Anomalous transport in anomaly free currents from symmetry breaking

Karl Landsteiner

Instituto de Física Teórica UAM-CSIC

EXCELENCIA SEVERO OCHOA

Campus Internacional excelencia UAM CSIC

Based on:

JHEP 02 (2026) 219, e-Print: 2511.08374 [hep-th]

In collaboration with:
E. Megias (Univ. Granada), N. Rai, A. Tamang (Manipal Inst. Techn.)

International Conference on Symmetry Breaking Phenomena in Quantum Field Theory

15-18 May 2026, Hefei, China

Outline

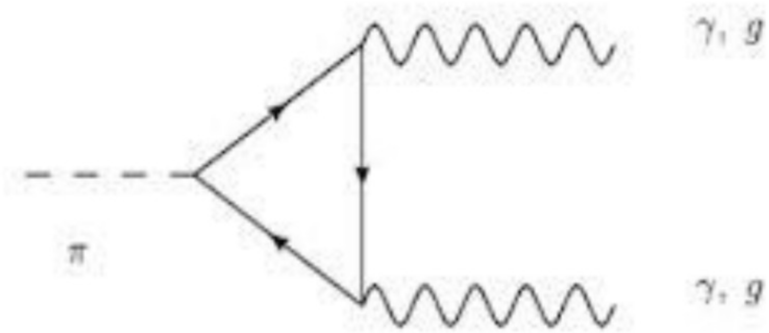
- Anomaly induced transport
- Holography
- Holographic model with anomalies and symmetry breaking
- Discussion and Outlook

Anomaly induced transport

- **Chiral anomalies**
  Conflict between two core concepts of theoretical physics:

Symmetry ⟷ vs **Quantum mechanics**

- Pion decay into photons or gravitons



$$D_\mu J_5^\mu = \frac{1}{8\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{192\pi^2} R_{\alpha\beta\mu\nu}\tilde{R}^{\alpha\beta\mu\nu}$$

[Adler],[Bell,Jackiw] 1969
[Kimura] 1969

Anomaly induced transport

- **Chiral anomalies**

$$\nabla_\mu J^\mu_a = \frac{d_{abc}}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a + \frac{b_a}{348\pi^2} R^{\alpha\beta}_{\mu\nu} \tilde{R}^{\mu\nu}_{\alpha\beta}$$

- **Anomaly coefficients:**

$$d_{abc} = \mathrm{str}\left(T_a T_b T_c\right)_r - \mathrm{str}\left(T_a T_b T_c\right)_\ell$$

$$b_a = \mathrm{str}\left(T_a\right)_r - \mathrm{str}\left(T_a\right)_\ell$$

Anomaly induced transport

- Chiral Magnetic and Vortical effects

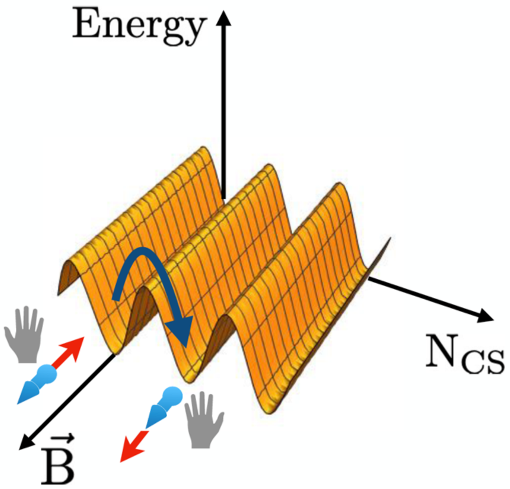
$$\vec{J}_A = d_{ABC}\frac{\mu_B}{4\pi^2}\vec{B}_C + \left(d_{ABC}\frac{\mu_B\mu_C}{4\pi^2} + b_A\frac{T^2}{12}\right)\vec{\Omega}$$

$$\vec{J}_\epsilon = \left(d_{ABC}\frac{\mu_B\mu_C}{8\pi^2} + b_A\frac{T^2}{24}\right)\vec{B}_A + \left(d_{ABC}\frac{\mu_A\mu_B\mu_C}{6\pi^2} + b_A\frac{\mu_A T^2}{6}\right)\vec{\Omega}$$

