



# Wave Propagation as a Probe of the Chiral Anomaly in Weyl Matter

Pavlo O. Sukhachov

Conference on Symmetry Breaking Phenomena in Quantum Field Theory,  
May 17, 2026

# Chapter 0

---



2016 - Ph.D., Theoretical physics, BITP, Kyiv, Ukraine

## Topics:

- **Interaction effects on 3D Dirac and Weyl systems: magnetic catalysis** [P. O. Sukhachov, Ukr. J. Phys. 59, 696 (2014); J. Phys. Stud. 18, 2071 (2014); E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, and P. O. Sukhachov, Phys. Rev. B 94, 115429 (2016)]
- **Quantum oscillations** [E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, and P. O. Sukhachov, Phys. Rev. B 90, 115131 (2014)]
- **Surface states** [E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, and P. O. Sukhachov, Phys. Rev. B 91, 121101(R) (2015); Phys. Rev. B 91, 235138 (2015); Phys. Rev. B 93, 235127 (2016); Phys. Rev. B 93, 235127 (2016)]
- **Chiral separation and chiral magnetic effect** [E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, and P. O. Sukhachov, Phys. Rev. B 92, 245440 (2015)]

# Chapter 0

2016 - Ph.D., Theoretical physics, BITP, Kyiv, Ukraine

## Topics:

- **Interaction effects on 3D Dirac and Weyl systems: magnetic catalysis** [P. O. Sukhachov, Ukr. J. Phys. 59, 696 (2014); J. Phys. Stud. 18, 2071 (2014); E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, and P. O. Sukhachov, Phys. Rev. B 94, 115429 (2016)]
- **Quantum oscillations** [E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, and P. O. Sukhachov, Phys. Rev. B 90, 115131 (2014)]
- **Surface states** [E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, and P. O. Sukhachov, Phys. Rev. B 91, 121101(R) (2015); Phys. Rev. B 91, 235138 (2015); Phys. Rev. B 93, 235127 (2016); Phys. Rev. B 93, 235127 (2016)]
- **Chiral separation and chiral magnetic effect** [E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, and P. O. Sukhachov, Phys. Rev. B 92, 245440 (2015)]



Four “Musketeers”

# Canadian chapter

---



2016 –2019, Postdoctoral Fellow/Associate in the Department of Applied Mathematics of the Western University, London, Canada

# Canadian chapter

---



2016 –2019, Postdoctoral Fellow/Associate in the Department of Applied Mathematics of the Western University, London, Canada

## Topics:

- Chiral kinetic theory and consistent chiral kinetic theory
- Collective modes in Dirac and Weyl plasma
- Axial or pseudo-electromagnetic fields
- Electric and chiral transport in Weyl semimetals
- Hydrodynamics of Weyl quasiparticles: transport and collective modes

# Canadian chapter



2016 –2019, Postdoctoral Fellow/Associate in the Department of Applied Mathematics of the Western University, London, Canada

## Topics:

- Chiral kinetic theory and consistent chiral kinetic theory
- Collective modes in Dirac and Weyl plasma
- Axial or pseudo-electromagnetic fields
- Electric and chiral transport in Weyl semimetals
- Hydrodynamics of Weyl quasiparticles: transport and collective modes

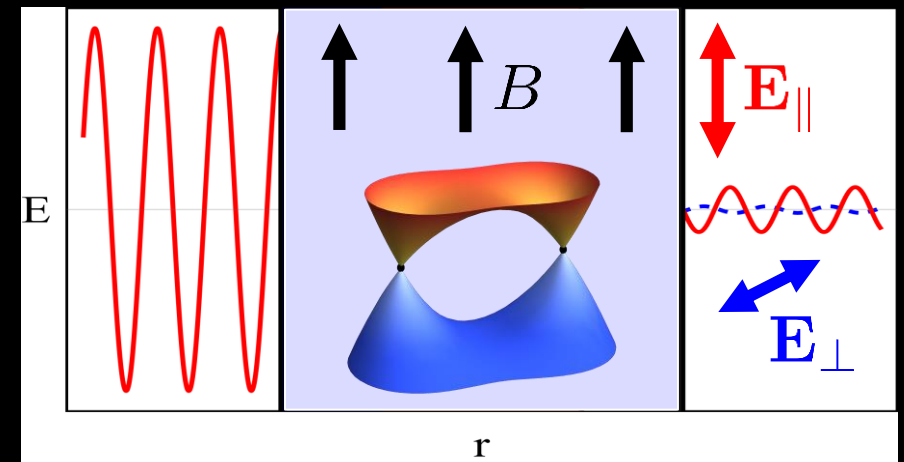
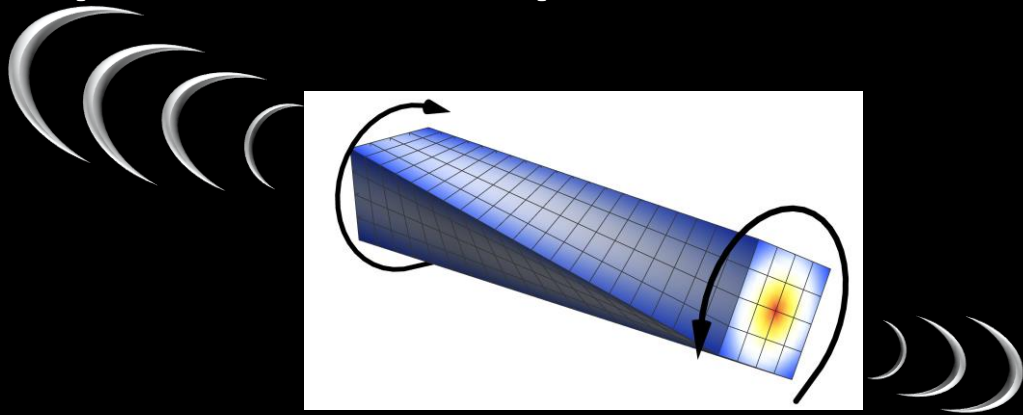


In total, 24 publication, including a book

[E. V. Gorbar, **V. A. Miransky**, I. A. Shovkovy, and P. O. Sukhachov, Electronic properties of Dirac and Weyl semimetals (World Scientific, 2021)]

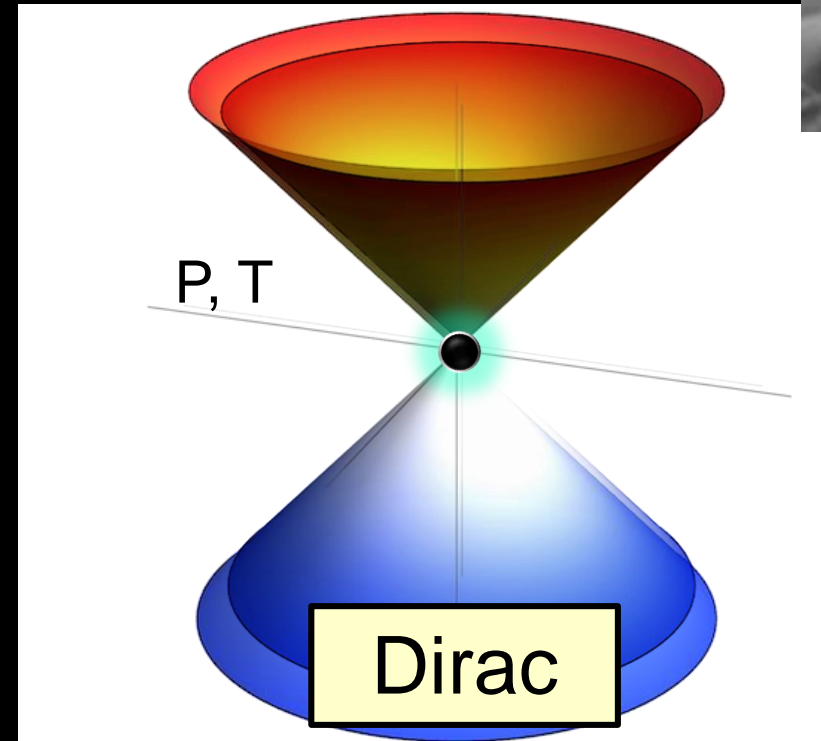
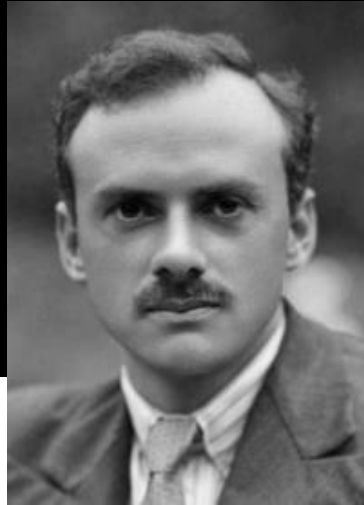
# Outline

1. Quantum anomalies → **reduction of sound attenuation** in Weyl semimetals subject to magnetic fields.
2. Static strains → **sound attenuation dichroism** – nonreciprocal propagation of sound.
3. The chiral anomaly → **nonlocal response** to light.
4. Anomalous nonlocal regime → **enhancement of electromagnetic wave penetration depth.**



# Dirac and Weyl Hamiltonians

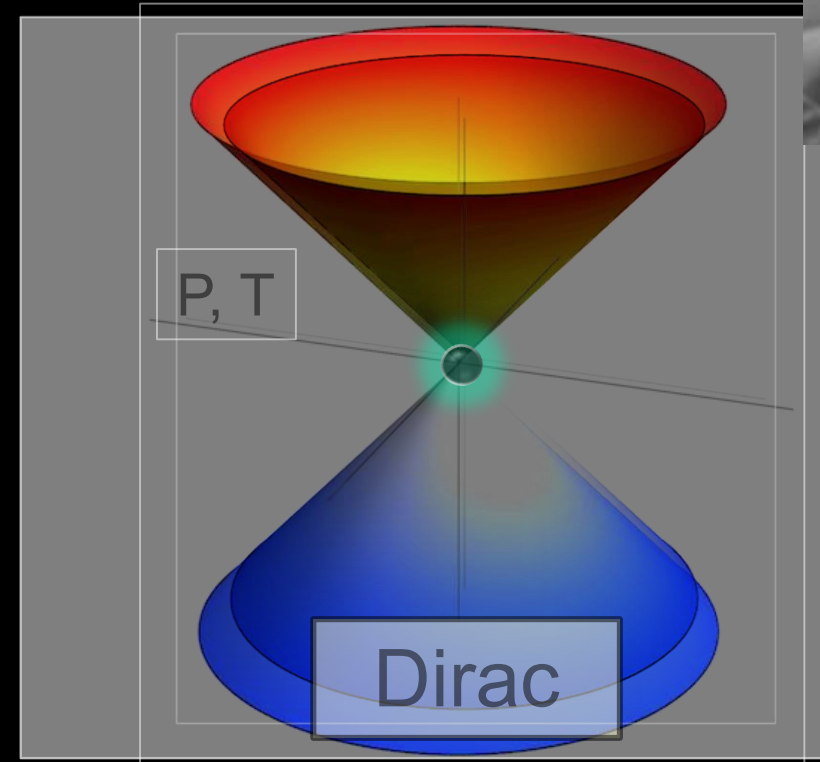
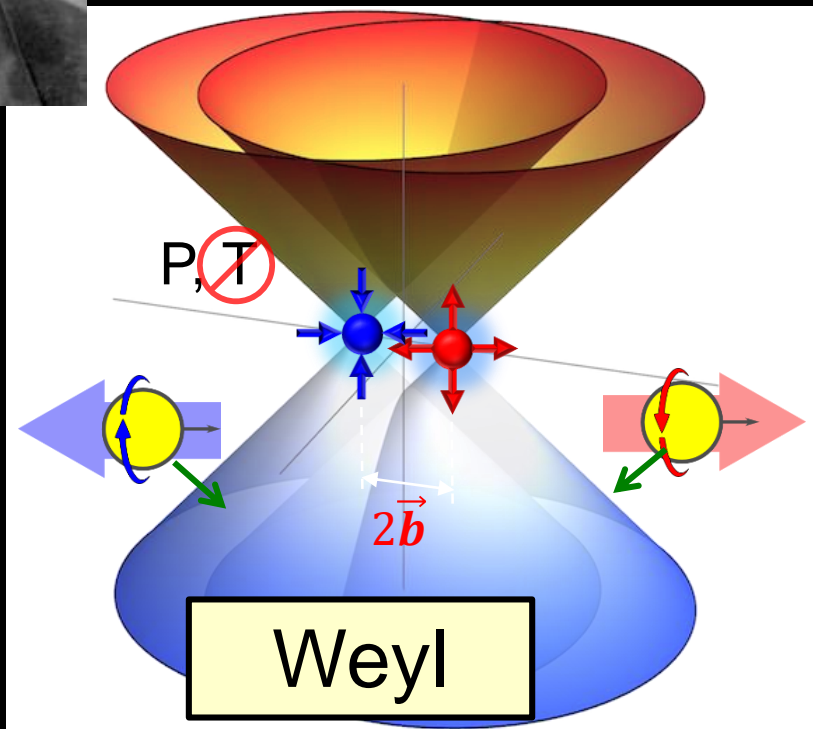
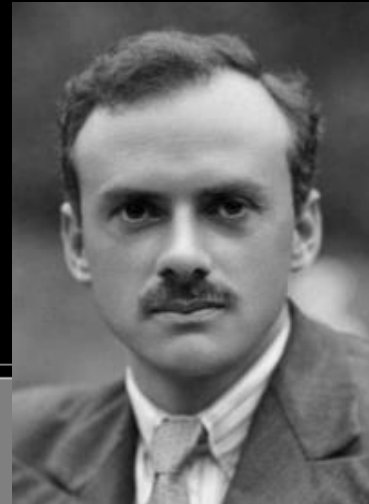
$$H(\mathbf{k}) = \begin{pmatrix} v\boldsymbol{\sigma} \cdot \mathbf{k} & 0 \\ 0 & -v\boldsymbol{\sigma} \cdot \mathbf{k} \end{pmatrix}$$



# Dirac and Weyl Hamiltonians



$$H(\mathbf{k}) = \begin{pmatrix} v\boldsymbol{\sigma} \cdot (\mathbf{k} - \mathbf{b}) & 0 \\ 0 & -v\boldsymbol{\sigma} \cdot (\mathbf{k} + \mathbf{b}) \end{pmatrix}$$

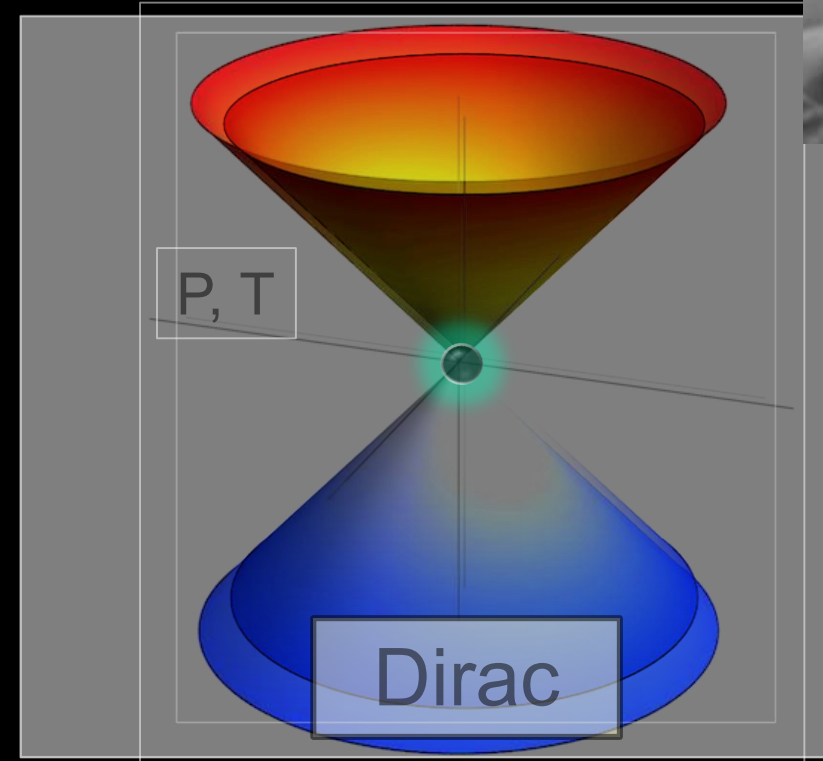
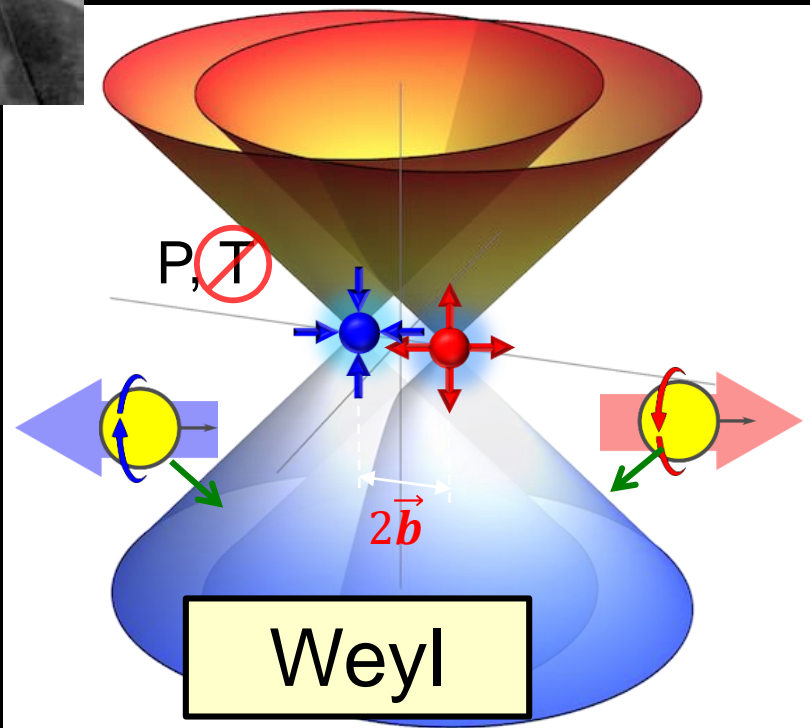


# Dirac and Weyl Hamiltonians



$$H(\mathbf{k}) = \begin{pmatrix} v\boldsymbol{\sigma} \cdot (\mathbf{k} - \mathbf{b}) & 0 \\ 0 & -v\boldsymbol{\sigma} \cdot (\mathbf{k} + \mathbf{b}) \end{pmatrix}$$

Chiral shift parameter



# Electrons in solids: Primer

---

❖ Hamiltonian:  $\hat{H} = \frac{\hat{\mathbf{p}}^2}{2m} + V(\mathbf{r}) \quad V(\mathbf{r}) = V(\mathbf{r} + \mathbf{a}_i)$

# Electrons in solids: Primer

---

❖ Hamiltonian:  $\hat{H} = \frac{\hat{\mathbf{p}}^2}{2m} + V(\mathbf{r})$      $V(\mathbf{r}) = V(\mathbf{r} + \mathbf{a}_i)$

❖ Bloch theorem:  $\psi_{\mathbf{k},n}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k},n}(\mathbf{r})$

Determines quantum geometry (topology)

# Electrons in solids: Primer

---

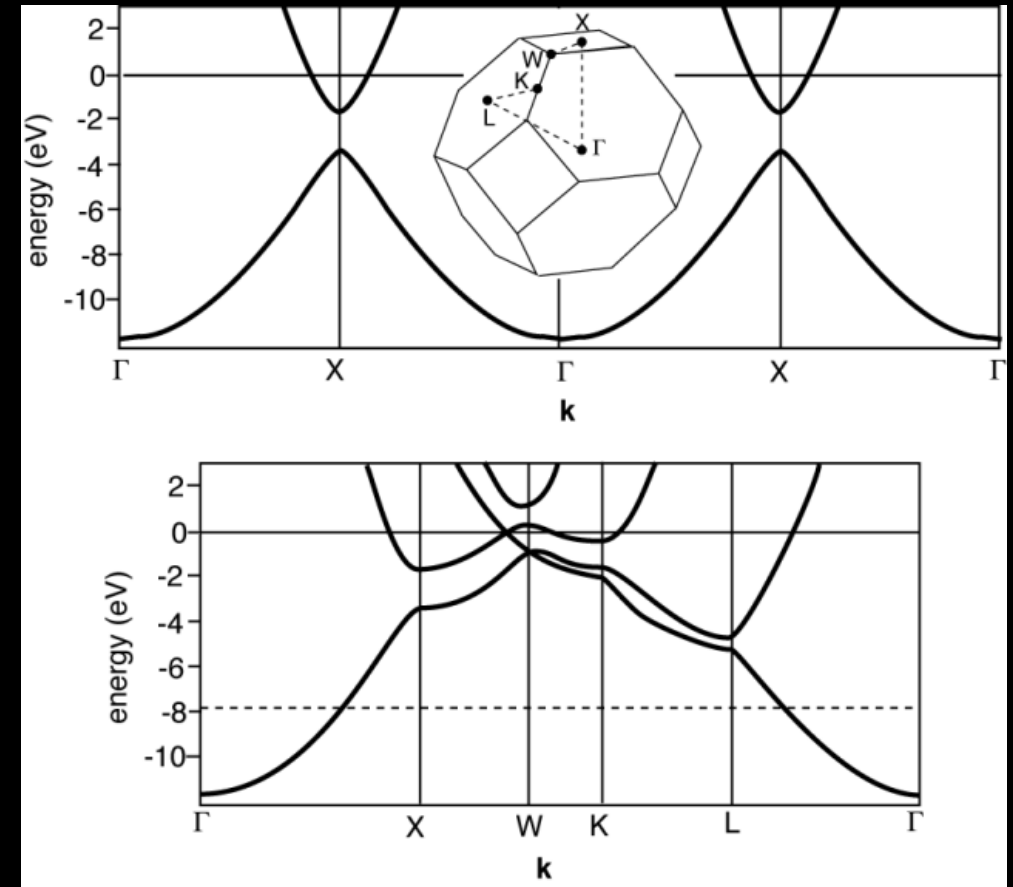
❖ Hamiltonian:  $\hat{H} = \frac{\hat{\mathbf{p}}^2}{2m} + V(\mathbf{r}) \quad V(\mathbf{r}) = V(\mathbf{r} + \mathbf{a}_i)$

❖ Bloch theorem:  $\psi_{\mathbf{k},n}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k},n}(\mathbf{r})$

❖ Band structure:  $\epsilon_n(\mathbf{k})$

# Electrons in solids: Primer

- ❖ Hamiltonian:  $\hat{H} = \frac{\hat{\mathbf{p}}^2}{2m} + V(\mathbf{r}) \quad V(\mathbf{r}) = V(\mathbf{r} + \mathbf{a}_i)$
- ❖ Bloch theorem:  $\psi_{\mathbf{k},n}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k},n}(\mathbf{r})$
- ❖ Band structure:  $\epsilon_n(\mathbf{k})$



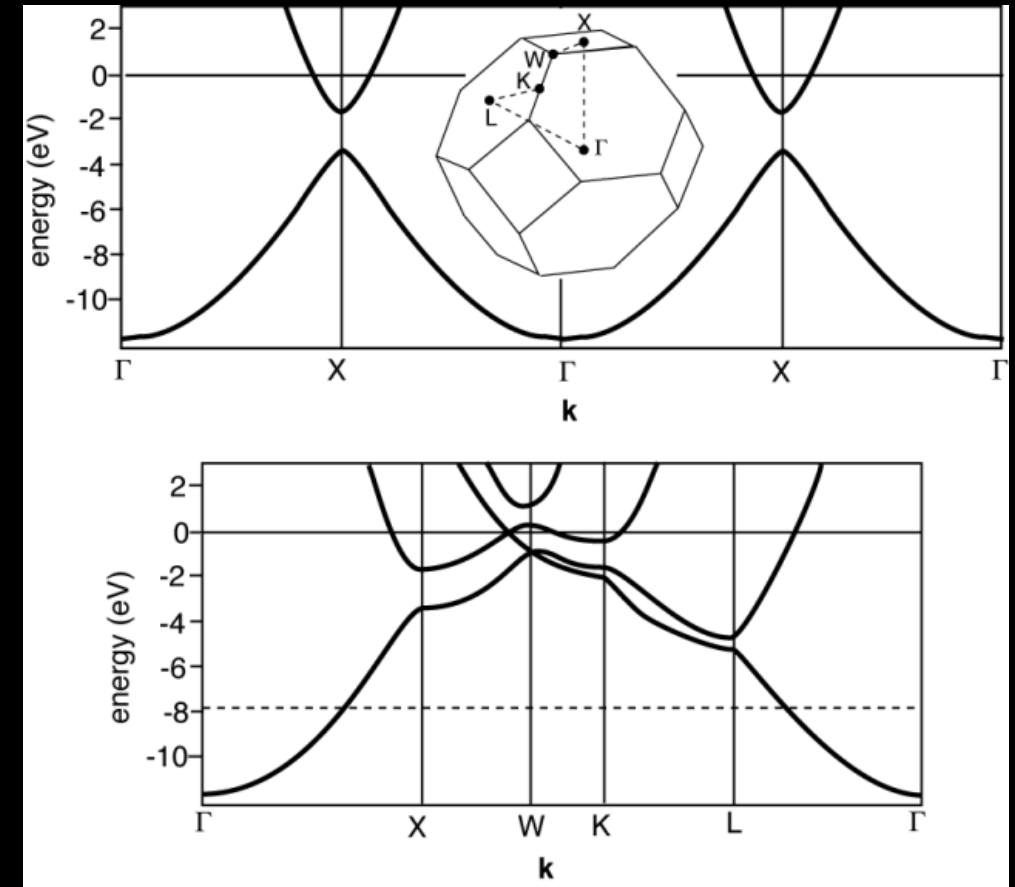
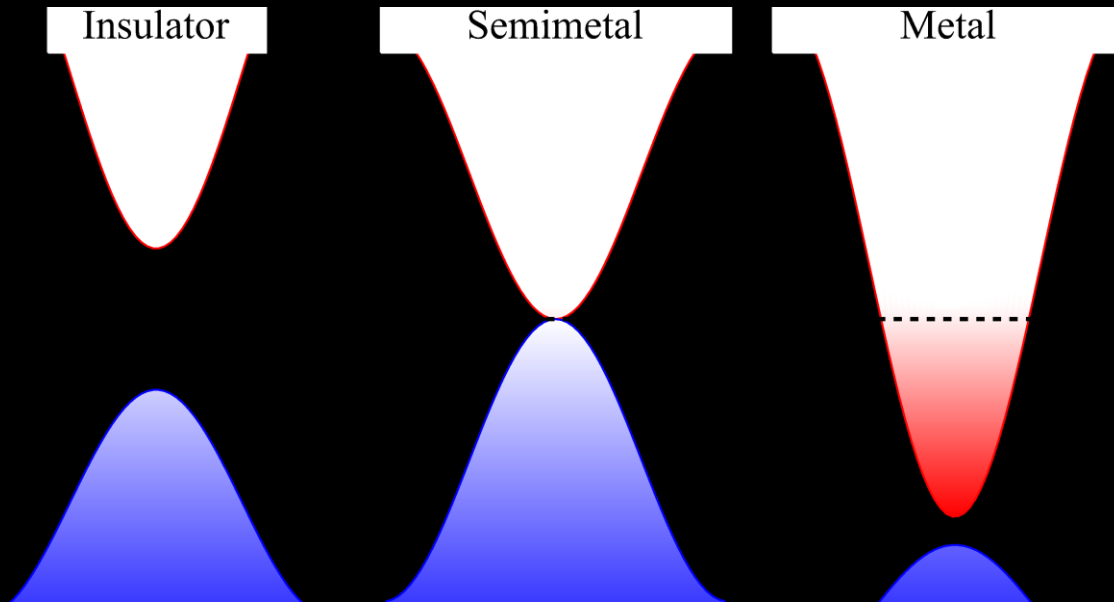
Band structure of Al

[B. Segall, Phys. Rev. 124, 1797 (1961)]

# Electrons in solids: Primer

- ❖ Hamiltonian:  $\hat{H} = \frac{\hat{\mathbf{p}}^2}{2m} + V(\mathbf{r}) \quad V(\mathbf{r}) = V(\mathbf{r} + \mathbf{a}_i)$
- ❖ Bloch theorem:  $\psi_{\mathbf{k},n}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k},n}(\mathbf{r})$
- ❖ Band structure:  $\epsilon_n(\mathbf{k})$

Quasi-momentum defined in the Brillouin zone



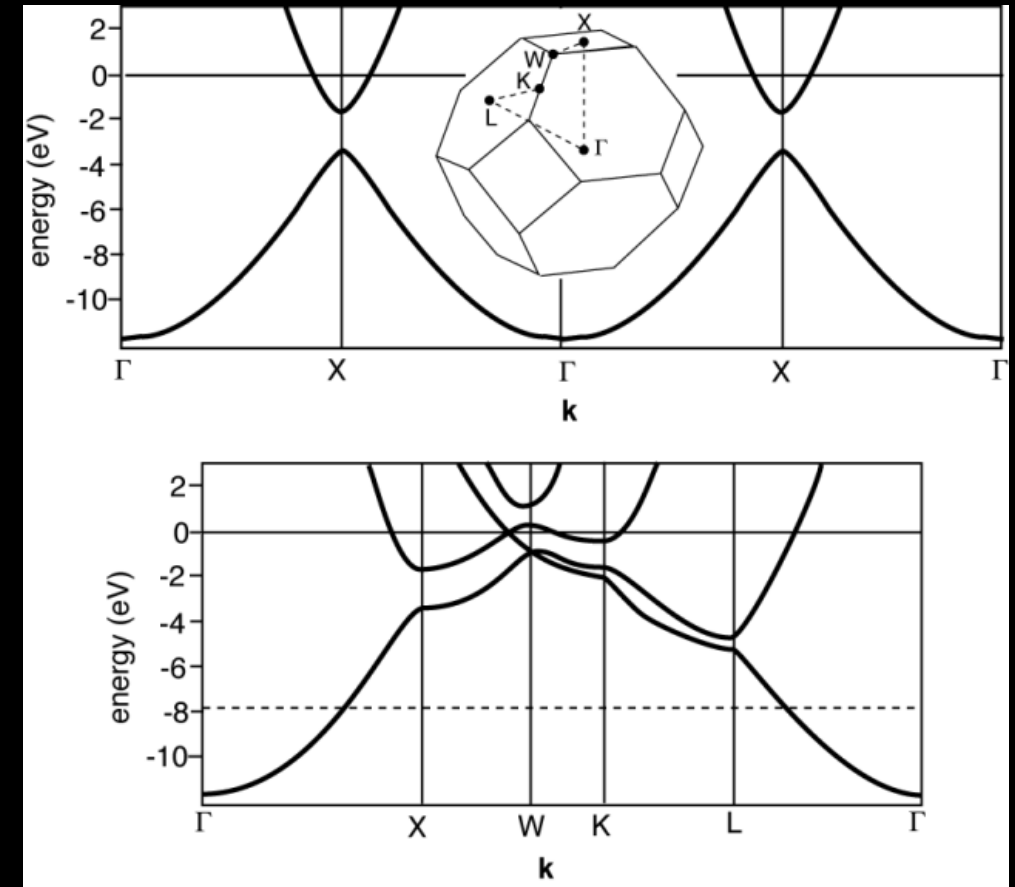
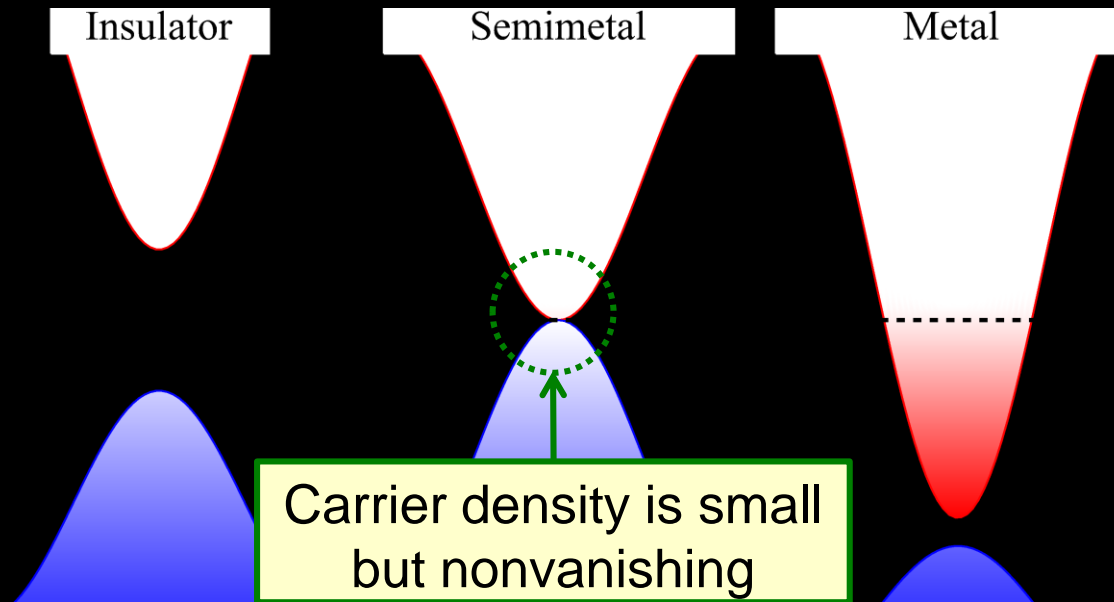
Band structure of Al

