Quantum Kinetic Theory for QCD

Shu Lin

Sun Yat-Sen University

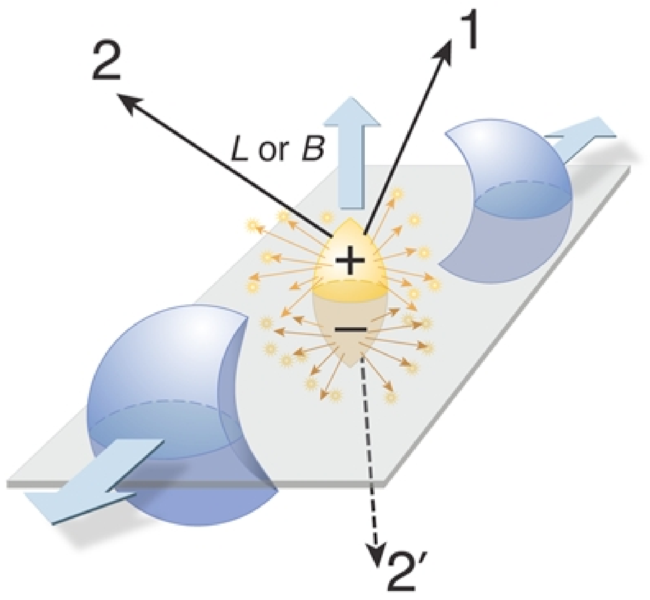
The International Conference on Symmetry Breaking Phenomena in Quantum Field Theory, Hefei, 2026/5/15-19

SL, 2603.04263,
PRD 2022

Outline

- Spin phenomena in heavy ion collisions: uncertainties and opportunities
- Quantum kinetic theory for QCD
- Applications to spin polarization and orbit-spin conversion
- Summary and outlook

Beginning of spin phenomena in HIC



$$L_{ini} \sim 10^5\hbar \rightarrow S_{final}$$

Liang, Wang, PRL 2005, PLB 2005

spin-orbital coupling via parton scattering

quark polarization P_q

recombination

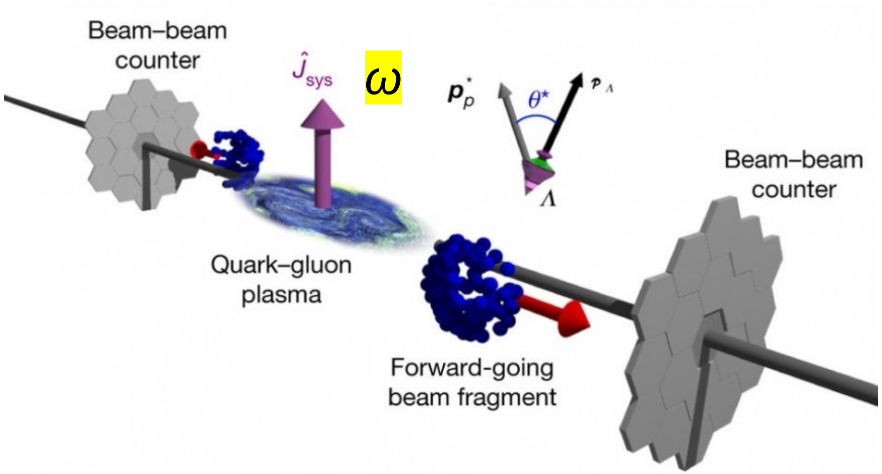
$qqq \longrightarrow B + X$ Baryon spin polarization

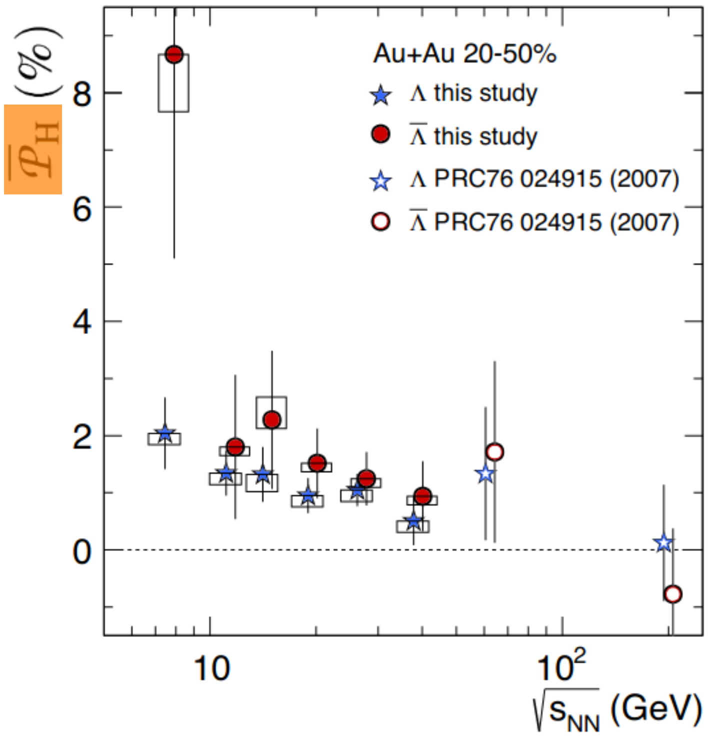
$q\bar{q} \longrightarrow V + X$ Vector meson spin alignment

Global polarization of Λ: first exmaple

$$\Lambda \to p + \pi^-$$ weak decay

$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}[1 + \alpha_H P_H \cos\theta^*]$$





