Gap generation in dice model: an avatar of magnetic catalysis

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V.A. Miransky, 1987-2020, PhD supervisor (1993) , 51 joint papers

International Conference on Symmetry Breaking in QFT, Hefei, China

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V.A. Miransky – life devoted to physics

Elementary particles, gauge theories

1) Dynamical chiral symmetry breaking, strongly coupled QED

Dynamical symmertry breaking and particle mass generation in gauge field theories, P.I. Fomin, V.P. Gusynin, V.A. Miransky, Yu.A. Sitenko, *Riv. Nuovo Cim.* **6** (1983) 1

2) $\bar{t}t$ – condensate

Is the t quark responsible for the mass of W and Z bosons? V.A. Miransky, M. Tanabashi, K. Yamawaki, *Mod. Phys. Lett. A* **4** (1989) 1043

3) Magnetic catalysis

Catalysis of dynamical flavor symmetry breaking by a magnetic field, V.P. Gusynin, V.A. Miransky, I.A. Shovkovy, *Phys. Rev. Lett.* **73** (1994) 3499

4) Conformal phase transition, Miransky scaling

Conformal phase transition in gauge theories, V.A. Miransky, K. Yamawaki, *Phys. Rev. D* **55** (1997) 5051

5) Color superconductivity

Schwinger-Dyson approach to color superconductivity in dense QCD, D.K. Hong, V.A. Miransky, I.A. Shovkovy, L.C.R. Wijewardhana, *Phys. Rev. D* **61** (2000) 056001

6) Reduced QED

Dynamical symmetry breaking on a brane in reduced QED, E.V. Gorbar, V.P. Gusynin, V.A. Miransky, *Phys. Rev. D* **64** (2002) 105028

7) Chiral asymmetry in gauge theories in a magnetic field

Radiative corrections to chiral separation effect in QED, E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, Xinyang Wang, *Phys. Rev. D* **88** (2013) 025025

Condensed matter

8) Gap generation in graphene

Magnetic field driven metal insulator phase transition in planar systems, E.V. Gorbar, V.P. Gusynin, V.A. Miransky, I.A. Shovkovy, *Phys. Rev. B* **66** (2002) 045108

9) Chiral anomaly in Dirac and Weyl semimetals

Chiral anomaly, dimensional reduction, and magnetoresistivity of Weyl and Dirac semimetals, E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, *Phys. Rev. B* **89** (2014) 085126

10) Consistent chiral anomaly in magnetic and pseudomagnetic fields in Weyl semimetals

Consistent chiral kinetic theory in Weyl materials: chiral magnetic plasmons, E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, P.O. Sukhachov, *Phys. Rev. Lett.* **118** (2017) 127601

Monographs

1) Models of strongly interacting elementary particles, V.P. Shelest, V.A. Miransky, G.M. Zinovjev, Atomizdat, Vol.1, 1975, Vol. 2, 1976.

2) Dynamical symmetry breaking in quantum field theories, V.A. Miransky, World Scientific, 1994.

3) Electronic properties of Dirac and Weyl semimetals, E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, P.O. Sukhachov, World Scientific, 2021.

Bound state formation in non-relativistic physics

Softer energy-momentum dispersion (larger *n*) favors bound state formation

$$\Psi(\vec{p}) = g \int \frac{d^3k}{(2\pi)^3} \frac{V\left(\vec{k} - \vec{p}\right)}{a|\vec{k}|^n + \Delta} \Psi\left(\vec{k}\right), \qquad E = -\Delta < 0$$

$\Delta \to 0, \quad \int \frac{d^3k}{(2\pi)^3} \frac{1}{|\vec{k}|^2}$ converges in *IR* $\rightarrow$ *critical coupling* $g_{cr} > 0$

critical coupling is absent for softer dispersion n≥3, i.e., $g_{cr} = 0$

Magnetic catalysis of gap generation

G. Jona-Lasinio, F.M. Marchetti, *On the pairing structure of the vacuum induced by a magnetic field in 2+1 – dimensional Dirac theory*, Phys. Lett. B 459 (1999) 208

