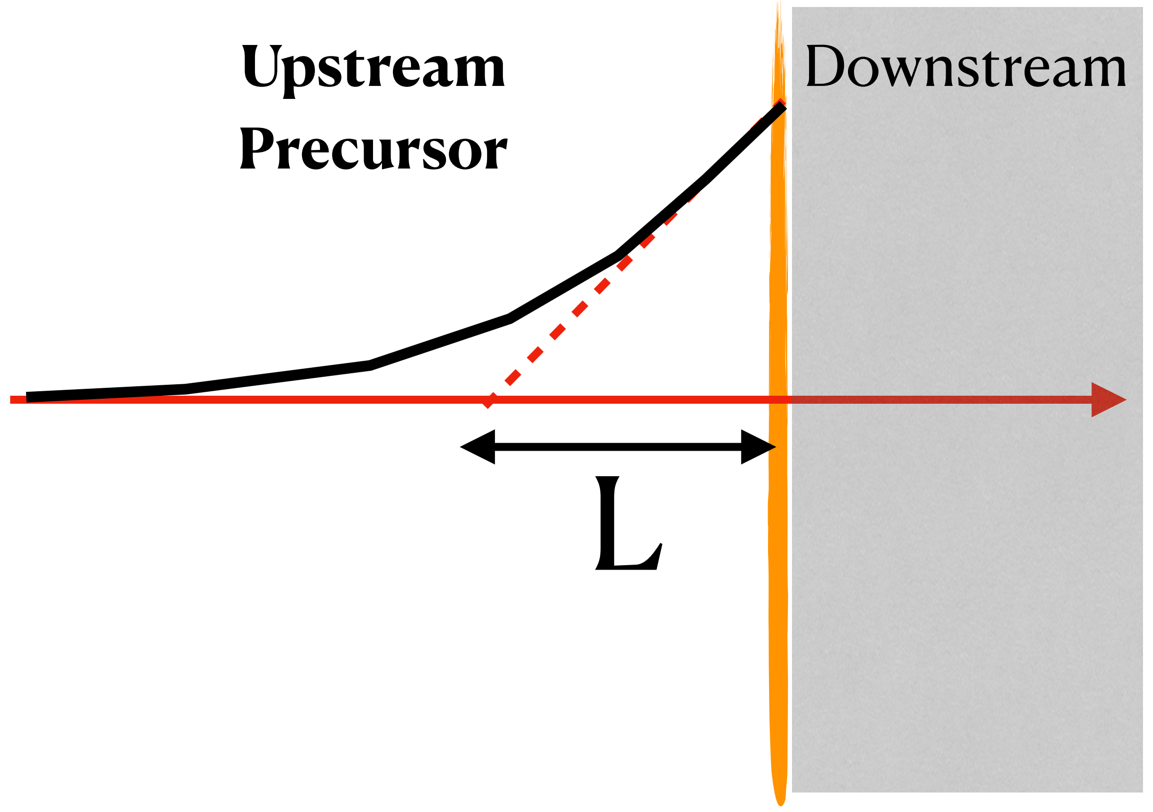
- The expected gamma ray spectrum of gamma rays from around Cas-A is already in marginal conflict with LHAASO upper limits for spectrum E^{-2}, despite the fact that at present E_{max}<<PeV
- For spectrum $E^{-2.3}$, all predictions are in agreement with LHAASO upper limits, but low current at E_{max}
- In this latter case E_{max}~30 TeV, despite Cas-A being at its highest point in terms of particle acceleration

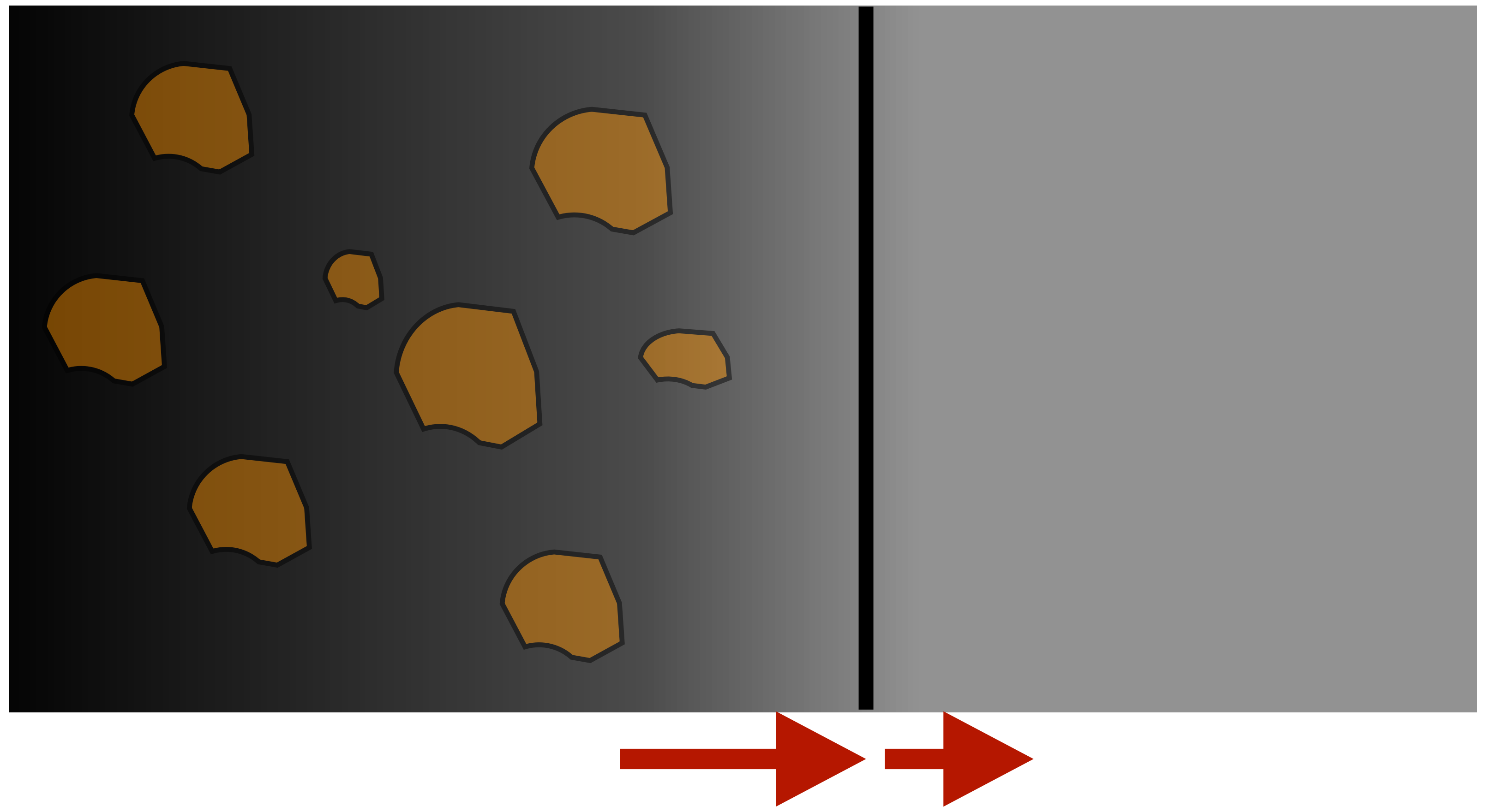


PB 2025

AMPLIFICATION DUE TO ACOUSTIC INSTABILITY

Effective magnetic field amplification could be achieved because of a combination of **acoustic instability** (Drury & Falle 1986, Drury & Downes 2012, Downes & Drury 2014) and **kinematic dynamo** (Beresnyak+ 2009)





