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# Nuclear Matter Properties and Neutron Star Structures from an Extended Linear Sigma Model


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Hefei, Anhui, May 18th

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# Motivation

$$\mathscr{L} = \bar{\psi}\left[i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu}\right]\psi + \ldots$$

The parameterizations in past studies on nuclear matter and neutron stars are lack of considerations of QCD symmetry patterns, such as chiral symmetry

# Nuclei structures (low/intermediate densities)

- Hadron interactions around saturation density $n_0 = 0.16\mathrm{fm}^{-3}$ are crucial to nuclei structures, e.g. $^{24}$Mg, $^{90}$Zr, $^{116}$Sn and $^{208}$ Pb

$$E(n,\alpha) = E_0(n) + E_{\mathrm{sym}}(n)\alpha^2 + O\left(\alpha^4\right)$$

$$\alpha = \left(n_n - n_p\right)/\left(n_n + n_p\right)$$

$$n = n_n + n_p$$

$$E_0(n) = E_0\left(n_0\right) + \frac{K_0}{2!}\chi^2 + \frac{J_0}{3!}\chi^3 + O\left(\chi^4\right)$$

$$E_{\mathrm{sym}}(n) = E_{\mathrm{sym}}\left(n_r\right) + L\left(n_r\right)\chi_r + O\left(\chi_r^2\right)$$

$$\chi \equiv \left(n - n_0\right)/3n_0$$

$$\mathscr{L} = \bar{\psi}\left[i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu}\right]\psi$$


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A. W. Steiner and B. A. Li, Phys. Rev. C 72, 041601 (2005).
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S. Karataglidis, K. Amos, B. A. Brown, and P. K. Deb, Phys. Rev. C 65, 044306 (2002).
R. J. Furnstahl, Nucl. Phys. A 706, 85 (2002).
B. A. Brown, Phys. Rev. Lett. 85, 5296 (2000).



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M. Farine, J. M. Pearson, and F. Tondeur, Nucl. Phys. A 615, 135 (1997).


$$n_0 \to 0.155 \pm 0.050\ \left(\mathrm{fm}^{-3}\right)$$
$$E_0(n_0) \to -15.0 \pm 1.0(\mathrm{MeV})$$
$$E_{\mathrm{sym}}(n_0) \to 30.9 \pm 1.9(\mathrm{MeV})$$
$$K_0 \to 230 \pm 30(\mathrm{MeV})$$
$$L_0 \to 52.5 \pm 17.5(\mathrm{MeV})$$
$$J_0 \to -700 \pm 500(\mathrm{MeV})$$

# Neutron star (high densities)

The density in the cores of NSs always reach nearly $8n_0$

M-R relations/ tidal deformations are sensitive to the EOS behavior throughout the whole density regions

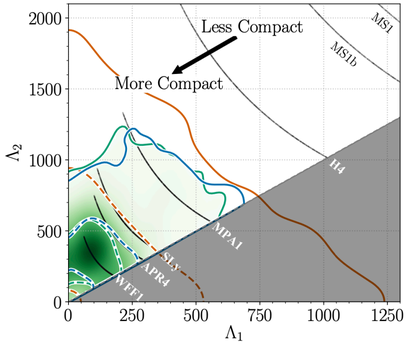



FIG. 1. Marginalized posterior for the tidal deformabilities of the two binary components of GW170817. The green shading shows the posterior obtained using the $\Lambda_a(\Lambda_s, q)$ EOS-insensitive relation to impose a common EOS for the two bodies, while the green, blue, and orange lines denote 50% (dashed) and 90% (solid) credible levels for the posteriors obtained using EOS-insensitive relations, a parametrized EOS without a maximum mass requirement, and independent EOSs (taken from [52]), respectively. The gray shading corresponds to the unphysical region $\Lambda_2 < \Lambda_1$ while the seven black scatter regions give the tidal parameters predicted by characteristic EOS models for this event [113, 115, 121–125].

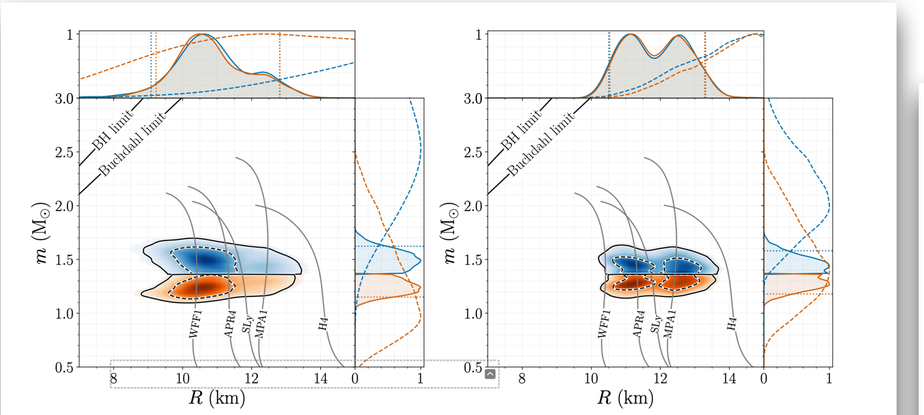



FIG. 3. Marginalized posterior for the mass $m$ and areal radius $R$ of each binary component using EOS-insensitive relations (left panel) and a parametrized EOS where we impose a lower limit on the maximum mass of $1.97\,M_\odot$ (right panel). The top blue (bottom orange) posterior corresponds to the heavier (lighter) NS. Example mass-radius curves for selected EOSs are overplotted in gray. The lines in the top left denote the Schwarzschild BH ($R = 2m$) and Buchdahl ($R = 9m/4$) limits. In the one-dimensional plots, solid lines are used for the posteriors, while dashed lines are used for the corresponding parameter priors. Dotted vertical lines are used for the bounds of the 90% credible intervals.

## TOV equation

TABLE I. Source properties for GW170817: we give ranges encompassing the 90% credible intervals for different assumptions of the waveform model to bound systematic uncertainty. The mass values are quoted in the frame of the source, accounting for uncertainty in the source redshift.

| | Low-spin priors ($\lvert\chi\rvert \leq 0.05$) | High-spin priors ($\lvert\chi\rvert \leq 0.89$) |
|---|---|---|
| Primary mass $m_1$ | $1.36$–$1.60\ M_\odot$ | $1.36$–$2.26\ M_\odot$ |
| Secondary mass $m_2$ | $1.17$–$1.36\ M_\odot$ | $0.86$–$1.36\ M_\odot$ |
| Chirp mass $\mathcal{M}$ | $1.188^{+0.004}_{-0.002} M_\odot$ | $1.188^{+0.004}_{-0.002} M_\odot$ |
| Mass ratio $m_2/m_1$ | 0.7–1.0 | 0.4–1.0 |
| Total mass $m_{\rm tot}$ | $2.74^{+0.04}_{-0.01} M_\odot$ | $2.82^{+0.47}_{-0.09} M_\odot$ |
| Radiated energy $E_{\rm rad}$ | $> 0.025 M_\odot c^2$ | $> 0.025 M_\odot c^2$ |
| Luminosity distance $D_{\rm L}$ | $40^{+8}_{-14}$ Mpc | $40^{+8}_{-14}$ Mpc |
| Viewing angle $\Theta$ | $\leq 55°$ | $\leq 56°$ |
| Using NGC 4993 location | $\leq 28°$ | $\leq 28°$ |
| Combined dimensionless tidal deformability $\tilde{\Lambda}$ | $\leq 800$ | $\leq 700$ |
| Dimensionless tidal deformability $\Lambda(1.4M_\odot)$ | $\leq 800$ | $\leq 1400$ |

B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 119, 161101 (2017)
B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 121, 161101 (2018)

Polytropic process +estimations of strong interaction + TOV equation

R. C. Tolman, Phys. Rev. 55, 364 (1939).
J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. 55, 374 (1939).

# $\delta$ meson effects

$\tilde{g}_{\sigma^2\delta^2}\sigma^2\vec{\delta}^2$ $\tilde{g}_{\sigma\delta^2}\sigma\vec{\delta}^2$ $g_\delta\vec{\delta}\bar{\psi}\vec{\tau}\psi$

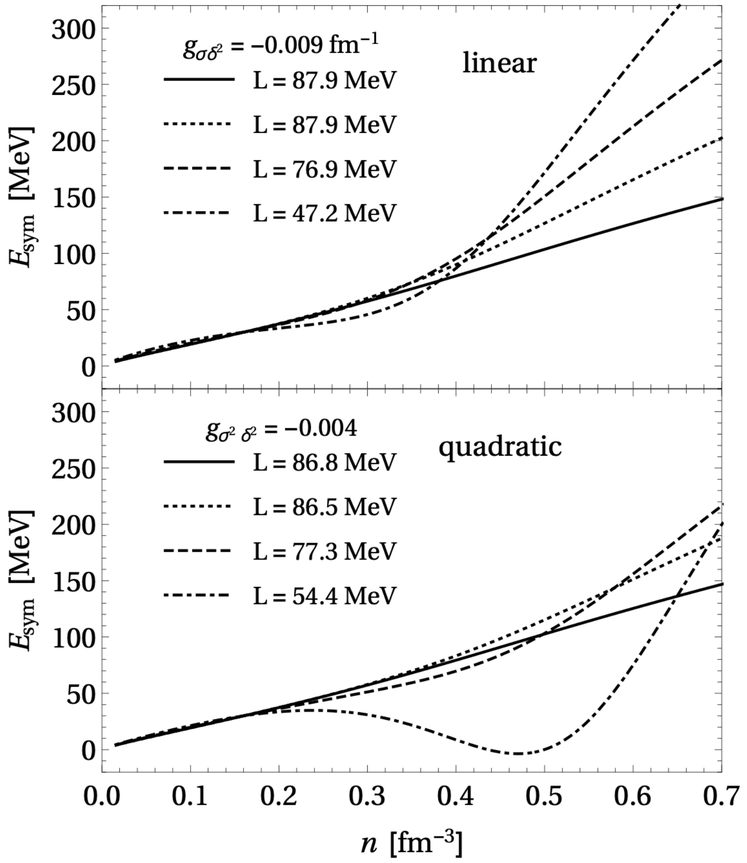



FIG. 2. Symmetry energy $E_{sym}$ for negative value of $g_\alpha$ with respect to various values of $C_\delta^2$ for both types of coupling.