(2+1)-dimensional NJL model, gap equation

$$m = 4iG \int \frac{d\omega d^2k}{(2\pi)^3} \frac{m}{\omega^2 - \vec{k}^2 - m^2 + i\epsilon}, \qquad G_{cr} = \frac{4\pi^{3/2}}{\Lambda}$$

In a magnetic field, gap equation in LLL approximation

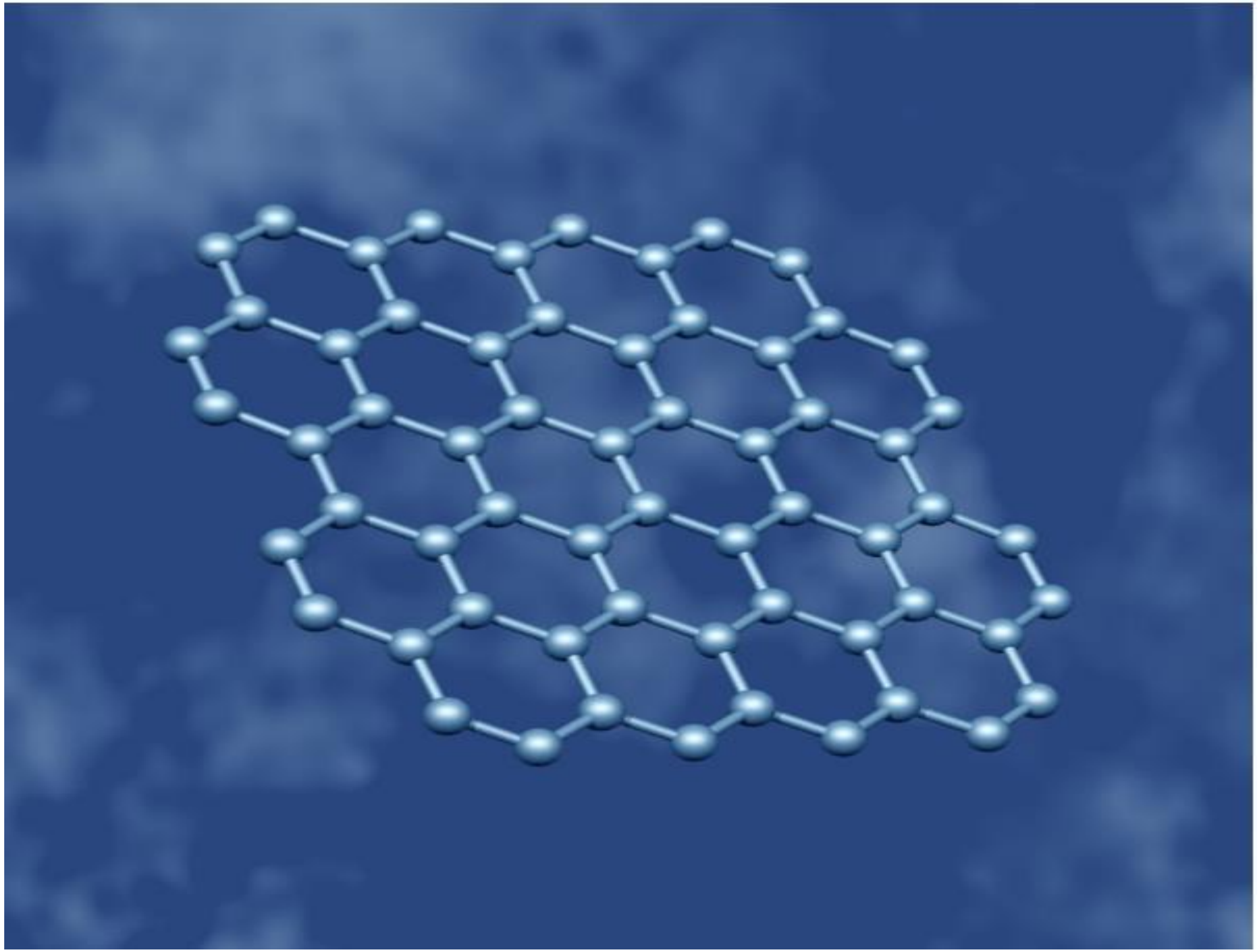
$$m = 4iG \int \frac{d\omega d^2k}{(2\pi)^3} \frac{me^{-k^2/(eB)}}{\omega^2 - m^2 + i\epsilon},$$