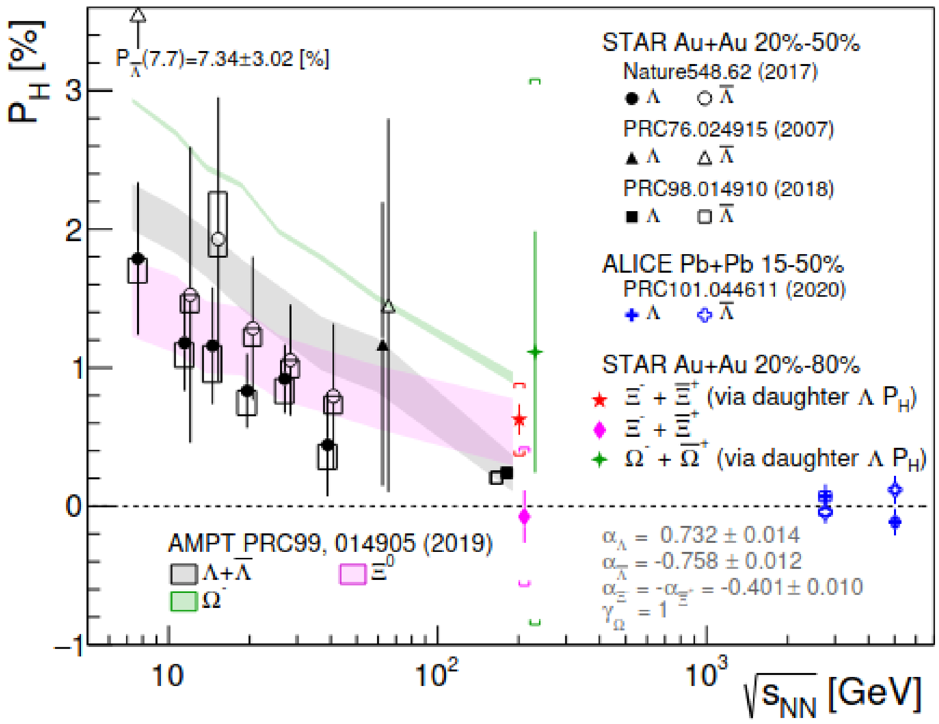
STAR, Nature 2017

$$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$$

Becattini et al, PRC 2017

Spin couples to QGP vorticity

More measured polarized baryons



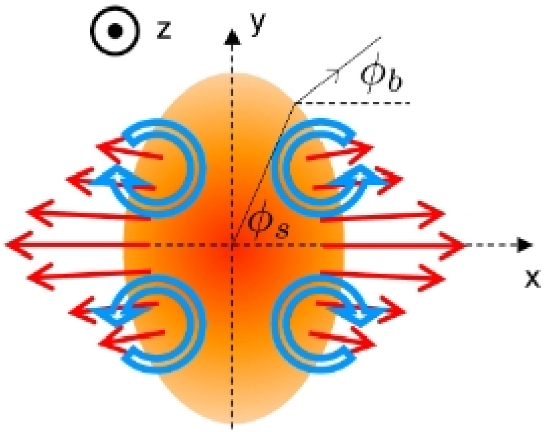
STAR, PRL 2021

$$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$$

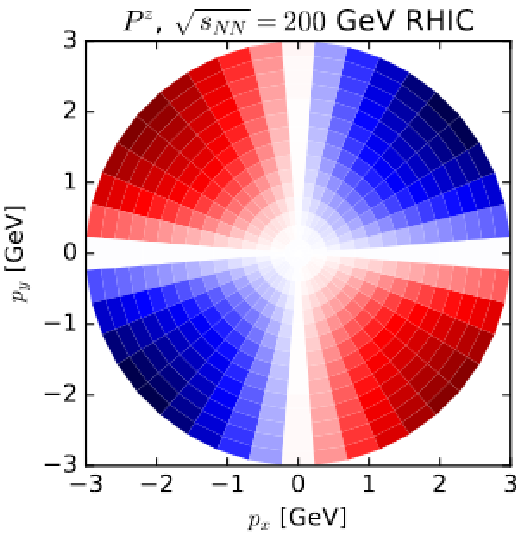
$$\mathbf{P} = \frac{\langle \mathbf{s} \rangle}{s} \approx \frac{(s+1)}{3}\frac{\omega}{T},$$

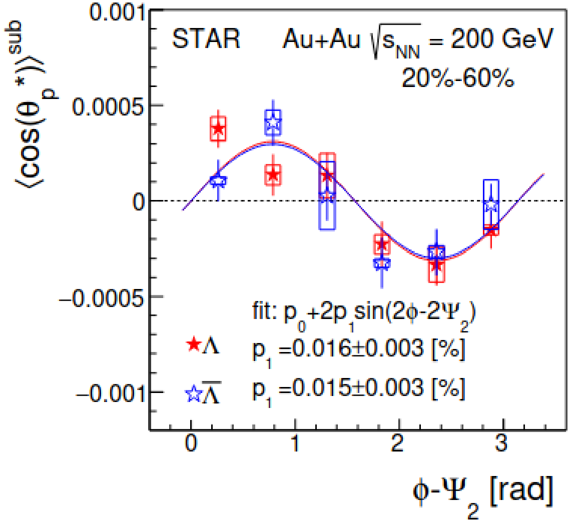
Spin-vorticity coupling structure independent
dictated by Einstein equivalence principle

Local polarization of Λ: projection along beam



$P^z \propto \omega^z$



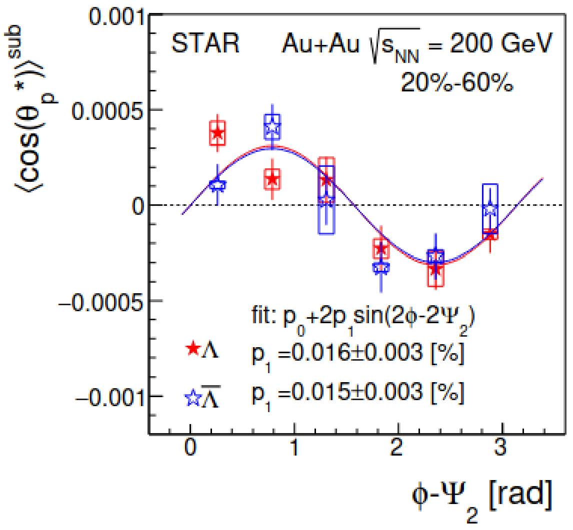


Becattini, Karpenko, PRL 2018
Wei, Deng, Huang, PRC 2019
Wu, Pang, Huang, Wang, PRR 2019
Fu, Xu, Huang, Song, PRC 2021

STAR, PRL 2019

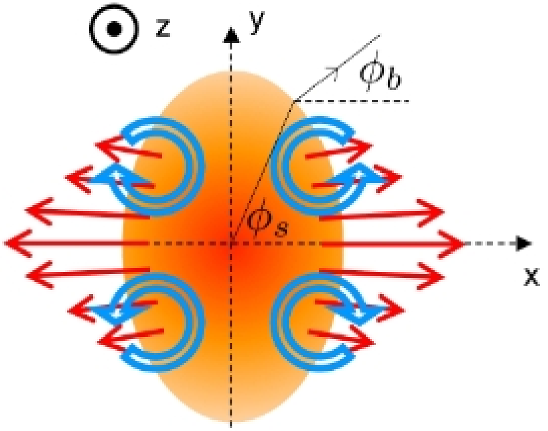
wrong sign

Local polarization of Λ: vorticity + shear



STAR, PRL 2019

vorticity



$P^z \propto \omega^z$

wrong sign

universal

shear (free theory)

