

A study of neutron star property based on parity doublet models

Masayasu Harada (Nagoya University)

@ International Conference on Symmetry Breaking Phenomena in Quantum Field Theory
(May 16, 2026)

Based on

- Y. Motohiro, Y. Kim, M. Harada, Phys. Rev. C 92, 025201 (2015);
Erratum: Phys. Rev. C 95, 059903 (2017).
- T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 103, 045205 (2021).
- T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 104, 065201 (2021).
- T. Minamikawa, B. Gao, T. Kojo and M. Harada, Symmetry 15, 745 (2023).
- B. Gao, Y. Yan and M. Harada, Phys. Rev. C 109, 065807 (2024).
- Y.-K. Kong, T. Minamikawa and M. Harada, Phys. Rev. C 108, 055206 (2023).
- Y.-K. Kong and M. Harada, Nuclear Physics Review, 2024, 41(3): 787-793.
- B. Gao, W.L. Yuan, M. Harada and Y.L. Ma, Phys. Rev. C 110, 045802 (2024).
- M. Kawaguchi, M. Harada and Y.L. Ma, Phys. Lett. B 876, 140400 (2026).
- X. Liu, B. Gao and M. Harada, in preparation.
- M. Kanazawa and M. Harada, in preparation

Memory of Prof. Vladimir Miransky

Masayasu Harada
(Nagoya Univ.)

Memory of Volodya

- 1988: He stayed in Nagoya for 1 year visiting Prof. Yamawaki.
I was a first-year master student.
- 1990: SCGT90 Workshop in Nagoya
- 1993: Volodya and I stayed in Kyoto Univ.
- 1996: SCGT96 Workshop in Nagoya
- ⋮
- 2004: DSB04 Workshop in Nagoya.
- 2009: SCGT09 Workshop in Nagoya.
- 2012: SCGT12 Workshop in Nagoya
- 2014: SCGT14mini Workshop in Nagoya
- 2015: SCGT15 Workshop in Nagoya

SCGT90 Workshop (1990, Nagoya)



Harada

Volodya

SCGT96 Workshop in Nagoya

Volodya



DSB04 Workshop in Nagoya

Volodya



SCGT09 Workshop in Nagoya

Volodya



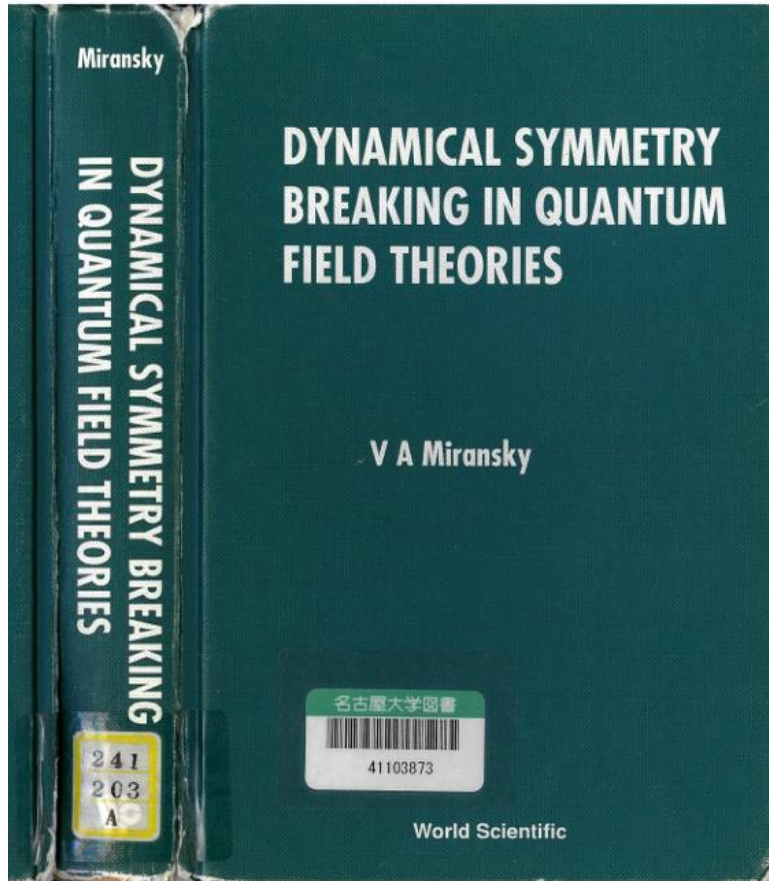
- I do not remember why I was not in this photo, although I was a co-chair of this workshop.

SCGT12 Workshop in Nagoya

Volodya



Volodya's Book: Dynamical Symmetry Breaking in Quantum Field Theories (written in Kyoto, 1993)



Preface

I also would like to express my thanks to V. Elias, M. Harada, A. Kovner, T. Kugo, M. G. Mitchard and B. Rosenstein for their remarks and suggestions concerning the book.

Figures

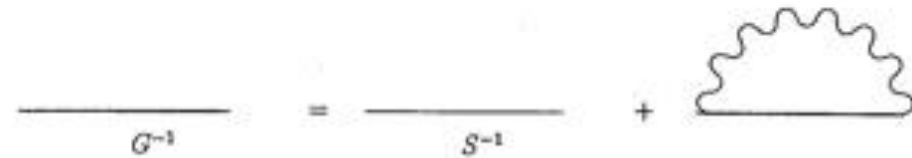


Fig. 8.6. Graphical representation of the Schwinger-Dyson equation in QED. A thin solid

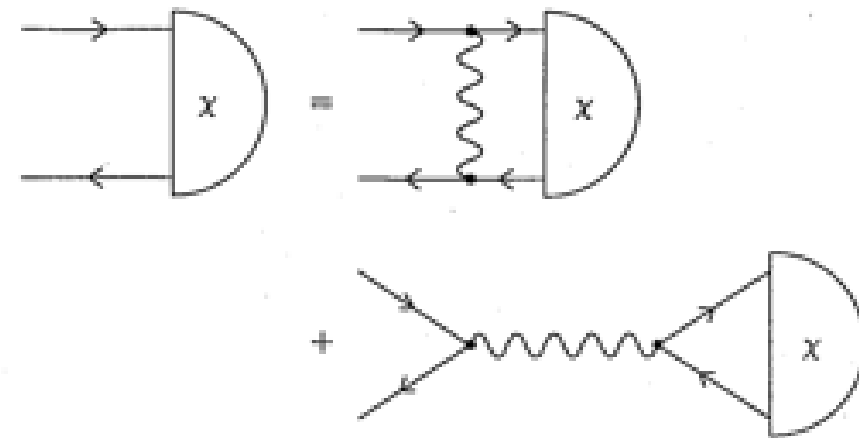


Fig. 10.1. Graphical representation of the homogeneous Bethe-Salpeter equation with the kernel (8.113) in QED.

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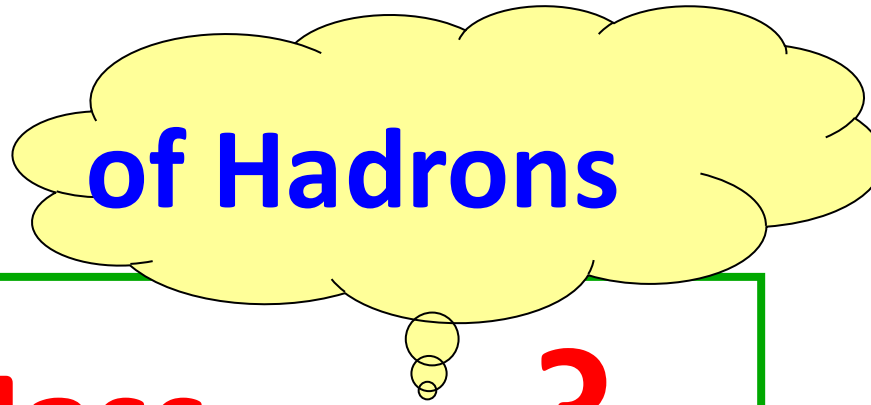
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1. Introduction

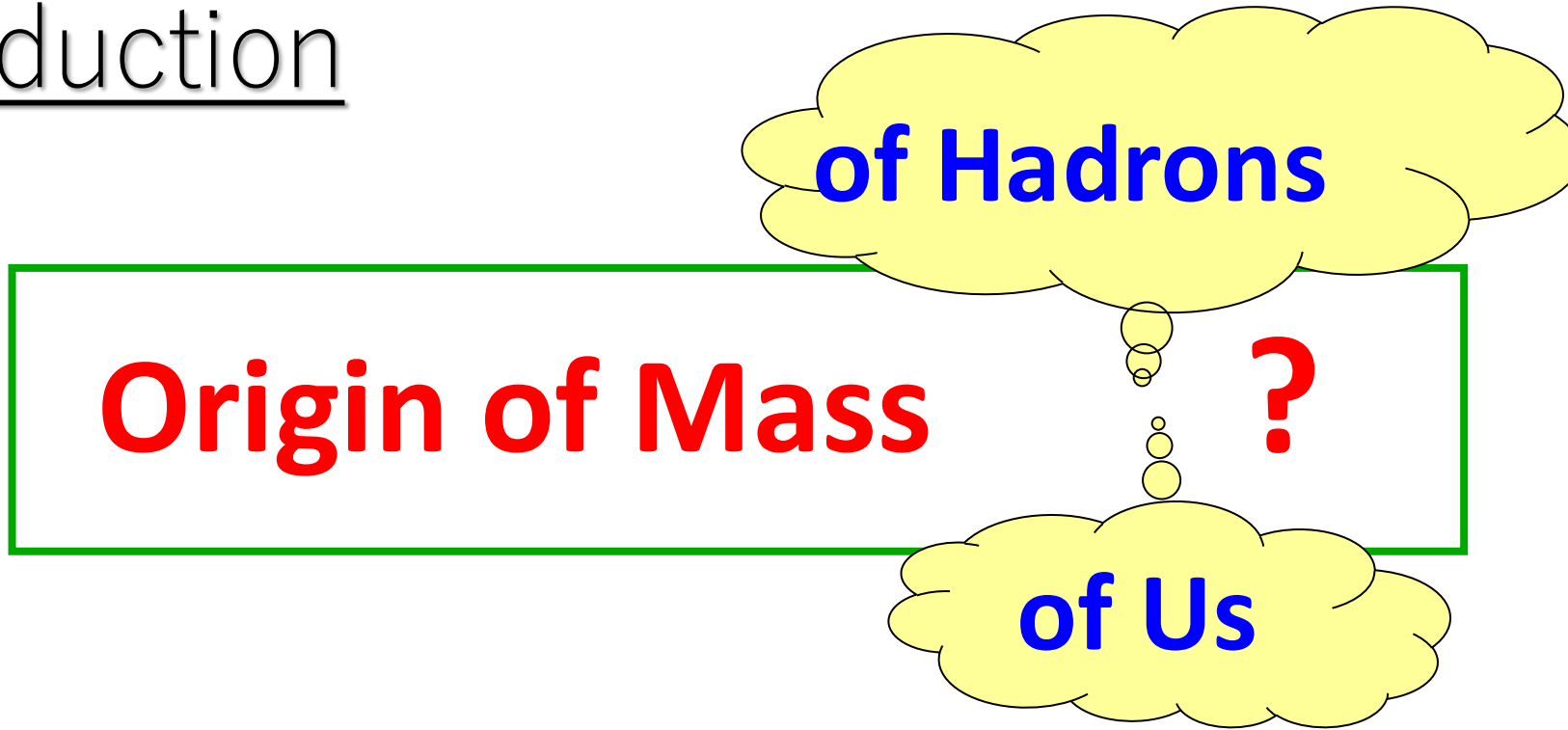
Origin of Mass ?