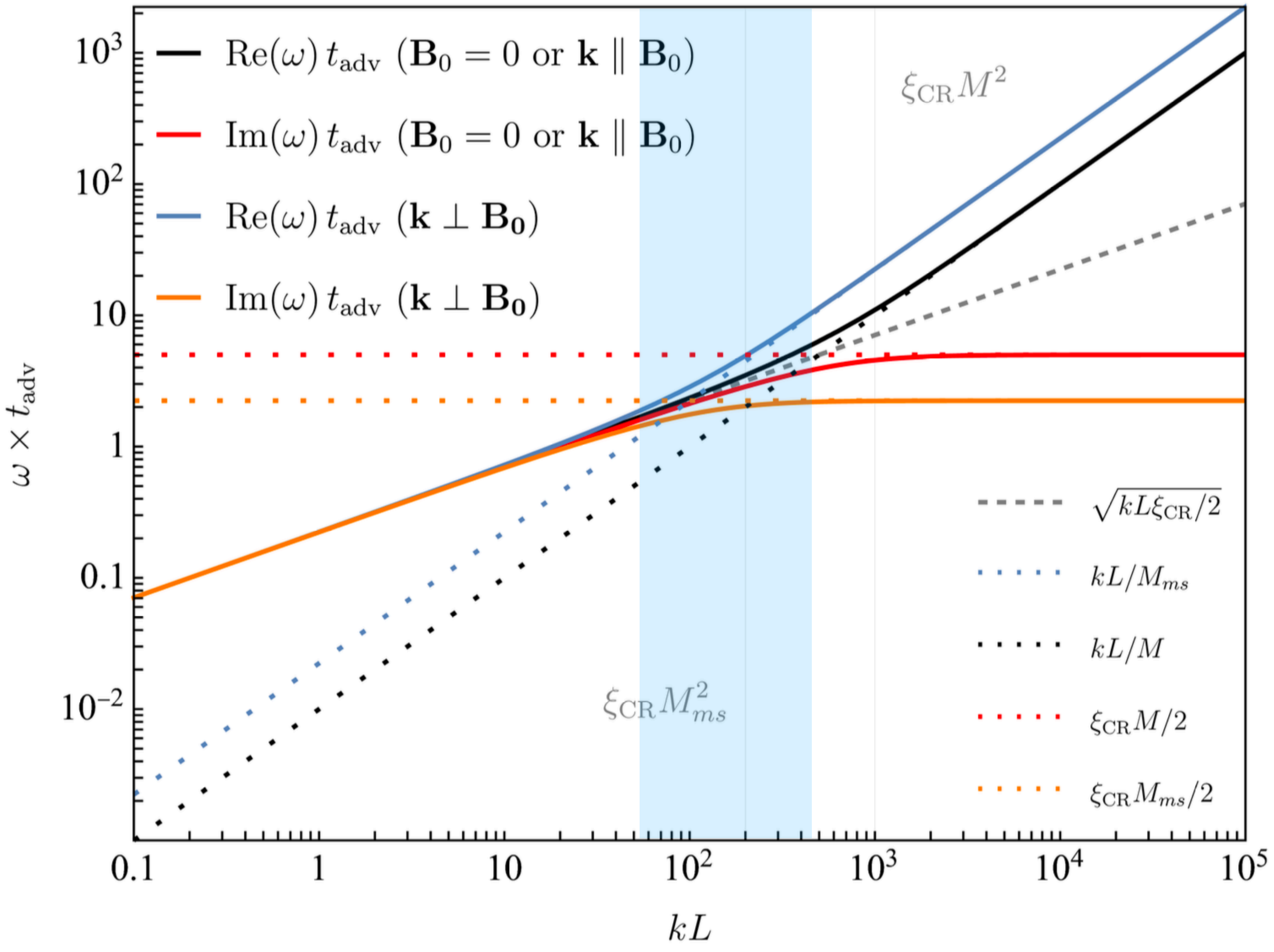
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0\,,$$
$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{\nabla P}{\rho} - \frac{\nabla P_{\mathrm{CR}}}{\rho} + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi\rho}\,,$$
$$\frac{\partial}{\partial t}\left(\frac{P}{\rho^{\gamma}}\right) + (\mathbf{u} \cdot \nabla)\left(\frac{P}{\rho^{\gamma}}\right) = 0\,,$$
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})\,,$$
$$\nabla \cdot \mathbf{B} = 0\,,$$

Perturbing

$$\omega^2 = c_s^2 k^2 + \frac{i\mathbf{k} \cdot \nabla P_{\mathrm{CR}}}{\rho_0}\,, \qquad (\mathbf{k} \parallel \mathbf{B}_0)$$
$$\omega^2 = c_{ms}^2 k^2 + \frac{i\mathbf{k} \cdot \nabla P_{\mathrm{CR}}}{\rho_0}\,, \qquad (\mathbf{k} \perp \mathbf{B}_0)$$

$c_{ms}^2 = c_s^2 + v_A^2$ and $v_A = B_0/\sqrt{4\pi\rho_0}$

AMPLIFICATION DUE TO ACOUSTIC INSTABILITY



Capanema, PB, Sobacchi 2026

$$P_{\rm CR}(x) = \xi_{\rm CR}\, \rho_0 u_0^2\, \frac{x}{L}$$

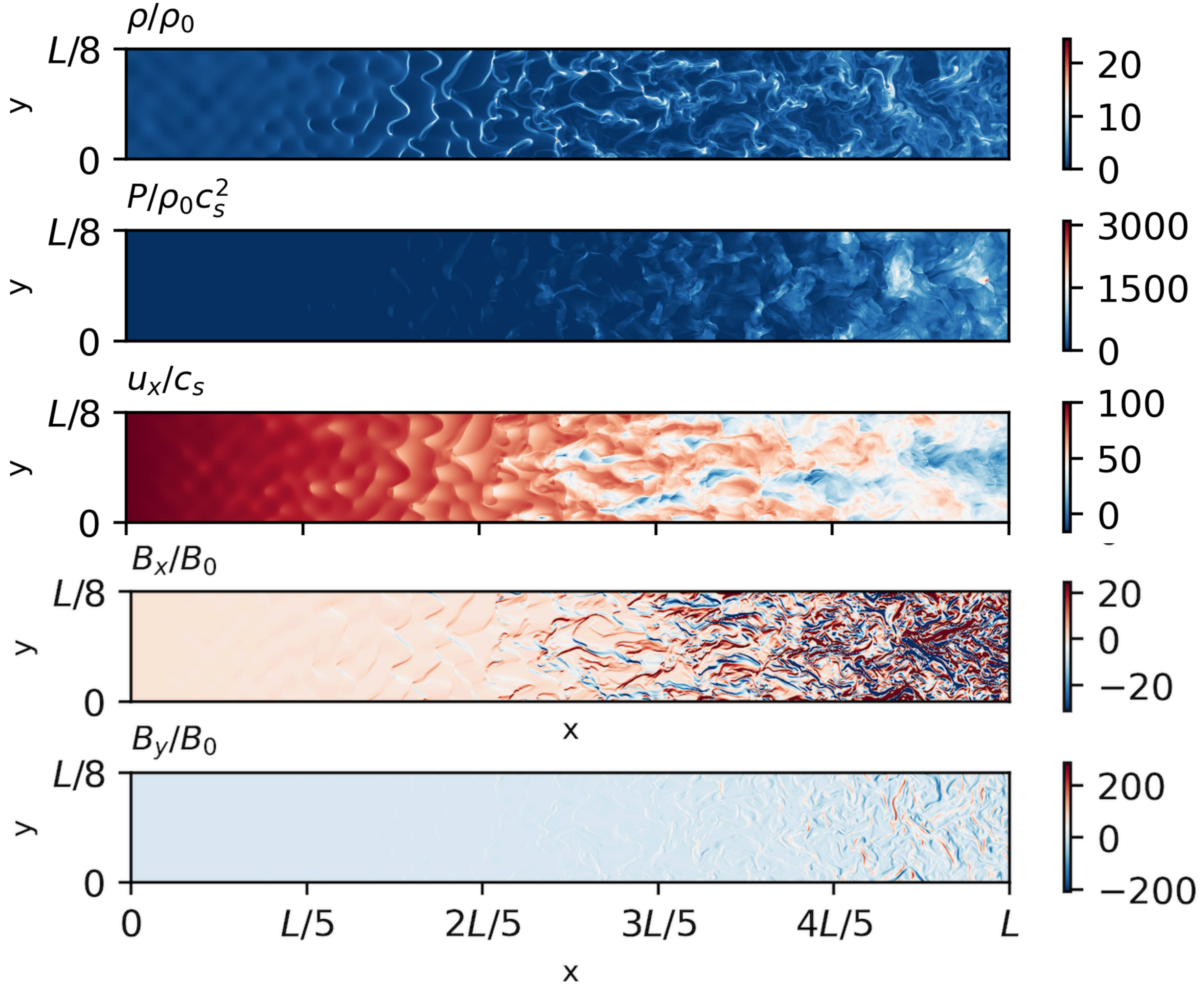
$$\nabla P_{\rm CR}(x) = \xi_{\rm CR}\, \frac{\rho_0 u_0^2}{L}\, \hat{\mathbf{x}}$$

$$\Gamma \to \begin{cases} \sqrt{\dfrac{k\,\xi_{\rm CR} u_0^2}{2L}}\,, & kL \ll \xi_{\rm CR} M^2 \\ \dfrac{\xi_{\rm CR} M u_0}{2L}\,, & kL \gg \xi_{\rm CR} M^2 \end{cases}$$

The time at disposal of this process for amplifying the magnetic field is the advection time on the scale of the precursor

The number of e-folds that can be achieved can be estimated as **$\sim\xi_{\rm CR}$ M/2** and we want this to be >>1