$P^i \propto \epsilon^{ijk}\, \hat{p}_k\, \hat{p}_l\, \sigma_{jl}$

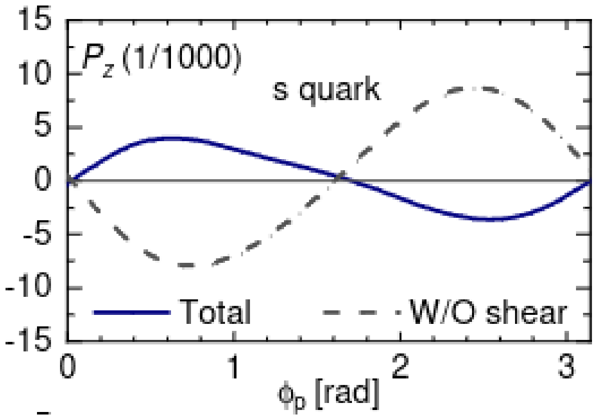
$P^z \propto \left(\langle p_y^2\rangle - \langle p_x^2\rangle\right) \sigma_{xy}$

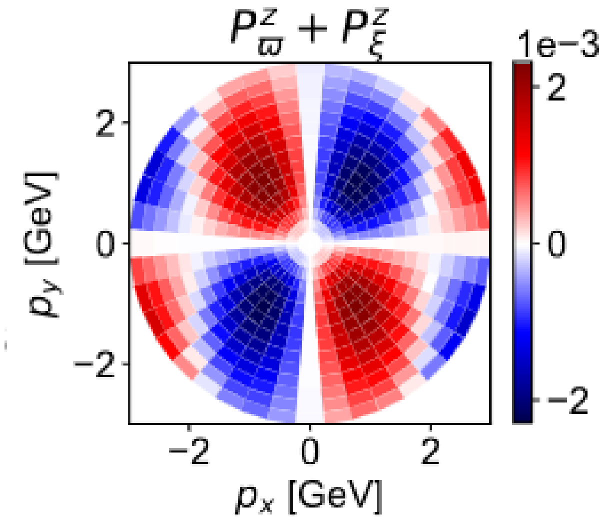
right sign

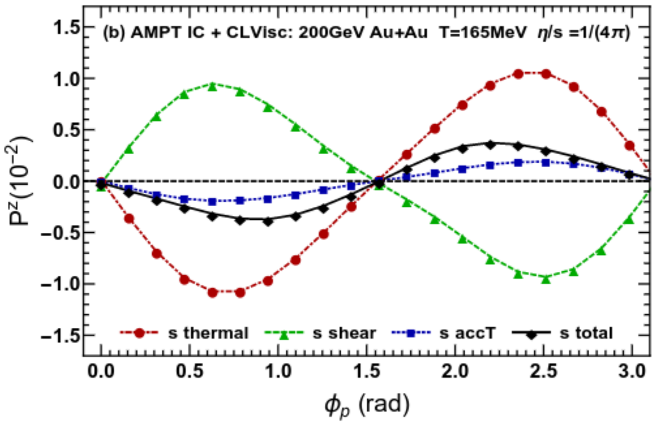
non-universal

Hidaka, Pu, Yang, PRD 2018
Liu, Yin, JHEP 2021
Becattini, et al, PLB 2021

Phenomenology: vorticity + shear







Fu, Liu, Pang, Song, Yin, PRL 2021
Becattini, et al, PRL 2021
Yi, Pu, Yang, PRC 2021

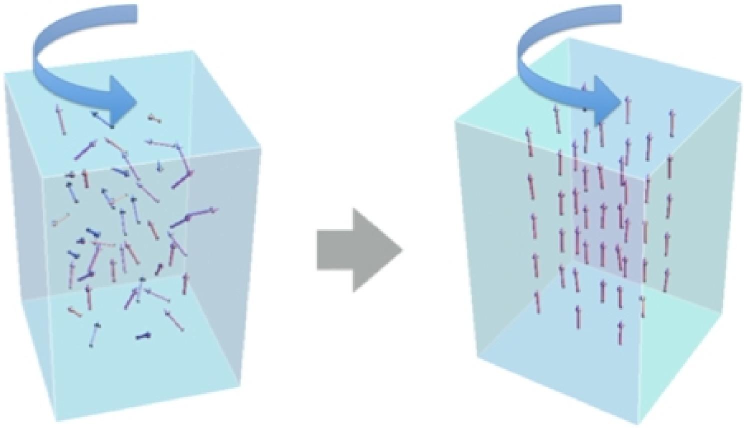
competition between shear and vorticity

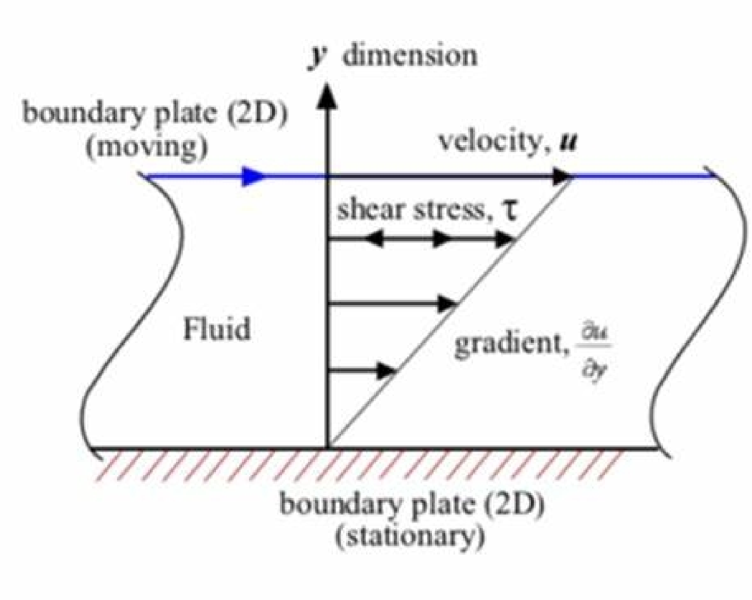
shear contribution may or not reverse the sign

Uncertainties in local polarization: interaction effect

$P^z \propto \omega^z$ ~ B

$P^i \propto \epsilon^{ijk} \hat{p}_k \hat{p}_l \sigma_{jl}$ ~ E





Equilibrium: collision vanishes

Steady state: collision nonvanishing

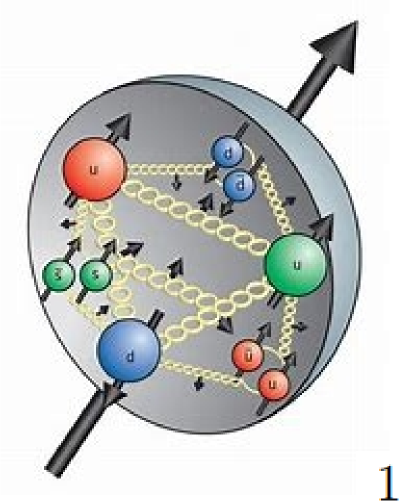
Uncertainties in local polarization: structure effect

