Noether's Theorem and Holographic Gravitational Anomaly[1]

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[1] The talk is based on ongoing works with Z.e. Gao, W. Jia(CUHK), X.S. Wang, J.q. Wu, H. Zhang(BNU).

Outline

Topological Massive Gravity

- (Bulk) Action

$$S_{\mathrm{TMG}} = \frac{1}{16\pi G}\int_M (R+2)\boldsymbol{\varepsilon}_M + \frac{1}{16\pi G\cdot 2\mu}\int_M \left(\boldsymbol{\Gamma}\wedge \mathrm{d}\boldsymbol{\Gamma} + \frac{2}{3}\boldsymbol{\Gamma}\wedge\boldsymbol{\Gamma}\wedge\boldsymbol{\Gamma}\right) \quad (1)$$

  Motivation: 1. 3D gravity with local dynamics; 2. dual CFT with gravitational anomaly,(Basu, Wen, Xu 2025) e.g. quantum Hall effect, heterotic string, Kerr/CFT; 3. chiral gravity and log gravity(Li, Song, Strominger 2008; Maloney, Song, Strominger 2010); 4. more solution: warped AdS.

- EOM $E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - g_{\mu\nu} + \frac{1}{\mu}C_{\mu\nu}$ including the Cotton tensor(Garcia, Hehl, Heinicke, Macias 2004) $C_{\mu\nu} = \varepsilon_{\mu\rho\sigma}\nabla^\rho\left(R^\sigma{}_\nu - \frac{1}{4}Rg^\sigma{}_\nu\right)$ satisfying $\varepsilon^{\lambda\mu\nu}C_{\mu\nu} = \nabla^\mu C_{\mu\nu} = g^{\mu\nu}C_{\mu\nu} = 0$.
- Central charges(Li, Song, Strominger 2008)

$$c_L = \frac{3}{2G}\left(1-\frac{1}{\mu}\right), \quad c_R = \frac{3}{2G}\left(1+\frac{1}{\mu}\right) \quad (2)$$

  Chiral point $\mu = 1$ (Li, Song, Strominger 2008; Maloney, Song, Strominger 2010)

Asymptotic Boundary Conditions and Holographic Renormalization

- Asymptotic boundary conditions(as a part of the defination of the theory) in Poincaré coordinate

$$g_{zz} = \frac{1}{z^2} + \mathcal{O}(z^0), \quad g_{za} = \mathcal{O}\left(\frac{1}{z}\right), \quad g_{ab} = \frac{1}{z^2} {g^{(0)}}_{ab} + \mathcal{O}(z^0) \tag{3}$$

- Holographically renormalized action with boundary terms cancelling the divergence and ensuring that the variational problem is well-defined under Dirichlet boundary conditions

$$\begin{aligned} S_{\mathrm{TMG}} = & \lim_{z\to 0} \frac{1}{16\pi G} \int_M (R+2)\boldsymbol{\varepsilon}_M + \frac{1}{16\pi G \cdot 2\mu} \int_M \left(\boldsymbol{\Gamma} \wedge \mathrm{d}\boldsymbol{\Gamma} + \frac{2}{3} \boldsymbol{\Gamma} \wedge \boldsymbol{\Gamma} \wedge \boldsymbol{\Gamma}\right) \\ & + \lim_{z\to 0} \frac{1}{16\pi G} \int_\Gamma \left(2K - 2 + \log z \tilde{R} - 2z \log z \gamma^{ab} D_a g_{zb}\right) \boldsymbol{\varepsilon}_\Gamma \\ & + \frac{1}{16\pi G \cdot 2\mu} \int_\Gamma \left(z {g^{(0)}}_{ac} {\Gamma^{(0)c}}_{bd} \gamma^{(0)de} g_{ze} - z \log z {D^{(0)}}_a g_{zb}\right) \mathrm{d}x^a \wedge \mathrm{d}x^b \end{aligned} \tag{4}$$

Outline

Noether's Theorem in Textbook

- Noether's theorem, or how we deal with symmetries in **particle mechanics**
- Action

$$S = \int_{[t_i, t_f]} L(q^a, \dot{q}^a)\, \mathrm{d}t \tag{5}$$

- Variation of the action

$$\delta S = \int_{[t_i, t_f]} \left(\frac{\partial L}{\partial q^a} - \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{q}^a} \right) \delta q^a \, \mathrm{d}t + \left(\frac{\partial L}{\partial \dot{q}^a} \delta q^a \right) \Big|_{t_i}^{t_f} \tag{6}$$