[B. Segall, Phys. Rev. 124, 1797 (1961)]

# Electrons in solids: Primer

- ❖ Hamiltonian:  $\hat{H} = \frac{\hat{\mathbf{p}}^2}{2m} + V(\mathbf{r}) \quad V(\mathbf{r}) = V(\mathbf{r} + \mathbf{a}_i)$
- ❖ Bloch theorem:  $\psi_{\mathbf{k},n}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k},n}(\mathbf{r})$
- ❖ Band structure:  $\epsilon_n(\mathbf{k})$

Quasi-momentum defined in the Brillouin zone



Band structure of Al

[B. Segall, Phys. Rev. 124, 1797 (1961)]

# Linear band crossings: Weyl semimetals

---

Hamiltonian (two bands  $\rightarrow$  2x2):

$$H(\mathbf{k}) = \epsilon_0(\mathbf{k}) + \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

# Linear band crossings: Weyl semimetals

---

Hamiltonian (two bands  $\rightarrow$  2x2):

$$H(\mathbf{k}) = \epsilon_0(\mathbf{k}) + \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

At the band-  
crossing point



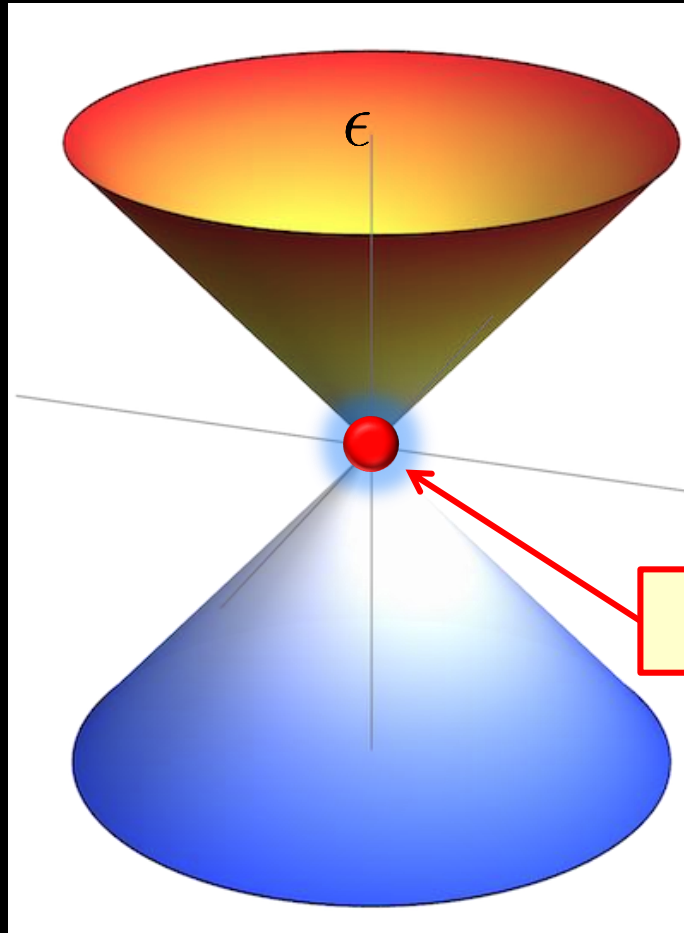
Linearization + shift

$$H_W(\mathbf{k}) = \pm v \boldsymbol{\sigma} \cdot \delta \mathbf{k}$$

# Linear band crossings: Weyl semimetals

Hamiltonian (two bands  $\rightarrow$  2x2):

$$H(\mathbf{k}) = \epsilon_0(\mathbf{k}) + \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$



At the band-  
crossing point



Linearization + shift

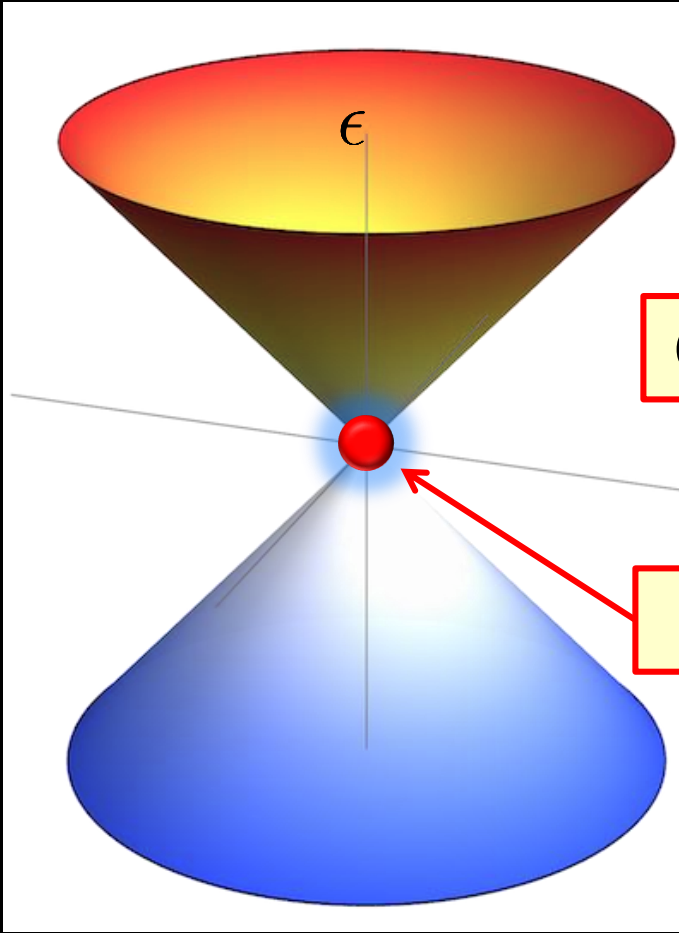
$$H_W(\mathbf{k}) = \pm v \boldsymbol{\sigma} \cdot \delta \mathbf{k}$$

Weyl node

# Linear band crossings: Weyl semimetals

Hamiltonian (two bands  $\rightarrow$  2x2):

$$H(\mathbf{k}) = \epsilon_0(\mathbf{k}) + \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$



At the band-crossing point



Linearization + shift

$$H_W(\mathbf{k}) \Rightarrow \pm v \boldsymbol{\sigma} \cdot \delta \mathbf{k}$$

Chirality

Weyl Hamiltonian

[H. Weyl, Z. Phys. 56, 330 (1929)]

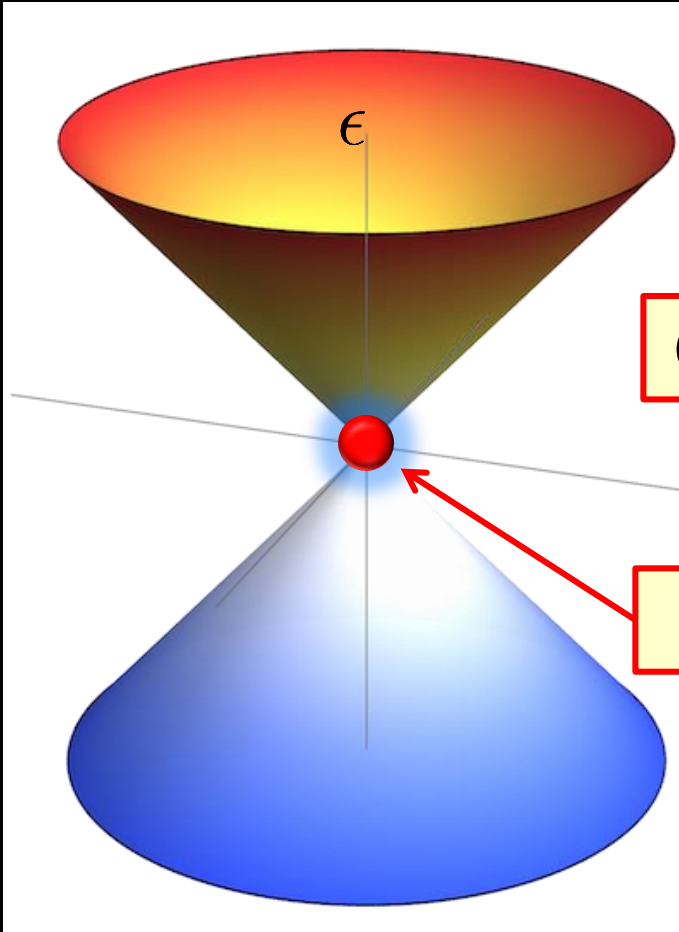
Weyl node



# Linear band crossings: Weyl semimetals

Hamiltonian (two bands  $\rightarrow$  2x2):

$$H(\mathbf{k}) = \epsilon_0(\mathbf{k}) + \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$



At the band-  
crossing point



Linearization + shift

$$H_W(\mathbf{k}) \Rightarrow \pm v \boldsymbol{\sigma} \cdot \delta \mathbf{k}$$

Chirality

Weyl Hamiltonian

[H. Weyl, Z. Phys. 56, 330 (1929)]

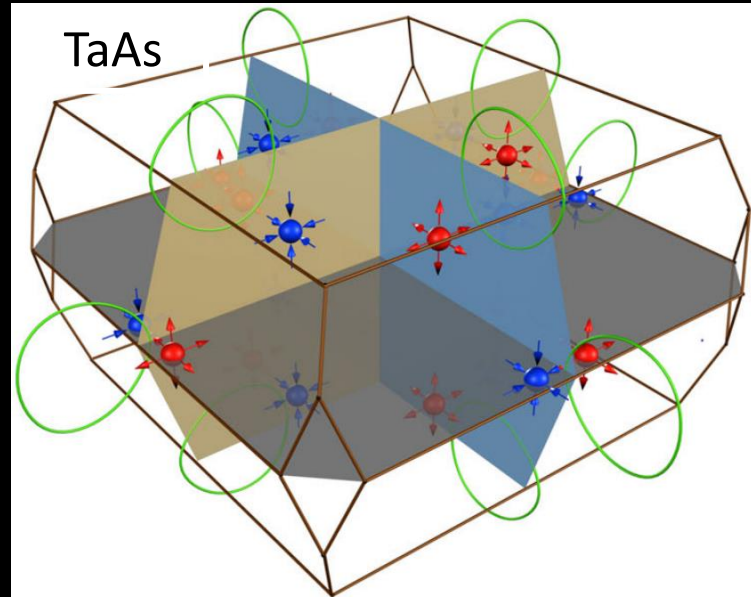
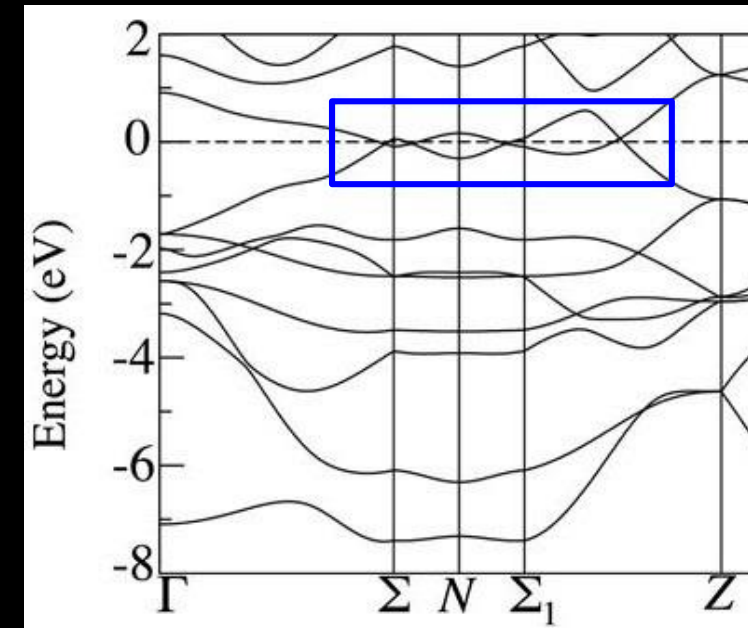
Weyl node

First mention of linear  
energy spectrum in solids:

[C. Herring, Phys. Rev. 52, 365 (1937)]



# 3D Weyl materials: ab initio results and experiment



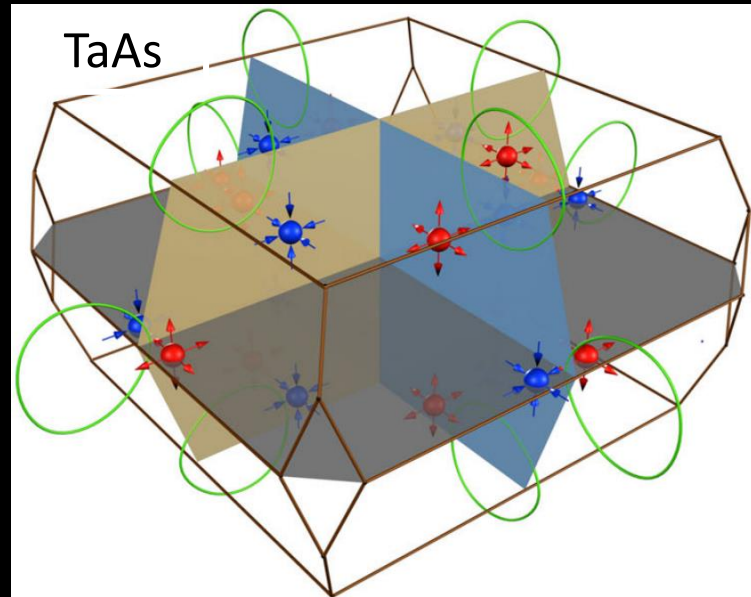
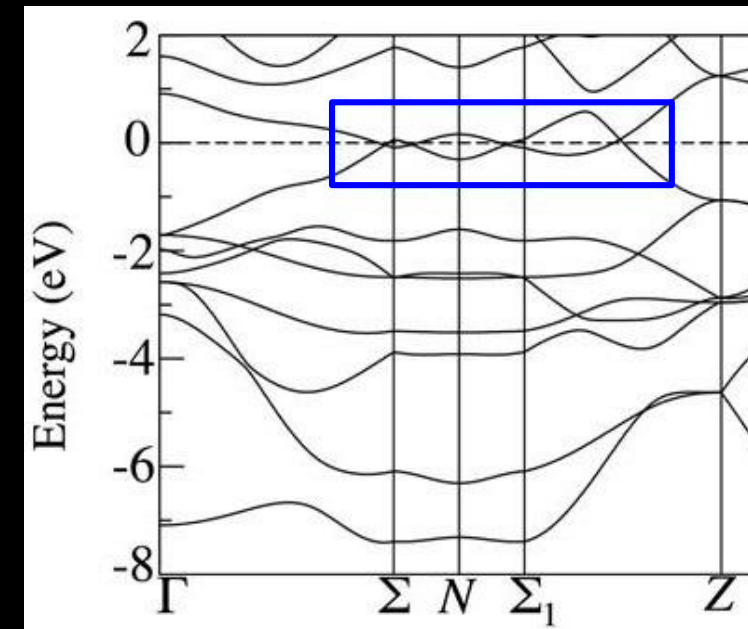
[H. Weng, et al., Phys. Rev. X **5**, 011029 (2015)]

- TaAs, TaP, NbAs, NbP

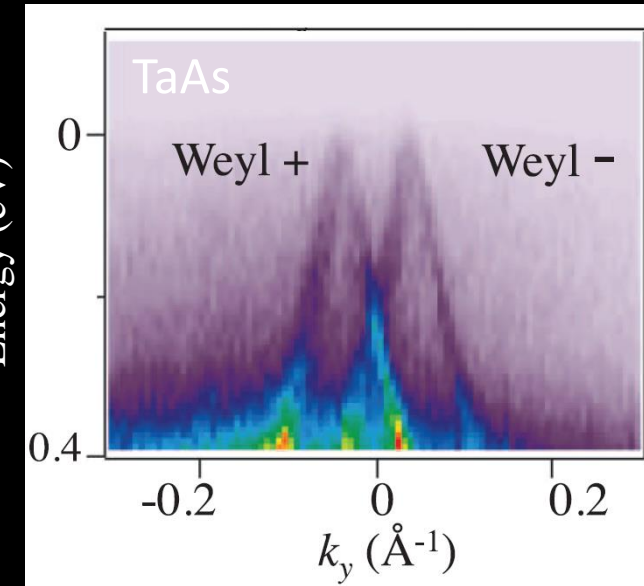
[S.-Y. Xu, et al., Nat. Phys. **11**, 748 (2015); S.-Y. Xu, et al., Science **349**, 613 (2015); B. Q. Lv, et al., Phys. Rev. X **5**, 031013 (2015); B. Q. Lv, et al., Nat. Phys. **11**, 724 (2015)]

- $\text{Co}_3\text{Sn}_2\text{S}_2$ ,  $\text{Co}_3\text{Sn}_2\text{S}_2$  [Q. Xu, et al., Phys. Rev. B **97**, 235416 (2018); E. Liu, et al., Nat. Phys. **14**, 1125 (2018)]
- $\text{MoTe}_2$ ,  $\text{WTe}_2$  [J. Jiang, et al., Nat. Commun. **8**, 13973 (2017); Y. Wu, et al., Phys. Rev. B **94**, 121113(R) (2016)]
- CrSb [C. Li, M. Hu, Z. Li, Communications Physics, **8**, 311 (2025)]

# 3D Weyl materials: ab initio results and experiment



[H. Weng, et al., Phys. Rev. X **5**, 011029 (2015)]

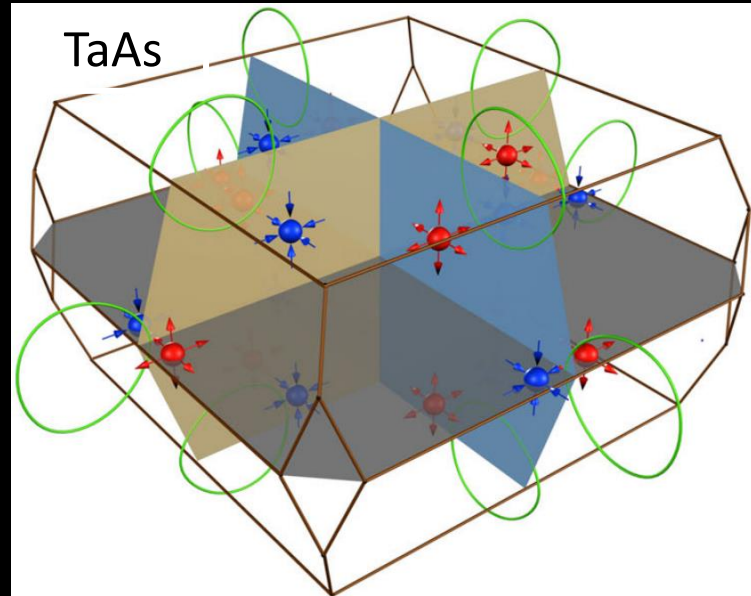
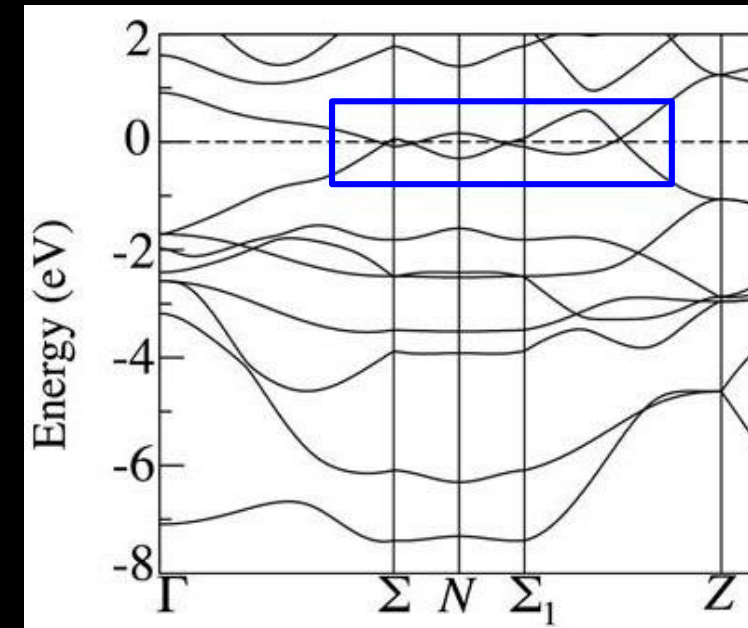


- TaAs, TaP, NbAs, NbP

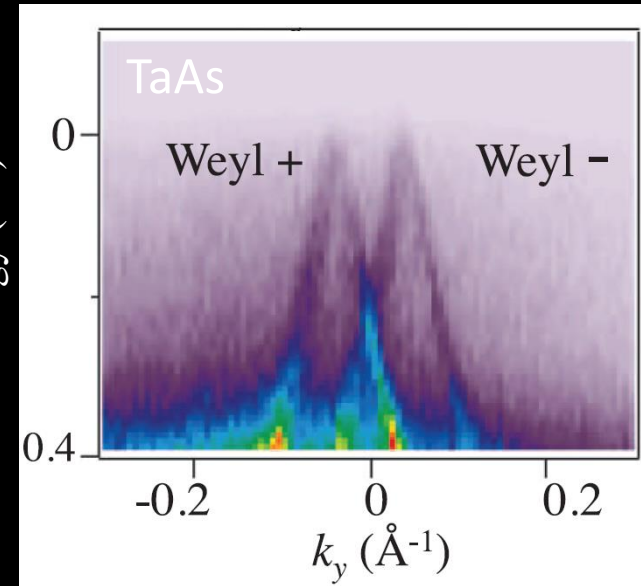
[S.-Y. Xu, et al., Nat. Phys. **11**, 748 (2015); S.-Y. Xu, et al., Science **349**, 613 (2015); B. Q. Lv, et al., Phys. Rev. X **5**, 031013 (2015); B. Q. Lv, et al., Nat. Phys. **11**, 724 (2015)]

- $\text{Co}_3\text{Sn}_2\text{S}_2$ ,  $\text{Co}_3\text{Sn}_2\text{S}_2$  [Q. Xu, et al., Phys. Rev. B **97**, 235416 (2018); E. Liu, et al., Nat. Phys. **14**, 1125 (2018)]
- $\text{MoTe}_2$ ,  $\text{WTe}_2$  [J. Jiang, et al., Nat. Commun. **8**, 13973 (2017); Y. Wu, et al., Phys. Rev. B **94**, 121113(R) (2016)]
- CrSb [C. Li, M. Hu, Z. Li, Communications Physics, **8**, 311 (2025)]

# 3D Weyl materials: ab initio results and experiment



[H. Weng, et al., Phys. Rev. X **5**, 011029 (2015)]



- TaAs, TaP, NbAs, NbP

[S.-Y. Xu, et al., Nat. Phys. **11**, 748 (2015); S.-Y. Xu, et al., Science **349**, 613 (2015); B. Q. Lv, et al., Phys. Rev. X **5**, 031013 (2015); B. Q. Lv, et al., Nat. Phys. **11**, 724 (2015)]

- $\text{Co}_3\text{Sn}_2\text{S}_2$ ,  $\text{Co}_3\text{Sn}_2\text{S}_2$  [Q. Xu, et al., Phys. Rev. B **97**, 235416 (2018); E. Liu, et al., Nat. Phys. **14**, 1125 (2018)]
- $\text{MoTe}_2$ ,  $\text{WTe}_2$  [J. Jiang, et al., Nat. Commun. **8**, 13973 (2017); Y. Wu, et al., Phys. Rev. B **94**, 121113(R) (2016)]
- CrSb [C. Li, M. Hu, Z. Li, Communications Physics, **8**, 311 (2025)]

# Topology of electron states: Berry curvature

❖ Adiabatic evolution [M.V. Berry, Proc. R. Soc. A **392**, 45 (1984)]:

$$H = H(\mathbf{k}(t))$$

$$H(\mathbf{k}) |u_n(\mathbf{k})\rangle = \epsilon_n(\mathbf{k}) |u_n(\mathbf{k})\rangle$$

❖ Closed trajectory in the parameter space → rate-independent phase factor:

$$e^{-i\gamma} |u_n(\mathbf{k})\rangle$$

❖ The Berry phase and the Berry connection:

$$\gamma = \oint d\mathbf{k} \cdot \mathcal{A}(\mathbf{k}), \quad \mathcal{A}(\mathbf{k}) = i \langle u_n(\mathbf{k}) | \partial_{\mathbf{k}} u_n(\mathbf{k}) \rangle$$

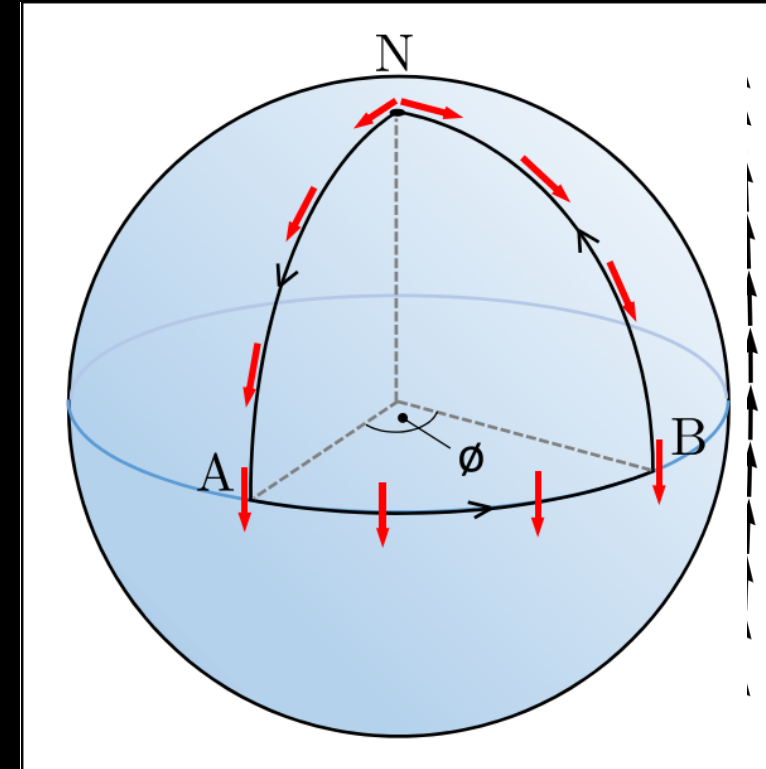
❖ The Berry curvature:

$$\Omega = \partial_{\mathbf{k}} \times \mathcal{A}(\mathbf{k})$$

$\mathcal{A}(\mathbf{r})$  Vector potential

$$\mathbf{B} = \nabla_{\mathbf{r}} \times \mathcal{A}(\mathbf{r})$$

Magnetic field



[A.T. Boothroyd, Contemp. Phys. **63**, 305 (2022)]

# Topology of electron states: Berry curvature

rate-independent phase factor:

Adiabatic evolution  $H(\mathbf{k}, t)$  Berry, Proc. R. Soc. A **392**, 45 (1984)  $i\hbar \frac{d}{dt} |u_n(\mathbf{k})\rangle = \epsilon_n(\mathbf{k}) |u_n(\mathbf{k})\rangle$

❖ Closed trajectory in the parameter space  $\rightarrow$  rate-independent phase factor:

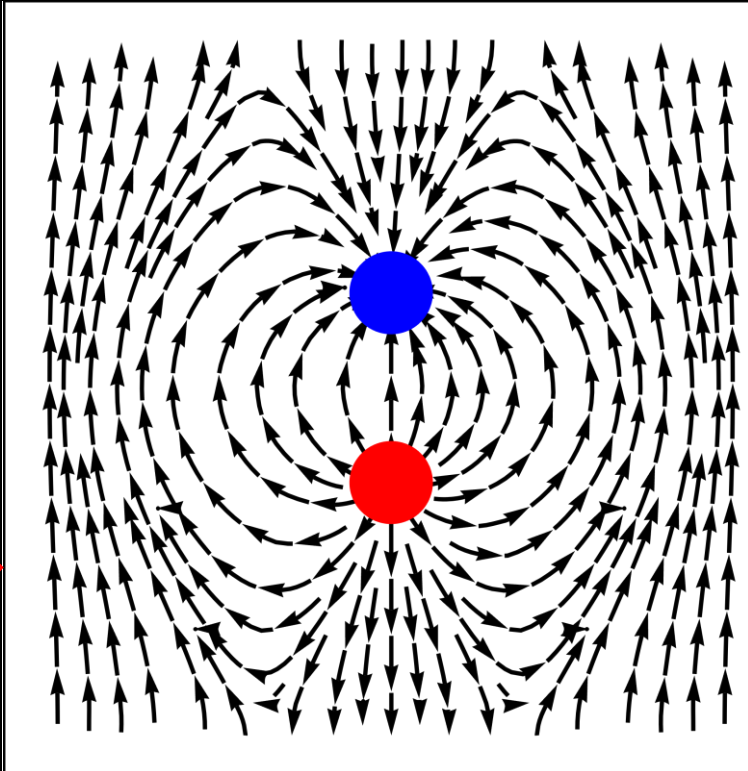
❖ The Berry phase and the Berry connection:

❖ The Berry phase and the Berry connection:  $\gamma = \oint d\mathbf{k} \cdot \mathcal{A}(\mathbf{k})$ ,  $\mathcal{A}(\mathbf{k}) = i \langle u_n(\mathbf{k}) | \partial_{\mathbf{k}} u_n(\mathbf{k}) \rangle$

❖ The Berry curvature:

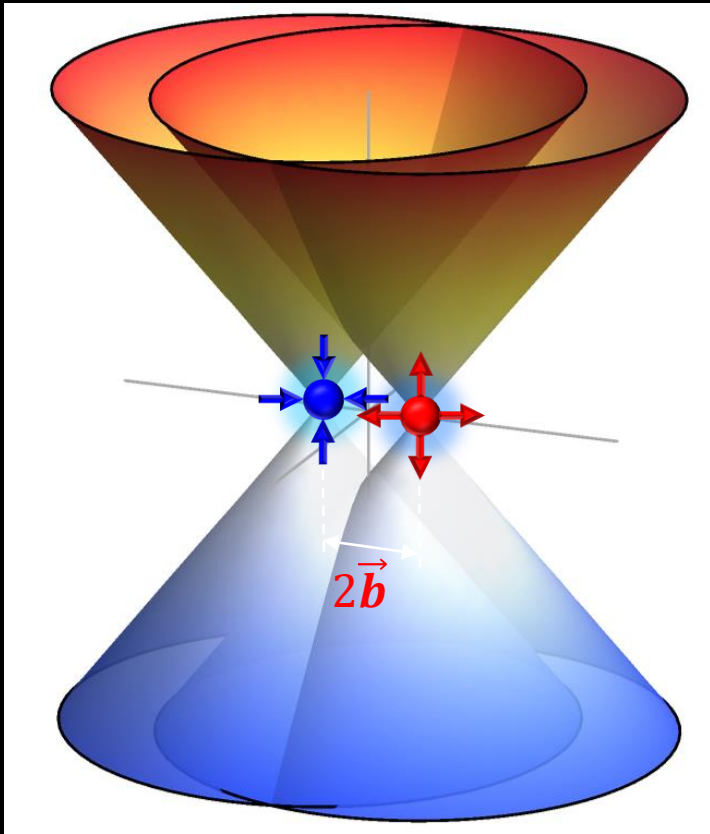
❖ The Berry curvature:  $\Omega = \partial_{\mathbf{k}} \times \mathcal{A}(\mathbf{k})$