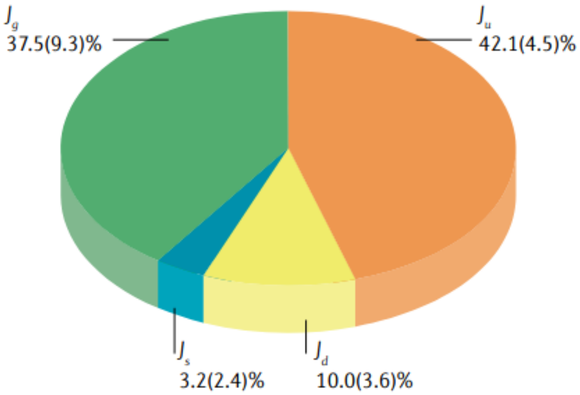
simplified structure

Λ: point particle

Λ: quark model

reality





$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

~40%

Alexandrou et al, PRD 2020

Polarization as a probe of hadron structure

hadron structure

quark polarization in QGP → hadron polarization

(theories) (data)

current phenomenology uses:
collisionless quantum kinetic theory

more accurate description: QCD based
quantum kinetic theory

Current status of Quantum kinetic theory

Collisionless QKT	Many works before 2020, reviewed in PPNP 2022
Collisions:	
QED or QED-like QKT	SL, PRD 2022 Yang, Hattori, Hidaka, JHEP 2020 Hattori, Hidaka, Yamamoto, Yang, JHEP 2021 Fang, Pu, Yang, PRD 2022, 2024, 2025 Z. Wang, SL, JHEP 2022, PRDs 2025 Sheng, Weickgnnant, Speranza, Rischke, Wang, PRD 2021
QCD QKT for heavy flavor	Li, Yee, PRD 2019 Hongo, Huang, Kaminski, Stephanov, Yee, JHEP 2022 Yang, Yao, PRD 2024
QCD QKT for quark/gluon	SL, 2603.04263

Early kinetic theory for QCD

$$(\partial_t + \hat{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}})\, f_s(\boldsymbol{x}, \boldsymbol{p}, t) = -C_s^{2\leftrightarrow 2}[f] - C_s^{\text{“}1\leftrightarrow 2\text{”}}[f]$$

Arnold, Moore, Yaffe, early 00s

$f_s(\boldsymbol{x}, \boldsymbol{p}, t)$: distributions of quarks and transverse gluons

$C_s^{2\leftrightarrow 2}[f]$: elastic collisions

$C_s^{\text{“}1\leftrightarrow 2\text{”}}[f]$: inelastic collisions

Boltzmann equation (spin-averaged), widely used for hydrodynamic (spin-independent) transport coefficients

Routes toward a kinetic theory

1. One may begin with the full hierarchy of Schwinger-Dyson equations for (gauge-invariant) correlation functions in a weakly non-equilibrium state in the underlying quantum field theory. For weak coupling, one may systematically justify, and then in-

SL, 2603.04263

2. One may consider the diagrammatic expansion for the equilibrium correlator appearing in the Kubo relation (1.8) for some particular transport coefficient. After carefully

Gagnon, Jeon, PRD 2007

3. One may directly argue (by examining equilibrium finite temperature correlators) that, for sufficiently weak coupling, the underlying high temperature quantum field theory has well-defined quasi-particles, that these quasi-particles are weakly interacting with

Given the complexities of real-time, finite-temperature diagrammatic analysis in gauge theories (especially non-Abelian theories), we find the last approach to be the most physically transparent and compelling. But this is clearly a matter of taste.

Arnold, Moore, Yaffe, JHEP 2000

DOF and hierarchy of scales

DOF: quarks/gluons w/ spin

$$\Lambda \gg g\Lambda \gg g^2\Lambda \gg g^4\Lambda \gtrsim \partial_X \sim \mathcal{P}_q$$

typical energy	chromo-E screening	chromo-B screening	inverse MFP	hydro gradient

↑ assumption polarization induced by hydro gradients

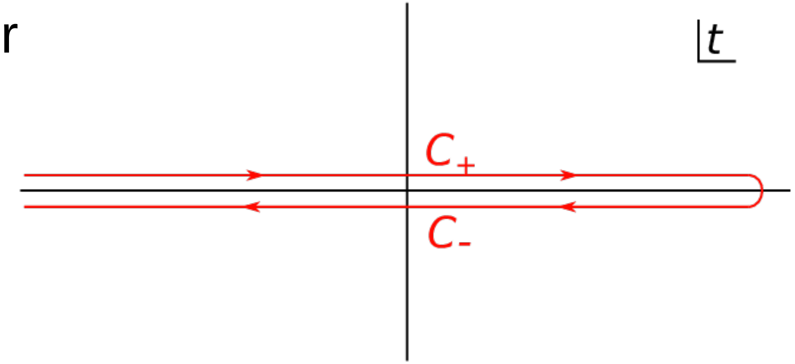
- coarse-graining within fluid element → localized collision
- mean field screened
- gradient expansion: 0^{th} order Boltzmann, 1^{st} order polarization

Derivation of the QKT

Dyson-Schwinger equation on the SK contour

↓ truncation to two-point function

Kadanoff-Baym equation on the SK contour



Chou, Su, Hao, Yu, Phys. Rept. 1985
Blaizot, Iancu, Phys. Rept. 2002

$$\left(i\partial\!\!\!/_x - m\right) S^<(x,y) = \int_z \left(\Sigma_R(x,z)S^<(z,y) + \Sigma^<(x,z)S_A(z,y)\right)$$

$$\tilde{S}^<(X = \frac{x+y}{2}, P) = \int d^4(x-y)e^{iP\cdot(x-y)}\langle S^<(x,y)\rangle \propto f(X,P)$$

- off-equilibrium state
- higher point functions fixed by two-point function
- preserve conservation laws

many-body effect

K-B equation for quark

$$\frac{i}{2}\not\partial S^< + (\not P - m)S^< = \mathrm{Re}[\Sigma_R]S^< + \Sigma^<\mathrm{Re}[S_R] + \frac{i}{2}\left(\Sigma^> S^< - \Sigma^> S^<\right)$$

thermal mass/width

collision

real part: quasi-particle dispersion

imag part: kinetic equation

diag → $\mathrm{tr}\left(\not\partial S^<\right) = \mathrm{tr}\left(\Sigma^> S^< - \Sigma^> S^<\right)$