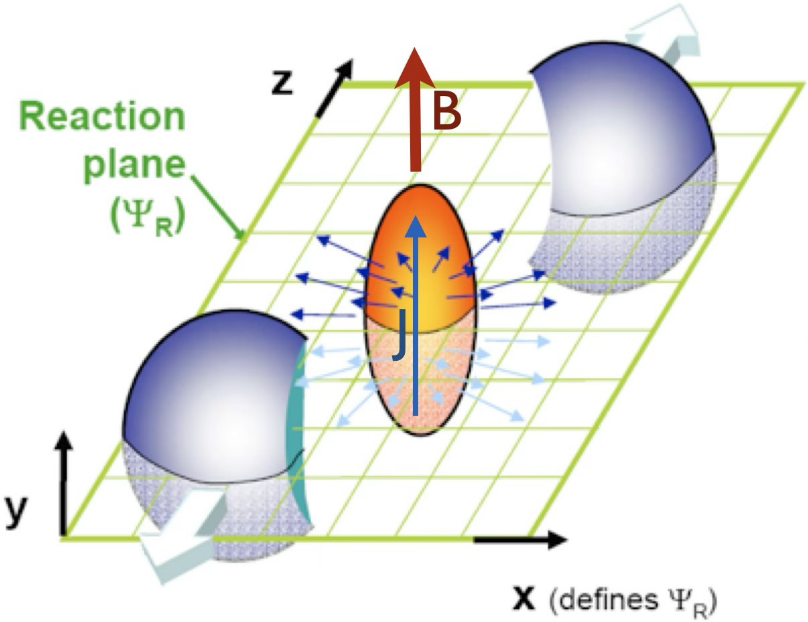
[Vilenkin] 1980
[Alekseev, Fröhlich, Chaianov] .
[Giovannini, Shaposhnikov] .
[Fukushima, Kharzeev, Warringa] .
[Newman]
[Son, Surowka]
[Newman, Oz]
[Erdmenger, Haack, Kaminski, Yarom]
[Banerjee, Bhattacharya, Bhattacharyya, Dutta, Loganayagam, Surowka]
[Megias, Landsteiner, Pena-Benitez]
[Megias, Melgar, Landsteiner, Pena-Benitez] 2012

Anomaly induced transport

- **Realizations:** **Heavy Ion Collisions ?**



$$\partial_\mu J_5^\mu = \frac{g^2 N_f}{16\pi^2}(G_{\mu\nu}\tilde{G}^{\mu\nu})$$



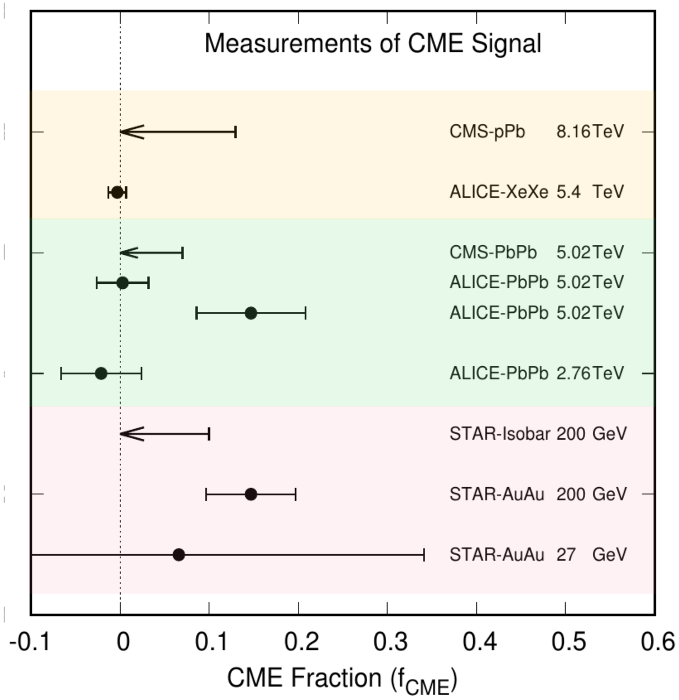
$$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$$

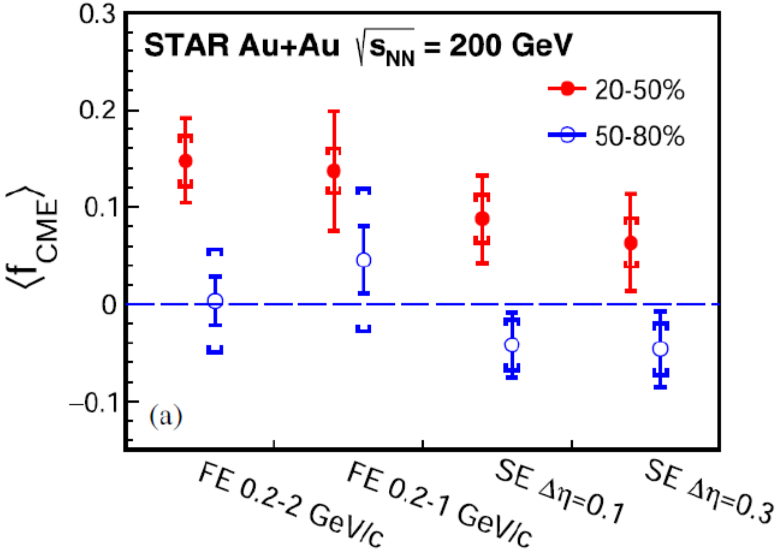
McLerran, Kharazeev, Warringa] 2008
[Fukushima, Kharazeev, Warringa] 2008

- Analogy to electroweak baryogenesis!
- Zhakharov conditions!