1. Introduction

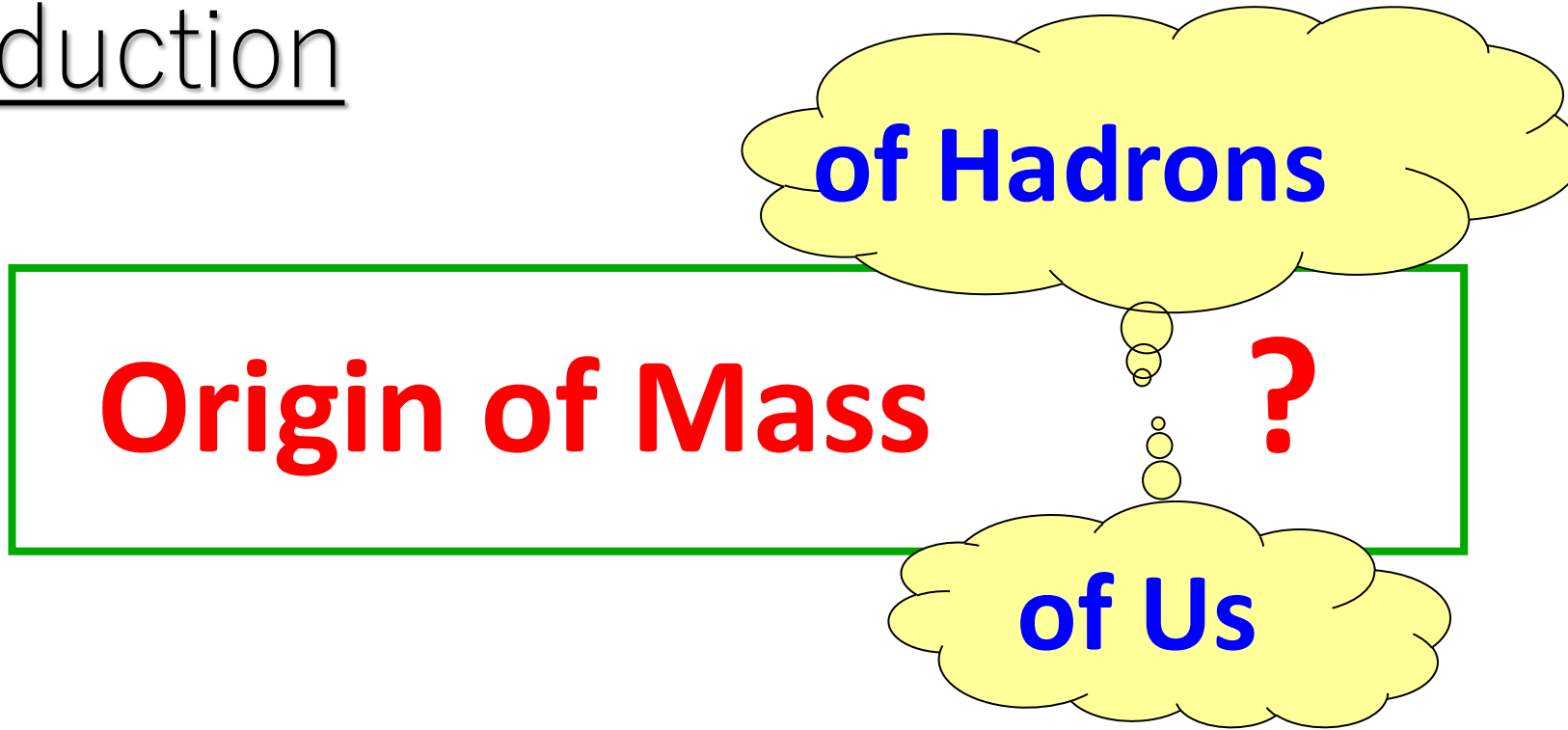


Origin of Mass ?

1. Introduction



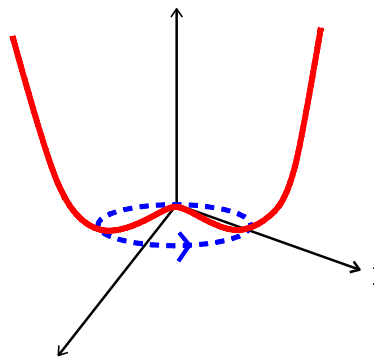
1. Introduction



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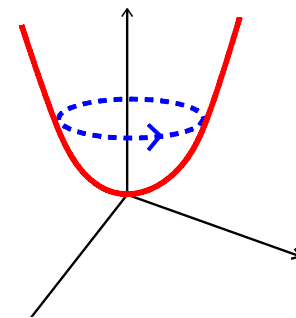
One of the Interesting problems of QCD

Spontaneous chiral symmetry breaking



chiral symmetry
broken phase at
vacuum

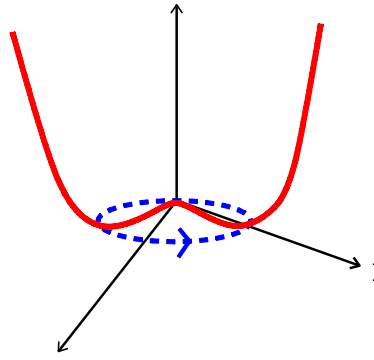
$$\langle \bar{q}q \rangle \neq 0 \text{ (chiral condensate)}$$



chiral symmetric
phase at high T
and/or density

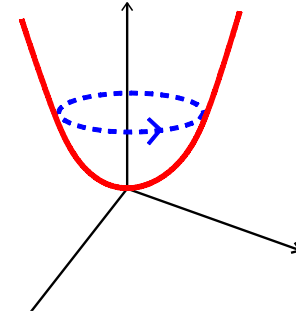
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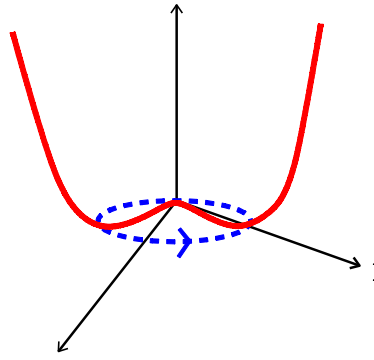


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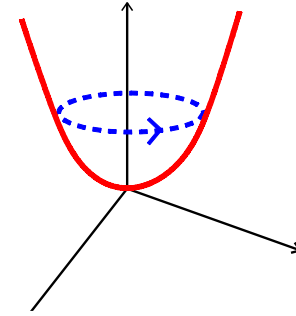
- The spontaneous chiral symmetry breaking is expected to generate a part of hadron masses.
- It causes mass difference between chiral partners.

Spontaneous chiral symmetry breaking



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chiral symmetric
phase at high T
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$$\langle \bar{q}q \rangle = 0$$

- The spontaneous chiral symmetry breaking is expected to generate a part of hadron masses.
- It causes mass difference between chiral partners.

- How much mass of nucleon is from the spontaneous chiral symmetry breaking ?
- What is the chiral partner of the nucleon ?

Parity Doublet models for nucleons

- How much mass of nucleon is from the spontaneous chiral symmetry breaking ?

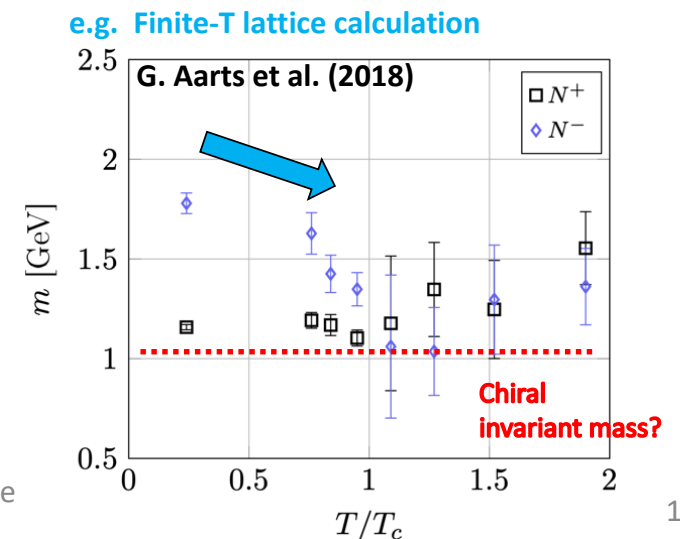
- A Parity doublet model for light baryons
 - In [C.DeTar, T.Kunihiro, PRD39, 2805 (1989)], N(1535) is regarded as the chiral partner to the N(939) having the chiral invariant mass.

$$m_N = m_0 + m_{\langle \bar{q}q \rangle} \leftarrow \text{spontaneous chiral symmetry breaking}$$

m_0
chiral invariant mass

- A Lattice QCD analysis at non-zero T supports parity doublet structure.

⇒ Constraint to m_0 from phenomenological analysis ?