- Hamiltonian from Legendre transformation $H = \frac{\partial L}{\partial \dot{q}^a} \dot{q}^a - L$ which is an on-shell conserved charge

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{q}^a} \dot{q}^a - \frac{\partial L}{\partial \dot{q}^a} \ddot{q}^a - \dot{L} \approx 0. \tag{7}$$

Noether's Theorem in Textbook

- Noether's theorem, or how we deal with symmetries in **field theory**(Peskin, Schroeder 1995)
- A transformation becomes a symmetry

$$\varphi^a(x) \mapsto \varphi'^a(x) = \varphi^a(x) + \delta\varphi^a(x) = \varphi^a(x) + \varepsilon^r f_r^a\big(\varphi, \partial_\mu \varphi\big) \Rightarrow \delta L(\varphi, \partial\varphi) = \partial_\mu(\varepsilon^r K_r^\mu) \quad (8)$$

- From the variation of the Lagrangian

$$\delta L = \left[\frac{\partial L}{\partial \varphi^a}\delta\varphi^a - \partial_\mu \frac{\partial L}{\partial\big(\partial_\mu \varphi^a\big)}\right]\delta\varphi^a + \partial_\mu\left[\frac{\partial L}{\partial\big(\partial_\mu \varphi^a\big)}\delta\varphi^a\right] \quad (9)$$

then the transformation of the Lagrangian can be calculated as rewriting $\delta\varphi^a$ into $\varepsilon^r f_r^a$.
If this transformation is a symmetry, then we have

$$\left[\frac{\partial L}{\partial \varphi^a}\delta\varphi^a - \partial_\mu \frac{\partial L}{\partial\big(\partial_\mu \varphi^a\big)}\right]\varepsilon^r f_r^a + \partial_\mu\left[\frac{\partial L}{\partial\big(\partial_\mu \varphi^a\big)}\varepsilon^r f_r^a - \varepsilon^r K_r^\mu\right] = 0. \quad (10)$$

- Conserved current $J_r^\mu := \frac{\partial L}{\partial\big(\partial_\mu \varphi^a\big)}\varepsilon^r f_r^a - \varepsilon^r K_r^\mu$ and conserved charge $Q_r|_\Sigma := \int_\Sigma * J_r$.

Noether's Theorem for AdS Gravity

- Noether's theorem with **anomalies**, or how we deal with **asymptotic symmetries in AdS gravity**
- Variation of the TMG action $S = \int_M \boldsymbol{L} + \int_\Gamma \boldsymbol{l}$

$$\delta S = -\frac{1}{16\pi G}\int_M E^{\mu\nu}\delta g_{\mu\nu}\boldsymbol{\varepsilon}_M - \frac{1}{4\pi}\int_\Gamma T^{ab}\delta g^{(0)}{}_{ab}\boldsymbol{\varepsilon}^{(0)} + \int_{\Sigma_f - \Sigma_i}\boldsymbol{\Theta} - \int_{\partial\Sigma_f - \partial\Sigma_i}\boldsymbol{C} \tag{11}$$

- Transformation $X_\xi = \int_M \mathrm{d}^3x \mathcal{L}_\xi g_{\mu\nu}\frac{\delta}{\delta g_{\mu\nu}}$ with $\xi^z = z\xi^{(1)z} + \mathcal{O}(z^3), \xi^a = \xi^{(0)a} + \mathcal{O}(z^2)$ becomes an asymptotic symmetry when $\xi^{(1)z} = D^{(0)}{}_a\xi^{(0)a}/2, D^{(0)}{}_a\xi^{(0)}{}_b + D^{(0)}{}_b\xi^{(0)}{}_a - g^{(0)}{}_{ab}D^{(0)}{}_c\xi^{(0)c} = 0.$

$$X_\xi \cdot \delta S = \int_\Gamma \boldsymbol{\nu}_\xi + \int_{\Sigma_f - \Sigma_i}(\xi\cdot\boldsymbol{L} + \boldsymbol{\Xi}_\xi) - \int_{\partial\Sigma_f - \partial\Sigma_i}\boldsymbol{\mu}_\xi \tag{12}$$