AMPLIFICATION DUE TO ACOUSTIC INSTABILITY

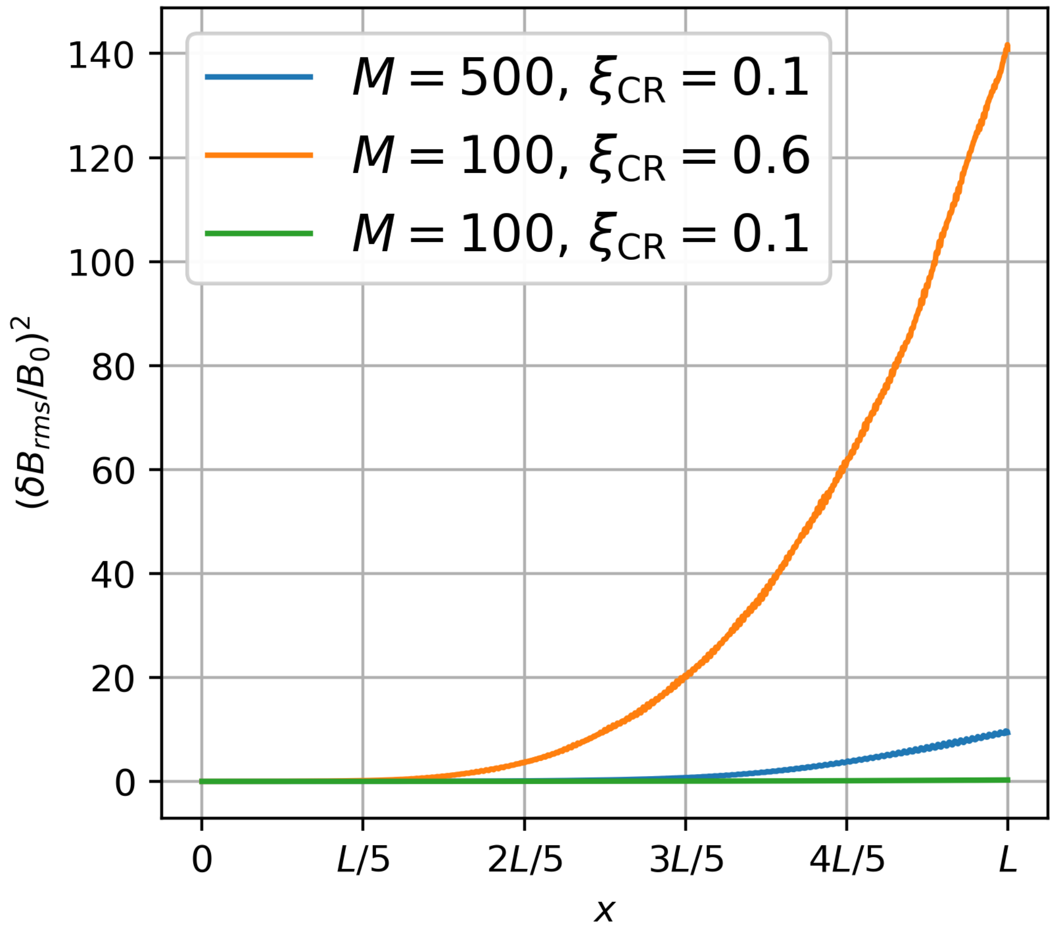


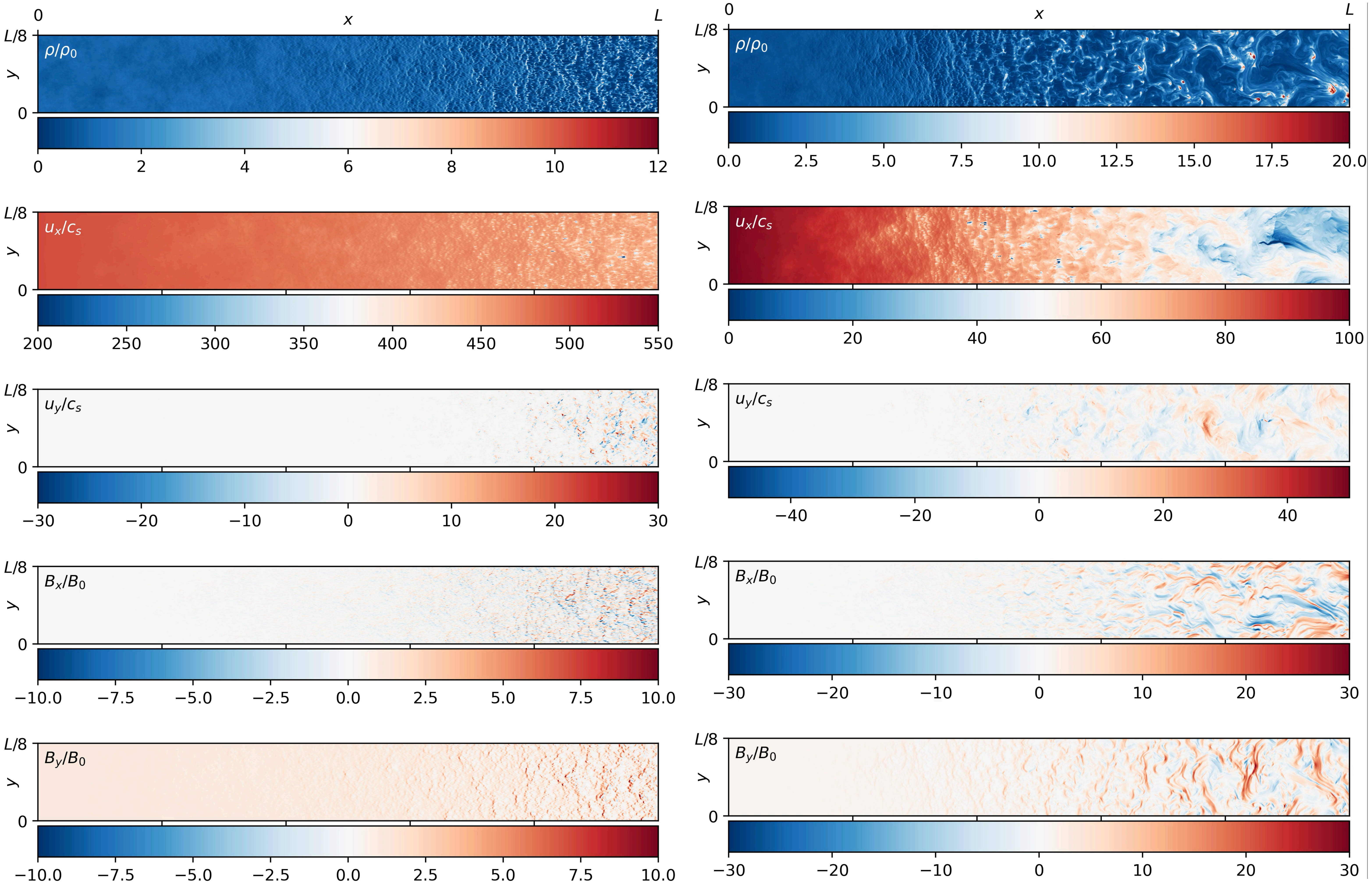
Capanema, PB, Sobacchi 2026

- In the linear phase of the instability the density perturbations grow and velocity perturbations develop, eventually reaching non linear levels, $\delta\rho/\rho \gtrsim 1$
- These structures often develop into shocklets due to the term $\partial \mathbf{u}/\partial t \sim \nabla P_{CR}/\rho$
- At the same time vorticity grows: $\partial \boldsymbol{\omega}/\partial t \sim (\nabla\rho \times \nabla P_{CR})/\rho^2$
- Turbulence leads to magnetic field amplification due to both compression and twisting of magnetic field lines

$M = 500,\ \xi_{\rm CR} = 0.1,\ \delta\rho_{0,rms}/\rho_0 = 0.1,\ v_A = c_s$

$M = 100,\ \xi_{\rm CR} = 0.6,\ \delta\rho_{0,rms}/\rho_0 = 0.1,\ v_A = c_s$





Capanema, PB, Sobacchi 2026 (see also previous results by Drury and Downes 2012, Downes &Drury 2014, Del Valle+2026)

