Energy Loss and Distributions in Homogeneous Chiral Media

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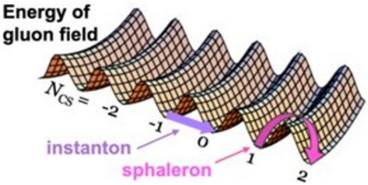
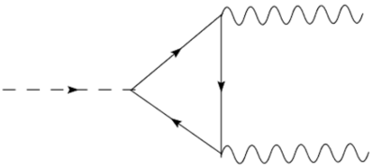
Overview

- What is chiral matter
- Sources of chiral imbalance
- Possible experimental applications
    - Source of high-intensity radiation and leads to energy loss
- Color Chiral Cherenkov Radiation
- Particle cascades
- Additional considerations and related work
- in progress and summary

- Sources of chiral imbalance may come from changes in N_{cs}.
- **QCD Vacuum**

  Chiral anomaly breaks the axial symmetry ($\psi \rightarrow e^{i\gamma^5\theta}\psi$) of the system, creating a non-conserved axial current.





Weyl and Dirac Semimetals

The effects of the chiral anomaly have been seen in Weyl semimetals

We have worked on implications due to the quantum hall effect

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Chiral magnetic effect in $ZrTe_5$

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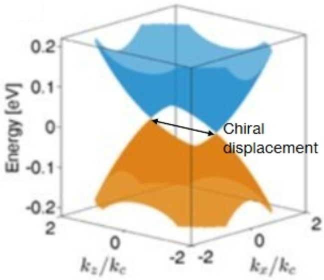
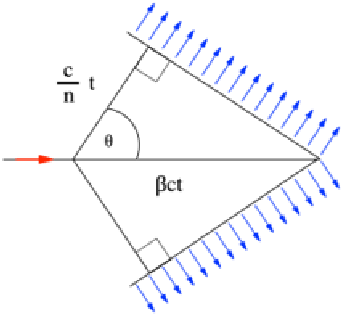


Testing for the Chiral Anomaly

- One way of studying the effects for chiral media is through the interaction between a fast-moving particle and a chiral medium.
- While traveling through the media the particle can lose energy via collisional or radiative energy loss.
- This has a number of advantages in various areas such as QGP and Axionic matter.

Cherenkov radiation



Quark-Gluon Plasma

One advantage comes from globally chiral-neutral media such as Quark-Gluon Plasma:

- The enhancement due to any chiral magnetic effect makes the rate of energy loss of a particle sensitive to *local* chiral imbalances in the plasma.
- Overall the jets produced will be chiral neutral; however, the chirality of radiated photons may oscillate depending on the local chiral imbalances in the plasma.

Maxwell's Equations

The effect of the anomaly in QED can be phenomenologically described using the pseudoscalar field Θ.

In terms of $b^\mu = (b_0, -\mathbf{b}) = c_A \partial^\mu \theta$:

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{c_A}{2}\,\theta\,\mathrm{Tr}(\tilde{F}_{\mu\nu}F^{\mu\nu})$$

$$\nabla \cdot \mathbf{E} = \rho + \mathbf{b} \cdot \mathbf{B}$$
$$\nabla \times \mathbf{B} - \partial_t \mathbf{E} = \mathbf{J} + b_0 \mathbf{B} - \mathbf{b} \times \mathbf{E}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} + \partial_t \mathbf{B} = 0$$

Frank Wilczek (1987)   A.A. Zyuzin, A.A. Burkov (2012)

Maxwell's Equations

The effect of the anomaly in QED can be phenomenologically described using the pseudoscalar field Θ.

Focusing on the Chiral Magnetic Effect:

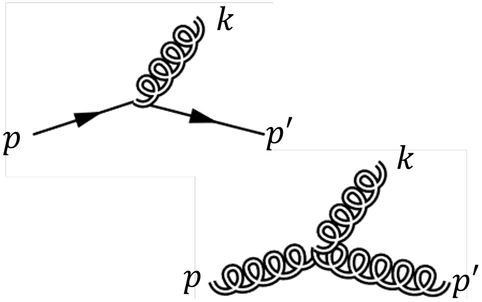
$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{c_A}{2}\,\theta\,\mathrm{Tr}(\tilde{F}_{\mu\nu}F^{\mu\nu})$$

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \rho \\ \nabla \times \mathbf{B} - \partial_t \mathbf{E} &= \mathbf{J} + b_0 \mathbf{B} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} + \partial_t \mathbf{B} &= 0 \end{aligned}$$

Frank Wilczek (1987)   A.A. Zyuzin, A.A. Burkov (2012)

QCD

- We extend our treatment to the realm of QCD, in which we consider collisional energy loss for quark scattering and gluon radiation.
- In particular we consider the impact of the anomaly on a quark radiating a gluon, and a gluon radiating a gluon.