Field lines of the Berry curvature for a Weyl semimetal  $\rightarrow$  dipole structure



# Strain-induced axial gauge fields

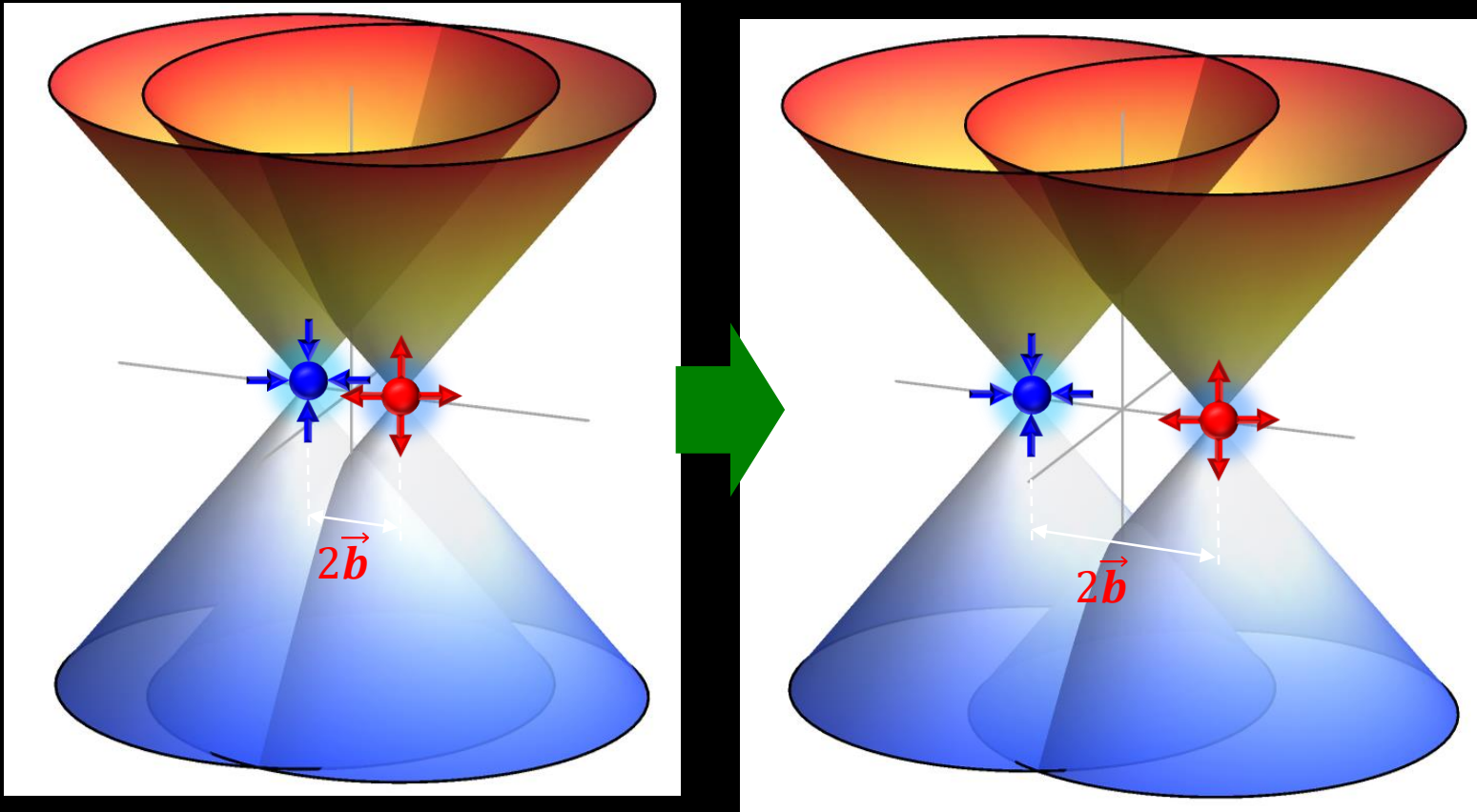
$$H_0(\mathbf{k}) = \begin{pmatrix} v_F \boldsymbol{\sigma} \cdot (\mathbf{k} - \mathbf{b}) & 0 \\ 0 & -v_F \boldsymbol{\sigma} \cdot (\mathbf{k} + \mathbf{b}) \end{pmatrix}$$



# Strain-induced axial gauge fields

$$H_0(\mathbf{k}) = \begin{pmatrix} v_F \boldsymbol{\sigma} \cdot (\mathbf{k} - \mathbf{b}) & 0 \\ 0 & -v_F \boldsymbol{\sigma} \cdot (\mathbf{k} + \mathbf{b}) \end{pmatrix}$$

$\mathbf{b} \rightarrow \mathbf{b}(\mathbf{r})$



# Strain-induced axial gauge fields

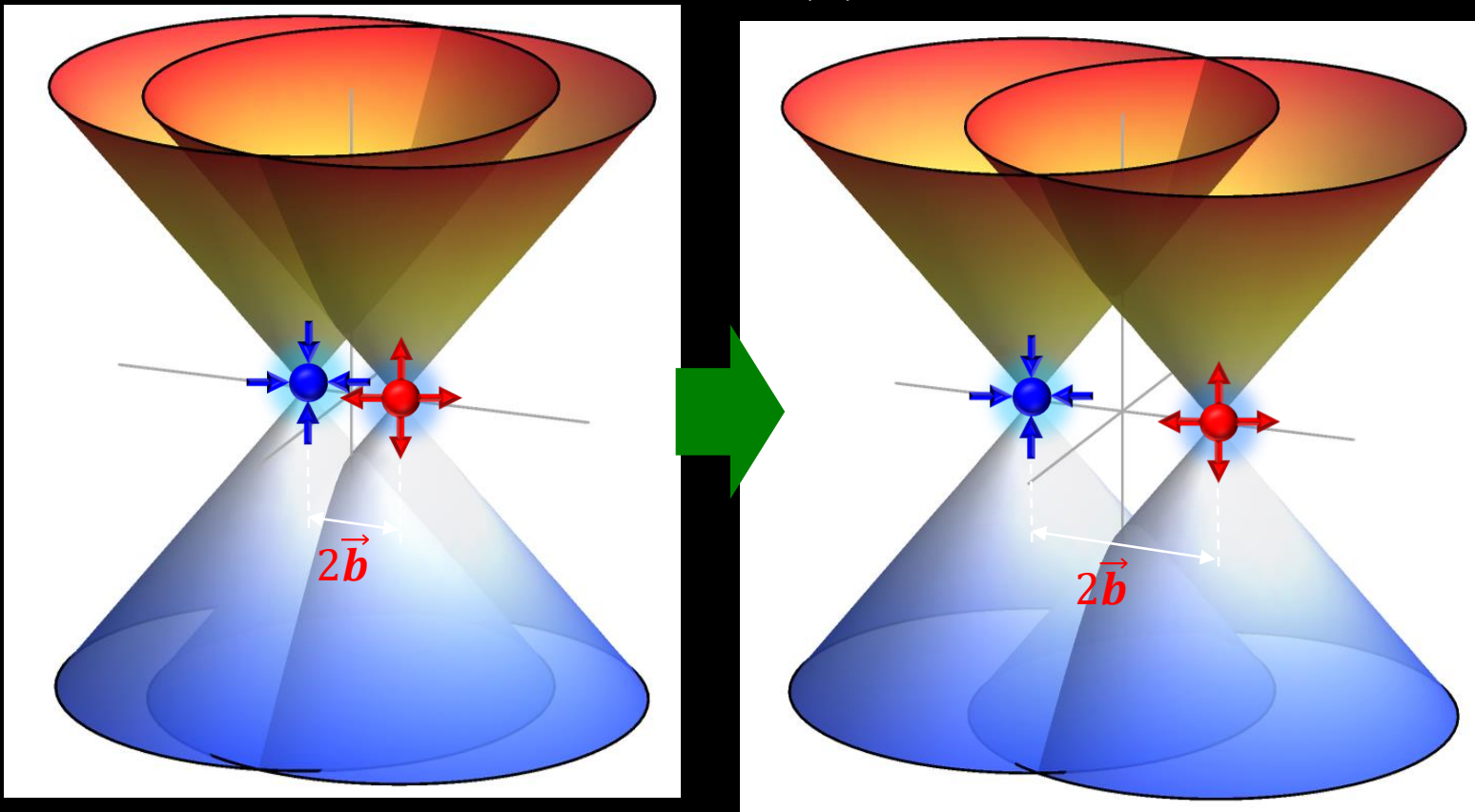
$$H_0(\mathbf{k}) = \begin{pmatrix} v_F \boldsymbol{\sigma} \cdot (\mathbf{k} - \mathbf{b}) & 0 \\ 0 & -v_F \boldsymbol{\sigma} \cdot (\mathbf{k} + \mathbf{b}) \end{pmatrix}$$

$\mathbf{b} \rightarrow \mathbf{b}(\mathbf{r})$

[A. Cortijo et al., PRL 115, 177202 (2015); R. Ilan, A.G. Grushin, and D.I. Pikulin, Nat. Rev. Phys. 2, 29 (2020)]

Pseudomagnetic (axial) field

$$\pm \mathbf{b}(\mathbf{r}) \equiv \pm e \mathbf{A}_5(\mathbf{r}) \leftrightarrow e \mathbf{A}(\mathbf{r})$$



# Strain-induced axial gauge fields

$$H_0(\mathbf{k}) = \begin{pmatrix} v_F \boldsymbol{\sigma} \cdot (\mathbf{k} - \mathbf{b}) & 0 \\ 0 & -v_F \boldsymbol{\sigma} \cdot (\mathbf{k} + \mathbf{b}) \end{pmatrix}$$

$\mathbf{b} \rightarrow \mathbf{b}(\mathbf{r})$

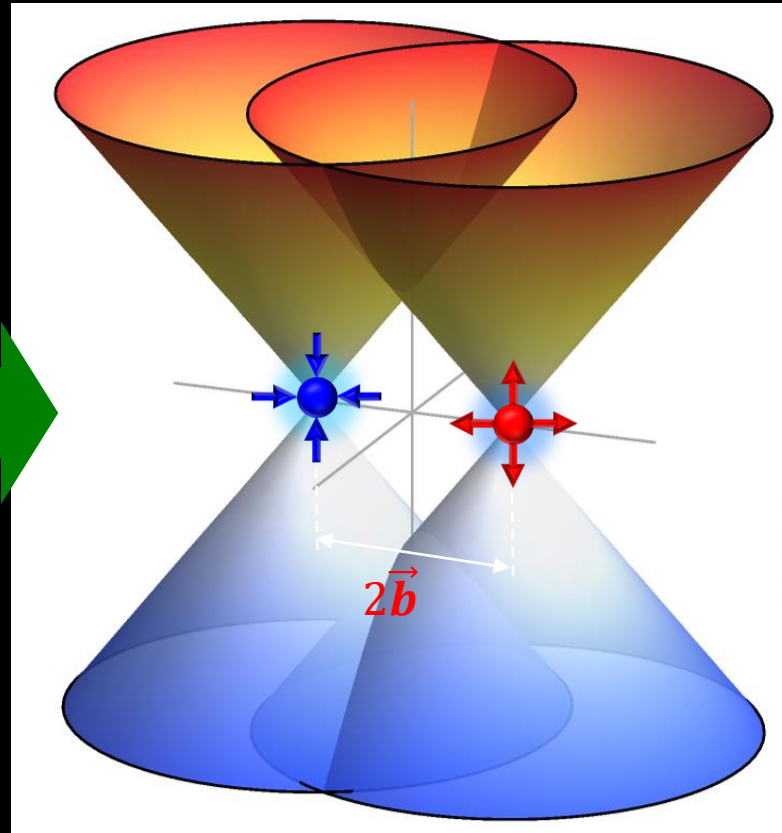
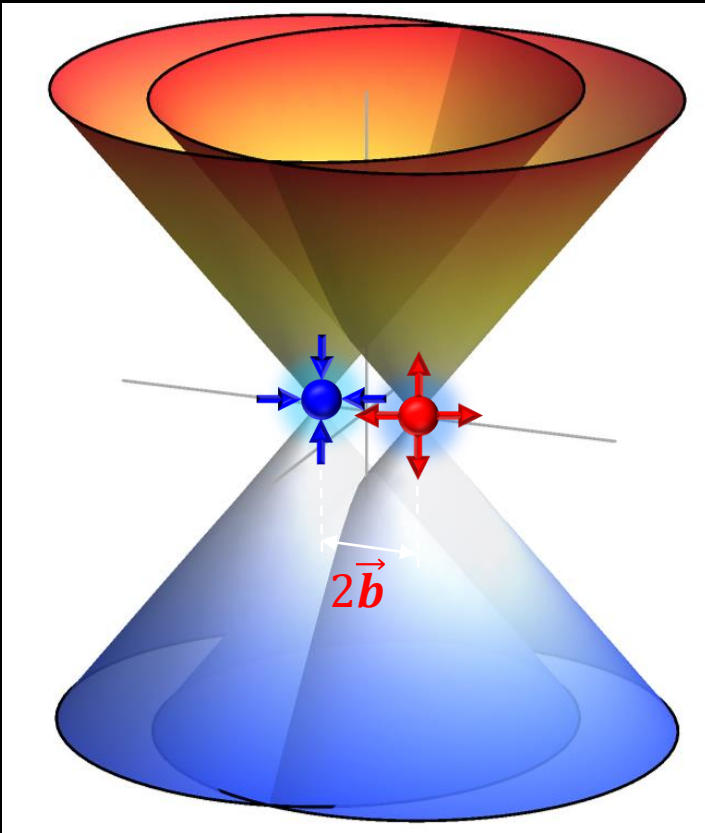
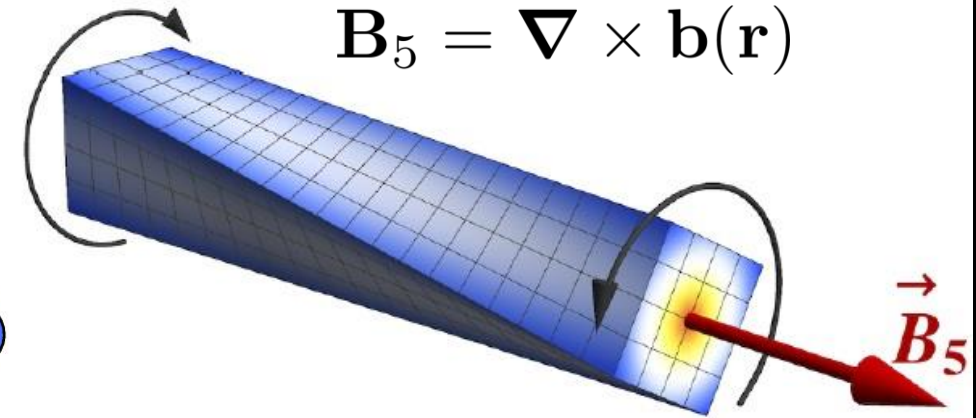
[A. Cortijo et al., PRL 115, 177202 (2015); R. Ilan, A.G. Grushin, and D.I. Pikulin, Nat. Rev. Phys. 2, 29 (2020)]

Pseudomagnetic (axial) field

$$\pm \mathbf{b}(\mathbf{r}) \equiv \pm e \mathbf{A}_5(\mathbf{r}) \leftrightarrow e \mathbf{A}(\mathbf{r})$$



$$\mathbf{B}_5 = \nabla \times \mathbf{b}(\mathbf{r})$$



# Quantum (chiral) anomaly

---

**Anomaly** = breakdown of classical symmetry by quantum effects

# Quantum (chiral) anomaly

---

**Anomaly** = breakdown of classical symmetry by quantum effects

**Classical** chiral systems: numbers of left- and right-handed particles are **separately** conserved

# Quantum (chiral) anomaly

---

**Anomaly** = breakdown of classical symmetry by quantum effects

**Classical** chiral systems: numbers of left- and right-handed particles are **separately** conserved



[P. Danaisawadi, et al. Sci. Rep. 6, 23832 (2016)]

# Quantum (chiral) anomaly

**Anomaly** = breakdown of classical symmetry by quantum effects

**Classical** chiral systems: numbers of left- and right-handed particles are **separately** conserved



Currents of left- and right-handed particles are **separately** conserved



[P. Danaisawadi, et al. Sci. Rep. 6, 23832 (2016)]

# Quantum (chiral) anomaly

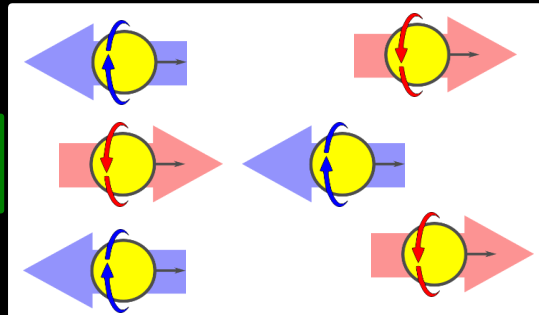
**Anomaly** = breakdown of classical symmetry by quantum effects

**Classical** chiral systems: numbers of left- and right-handed particles are **separately** conserved



[P. Danaisawadi, et al. Sci. Rep. 6, 23832 (2016)]

**Quantum systems:**



# Quantum (chiral) anomaly

**Anomaly** = breakdown of classical symmetry by quantum effects

Classical chiral systems: numbers of left- and right-handed particles are **separately** conserved

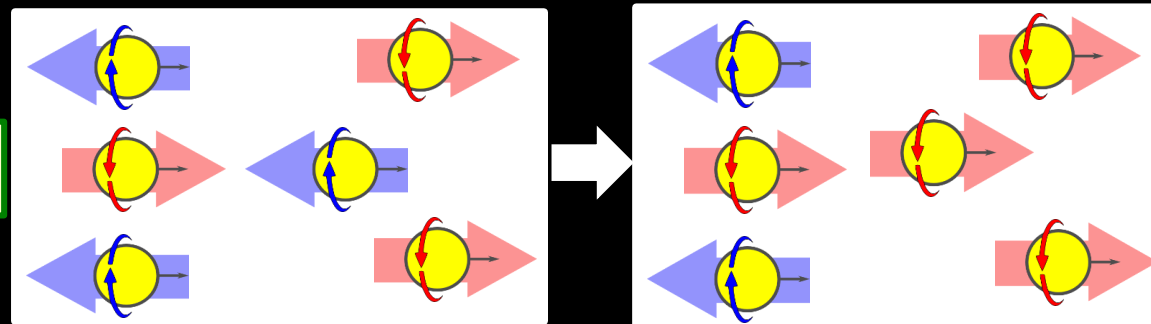


[P. Danaisawadi, et al. Sci. Rep. 6, 23832 (2016)]

**Chiral Anomaly**

**E, B**

**Quantum systems:**



# Quantum (chiral) anomaly

**Anomaly** = breakdown of classical symmetry by quantum effects

Classical chiral systems: numbers of left- and right-handed particles are **separately** conserved

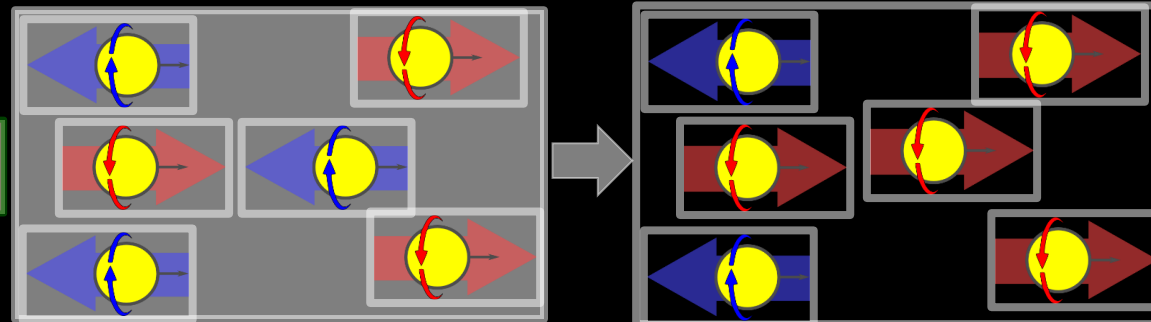


[P. Danaisawadi, et al. Sci. Rep. 6, 23832 (2016)]

**Chiral Anomaly**

**E, B**

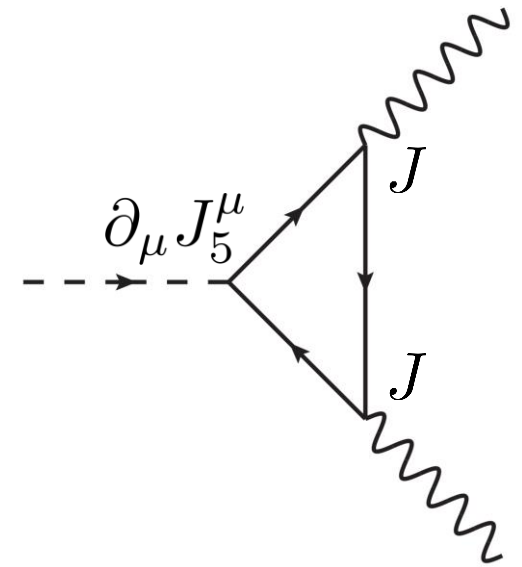
Quantum systems:



High-energy physics:

$$\pi^0 \rightarrow \gamma + \gamma$$

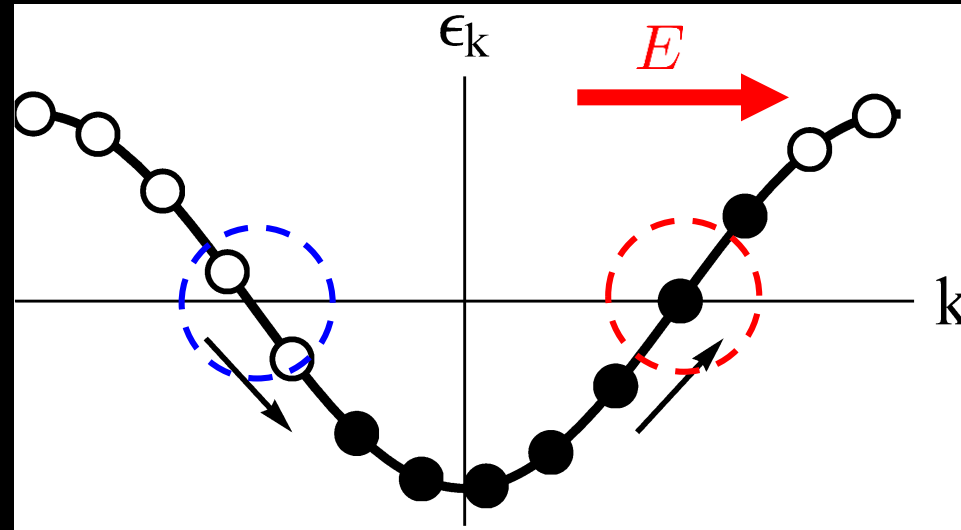
Allowed by the chiral anomaly!



[S.L. Adler, Phys. Rev. **177**, 2426 (1969), J.S. Bell and R. Jackiw, Nuovo Cim. A **60**, 47 (1969)]

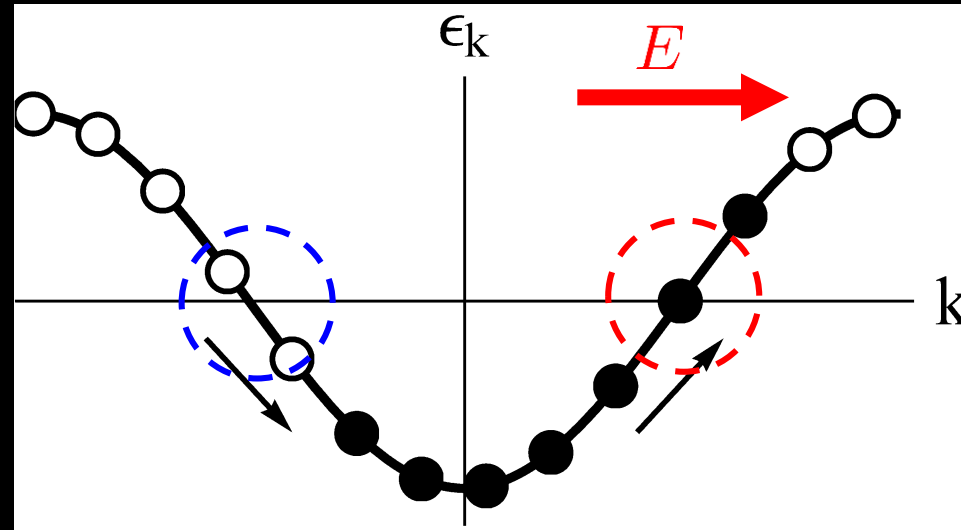
# Quantum (chiral) anomaly: solid state viewpoint

1D crystal:

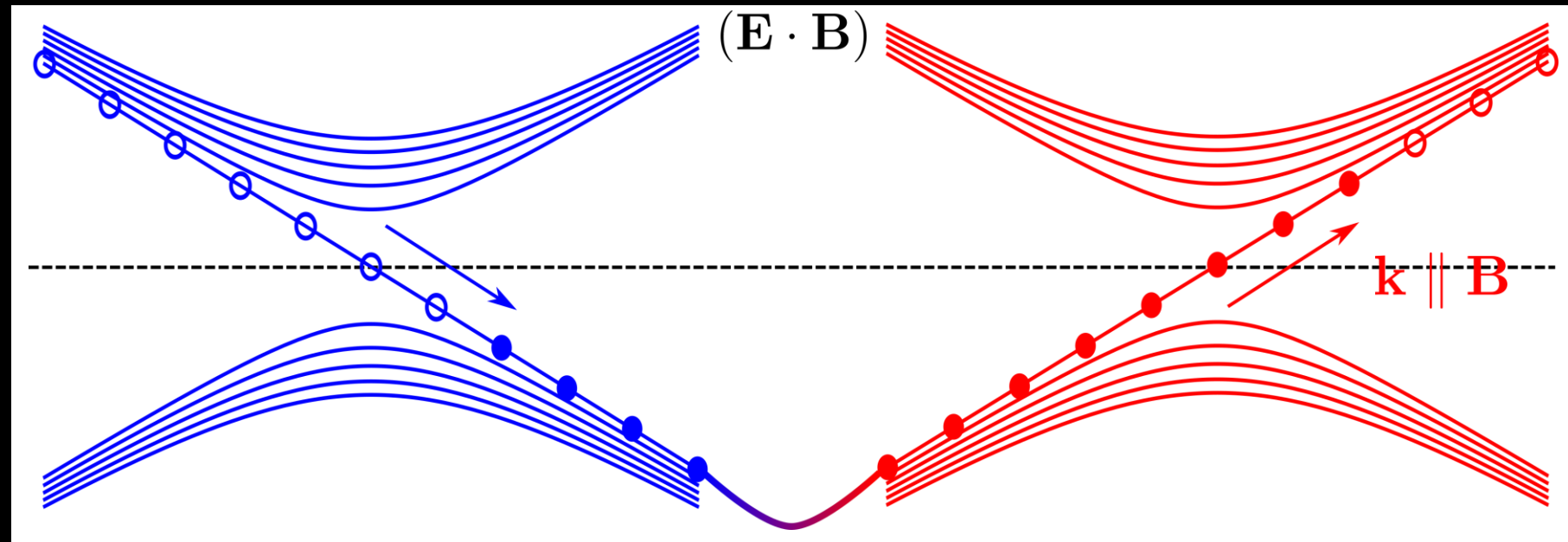
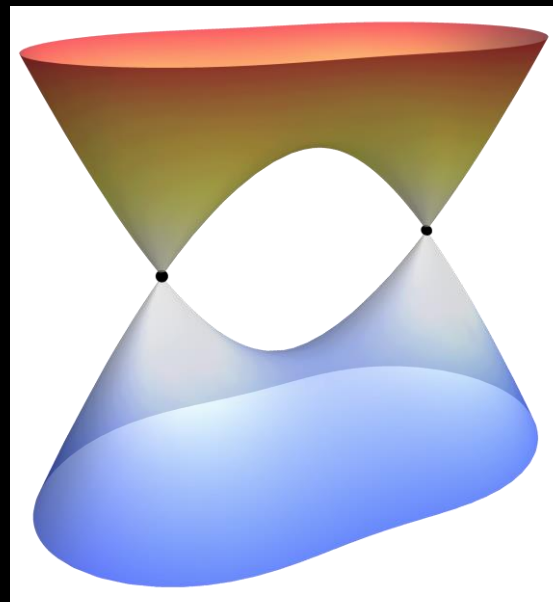


# Quantum (chiral) anomaly: solid state viewpoint

1D crystal:



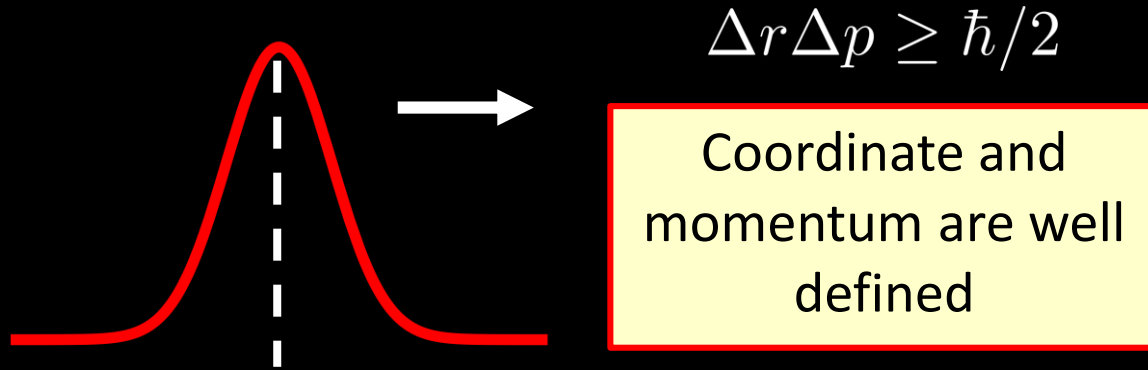
3D crystal:



# Semiclassical approach

[N.W. Ashcroft and N.D. Mermin, Solid State Physics (1976)]

Wave packet:



Equations of motion:

$$\partial_t \mathbf{r} = \mathbf{v}_p,$$

$$\partial_t \mathbf{p} = -e\mathbf{E} - \frac{e}{c} [(\partial_t \mathbf{r}) \times \mathbf{B}],$$

$$\mathbf{v}_p = \partial_{\mathbf{p}} \epsilon$$

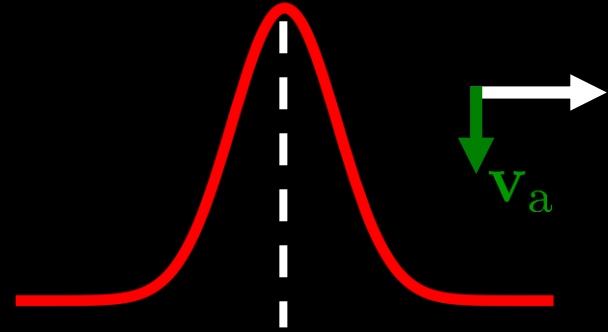
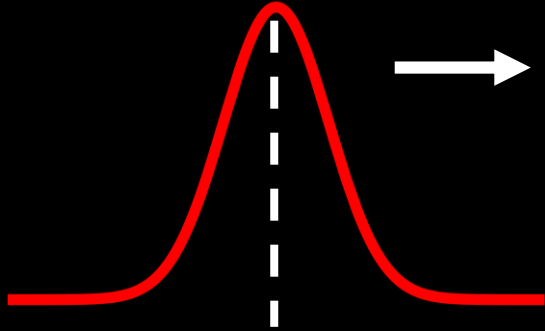
# Semiclassical approach