Anomaly induced transport

- **Realizations: Heavy Ion Collisions ?**



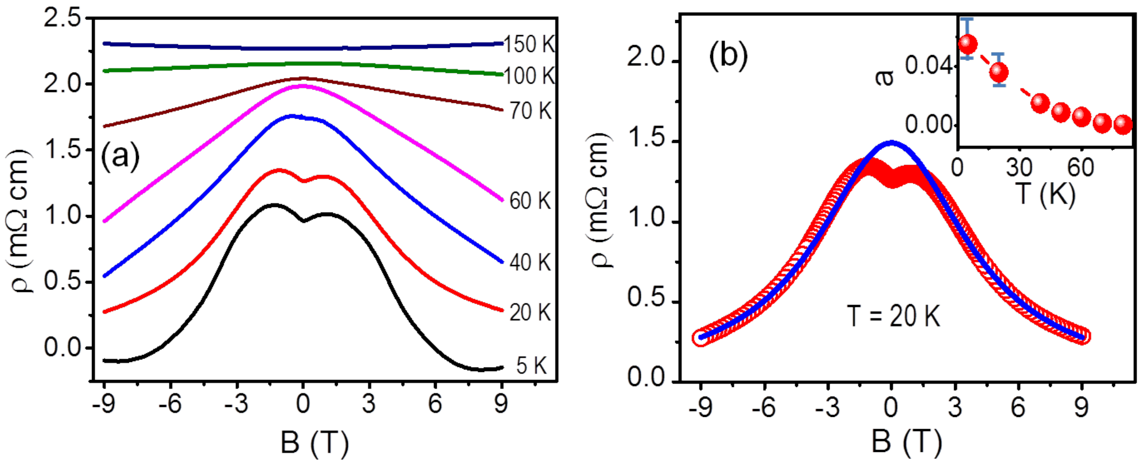


[Kharzeev, Liao, Tribedy] 2024

Anomaly induced transport

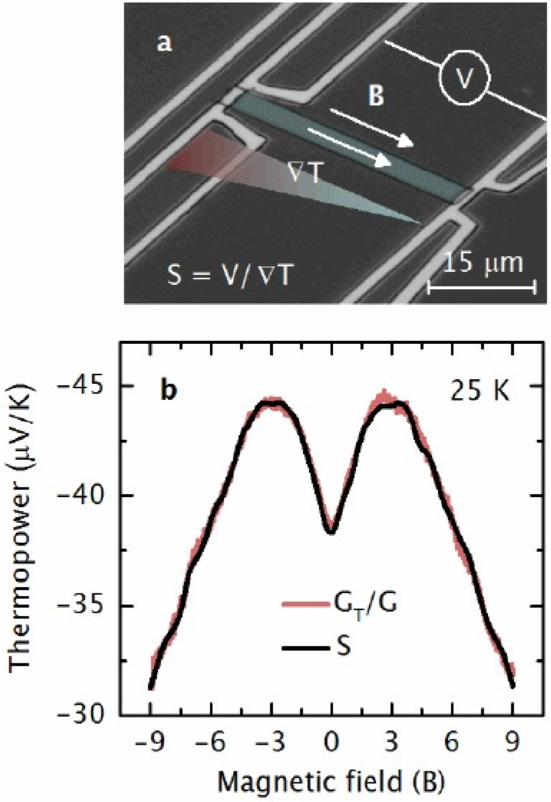
- **Realizations: Weyl semi-metals !**

CME in $ZrTe_5$



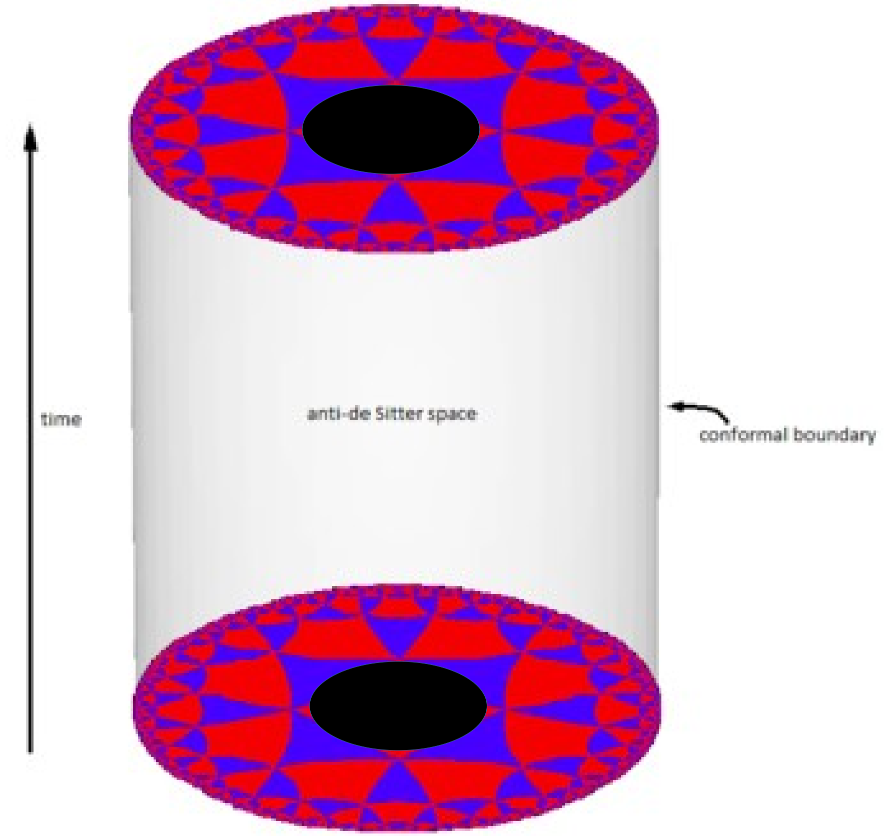
[Li, Kharzeev, Zhang, Huang, Pletikosic,Fedorov, Zhong,Schneeloch,Gu, Valla] 2015