Boltzmann

off-diag → spin kinetic equation

Gradient expansion

$$\frac{i}{2}\partial\!\!\!/ S^< + (P\!\!\!/ - m)S^< = \mathrm{Re}[\Sigma_R]S^< + \Sigma^<\mathrm{Re}[S_R] + \frac{i}{2}(\Sigma^> S^< - \Sigma^> S^<)$$

$$S^< = S^{<(0)} + S^{<(1)} + \cdots .$$

$$S^<(X,P) = S^{<(0)}(X,P) + S^{<(1)}_{\mathrm{un}}(X,P) = 2\pi\epsilon(P\cdot u)\delta(P^2 - m^2)\,(P\!\!\!/ + m)\,(-f_q(X,P))$$

$$+ S^{<(1)}_{\mathrm{pol}}(X,P) = \gamma^5\gamma_\mu\mathcal{A}^\mu + \frac{i[\gamma_\mu,\gamma_\nu]}{4}\mathcal{S}^{\mu\nu}$$

$$f_q = f_q^{\mathrm{leq}} + \delta f_q.$$

$$\partial\!\!\!/ S^{<(0)} = (\Sigma^> S^< - \Sigma^> S^<)^{(1)}$$

$$f_q^{\mathrm{leq}}(X,P) = (\exp((P\cdot u(X) - \mu(X)/T(X)) + 1)^{-1}$$

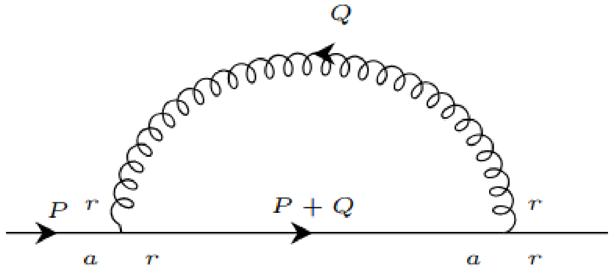
hydrodynamic gradient

Power counting in coupling

$$\frac{i}{2}\partial\!\!\!/ S^{<} + (P\!\!\!/ - m)S^{<} = \mathrm{Re}[\Sigma_R]S^{<} + \Sigma^{<}\mathrm{Re}[S_R] + \frac{i}{2}\left(\Sigma^{>}S^{<} - \Sigma^{>}S^{<}\right)$$

thermal mass/width — collision

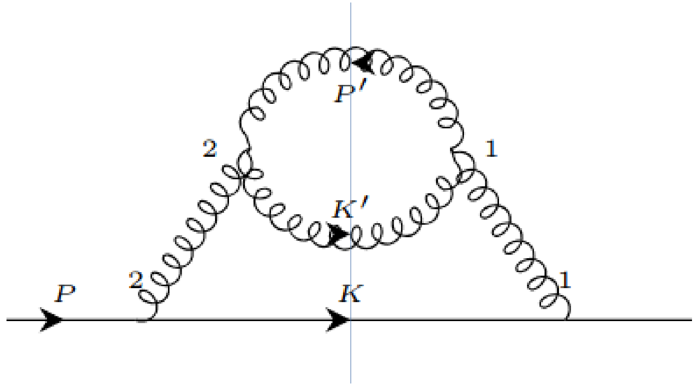
dispsersion correction



$\sim O(g^2)$ parametrically suppressed

collision

$$\partial\!\!\!/ S^{<(0)} = \left(\Sigma^{>}S^{<} - \Sigma^{>}S^{<}\right)^{(1)} \quad \sim O(\partial\, g^0)$$



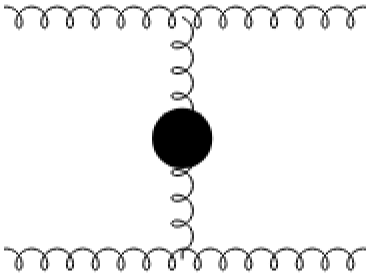
naively $\sim O(g^4)$

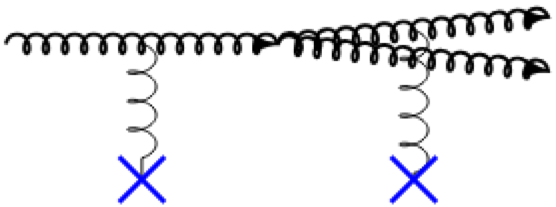
enhanced by $\delta f \sim \dfrac{\partial_X f_{\mathrm{leq}}}{g^4} \sim \tau_R \partial_X f_{\mathrm{leq}}$