N. Zabari, S. Kubis, and W. Wójcik, Phys. Rev. C 99, 035209 (2019).

T. Miyatsu, M.-K. Cheoun, and K. Saito, Astrophys. J. 929, 82 (2022).

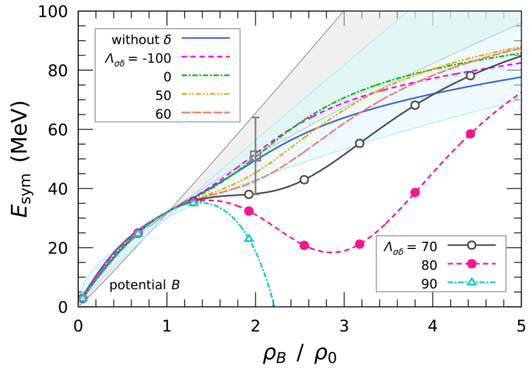



**Figure 4.** Same as Figure 3 but with the $\sigma$-$\delta$ mixing. The coupling constants in potential $B$ are used.

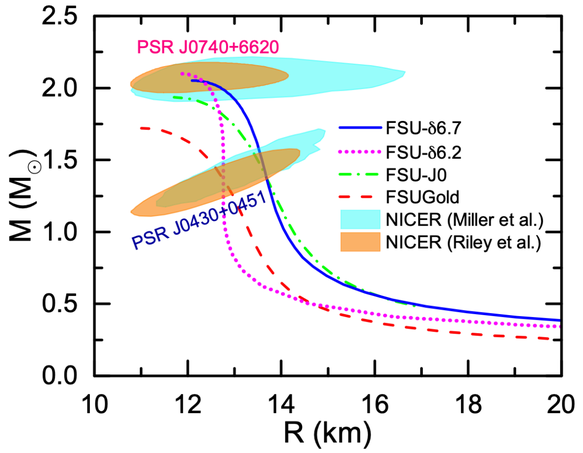



**Figure 2.** M-R relation for static NSs from FSUGold, FSU-J0, FSU-$\delta$6.7 and FSU-$\delta$6.2. The NICER (XMM-Newton) constraints with 68% C.L for PSR J0030+0451 (Riley et al. 2019; Miller et al. 2019) and PSR J0740+6620 (Riley et al. 2021; Miller et al. 2021) are also included for comparison.

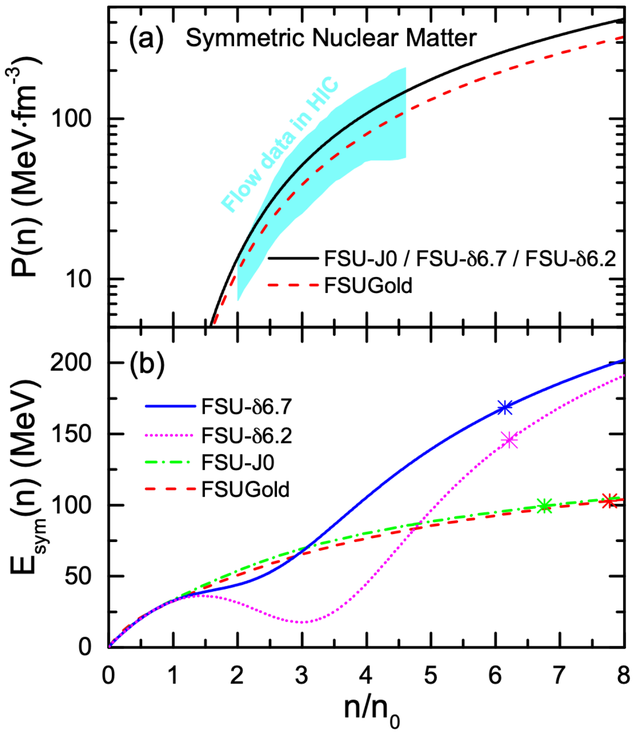



**Figure 1.** Pressure vs density (a) and density dependence of the symmetry energy (b) predicted by FSUGold, FSU-J0, FSU-$\delta$6.7 and FSU-$\delta$6.2. The shaded area in panel (a) represents the constraints from flow data in heavy-ion collisions (Danielewicz et al. 2002). The central density of maximum mass NS configuration is indicated by '$*$' in panel (b).

F. Li, B.-J. Cai, Y. Zhou, W.-Z. Jiang, and L.-W. Chen, Astrophys. J. 929, 183 (2022).

# Hyperon effects (Strangeness)

$$\mathscr{L}_b = \sum_b \bar{\psi}_b \left[ \gamma^\mu \left( i\partial_\mu - g_{\omega b}\omega_\mu - g_{\rho b}\tau_b \cdot \varrho_\mu \right) - \left( m_b - g_{\sigma b}\sigma \right) \right] \psi_b$$

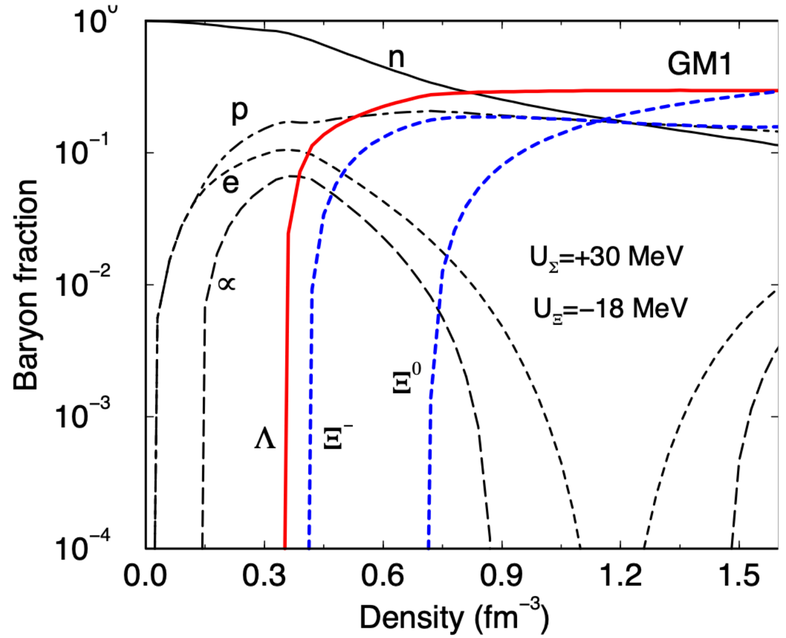



FIG. 31 Neutron star matter fractions of baryons and leptons, calculated as a function of density. From Schaffner-Bielich, 2008.

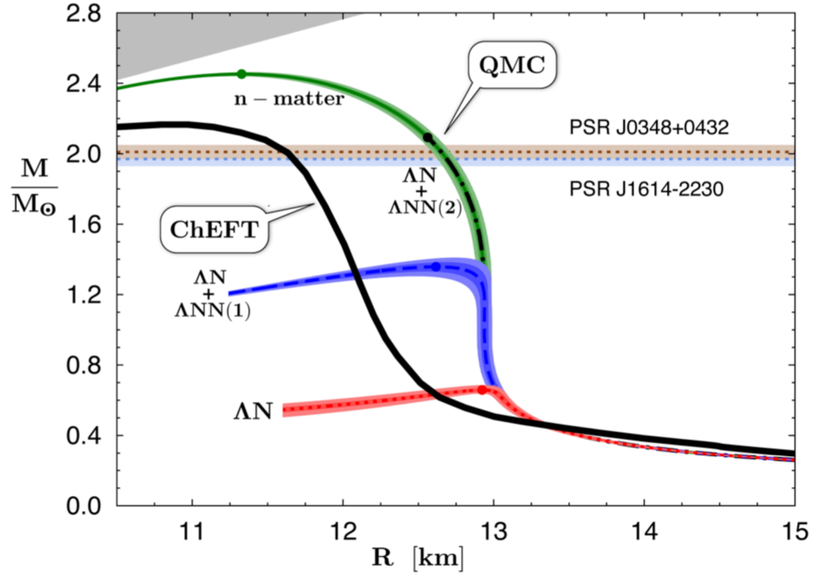



FIG. 33 Mass-radius relationship for various EoS scenarios of neutron stars, including nucleons and leptons only (Hell and Weise, 2014) as well as upon including Λ hyperons (Lonardoni *et al.*, 2015). From Weise, 2015.