CthME in NbP



[Gooth, Niemann, Meng, Grushin, Landsteiner, Gotsmann, Menges, Schmidt, Shekhar, Suß, Huhne, Rellinghaus, Felser, Yan, Nielsch] 2017

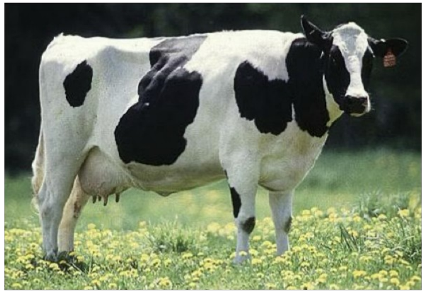
(The value of) Holography

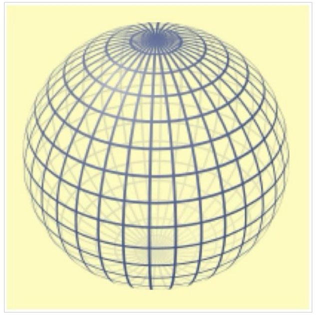


Gravity in asymptotically AdS = QFT

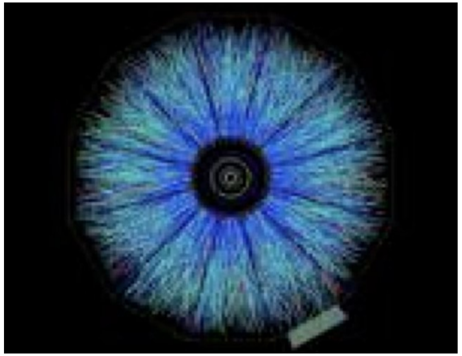
Holographic Dictionary	
Metric	Energy Momentum Tensor
Gauge field	Conserved current = symmetry
Scalar field	Scalar operator
Boundary value	**Coupling**
Black Hole	**Temperature**

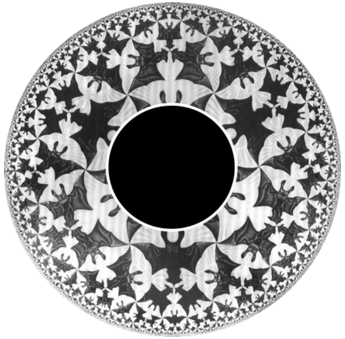
(The value of) Holography





"AdS is the hyperbolic cow of sQGP"



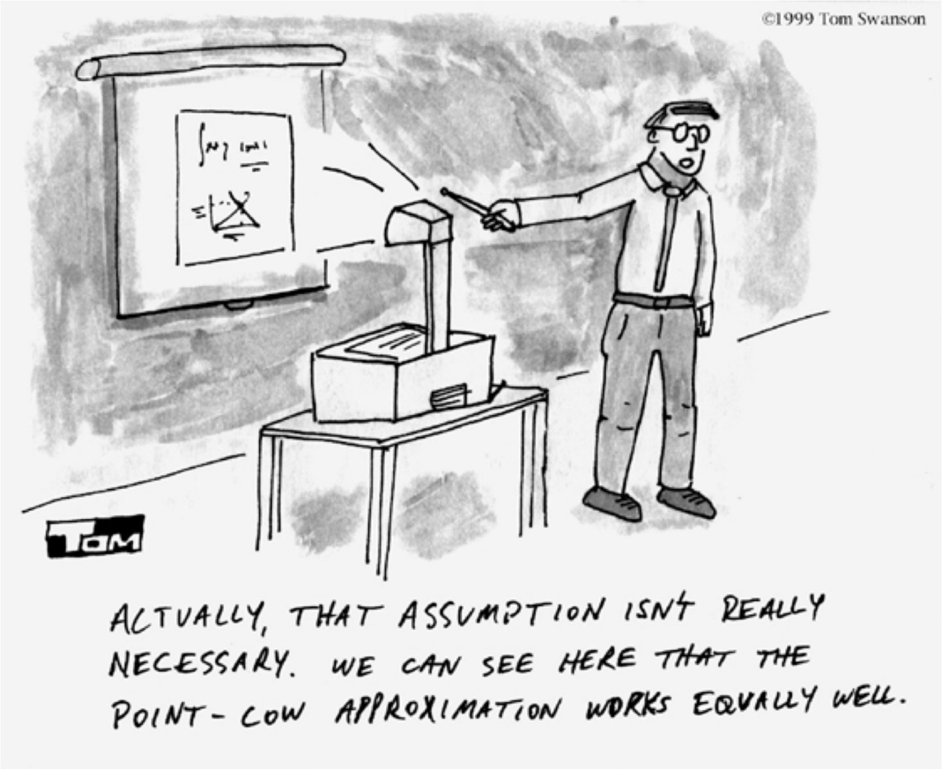


$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Son, Starinets, Policastro , Kovtun ...

(The value of) Holography

Anomalies are better than that:



- CVE thought to be impossible

$$\vec{J} = \frac{\mu\mu_5}{2\pi^2}\vec{\Omega}$$

- Naively gravitational anomaly 4-th order in derivatives

$$\vec{J}_5 = \frac{T^2}{12\pi}\vec{\Omega}$$

- Strict equilibrium = smooth Euclidean section vanishing CME

$$A_5^0 = \mu_5$$

$$\vec{J} = \frac{\mu_5 - A_5^0}{2\pi^2}\vec{B}$$

Anomalous transport and symmetry breaking

Holography is a discovery tool for (anomalous) transport phenomena!