[N.W. Ashcroft and N.D. Mermin, Solid State Physics (1976)]

Wave packet:

$$\Delta r \Delta p \geq \hbar/2$$

Coordinate and  
momentum are well  
defined



[D. Xiao, M.-C. Chang, and Q. Niu, Rev. Mod. Phys. **82**, 1959 (2010)]

[D.T. Son and N. Yamamoto, Phys. Rev. D **87**, 085016 (2013)]

[M.A. Stephanov and Y. Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

Equations of motion:

$$\partial_t \mathbf{r} = \mathbf{v}_p,$$

$$\partial_t \mathbf{p} = -e\mathbf{E} - \frac{e}{c} [(\partial_t \mathbf{r}) \times \mathbf{B}],$$

$$\mathbf{v}_p = \partial_{\mathbf{p}} \epsilon$$

$$\partial_t \mathbf{r} = \mathbf{v}_p + [(\partial_t \mathbf{p}) \times \boldsymbol{\Omega}(\mathbf{p})],$$

$$\partial_t \mathbf{p} = -e\mathbf{E} - \frac{e}{c} [(\partial_t \mathbf{r}) \times \mathbf{B}],$$

$$\mathbf{v}_p = \partial_{\mathbf{p}} \epsilon$$

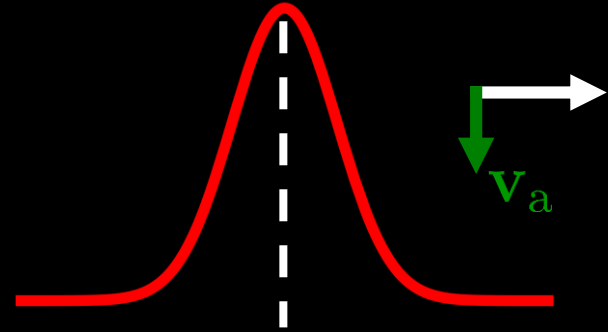
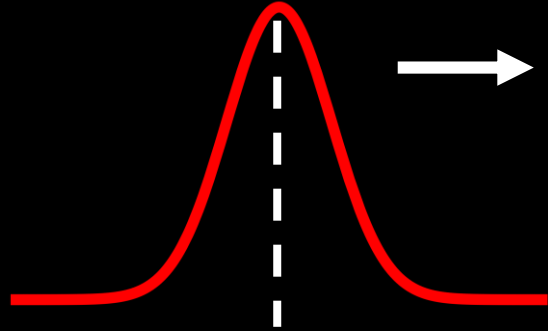
# Semiclassical approach

[N.W. Ashcroft and N.D. Mermin, Solid State Physics (1976)]

Wave packet:

$$\Delta r \Delta p \geq \hbar/2$$

Coordinate and momentum are well defined



[D. Xiao, M.-C. Chang, and Q. Niu, Rev. Mod. Phys. **82**, 1959 (2010)]

[D.T. Son and N. Yamamoto, Phys. Rev. D **87**, 085016 (2013)]

[M.A. Stephanov and Y. Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

Equations of motion:

$$\partial_t \mathbf{r} = \mathbf{v}_p,$$

$$\partial_t \mathbf{p} = -e\mathbf{E} - \frac{e}{c} [(\partial_t \mathbf{r}) \times \mathbf{B}],$$

$$\mathbf{v}_p = \partial_{\mathbf{p}} \epsilon$$

$$\partial_t \mathbf{r} = \mathbf{v}_p + [(\partial_t \mathbf{p}) \times \boldsymbol{\Omega}(\mathbf{p})],$$

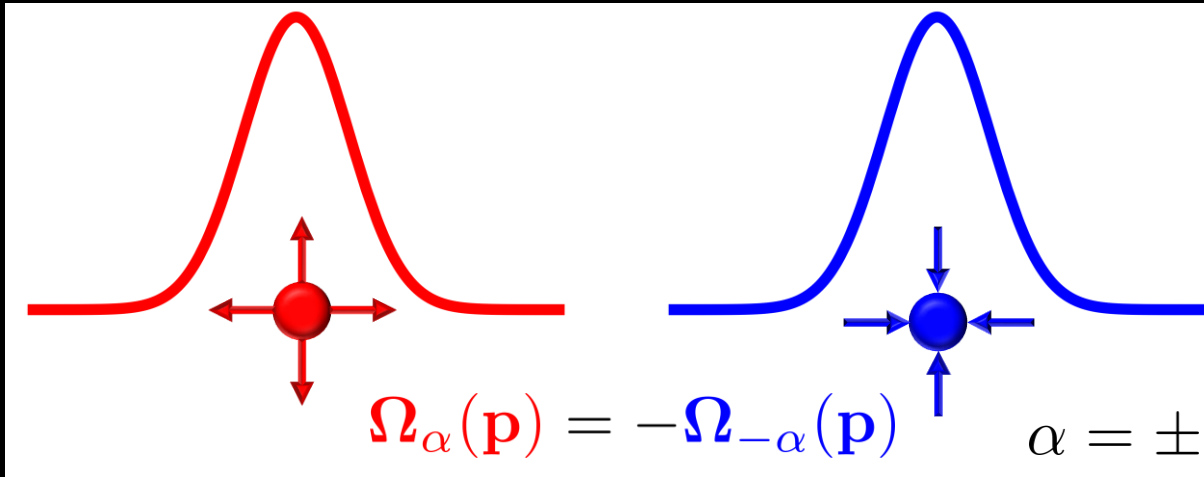
$$\partial_t \mathbf{p} = -e\mathbf{E} - \frac{e}{c} [(\partial_t \mathbf{r}) \times \mathbf{B}],$$

$$\mathbf{v}_p = \partial_{\mathbf{p}} \epsilon$$

$$\mathcal{A}(\mathbf{r}) \longleftrightarrow \mathcal{A}(\mathbf{p})$$

# Transport effect of chiral anomaly

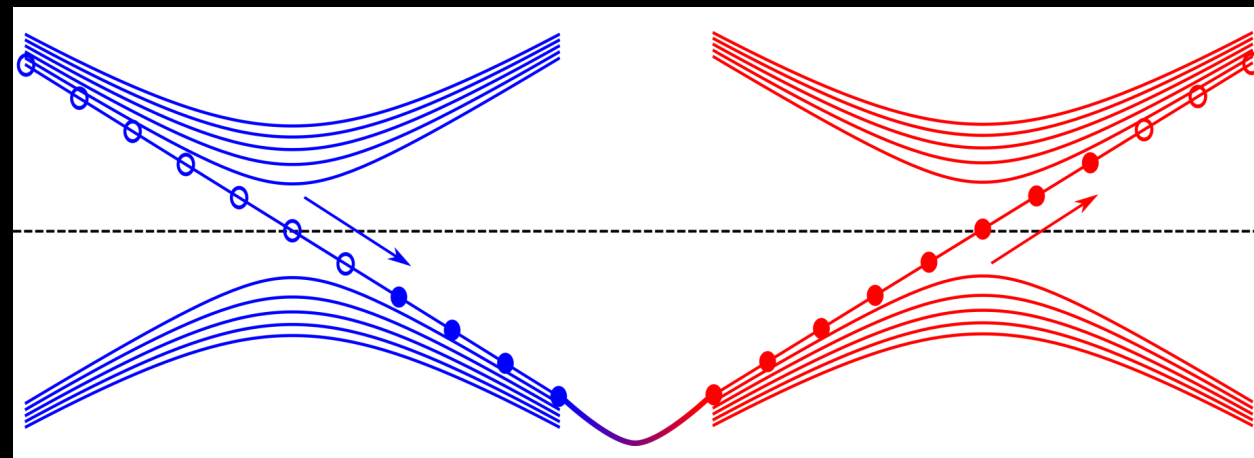
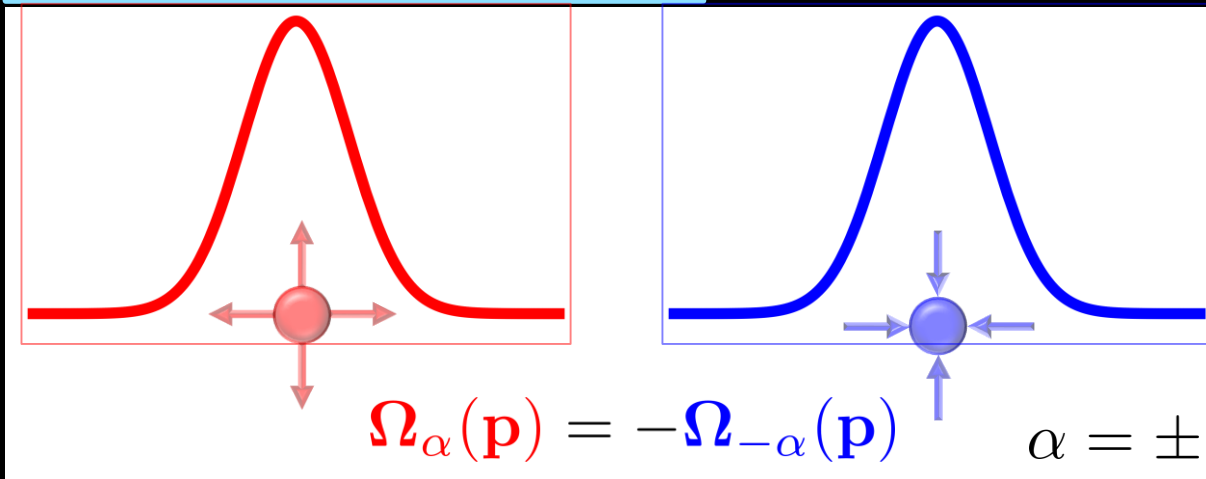
Multiple Weyl nodes:



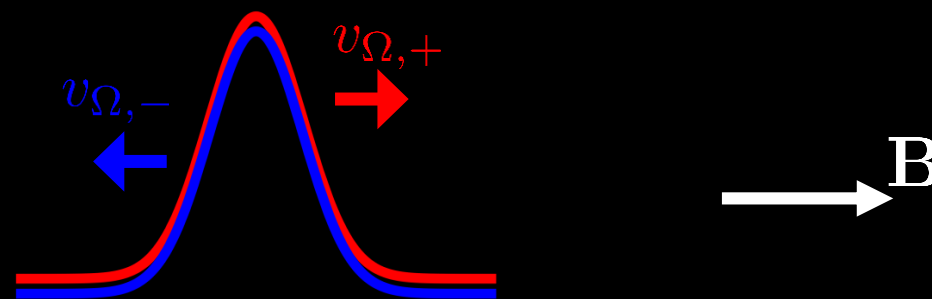
$$\partial_t \mathbf{r}_{\alpha} = \mathbf{v}_{\alpha} - \frac{e}{c} \mathbf{B} (\mathbf{v}_{\alpha} \cdot \boldsymbol{\Omega}_{\alpha}) + \dots$$

# Transport effect of chiral anomaly

Multiple Weyl nodes:

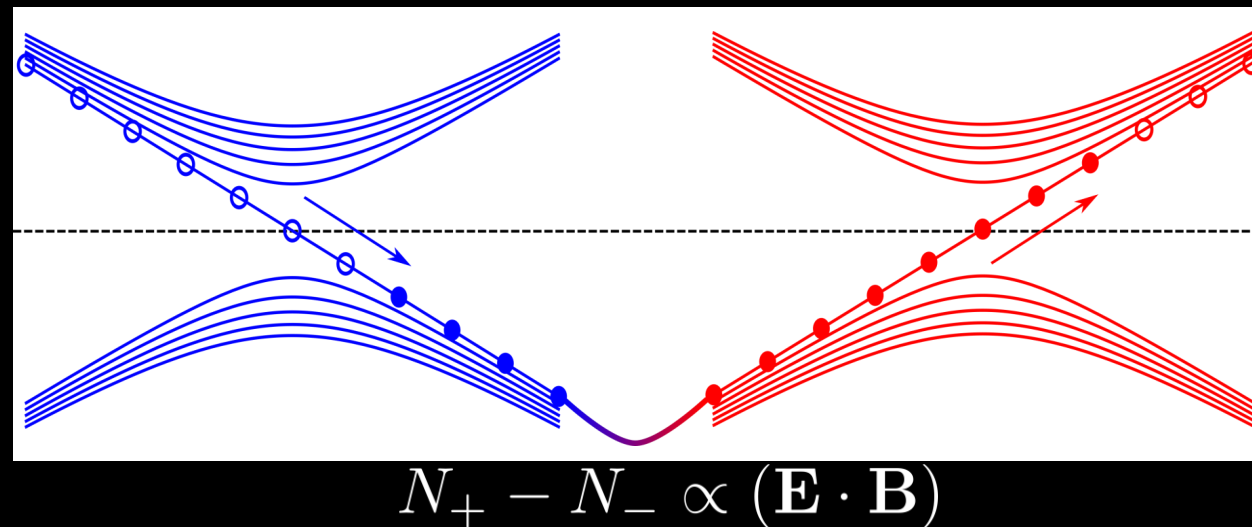
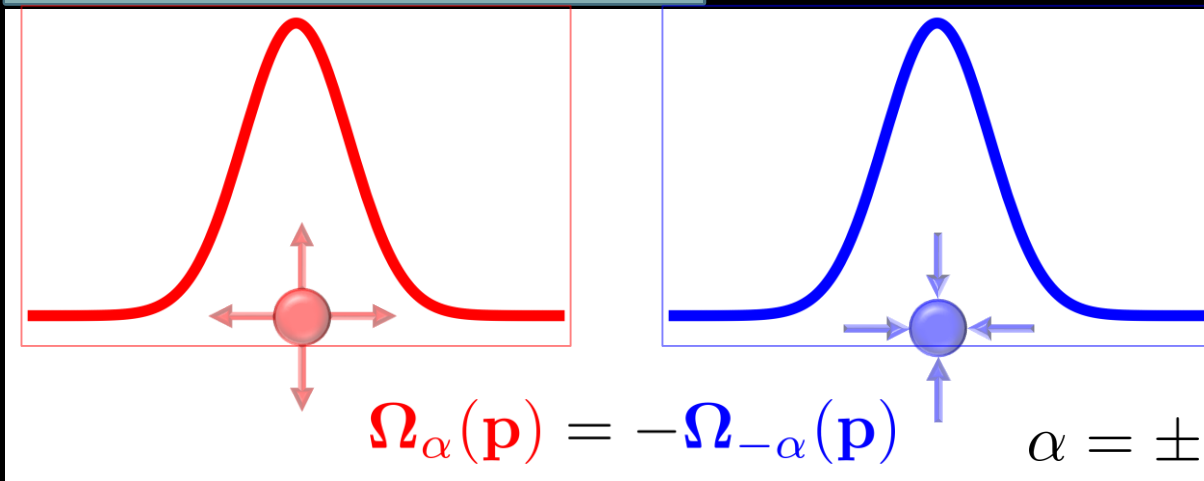


$$\partial_t \mathbf{r}_{\alpha} = \mathbf{v}_{\alpha} - \frac{e}{c} \mathbf{B} (\mathbf{v}_{\alpha} \cdot \boldsymbol{\Omega}_{\alpha}) + \dots$$



# Transport effect of chiral anomaly

Multiple Weyl nodes:

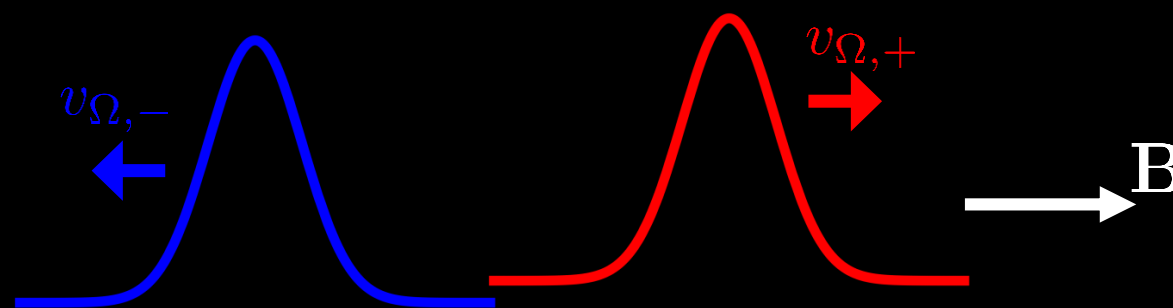


$$\partial_t \mathbf{r}_{\alpha} = \mathbf{v}_{\alpha} - \frac{e}{c} \mathbf{B} (\mathbf{v}_{\alpha} \cdot \boldsymbol{\Omega}_{\alpha}) + \dots$$

Chiral magnetic effect:

$$\mathbf{j} \propto (N_+ - N_-) \mathbf{B}$$

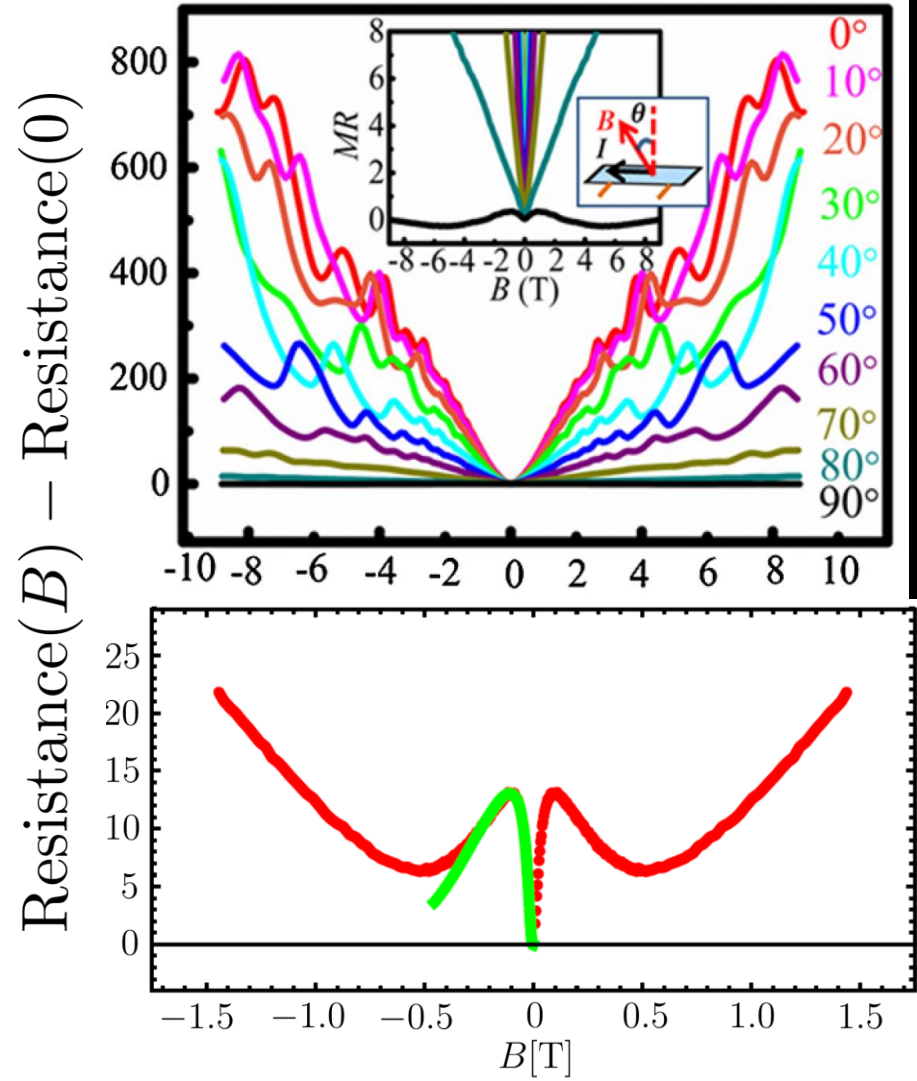
[A. Vilenkin, Phys. Rev. D **22**, 3080 (1980); K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D **78**, 074033 (2008)]



Separation of wave packets  $\rightarrow$  electric current

# Transport in magnetic fields

## Weyl semimetals (TaAs)

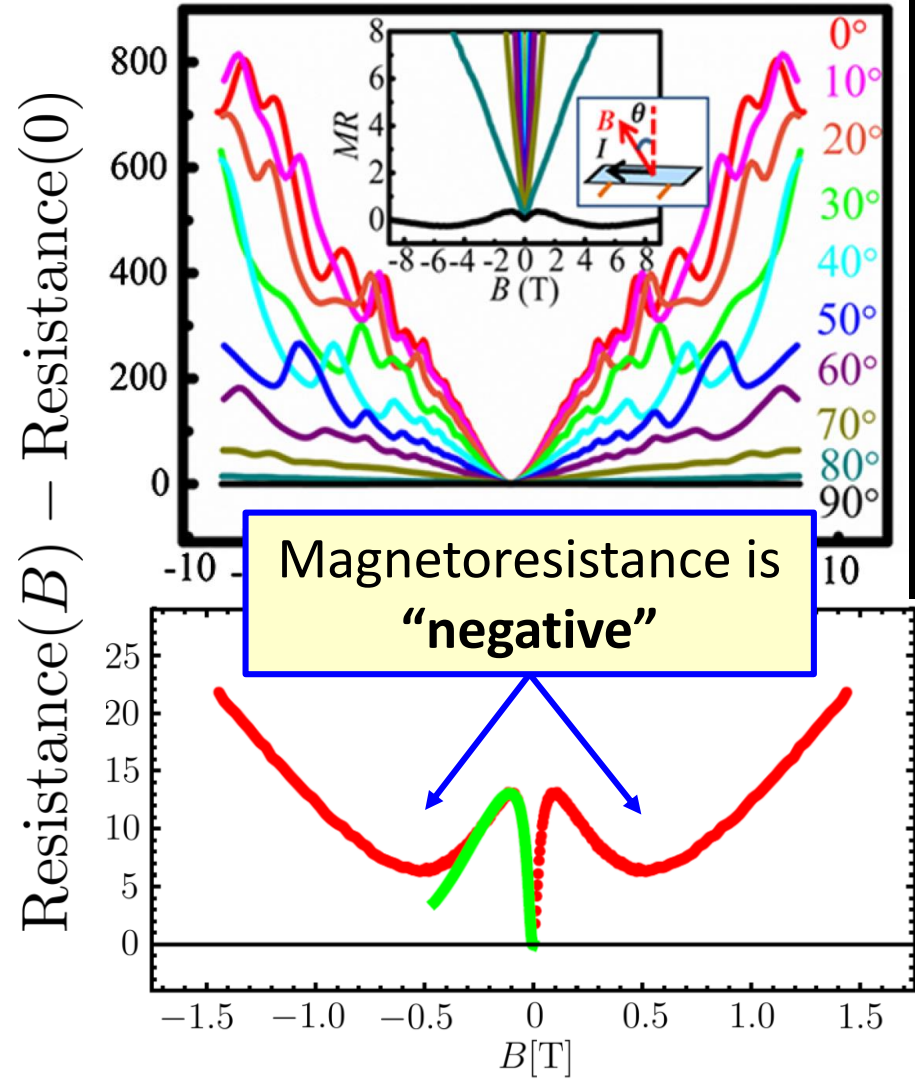


[X. Huang, L. Zhao, Y. Long, et al.,  
Phys. Rev. X **5**, 031023  
(2015)]

[C.-L. Zhang, S.-Y. Xu, I. Belopolski, Z. Yuan, et al.,  
Nat. Commun. **7**, 10735  
(2016)]

# Transport in magnetic fields

## Weyl semimetals (TaAs)

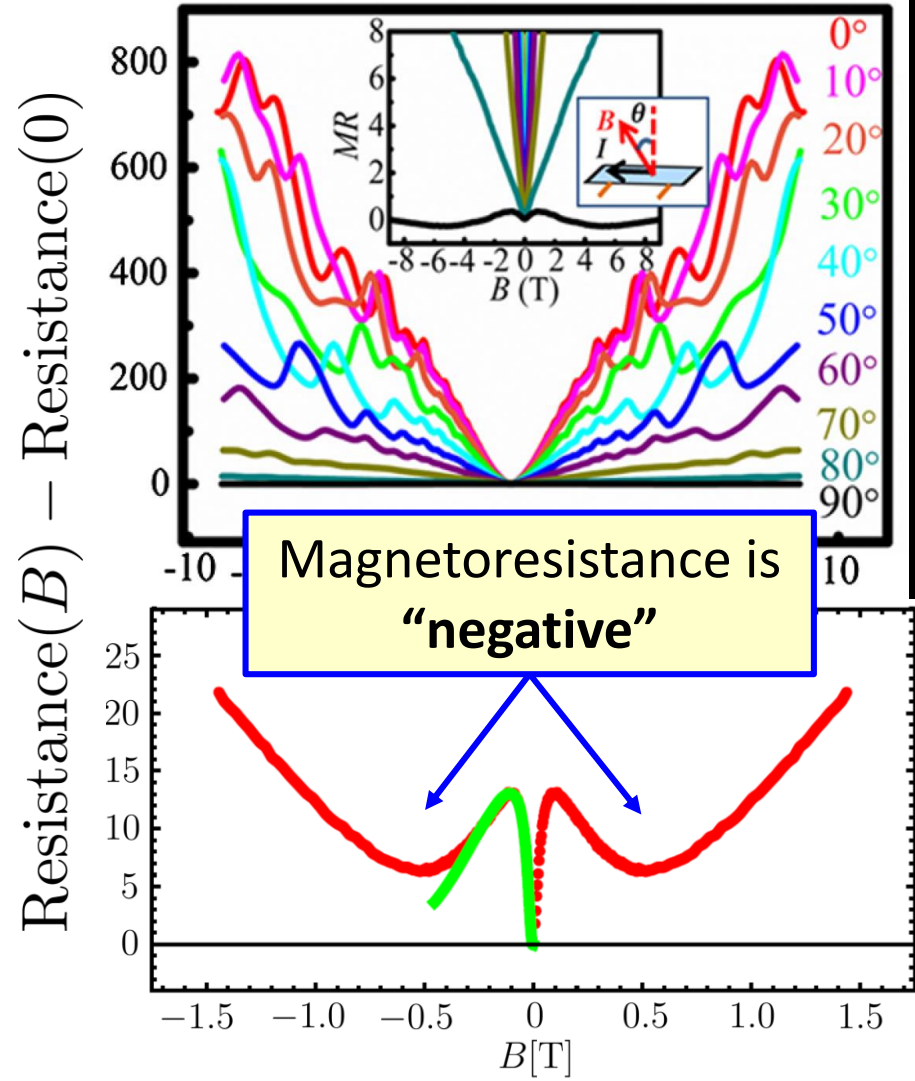


[X. Huang, L. Zhao, Y. Long, et al., Phys. Rev. X 5, 031023 (2015)]

[C.-L. Zhang, S.-Y. Xu, I. Belopolski, Z. Yuan, et al., Nat. Commun. 7, 10735 (2016)]

# Transport in magnetic fields

## Weyl semimetals (TaAs)

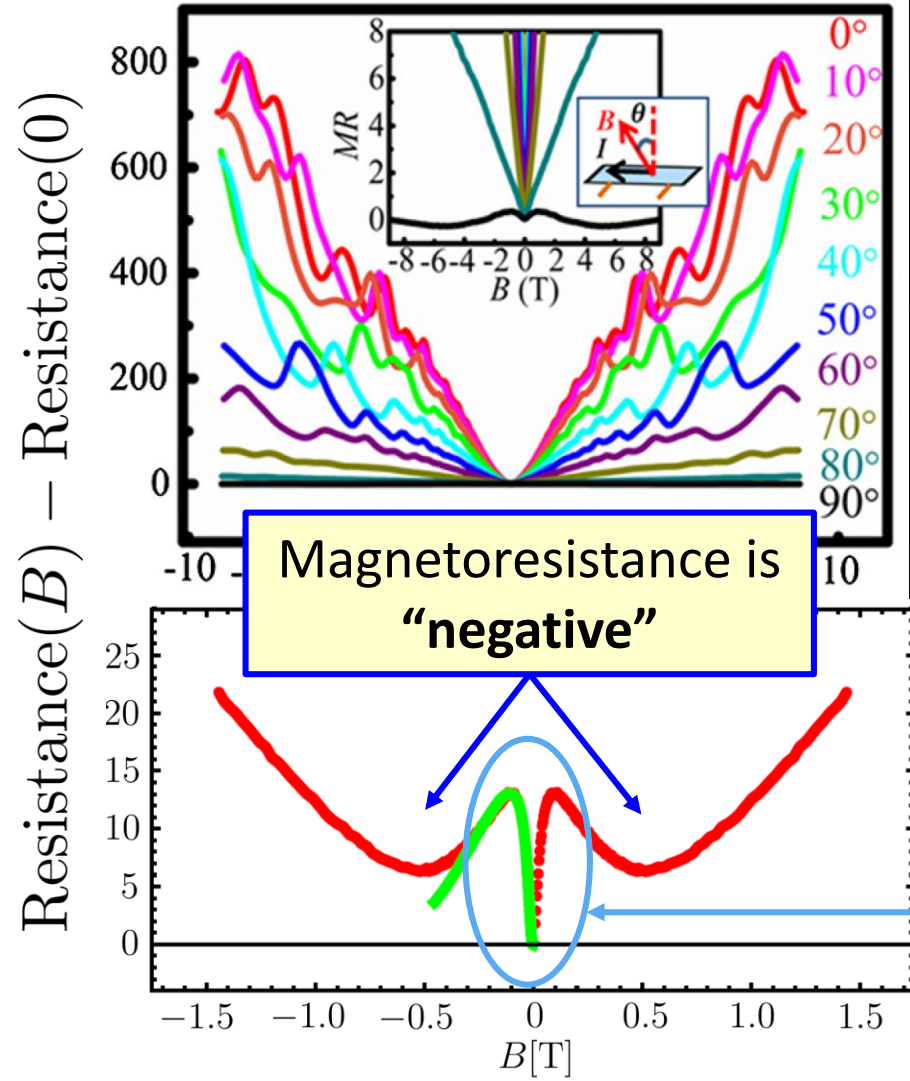


[X. Huang, L. Zhao, Y. Long, et al., Phys. Rev. X 5, 031023 (2015)]

[C.-L. Zhang, S.-Y. Xu, I. Belopolski, Z. Yuan, et al., Nat. Commun. 7, 10735 (2016)]