Chemical potential equilibrium, e.g. $\mu_\Lambda = \mu_p + \mu_e$

A. Gal, E. V. Hungerford, and D. J. Millener, Rev. Mod. Phys. 88, 035004 (2016).

# Method to handle dense system

## 1. Relativistic mean field approximation

J. D. Walecka, Ann. Phys. 83, 491 (1974).

$$\mathscr{L} = \mathscr{L}_{\mathrm{N}} + \mathscr{L}_{\sigma} + \mathscr{L}_{\omega} + \mathscr{L}_{\mathrm{I}}$$

$$\mathscr{L}_{\mathrm{N}} = \bar{\psi}\left(\mathrm{i}\gamma_{\mu}\partial^{\mu} - m_{\mathrm{N}}\right)\psi$$

$$\mathscr{L}_{\sigma} = \frac{1}{2}\left(\partial_{\mu}\phi\partial^{\mu}\phi - m_{\sigma}^{2}\phi^{2}\right)$$

$$\mathscr{L}_{\omega} = -\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_{\omega}^{2}\omega_{\mu}\omega^{\mu}$$

$$\mathscr{L}_{\mathrm{I}} = \mathscr{L}_{\sigma\mathrm{N}} + \mathscr{L}_{\omega\mathrm{N}} = g_{\sigma}\phi\bar{\psi}\psi - g_{\omega}\omega^{\mu}\bar{\psi}\gamma_{\mu}\psi$$

$$\left(\mathrm{i}\gamma_{\mu}\partial^{\mu} - g_{\omega}\gamma_{0}\omega^{0} - m_{\mathrm{N}}\right)\psi = 0, \quad m_{\mathrm{N}}^{*} = m_{\mathrm{N}} - g_{\sigma}\phi,$$

$$\phi = \frac{g_{\sigma}}{m_{\sigma}^{2}}\rho_{\mathrm{s}}, \quad \rho_{\mathrm{s}} = \langle\bar{\psi}\psi\rangle,$$

$$\omega^{0} = \frac{g_{\omega}}{m_{\omega}^{2}}\rho_{\mathrm{B}}, \quad \rho_{\mathrm{B}} = \left\langle\psi^{\dagger}\psi\right\rangle,$$

## 2. Relativistic Hartree-Fock method

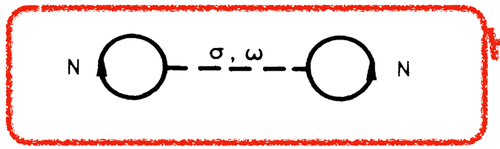
$$\mathscr{L}_{I} = -g_{\sigma}\bar{\psi}\sigma\psi - g_{\omega}\bar{\psi}\gamma_{\mu}\omega^{\mu}\psi + \frac{f_{\omega}}{2M}\bar{\psi}\sigma_{\mu\nu}\partial^{\nu}\omega^{\mu}\psi$$

$$-g_{\rho}\bar{\psi}\gamma_{\mu}\rho^{\mu}\cdot\tau\psi + \frac{f_{\rho}}{2M}\bar{\psi}\sigma_{\mu\nu}\partial^{\nu}\rho^{\mu}\cdot\tau\psi$$

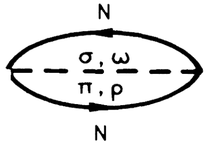
$$-e\bar{\psi}\gamma_{\mu}\frac{1}{2}\left(1+\tau_{3}\right)A^{\mu}\psi + \mathscr{L}_{\pi\mathrm{NN}}$$

A. Bouyssy, J.-F. Mathiot, N. V. Giai, and S. Marcos, Phys Rev C 36, 380 (1987).

$$\Sigma(\mathbf{p}) = \Sigma_{S}(p) + \gamma_{0}\Sigma_{0}(p) + \gamma\cdot\hat{\mathbf{p}}\Sigma_{V}(p)$$




(a)




(b)



**Can we extend an EFT/model taking care of QCD symmetry patterns into dense nucleon systems?**

# An extended linear sigma model in nuclear matter

Phys. Rev. D 109 (2024) 7, 7, **YM** and Y. L. Ma
Phys. Rev. D 112 (2025) 5, 054026, **YM** and Y. L. Ma

P-wave problems in light scalar meson sectors below 1GeV

↓

A tetra-quark picture to include $\delta$ ($a_0(980)$) meson and hyperon

F. E. Close and N. A. Tornqvist, J. Phys. G 28, R249 (2002)

# Freedoms to be considered

Highlights of the parametrization:

I. Include meson exchanges, e.g. $f_0(500)(\sigma)$ and $a_0(980)(\delta)$

II. Include baryon freedoms, e.g. nucleon and hyperon

$$R^\mu = V^\mu - A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N^\mu + \rho^{\mu 0}}{\sqrt{2}} - \frac{f_{1N}^\mu + a_1^{\mu 0}}{\sqrt{2}} & \rho^{\mu +} - a_1^{\mu +} & K^{*\mu +} - K_1^{\mu +} \\ \rho^{\mu -} - a_1^{\mu -} & \frac{\omega_N^\mu - \rho^{\mu 0}}{\sqrt{2}} - \frac{f_{1N}^\mu - a_1^{\mu 0}}{\sqrt{2}} & K^{*\mu 0} - K_1^{\mu 0} \\ K^{*\mu -} - K_1^{\mu -} & \bar{K}^{*\mu 0} - \bar{K}_1^{\mu 0} & \omega_S^\mu - f_{1S}^\mu \end{pmatrix}$$

$$L^\mu = V^\mu + A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N^\mu + \rho^{\mu 0}}{\sqrt{2}} + \frac{f_{1N}^\mu + a_1^{\mu 0}}{\sqrt{2}} & \rho^{\mu +} + a_1^{\mu +} & K^{*\mu +} + K_1^{\mu +} \\ \rho^{\mu -} + a_1^{\mu -} & \frac{\omega_N^\mu - \rho^{\mu 0}}{\sqrt{2}} + \frac{f_{1N}^\mu - a_1^{\mu 0}}{\sqrt{2}} & K^{*\mu 0} + K_1^{\mu 0} \\ K^{*\mu -} + K_1^{\mu -} & \bar{K}^{*\mu 0} + \bar{K}_1^{\mu 0} & \omega_S^\mu + f_{1S}^\mu \end{pmatrix}$$

$$\sigma = \cos\theta_0 \sigma' + \sin\theta_0 \hat{\sigma}'$$

$$a_0 = \cos\theta_8 a_0' + \sin\theta_8 \hat{a}_0{}'$$

$$f_0 = \cos\theta_8 f_0' + \sin\theta_8 \hat{f}_0{}'$$

$$\Phi = S + iP = \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_S^+ + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_S^0 + iK^0 \\ K_S^- + iK^- & \bar{K}_S^0 + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

$\langle \sigma' \rangle = \sqrt{3}\alpha$ and $\langle \hat{\sigma}' \rangle = \sqrt{3}\beta$

$a_0$ mesons may be crucial to NS tidal deformations and neutron skin of nucleus

D. Adhikari et al. (PREX), Phys. Rev. Lett. 126, 172502 (2021)
B. T. Reed, F. J. Fattoyev, C. J. Horowitz, and J. Piekarewicz, Phys. Rev. Lett. 126, 172503 (2021)
F. Li, B. J. Cai, Y. Zhou, W. Z. Jiang, and L. W. Chen, Astrophys. J. 929, 183 (2022)

$$B_N \equiv \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

High densities may lead to hyperon cores of NSs

N. K. Glendenning, Astrophys. J. 293, 470 (1985)
N. K. Glendenning and S. A. Moszkowski, Phys. Rev. Lett. 67, 2414 (1991).
S. Weissenborn, D. Chatterjee, and J. Schaffner-Bielich, Phys. Rev. C 85, 065802 (2012), [Erratum: Phys. Rev. C 90, 019904 (2014)]

# The chiral transformations


$SU(3)_R \otimes SU(3)_L$

$$\Phi' \to g_L \Phi' g_R^\dagger, \quad \hat{\Phi}' \to g_L \hat{\Phi}' g_R^\dagger$$

$$L_\mu \to g_L L_\mu g_L^\dagger, \quad R_\mu \to g_R R_\mu g_R^\dagger$$

$$N_R^{(RR)} \to g_R N_R^{(RR)} g_R^\dagger$$

$$N_R^{(LL)} \to g_R N_R^{(LL)} g_L^\dagger$$

Scalars Vectors Baryons

2-quark $U(1)_A$ 4-quark

$$\Phi' \to e^{2i\nu} \Phi', \quad \hat{\Phi}' \to e^{-4i\nu} \hat{\Phi}',$$

$$L_\mu \to L_\mu, \quad R_\mu \to R_\mu$$

$$N_R^{(RR)} \to e^{-3i\nu} N_R^{(RR)}, \quad N_L^{(RR)} \to e^{-i\nu} N_L^{(RR)}$$

$$N_R^{(LL)} \to e^{i\nu} N_R^{(LL)}, \quad N_L^{(LL)} \to e^{3i\nu} N_L^{(LL)}$$

di-quark approximation

# Power counting rules

① The operators are limited within dimension-4, for the higher dimensional operators are suppressed by the cutoff scale;

② The quark number of an operator is limited within 8 and the number of flavor space traces is limited to only 1, for its suppression by $N_c$;

③ The explicit symmetry breaking caused by quark mass is treated as perturbation, which is crucial to strangeness in NS.

A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 77, 034006 (2008).
A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 72, 034001 (2005)
A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 79, 074014 (2009)
D. Parganlija, F. Giacosa, and D. H. Rischke, Phys. Rev. D 82, 054024 (2010)

# The Lagrangian at the leading order for RMF

$$\mathscr{V}_{\mathrm{M}} = c_2 \operatorname{Tr} S'^2 - d_2 \operatorname{Tr} \hat{S}'^2 - c_4 \operatorname{Tr} S'^4 - 2e_3 \epsilon_{abc}\epsilon_{def} S'_{ad} S'_{be} \hat{S}'_{cf}$$

Quark configuration mixing

$$\mathscr{V}_{\mathrm{V}} = \tilde{h}_2 \operatorname{Tr}\left(S'^2 V^2\right) + \tilde{g}_3 \operatorname{Tr} V^4 +$$

$$a_1 \epsilon_{abc}\epsilon_{def} V_{ad} V_{be} \left(S'^2\right)_{cf}$$

Masses by spontaneous chiral symmetry breaking

$$\mathscr{L}_{\mathrm{B}}^{\mathrm{RMF}} = \operatorname{Tr}\left(\bar{B} i\gamma_\mu \partial^\mu B\right) + c \operatorname{Tr}\left(\bar{B}\gamma^0 V B\right) - g \operatorname{Tr}\left(\bar{B} S' B\right)$$

$$+ h\epsilon_{abc}\epsilon_{def} \bar{B}_{ad}\gamma^0 B_{be} V_{cf}$$

$$- e\epsilon_{abc}\epsilon_{def} \bar{B}_{ad}\gamma^0 B_{be} S'_{cf}$$

Only two quark configuration couples to baryon at the leading order

# Hadron masses and NM properties

The units are in MeV, except $n_0$ is in $\mathrm{fm}^{-3}$

Y. Sugahara and H. Toki, Nucl. Phys. A579,557 (1994).
F. Li, B. J. Cai, Y. Zhou, W. Z. Jiang, and L. W. Chen, Astrophys. J. 929, 183 (2022).