solution $m = \frac{G|eB|}{2\pi}$ exists for any G,

$G_{cr}=0$ – magnetic catalysis

Key feature: dispersionless Landau levels (i.e., flat bands) $E_n = \sqrt{m^2 + 2|eB|n}$, n=0,1,2,…

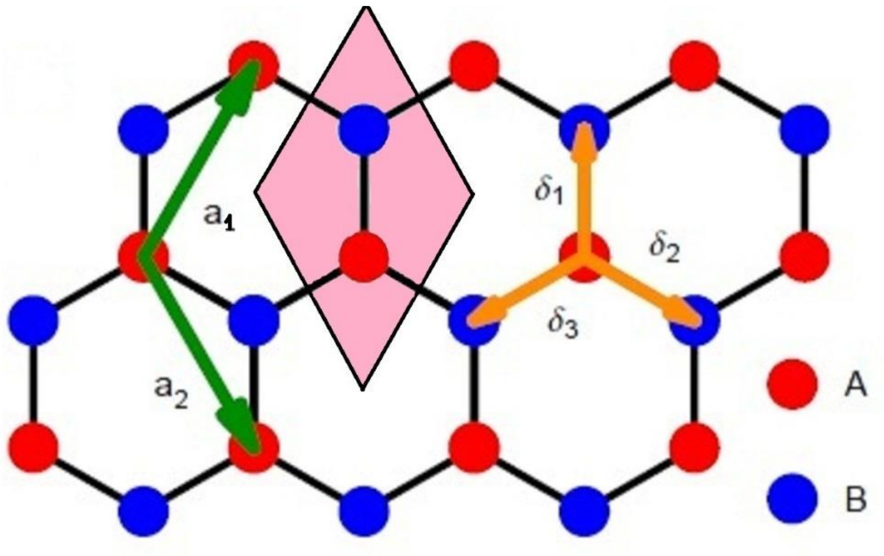
Graphene – 2D Dirac material



Reduced QED framework:

Dirac fermions propagate in a plane, electromagnetic fields propagate in (3+1) dimensions