# Transport in magnetic fields

## Weyl semimetals (TaAs)



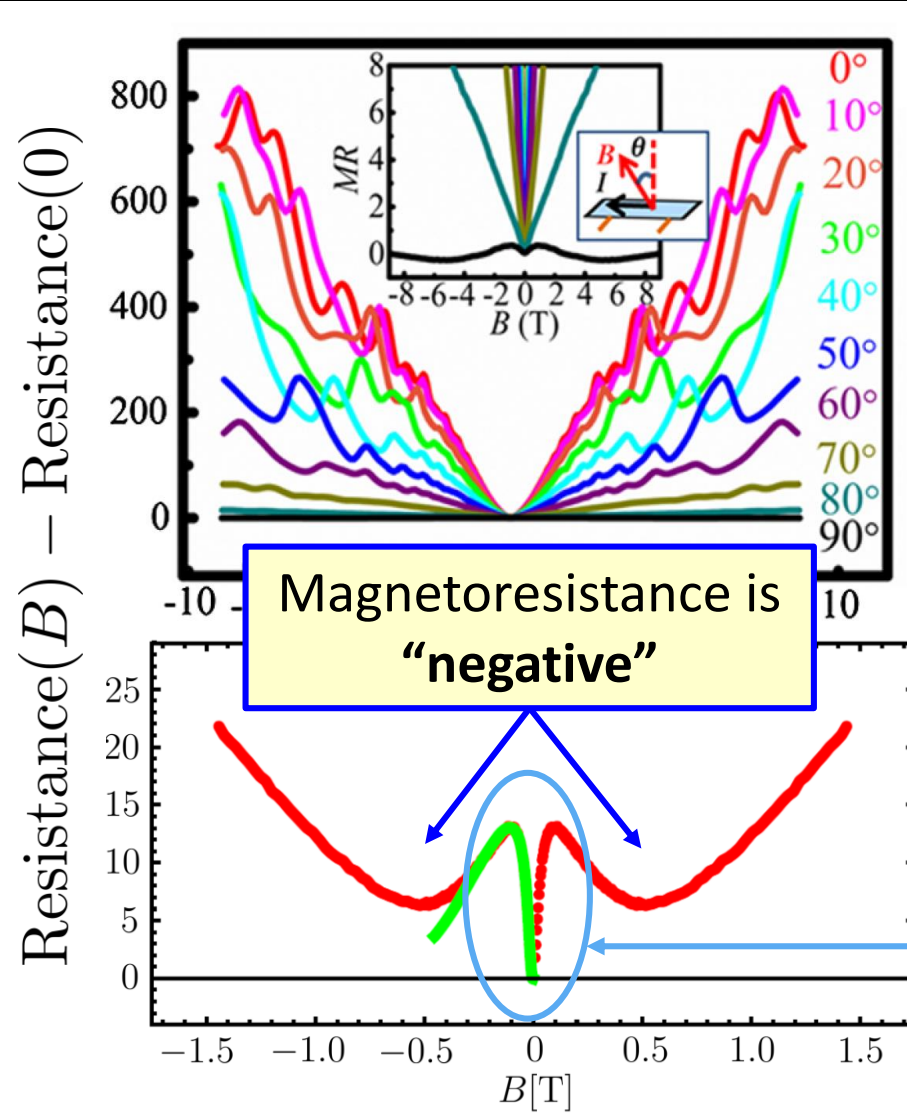
[X. Huang, L. Zhao, Y. Long, et al., Phys. Rev. X 5, 031023 (2015)]

[C.-L. Zhang, S.-Y. Xu, I. Belopolski, Z. Yuan, et al., Nat. Commun. 7, 10735 (2016)]

Weak antilocalization

# Transport in magnetic fields

## Weyl semimetals (TaAs)



[X. Huang, L. Zhao, Y. Long, et al., Phys. Rev. X **5**, 031023 (2015)]

[C.-L. Zhang, S.-Y. Xu, I. Belopolski, Z. Yuan, et al., Nat. Commun. **7**, 10735 (2016)]

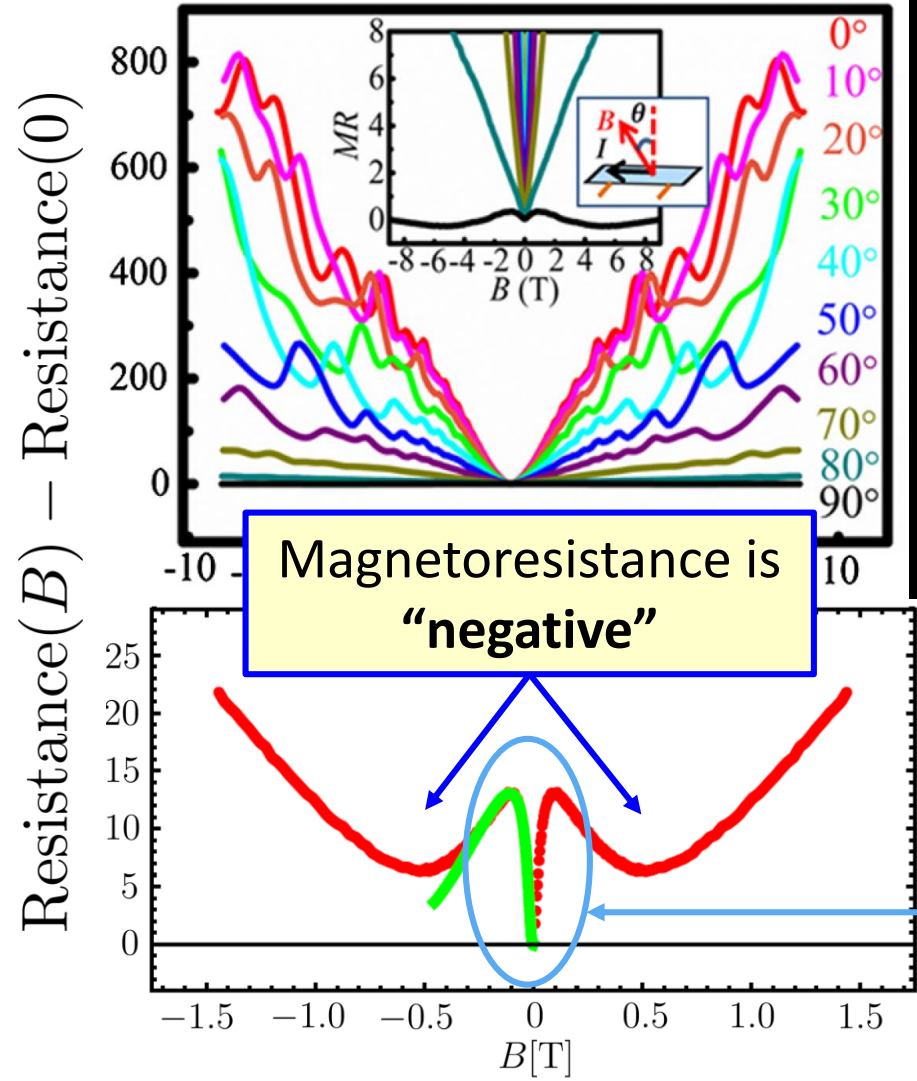
## Other mechanisms:

- Current jetting [R. D. dos Reis, et al., New J. Phys. **18**, 085006 (2016)]
- Weak antilocalization-localization crossover
- Scattering off charged disorder

[P. Goswami, J. H. Pixley, and S. Das Sarma, Phys. Rev. B **92**, 075205 (2015)]

# Transport in magnetic fields

## Weyl semimetals (TaAs)



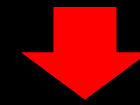
[X. Huang, L. Zhao, Y. Long, et al., Phys. Rev. X **5**, 031023 (2015)]

[C.-L. Zhang, S.-Y. Xu, I. Belopolski, Z. Yuan, et al., Nat. Commun. **7**, 10735 (2016)]

Weak antilocalization

## Other mechanisms:

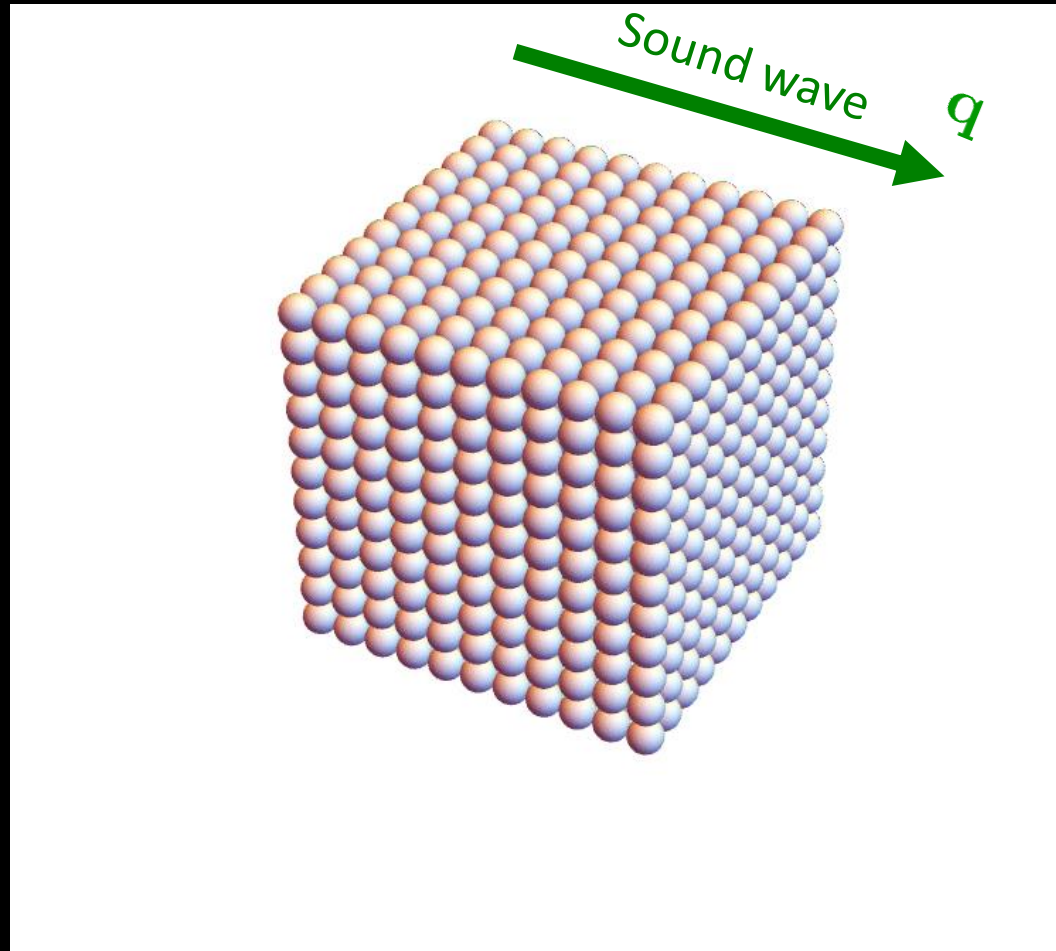
- Current jetting [R. D. dos Reis, et al., New J. Phys. **18**, 085006 (2016)]
- Weak antilocalization-localization crossover
- Scattering off charged disorder [P. Goswami, J. H. Pixley, and S. Das Sarma, Phys. Rev. B **92**, 075205 (2015)]



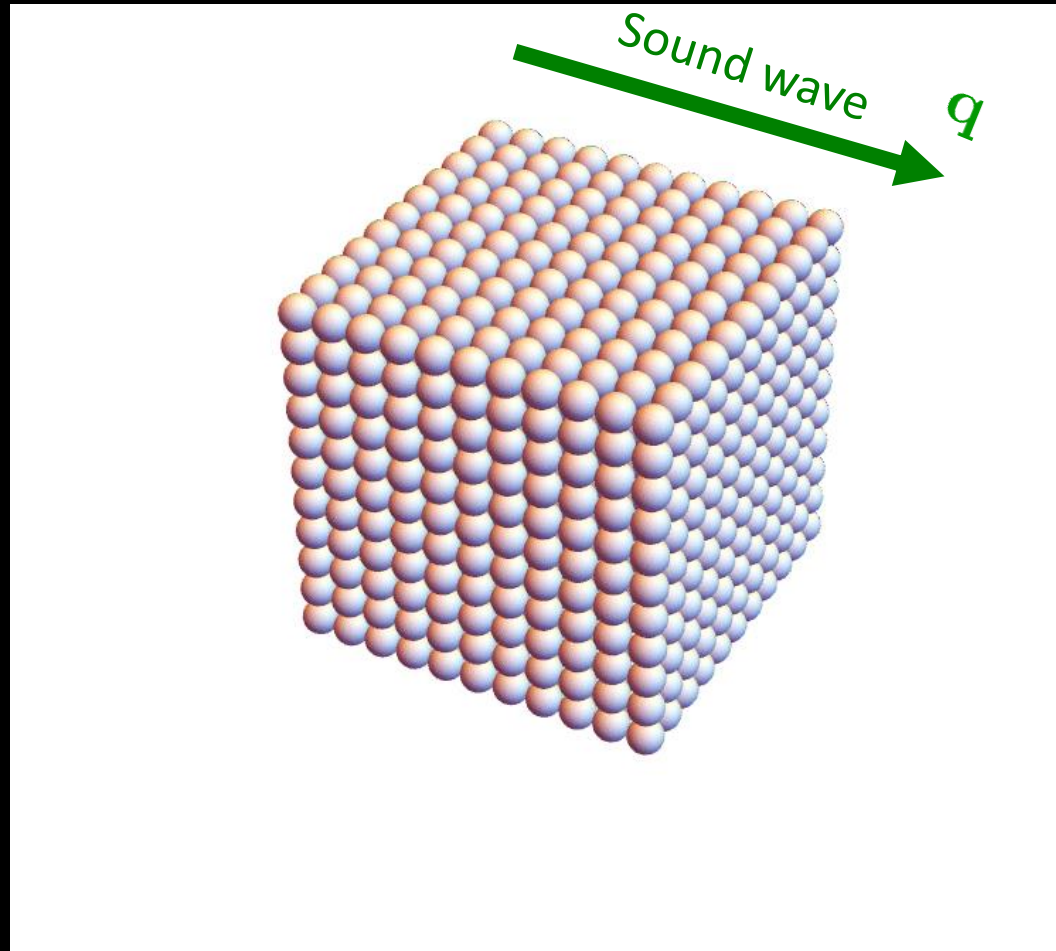
- **New approaches** are needed to verify chiral anomaly
- Novel effects await in the **ac regime**

# Sound propagation

---



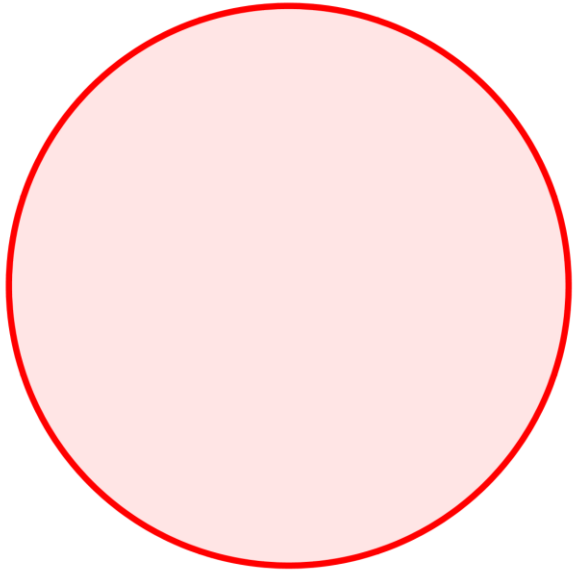
# Sound propagation



**Deformation potential:**  
Sound  $\rightarrow$  modifies the  
potential acting on electrons

# Sound propagation

---

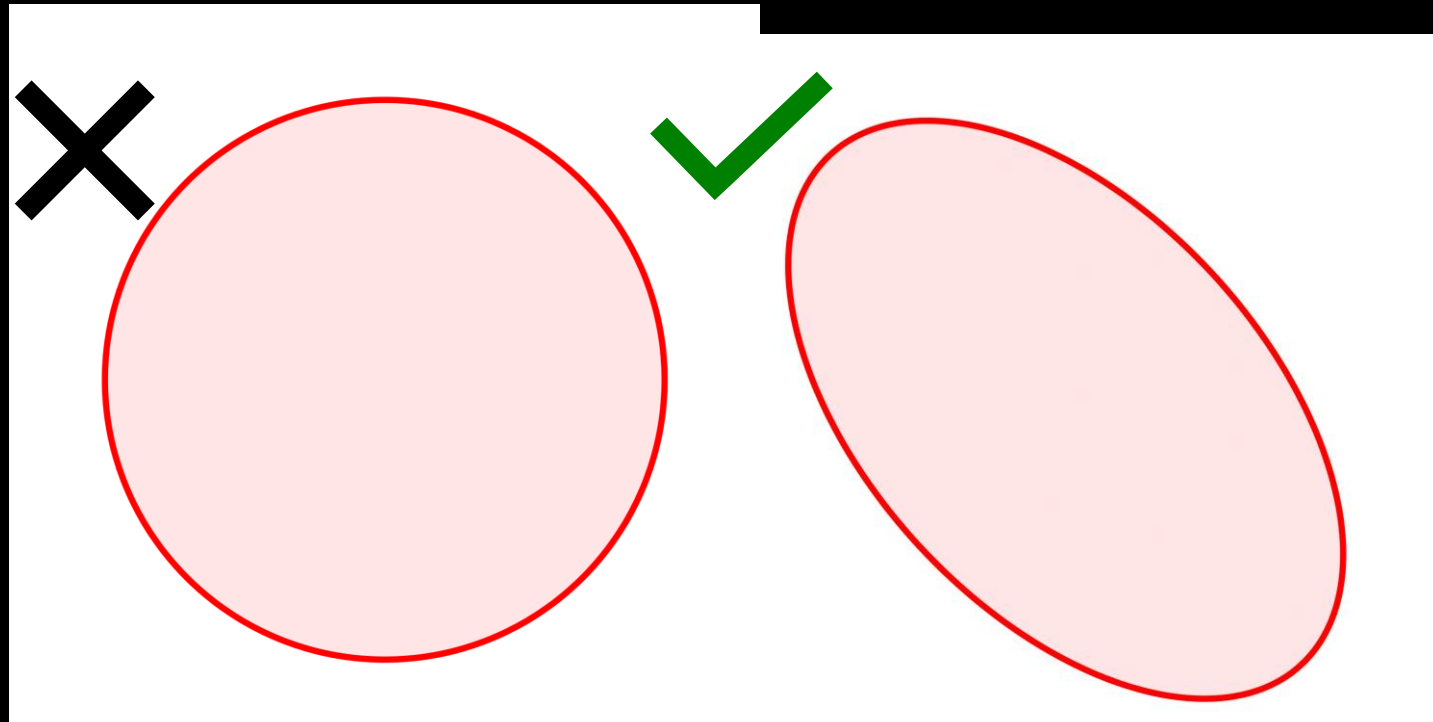


**Momentum-independent  
deformation potential**

$$U(k_F, t, \mathbf{r})$$

# Sound propagation

Strong screening  $\Rightarrow$  local charge neutrality  $\Rightarrow$  only **volume-preserving** deformations allowed



**Momentum-independent**  
deformation potential

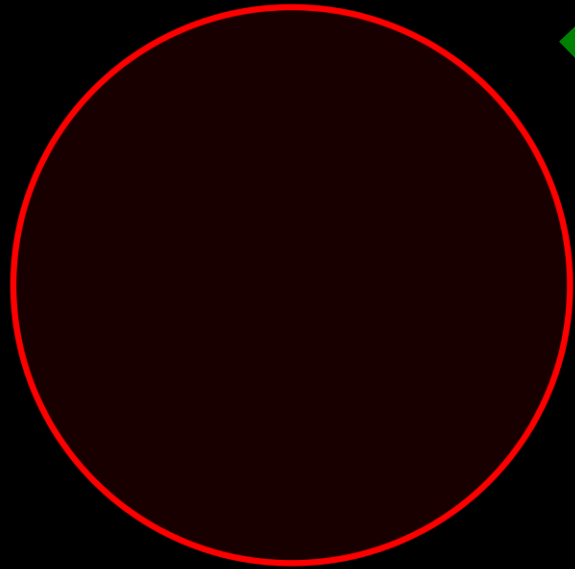
$$U(k_F, t, \mathbf{r})$$

**Momentum-dependent**  
deformation potential

$$U(k_F \cos(2\theta), t, \mathbf{r})$$

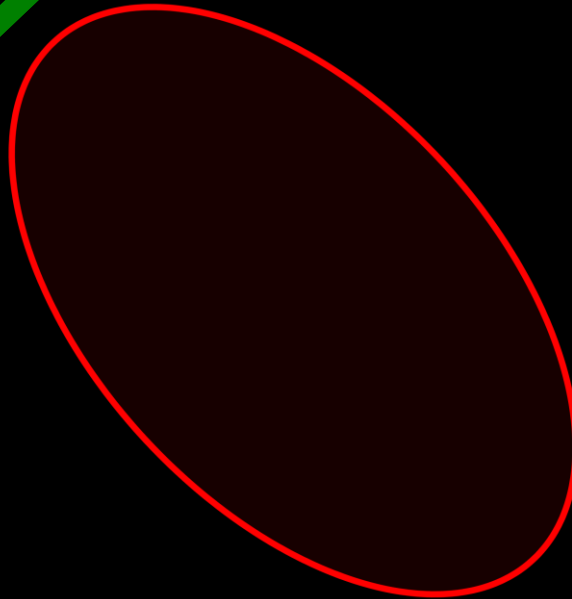
# Sound propagation

Strong screening  $\Rightarrow$  local charge neutrality  $\Rightarrow$  only **volume-preserving** deformations allowed



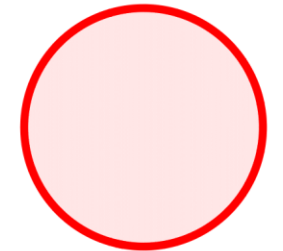
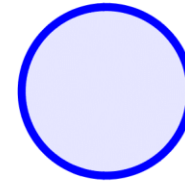
Momentum-independent  
deformation potential

$$U(k_F, t, \mathbf{r})$$



Momentum-dependent  
deformation potential

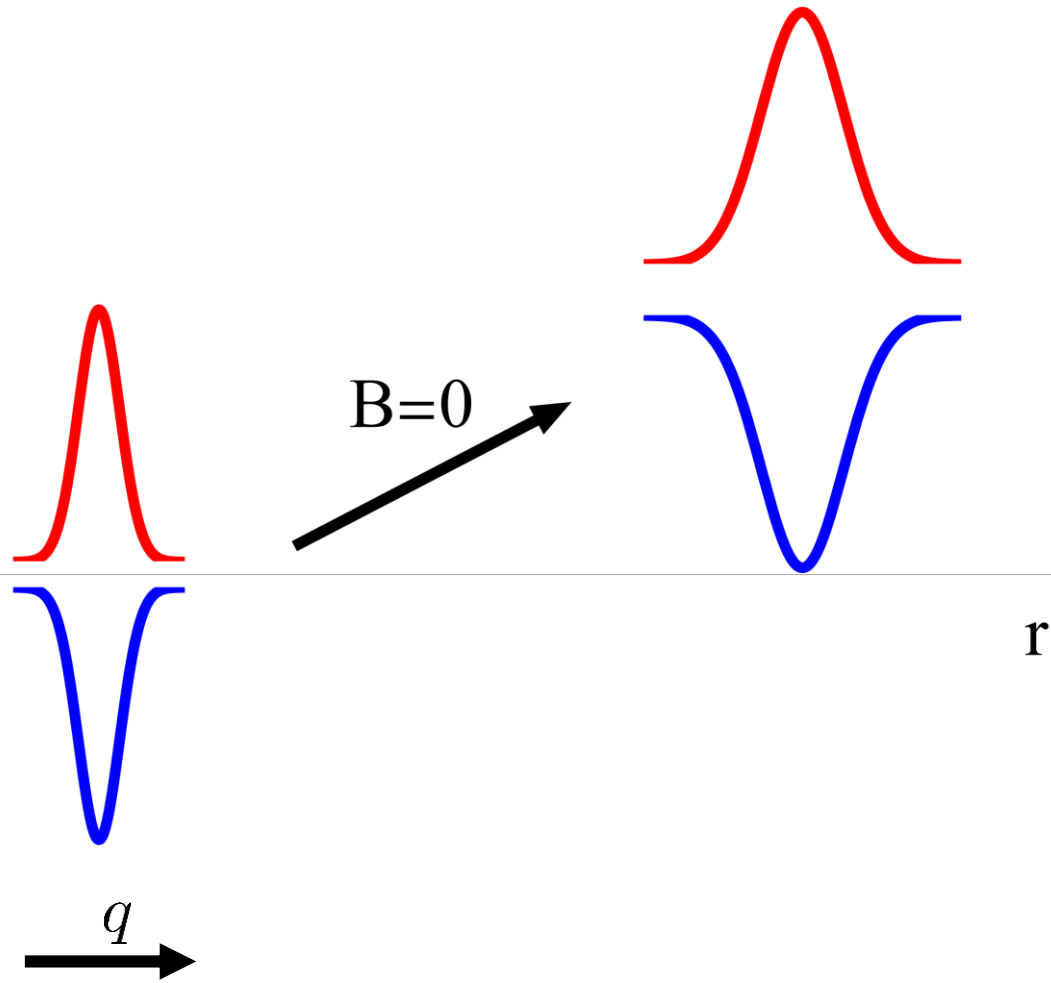
$$U(k_F \cos(2\theta), t, \mathbf{r})$$



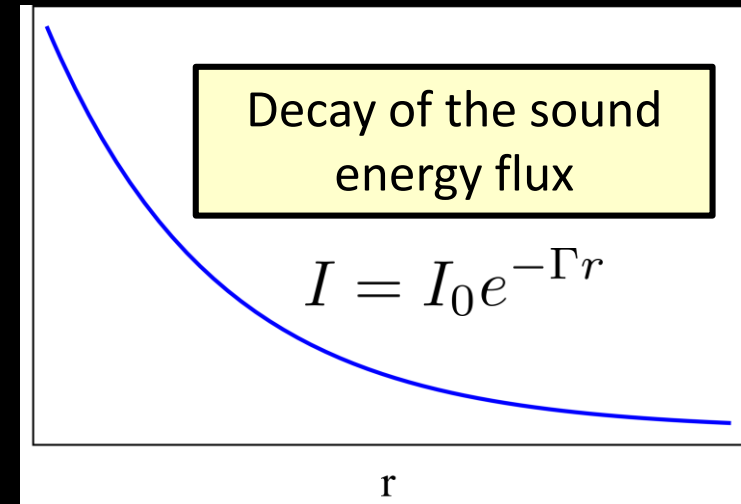
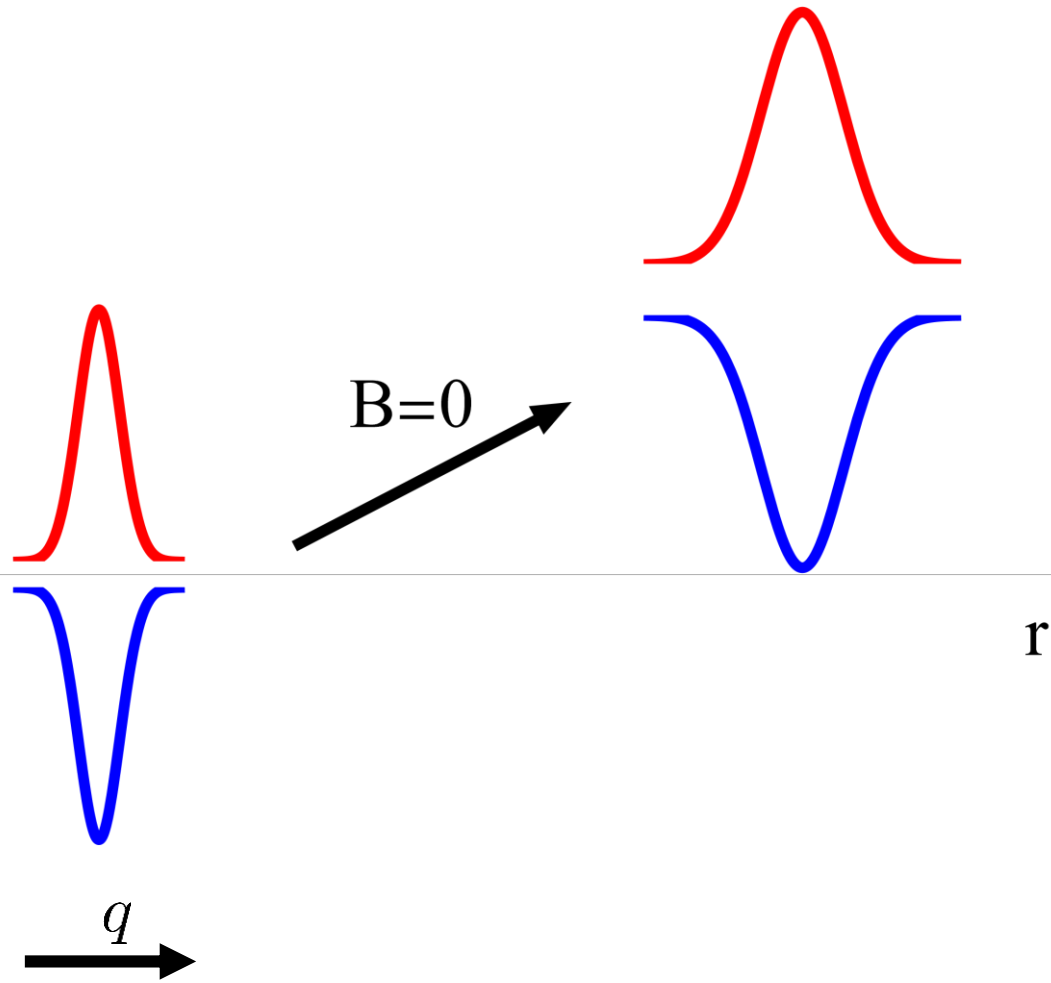
Valley-dependent  
deformation potential

$$U_+(k_F, t, \mathbf{r}) = -U_-(k_F, t, \mathbf{r})$$

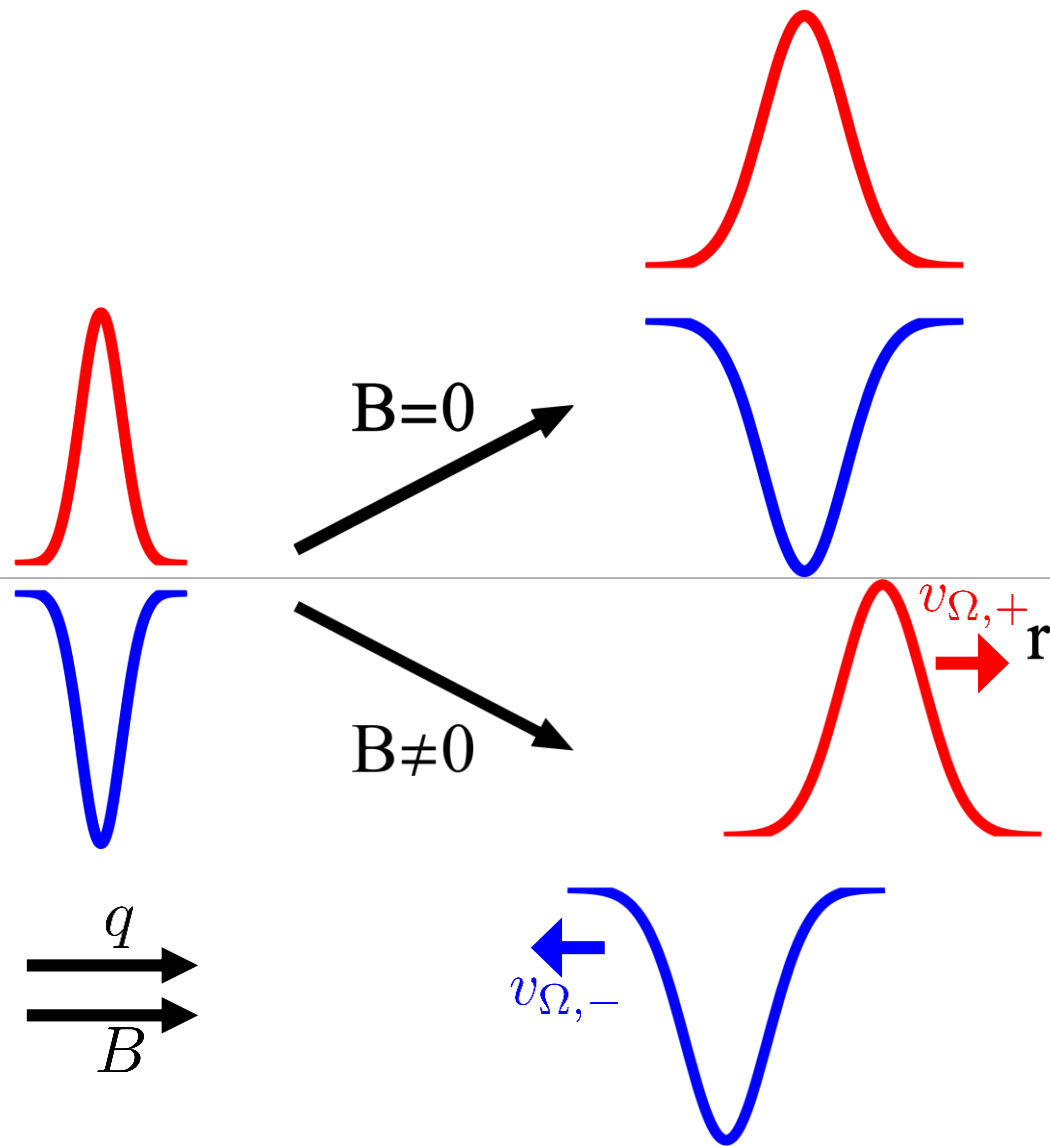
# Symmetric Weyl nodes in magnetic field