Goal: what happens if an anomalous and a non-anomalous symmetry are broken by a ***common*** symmetry breaking parameter?

Model:

$$S_{\rm kin} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R + 2\Lambda - \frac{1}{4}F_V^2 - \frac{1}{4}F_A^2 - \frac{1}{4}F_W^2\right)$$

$$S_{\rm CS} = \frac{1}{16\pi G_5}\int d^5\sqrt{-g}\epsilon^{mnpqr}A_m\left(\frac{\kappa}{3}F_{A,np}F_{A,qr} + \kappa F_{V,np}F_{V,qr} + R^{ab}_{np}R_{abqr}\right)$$

$$S_\partial = \frac{1}{8\pi G_5}\int\sqrt{h}\,(K - 4n_m\epsilon^{mnpqr}A_nK_{pl}D_qK^l_r)$$

Matching to axial anomalies of Dirac fermions:

$$\frac{\kappa}{16\pi G_5} = -\frac{N_f}{16\pi^2} \qquad\qquad \frac{\lambda}{16\pi G_5} = -\frac{N_f}{384\pi^2}$$

Anomalous transport and symmetry breaking

Symmetries:

- J_V ...Vector - consistent current is anomaly free
  covariant current has anomaly
- J_A … Axial – axial anomaly
- J_W … “Anomaly free” current
- T …. Energy-momentum tensor (similar to J_V)

Symmetry breaking: $$S_\phi = \frac{1}{16\pi G_5}\int \sqrt{-g}\left(-|D\phi|^2 - m^2\phi^2\right) \qquad D_n\phi = \left(\partial_n - i(A_n - W_n)\right)\phi$$

$$m^2 = -3 \qquad \phi(r) \approx \frac{M}{r} + \cdots + \frac{\mathcal{O}}{r^3} + \ldots$$

QFT intuition: $$S_M = \int d^4x M\bar{\Psi}\Psi$$

Anomalous transport and symmetry breaking

Symmetries:

- J_V ...Vector - consistent current is anomaly free
  covariant current has anomaly
- J_A … Axial – axial anomaly
- J_W … Anomaly free current
- T …. energy momentum tensor, as J_V

Symmetry breaking: $S_\phi = \frac{1}{16\pi G_5}\int \sqrt{-g}\left(-|D\phi|^2 - m^2\phi^2\right)$ $\quad D_n\phi = \left(\partial_n - i(A_n - W_n)\right)\phi$

$m^2 = -3$ $\quad \phi(r) \approx \frac{M}{r} + \cdots + \frac{\mathcal{O}}{r^3} + \dots$

QFT interpretation: $S_M = \int d^4x M \bar{\Psi}\Psi$

Anomalous transport and symmetry breaking

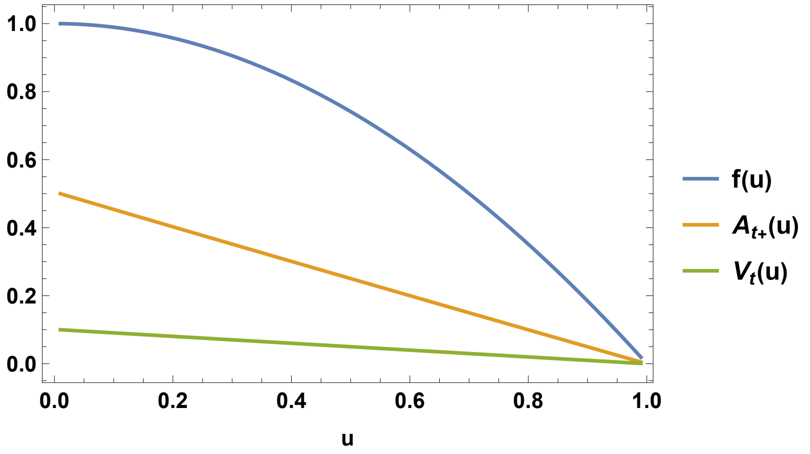
Ansatz: $$ds^2 = \frac{1}{u^2}\left(-f(u)dt^2 + e^{\chi(u)}\frac{du^2}{f(u)} + d\vec{x}^2\right) \qquad \phi = \phi(u)\ , \qquad \lim_{u\to 0}\frac{\phi(u)}{\sqrt{u}} = M$$