$\boldsymbol{\nu}_\xi$ is configuration-independent

$$\boldsymbol{\nu}_\xi|_\Gamma = \frac{1}{32\pi G}\left[\frac{1}{\mu}\varepsilon^{(0)cd}\partial_b\partial_c\Gamma^{(0)b}{}_{da}\xi^{(0)a} + \left(-2R^{(0)} + \frac{1}{\mu}\varepsilon^{(0)}{}_a{}^bD^{(0)}{}_c\Gamma^{(0)a}{}_{bc}\right)\xi^{(1)z}\right]\cdot\boldsymbol{\varepsilon}^{(0)}\Big|_\Gamma \tag{13}$$

Noether's Theorem for AdS Gravity

- Compare these two calculations, we can get 3 claims from Noether's theorem.

$$
\begin{aligned}
X_\xi \cdot \delta S \approx & -\frac{1}{4\pi}\int_\Gamma T^{ab}\big(X_\xi \cdot \delta g^{(0)}{}_{ab}\big)\boldsymbol{\varepsilon}^{(0)} + \int_{\Sigma_f-\Sigma_i} X_\xi \cdot \boldsymbol{\Theta} - \int_{\partial\Sigma_f-\partial\Sigma_i} X_\xi \cdot \boldsymbol{C} \\
= & \frac{1}{2\pi}\int_\Gamma \big(D^{(0)b}T_{ab}\xi^{(0)a} + T\xi^{(1)z}\big)\boldsymbol{\varepsilon}^{(0)} + \frac{1}{2\pi}\int_{\partial\Sigma_f-\partial\Sigma_i} T^{ab}\xi^{(0)}{}_b\varepsilon^{(0)}{}_{aa_1}\,\mathrm{d}x^{a_1} \\
& + \int_{\Sigma_f-\Sigma_i} X_\xi \cdot \boldsymbol{\Theta} - \int_{\partial\Sigma_f-\partial\Sigma_i} X_\xi \cdot \boldsymbol{C} \equiv \int_\Gamma \boldsymbol{\nu}_\xi + \int_{\Sigma_f-\Sigma_i}\big(\xi\cdot\boldsymbol{L} + \boldsymbol{\Xi}_\xi\big) - \int_{\partial\Sigma_f-\partial\Sigma_i}\boldsymbol{\mu}_\xi
\end{aligned}
\tag{14}
$$

0. Noether charge $H_\xi := \int_\Sigma \big(X_\xi\cdot\boldsymbol{\Theta} - \xi\cdot\boldsymbol{L} - \boldsymbol{\Xi}_\xi\big) + \int_{\partial\Sigma}\big(-X_\xi\cdot\boldsymbol{C} + \boldsymbol{\mu}_\xi\big) \equiv \int_M E^{\mu\nu} + \int_\Gamma T^{ab}$
1. Conservation up to the configuration-independent anomaly $H_\xi|_{\Sigma_f-\Sigma_i} \approx \boldsymbol{\nu}_\xi$.
2. Symmetry is physical(like the case of field theory) $X_\xi \cdot \delta E^{\mu\nu} \approx 0$.
3. Conserved charge generates the asymptotic symmetry, or the $X_\xi \cdot \delta\big(\int_\Sigma \boldsymbol{\Theta} - \int_{\partial\Sigma}\boldsymbol{C}\big) \approx -\delta H_\xi$.

Outline

Conserved Charges from Noether's Theorem and Stress Tensor

$$
\begin{aligned}
H_\xi &:= \int_\Sigma \left(X_\xi \cdot \boldsymbol{\Theta} - \xi \cdot \boldsymbol{L} - \boldsymbol{\Xi}_\xi \right) + \int_{\partial\Sigma} \left(-X_\xi \cdot \boldsymbol{C} + \boldsymbol{\mu}_\xi \right) \\
&= \frac{1}{16\pi G} \int_\Sigma E^{\mu\nu} \xi_\nu \varepsilon_{\mu\mu_1\mu_2} \, \mathrm{d}x^{\mu_1} \wedge \mathrm{d}x^{\mu_2} - \frac{1}{2\pi} \int_{\partial\Sigma} T^{ab} \xi^{(0)}{}_b \varepsilon^{(0)}{}_{aa_1} \, \mathrm{d}x^{a_1}
\end{aligned}
\tag{15}
$$

i.e. Noether charge = conserved charge induced by the stress tensor(Skenderis, Taylor, Rees 2009).