Hexagonal lattice, band structure, Dirac quasiparticles at low energy

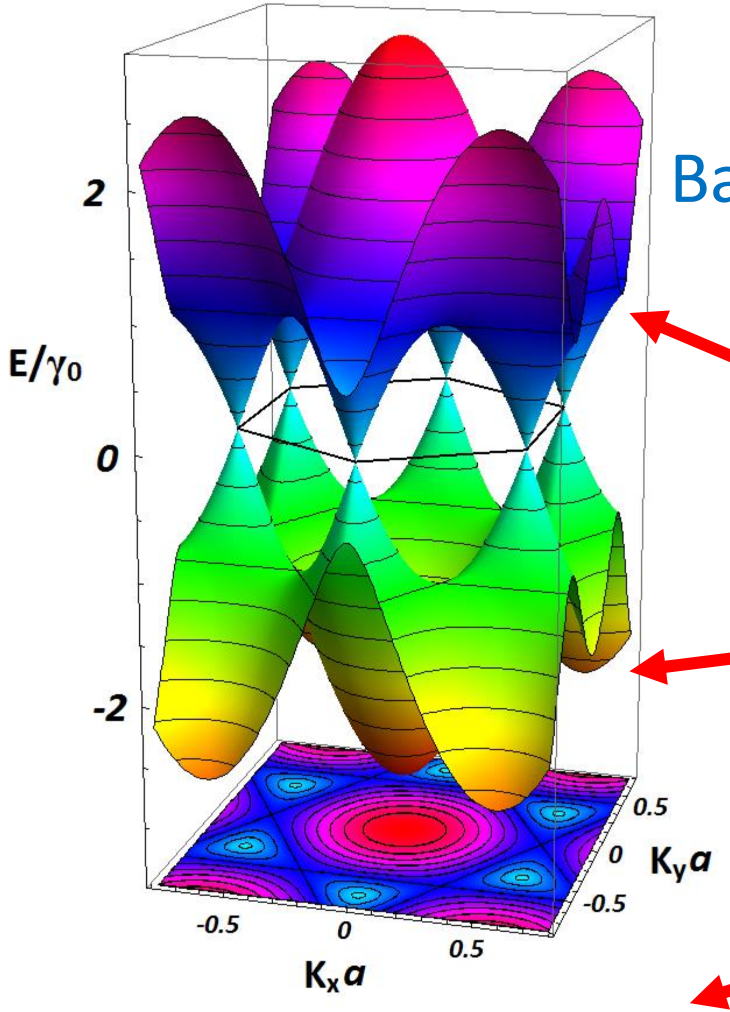


$\vec{a}_1, \vec{a}_2$ – translation vectors

A, B – sublattices

$\hbar v_F(\hat{\sigma} \boldsymbol{k})\psi = E\psi$ – 2D Dirac fermions

G.W. Semenoff, *Condensed-matter simulation of a three-dimensional anomaly*, Phys. Rev. Lett. 53 (1984) 2449



Band structure

conduction band

valence band

gap equation

$$\Delta = \frac{i\alpha_g}{2}\int \frac{d\omega d^2k}{(2\pi)^3}\frac{\Delta}{\omega^2 - v_F^2 k^2 - \Delta^2 + i\epsilon},$$ $\alpha_{g,cr}$ is not zero

Flat band

quenched kinetic energy (constant energy dispersion)

H = K + V(r-r')

Kinetic energy K>0

ordinary quasiparticles

H = K + V(r-r')
K=const – flat band

Superconductivity, strong correlations

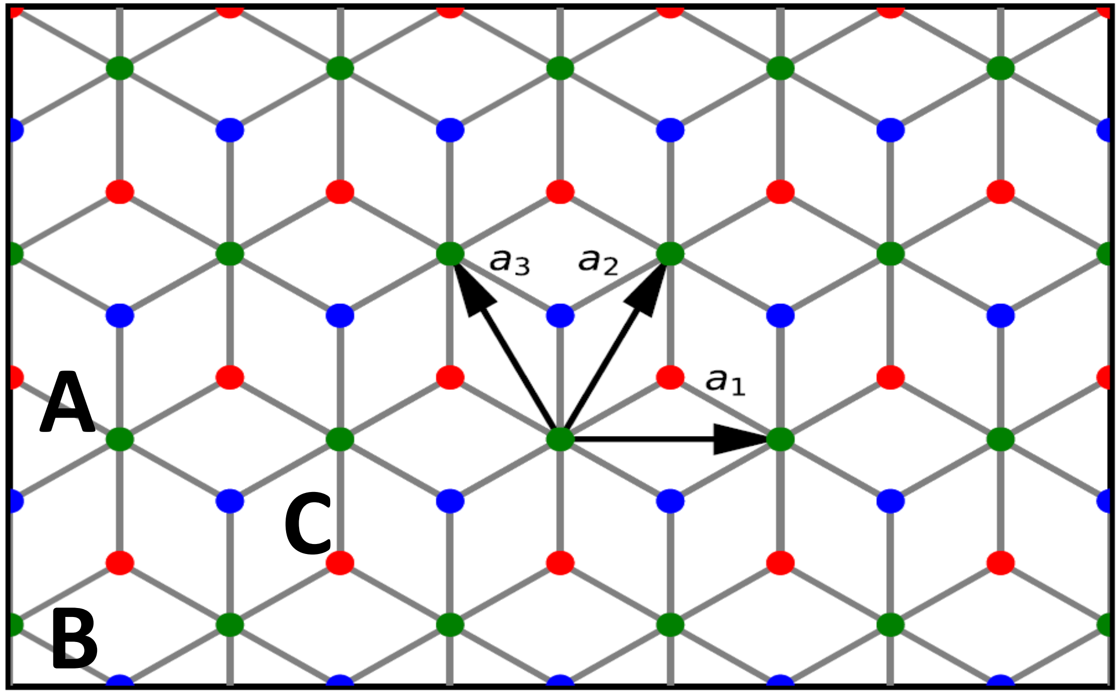
V(r-r') plays crucial role, bound state formation for arbitrary weak attraction