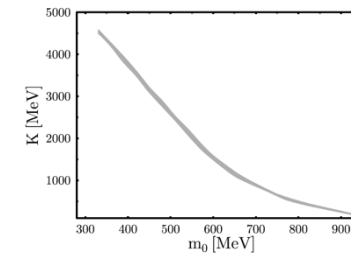
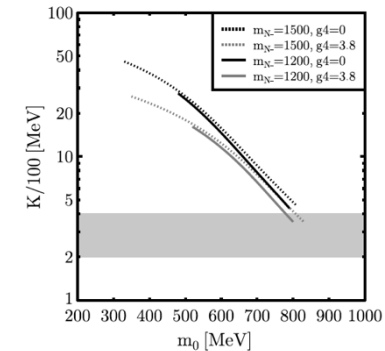


Constraint to m_0 at vacuum

- analysis based on a **simple linear sigma model**
 - C.DeTar, T.Kunihiro, PRD39, 2805 (1989)
 - D.Jido, M.Oka, A.Hosaka, PTP106, 873 (2001)
 - $N(1535)$ is regarded as the chiral partner to $N(939)$.
 - use $\Gamma_{N^* \rightarrow \pi N}$ as an input to determine $m_0 = 270 \text{ MeV}$
- Inclusion of **derivative interactions** in an extended model
 - T. Yamazaki and M. Harada, PRD99, 034012 (2019)
 - $g_A = 1.26$ and $\Gamma_{N^* \rightarrow \pi N}$ can be explained for $300 \lesssim m_0 \lesssim 1000 \text{ MeV}$.

Constraint to m_0 in dense matter

- a simple linear sigma model with $\sigma^4 \ln \sigma^2$ term, ω & ρ mesons
T.Hatsuda & M.Prakash, PLB224, 11 (1989)
 - use $m_0 = 270 \text{ MeV}$ (De Tar-Kunihiro, Jido et al.) as an input
 - obtain $K_0(\text{incompressibility}) \approx 143 \text{ MeV}$
- add ω^4 term, no $\sigma^4 \ln \sigma^2$ term
D.Zschiesche et al., PRC75, 055202 (2007)
 - use $K_0 \lesssim 400 \text{ MeV}$ to obtain $m_0 \gtrsim 800 \text{ MeV}$
- add $\omega^4, \rho^4, \omega^2 \rho^2$ terms, no $\sigma^4 \ln \sigma^2$ term
V.Dexheimer et al., PRC77, 025803 (2008)
 - use $K_0 \lesssim 400 \text{ MeV}$ to obtain $m_0 \gtrsim 800 \text{ MeV}$
- add σ^6 term, no $\omega^4, \rho^4, \omega^2 \rho^2, \sigma^4 \ln \sigma^2$ terms
Y.Motohiro, Y.Kim, M.Harada, Phys. Rev. C 92, 025201 (2015)
 - $K_0 \approx 240 \text{ MeV}$ for $500 \lesssim m_0 \lesssim 900 \text{ MeV}$



Constraint to m_0 from Neutron Star observation

- PDM with σ^6 term

T. Yamazaki and M. Harada, Phys. Rev. C 100, 025205 (2019)

- use binary dimensionless tidal deformability $\tilde{\Lambda} < 800$ from GW170817
- obtain $m_0 > 500 \text{ MeV}$

- PDM (low density) - NJL (high density) **crossover model**

T. Minamikawa, T. Kojo and M.H., Phys. Rev. C 103, 045205 (2021).

- use $M_{max} \gtrsim 2M_\odot$ & GW170817 & NICER
- obtain $600 \lesssim m_0 \lesssim 900 \text{ MeV}$

- PDM - NJL **crossover model with iso-triplet a_0 meson**

Y.-K. Kong, T. Minamikawa and M. Harada, Phys. Rev. C 108, 055206 (2023).

- use $M_{max} \gtrsim 2M_\odot$ & GW170817 & NICER
- obtain $560 \lesssim m_0 \lesssim 840 \text{ MeV}$

- PDM - NJL **1st-order model**

B. Gao, W. L. Yuan, M.H. and Y.L. Ma, Phys. Rev. C 110, 045802 (2024).

- Constraint to m_0 is not affected very much.

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Outline

1. Introduction
2. Parity Doublet Model
3. PDM-NJL crossover model & Constraints to m_0 from neutron star observation
4. Constraints from the supernova remnant HESS J1731-347
5. Study of 1st order phase transition
6. Effect of iso-triplet scalar $a_0(980)$ meson
7. Recent Analyses
8. Summary

2. Parity Doublet Model (PDM)

C.DeTar, T.Kunihiro, PRD39, 2805 (1989)
D.Jido, M.Oka, A.Hosaka, PTP106, 873 (2001)

□ **N(1535) = chiral partner** to N(939)

□ N(939) Normal assignment $\psi_1 = \psi_{1l} + \psi_{1r}$

➤ The transformation property under the chiral group is assigned with the chirality.

$$\gamma_5 \psi_{1l} = -\psi_{1l}; \quad \psi_{1l} \rightarrow g_L \psi_{1l}; \quad g_L \in \text{SU}(2)_L$$

$$\gamma_5 \psi_{1r} = +\psi_{1r}; \quad \psi_{1r} \rightarrow g_R \psi_{1r}; \quad g_R \in \text{SU}(2)_R$$

□ N(1535) Mirror assignment $\psi_2 = \psi_{2l} + \psi_{2r}$

➤ The chiral transformation is assigned oppositely to the chirality

$$\gamma_5 \psi_{2l} = -\psi_{2l}; \quad \psi_{2l} \rightarrow g_R \psi_{2l}; \quad g_R \in \text{SU}(2)_R$$

$$\gamma_5 \psi_{2r} = +\psi_{2r}; \quad \psi_{2r} \rightarrow g_L \psi_{2r}; \quad g_L \in \text{SU}(2)_L$$

Linear sigma model (2 flavor)

□ A scalar-pseudoscalar field : $M = \sigma + i\vec{\pi} \cdot \vec{\tau}$

➤ Transforms as $M \rightarrow g_L M g_R^\dagger$; VEV $\langle M \rangle = \sigma_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

□ A linear sigma model with 2 baryons

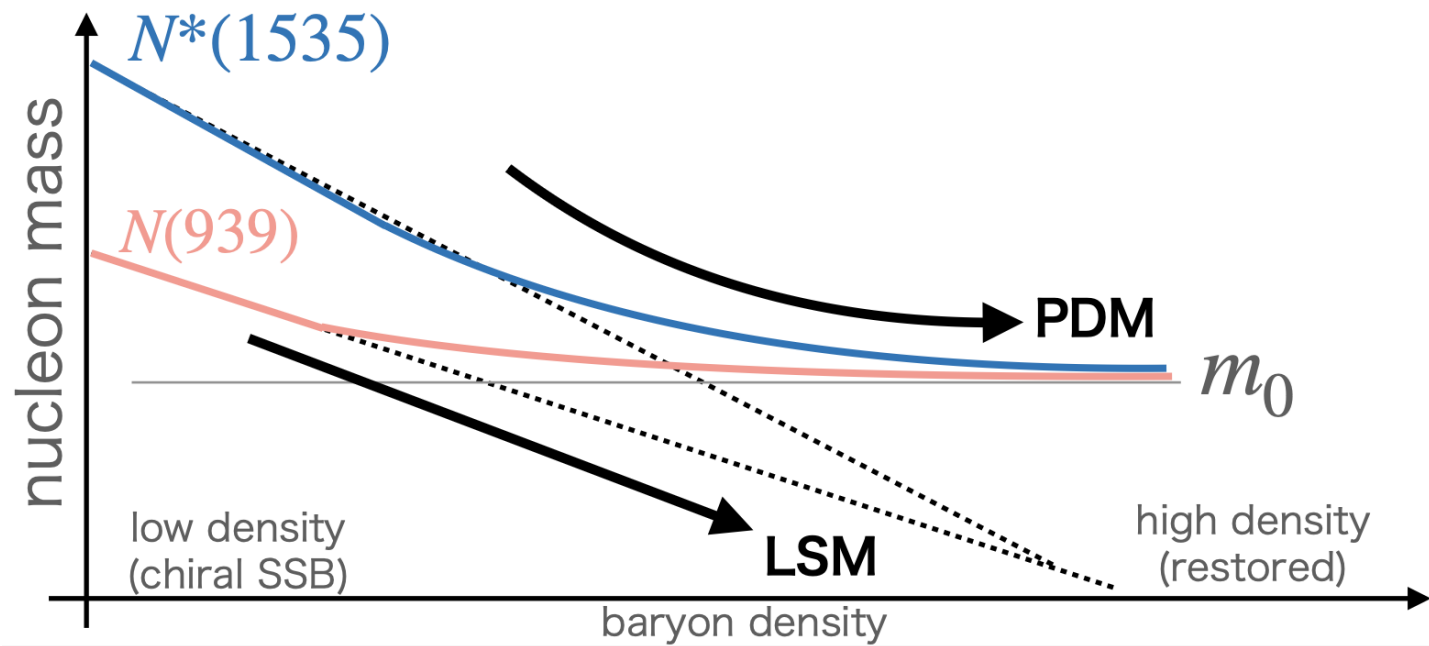
$$\begin{aligned} \mathcal{L}_N = & \bar{\psi}_{1r} i\gamma^\mu \partial_\mu \psi_{1r} + \bar{\psi}_{1l} i\gamma^\mu \partial_\mu \psi_{1l} + \bar{\psi}_{2r} i\gamma^\mu \partial_\mu \psi_{2r} + \bar{\psi}_{2l} i\gamma^\mu \partial_\mu \psi_{2l} \\ & - m_0 [\bar{\psi}_{1l} \psi_{2r} - \bar{\psi}_{1r} \psi_{2l} - \bar{\psi}_{2l} \psi_{1r} + \bar{\psi}_{2r} \psi_{1l}] \\ & - g_1 [\bar{\psi}_{1r} M^\dagger \psi_{1l} + \bar{\psi}_{1l} M \psi_{1r}] - g_2 [\bar{\psi}_{2r} M \psi_{2l} + \bar{\psi}_{2l} M^\dagger \psi_{2r}] \end{aligned}$$

- **chiral invariant mass m_0** : baryons can have masses even without spontaneous chiral symmetry breaking.
- Two masses are separated with each other by the effect of spontaneous chiral symmetry breaking.

Schematic density dependence of masses in PDM

● $m_{\pm} = \frac{1}{2} \left[\sqrt{(g_1 + g_2)^2 \sigma^2 + 4m_0^2} \mp (g_1 - g_2)\sigma \right]$

➤ $m_+ = m(N(939)), m_- = m(N(1535))$



- Both masses decrease with density and approach m_0 associated with chiral symmetry restoration.
- In the ordinary linear sigma model, the nucleon mass approaches 0.