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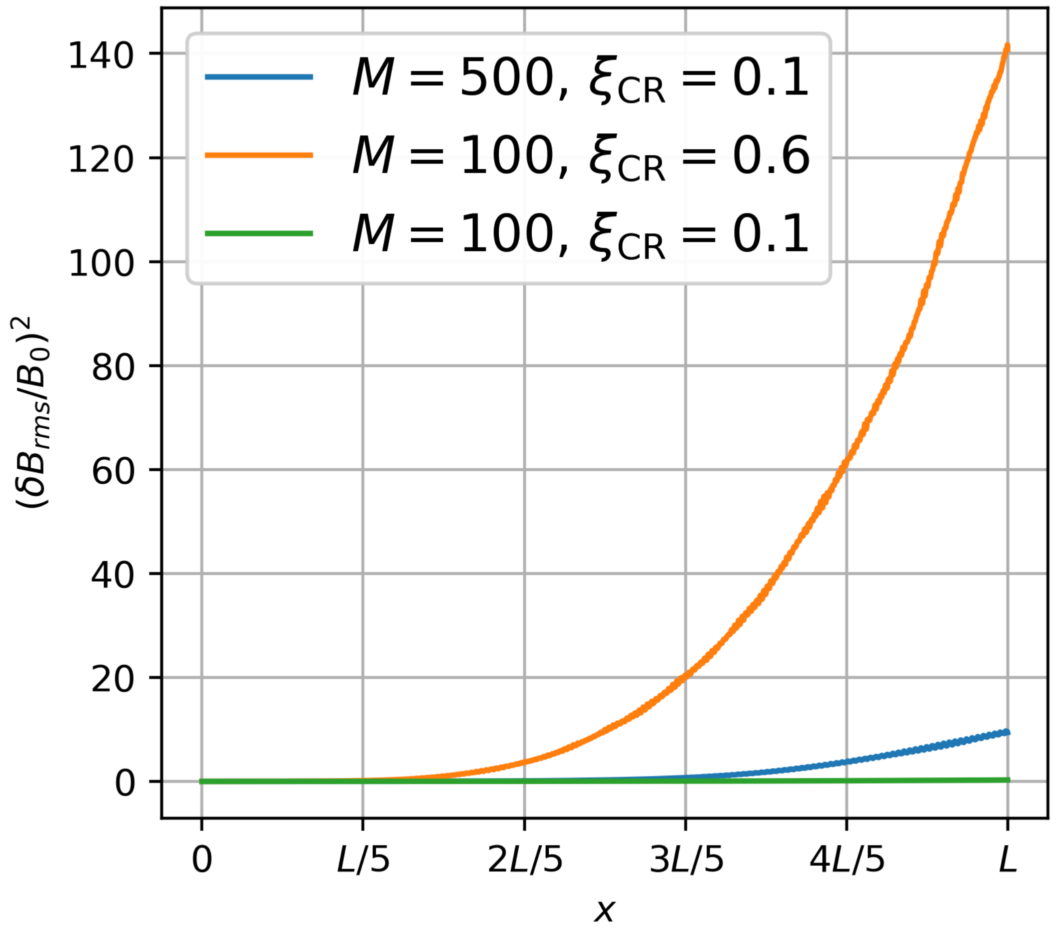
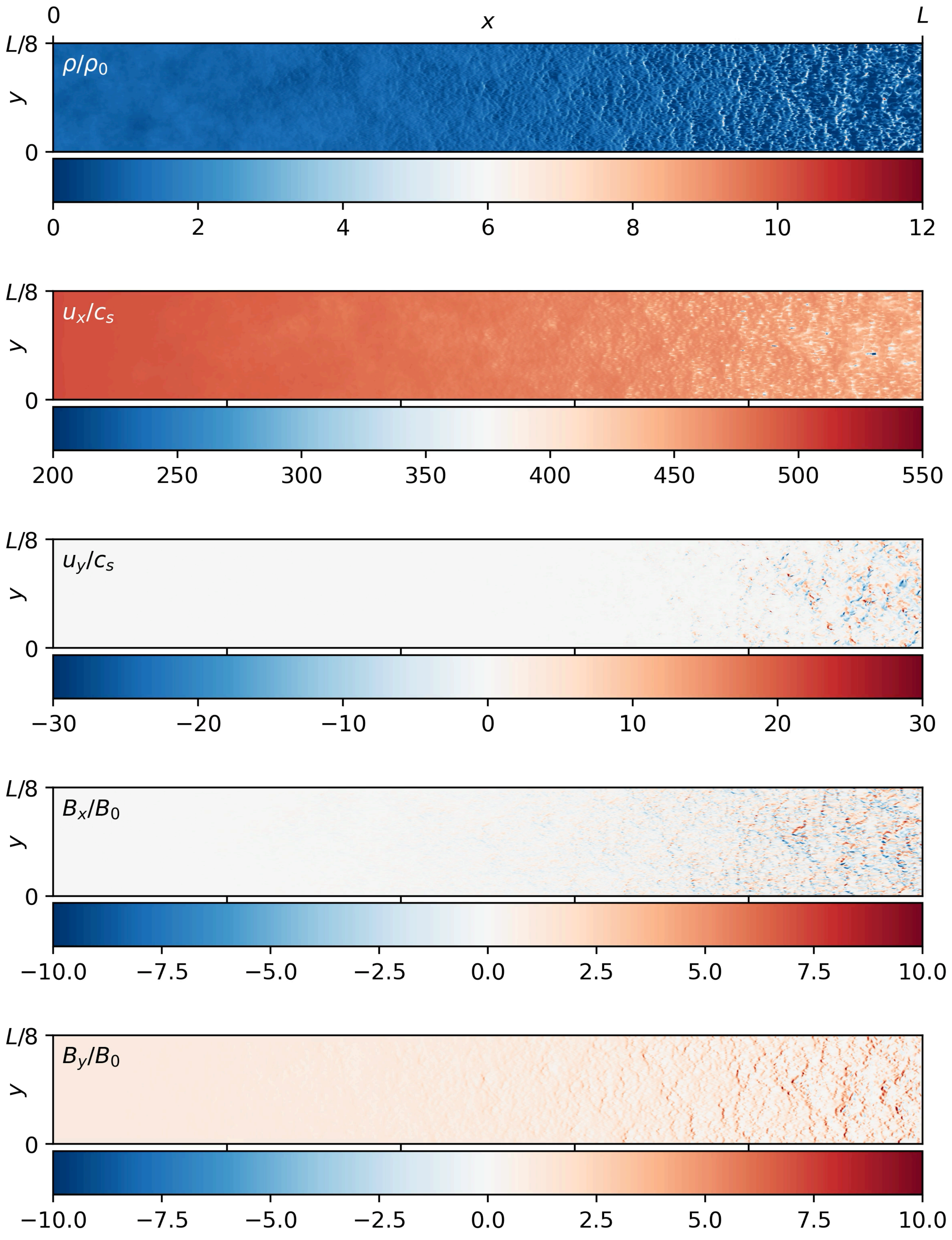
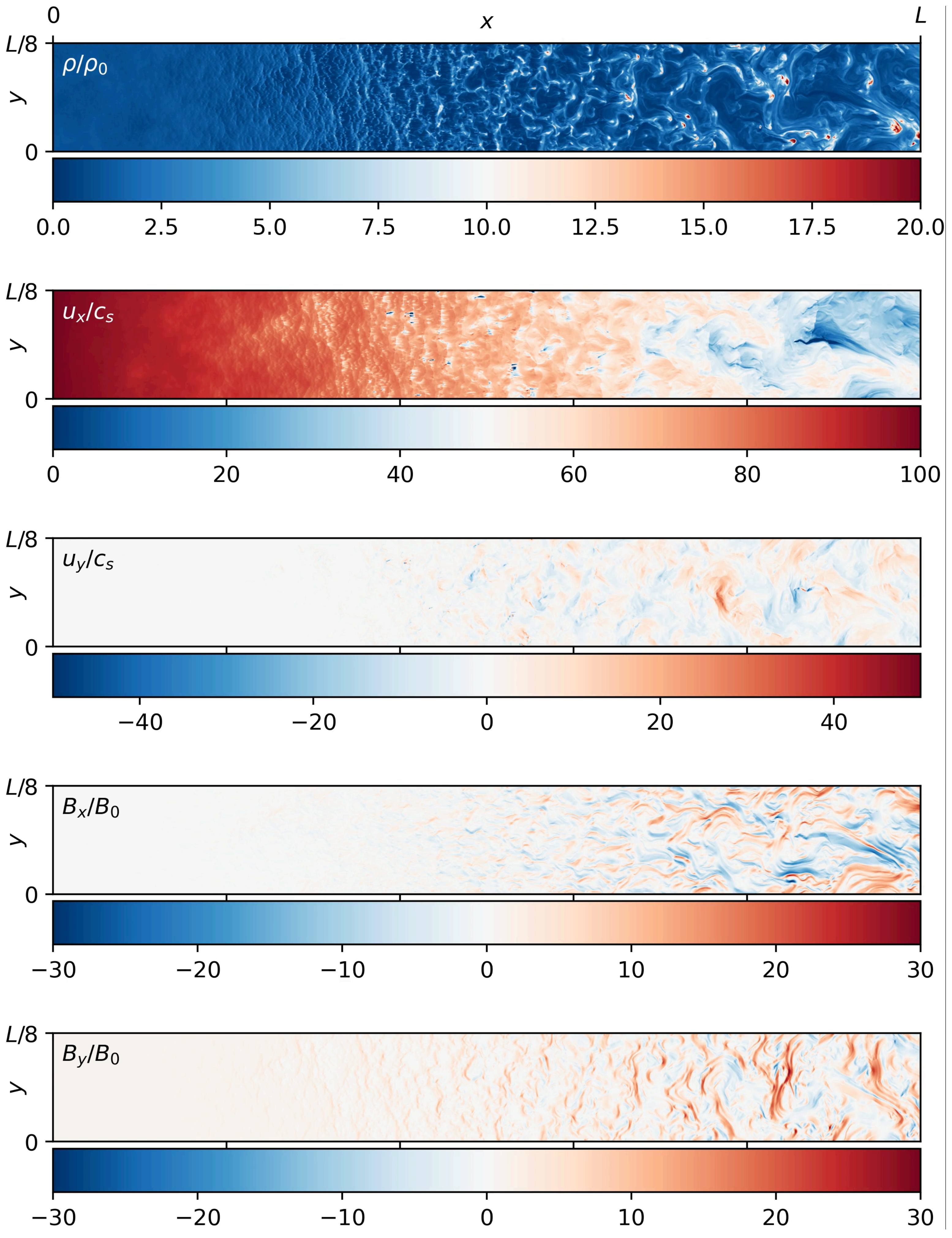