Color Chiral Cherenkov Radiation

- The radiation rate for a particle traveling through a homogeneous medium can be computed.
- After some computation we find that for relativistic particles the frequency dependence of the differential rate is:

$$dW_{a\to bc} = \frac{g^2 C_{a\to bc}}{2(2\pi)^2} \sum_{ss'} \delta(\omega + E' - E)\, \delta^{(3)}(\mathbf{k} + \mathbf{p}' - \mathbf{p}) \times \frac{1}{8EE'\omega} \left|M_{a\to bc}\right|^2 d^3p'\, d^3k$$

where $x_\pm$ indicate the bounds on the ratio of frequencies $x = \omega/E$:

$$x_+ = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4m^2}{s}}\right)$$

$$x_- = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4m^2}{s}}\right)$$

Three Vertex

For a gluon in a chiral medium

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) - \frac{c_A}{2}\theta\,\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

The anomaly af

$$S_\theta = c_A \int \partial_\mu \theta \epsilon^{\mu\nu\rho\sigma} \left(\frac{1}{2}A^a_\nu \partial_\rho A^a_\sigma - g\frac{2i}{3}\frac{1}{4} i f^{abc} A^a_\nu A^b_\rho A^c_\sigma\right) d^4x$$

one will find t anomalous triple-gluon vertex $= g b_\mu \epsilon^{\mu\nu\rho\sigma} f^{abc}$

Energy Loss

- For the rate of energy loss we use:

$$-\frac{dE}{dx} = \int_0^E d\omega\, \omega\, \frac{dW}{d\omega}$$

- This is shown in the figure.
- In the limit $b_0 E >> m^2, \omega_p^2$:

$$-\frac{dE_q}{dz} \propto b_0 E$$

$$-\frac{dE_{g_R}}{dz} \propto b_0 E \ln \frac{b_0 E}{\omega_p^2}$$

$$-\frac{dE_{g_R}}{dz} \propto b_0 E (\ln \frac{b_0 E}{\omega_p^2} - C)$$

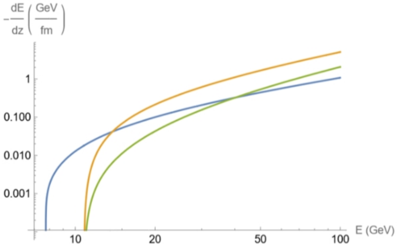


FIG. 3. The rate of energy loss for $q \to gg$, $g_R \to gg$, and $g_L \to gg$ are blue, orange, and green respectively. Parameters: $g = 2$, $T = 300$ MeV, $b_0 = 50$ MeV. The blue line is a contribution of one quark.

Multiple Scattering

- In the next section, we will look at **cascades** due to the anomaly in the context of both QED and QCD.
- In either case, we consider *ultrarelativistic* processes.

QED Cascade

- Some given distributions of polarized photons ($\gamma_\pm$), electrons (e), and positrons ($\bar{e}$) in a dilute chiral medium will evolve according to two processes:
  - pair production $\gamma_- \to e + \bar{e}$ and decay $e \to e + \gamma_+$
- The distributions of left and right photons evolve recursively, where $dW_{a\to bc}/dx$ is the inclusive rate and $W^{\mathrm{tot}}_{a\to bc}$ is the total rate:

$$\gamma_+(y, t{+}\Delta t) = \gamma_+(y, t) + \Delta t \int_0^1 dz \int_0^1 dx \, \frac{dW_{e\to\gamma_+ e}}{dx} \left[e(z,t) + \bar{e}(z,t)\right] \delta(y - xz)$$

$$\gamma_-(y, t{+}\Delta t) = \left(1 - W^{\mathrm{tot}}_{\gamma_-\to e\bar{e}} \, \Delta t\right) \gamma_-(y, t)$$

The distributions of e and $\bar{e}$ evolve in a similar yet more complicated manner.