long relaxation time & large deviation from leq

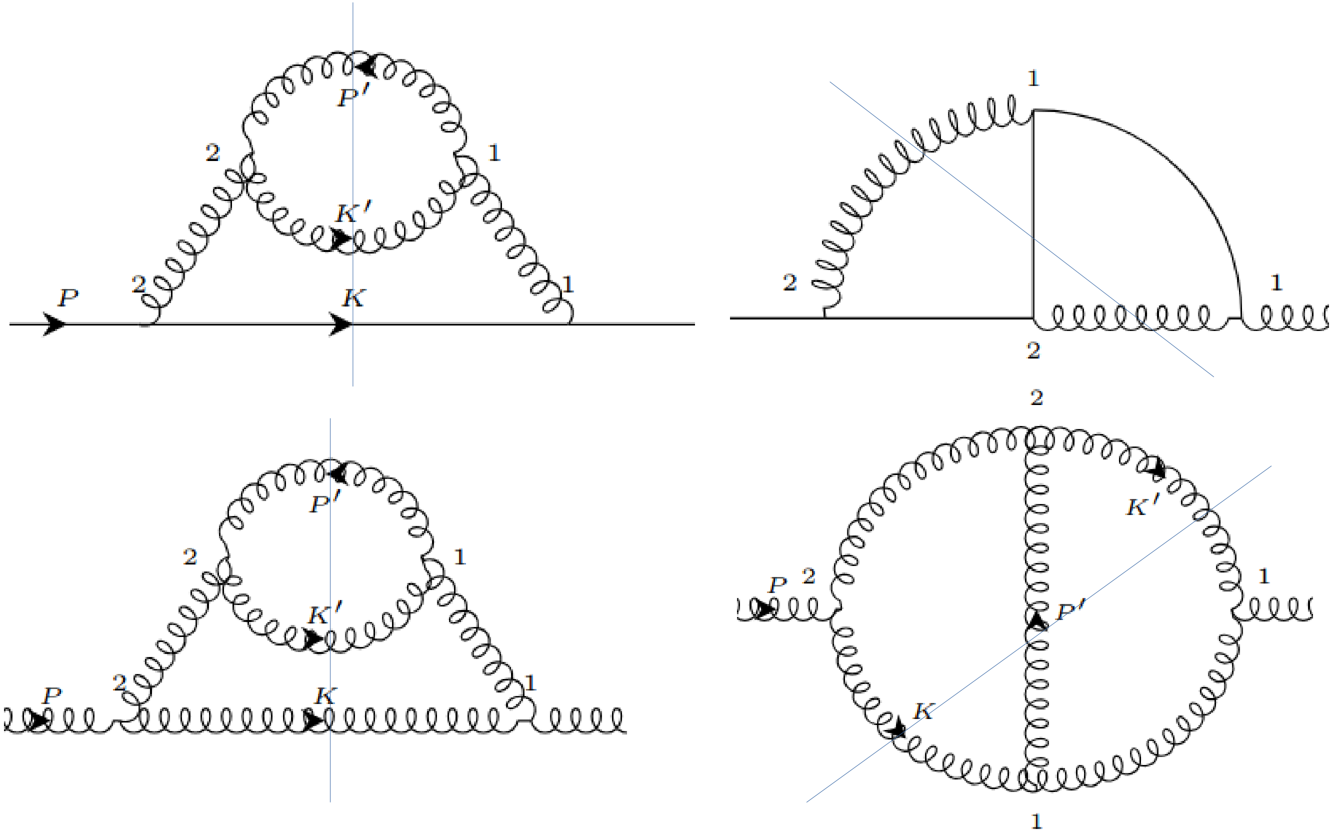
Where dispersion correction not ignored

- screening IR divergence (elastic collision)
- regulate collinear divergence (inelastic collision)





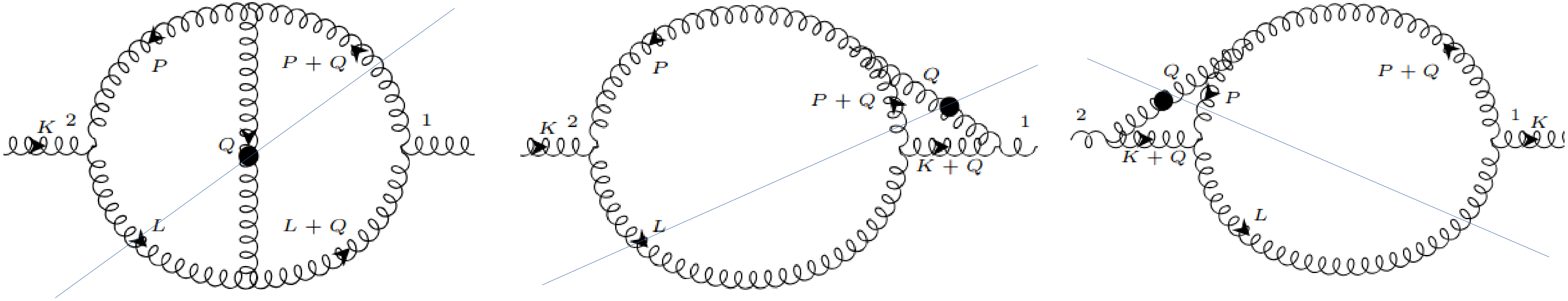
Self-energies: elastic collisions



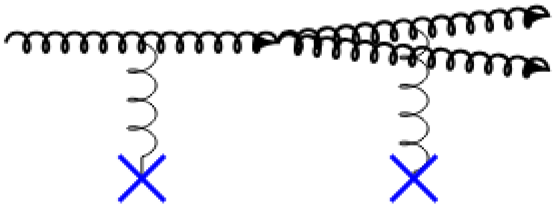
+ many other 2-loop diagrams

binary collisions

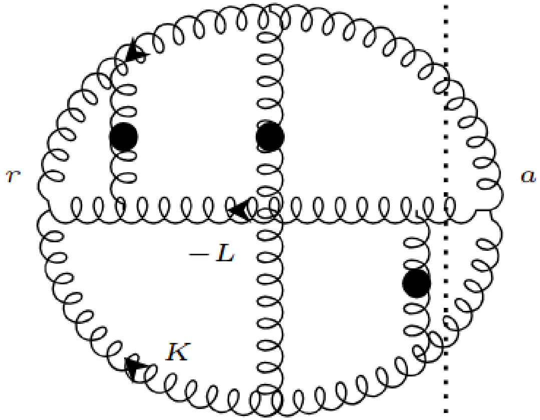
Self-energies: inelastic collisions



medium-induced
collinear radiation

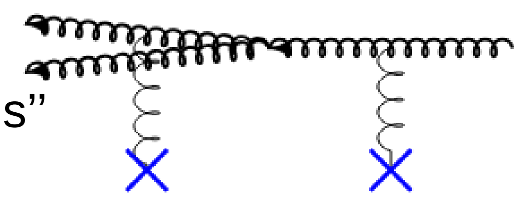


multiple interaction
with medium



3-body
“bound state”

3g-vertex in spin basis



$$\epsilon^s(L) = \frac{1}{\sqrt{2}}\left(1, is, -\frac{l^s}{l_{\parallel}}\right) \qquad s = \pm 1$$

$$l^s \equiv l_1 + isl_2$$

quantization axis:
collinear direction z

$$l_{\parallel} \sim O(1) \quad l_{\perp} \sim O(g)$$

$$\epsilon_i^s(P)\epsilon_j^{-s'*}(L)\epsilon_k^{s''}(K)\left[\delta_{ik}(-p+k)_j + \delta_{kj}(-k-l)_i + \delta_{ji}(l+p)_k\right]$$

$$= \delta_{s,-s''}\sqrt{2}\left[\frac{p_{\parallel}k_{\perp} - k_{\parallel}p_{\perp}}{l_{\parallel}}\right]^{s'} + \delta_{-s',s''}\sqrt{2}\left[\frac{k_{\parallel}l_{\perp} - l_{\parallel}k_{\perp}}{p_{\parallel}}\right]^{s} + \delta_{s,-s'}\sqrt{2}\left[\frac{l_{\parallel}p_{\perp} - p_{\parallel}l_{\perp}}{k_{\parallel}}\right]^{s''}$$