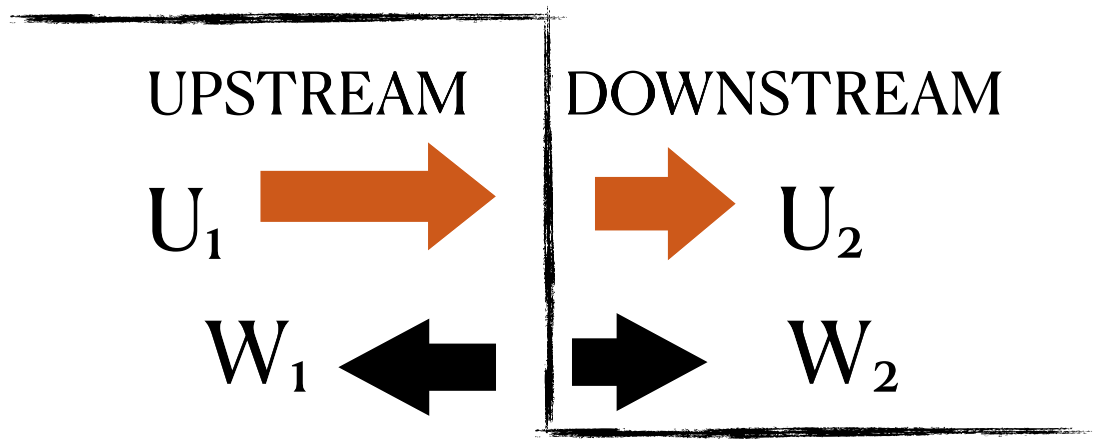


Capanema, PB, Sobacchi 2026 (see also previous results by Drury and Downes 2012, Downes &Drury 2014, Del Valle+2026)

EFFECT OF MFA ON THE SPECTRUM OF ACCELERATED PARTICLES

- THE ACTION OF COSMIC RAYS IS IN GENERAL OF INCREASING THE COMPRESSION FACTOR AT THE SHOCK DUE TO THE CHANGE OF ADIABATIC INDEX (AND OTHER EFFECTS, **PRECURSOR**) —> SPECTRUM SHOULD BECOME HARDER THAN STANDARD DSA

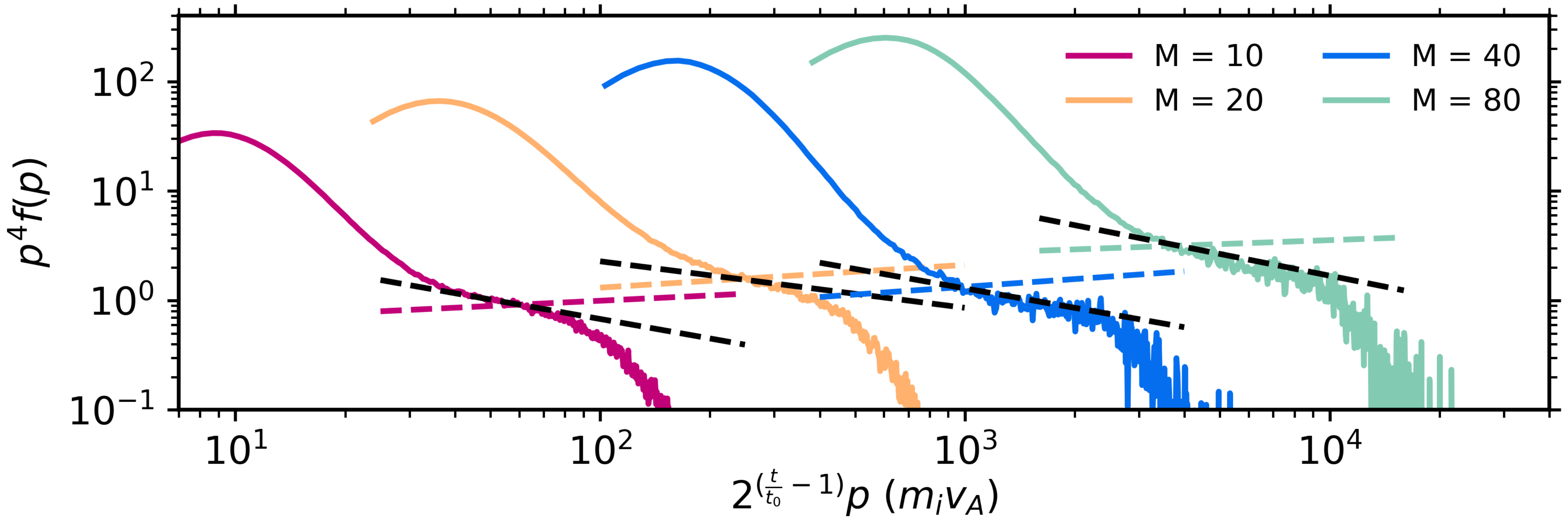
- HOWEVER, THE AMPLIFICATION OF MAGNETIC FIELD MAKES ANOTHER EFFECT APPEAR:



THE VELOCITY OF THE WAVES UPSTREAM IS $U_1 - W_1 \approx U_1$

IN HYBRID SIMULATIONS THE DOWNSTREAM WAVES ARE SEEN TO MOVE IN THE SAME DIRECTION AS THE PLASMA, WITH APPROXIMATELY THE ALFVEN SPEED IN THE AMPLIFIED FIELD (POSTCURSOR)

Haggerty & Caprioli 2020; Caprioli, Haggerty & PB 2020



$$W_2 \approx \frac{\delta B}{\sqrt{4\pi\rho}} = \alpha U_2 \quad \Rightarrow \quad q \approx \frac{3R}{R-1-\alpha}$$

SPECTRUM BECOMES STEEPER

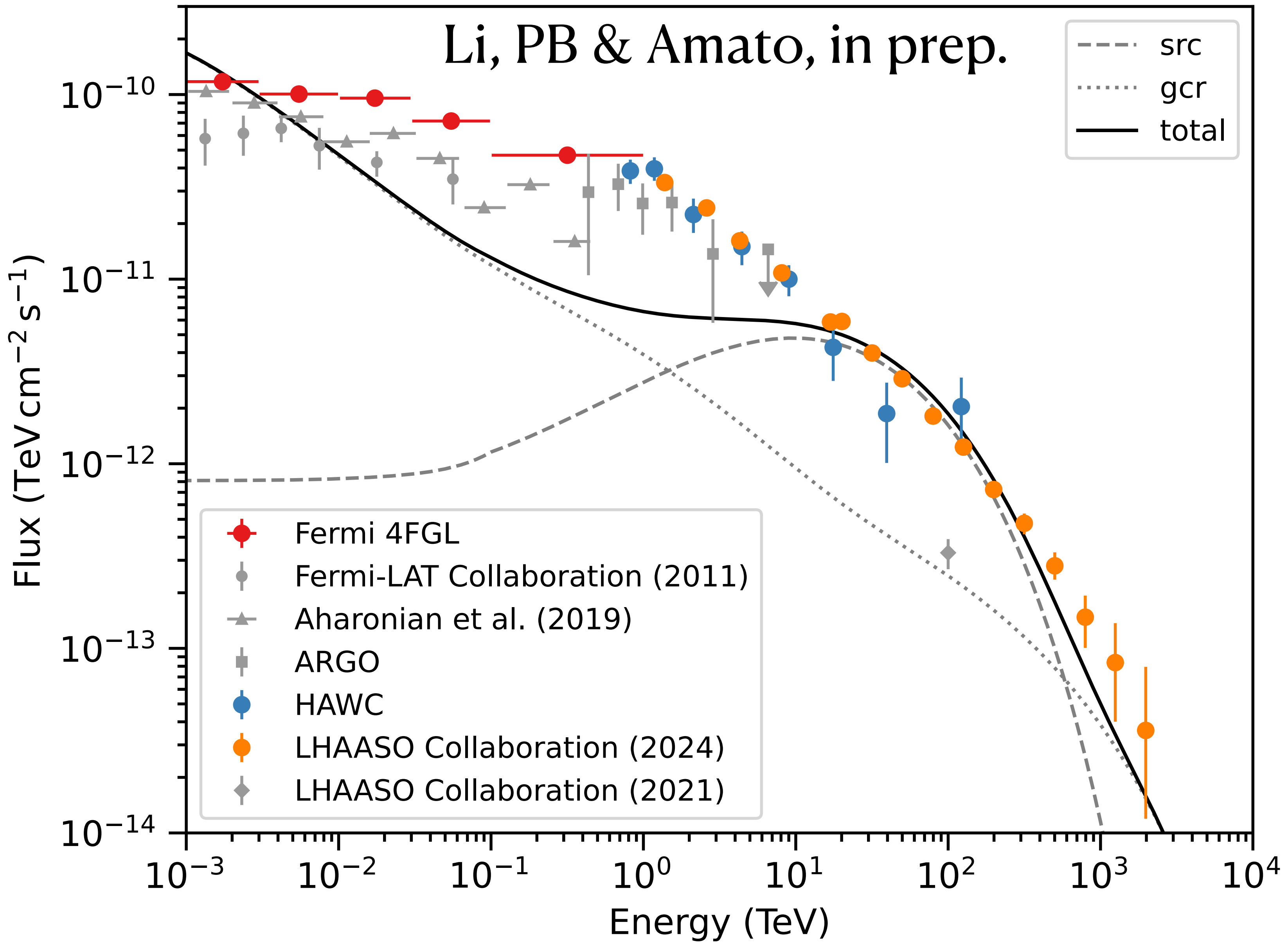
In turn this reduces the current in the form of escaping particles, and hence the $E_{\max}$ - self-regulation?