QED Cascade

- Such descriptions of the distributions are possible due to the factorizability of multiple-particle production rates such as $W_{e\to e\gamma\gamma}$ into individual $W_{e\to e\gamma}$ rates.
- $W_{e\to e\gamma\gamma}$ can be factored because instabilities in the fermion propagators force an on-mass-shell condition such that (where τ is the chiral relaxation time):

$$\frac{1}{\left[(p-k_1)^2 - m^2 + 2iE_q/\tau\right]^2} \approx \frac{\pi\tau}{2E_q}\delta\big((p-k_1)^2 - m^2\big)$$

- This leads to:

$$W_{e\to e\gamma\gamma} = \tau \int_0^E d\omega_1 \frac{dW_{e(p)\to e(q)\gamma(k_1)}}{d\omega_1} \int_0^{E_q} d\omega_2 \frac{dW_{e(q)\to e(p')\gamma(k_2)}}{d\omega_2}$$

QED Cascade

The corresponding splitting functions in the chiral limit, where the delta-function coefficients are virtual corrections to the appropriate production rates:

$$P_{e\gamma_-}(x) = x^2 + (1-x)^2$$

$$P_{\gamma_+ e}(x) = \frac{1+(1-x)^2}{x}$$

$$P_{\gamma_-\gamma_-}(x) = -\frac{2}{3}\,\delta(1-x)$$

$$P_{ee}(x) = \left[\frac{1+x^2}{1-x}\right]_+ + \frac{3}{2}\,\delta(1-x)$$

where the plus-distribution is defined as:

$$\int_x^1 dz\,\frac{f(z)}{(1-z)_+}$$

$$= \int_x^1 dz\,\frac{f(z)-f(1)}{1-z} + f(1)\ln(1-x)$$

QED Cascade

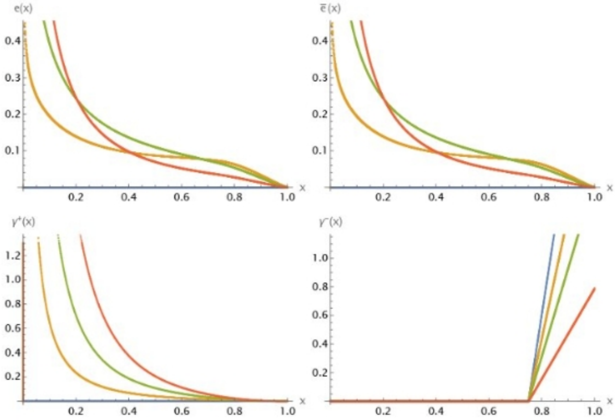
From the splitting functions one may construct the evolution equations:

$$\frac{de(y,t)}{dt} = \frac{1}{\tau}\int_y^1 \frac{dx}{x}\left[P_{ee}(x)\, e\left(\frac{y}{x},t\right) + P_{e\gamma_-}(x)\,\gamma_-\left(\frac{y}{x},t\right)\right]$$

$$\frac{d\gamma_+(y,t)}{dt} = \frac{1}{\tau}\int_y^1 \frac{dx}{x}\left[P_{\gamma_+ e}(x)\, e\left(\frac{y}{x},t\right) + P_{\gamma_+ e}(x)\,\bar{e}\left(\frac{y}{x},t\right)\right]$$

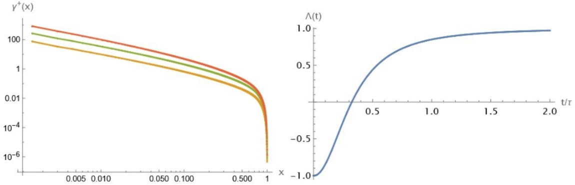
$$\frac{d\gamma_-(y,t)}{dt} = \frac{1}{\tau}\int_y^1 \frac{dx}{x} P_{\gamma_-\gamma_-}(x)\,\gamma_-\left(\frac{y}{x},t\right)$$

QED Cascade



Chiral Cherenkov cascade in χQED with the specified initial conditions. Blue line: $t = 0$, yellow line: $t = \tau/2$, green line: $t = \tau$, red line: $t = 2\tau$.