Zero critical coupling for gap generation in c.m. systems with flat band?

graphene in a magnetic field

$\Delta \sim v_F \sqrt{|eB|/c}$

Dice lattice



B. Sutherland, Phys. Rev. B **34**, 5208 (1986)

- 3 sublattices: A, B, C
- Hopping parameter t
- All distances are the same

$$|\bar{a}_1| = |\bar{a}_2| = |\bar{a}_3|$$

$$\boldsymbol{\delta}_1 = \frac{\boldsymbol{a}_1 + \boldsymbol{a}_2}{3}, \quad \boldsymbol{\delta}_2 = \frac{\boldsymbol{a}_3 - \boldsymbol{a}_1}{3}, \quad \boldsymbol{\delta}_3 = -\frac{\boldsymbol{a}_2 + \boldsymbol{a}_3}{3}$$

Equations of motion:

$$\varepsilon \Psi_A(\mathbf{r}) = -t_1 \sum_j \Psi_C(\mathbf{r} - \delta_j)$$

$$\varepsilon \Psi_C(\mathbf{r}) = -t_1 \sum_j \Psi_A(\mathbf{r} + \delta_j) - t_2 \sum_j \Psi_B(\mathbf{r} - \delta_j)$$

$$\varepsilon \Psi_B(\mathbf{r}) = -t_2 \sum_j \Psi_C(\mathbf{r} + \delta_j)$$

$$\Psi^T = (\Psi_A, \Psi_C, \Psi_B)$$

Energy spectrum

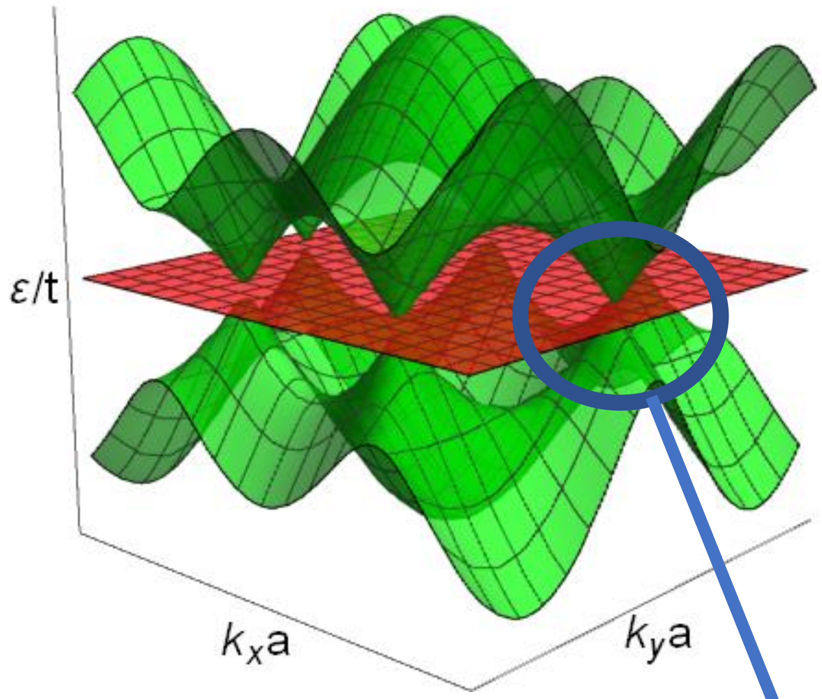
$$H\Psi = \varepsilon\Psi$$

$$H = \begin{pmatrix} 0 & f_k \cos\Theta & 0 \\ f_k^* \cos\Theta & 0 & f_k \sin\Theta \\ 0 & f_k^* \sin\Theta & 0 \end{pmatrix}$$