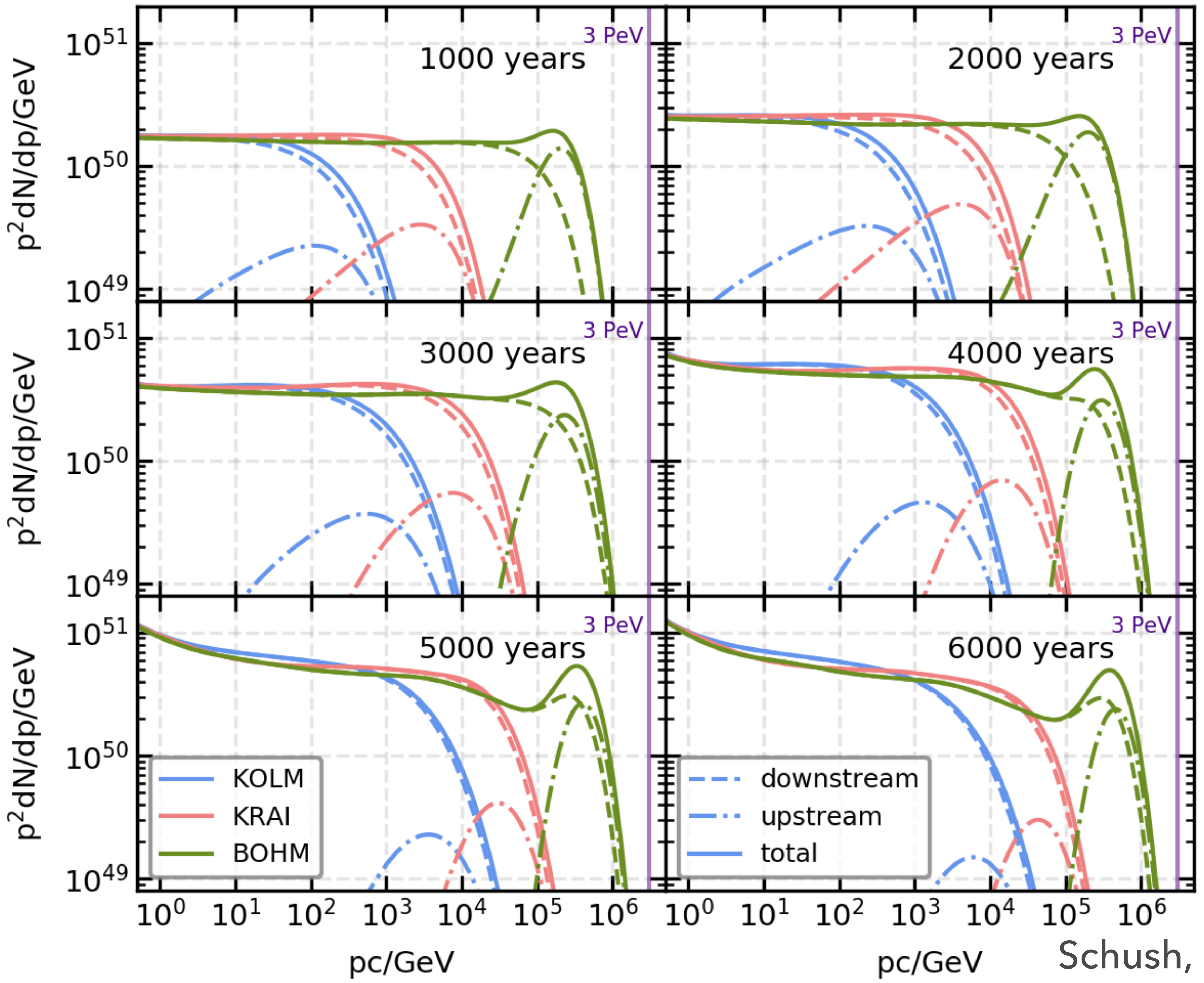
GENERAL IMPLICATIONS

- The excitation of the non-resonant instability has profound implications for E_{max} in SNRs
- Because of the scaling $\mathbf{E_{max} \propto v_s^3 \rho^{1/2}}$ the maximum energy is the highest in the early phases of core collapse SNe when however the mass processed is small, hence spectrum suppressed
- **In the end, it appears to be difficult for the non-resonant instability to have a large impact on the effective E_{max} in SNRs, which remains smaller than the knee energy, at least for SNR in their standard environments**
- Particles might reach higher E_{max} due to acoustic instability+kinematic dynamo, [erhaps in collaboration with the non resonant instability, that also causes large $\delta\rho/\rho$
- **The escape process shapes the whole source spectrum that we use in transport calculations! ... and it is model dependent...**

CAN WE EXCLUDE SNRS AS PEVATRONS?

- No, what we can reasonably state is that the bulk of SNRs (type Ia and IIn) are not PeVatrons, in the sense that their E_{max} at the beginning of the Sedov-Taylor phase is <<PeV. **The fact that at earlier times they can be PeVatrons is irrelevant from the point of view of the origin of CRs**
- It has been speculated that some type IIn SNR may occur in extremely large density media, due to **massive eruptions** during the late stages of the pre-SN star (Ekanger+2026), but it is not clear how frequent this phenomenon may be and how much of the mass is processed during such encounters
- It has been speculated that E_{max}~PeV in SN explosions in the core of star clusters (View+2022). The gamma ray emission from Cygnus OB2 could be due to such a SNR (Haarer+2025). Even assuming that E_{max}~PeV, a substantial part of the observed gamma rays would have to be of some other origin. A leptonic origin would require a large K_{ep} (Li, PB &Amato, 2026). **Reaching PeV always requires Bohm diffusion**



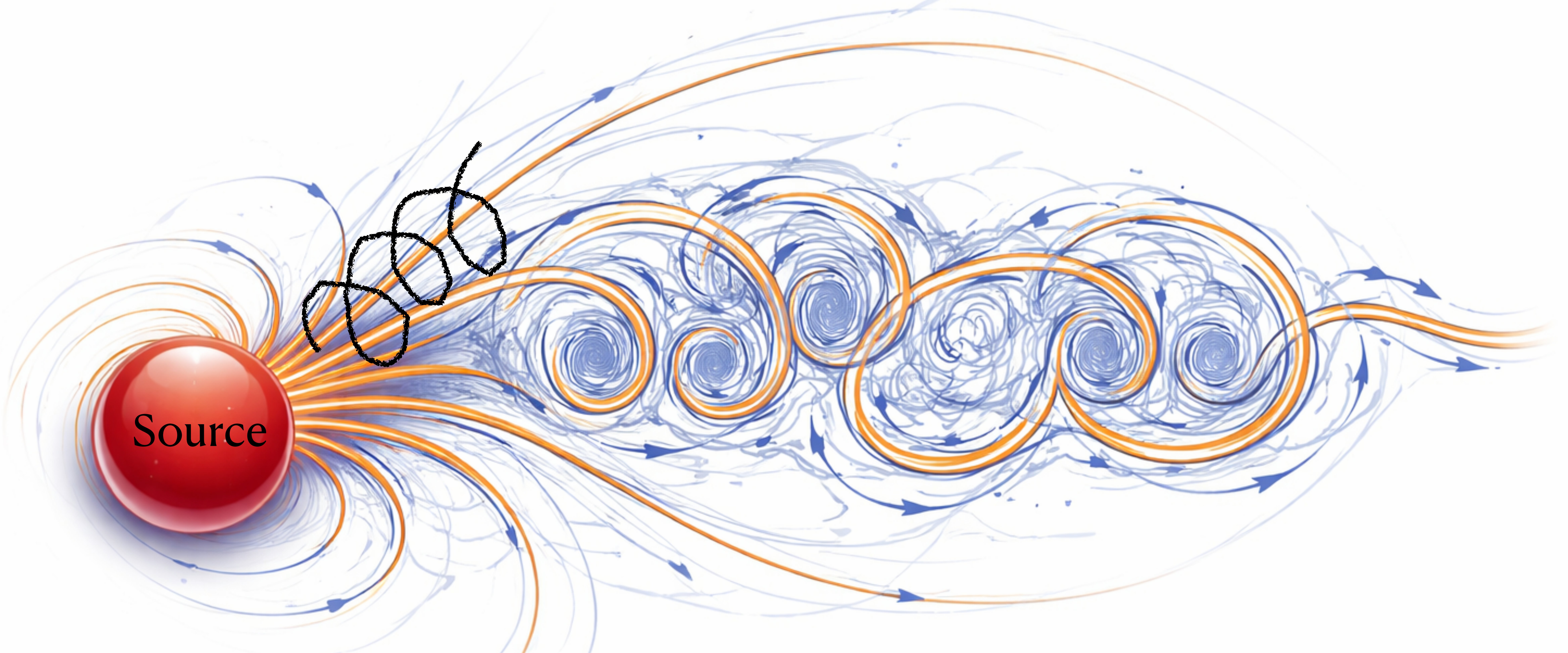


Schush, PB & Brose 2025

Outside Sources and before becoming Cosmic Rays

On the scale of the coherence scale of the Galactic magnetic field, particles do not yet experience Galactic transport, and in particular transport is not three dimensional!