| | $g_{\sigma NN}$ | $g_{a_0 NN}$ | $g_{fNN}$ | $g_{\omega NN}$ | $g_{\rho NN}$ |
|---|---|---|---|---|---|
| TM1 | −10.0 | — | — | −12.6 | −4.63 |
| FSU-$\delta$6.7 | 10.2 | 6.7 | — | −13.4 | −7.27 |
| el-g30g | −6.17 | 5.13 | 2.95 | −6.09 | 5.30 |
| el-g30e | −6.20 | −5.03 | 3.00 | 6.09 | 5.30 |
| el-g30eg | −5.97 | −0.671 | 2.68 | 6.06 | 3.45 |
| el-g350eg | −6.12 | −0.852 | 2.85 | 6.37 | 3.71 |
| el-g3100eg | −6.36 | −0.442 | 3.19 | 6.73 | 3.85 |
| el-g3150eg | −6.38 | −0.413 | 3.20 | 7.09 | 4.04 |

Increasing the four-vector couplings

| | $n_0$ | $E_0$ | $E_{\mathrm{sym}}(n_0)$ | $J(n_0)$ | $L(n_0)$ | $K(n_0)$ |
|---|---|---|---|---|---|---|
| Empirical | $0.155 \pm 0.050$ | $-16.0 \pm 1.0$ | $31.7 \pm 3.2$ | $-200 \pm 600$ | $58.7 \pm 28.1$ | $250 \pm 50$ |
| el-g30g | 0.155 | −14.6 | 30.9 | 451 | 83.6 | 418 |
| el-g30e | 0.155 | −14.6 | 31.6 | 479 | 85.8 | 419 |
| el-g30eg | 0.155 | −14.6 | 30.1 | 421 | 82.2 | 415 |
| el-g350eg | 0.155 | −15.2 | 30.9 | −392 | 80.7 | 370 |
| el-g3100eg | 0.155 | −15.4 | 31.4 | −1020 | 71.7 | 317 |
| el-g3150eg | 0.155 | −15.6 | 31.6 | −1470 | 63.7 | 253 |

| | $m_N$ | $m_\sigma$ | $m_{f_0(a_0)}$ | $m_{f_0'(a_0')}$ | $m_{\sigma'}$ | $m_\rho$ | $m_\rho$ |
|---|---|---|---|---|---|---|---|
| Empirical | 938-940 | 400-800 | 960-1010 | 1250-1500 | 1430-1530 | 761-765 | 782-783 |
| el-g30g | 939 | 522 | 989 | 1480 | 1510 | 763 | 783 |
| el-g30e | 939 | 525 | 991 | 1480 | 1510 | 763 | 783 |
| el-g30eg | 939 | 498 | 983 | 1500 | 1520 | 763 | 783 |
| el-g350eg | 939 | 485 | 991 | 1470 | 1510 | 763 | 783 |
| el-g3100eg | 939 | 502 | 994 | 1470 | 1510 | 763 | 783 |
| el-g3150eg | 939 | 485 | 991 | 1470 | 1510 | 763 | 783 |

A. Sedrakian, J. J. Li, and F. Weber, Prog. Part. Nucl. Phys. 131, 104041 (2023)
J. Piekarewicz, Phys. Rev. C 69, 041301 (2004).
M.I. Tews, J. M. Lattimer, A. Ohnishi, and E. E. Kolomeitsev, Astrophys. J. 848, 105 (2017).
M. Dutra, et al., Phys. Rev. C 85, 035201 (2012).
R. L. Workman, et al. (Particle Data Group), PTEP 2022, 083C01 (2022).

# Comparison with Walecka-type models

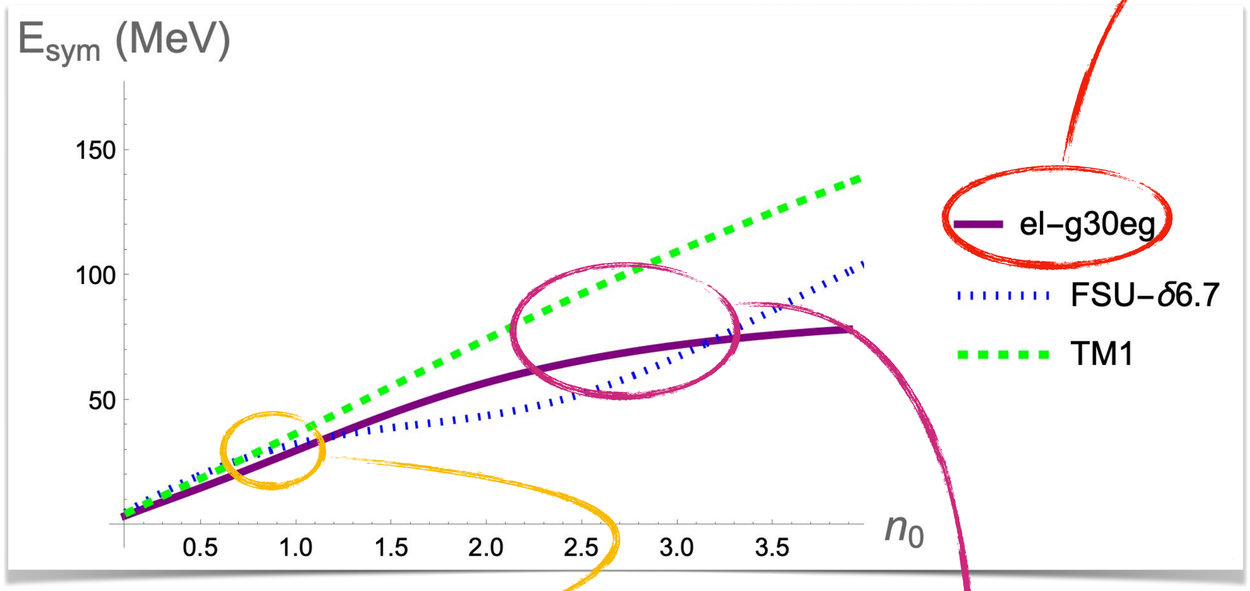
**Saturation density**

A. Sedrakian, J. J. Li, and F. Weber, Prog. Part. Nucl. Phys. 131, 104041 (2023)

| | Empirical | ELSM | TM1 | FSU – $\delta$6.7 |
|---|---|---|---|---|
| $n_0(\mathrm{fm}^{-3})$ | 0.155±0.005 | 0.155 | 0.145 | 0.148 |