Thermodynamic Potential

Y.Motohiro, Y.Kim, M.Harada, Phys. Rev. C 92, 025201 (2015)
Erratum: Phys. Rev. C 95, 059903 (2017).

- ρ and ω mesons are added based on the hidden local symmetry.
- σ^6 term is added to the scalar potential
- Mean field approximation

$$\Omega_h(\mu_B, \mu_Q) = \Omega_B(\mu_B, \mu_Q; \varphi) + V(\varphi) - V(\varphi_{\text{vac}}). ; \varphi = \{\sigma, \omega, \rho\}$$

$$\varphi = \{\sigma, \omega, \rho\}; \varphi_{\text{vac}} = \{\sigma = f_\pi, \omega = 0, \rho = 0\}$$

$$V(\varphi) = -\frac{1}{2}\bar{\mu}^2\sigma^2 + \frac{1}{4}\lambda_4\sigma^4 - \frac{1}{6}\lambda_6\sigma^6 - m_\pi^2 f_\pi \sigma - \frac{1}{2}m_\omega^2\omega^2 - \frac{1}{2}m_\rho^2\rho^2,$$

$$\Omega_B(\mu_B, \mu_Q; \varphi) = -2 \sum_{i=\pm} \sum_{\alpha=p,n} \int_{\mathbf{p}} (\mu_\alpha^* - E_{\mathbf{p}}^i) \Theta(\mu_\alpha^* - E_{\mathbf{p}}^i)$$

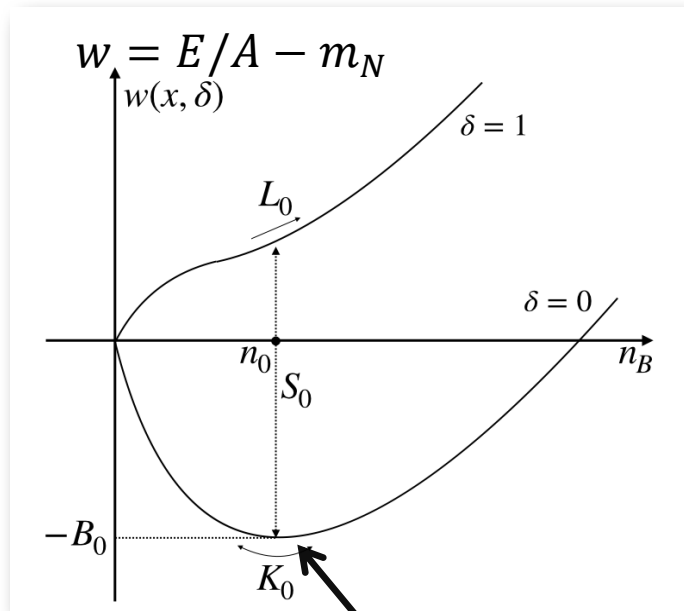
$$\mu_p^* = \mu_B - g_{\omega NN}\omega - \frac{1}{2}g_{\rho NN}\rho + \mu_Q, \quad E_{\mathbf{p}}^i = \sqrt{\mathbf{p}^2 + m_i^2}$$

$$\mu_n^* = \mu_B - g_{\omega NN}\omega + \frac{1}{2}g_{\rho NN}\rho.$$

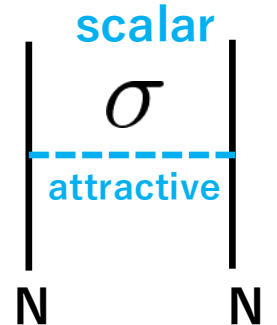
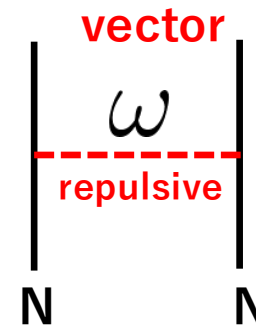
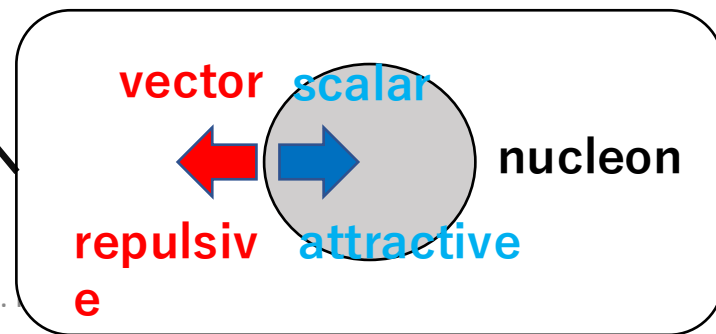
Nuclear Matter at normal nuclear density

Y. Motohiro, Y. Kim, M. Harada, Phys. Rev. C 92, 025201 (2015); Erratum: Phys. Rev. C 95, 059903 (2017).

We determine the 4 parameters from the saturation properties as inputs for a given value of the chiral invariant mass m_0 .



- Nuclear saturation density
 - $\rho(\mu_B^* = 923\text{MeV}) = n_0 = 0.16\text{fm}^{-3}$
- Binding energy at normal nuclear density
 - $w = \left[\frac{E}{A} - m(939) \right]_{n_0} = -16\text{MeV}$
- Incompressibility
 - $K_0 = 9\rho_0^2 \left. \frac{\partial^2(E/A)}{\partial \rho^2} \right|_{n_0} = 240\text{MeV}$
- Symmetry energy
 - $S_0 = 31\text{ MeV}$

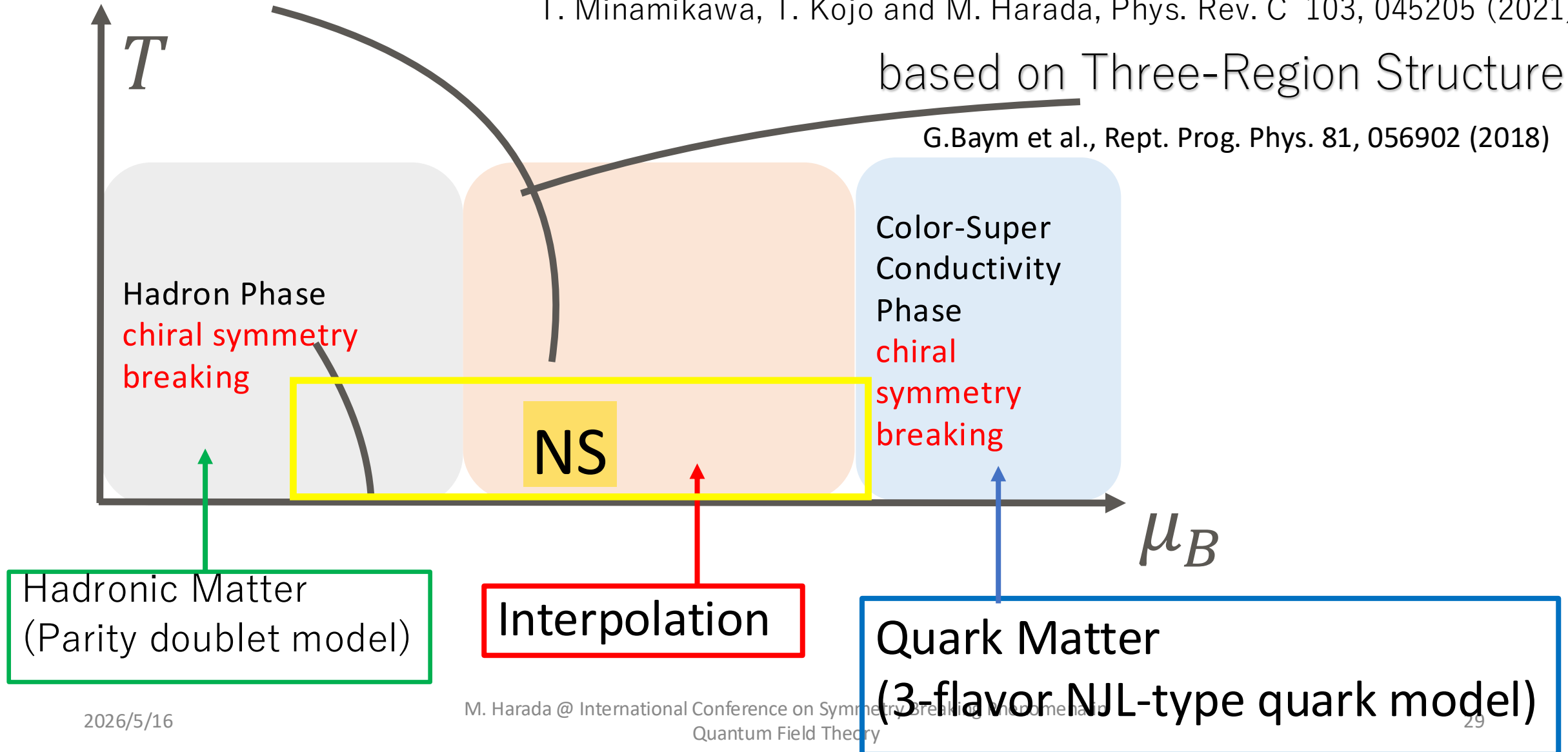


3. PDM-NJL crossover model & Constraints to m_0 from neutron star observation

T. Minamikawa, T. Kojo and M. Harada, Phys. Rev. C 103, 045205 (2021).

based on Three-Region Structure

G. Baym et al., Rept. Prog. Phys. 81, 056902 (2018)



Quark Matter (High density region)

- The **Color-Super Conductivity** is expected to occur in the high density limit of QCD, in which two quarks make a Cooper pair **breaking the color symmetry and the chiral symmetry**.
- In the present analysis, we use a model of NJL-type including the following **4-point interaction terms**:
 - Attractive force between two quarks

$$H \sum_{A,A'=2,5,7} [(\bar{q}i\gamma_5\tau_A\lambda_{A'}C\bar{q}^T) (q^T Ci\gamma_5\tau_A\lambda_{A'}q) + (\bar{q}\tau_A\lambda_{A'}C\bar{q}^T) (q^T C\tau_A\lambda_{A'}q)]$$