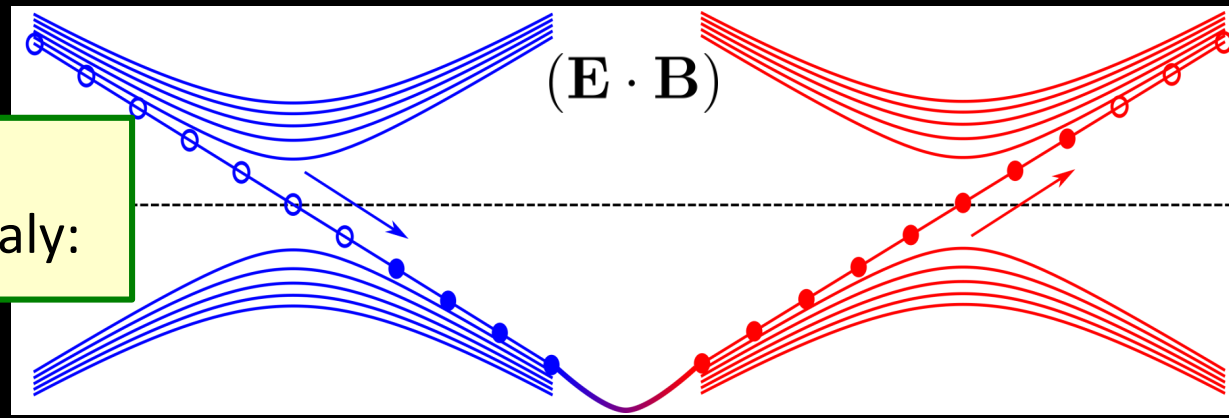
# Symmetric Weyl nodes in magnetic field



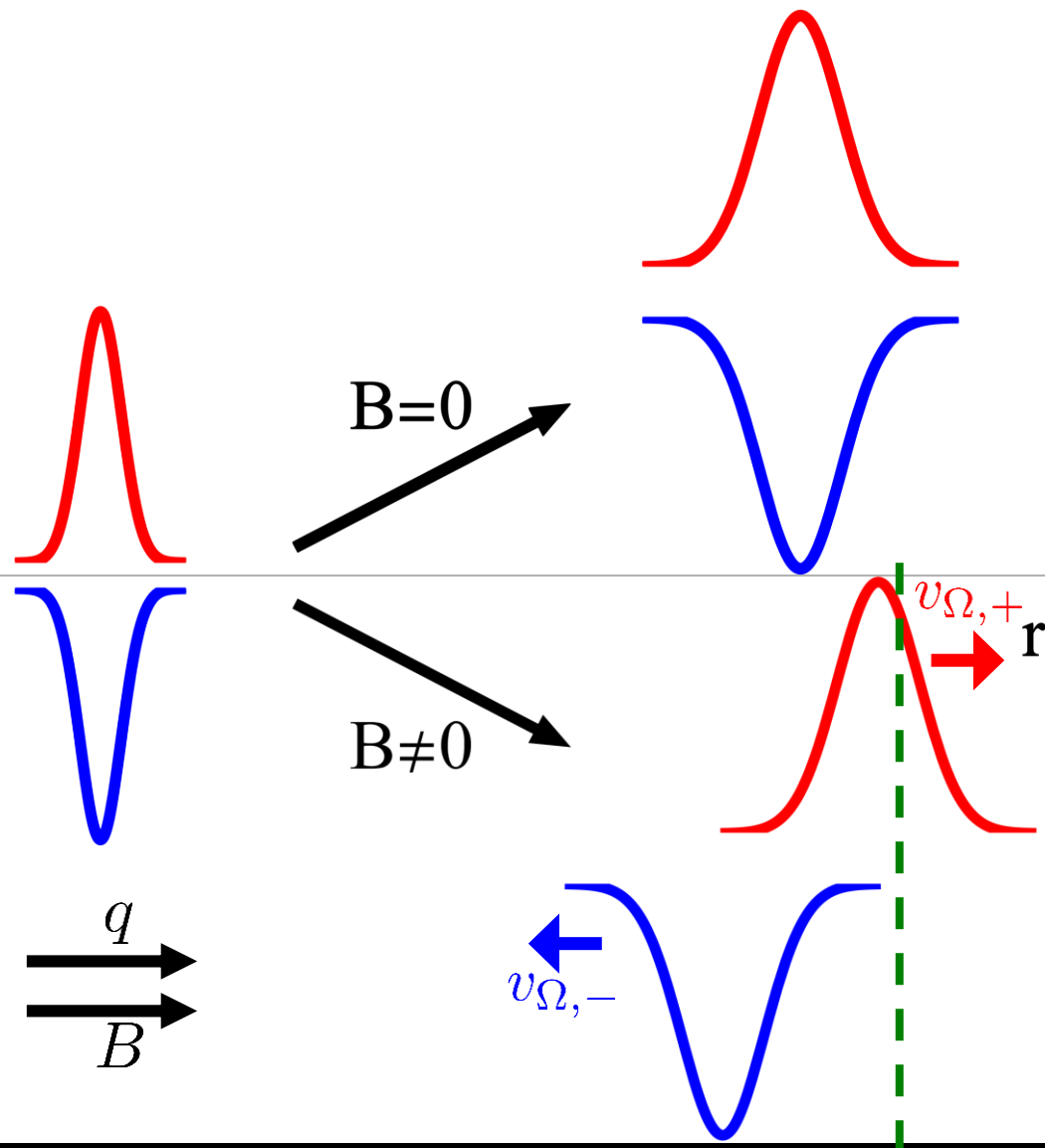
# Symmetric Weyl nodes in magnetic field



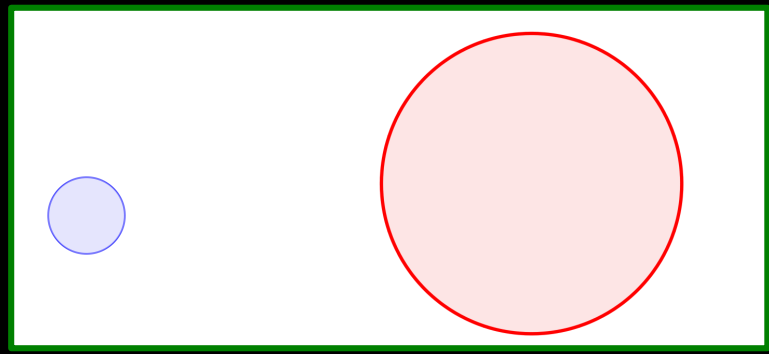
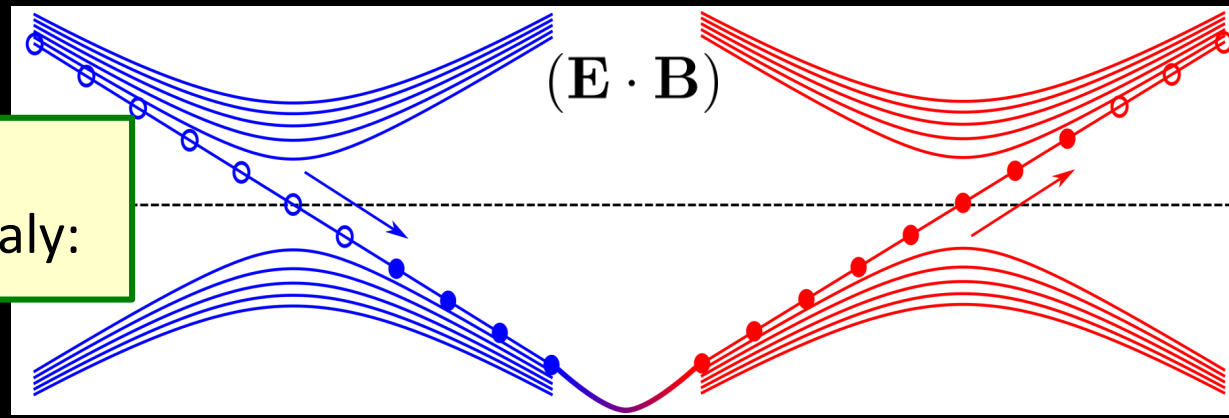
Chiral anomaly:



# Symmetric Weyl nodes in magnetic field

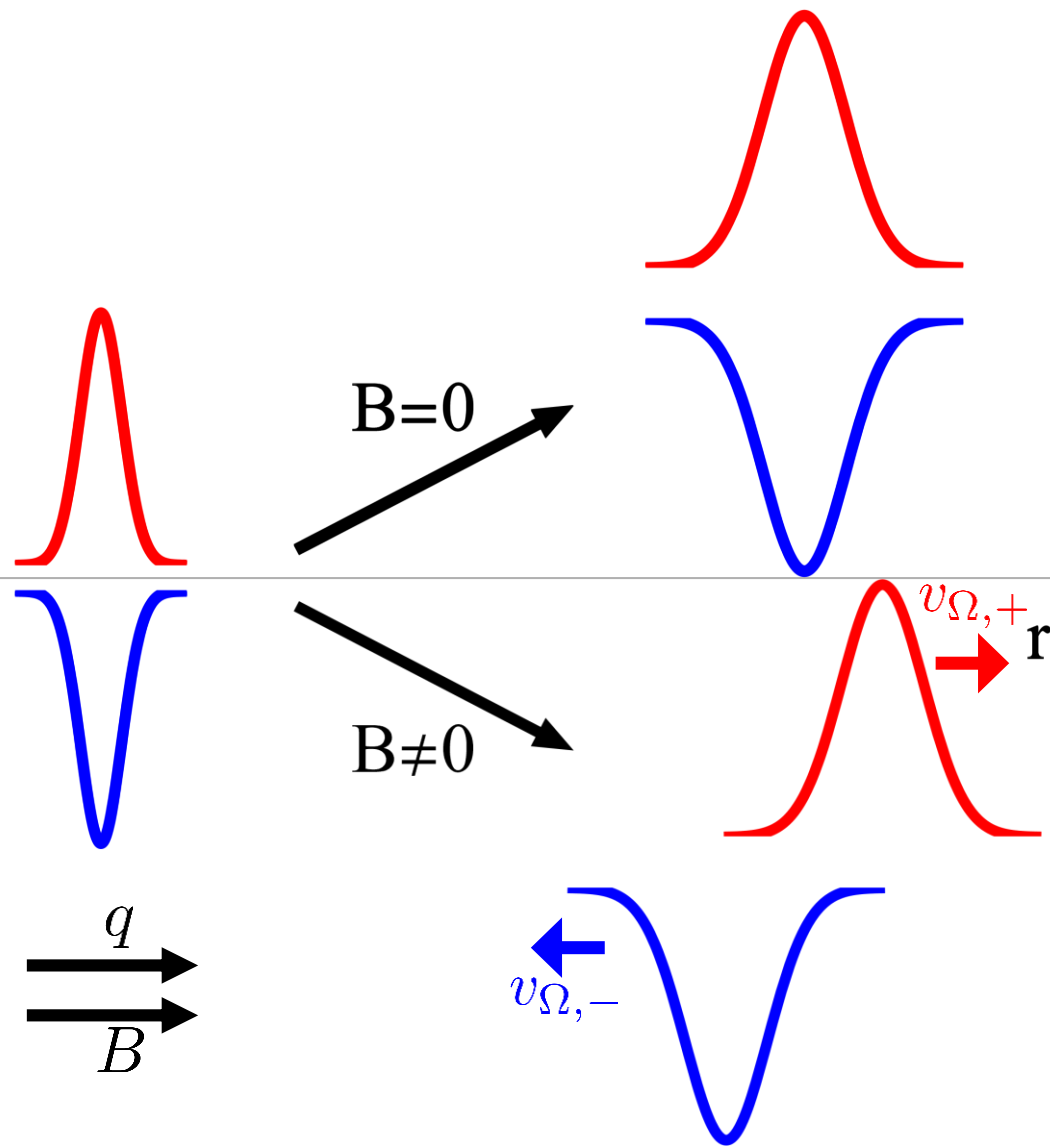


Chiral anomaly:

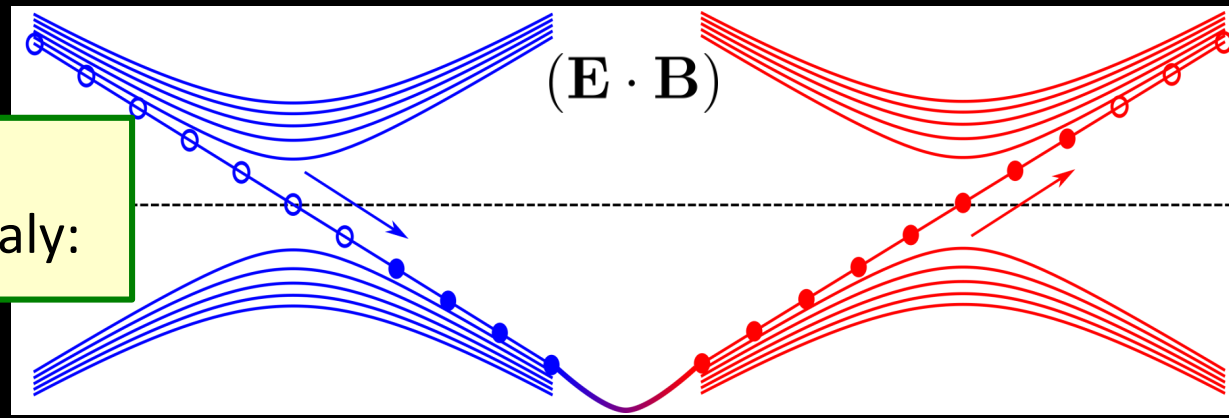


No longer locally neutral!

# Symmetric Weyl nodes in magnetic field



Chiral anomaly:



Anomaly-induced drift of chiral wave packets violates local charge neutrality  $\rightarrow$  screening of the valley-odd deformation potential

Magnetic field reduces the sound attenuation

# Sound attenuation coefficient

$$\Gamma = \frac{Q}{VI},$$

Sound attenuation  
coefficient

$$Q = \frac{1}{T} \int_0^T dt \left\langle \frac{d}{dt} \hat{H} \right\rangle,$$

Dissipated energy

$$I = v_s \frac{\rho_m \langle |\partial_t \mathbf{u}|^2 \rangle_T}{2} = v_s \frac{\rho_m \omega^2 u_0^2}{2}.$$

Sound energy flux

# Sound attenuation coefficient

$$\Gamma = \frac{Q}{VI},$$

Sound attenuation coefficient

$$Q = \frac{1}{T} \int_0^T dt \left\langle \frac{d}{dt} \hat{H} \right\rangle,$$

Dissipated energy

$$I = v_s \frac{\rho_m \langle |\partial_t \mathbf{u}|^2 \rangle_T}{2} = v_s \frac{\rho_m \omega^2 u_0^2}{2}.$$

Sound energy flux

$$\frac{Q}{V} = - \sum_{\alpha}^{N_w} \frac{1}{2e} \operatorname{Re} \left\{ i\omega \left( \lambda_{ij}^{(\alpha)} u_{ij} \right)^* N_{\alpha} \right\}.$$

# Sound attenuation coefficient

$$\Gamma = \frac{Q}{VI},$$

Sound attenuation coefficient

$$Q = \frac{1}{T} \int_0^T dt \left\langle \frac{d}{dt} \hat{H} \right\rangle,$$

Dissipated energy

$$I = v_s \frac{\rho_m \langle |\partial_t \mathbf{u}|^2 \rangle_T}{2} = v_s \frac{\rho_m \omega^2 u_0^2}{2}.$$

Sound energy flux

$$\frac{Q}{V} = - \sum_{\alpha}^{N_W} \frac{1}{2e} \operatorname{Re} \left\{ i\omega \left( \lambda_{ij}^{(\alpha)} u_{ij} \right)^* N_{\alpha} \right\}.$$

$$\Gamma = - \frac{q}{e v_s \rho_m \omega u_0} \operatorname{Re} \left\{ \lambda N + \lambda^{(5)} N_5 \right\}, \quad \text{at } \nu_{\alpha} = \nu, \quad \lambda_{ij}^{(\alpha)} = \lambda_{ij} + \chi_{\alpha} \lambda_{ij}^{(5)}.$$

Sound attenuation is determined by the non-equilibrium part of the charge densities

# Kinetic equations

---

Diffusive approximation:  $\tau_{\alpha,\alpha} \ll \tau_{\alpha,\beta}$  and  $\tau_{\alpha,\alpha}\omega \ll 1$

$$\partial_t N_\alpha + (\nabla \cdot \mathbf{j}_\alpha) - e^2 \nu_\alpha \left( \left[ \mathbf{E} + \frac{1}{e} \lambda_{ij}^{(\alpha)} \nabla u_{ij} \right] \cdot \mathbf{v}_{\Omega,\alpha} \right) = - \sum_{\beta}^{N_W} T_{\alpha,\beta} N_\beta - u_{ij} \sum_{\beta}^{N_W} \frac{\lambda_{ij}^{(\alpha)} - \lambda_{ij}^{(\beta)}}{\tau_{\alpha,\beta}}.$$

$$\mathbf{v}_{\Omega,\alpha} = \chi_\alpha \frac{e}{4\pi^2 c \hbar^2 \nu_\alpha} \mathbf{B}_\alpha.$$

# Kinetic equations

Diffusive approximation:  $\tau_{\alpha,\alpha} \ll \tau_{\alpha,\beta}$  and  $\tau_{\alpha,\alpha}\omega \ll 1$

$$\partial_t N_\alpha + (\nabla \cdot \mathbf{j}_\alpha) - e^2 \nu_\alpha \left( \left[ \mathbf{E} + \frac{1}{e} \lambda_{ij}^{(\alpha)} \nabla u_{ij} \right] \cdot \mathbf{v}_{\Omega,\alpha} \right) = - \sum_{\beta}^{N_W} T_{\alpha,\beta} N_\beta - u_{ij} \sum_{\beta}^{N_W} \frac{\lambda_{ij}^{(\alpha)} - \lambda_{ij}^{(\beta)}}{\tau_{\alpha,\beta}}.$$

Anomalous term

$$\mathbf{v}_{\Omega,\alpha} = \chi_\alpha \frac{e}{4\pi^2 c \hbar^2 \nu_\alpha} \mathbf{B}_\alpha.$$

# Kinetic equations

Diffusive approximation:  $\tau_{\alpha,\alpha} \ll \tau_{\alpha,\beta}$  and  $\tau_{\alpha,\alpha}\omega \ll 1$

$$\partial_t N_\alpha + (\nabla \cdot \mathbf{j}_\alpha) - e^2 \nu_\alpha \left( \left[ \mathbf{E} + \frac{1}{e} \lambda_{ij}^{(\alpha)} \nabla u_{ij} \right] \cdot \mathbf{v}_{\Omega,\alpha} \right) = - \sum_{\beta}^{N_W} T_{\alpha,\beta} N_\beta - u_{ij} \sum_{\beta}^{N_W} \frac{\lambda_{ij}^{(\alpha)} - \lambda_{ij}^{(\beta)}}{\tau_{\alpha,\beta}}.$$

Anomalous term

$$\mathbf{v}_{\Omega,\alpha} = \chi_\alpha \frac{e}{4\pi^2 c \hbar^2 \nu_\alpha} \mathbf{B}_\alpha.$$

Inter-valley transfer induced by deformation

# Kinetic equations

Diffusive approximation:  $\tau_{\alpha,\alpha} \ll \tau_{\alpha,\beta}$  and  $\tau_{\alpha,\alpha}\omega \ll 1$

$$\partial_t N_\alpha + (\nabla \cdot \mathbf{j}_\alpha) - e^2 \nu_\alpha \left( \left[ \mathbf{E} + \frac{1}{e} \lambda_{ij}^{(\alpha)} \nabla u_{ij} \right] \cdot \mathbf{v}_{\Omega,\alpha} \right) = - \sum_{\beta}^{N_W} T_{\alpha,\beta} N_\beta - u_{ij} \sum_{\beta}^{N_W} \frac{\lambda_{ij}^{(\alpha)} - \lambda_{ij}^{(\beta)}}{\tau_{\alpha,\beta}}.$$

Anomalous term

$$\mathbf{v}_{\Omega,\alpha} = \chi_\alpha \frac{e}{4\pi^2 c \hbar^2 \nu_\alpha} \mathbf{B}_\alpha.$$

$$T_{\alpha,\beta} = \delta_{\alpha,\beta} \sum_{\gamma}^{N_W} \frac{1}{\tau_{\alpha,\gamma}} - \frac{1}{\tau_{\beta,\alpha}}.$$

Inter-valley transfer induced by deformation

# Kinetic equations

Diffusive approximation:  $\tau_{\alpha,\alpha} \ll \tau_{\alpha,\beta}$  and  $\tau_{\alpha,\alpha}\omega \ll 1$

$$\partial_t N_\alpha + (\nabla \cdot \mathbf{j}_\alpha) - e^2 \nu_\alpha \left( \left[ \mathbf{E} + \frac{1}{e} \lambda_{ij}^{(\alpha)} \nabla u_{ij} \right] \cdot \mathbf{v}_{\Omega,\alpha} \right) = - \sum_{\beta}^{N_W} T_{\alpha,\beta} N_\beta - u_{ij} \sum_{\beta}^{N_W} \frac{\lambda_{ij}^{(\alpha)} - \lambda_{ij}^{(\beta)}}{\tau_{\alpha,\beta}}.$$

Anomalous term

$$\mathbf{v}_{\Omega,\alpha} = \chi_\alpha \frac{e}{4\pi^2 c \hbar^2 \nu_\alpha} \mathbf{B}_\alpha.$$

$$T_{\alpha,\beta} = \delta_{\alpha,\beta} \sum_{\gamma}^{N_W} \frac{1}{\tau_{\alpha,\gamma}} - \frac{1}{\tau_{\beta,\alpha}}.$$

Inter-valley transfer induced by deformation

Currents in each of the nodes:

$$\mathbf{j}_\alpha = -D_\alpha \nabla N_\alpha + \sigma_\alpha \mathbf{E} + e \nu_\alpha D_\alpha \lambda_{ij}^{(\alpha)} \nabla u_{ij} + \frac{e}{\nu_\alpha} \mathbf{v}_{\Omega,\alpha} N_\alpha.$$

Anomalous part of the current

# Kinetic equations

Diffusive approximation:  $\tau_{\alpha,\alpha} \ll \tau_{\alpha,\beta}$  and  $\tau_{\alpha,\alpha}\omega \ll 1$

$$\partial_t N_\alpha + (\nabla \cdot \mathbf{j}_\alpha) - e^2 \nu_\alpha \left( \left[ \mathbf{E} + \frac{1}{e} \lambda_{ij}^{(\alpha)} \nabla u_{ij} \right] \cdot \mathbf{v}_{\Omega,\alpha} \right) = - \sum_{\beta}^{N_W} T_{\alpha,\beta} N_\beta - u_{ij} \sum_{\beta}^{N_W} \frac{\lambda_{ij}^{(\alpha)} - \lambda_{ij}^{(\beta)}}{\tau_{\alpha,\beta}}$$

Anomalous term

$$\mathbf{v}_{\Omega,\alpha} = \chi_\alpha \frac{e}{4\pi^2 c \hbar^2 \nu_\alpha} \mathbf{B}_\alpha.$$

$$T_{\alpha,\beta} = \delta_{\alpha,\beta} \sum_{\gamma}^{N_W} \frac{1}{\tau_{\alpha,\gamma}} - \frac{1}{\tau_{\beta,\alpha}}$$

Inter-valley transfer induced by deformation

Currents in each of the nodes:

$$\mathbf{j}_\alpha = -D_\alpha \nabla N_\alpha + \sigma_\alpha \mathbf{E} + e \nu_\alpha D_\alpha \lambda_{ij}^{(\alpha)} \nabla u_{ij} + \frac{e}{\nu_\alpha} \mathbf{v}_{\Omega,\alpha} N_\alpha.$$

Anomalous part of the current

Gauss law:

$$\nabla \cdot \mathbf{E} = 4\pi \sum_{\alpha}^{N_W} N_\alpha.$$

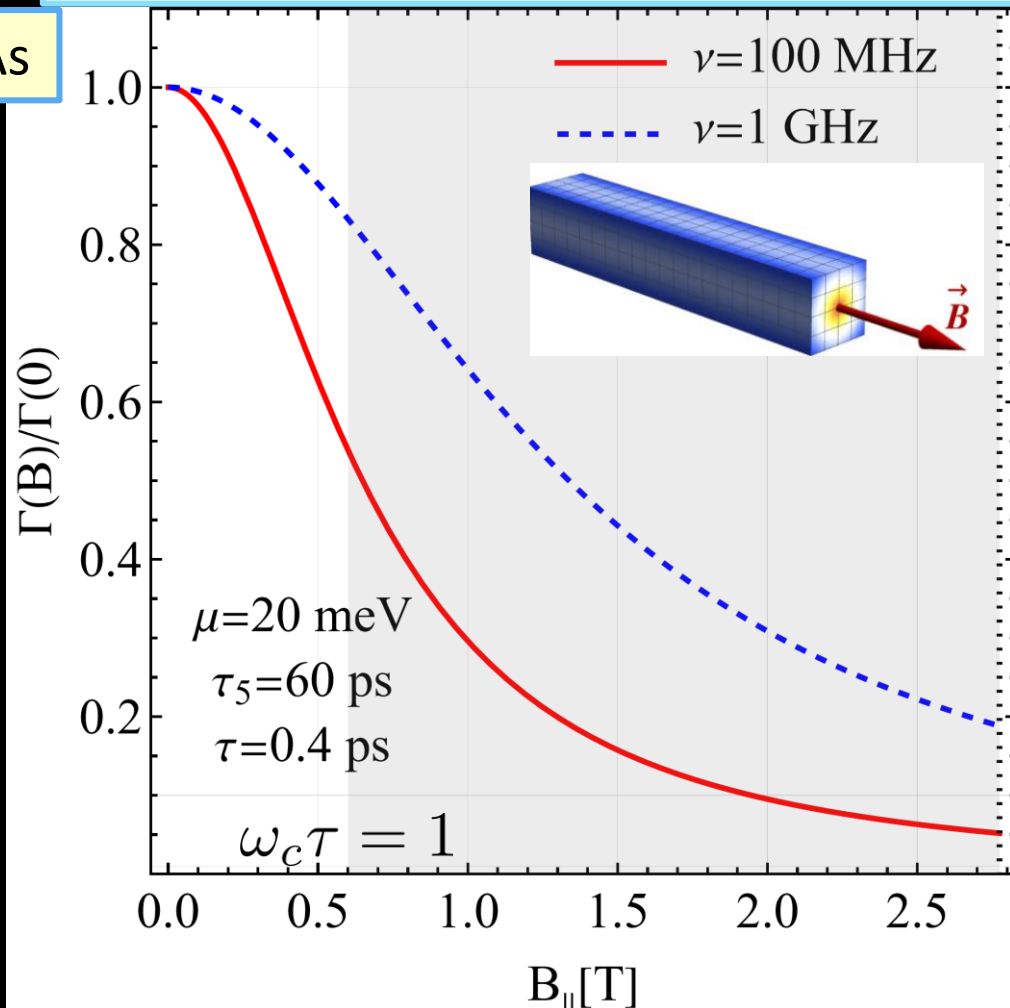
Inclusion of the Gauss law is the key difference from the previous studies

[B. Z. Spivak and A. V. Andreev, Phys. Rev. B **93**, 085107 (2016).]