Small $g_3$ and $g_\delta$

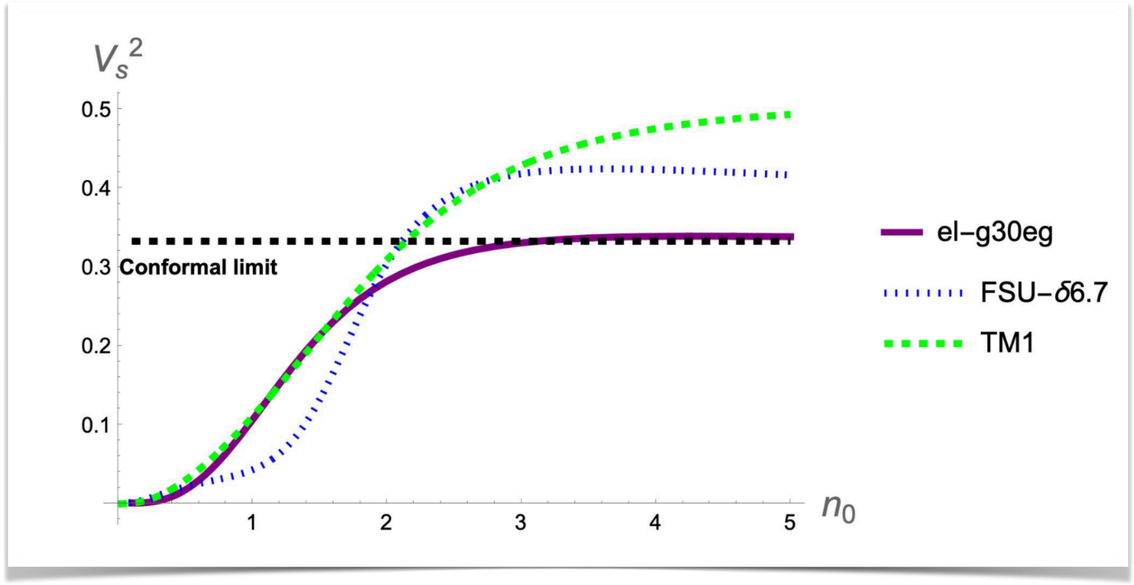
## Symmetric nuclear matter




$L\left(2/3n_0\right) \geq 49\mathrm{MeV}$

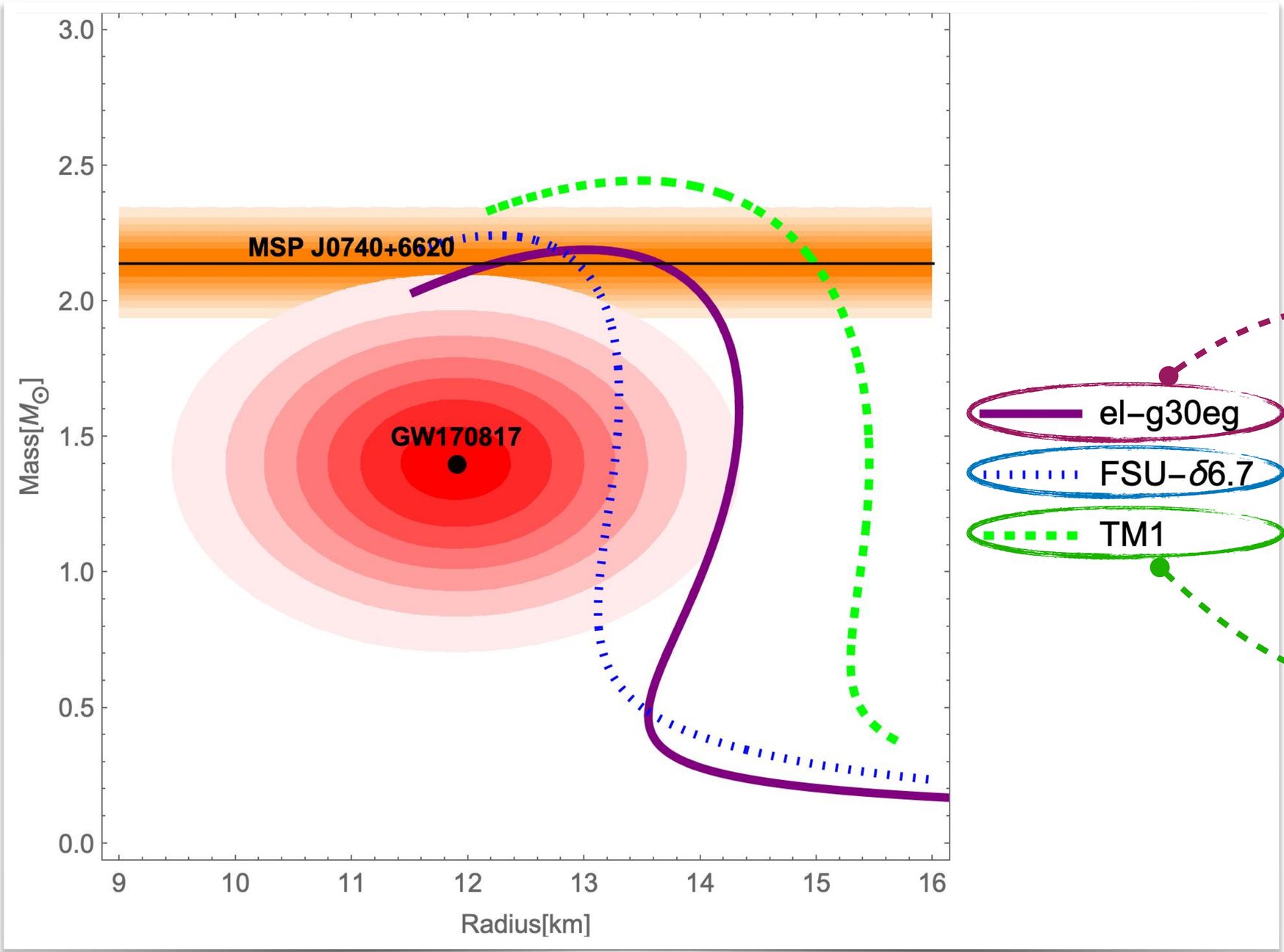
GW170817 $\Lambda_{1.4} \leq 580$

## Pure neutron matter

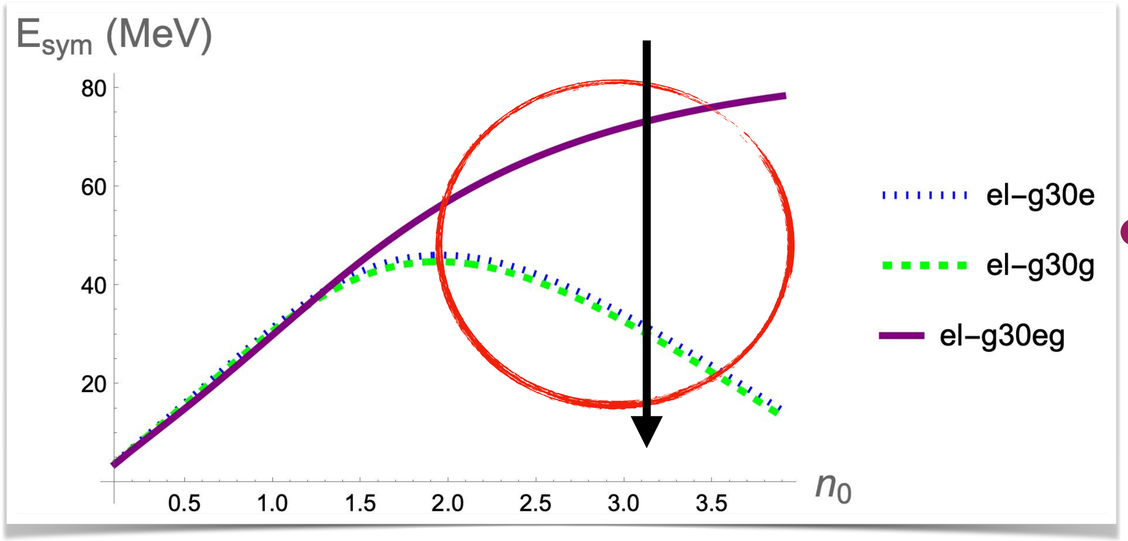


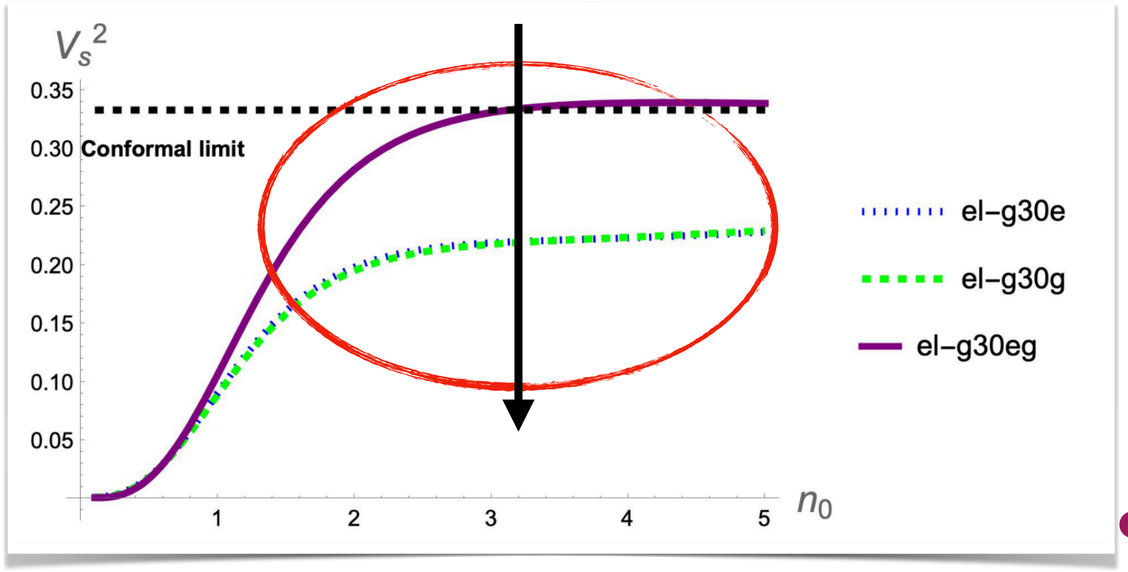

D. Adhikari et al., PREX, Phys. Rev. Lett. 126, 172502 (2021)
B. T. Reed, F. J. Fattoyev, C. J. Horowitz, and J. Piekarewicz, Phys. Rev. Lett. 126, 172503 (2021)

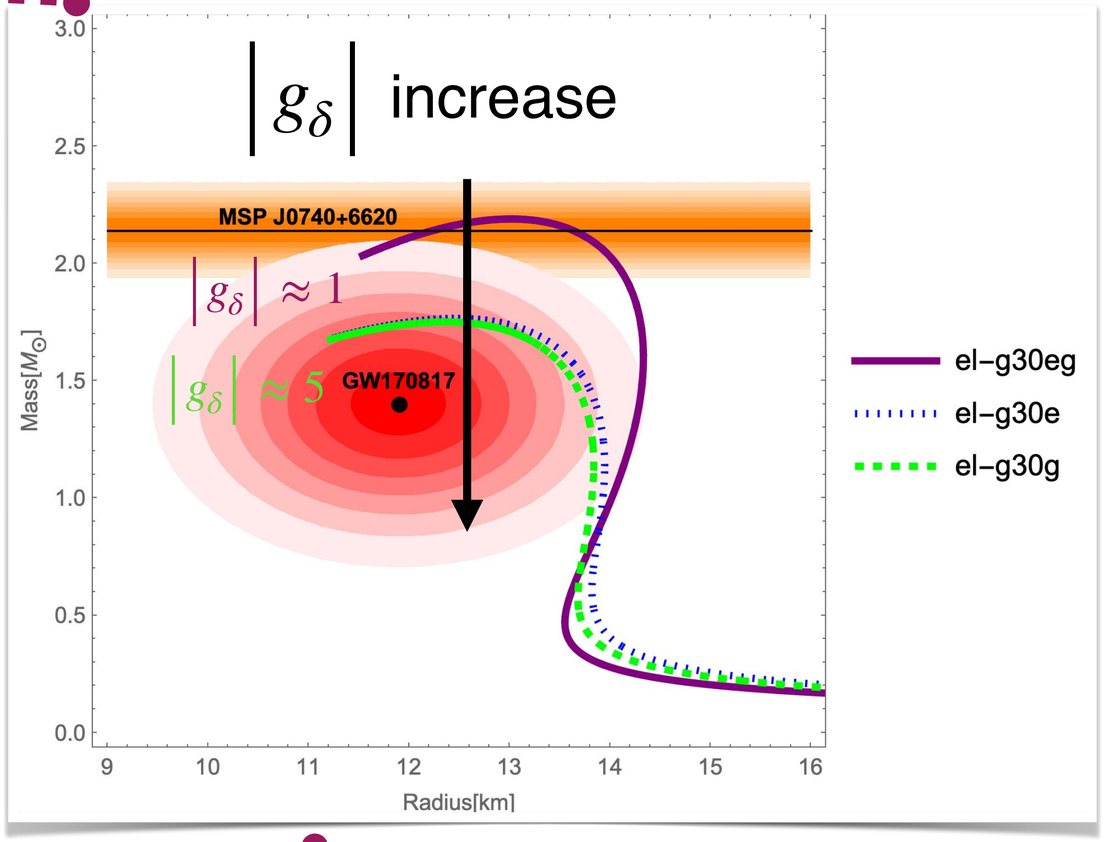
B. P. Abbott et al., LIGO Scientific, Virgo, Phys. Rev. Lett. 121, 161101 (2018)