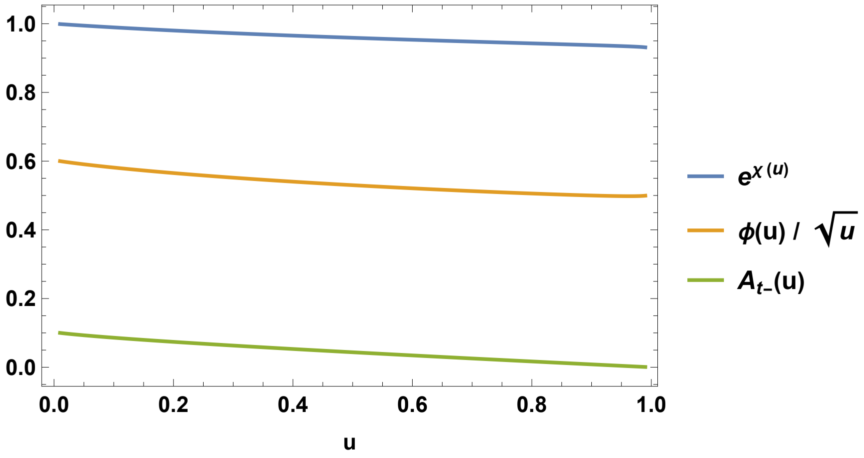
$$S = S_t(u)dt\ , \qquad \lim_{u\to 0} S_t(u) = \mu_S\ , \qquad S \in \{V, A, W\}$$

M=0: $$f(u) = 1 - (1 + \sum_S Q_S^2)u^2 + \sum_S Q_S^2 u^3 \qquad S_t(u) = \mu_S(1-u)$$

$$\pi T = 1 - \frac{\sum_S \mu_S^2}{6} \qquad Q_S = \frac{\mu_S}{\sqrt{3}}$$

M≠0: numerical solutions $M = 0.7T\ , \quad \mu_V = 0.12T\ , \quad \mu_A = 0.35T\ , \quad \mu_W = 0.23T$





Anomalous transport and symmetry breaking

Response and Kubo formulas:

$$\begin{aligned}\vec{J}_v &= \sigma^B_{vv}\,\vec{B}_v + \sigma^B_{va}\,\vec{B}_a + \sigma^B_{vw}\,\vec{B}_w + \sigma^V_v\,\vec{\Omega}\,,\\ \vec{J}_a &= \sigma^B_{av}\,\vec{B}_v + \sigma^B_{aa}\,\vec{B}_a + \sigma^B_{aw}\,\vec{B}_w + \sigma^V_a\,\vec{\Omega}\,,\\ \vec{J}_w &= \sigma^B_{wv}\,\vec{B}_v + \sigma^B_{wa}\,\vec{B}_a + \sigma^B_{ww}\,\vec{B}_w + \sigma^V_w\,\vec{\Omega}\,,\\ \vec{J}_\varepsilon &= \sigma^B_{\varepsilon v}\,\vec{B}_v + \sigma^B_{\varepsilon a}\,\vec{B}_a + \sigma^B_{\varepsilon w}\,\vec{B}_w + \sigma^V_\varepsilon\,\vec{\Omega}\,.\end{aligned}$$

$$\sigma^B_{sb} = \lim_{p_k\to 0}\frac{i}{2p_k}\sum_{i,j}\epsilon_{ijk}\langle J^i_s J^j_b\rangle|_{\omega=0} \qquad \sigma^V_s = \lim_{p_k\to 0}\frac{i}{2p_k}\sum_{i,j}\epsilon_{ijk}\langle J^i_s J^j_\varepsilon\rangle|_{\omega=0}\,, \qquad (s\in\{v,a,w,\varepsilon\}\,,\; b\in\{v,a,w\})$$

M=0:

$$\sigma^B_{sv}(0) = \frac{1}{2\pi^2}\begin{cases}\mu_a\,, & (s=v)\\ \mu_v\,, & (s=a)\\ \mu_v\mu_a\,, & (s=\varepsilon)\end{cases}\,,$$

$$\sigma^B_{sa}(0) = \frac{1}{2\pi^2}\begin{cases}\mu_v\,, & (s=v)\\ \mu_a\,, & (s=a)\\ \frac{\mu_v^2+\mu_a^2}{2}+\frac{\pi^2}{6}T^2\,, & (s=\varepsilon)\end{cases}\,,$$

$$\sigma^V_s(0) = \frac{1}{2\pi^2}\begin{cases}\mu_v\mu_a\,, & (s=v)\\ \frac{\mu_v^2+\mu_a^2}{2}+\frac{\pi^2}{6}T^2\,, & (s=a)\\ \frac{\mu_a}{3}\left(3\mu_v^2+\mu_a^2+\pi^2T^2\right)\,, & (s=\varepsilon)\end{cases}\,.$$

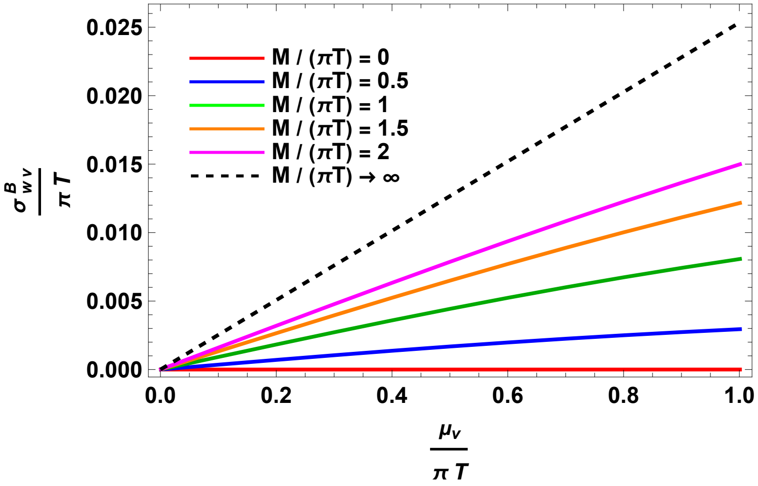
All conductivities related to non-anomalous symmetry vanish!