# Results of the chiral kinetic approach

Screening is strong even in semimetals  
( $\mu \approx 10 - 20$  meV)

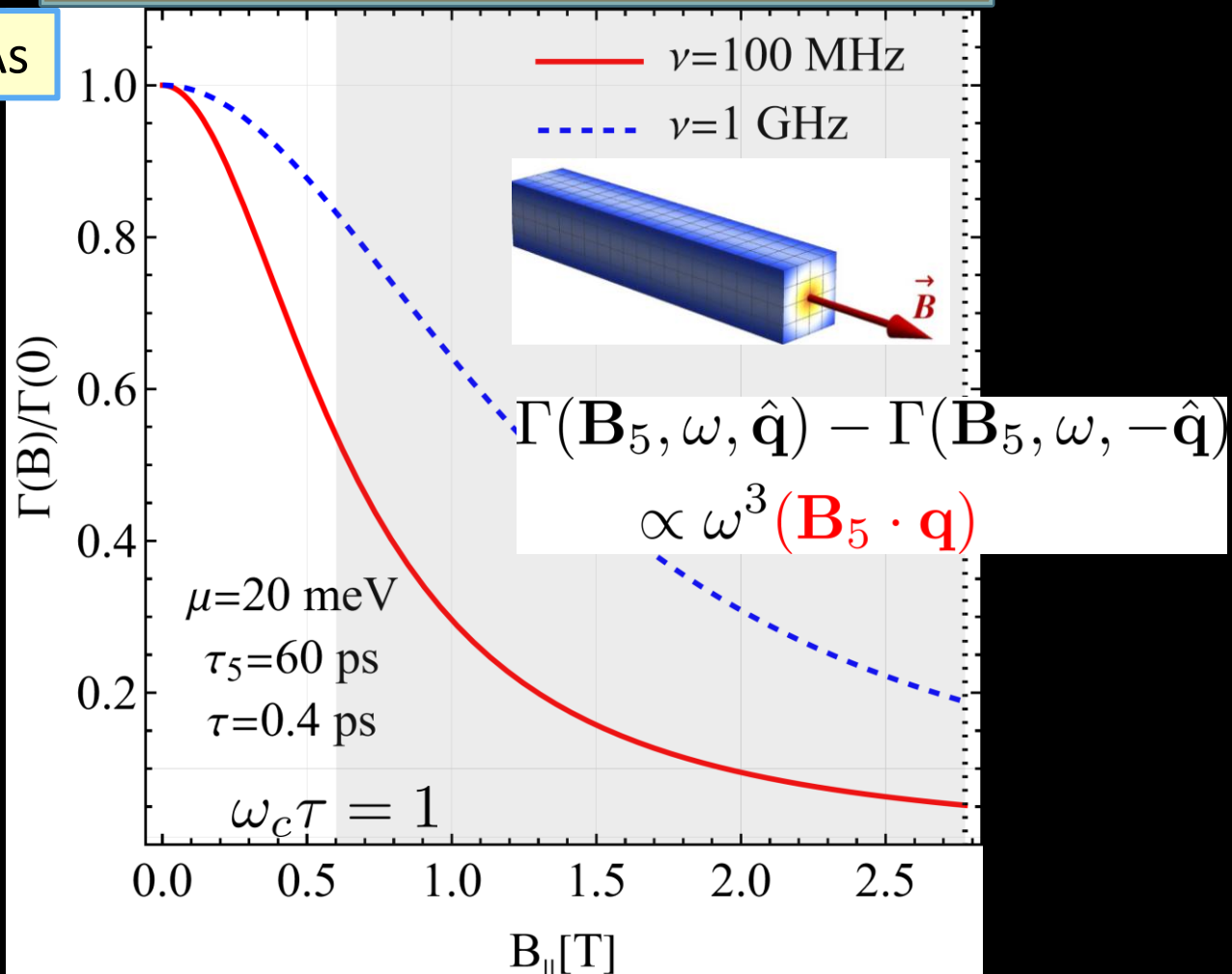
TaAs



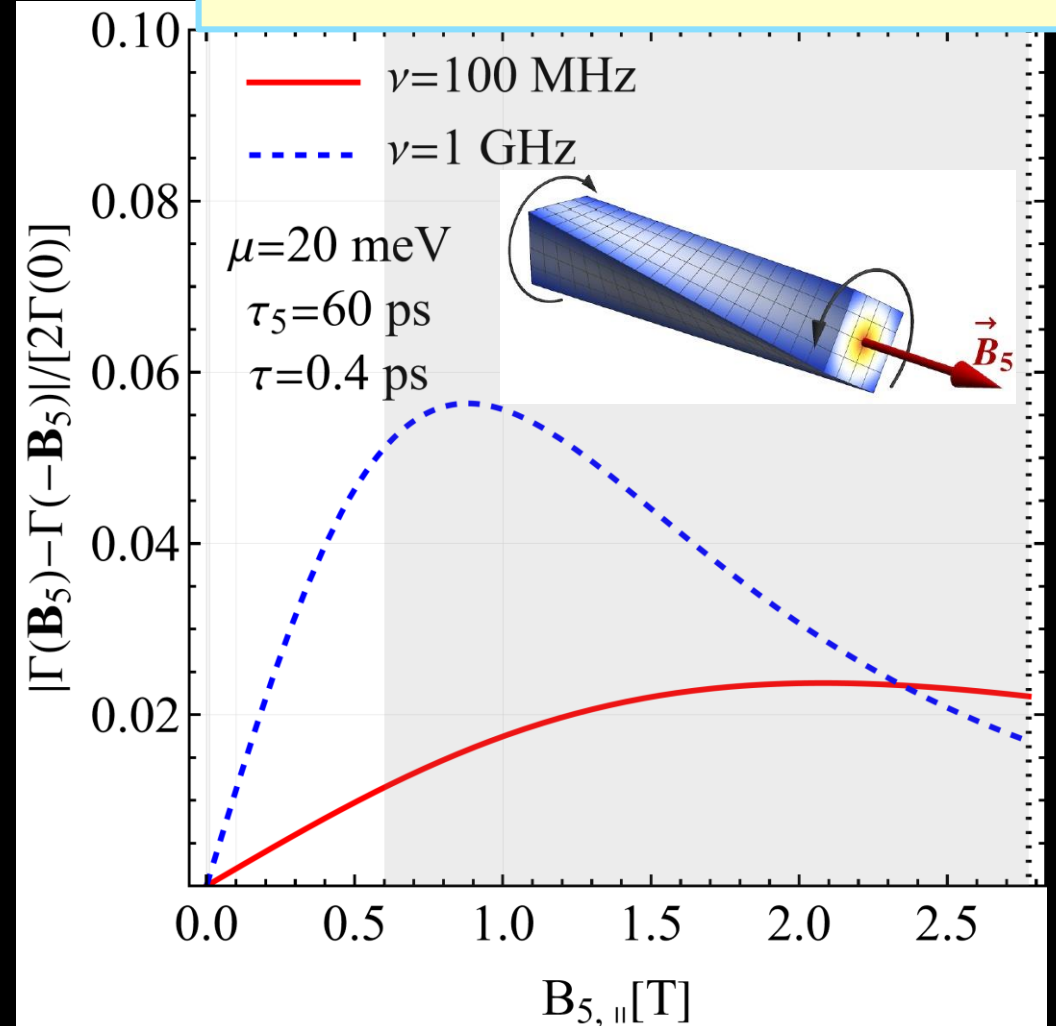
# Results of the chiral kinetic approach

Screening is strong even in semimetals  
 $(\mu \approx 10 - 20 \text{ meV})$

TaAs

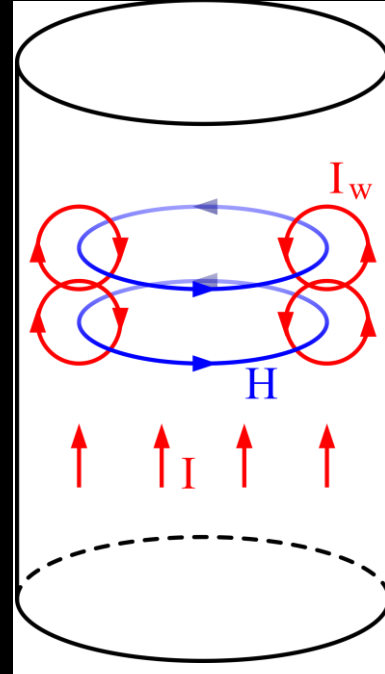
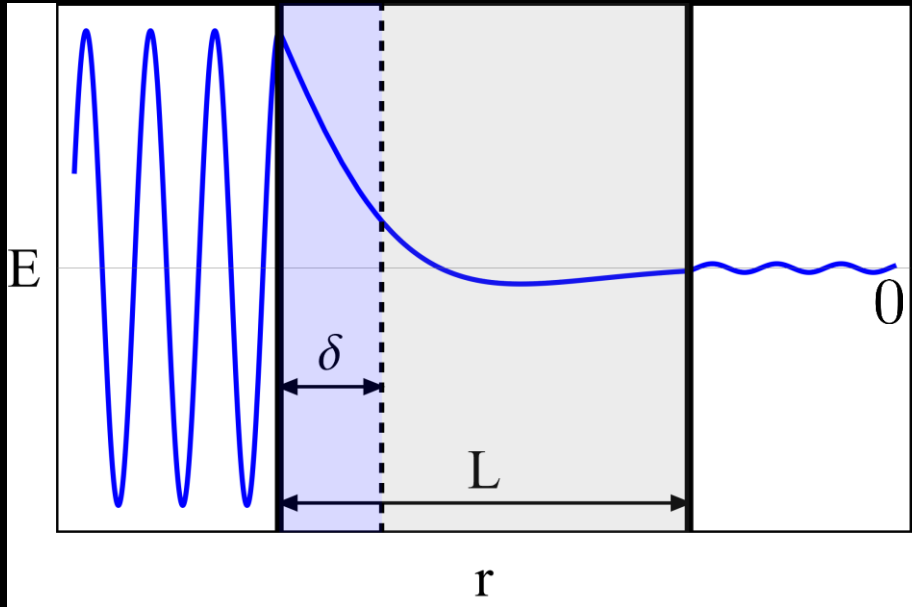


Pseudomagnetic field  $\rightarrow$  **sound attenuation dichroism**



# Light propagation: Skin effect

Conventional skin effect:

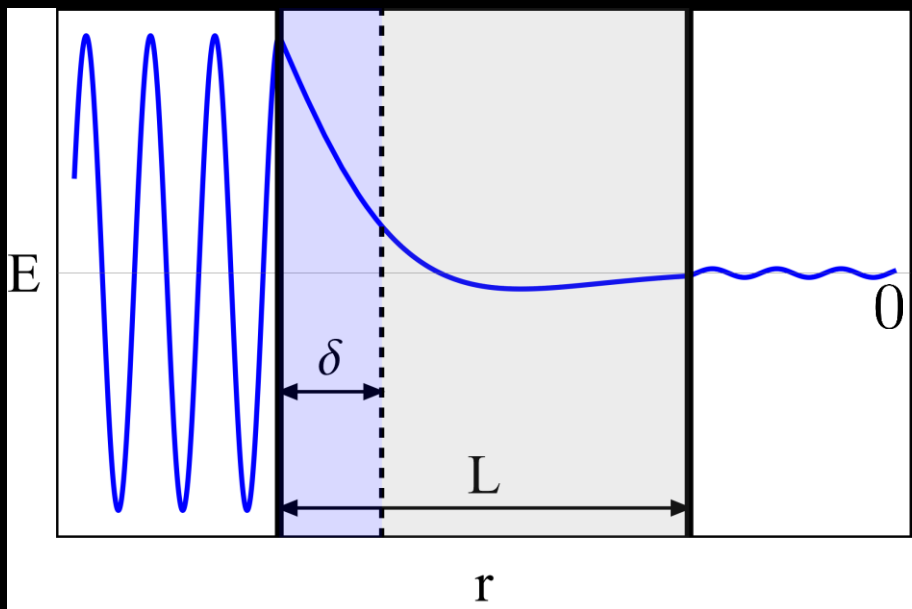


$$E \sim e^{-z/\delta(\omega)}, \quad \delta(\omega) = \frac{c}{\sqrt{2\pi\omega\sigma_0}}$$

$$l_{\text{mfp}} = v_F \tau \ll \delta(\omega)$$

# Light propagation: Skin effect

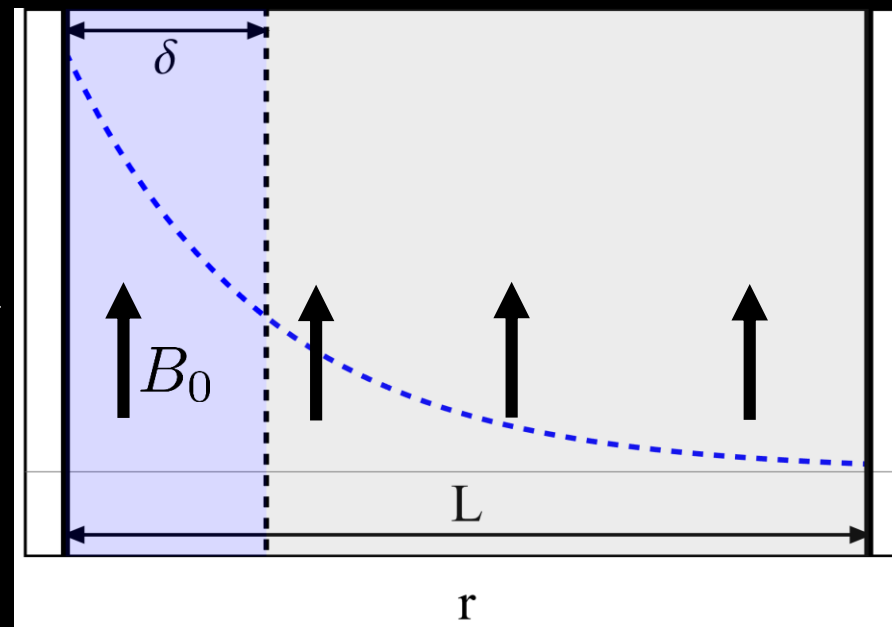
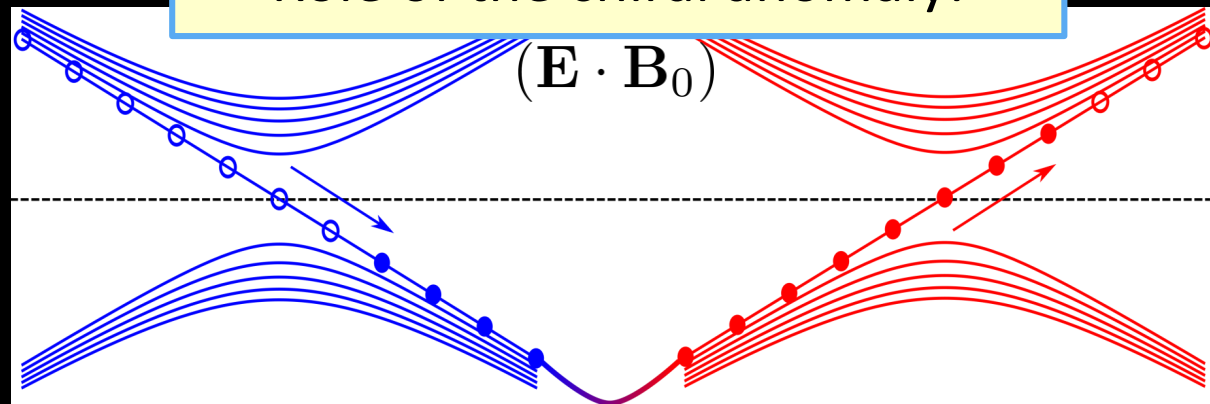
Conventional skin effect:



$$E \sim e^{-z/\delta(\omega)}, \quad \delta(\omega) = \frac{c}{\sqrt{2\pi\omega\sigma_0}}$$

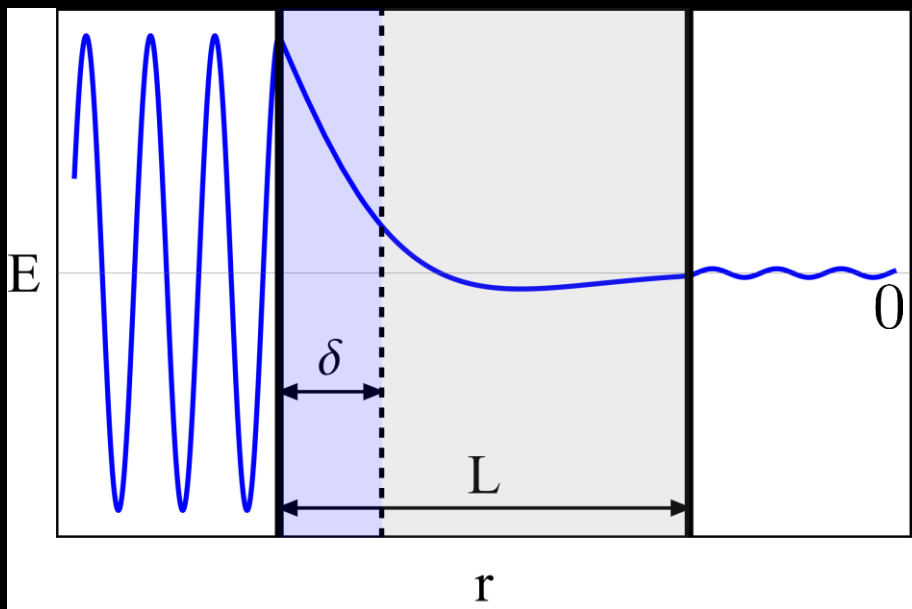
$$l_{\text{mfp}} = v_F \tau \ll \delta(\omega)$$

Role of the chiral anomaly:



# Light propagation: Skin effect

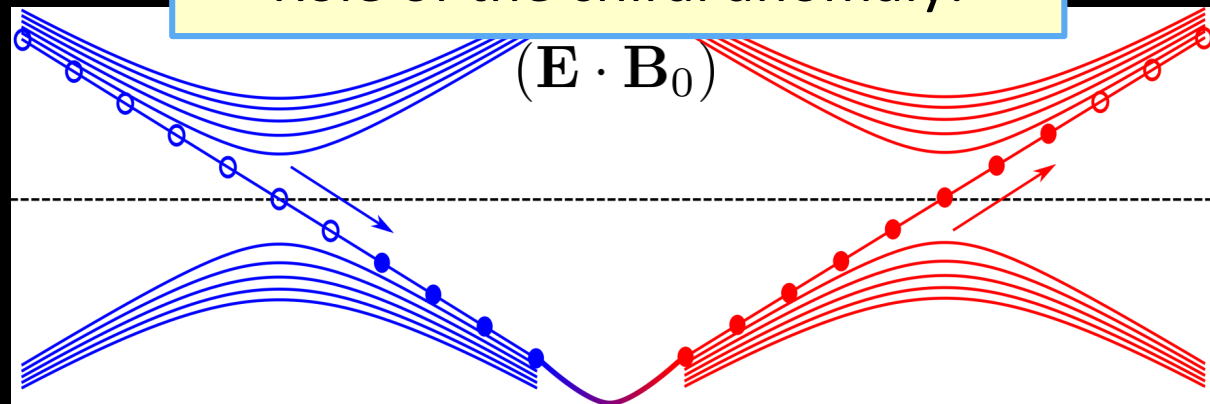
Conventional skin effect:



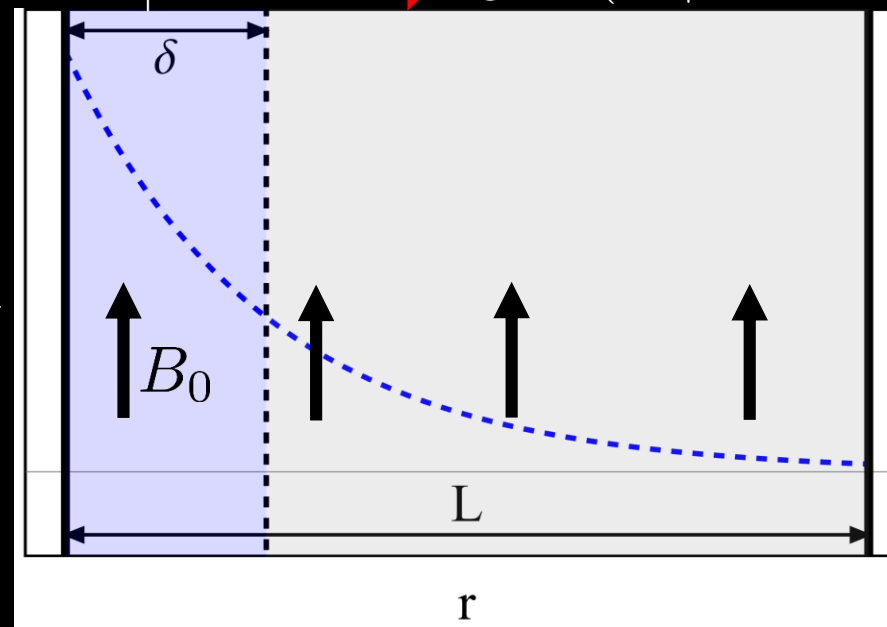
$$E \sim e^{-z/\delta(\omega)}, \quad \delta(\omega) = \frac{c}{\sqrt{2\pi\omega\sigma_0}}$$

$$l_{\text{mfp}} = v_F \tau \ll \delta(\omega)$$

Role of the chiral anomaly:

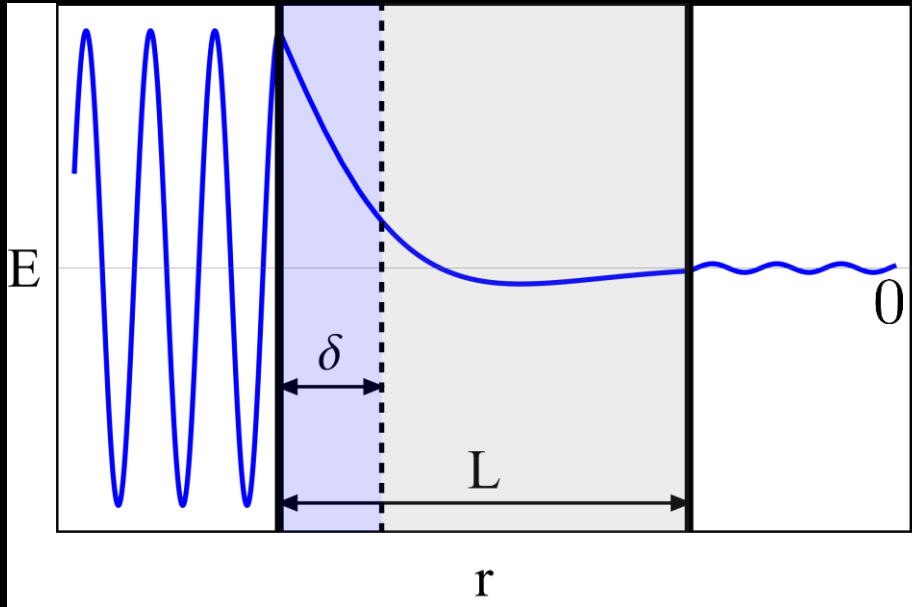


$$N_+ - N_- \rightarrow \mathbf{j} \sim (N_+ - N_-) \mathbf{B}_0$$



# Light propagation: Skin effect

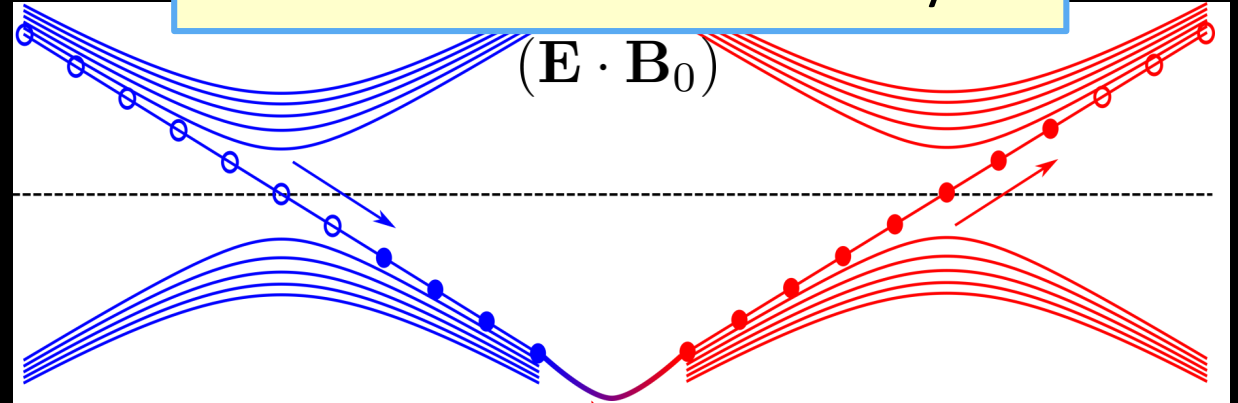
Conventional skin effect:



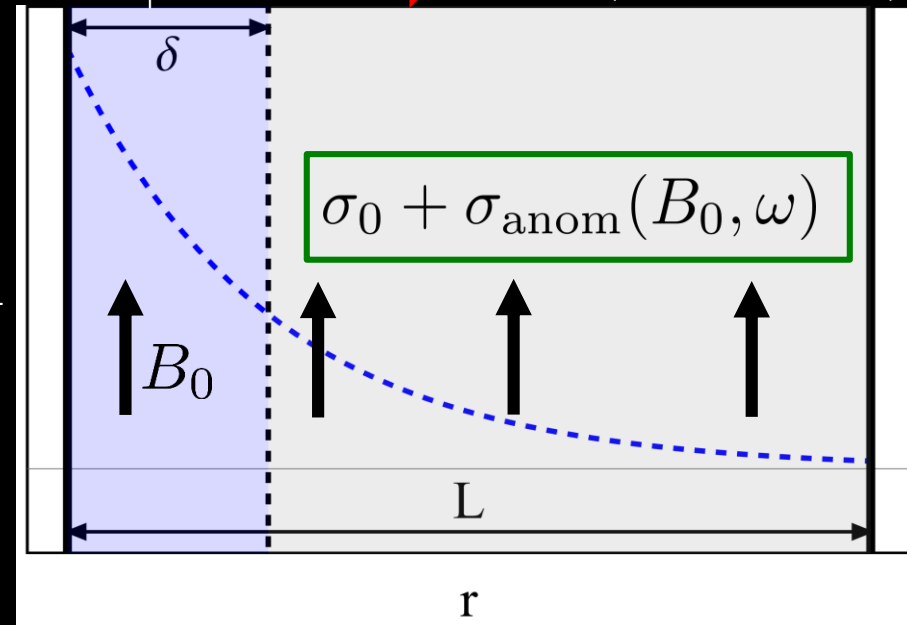
$$E \sim e^{-z/\delta(\omega)}, \quad \delta(\omega) = \frac{c}{\sqrt{2\pi\omega\sigma_0}}$$

$$l_{\text{mfp}} = v_F \tau \ll \delta(\omega)$$

Role of the chiral anomaly:



$$N_+ - N_- \rightarrow \mathbf{j} \sim (N_+ - N_-) \mathbf{B}_0$$



# Enhancement of light transmission

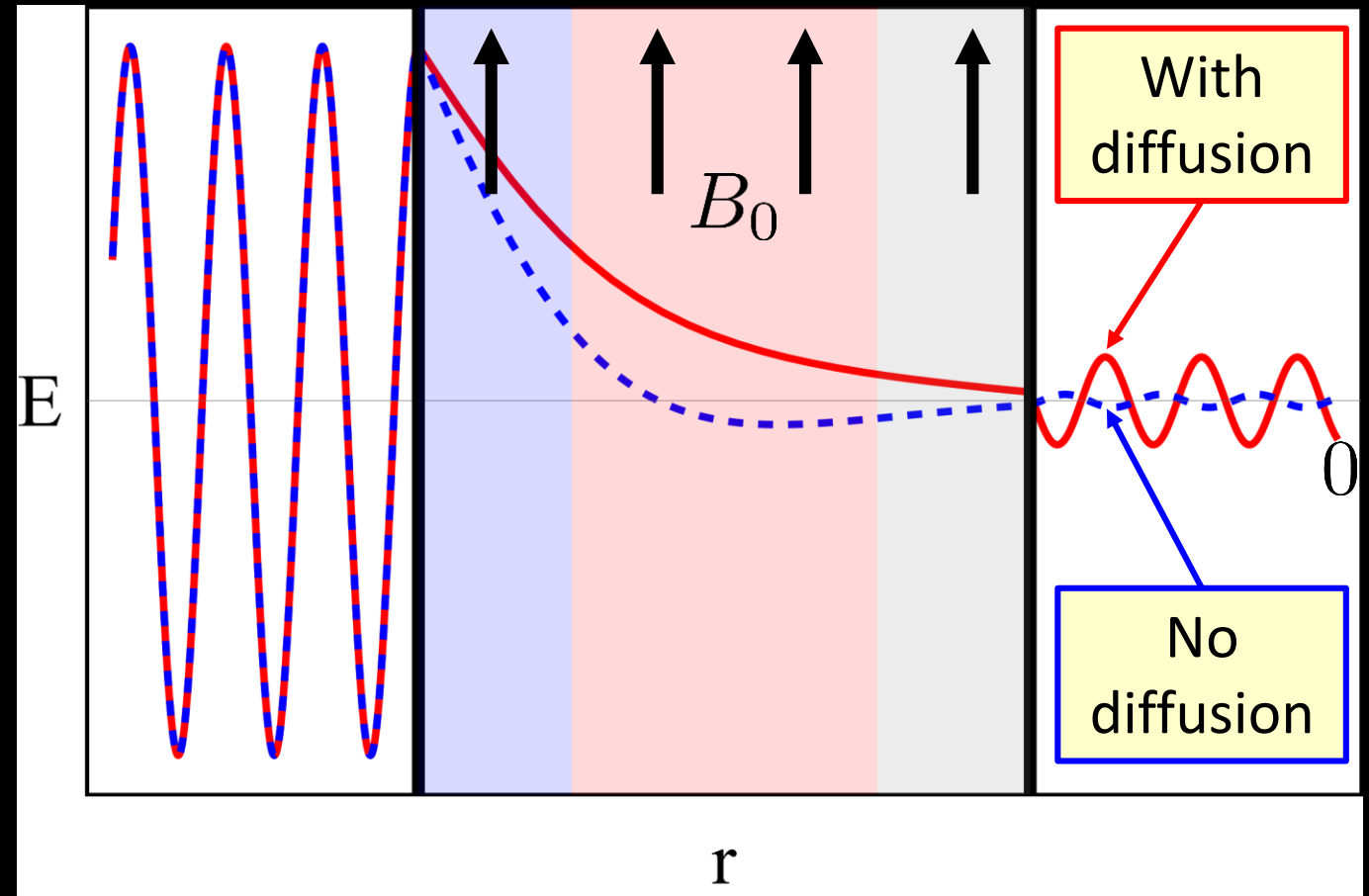
Chiral anomaly + diffusion of the chiral charge



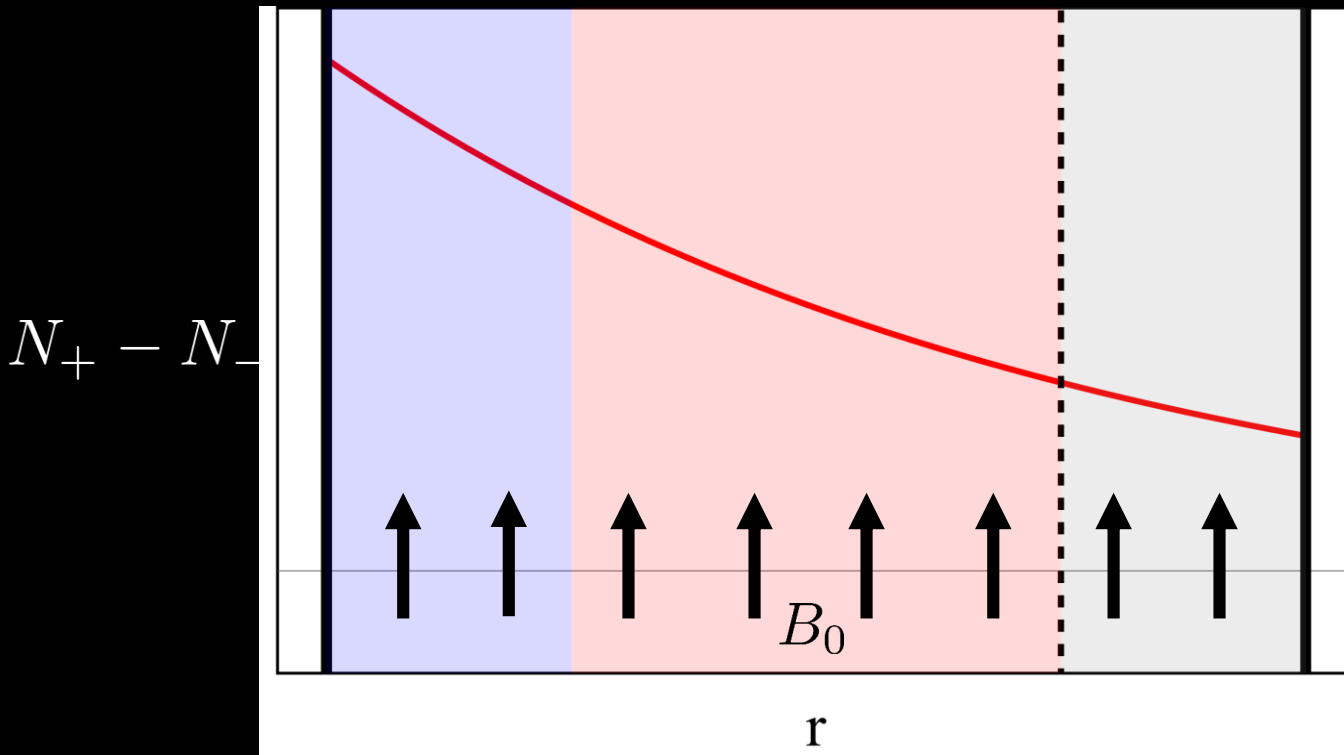
Anomalous nonlocal regime



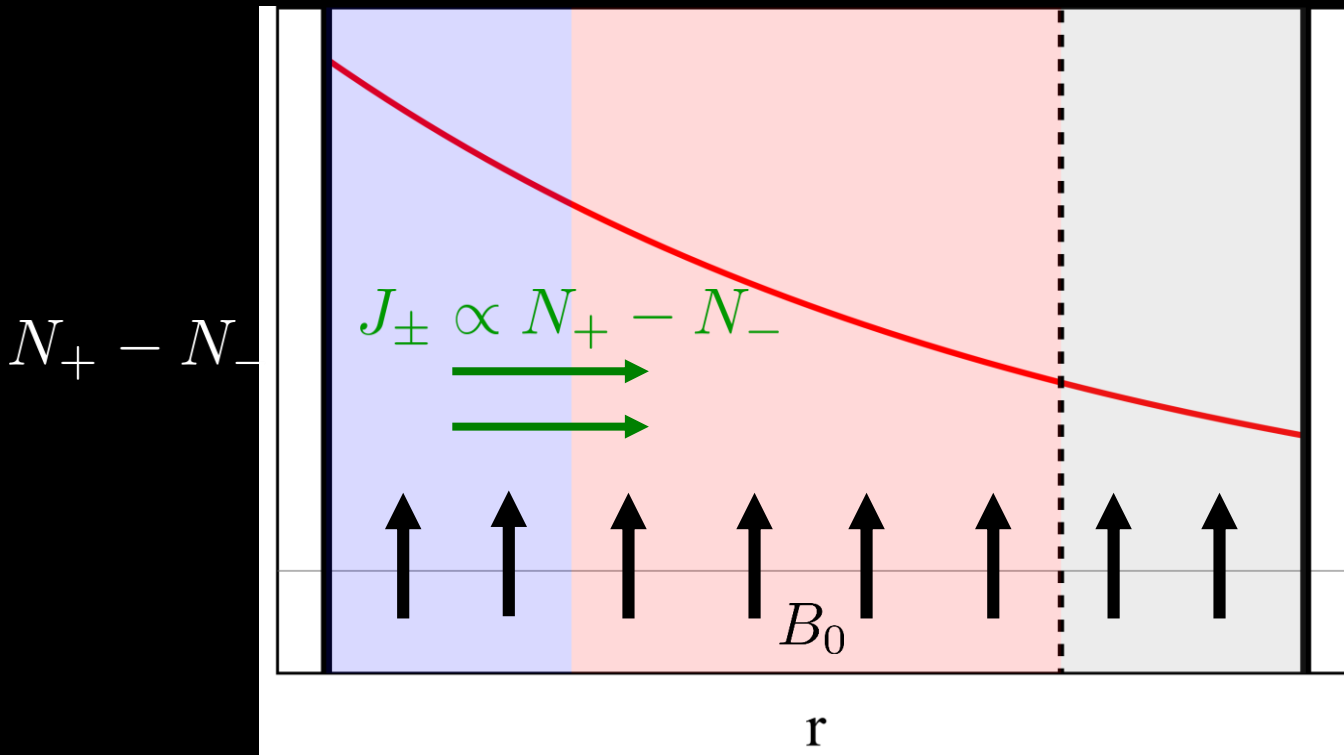
Transmission of electromagnetic field is **enhanced**