3.0
2.5
2.0
1.5
1.0
0.5
0.0
Mass[$M_\odot$]
9
10
11
12
13
14
15
16
Radius[km]
MSP J0740+6620
GW170817
el–g30eg
FSU–δ6.7
TM1
$g_\delta \sim 1$
$g_\delta \sim 5$
$g_\delta = 0$

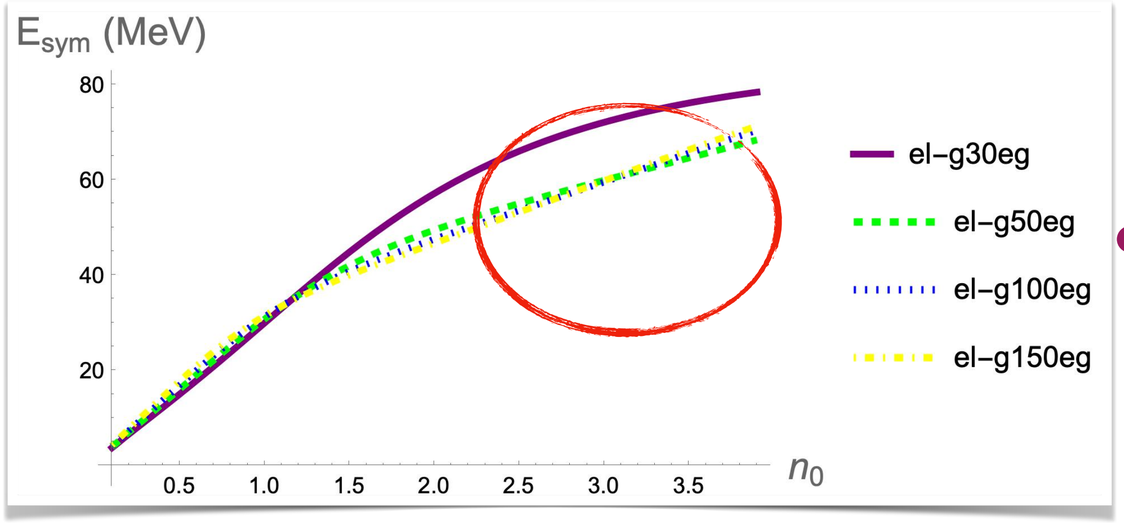
# Comparison among different $g_{aNN}$ cases

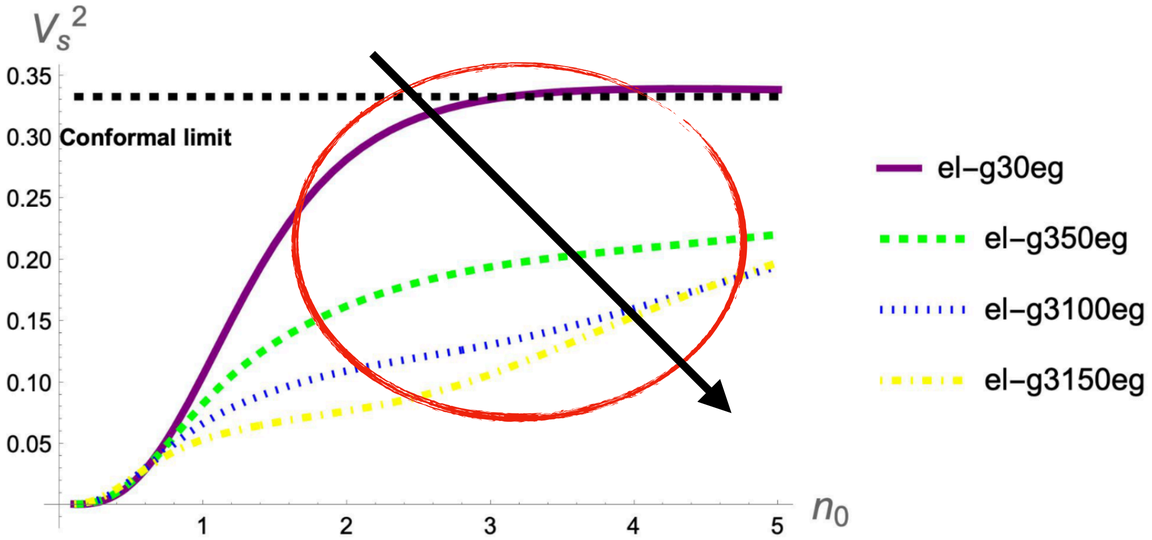


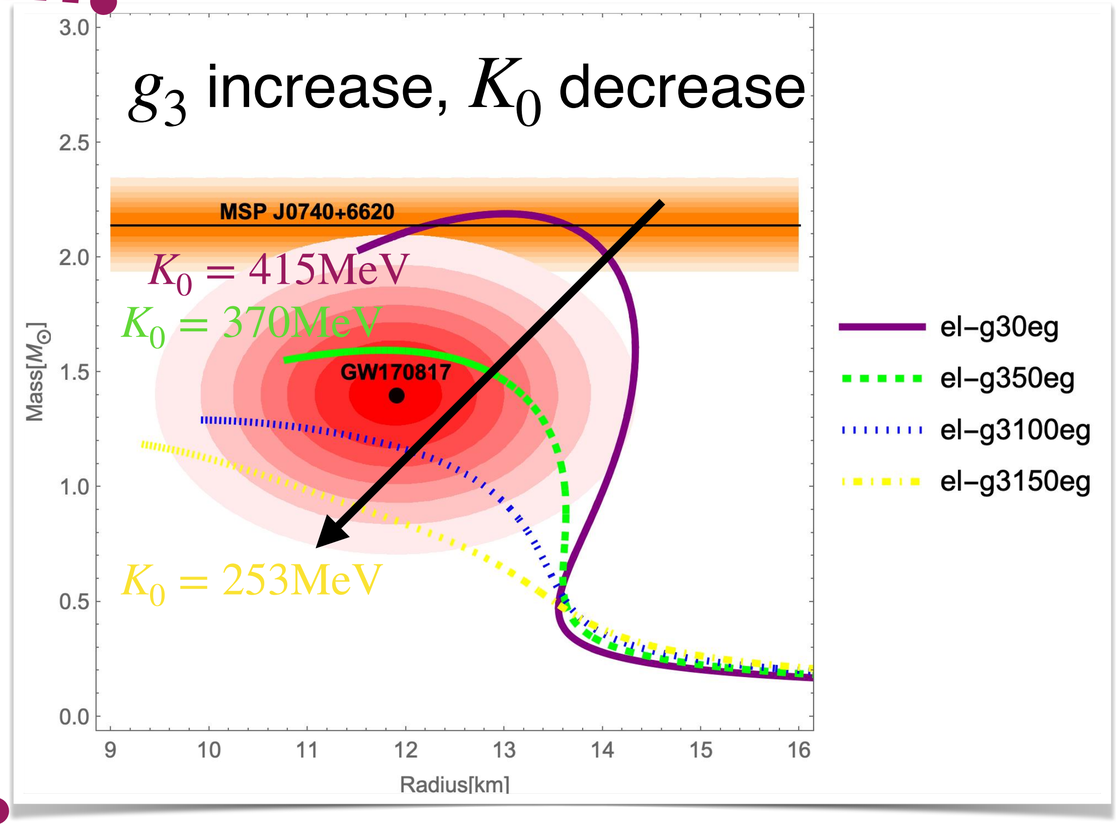
# Comparison among different $g_3$ cases





H. T. Cromartie et al. (NANOGrav), Nature Astron. 4, 72 (2019).
B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 121, 161101 (2018),

# Quite different parameter space from EFT view

MeV

| | $C_{\sigma\delta^2}$ | $C_{\sigma^2\delta^2}$ |
|---|---|---|
| Zabari-19 | $\pm 1.77$ | $\pm 0.004$ |
| FSU-$\delta$6.7 | — | 2.63 |
| el-g30g | $-1860$ | $-9.40$ |
| el-g30e | $-1940$ | $-9.71$ |
| el-g30eg | $-1480$ | $-7.70$ |
| el-g350eg | $-1690$ | $-8.75$ |
| el-g3100eg | $-2190$ | $-11.0$ |
| el-g3150eg | $-2160$ | $-11.0$ |

Quite different parameter space!

| | $\sigma$ | | $\delta$ | |
|---|---|---|---|---|
| | $p_4$ | $p_2$ | $p_4$ | $p_2$ |
| el-g30g | 51.7% | 48.3% | 77.9% | 22.1% |
| el-g30e | 51.2% | 48.7% | 77.2% | 22.8% |
| el-g30eg | 54.2% | 45.8% | 81.5% | 18.5% |
| el-g350eg | 52.4% | 47.6% | 77.3% | 20.7% |
| el-g3100eg | 49.2% | 50.8% | 74.4% | 25.6% |
| el-g3150eg | 48.9% | 51.1% | 74.3% | 25.7% |

Slight difference of configurations
but large difference at macroscopic level

N. Zabari, S. Kubis, and W. Wójcik, Phys. Rev. C 99, 035209 (2019)
F. Li, B.-J. Cai, Y. Zhou, W.-Z. Jiang, and L.-W. Chen, Astrophys. J. 929, 183 (2022).