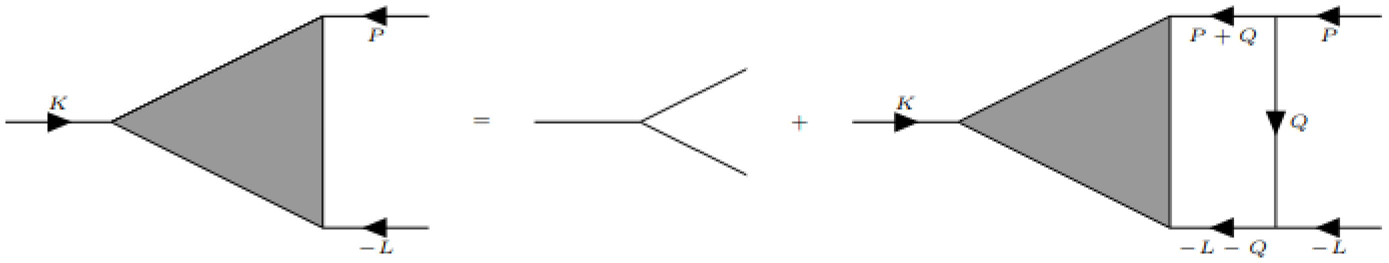
$$\Delta S = -s' \qquad \Delta L = s'$$

partial wave

$$q_1 + is'q_2 \sim Y^*_{1,-s'}$$

orbit-spin conversion
with local collisions

3-g bound state interpretation



$$i\delta E F(P,-L,K) = I_0(P,-L,K) + g^2\frac{C_A}{2}\int_Q 2\pi\delta(q_0-q_\parallel)D^*_{rr,\mu\nu}(Q)\hat{e}^\mu\hat{e}^\nu\Big([F(P+Q,-L-Q,K)-F(P,-L,K)]+$$

kinetic

$$[F(P+Q,-L,K-Q)-F(P,-L,K)]+[F(P,-L-Q,K+Q)-F(P,-L,K)]\Big)$$

pairwise potential

$$\delta E = \frac{p_\perp^2+m_g^2}{2p_\parallel}+\frac{l_\perp^2+m_g^2}{-2l_\parallel}+\frac{k_\perp^2+m_g^2}{2k_\parallel}$$

3-gluon 2D “bound state”

mass: $(p_\parallel,-l_\parallel,k_\parallel)$

momenta: $(\mathbf{p}_\perp,-\mathbf{l}_\perp,\mathbf{k}_\perp)$

Inelastic collision rate ~ spectral density of bound state

Application to spin polarization

$$\frac{i}{2}\not{\partial} S^{<(0)} + (\not{P} - m)S^{<(1)} = \frac{i}{2}\left(\Sigma^{>}S^{<} - \Sigma^{<}S^{>}\right)^{(1)}$$ constraint equation

$$\frac{i}{2}\not{\partial} S^{<(1)} + (\not{P} - m)S^{<(2)} = \frac{i}{2}\left(\Sigma^{>}S^{<} - \Sigma^{<}S^{>}\right)^{(2)}$$ kinetic equation

Non-dynamical part from constraint equation

$$S^{<(1)}_{\rm pol}(X,P) = \gamma^5\gamma_\mu \mathcal{A}^\mu + \frac{i[\gamma_\mu,\gamma_\nu]}{4}\mathcal{S}^{\mu\nu}$$

$$\mathcal{A}^\mu = -2\pi\epsilon(P\cdot u)\frac{\epsilon^{\mu\nu\rho\sigma}P_\rho u_\sigma \mathcal{D}_\nu f_q}{2(P\cdot u+m)}\delta(P^2-m^2)$$

free collisional

$$\mathcal{D}_\nu = \partial_\nu - \Sigma_\nu^{>} - \Sigma_\nu^{<}\frac{1-f_q}{f_q} \qquad g^4\delta f \sim O(\partial_X)$$

Early studies with QED:
SL, Wang, JHEP 2022, PRDs 2025
Fang, Pu, Yang, PRD 2024
Fang, Pu, PRD 2025

Dynamical part from kinetic equation

$$\mathcal{A}^a_\mu = 2\pi\delta(P^2-m^2)a_\mu f_A \quad \sim O(m)$$

SL, Wang, PRD 2025

Expectation from QCD based collision

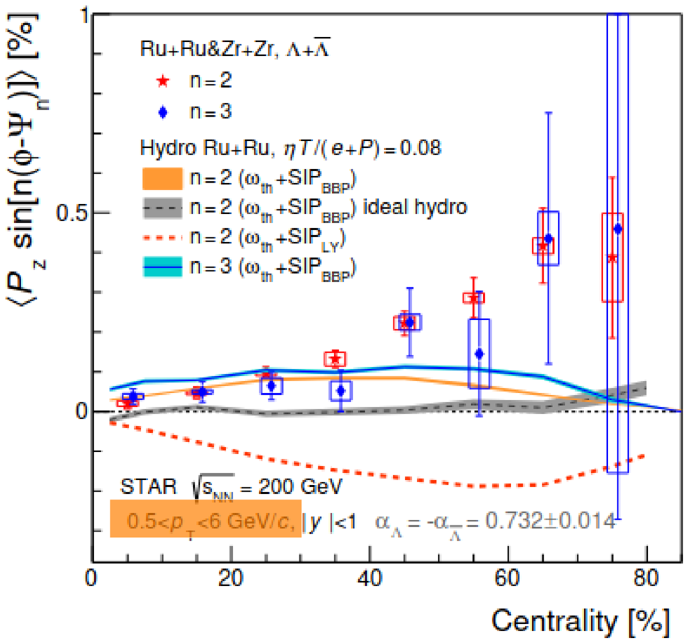
$$S^{<(1)}_{\rm pol}(X,P) = \gamma^5\gamma_\mu\mathcal{A}^\mu + \frac{i[\gamma_\mu,\gamma_\nu]}{4}\mathcal{S}^{\mu\nu}$$

$$\mathcal{A}^\mu = -2\pi\epsilon(P\cdot u)\frac{\epsilon^{\mu\nu\rho\sigma}P_\rho u_\sigma \mathcal{D}_\nu f_q}{2(P\cdot u+m)}\delta(P^2-m^2)$$

free collisional

$$\mathcal{D}_\nu = \partial_\nu - \Sigma^>_\nu - \Sigma^<_\nu \frac{1-f_q}{f_q}$$

s quark: inelastic collision important



Vortical and non-vortical hydro gradient

$$S^{<(1)} = -2\pi\epsilon(P\cdot u)\left[P^\mu f_q^A(P) + \frac{\epsilon^{\mu\nu\rho\sigma}P_\rho u_\sigma \mathcal{D}_\nu f_q}{2P\cdot u}\right]\delta(P^2 - m^2)$$

Vorticity: same as in free theory, no collisional contribution

$$f_q^A = \frac{P\cdot\Omega}{2P\cdot u}\frac{\partial f_q}{\partial(P\cdot u)} \qquad \mathcal{D}_\nu = \partial_\nu - \Sigma_\nu^> - \Sigma_\nu^< \frac{1-f_q}{f_q}$$

Non-vorticity: collisional contribution

$$\mathcal{D}_\nu = \partial_\nu - \Sigma_\nu^> - \Sigma_\nu^< \frac{1-f_q}{f_q} \sim O(\partial\, g^0)$$

$$\delta f \sim \frac{\partial_X}{g^4}$$

not suppressed by coupling

Gauge dependencies

QKT derived in Coulomb gauge, other gauges?