QED Cascade: Degree of Polarisation



Left panel: log plot for γ^+. Right panel: The degree of polarization $\Lambda(t)$ of the cascade.

- Given the degree of polarisation

$$\Lambda(t) = \frac{\int_0^1 \left[\gamma_+(x,t) - \gamma_-(x,t)\right] dx}{\int_0^1 \left[\gamma_+(x,t) + \gamma_-(x,t)\right] dx}$$

- one finds the system quickly becomes positively polarised.

- One may obtain similar evolution equations in QCD using the polarized splitting functions given a colour factor C_F, and the number of colours and flavours N_c, N_f.
- The main difference comes from the additional gluon splitting process.
- Summing over the functions corresponding to this process gives the expected unpolarized splitting function.

$$P_{g_+q}(x) = C_F\, P_{\gamma_+e}(x)$$
$$P_{qg_-}(x) = \tfrac{1}{2}\, P_{e\gamma_-}(x)$$
$$P_{qq}(x) = C_F\, P_{ee}(x)$$
$$P_{g_+g_+} = \frac{N_c}{[x(1-x)]_+}$$
$$P_{g_+g_-} = \frac{N_c\, x}{(1-x)^3}$$

$$P_{g_-g_-} = N_c\left[\frac{x^3}{(1-x)_+}\right] + \left(\frac{11}{6}N_c - \frac{N_f}{3}\right)\delta(1-x)$$

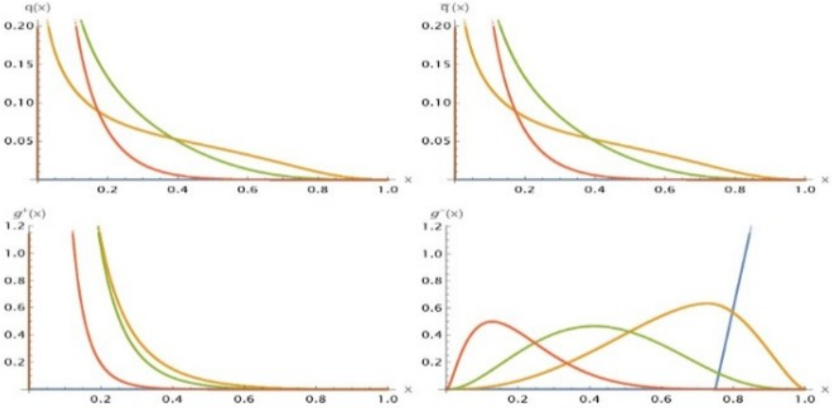
$$P_{gg}(x) = 2N_c\left[\frac{1-x}{x} + \frac{x}{1-x} + x(1-x)\right] + \left(\frac{11}{6}N_c - \frac{N_f}{3}\right)\delta(1-x)$$

- With the exception of g_+, the form of the evolution equations in QCD are identical to those in QED.
- The equation for g_+ gains terms that are proportional to g_+ and g_- such that:

$$\frac{dg_+(y,t)}{dt} = \frac{1}{\tau}\int_y^1 \frac{dx}{x}\left[N_f P_{g_+q}(x)\, q\left(\frac{y}{x},t\right) + N_f P_{g_+q}(x)\, \bar{q}\left(\frac{y}{x},t\right)\right.$$

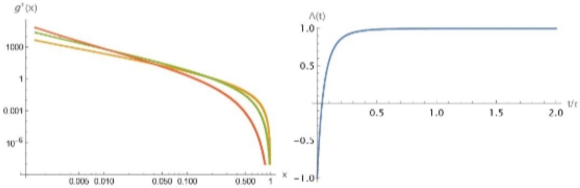
$$\left. + \mathrm{P}_{g_+g_+}(x)\, g_+\left(\tfrac{y}{x},t\right) + P_{g_+g_-}(x)\, g_-\left(\tfrac{y}{x},t\right)\right]$$