**Strangeness in this framework under RMF (Neglecting the 4-quark configurations but with explicit symmetry breaking terms at leading order)**


In collaboration with Lu-Qi Zhang (张璐琦)

$$\mathscr{V}_M = c_2 \,\mathrm{Tr}\, S'^2 - \mathrm{c}_4 \,\mathrm{Tr}\, S'^4 - \mathrm{b}_1 \,\mathrm{Tr}\left(\xi S'^3\right) - 2\mathrm{G}\,\mathrm{Tr}\left(\xi S'\right)$$

$$\mathscr{V}_V = \tilde{h}_2 \,\mathrm{Tr}\left(V^2 S'^2\right) + a_1 \epsilon^{ijk}\epsilon^{lmn}(V)_{il}(V)_{jm}\left(S'^2\right)_{kn} + b_2 \,\mathrm{Tr}\left(V^2 \xi S'\right)$$
$$+ b_3 \epsilon^{ijk}\epsilon^{lmn}(V)_{il}(V)_{jm}\left(\xi S'\right)_{kn} + \tilde{g}_3 \,\mathrm{Tr}\, V^4 + \tilde{a}_2 \epsilon^{ijk}\epsilon^{lmn}(V)_{il}(V)_{jm}(V^2)_{kn}$$

$$\mathscr{L}^B_{\mathrm{RMF}} = \mathrm{Tr}\left(\bar{B} i\gamma_\mu \partial^\mu B\right) + c\,\mathrm{Tr}\left[\bar{B}\gamma_0 V B\right] + c'\,\mathrm{Tr}\left[\bar{B}\gamma_0 B V\right]$$
$$+ h\epsilon^{ijk}\epsilon^{lmn}(\bar{B})_{il}\left(V\gamma_0\right)_{jm}(B)_{kn} - g\,\mathrm{Tr}[\bar{B}S'B]$$
$$- e\epsilon^{ijk}\epsilon^{lmn}(\bar{B})_{il}(S')_{jm}(B)_{kn} - b_4\,\mathrm{Tr}[\bar{B}\xi B] - b_5\epsilon^{ijk}\epsilon^{lmn}(\bar{B})_{il}(\xi)_{jm}(B)_{kn}$$

Mass splittings (beta equilibrium )

# Strangeness in this framework under RMF (Neglecting the 4-quark configurations)

arXiv:2512.23477 [nucl-th], (2025), YM, Y. L. Ma and L.-Q. Zhang

$$S = \frac{1}{\sqrt{2}}\left(\lambda_8 f_0 + \lambda_3 a_0\right) + \frac{1}{\sqrt{3}}\mathrm{I}\cdot\sigma$$

$$V = \frac{1}{2}\begin{pmatrix} \rho+\omega & 0 & 0 \\ 0 & -\rho+\omega & 0 \\ 0 & 0 & \sqrt{2}\phi \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

Neglected in EOS after the determination of mass spectrum

# Physical couplings and mass formula

$$g_{\rho NN} = -\frac{1}{2}(c+h)$$

$$g_{\omega NN} = -\frac{1}{2}(c-h)$$

$$g_{\omega\Lambda\Lambda} = -\frac{1}{6}\left(c+c'-4h\right)$$

$$g_{\sigma NN} = g_{\sigma\Lambda\Lambda} = -\frac{1}{\sqrt{3}}(g-e)$$

$$\xi = \xi_1\lambda_1 + \xi_3\lambda_3 + \xi_8\lambda_8$$

Non-zero $\langle a_0 \rangle$ and $\langle f_0 \rangle$

Gell-Mann Okubo Formula

$$m_{\Sigma^+} = \frac{1}{2}\left(-2m_n + 3m_\Lambda - 2m_{\Xi^-} + 3m_{\Sigma^0}\right)$$

$$m_{\Sigma^-} = \frac{1}{2}\left(2m_n - 3m_\Lambda + 2m_{\Xi^-} + m_{\Sigma^0}\right)$$

$$m_{\Xi^0} = -m_n - m_p + 3m_\Lambda - m_{\Xi^-} + m_{\Sigma^0}\,.$$

$$m_{a_0}^2 + m_{f_0}^2 = 2m_\sigma^2$$

# Hadron spectra and nuclear matter properties

| | Empirical | $\sigma_{\pi N}$-75$^{+}$ | $\sigma_{\pi N}$-100$^{-}$ | $\sigma_{\pi N}$-400$^{-}$ | $\sigma_{\pi N}$-600$^{-}$ |
|---|---|---|---|---|---|
| $n_0(\mathrm{fm}^{-3})$ | $0.1550 \pm 0.0500$ | 0.1592 | 0.1592 | 0.1592 | 0.1592 |
| $e_0(\mathrm{MeV})$ | $-15.00 \pm 1.00$ | $-16.00$ | $-16.00$ | $-16.02$ | $-16.00$ |
| $K(n_0)(\mathrm{MeV})$ | $230.0 \pm 30.0$ | 240.8 | 241.3 | 241.6 | 239.2 |
| $E_{\mathrm{sym}}(n_0)(\mathrm{MeV})$ | $30.90 \pm 1.90$ | 30.00 | 30.00 | 30.01 | 30.00 |
| $L(n_0)(\mathrm{MeV})$ | $52.50 \pm 17.50$ | 54.91 | 52.18 | 70.94 | 76.74 |

Align with the empirical values

$$U_\Lambda \approx -28 \text{ MeV}$$

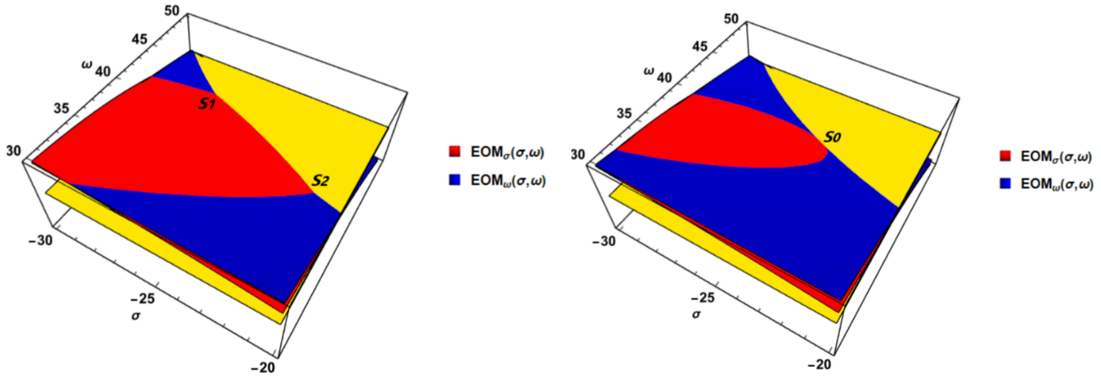
$$m_N = m + \sigma_{\pi N}(0)$$

Account for the contribution of spontaneous and explicit breaking

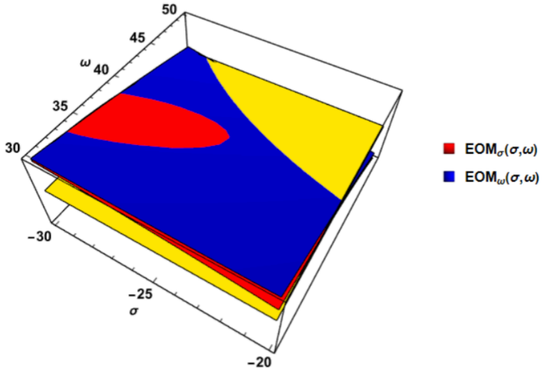
| | Empirical | $\sigma_{\pi N}$-75$^{+}$ | $\sigma_{\pi N}$-100$^{-}$ | $\sigma_{\pi N}$-400$^{-}$ | $\sigma_{\pi N}$-600$^{-}$ |
|---|---|---|---|---|---|
| $m_n$ | $939.565 \pm (5 \times 10^{-7})$ | 939.565 | 939.565 | 939.565 | 939.565 |
| $m_p$ | $938.272 \pm (2.9 \times 10^{-7})$ | 938.272 | 938.272 | 938.272 | 938.272 |
| $m_\Lambda$ | $1115.68 \pm (0.006)$ | 1115.59 | 1115.54 | 1115.57 | 1115.58 |
| $m_{\Sigma+}$ | $1189.37 \pm 0.06$ | 1192.30 | 1191.21 | 1191.74 | 1191.79 |
| $m_{\Sigma^0}$ | $1192.64 \pm 0.02$ | 1192.17 | 1192.18 | 1192.84 | 1191.98 |
| $m_{\Sigma-}$ | $1197.45 \pm 0.03$ | 1192.03 | 1191.15 | 1191.95 | 1192.16 |
| $m_{\Xi^0}$ | $1314.82 \pm 0.21$ | 1331.34 | 1330.65 | 1330.90 | 1330.90 |
| $m_{\Xi-}$ | $1321.70 \pm 0.09$ | 1329.77 | 1329.30 | 1329.82 | 1329.98 |
| $m_{a_0}$ | $995.000 \pm 25.000$ | 980.132 | 979.998 | 980.024 | 980.040 |
| $m_{f_0}$ | $995.000 \pm 15.000$ | 990.119 | 989.962 | 989.971 | 989.974 |
| $m_\sigma$ | $475.000 \pm 75.000$ | 985.183 | 984.993 | 986.010 | 985.019 |
| $m_\rho$ | $773.000 \pm 2.000$ | 775.261 | 775.260 | 775.260 | 775.260 |
| $m_\omega$ | $782.660 \pm 0.13$ | 782.659 | 782.660 | 782.660 | 782.660 |
| $m_\phi$ | $1019.46 \pm 0.02$ | 1019.46 | 1019.46 | 1019.46 | 1019.46 |
| $\sigma_{\pi N}(0)$ | $60.5000 \pm 28.5000$ | 75.0541 | $-104.048$ | $-402.087$ | $-625.663$ |
| $m$ | $878.419 \pm 29.1465$ | 863.946 | 1043.05 | 1341.09 | 1564.66 |