- On one hand, this equality can be obtained by comparing the two calculations of $X_\xi \cdot \delta S$, without the need for the explicit expression of the stress tensor.

$$
\begin{aligned}
X_\xi \cdot \delta S &\supset \frac{1}{2\pi} \int_{\partial\Sigma_f - \partial\Sigma_i} T^{ab} \xi^{(0)}{}_b \varepsilon^{(0)}{}_{aa_1} \, \mathrm{d}x^{a_1} + \int_{\Sigma_f - \Sigma_i} X_\xi \cdot \boldsymbol{\Theta} - \int_{\partial\Sigma_f - \partial\Sigma_i} X_\xi \cdot \boldsymbol{C} \\
X_\xi \cdot \delta S &\supset \int_{\Sigma_f - \Sigma_i} \left(\xi \cdot \boldsymbol{L} + \boldsymbol{\Xi}_\xi \right) - \int_{\partial\Sigma_f - \partial\Sigma_i} \boldsymbol{\mu}_\xi
\end{aligned}
\tag{16}
$$

- On the other hand, this equality can also be verified with the explicit expression of the stress tensor.

Conserved Charges from Covariant Phase Space(CPS) Formalism

1. Bulk Lagrangian only $S = \int_M \boldsymbol{L}$
2. Symplectic twins $\delta \boldsymbol{L} = \boldsymbol{E}_a \delta\varphi^a + \mathrm{d}\boldsymbol{\Theta}, \boldsymbol{\Omega} = \delta\boldsymbol{\Theta}$
3. Symmetry(in the phase space) X_ξ
4. Hamiltonian $\delta H_\xi = -\int_\Sigma X_\xi \cdot \boldsymbol{\Omega}$, brackets, …

- ADM mass & Black Hole entropy(Lee, Wald 1990; Wald 1993; Iyer, Wald 1994; 1995; Wald, Zoupas 2000; Harlow, Wu 2020)

- Conserved current

$$\boldsymbol{J}_\xi := X_\xi \cdot \boldsymbol{\Theta} - \left(\xi \cdot \boldsymbol{L} + \boldsymbol{\Xi}_\xi\right) \approx \mathrm{d}\boldsymbol{Q}_\xi \quad (17)$$

This charge is not the one in the CPS sense.

$$\delta \boldsymbol{J}_\xi \approx -X_\xi \cdot \boldsymbol{\Omega} + \mathrm{d}\left(\xi \cdot \boldsymbol{\Theta} + \boldsymbol{\Sigma}_\xi\right) \quad (18)$$

But of course, CPS has its own remedial measures.

$$\delta\, \mathrm{d}\left(\boldsymbol{Q}_\xi - \boldsymbol{C}'_\xi\right) \approx -X_\xi \cdot \boldsymbol{\Omega} \quad (19)$$

with $\delta \boldsymbol{C}'_\xi = \xi \cdot \boldsymbol{\Theta} + \boldsymbol{\Sigma}_\xi$. (Tachikawa 2007; Cheng, Hung, Liu, Zhou 2016; Jiang 2019)

- In short, CPS can only obtain variations of charges. The trouble is that there are configuration-independent parts in the TMG charge and stress tensor.

Conserved Charges from Covariant Phase Space(CPS) Formalism

1. Bulk Lagrangian only $S = \int_M \boldsymbol{L}$
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- ADM mass & Black Hole entropy(Lee, Wald 1990; Wald 1993; Iyer, Wald 1994; 1995; Wald, Zoupas 2000; Harlow, Wu 2020)

Two obvious advantages of Noether's theorem compared to CPS Formalism

- Take the boundary effect into account correctly by holographic renormalization.
- Obtain the complete and closed expression for the conserved charge directly.