QCD Cascade



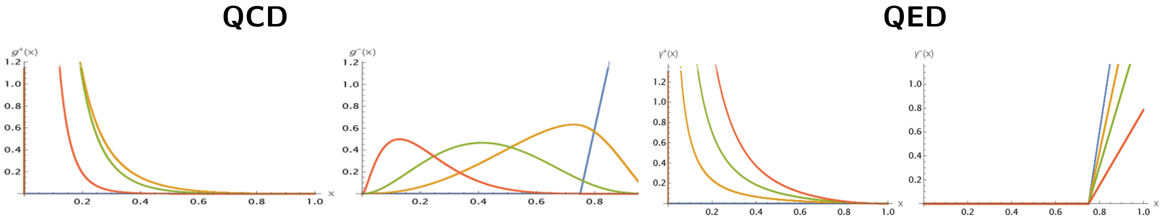
Chiral Cherenkov cascade in χQCD with the initial conditions: $g^{-}(x,0) = 32\,(x - 3/4)\,\mathrm{H}\,(x \geq 3/4)$, $q(x,0) = \bar{q}(x,0) = g^{+}(x,0) = 0$. Blue line: $t = 0$, yellow line: $t = \tau/2$, green line: $t = \tau$, red line: $t = 2\tau$.

QCD Cascade



Left panel: log plot for g^+. Right panel: The degree of polarization $\Lambda(t)$ of the cascade.

- Gluons become positively polarised faster than photons relative to τ.
- The concentration of right-handed gluons can be seen more clearly in the log plot.



- Unlike γ_-, g_- is redistributed towards small values of x due to the additional gluon splitting.
- The additional process also concentrates g_+ towards small x.

Additional Considerations

- The addition of mass can have interesting consequences for the cascade; we have considered this in the case of QED, but will need to expand to QCD.
- The evolution equations can be used to compute energy loss in a chiral medium.
- In addition to cascades in homogeneous chiral media we have considered the anomaly:
  - In condensed matter systems such as Weyl semi-metals.
  - Its effect on radiative energy loss.

Inhomogeneous Media

Jeremy Hansen, Kazuki Ikeda, Dmitri E. Kharzeev, Qiang Li, and Kirill Tuchin

- We also consider the anomalous hall effect do the current

$$j_{\mathrm{AH}} = \vec{b} \times \vec{E}$$

- This gives rise to the rate of photon emission as computed by my collaborator

$$\frac{dW}{d\Omega d\omega} = \frac{\alpha}{16\pi} \sum_{\lambda} \delta(\omega + E' - E) \frac{\vec{k}^3}{EE'\omega^2 \varepsilon_{ij} e_i^* e_j}$$

$$4e_i^* e_j \left[p_i p_j' + p_j p_i' + \delta_{ij}(EE' - \mathbf{p} \cdot \mathbf{p}' - m^2) \right]$$

Kirill Tuchin (2018)

- the rate of emission stems from the effect on the photon's energy which obeys Fresnel's equation

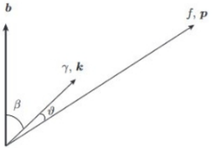
$$\left| \vec{k}_i \vec{k}_j - \vec{k}^2 \delta_{ij} + \omega^2 \varepsilon_{ij} \right| = 0$$

Where $\varepsilon_{ij} = \varepsilon \delta_{ij} - i\epsilon_{ijk} b_k / \omega$,and ε permittivity

Inhomogeneous Media

- the full solution to the Fresnel's equation gives

$$\omega^2 - \vec{k}^2 = \omega_p^2 + \frac{b^2\omega^2\sin(\beta)^2 - \lambda b\omega\sqrt{b^2\omega^2\sin(\beta)^4 + 4(\omega^2-\omega_p^2)^2\cos(\beta)^2}}{2(\omega^2-\omega_p^2)}$$



- Energy loss is maximum when the radiated particle (k) is aligned to the chiral displacement (b) such that

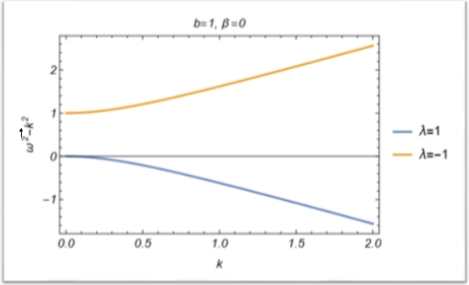
$$\omega^2 - \vec{k}^2 = \omega_p^2 - \lambda b\omega,\ \beta=0$$