Boltzmann equation (diagonal) gauge independent

$$S_{\rm un}^{<(1)}(X,P) = 2\pi\epsilon(P\cdot u)\delta(P^2-m^2)\left(\not{P}+m\right)\left(-\delta f_q(X,P)\right),$$

Spin polarization (off-diagonal) gauge dependent

$$\mathcal{A}^\mu = -2\pi\epsilon(P\cdot u)\frac{\epsilon^{\mu\nu\rho\sigma}P_\rho u_\sigma \mathcal{D}_\nu f_q}{2(P\cdot u+m)}\delta(P^2-m^2) \qquad \mathcal{D}_\nu = \partial_\nu - \Sigma_\nu^> - \Sigma_\nu^< \frac{1-f_q}{f_q}$$

gauge dependencies expected to cancel
between self-energy and vertex corrections

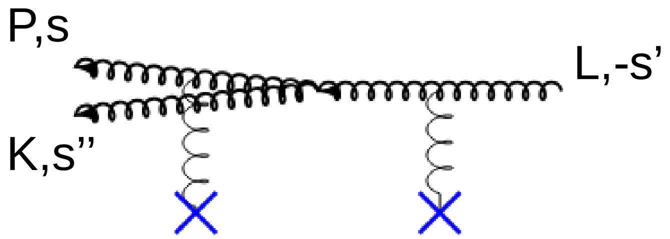
SL, Z.y. Wang, in preparation

Application to orbital-spin conversion

2 ⟷ 2 Non-local collision

Weickgnnant, Speranza, Sheng, Wang, Rischke, PRD 2021
Zhang, Fang, Wang, Wang, PRC 2019

1 ⟷ 2 Local collision



naturally expected
near equilbrium

Summary and outlook

- Derived QKT for QCD. 0^{th} order reproduces Boltzmann equation, 1^{st} order gives spin polarization
- Local collision can realize orbit-spin conversion
- Gauge dependencies in quark polarization, expected to cancel in hadron polarization

Thank you!

K-B equation for gluon

$$\left[-P^2\eta^{\mu\nu}+\frac{i}{2}\left(-2P\cdot\partial\eta^{\mu\nu}+\partial^\nu P^\mu-\frac{1}{\xi}P^{\mu\alpha}P^{\nu\beta}\partial_\beta P_\alpha\right)\right]D^{<(0)}_{\nu\rho}=\mathrm{Re}[\Pi^{T(0)}_R]P^{\mu\nu}_T D^{<(0)}_{\nu\rho}+\Pi^{<(0)}_T P^{\mu\nu}_T \mathrm{Re}[D^{R(0)}_{\nu\rho}]$$

thermal mass/width

$$+\frac{i}{2}\left(\Pi^{\mu\nu>(0)}D^{<(0)}_{\nu\rho}-\Pi^{\mu\nu<(0)}D^{>(0)}_{\nu\rho}\right)$$

collision

$$D^{<(0)}_{\mu\nu,ab}(X,P)=2\pi\epsilon(P\cdot u)\delta(P^2)P^T_{\mu\nu}f^{\mathrm{leq}}_g(X,P)\delta_{ab}$$

0th order: local equilibrium

transverse

spin average $2P\cdot\partial P^{\nu\rho}_T D^{<(0)}_{\nu\rho}=\left(\Pi^{\mu\nu>}D^<_{\nu\mu}-\Pi^{\mu\nu<}D^>_{\nu\mu}\right)^{(1)}$

→ 1st order: unpolarized part

$$D^{<(1)}_{\mu\nu,\mathrm{un}}(X,P)=2\pi\epsilon(p\cdot u)\delta(P^2)P^T_{\mu\nu}\delta f_g(X,P),$$

qqg-vertex in spin basis

quantization axis:
collinear direction z

$$\bar{u}_s(L)\gamma^i u_s(P)\epsilon^i_s = \left(\frac{2l_{\|}}{p_{\|}}\right)^{1/2} \frac{k_{\|}p_s - p_{\|}k_s}{k_{\|}}, \quad \Delta S = -s,\ \Delta L = s$$

$$Y_{1,s}(\hat{p}) \propto p_s$$

$$\bar{u}_s(L)\gamma^i u_s(P)\epsilon^i_{-s} = \left(\frac{2l_{\|}}{p_{\|}}\right)^{1/2} \frac{k_{\|}p_{-s} - p_{\|}k_{-s}}{k_{\|}}, \quad \Delta S = s,\ \Delta L = -s$$

$$\bar{u}_s(L)\gamma^i u_{-s}(P)\epsilon^i_s = \left(\frac{1}{2l_{\|}p_{\|}}\right)^{1/2} 2mk_{\|}s. \quad \Delta S = \Delta L = 0.$$

$$Y_{1,0}(\hat{p}) \propto p_{\|},$$

quark mass induces quark spin flip,
but conserve total spin

orbit-spin conversion
with local collisions

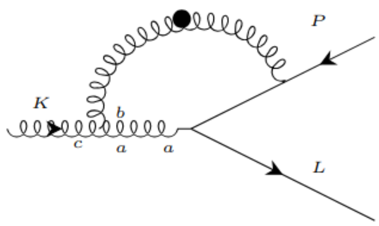
qqg bound state interpretation

kinetic

$$i\delta E F(P,-L,Q) = I_0(P,-L,K) + g^2 \int_Q 2\pi\delta(q_0 - q_{\|}) D^*_{rr,\mu\nu}(Q)\hat{e}^\mu \hat{e}^\nu \Big[$$

$$\left(C_F - \frac{1}{2}C_A\right) F(P+Q,-L-Q,K) + \frac{C_A}{2} F(P+Q,-L,K-Q) + \frac{C_A}{2} F(P,-L-Q,K+Q)\Big].$$

weighted pairwise potential



qqg 2D "bound state"

mass: $(p_{\|}, -l_{\|}, k_{\|})$

momenta: $(\mathbf{p}_\perp, -\mathbf{l}_\perp, \mathbf{k}_\perp)$

Inelastic collision rate ~ spectral density of bound state