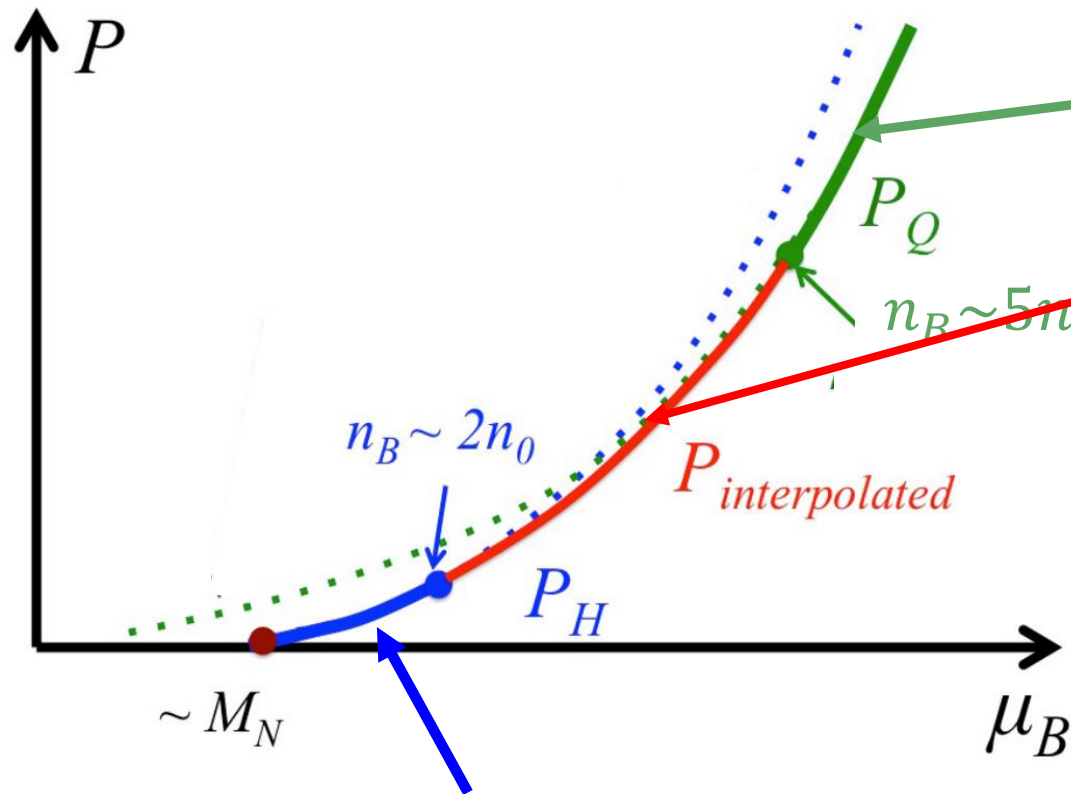
- Repulsive force between two quarks

$$-g_V(\bar{q}\gamma^\mu q)^2$$

Unified EOS for NS in the PDM-NJL crossover model

G. Baym et al., Rept. Prog. Phys. 81, 056902 (2018).

T. Minamikawa, T. Kojo and M.H., Phys. Rev. C 103, 045205 (2021).



Interpolation

We use 5-th order polynomial of μ for the pressure P .

6-coefficients are determined by connecting P to the second order at $2n_0$ and $5n_0$.

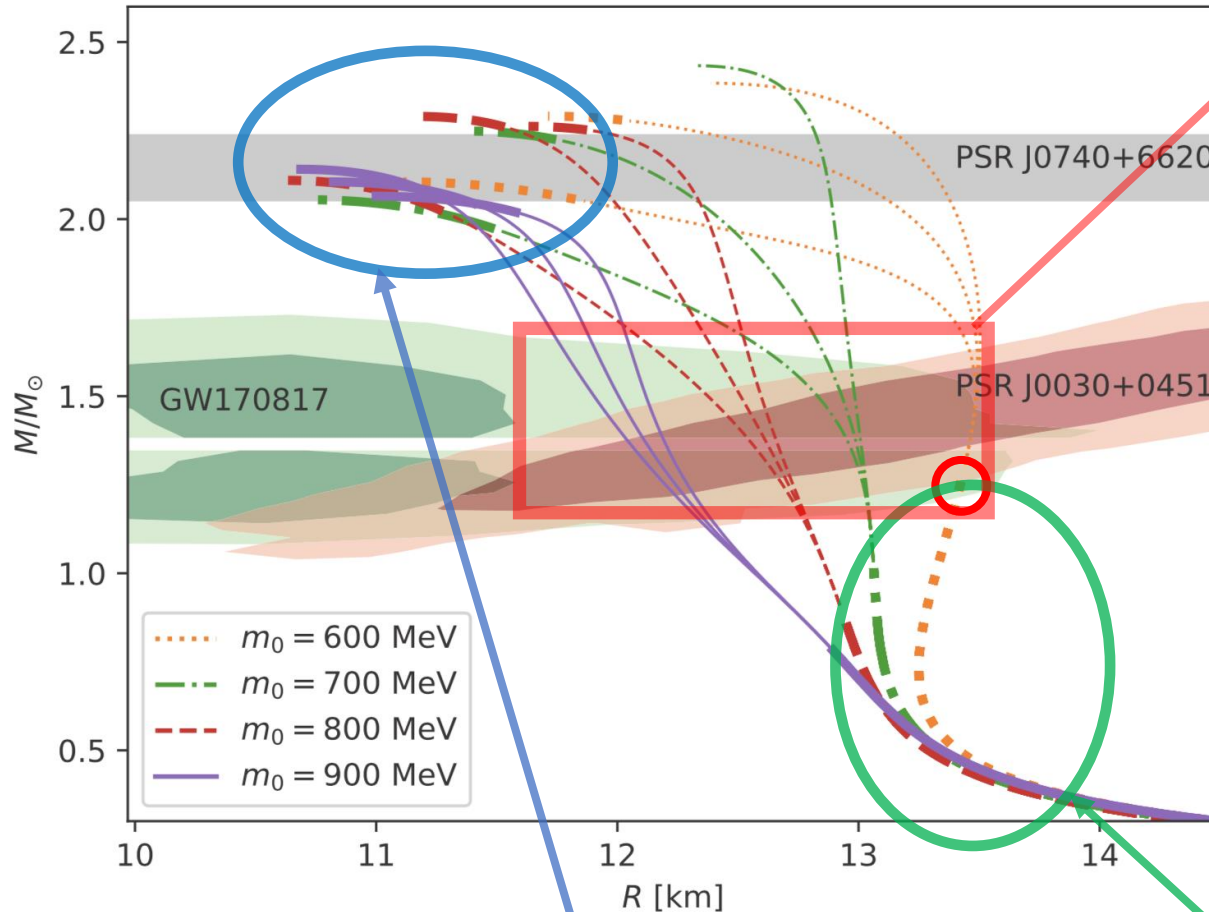
$$P_{had} = P_{int},$$

$$\frac{\partial P_{had}}{\partial \mu} = \frac{\partial P_{int}}{\partial \mu}, \quad \frac{\partial^2 P_{had}}{\partial \mu^2} = \frac{\partial^2 P_{int}}{\partial \mu^2}$$

Parity double model

M-R relation

T. Minamikawa, T. Kojo and M.H., Phys. Rev. C 103, 045205 (2021).



$600\text{MeV} \leq m_0 \leq 900\text{MeV}$

Mass formula

$$m_{\pm} = \sqrt{m_0^2 + \left(\frac{g_1 + g_2}{2}\right)^2 \sigma^2} \mp \frac{g_1 - g_2}{2} \sigma$$

m_0	interact ion	Attractive force by σ	Repulsive force by ω	EOS
small	strong	strong	strong	stiff
large	weak	weak	weak	soft

NSs include quark matter inside.

NSs are made from only hadronic matter.

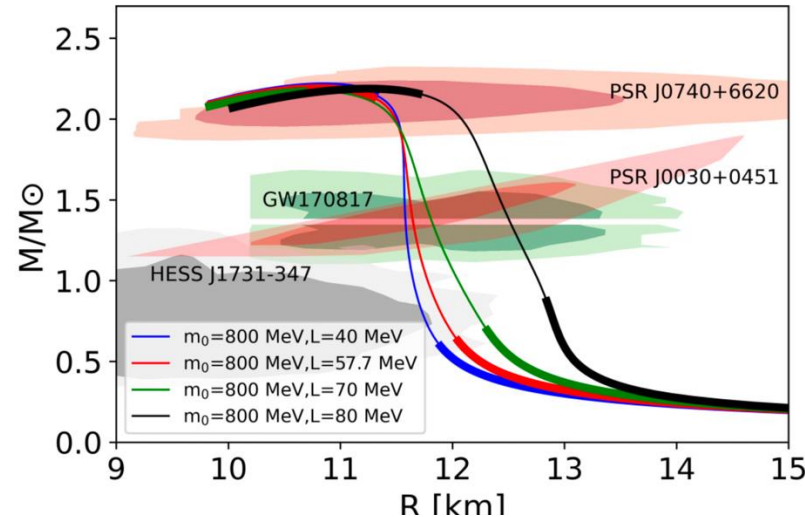
4. Constraints from supernova remnant HESS J1731-347

B. Gao, Y. Yan and M. Harada, Phys. Rev. C 109, 065807 (2024)

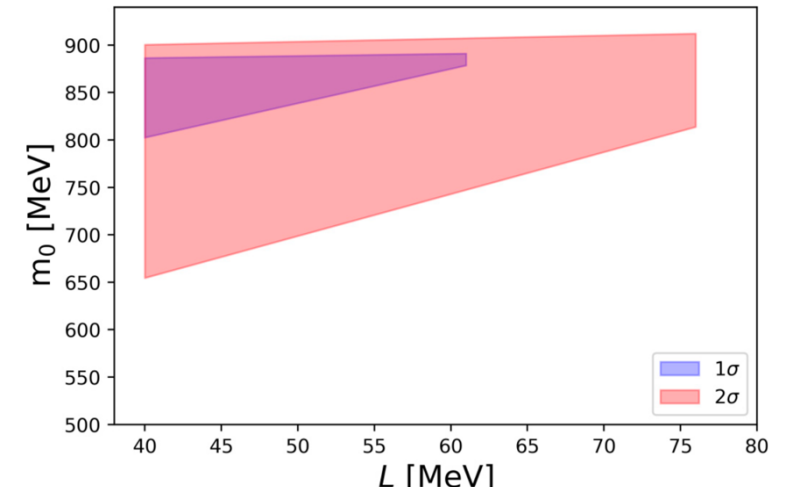
- We added the $\omega - \rho$ mixing term to the model for controlling the slope parameter L_0 .