The density of particles in the tube accessible to particles is >>Galactic and the gradients can be large as well —> Conditions for excitation of streaming instabilities



Dynamical Effects of CR near a source

Since in the ordinary ISM $E_{CR} \approx E_B \approx E_{th}$, it is clear that near a source the escaping CR must produce

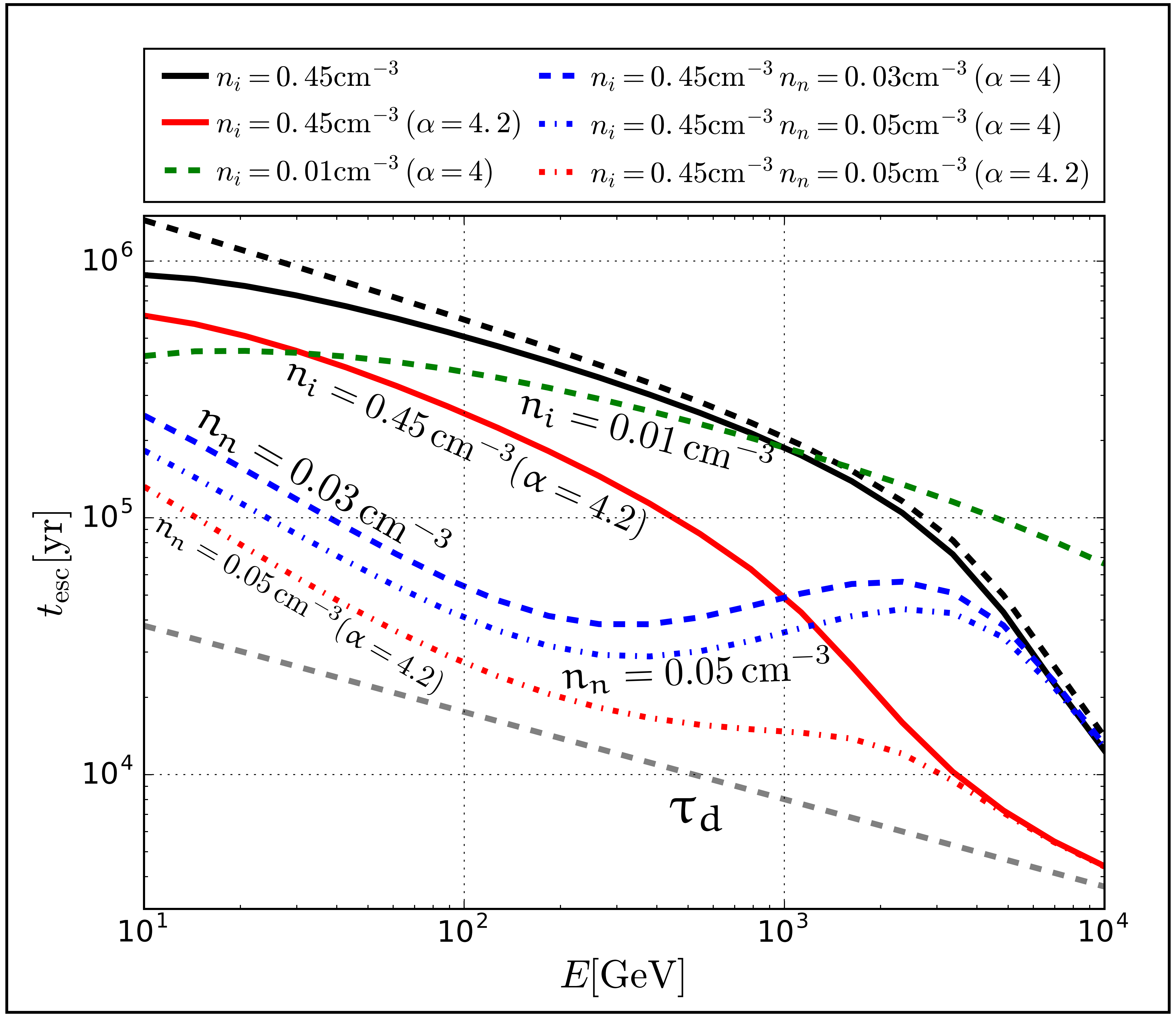
1. Local dynamical effects (gas evacuation, heating, vorticity, etc)

2. Magnetic field modification (amplification, shears, etc)

Notice that self-generation has a positive feed-back: larger gradients lead to stronger confinement which in turn lead to larger CR densities

This chain of events leads to some self-regulation of the whole process

Self-Confinement Near a SNR

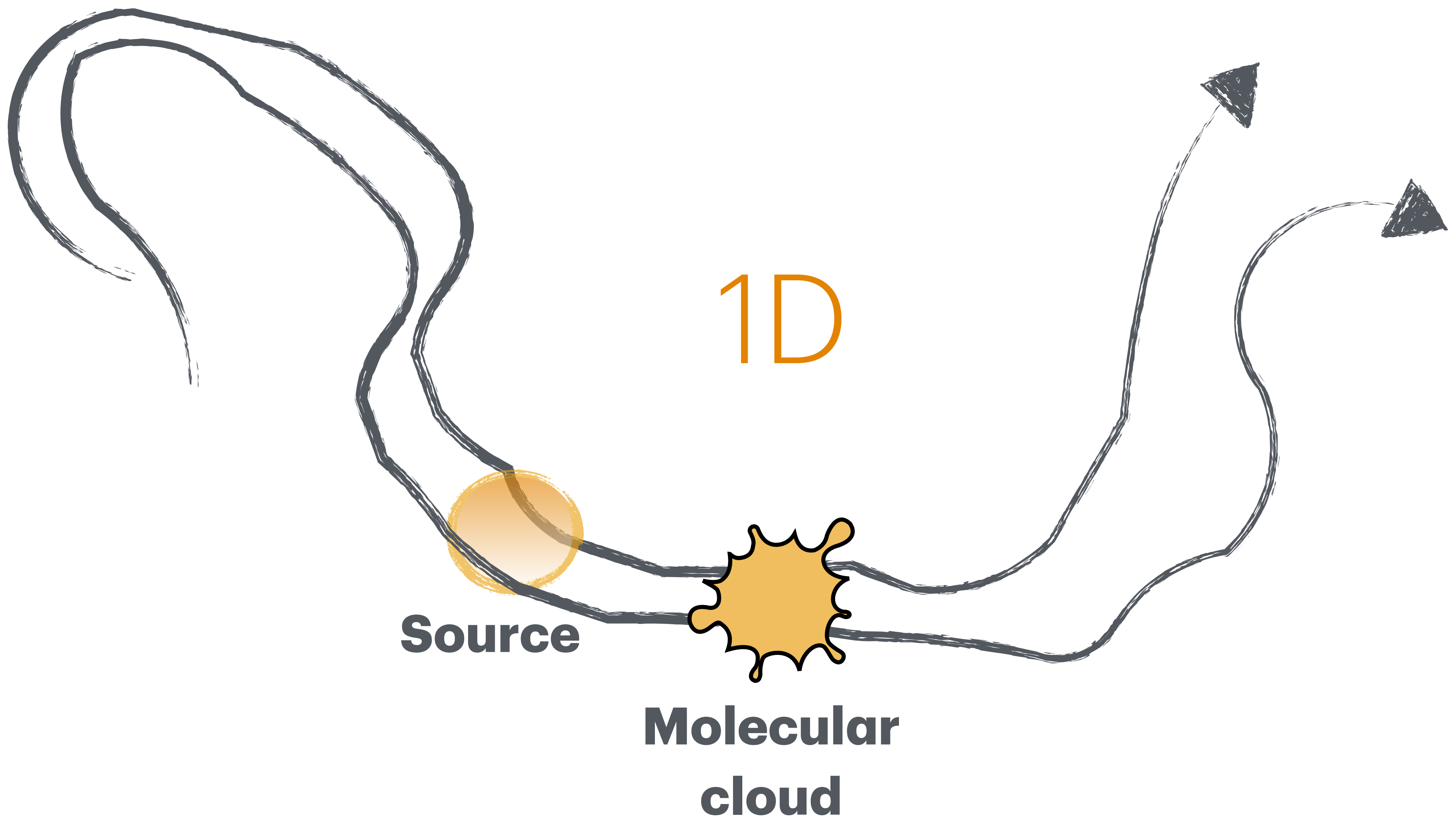


D'Angelo, PB, Amato 2016, 2018

Nava+2016, Recchia+, 2023

Depending on the total and ionized density in the circum-source medium, self-generation increases the confinement time by about one order of magnitude

GAMMA RAYS FROM OCCASIONAL NEARBY MOLECULAR CLOUDS



While confinement is easier if the surrounding medium is almost completely ionized, interactions of CR are easier in denser (hence partially ionized) plasma

The coexistence of these two conditions may occur when a SNR is located near a molecular cloud, that only acts as target, while confinement is guaranteed by the surrounding ionized medium