- When the anomaly is perpendicular to the outgoing radiation it gives no contribution for $\omega>\omega_p$ since

$$\omega^2 - \vec{k}^2 = \omega_p^2 + \frac{(1-\lambda)b\omega^2}{2(\omega^2-\omega_p^2)}$$

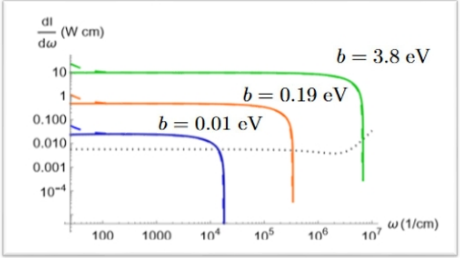
- This stems from the kinematic constraint

$$4m^2\omega^2 + (\omega^2-\vec{k}^2)(4(EE'-m^2)+\omega^2-\vec{k}^2) = 0$$



After integrating over angles one finds the intensity of radiation per frequency as an increasing function of chiral displacement

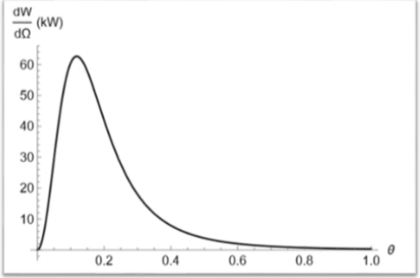
$$\frac{dP}{d\omega} = \frac{\alpha f}{2}\theta\left(\frac{E^2 f}{m^2} - \omega\right) \qquad \text{for} \qquad f = \lambda b \cos\beta$$



Intensity spectrum of Chiral Cherenkov radiation for electrons of energy $E = 3$ MeV, $\beta = 0$ in a Weyl semimetal, dotted line: bremsstrahlung in $Co_3Sn_2S_2$.

Alternatively, one can find the angular distribution for the intensity of radiation by integrating over frequencies

$$\frac{dW}{d\Omega} = \frac{\alpha f}{2\pi}\frac{1}{\vartheta^2 + m^2/E^2}$$



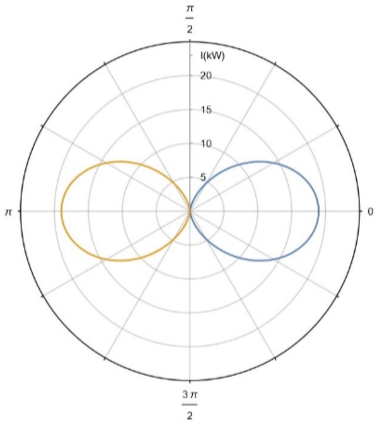
Angular distribution of Chiral Cherenkov radiation. $E = 3$ MeV, $\beta = 0$, $b = 3.9$ eV.

Inhomogeneous Media

- After integrating over angles and frequencies, and converting the rate of radiation produced to the rate of energy loss on obtains the total radiated power

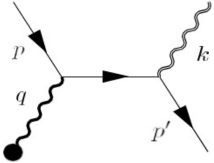
$$P = \frac{\alpha f^2 E^2}{2m^2} = \frac{\alpha b^2 \cos^2 \beta E^2}{2m^2}$$

- The two loops have opposite polarization



Chiral Cherenkov radiation intensity for incidence angle (β). $E = 3$ MeV. $b = 3.9$ eV.

Radiative Energy Loss



the anomaly may impact radiative energy loss

For an in-depth discussion refer to our papers

“Bremsstrahlung in chiral medium: Anomalous magnetic contribution to the Bethe-Heitler formula.”

“Electromagnetic bremsstrahlung and energy loss in chiral medium.”