$$-\lambda_{\omega\rho}(g_{\omega}\omega)^2(g_{\rho}\rho)^2$$

$m_0 = 800\text{MeV}$ and
 $L_0 = 40, 57.7, 60, 80\text{MeV}$



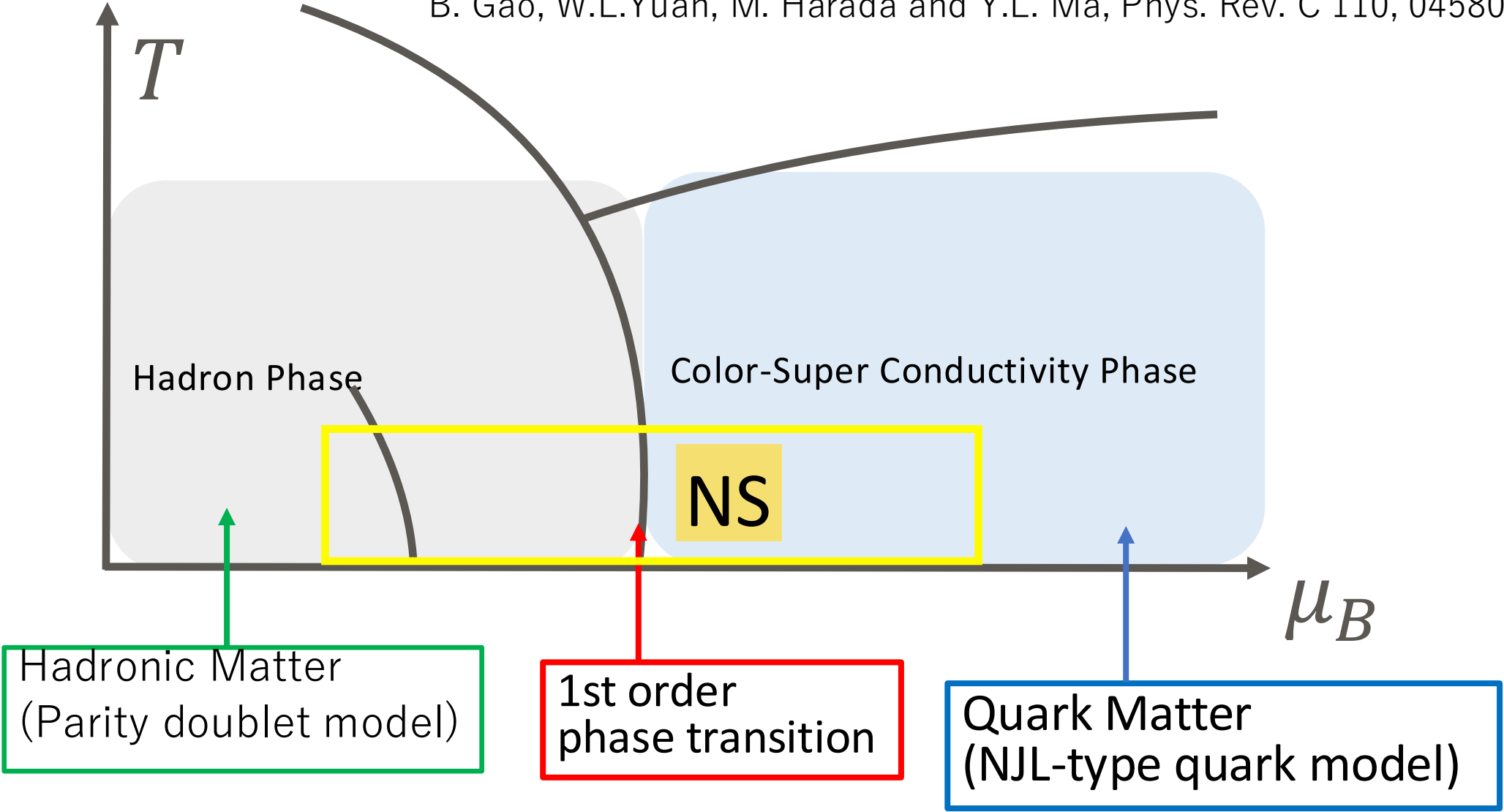
Constraints to m_0 for given L_0 .



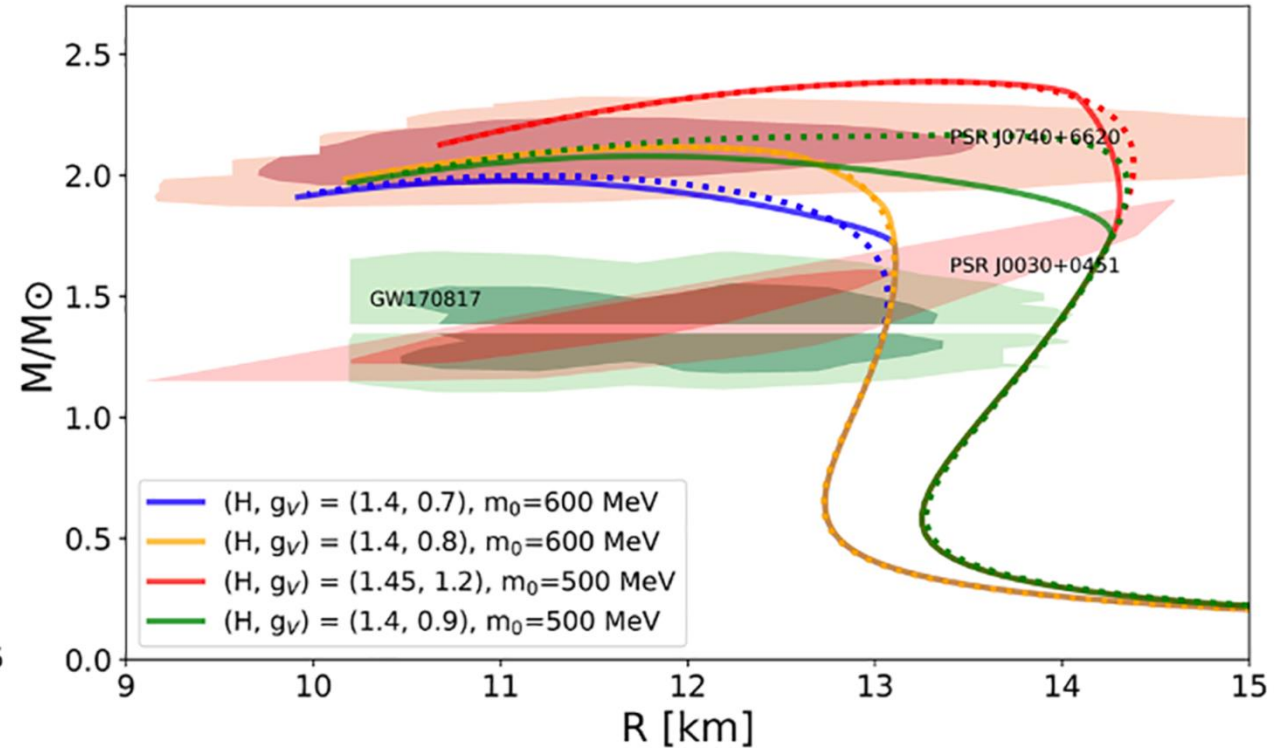
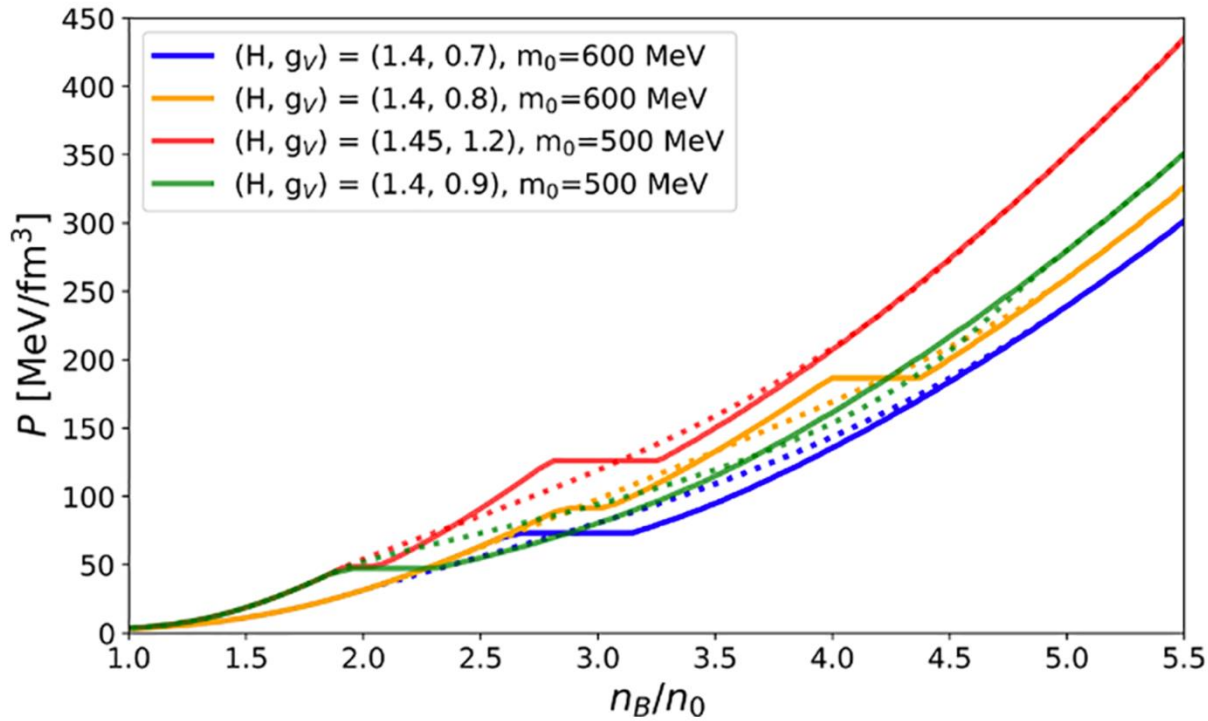
- For $m_0 = 800\text{MeV}$ and $L_0 = 40\text{MeV}$ (blue curve), it is possible to explain the HESS as a neutron star with satisfying constraints from LIGO/VIRGO and NICER.
- If HESS is a neutron star, HESS data is used to obtain constraint to m_0 :
e.g. $m_0 \gtrsim 850$ MeV for $L_0 = 57.7$ MeV