Molecular cloud touching the shock is not a good place to test diffusion and self-generation phenomena for many good reasons…

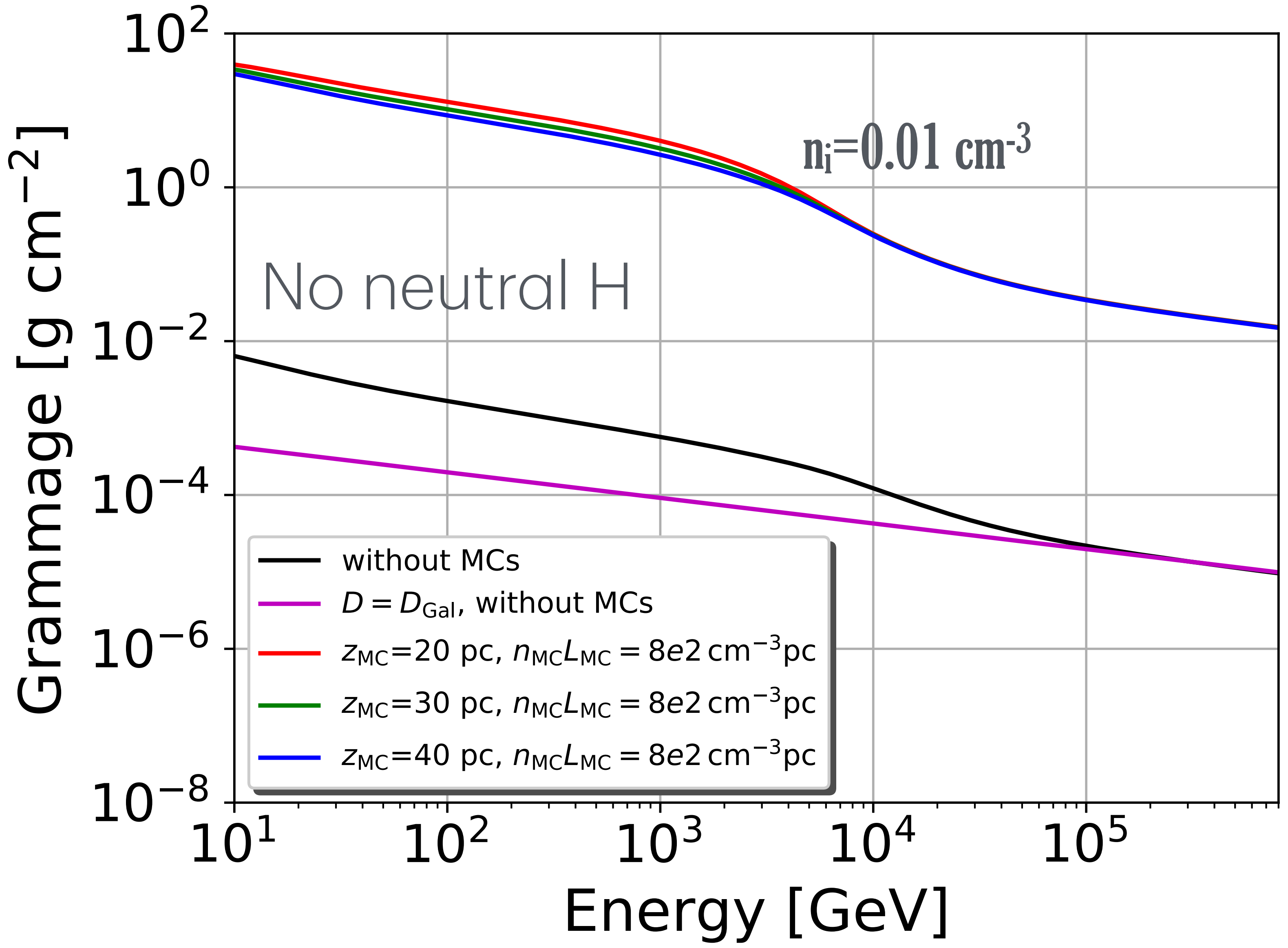
- *The shock velocity drops quickly making acceleration inefficient*
- *Only ionized gas takes part in the shock formation, low number of particles to become CR*
- *Large density of neutrals implies strong damping of waves —> weak acceleration*

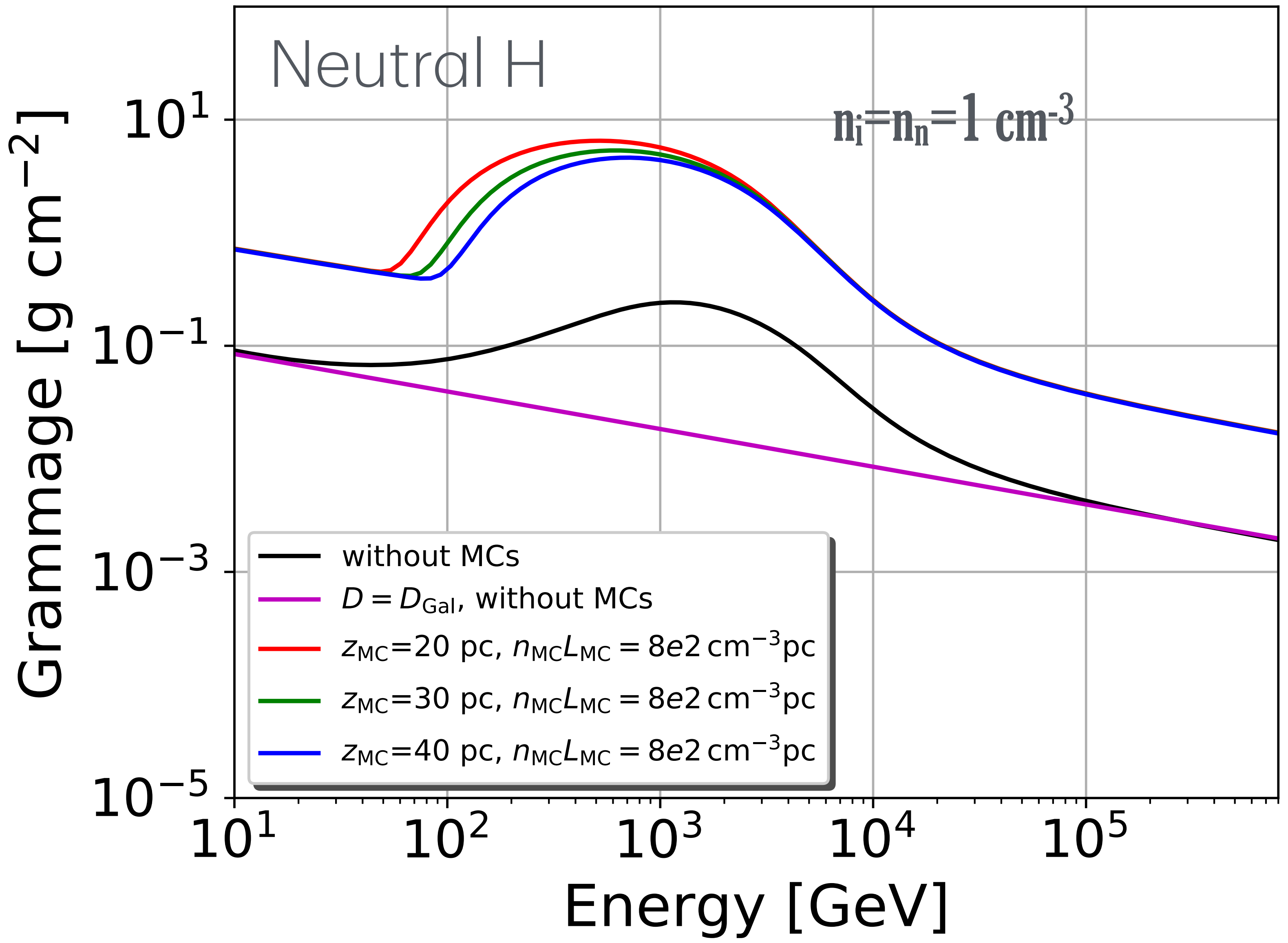
GRAMMAGE NEAR THE SOURCE

The grammage accumulated by CR near a source due to self-confinement depends on conditions (level of ionisation, coherence length)

Most importantly it depends upon the presence of molecular clouds in the neighbourhood of a source

...but it is clear that it is not a phenomenon that we can ignore at a time in which measurements are made at percent level





Bao, PB & Chen 2023

CONCLUSIONS

- Reaching high energies in SNR requires very effective **magnetic field amplification**
- Both non **resonant instability and acoustic instability**, possibly the combination of the two, could play an important role in accelerating to VHE
- These **instabilities** regulate the maximum energy of accelerated particles and in doing so they **shape the spectrum of particles** escaping into the ISM (Source spectrum in transport calculations)
- This source spectrum is **not a perfect power law**, it shows bumps and dips reflecting the time dependence of Emax and the different stages of SNR evolution
- It is quite likely that SNR may reach ≲100 TeV and **other sources** may be needed to pick up and reach the knee
- The gamma ray observations of SNR and **especially the regions around them** are crucial to make sense of this whole picture, as shown by the **case of Cas A**