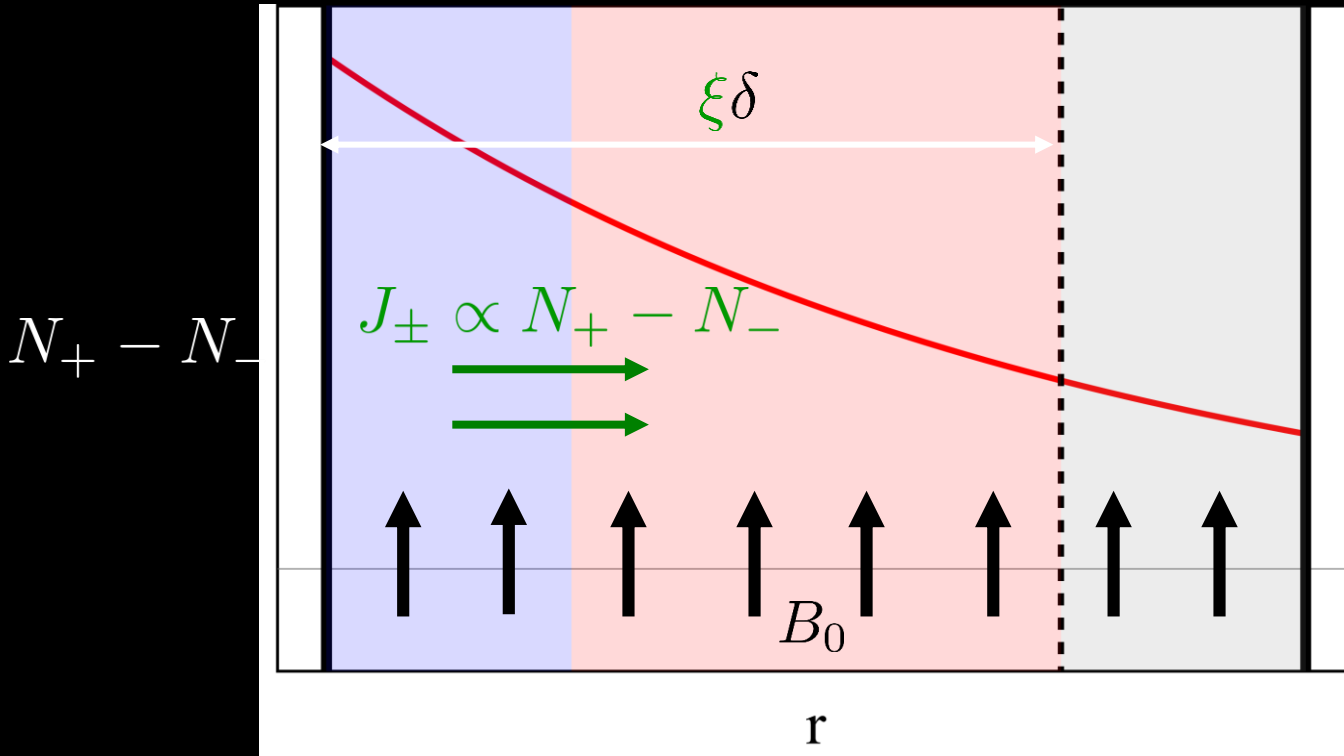
# Nonlocal regime



# Nonlocal regime



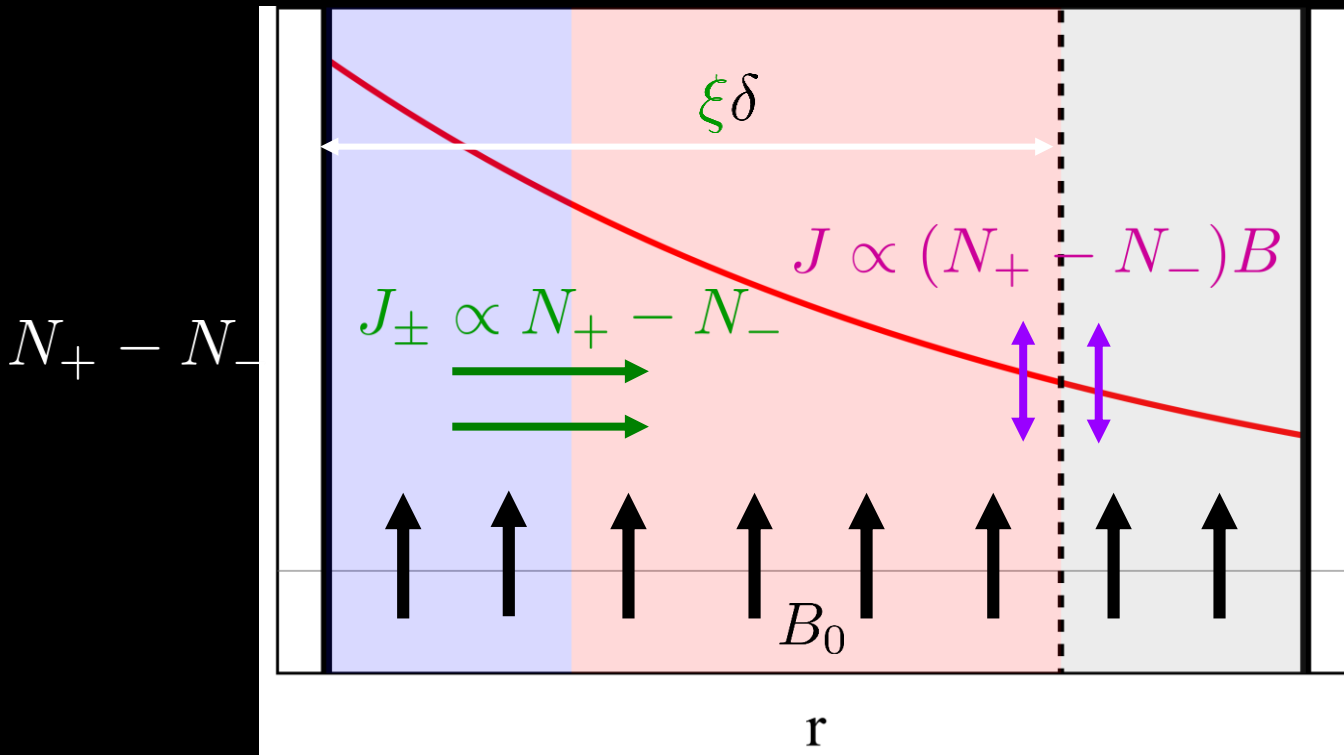
# Nonlocal regime



Nonlocality parameter:

$$\xi = \frac{\sqrt{4\pi\sigma_0 D}}{c}$$

# Nonlocal regime



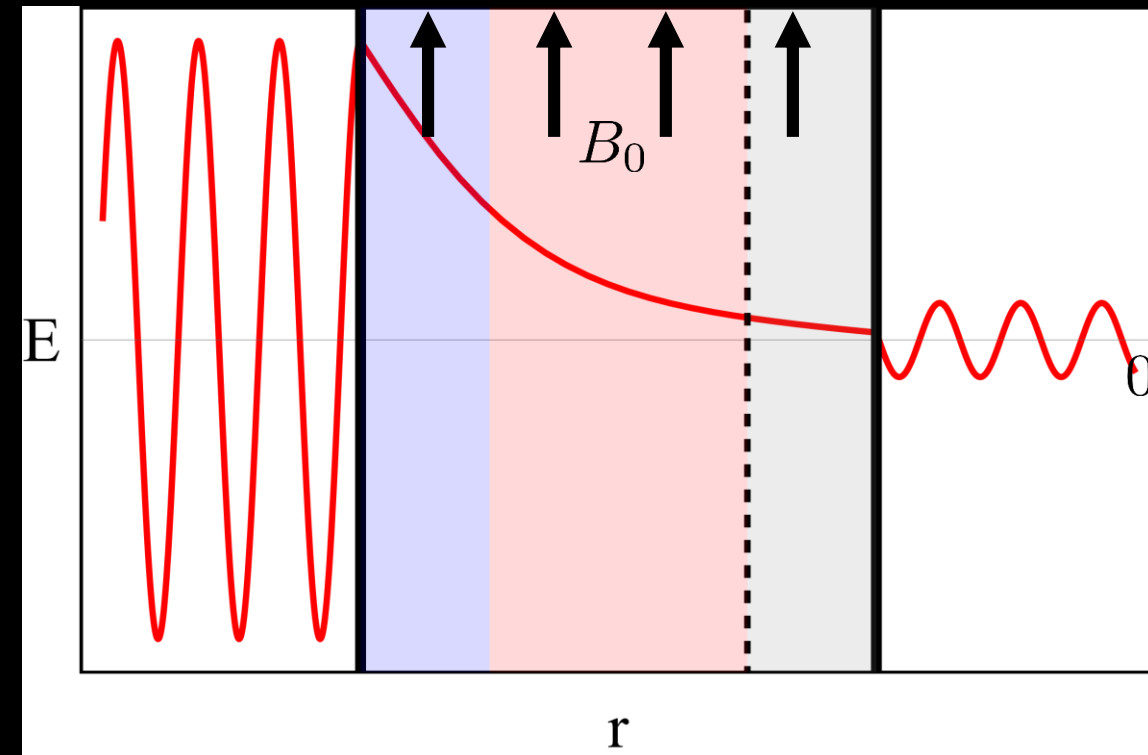
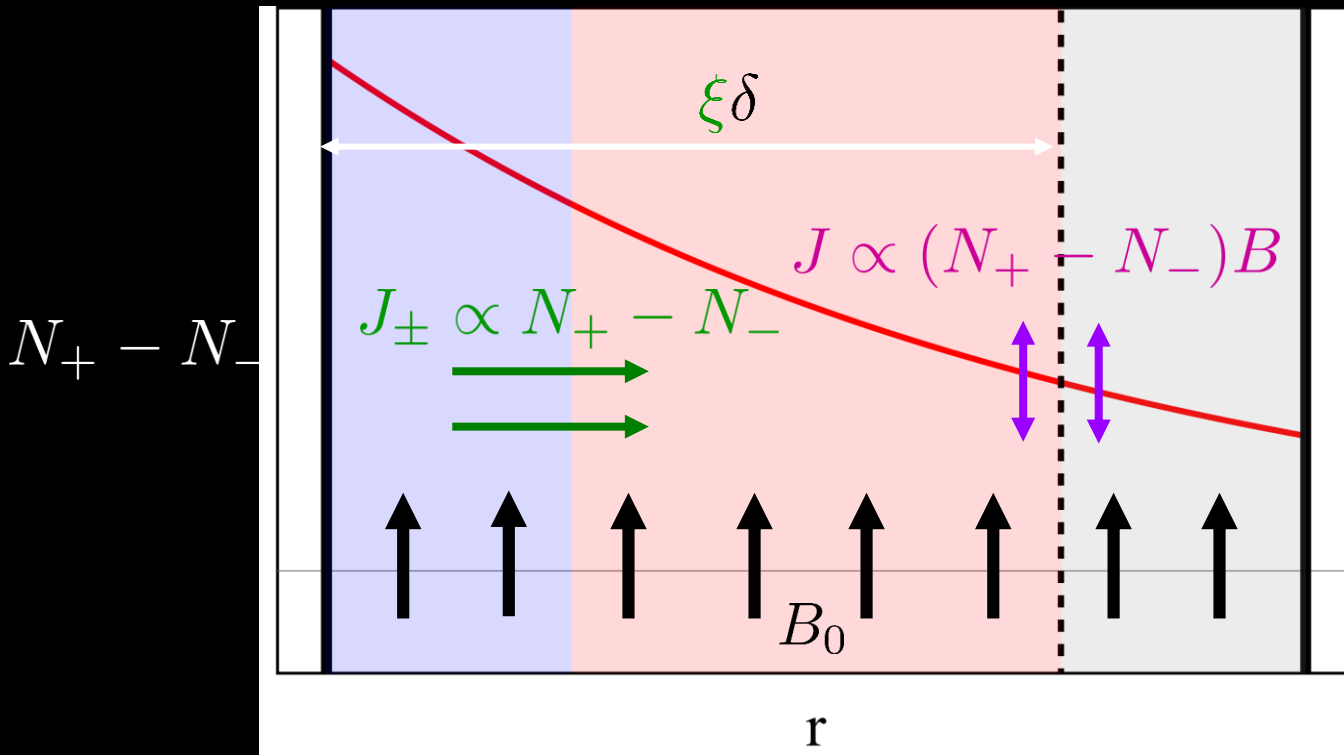
Nonlocality parameter:

$$\xi = \frac{\sqrt{4\pi\sigma_0 D}}{c}$$

# Nonlocal regime

Chiral imbalance diffuses away from the skin layer  $\rightarrow$  skin depth increases

Electromagnetic field penetration enhances



Nonlocality parameter:

$$\xi = \frac{\sqrt{4\pi\sigma_0 D}}{c}$$

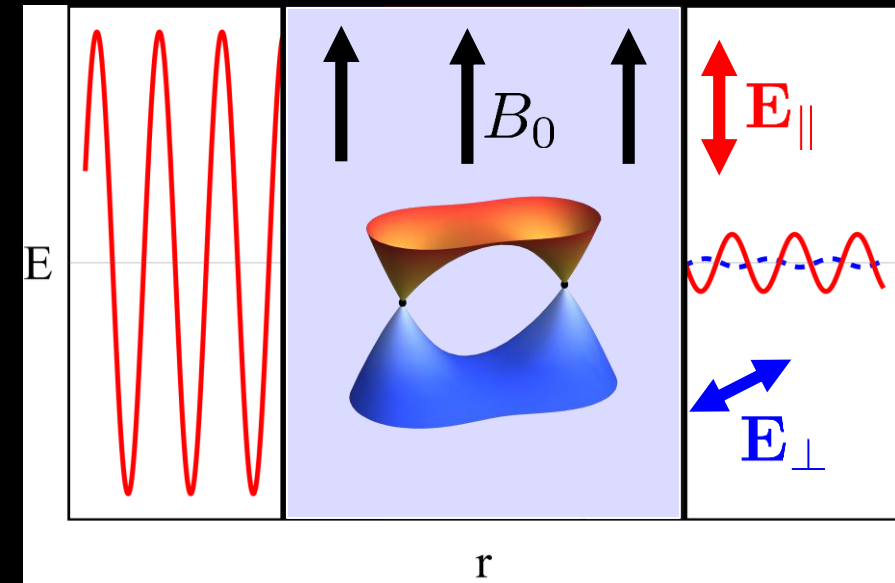
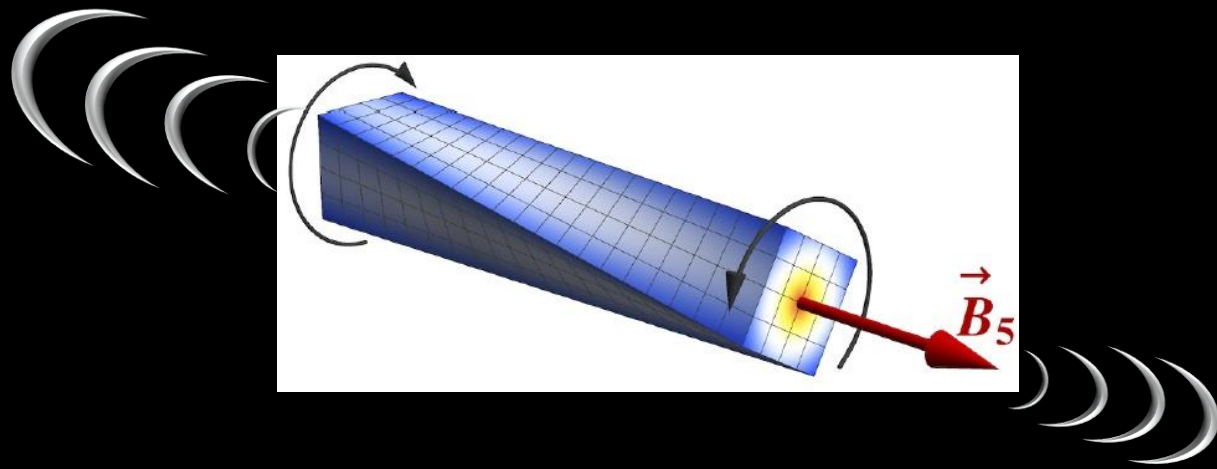
# Summary

---

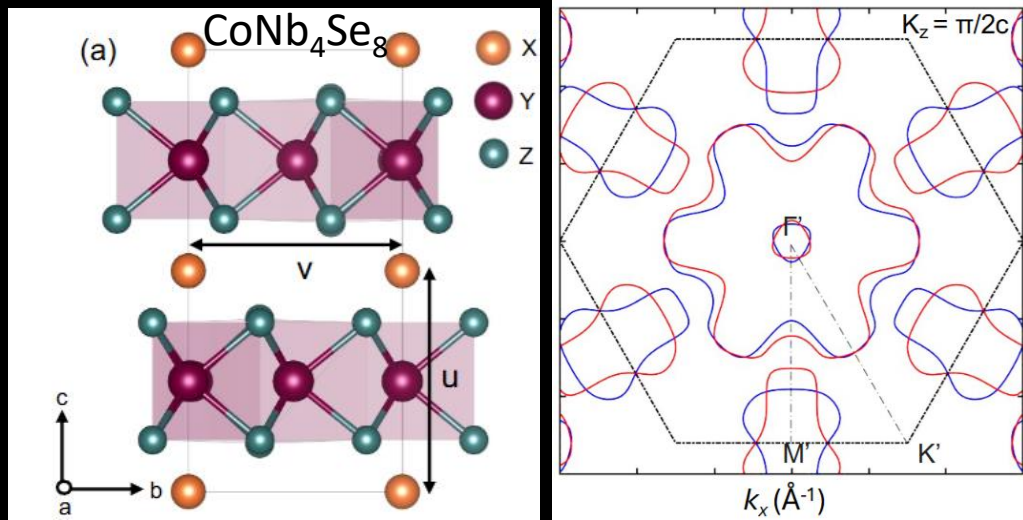
1. The chiral anomaly is manifested in **dynamical responses**
2. **The anomaly reduces the sound attenuation** in Weyl semimetals subject to magnetic fields
3. Strain-induced pseudomagnetic field or non-symmetric Weyl nodes → **sound attenuation dichroism.**
4. The chiral anomaly → **nonlocal regime** in a response to light.
5. Anomalous nonlocal regime → **enhancement of electromagnetic wave penetration depth**

# Possible applications

1. New experimental methods to **identify quantum anomalies**
2. Magnetic and pseudomagnetic fields as **control knobs for sound propagation and attenuation**
3. Sound attenuation dichroism as an **indicator of pseudomagnetic fields**
4. Enhancement of the penetration depth as a way to **measure light polarization**



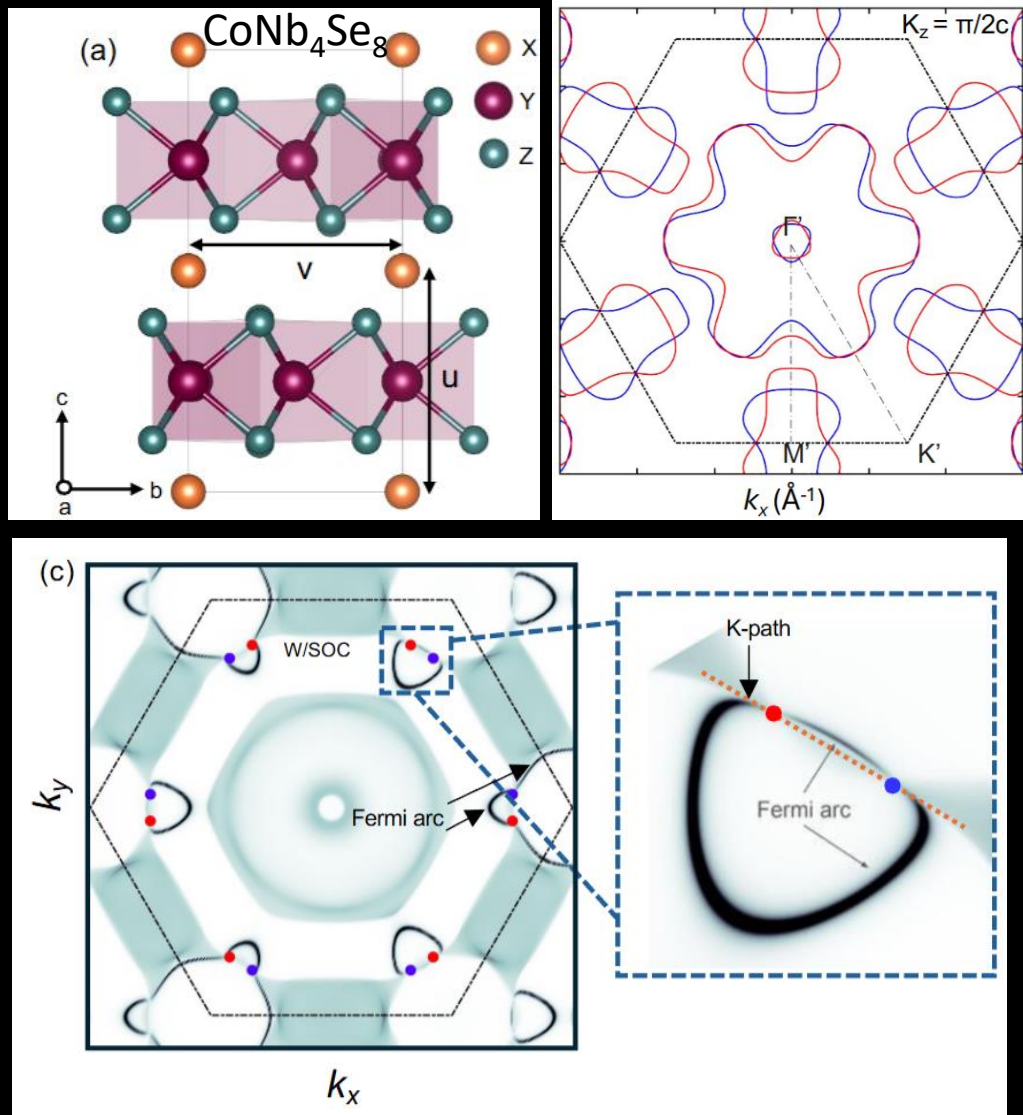
# New directions: Weyl altermagnets



## Definitions:

- Altermagnets: unconventional magnets with spin-split energy bands
- Intercalated TMDs  $\rightarrow$  platforms combining altermagnetism and Weyl physics

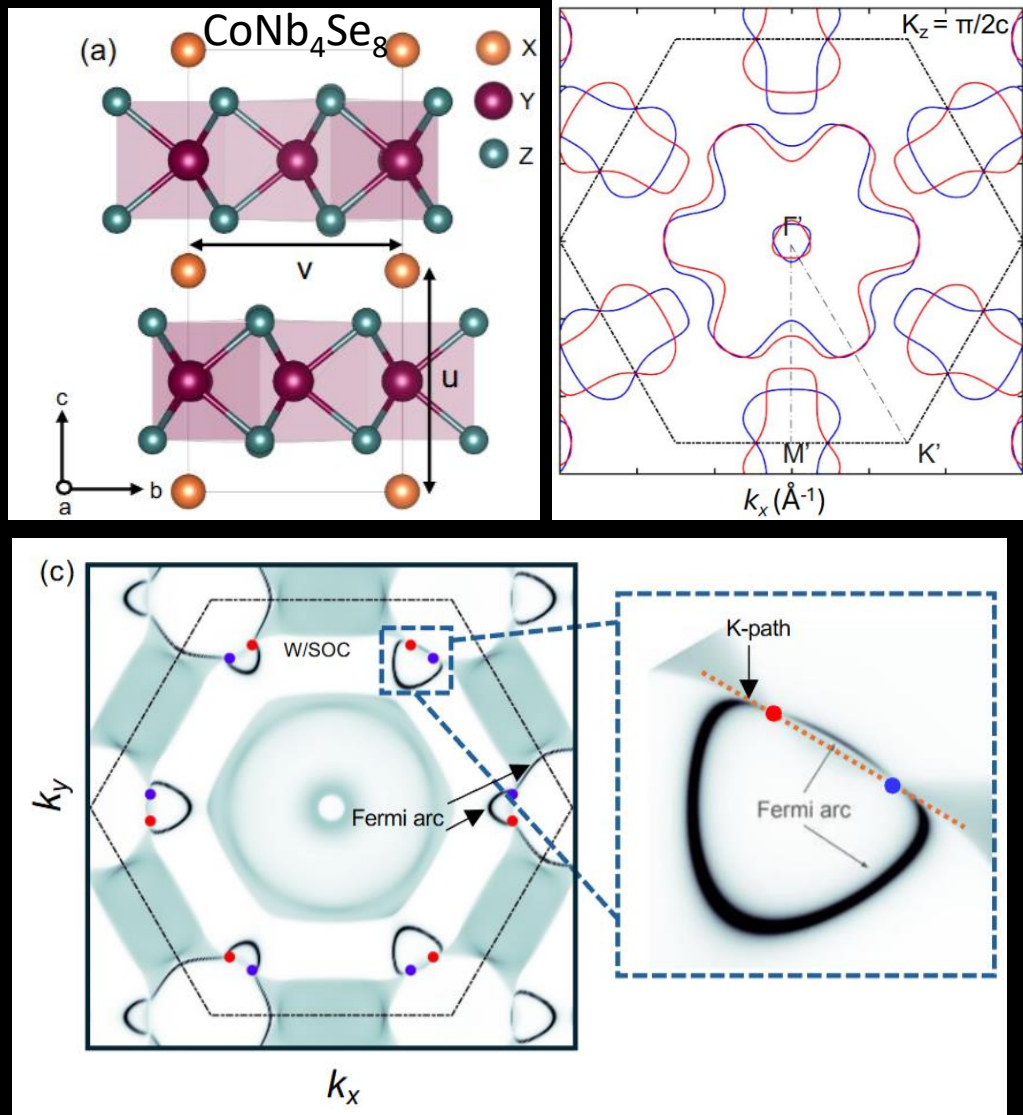
# New directions: Weyl altermagnets



## Definitions:

- Altermagnets: unconventional magnets with spin-split energy bands
- Intercalated TMDs  $\rightarrow$  platforms combining altermagnetism and Weyl physics

# New directions: Weyl altermagnets



## Definitions:

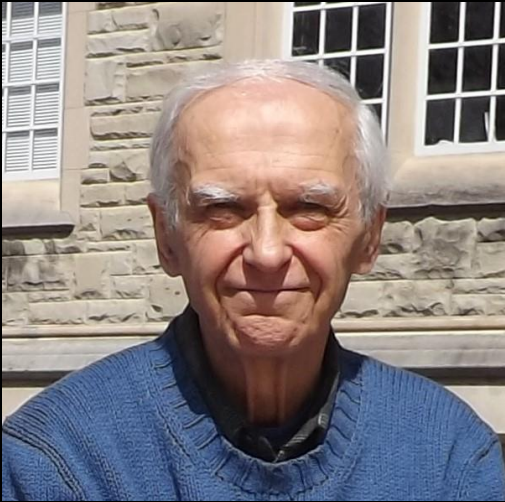
- Altermagnets: unconventional magnets with spin-split energy bands
- Intercalated TMDs  $\rightarrow$  platforms combining altermagnetism and Weyl physics

## Open questions:

- Interplay of altermagnetism and quantum anomalies
- Electric and spin transport properties of Weyl altermagnets
- Interaction effects: magnetic catalysis and superconductivity

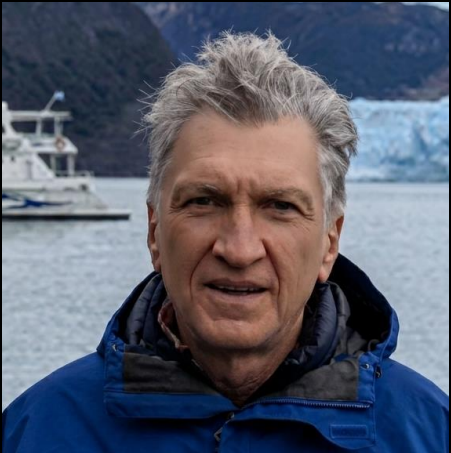
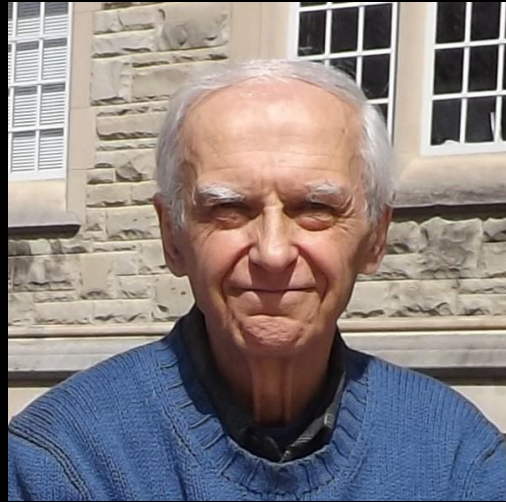
# Acknowledgements

---



# Acknowledgements

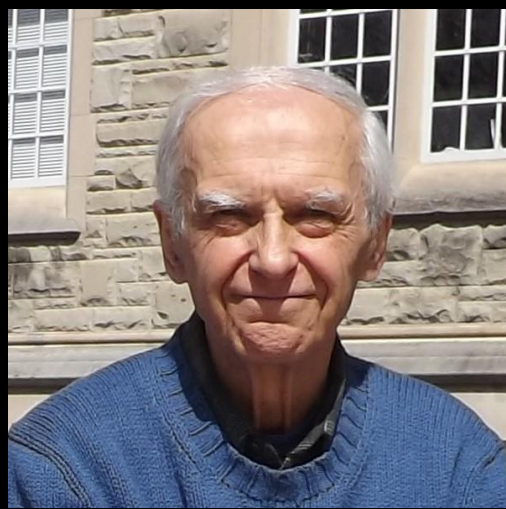
---



Yale University



# Acknowledgements



# Summary

---

1. The chiral anomaly is manifested in **dynamical responses**
2. **The anomaly reduces the sound attenuation** in Weyl semimetals subject to magnetic fields.
3. Strain-induced pseudomagnetic field or non-symmetric Weyl nodes → **sound attenuation dichroism.**
4. The chiral anomaly → **nonlocal regime** in a response to light.
5. Anomalous nonlocal regime → **enhancement of electromagnetic wave penetration depth**