5. Study of 1st order phase transition from PDM to NJL

B. Gao, W.L.Yuan, M. Harada and Y.L. Ma, Phys. Rev. C 110, 045802 (2024).



Comparison with crossover



- Difference

- M-R relation: sudden change for 1st order
- maximum mass of NS: smaller for 1st order
- **Constraint to m_0 is not affected.**

6. Effect of iso-triplet scalar $a_0(980)$ meson

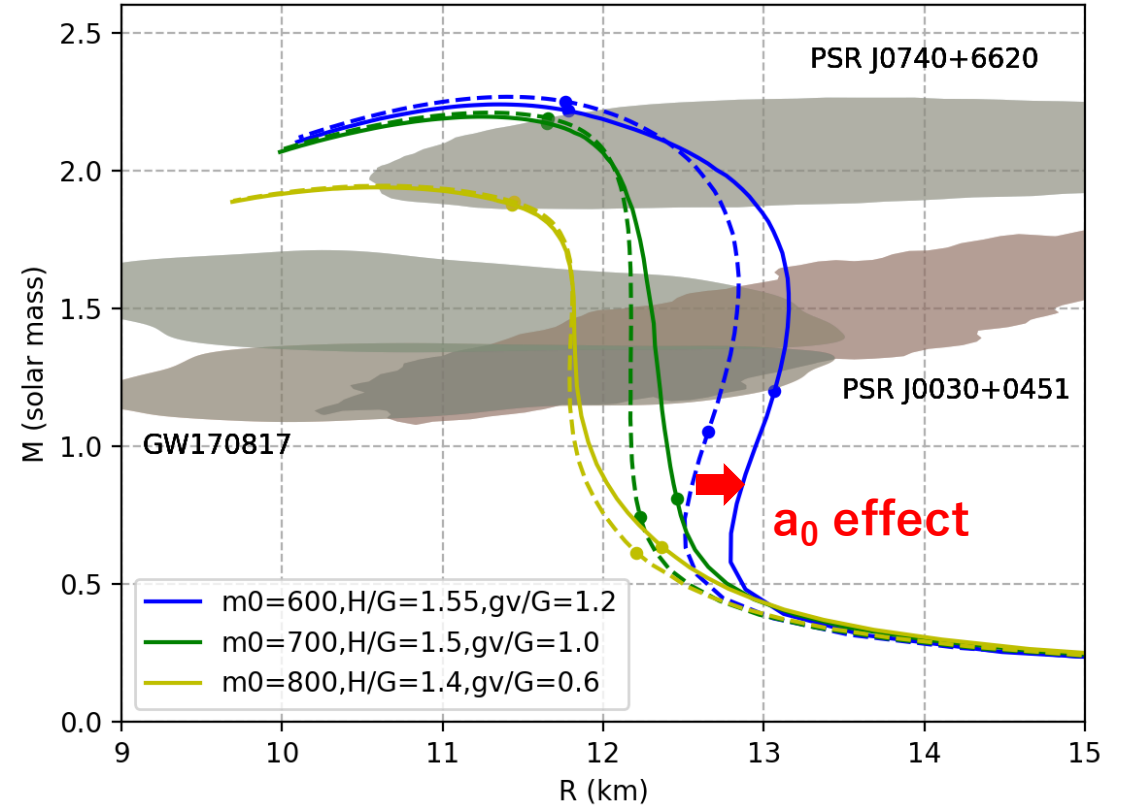
Y.-K. Kong, T. Minamikawa and M. Harada, Phys. Rev. C 108, 055206 (2023).

- We construct a PDM with $a_0(980)$ meson and study its effects in neutron stars.
- Effect of $a_0(980)$ increases the radius of intermediate mass NS

$$520 \text{ MeV} \lesssim m_0 \lesssim 850 \text{ MeV}$$



$$560 \text{ MeV} \lesssim m_0 \lesssim 840 \text{ MeV}$$



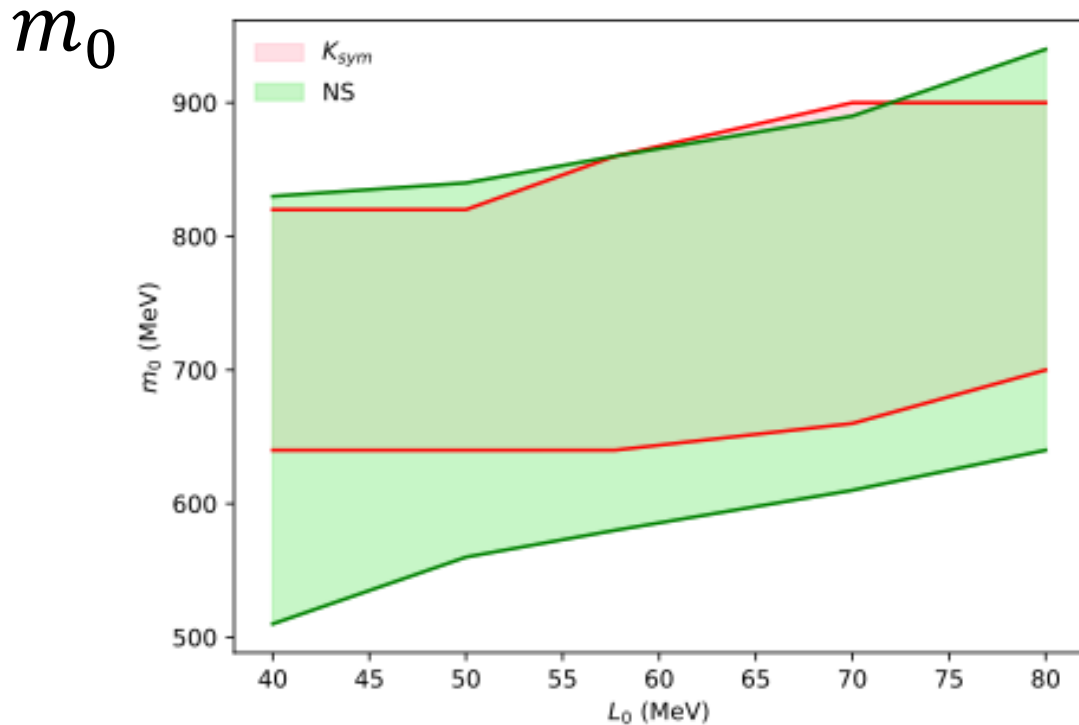
— PDM with a_0
 --- PDM without a_0

$L_0 = 50 \text{ MeV}$

Constraints to m_0 from K_{sym}

Y.K. Kong and Masayasu. Harada,
Nuclear Physics Review, 2024, 41(3): 787-793

$$S(n_B) = S_0 + \left(\frac{n_B - n_0}{n_0}\right) \frac{L_0}{3} + \left(\frac{n_B - n_0}{n_0}\right)^2 \frac{K_{sym}}{18} + \dots$$



$$K_{sym} = -107 \pm 88 \text{ MeV}$$

Empirical value from
LI B A, CAI B J, XIE W J, et al. Universe, 2021, 7(6).

Green area : constraint from NS
Pink area : constraint from K_{sym}

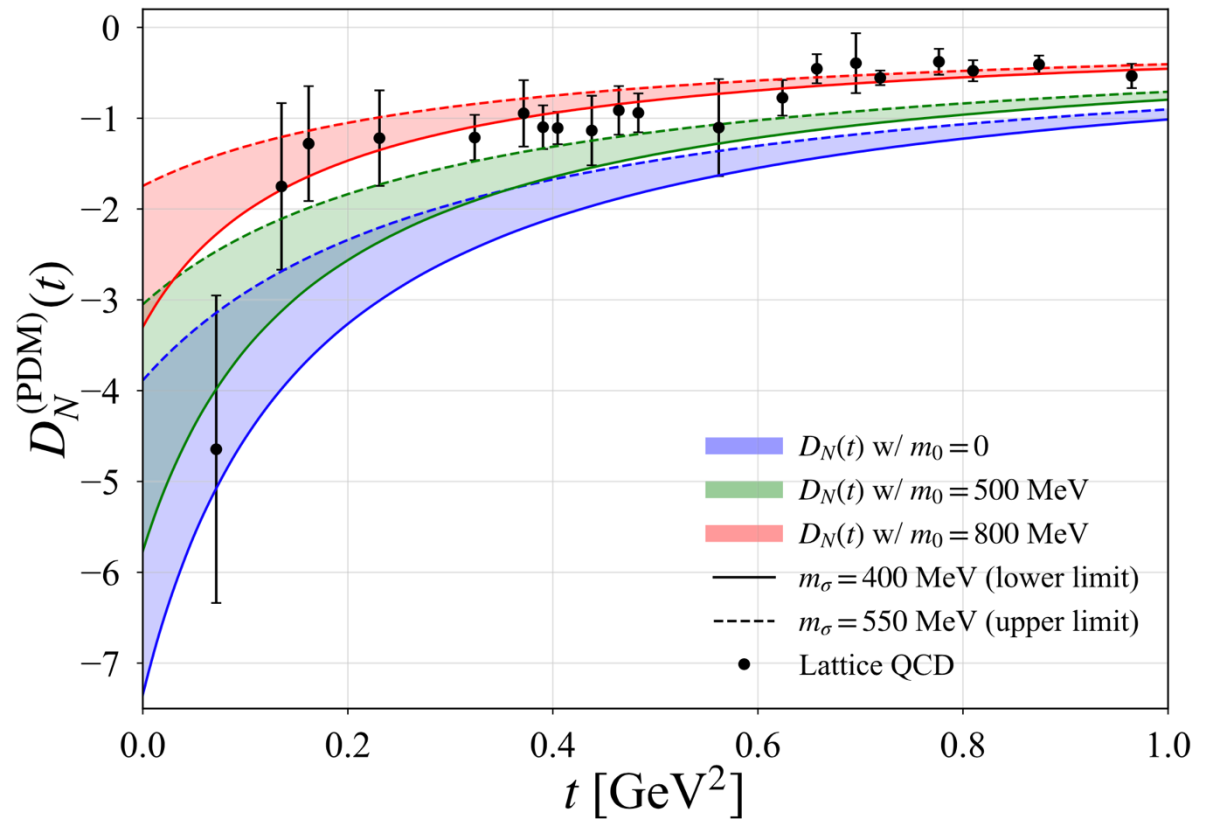
L_0

- K_{sym} seems to provide stronger constraint to m_0 .