- Conserved current

$$\boldsymbol{J}_\xi := X_\xi \cdot \boldsymbol{\Theta} - \left(\xi \cdot \boldsymbol{L} + \boldsymbol{\Xi}_\xi\right) \approx \mathrm{d}\boldsymbol{Q}_\xi \quad (17)$$

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$$\delta \boldsymbol{J}_\xi \approx -X_\xi \cdot \boldsymbol{\Omega} + \mathrm{d}\left(\xi \cdot \boldsymbol{\Theta} + \boldsymbol{\Sigma}_\xi\right) \quad (18)$$

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Outline

Gang of Six: Weyl, Gravitational, Diffeomorphism & Lorentz, Covariant & Consistent

Anomalies

Weyl/Conformal Anomaly ✓

$$\operatorname{tr} T \neq 0 \tag{20}$$

Gravitational Anomaly(Castro, Mühlmann 2020)

Diffeomorphism Anomaly

$$D^j T_{ij} \neq 0 \tag{21}$$

Lorentz Anomaly

$$T_{[ij]} \neq 0 \tag{22}$$

Covariant Anomaly

Consistent Anomaly ✓

- Covariant Anomaly: $D^j T_{ij}$ is covariant under diffeomorphism, but does not satisfy Wess-Zumino consistency condition, and can't be derived from variation of any effective action.
- Consistent Anomaly: $D^j T_{ij}$ satisfies W-Z consistency condition, but is not covariant under diffeomorphism, and can be derived from variation of some effective action. (Skenderis, Taylor, Rees 2009)

Anomalies from Noether's Theorem and Stress Tensor

$$\begin{aligned} D^{(0)b}T_{ab} &= \frac{1}{4G}\lim_{z\to 0}\frac{E_{za}}{z} + \frac{1}{16G\mu}\varepsilon^{(0)cd}\partial_b\partial_c\Gamma^{(0)b}{}_{cd} \\ g^{(0)ab}T_{ab} &= \frac{1}{4G}\lim_{z\to 0}E_{zz} - \frac{1}{8G}R^{(0)} + \frac{1}{16G\mu}\varepsilon^{(0)}{}_a{}^b D^{(0)c}\Gamma^{(0)a}{}_{bc} \end{aligned} \tag{23}$$

from which the left and right central charge c_L and c_R can be read out.

- On one hand, these anomalies can be obtained by comparing the two calculations of $X_\xi \cdot \delta S$, without the need for the explicit expression of the stress tensor.(Kraus, Larsen 2006)

$$\begin{aligned} X_\xi \cdot \delta S &\supset \frac{1}{2\pi}\int_\Gamma \left(D^{(0)b}T_{ab}\xi^{(0)a} + T\xi^{(1)z}\right)\varepsilon^{(0)} \\ X_\xi \cdot \delta S &\supset \int_\Gamma \nu_\xi \end{aligned} \tag{24}$$

- On the other hand, these anomalies can also be verified with the explicit expression of the stress tensor.(Solodukhin 2006; Skenderis, Taylor, Rees 2009)

Mixed Anomalies

- Since the Noether theorem provides a method for reading anomalies and avoids calculating the explicit expression of the stress tensor, one can attempt to change the form of the anomalies by adding boundary counterterms to the action.
- By adding the local counterterm $\ell \propto g^{(0)}{}_{ac}\Gamma^{(0)c}{}_{bd}g^{de}\Gamma^{(0)f}{}_{ef}\,\mathrm{d}x^a \wedge \mathrm{d}x^b$, the anomalies can be transformed into $g^{(0)ab}T_{ab} = -R^{(0)}/(8G)$, $D^{(0)b}T_{ab} = \ldots$, that is, the gravitational Chern-Simons term only contribute to the gravitational anomaly but do not contribute to the Weyl anomaly.
- There might be suitable local counterterm that would make the gravitational Chern-Simons term contribute only to the Weyl anomaly but not to the gravitational anomaly. However, obtaining such local counterterm is extremely difficult due to the complexity of the diffeomorphism of the affine connection $\Gamma^{(0)c}{}_{ab}$.
- Furthermore, the local counterterm can only convert gravitational anomalies and Weyl anomalies into each other, but it cannot eliminate both at the same time. This is similar to the standard story of mixed anomalies in the field theory.

ご清聴ありがとうございました。

- Topological Massive Gravity: Defination and Properties
- Noether's Theorem in Textbook and AdS Gravity
- 3 Conserved Charges
  - Noether's Theorem & Stress Tensor & Covariant Phase Space
- 2 Anomalies: Gravitational & Weyl
  - from Noether's Theorem & Stress Tensor

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