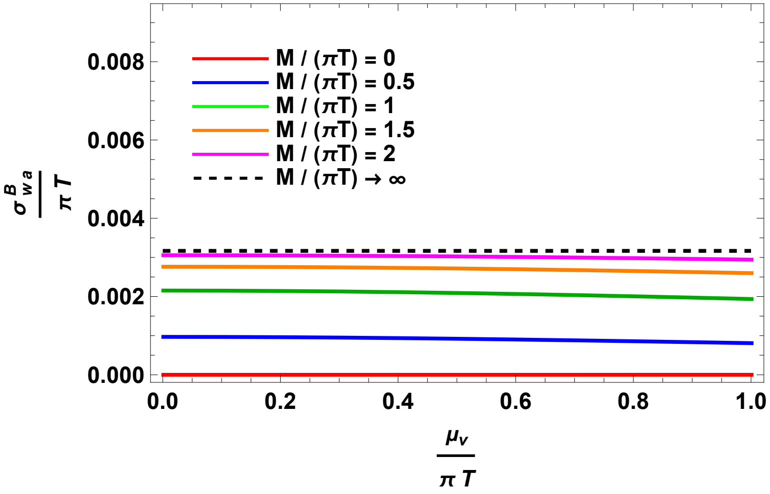
$$\sigma_{wx} = 0$$

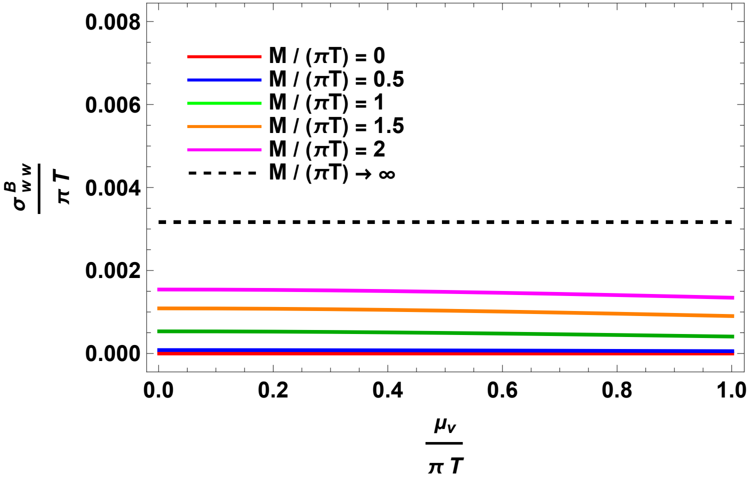
Anomalous transport and symmetry breaking

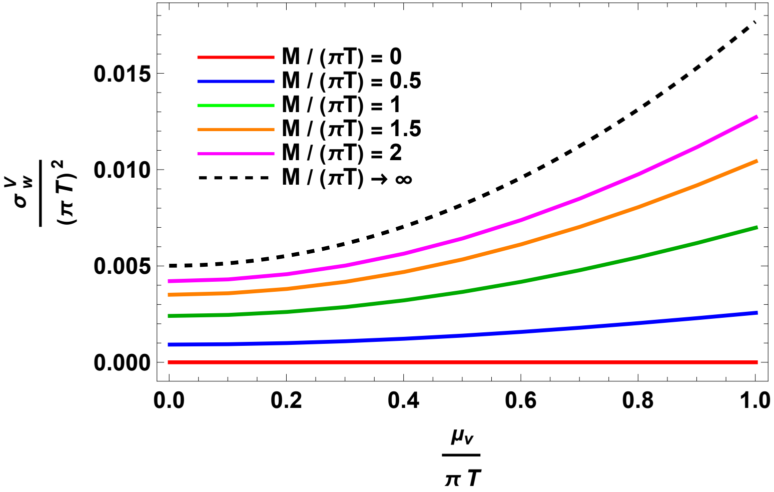
M≠0:

$$\vec{J}_w = \sigma^B_{wv}\,\vec{B}_v + \sigma^B_{wa}\,\vec{B}_a + \sigma^B_{ww}\,\vec{B}_w + \sigma^V_w\,\vec{\Omega}$$