7. Recent Analysis: Gravitational Form Factor

M. Kawaguchi, M. Harada and Y.L. Ma, Phys. Lett. B 876, 140400 (2026)

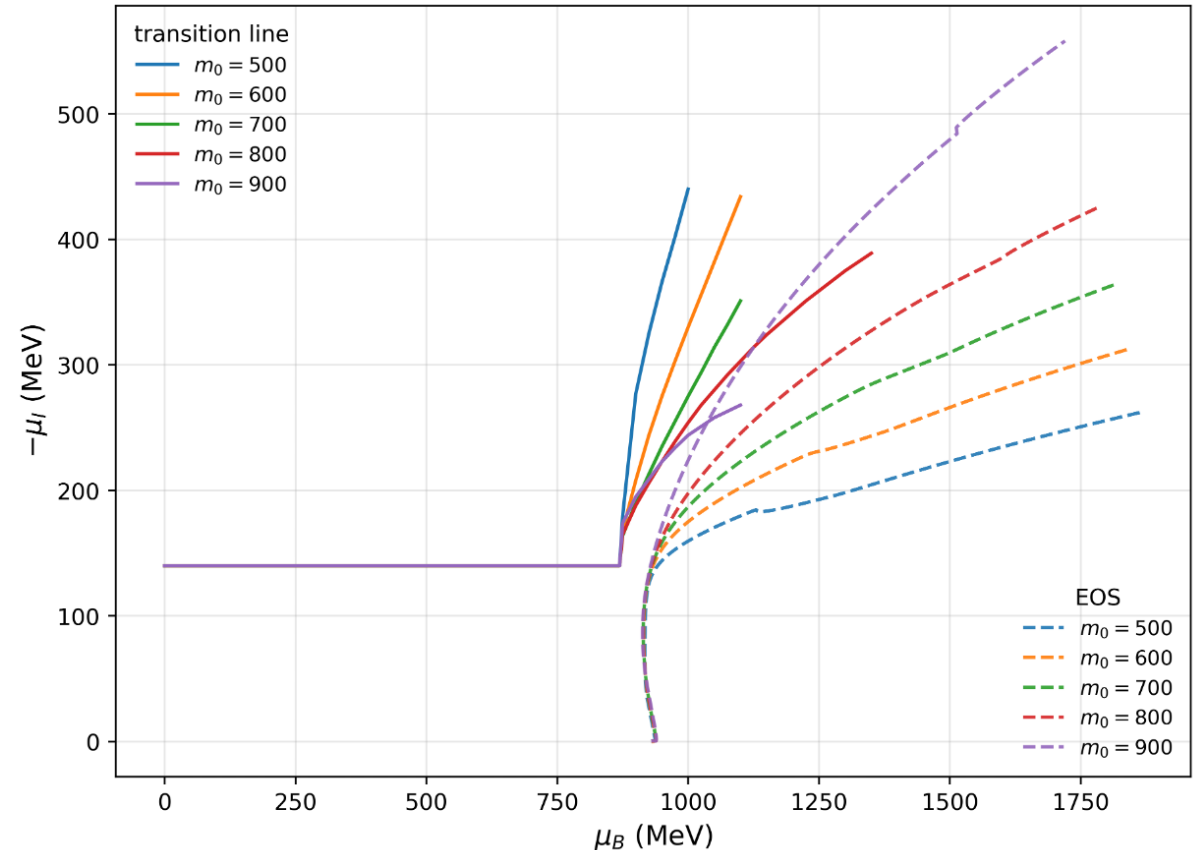
- We constructed a Gravitational Form Factor of Nucleon assuming the sigma meson dominance.
- We compared our prediction with the lattice data.
- We concluded that the preferred value of m_0 lies within the range $m_0 = 500 - 900$ MeV.



7. Recent Analysis: Pion Condensation

X. Liu, B. Gao and M. Harada, in preparation

- We are studying the pion condensation phase in the PDM.
- Our results show that $\mu_I^{cr} > m_\pi$, when the baryon appears in the matter.
- We conclude that the pion condensation phase is unlikely to occur in Neutron Stars.

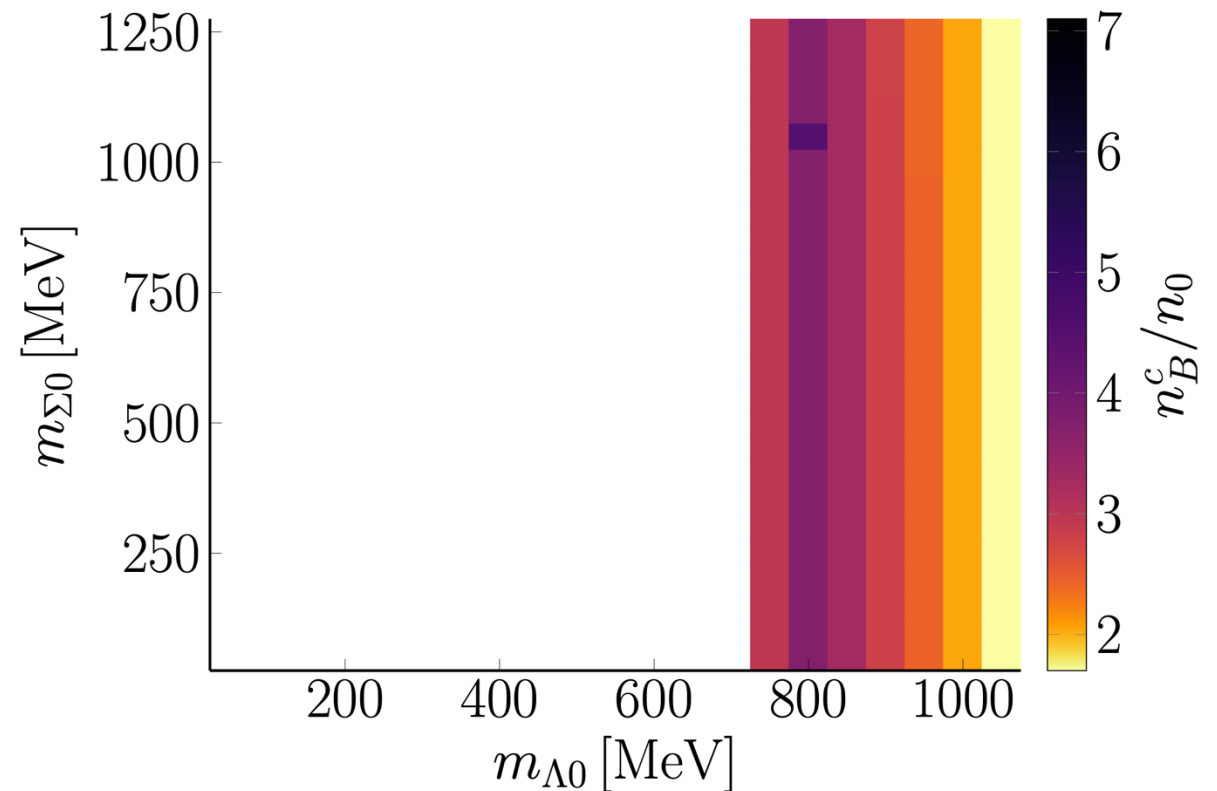


7. Recent Analysis: Hyperon chiral invariant mass

M. Kanazawa and M. Harada, in preparation

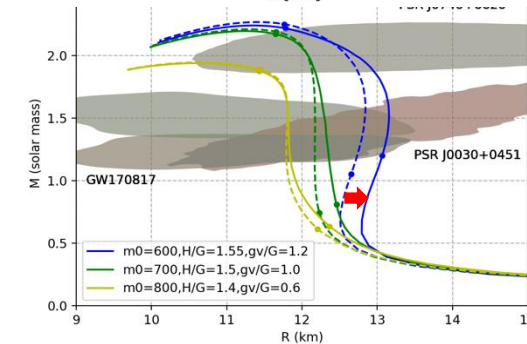
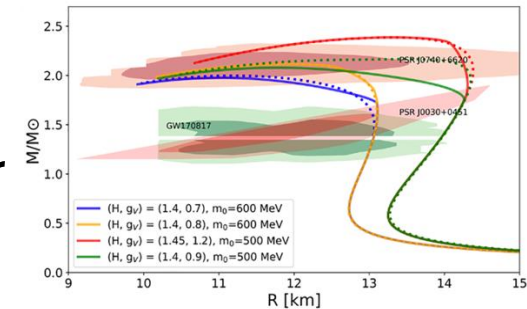
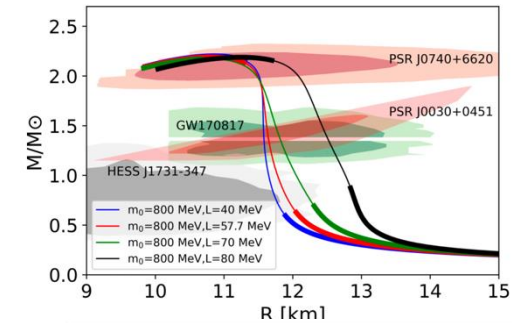
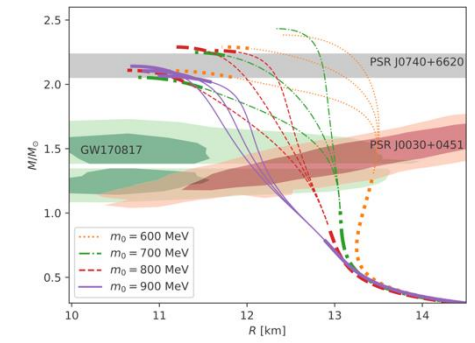
- We construct a model for hyperons Λ and Σ baryons based on $SU(2)_L \times SU(2)_R$ chiral symmetry with including their chiral invariant masses.
- We found that the Λ hyperon appears in matter for smaller μ_B for larger $m_{\Lambda 0}$.

Λ Onset Density ($m_{N0} = 500$ MeV)



8. Summary

- Chiral invariant mass (**microscopic** information) in the PDM is constrained by NS observational data (**macroscopic** information):
 - $600 \leq m_0 \leq 900 \text{ MeV}$
 - (roughly more than 50% of nucleon mass)
- If HESS is a neutron star, m_0 is strongly constrained:
 - e.g. $m_0 \gtrsim 850 \text{ MeV}$ for $L_0 = 57.7 \text{ MeV}$
- 1st order phase transition instead of crossover
 - constraint to m_0 is not affected very much.
- Effect of iso-triplet
 - $a_0(980)$ meson slightly modify the constraint.



Thank you very much for your attention !