In particular, we consider how the anomaly modifies the photon propagator such that

$$D_{\mu\nu}(q) = -i\frac{q^2 g_{\mu\nu} + i\epsilon_{\mu\nu\rho\sigma} b^\rho q^\sigma + b_\mu b_\nu}{q^4 + b^2 q^2 - (b\cdot q)^2}$$

Ralf Lehnert, Robertus Potting (2004)

And the impact of the anomaly on the corresponding dispersion for a homogeneous medium

$$\omega^2 = \boldsymbol{k}^2 + k^2 = \boldsymbol{k}^2 - \lambda b_0 |\boldsymbol{k}|$$

In case we find that under the right conditions, we find that the rate of energy loss may be enhanced by the chiral anomaly

Using the photon propagator in chiral medium in the static limit

$$D_{00}(\boldsymbol{q}) = \frac{i}{\boldsymbol{q}^2},$$

$$D_{0i}(\boldsymbol{q}) = D_{0i}(\boldsymbol{q}) = 0,$$

$$D_{ij}(\boldsymbol{q}) = -\frac{i\delta_{ij}}{\boldsymbol{q}^2 - b_0^2} - \frac{\epsilon_{ijk}q^k}{b_0(\boldsymbol{q}^2 - b_0^2)} + \frac{\epsilon_{ijk}q^k}{b_0\boldsymbol{q}^2}$$

Given how the source current couples to the propagator, the anomaly most apparently impacts the energy loss via magnetic moment

To find the vector potential we use the static current

$$J^0(\boldsymbol{x}) = e'\delta(\boldsymbol{x}), \qquad \boldsymbol{J}(\boldsymbol{x}) = \boldsymbol{\nabla} \times (M\delta(\boldsymbol{x}))$$

Electric Monopole

Magnetic Moment

Giving rise to

$$A^0(\boldsymbol{q}) = e'/\boldsymbol{q}^2$$

$$A^\ell(\boldsymbol{q}) = -\frac{1}{\boldsymbol{q}^2 - b_0^2}\left[i(M \times \boldsymbol{q})^\ell + \frac{b_0}{\boldsymbol{q}^2}(M \cdot \boldsymbol{q}q^\ell - \boldsymbol{q}^2 M^\ell)\right]$$

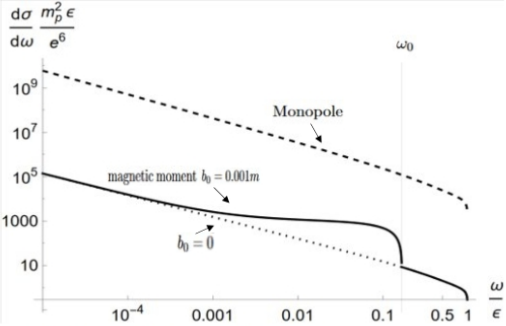
Regulating the divergence at **q**=b_0 with the finite resonate width Γ in the ultra-relativistic semi-soft photon limit ($b_0 \ll \omega \ll E$), we can derive the following approximation for the differential cross section integrated over directions.

$$\frac{d\sigma_M}{d\omega} \approx \frac{2e^4 M^2}{3(2\pi)^3\omega}\left[\frac{3b_0^2}{m^2}\ln\left(\frac{4\epsilon^4}{m^2\omega^2}\right) + \ln^2\frac{4\epsilon^2}{m^2} + \frac{2b_0^4\pi}{m^2\Gamma^2}\Theta(\omega_0-\omega)\right]$$

where $\omega_0 = \frac{2\epsilon^2 b_0}{2\epsilon b_0 + m^2}$

This can be compared to the traditional expression from the mono-pole contribution in this limit

$$\frac{d\sigma_e}{d\omega} \approx \frac{Z^2 e^6}{12(2\pi)^3 m^2\omega}\left(\ln\frac{2\epsilon^2}{m\omega} - \frac{1}{2}\right)$$



photon bremsstrahlung cross section
for $m = m_p, \Gamma = 0.01 b_0, \epsilon = 100m$, and $M = 5M_N$

In progess

- this work considers fermions with averaged polarization
- the presences of the anomaly may affect such polarized differently, suggesting that our work may be extended
- In particular certain polarizations may be suppressed by the anomaly
- other works include a hydrodynamic treatment of spin and orbital momentum coupling, which may have implications for my previous work

Summary

- The presence of a finite chiral conductivity has a potentially significant effect on the collisional and radiative energy loss including Weyl semimetals and QGP.
- In QED and QCD the chiral anomaly can lead to chiral Cherenkov radiation.
- Instabilities in intermediate particles lead to effective on-mass-shell conditions.
- Such conditions allow for the development of chiral evolution equations.
- In addition to the effects considered above we have considered the anomaly in a variety of contexts.

extra frame