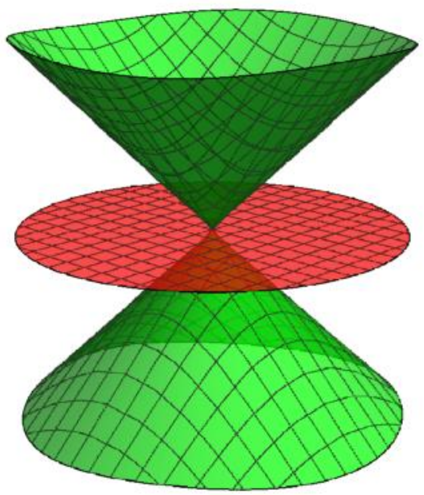
$$\varepsilon_\lambda(\mathbf{k}) = \lambda|f_k|,\ \lambda = \pm 1, 0$$

- $\lambda = \pm 1$ – dispersive bands
- $\lambda = 0$ – completely flat band

$$f_{\boldsymbol{k}} = -\sqrt{t_1^2 + t_2^2}\left(1 + e^{-i\boldsymbol{k}a_2} + e^{-i\boldsymbol{k}a_3}\right)$$



Near **K**, **K**´ points

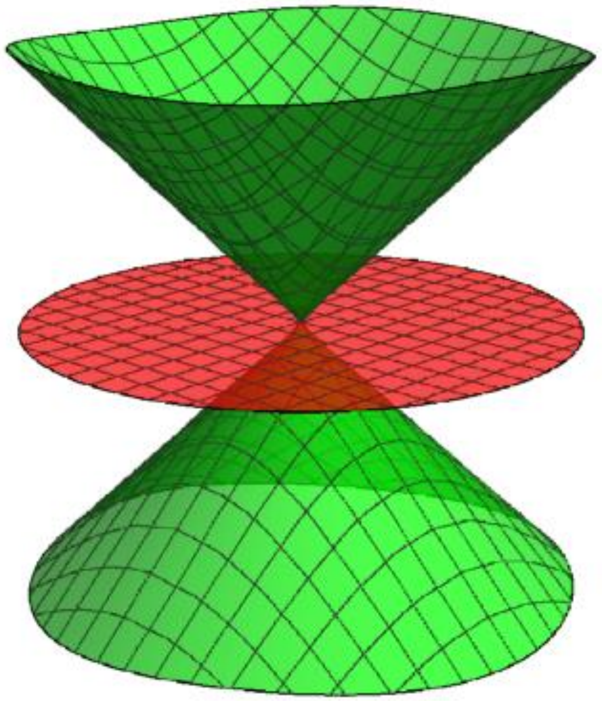


Low-energy effective Hamiltonian

- Linearization $\mathbf{k} = \mathbf{K}(\mathbf{K}') + \tilde{\mathbf{k}}$
- $f_{\mathbf{k}} = \hbar v_F(\pm k_x - i k_y)$ $v_F = 3ta/2\hbar$
- Effective low-energy Hamiltonian ($\Theta = \frac{\pi}{4}$)

$$H_K = \hbar v_F \begin{pmatrix} 0 & k_- \cos\Theta & 0 \\ k_+ \cos\Theta & 0 & k_- \sin\Theta \\ 0 & k_+ \sin\Theta & 0 \end{pmatrix}$$

$$k_\pm = k_x \pm i k_y$$



Energy dispersion:

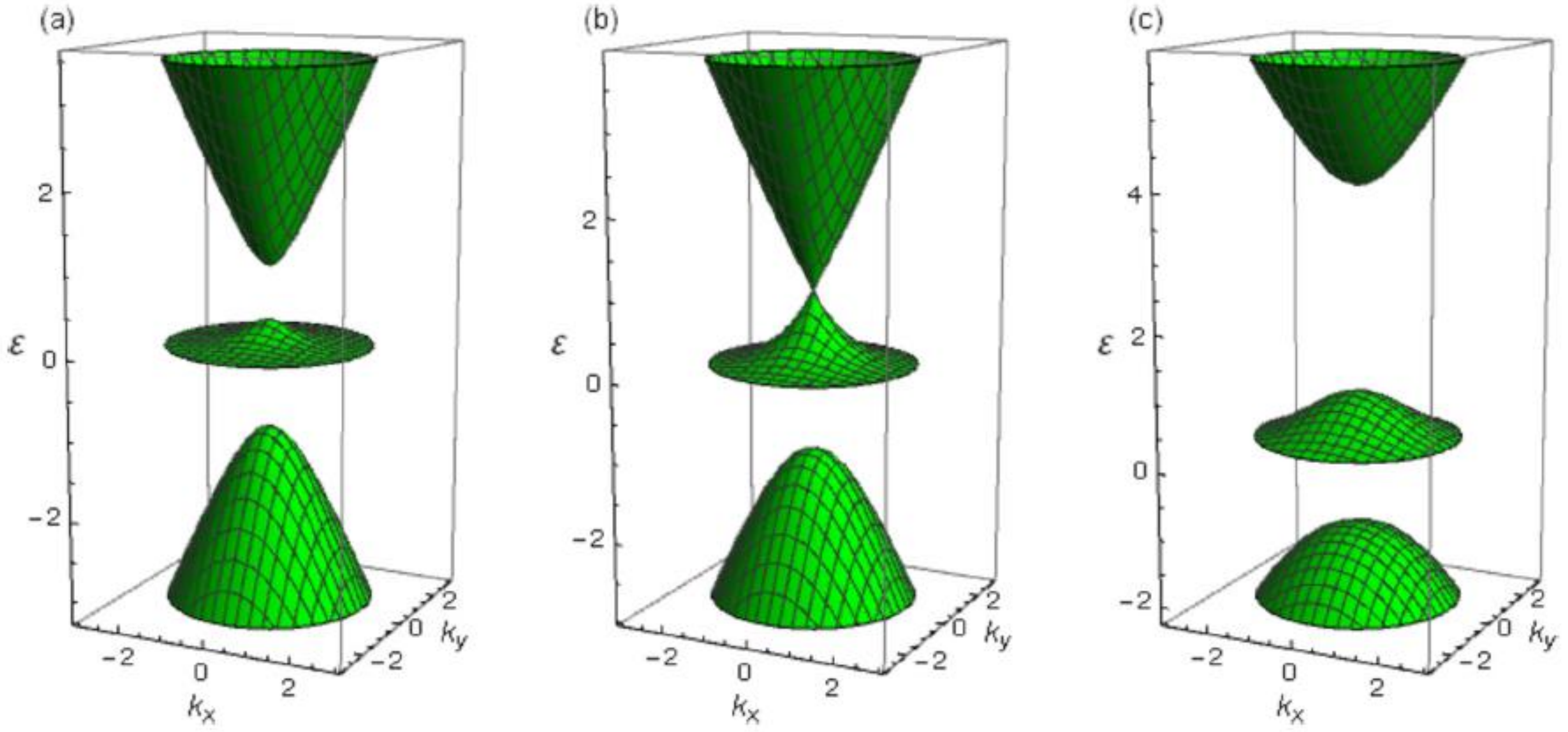
E(***k***)=$\pm v_F$ – Dirac cones,

E(***k***)= 0 – flat band

Intravalley gap

- $H_g = \begin{pmatrix} m & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & -m \end{pmatrix}$ - intravalley gaps