Anomalous transport and symmetry breaking

Chiral magnetic conductivities vs symmetry breaking

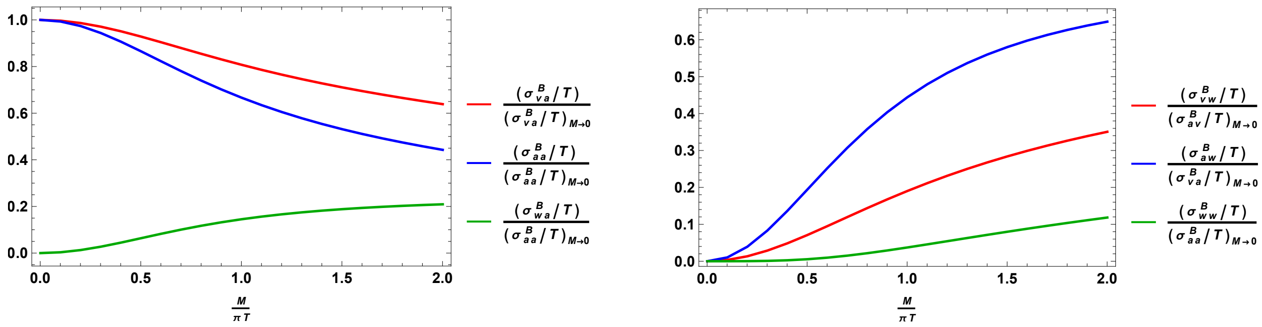


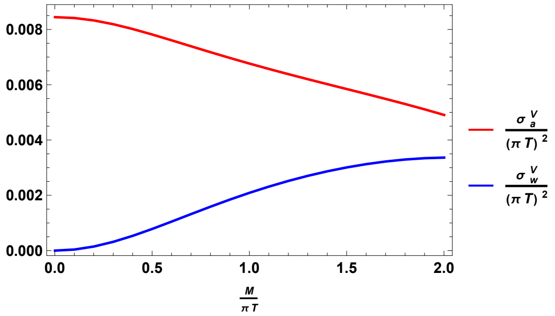
Figure 7. Plots of the chiral magnetic conductivities σ^B_{sa} (left panel) and σ^B_{sw} (right panel), where

Vortical conductivities: temperature part

$$\mu_v = \mu_a = \mu_w = 0$$

$$\vec{J}_a = \sigma^V_a \vec{\Omega}$$

$$\vec{J}_w = \sigma^V_w \vec{\Omega}$$



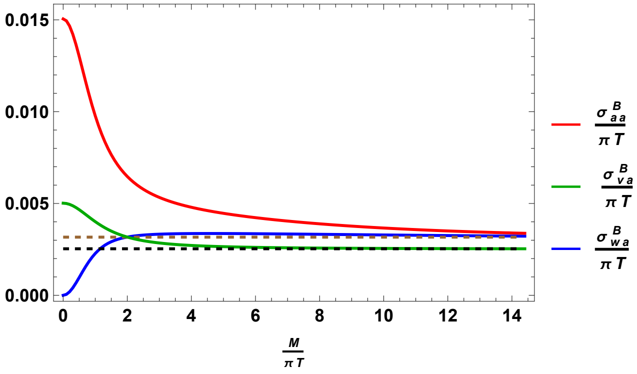
Anomalous transport and symmetry breaking

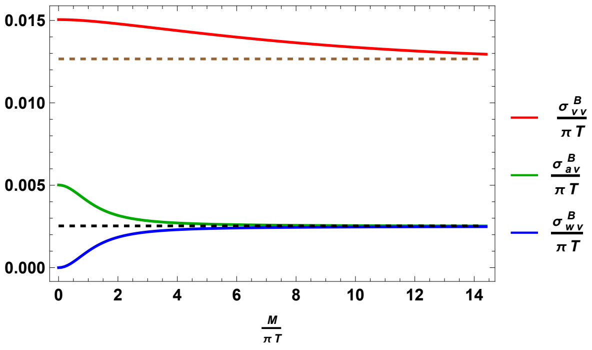
The covariant derivative $D_\mu \phi = \partial_\mu \phi - i(A_\mu - W_\mu)\phi = \partial_\mu \phi - i A^-_\mu \phi$ $\quad A^\pm_\mu = A_\mu \pm W_\mu$

This implies for the conductivities $\sigma_{AS} = \frac{1}{2}(\sigma_{+S} + \sigma_{-S})$ $\quad \sigma_{WS} = \frac{1}{2}(\sigma_{+S} - \sigma_{-S})$

The M→∞ limit the W-field symmetry is maximally broken: $\sigma_{-S} = 0$

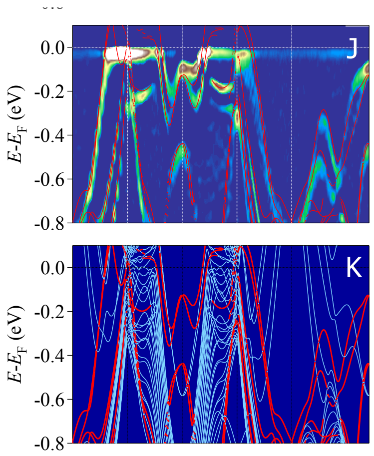
$$\sigma_{WV} = \frac{\mu_V}{4\pi^2}$$
$$\sigma_{WA} = \frac{\mu_A + \mu_W}{16\pi^2}$$
$$\sigma_{WW} = \frac{\mu_A + \mu_W}{16\pi^2}$$
$$\sigma^V_W = \frac{\mu_V^2 + (\mu_A + \mu_W)^2/4}{8\pi^2} + \frac{\pi^2 T^2}{24}$$





Discussion and Outlook

- Anomalies give rise to Chiral “transport” effects
- Conductivities are determined by anomaly coefficients
- Surprisingly non-anomalous currents can participate if symmetries are broken
- Realization in terms of holographic model explicit symmetry breaking
- What happens for spontaneous symmetry breaking? [work in progress]
- Weak coupling QFT calculation
- Real world applications?
  - TaAs: Weyl semi-metal with 24 Weyl nodes,
  - Possble complicated symmetry breaking patterns



谢谢