E.V.G., V.P. Gusynin, D.O. Oriekhov, *Gap generation and flat band catalysis in dice model with local interaction*, Phys. Rev. B 103, 155155 (2021)



a) $m_2 = 0.35\, m$

b) $m_2 = m$

c) $m_2 = 4\, m$

critical local four - fermion coupling

$U_{cr} = 8.9 \frac{v_F^2}{\Lambda}$

Intervalley gap

Inverse Green function $G^{-1}(\omega, \boldsymbol{k}) = \begin{pmatrix} \omega - H_K & F \\ F^{+} & \omega + H_{K'} \end{pmatrix}$

Intervalley gap $F = diag(\Delta, \Delta_2, \Delta)$

Gap equation in flat band approximation (the same form as in a magnetic field)

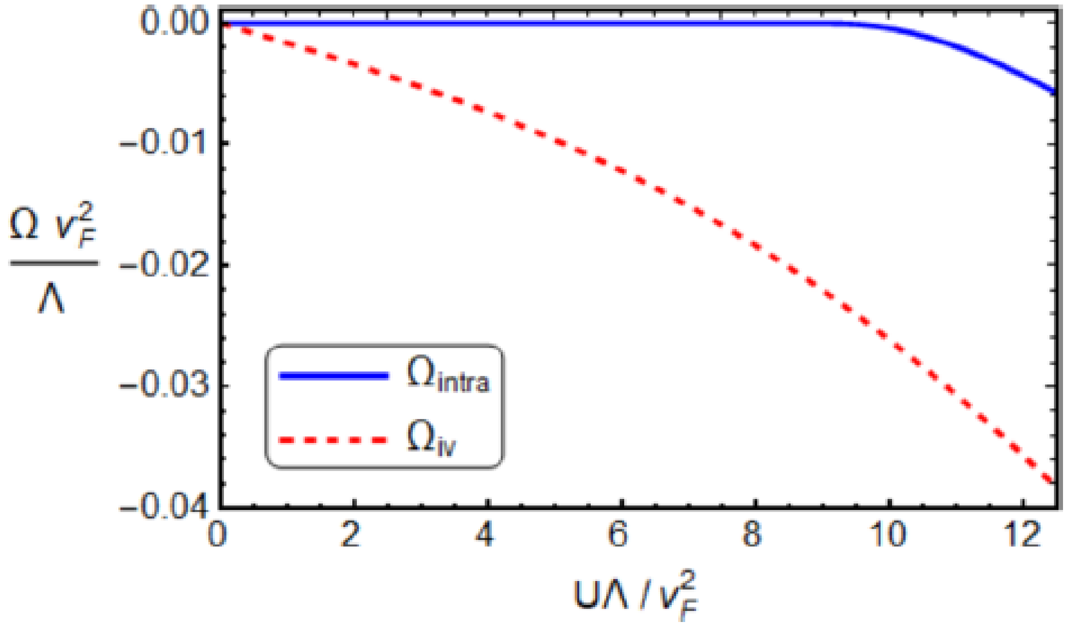
$$\Delta = \frac{iU}{\pi v_F^2} \int \frac{d\omega d^2 k}{(2\pi)^3} \frac{\Delta}{\omega^2 - \Delta^2 + i\epsilon}, \qquad \Delta_2 = 0$$

↓

$$\Delta = \frac{U\Lambda^2}{8\pi^2 v_F^2}$$

$U_{cr} = 0$ – zero critical coupling as in magnetic catalysis

Free energy density



Solution with intervalley gap always has lower energy

Dimensionless free energy density as a function of dimenionless coupling constant for intravalley Ω_{intra} and intervalley Ω_{iv} gaps

Conclusion and Outlook

- In the absence of a magnetic field, flat band systems provide a platform which realizes the main characteristics of magnetic catalysis
- Condensed matter systems with flat bands like magic angle twisted bilayer graphene form one of the most natural classes of physical systems with the strong interaction dominance and rich phase diagram

Thank you for attention!