# Problems in M-R relation and RMF approach for $\sigma_{\pi N}$-$75^{+}$

Solution plane for EOMs of mesons

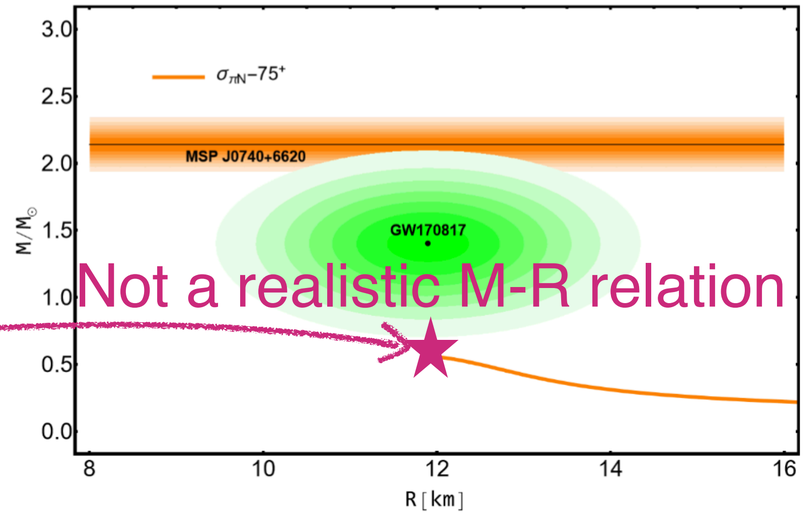



(a) EOMs at $n = 3.900n_0$. (b) EOMs at $n = 4.015n_0$.

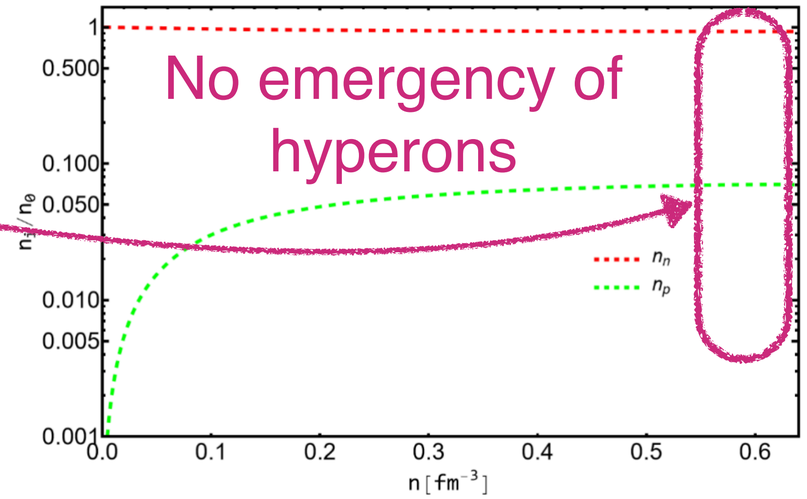



(c) EOMs at $n = 4.050n_0$.

No continuous solutions

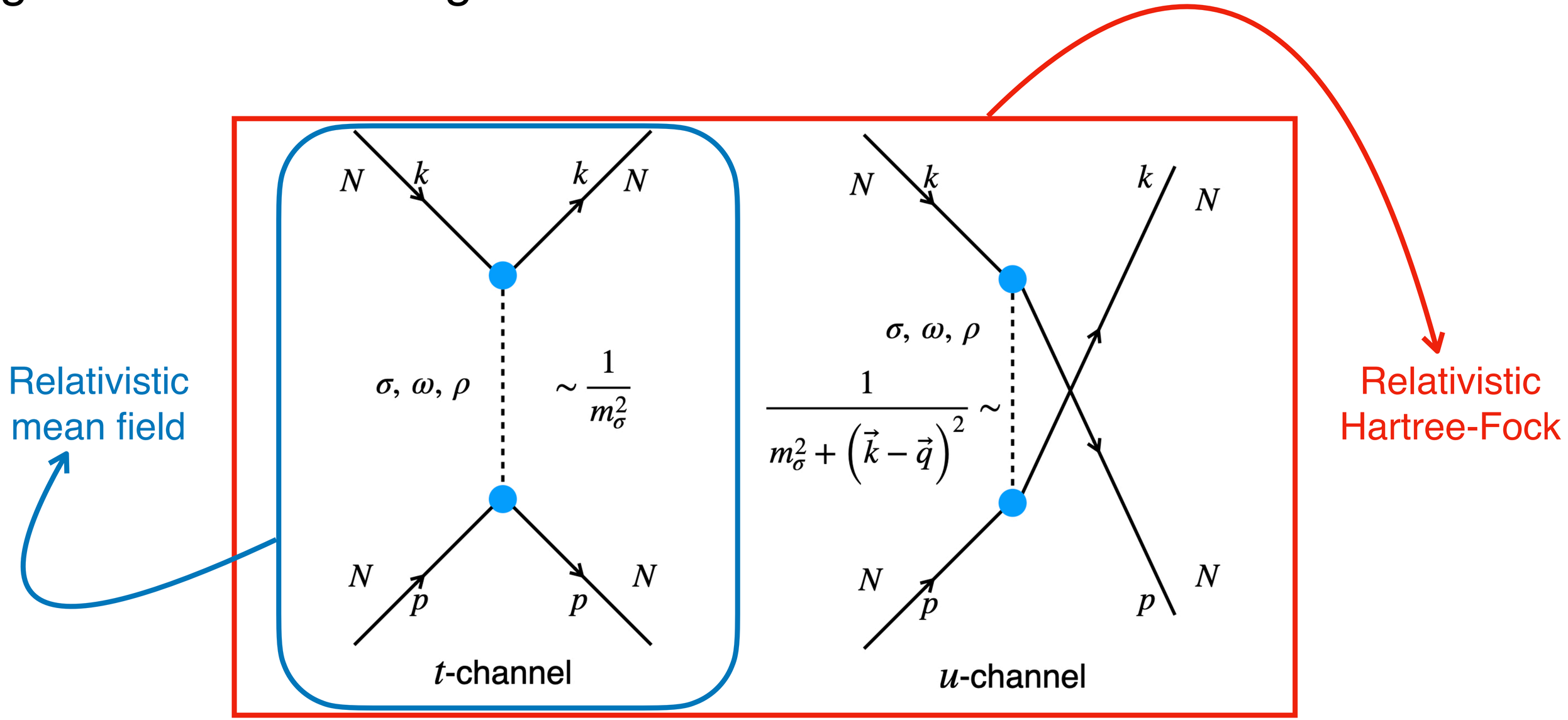



(a) M-R relation for $\sigma_{\pi N}$-$75^{+}$.

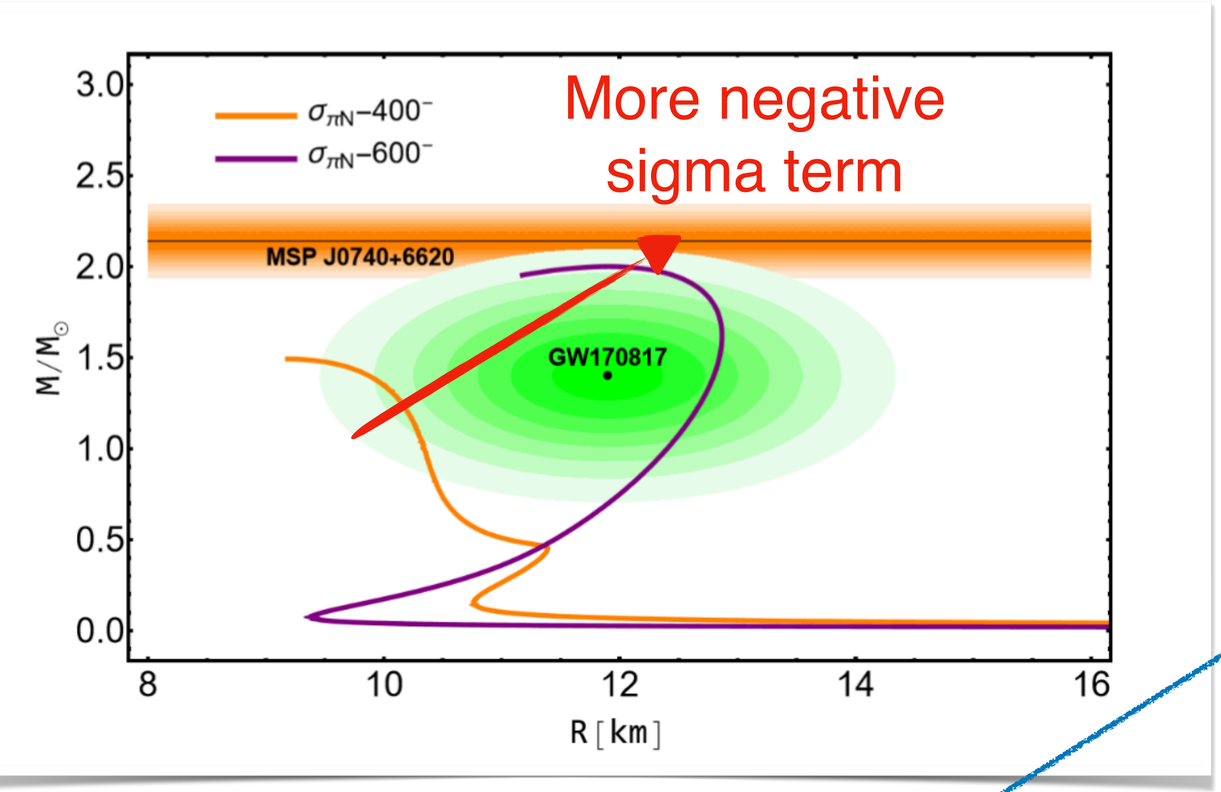



(b) Baryon fraction for $\sigma_{\pi N}$-$75^{+}$.

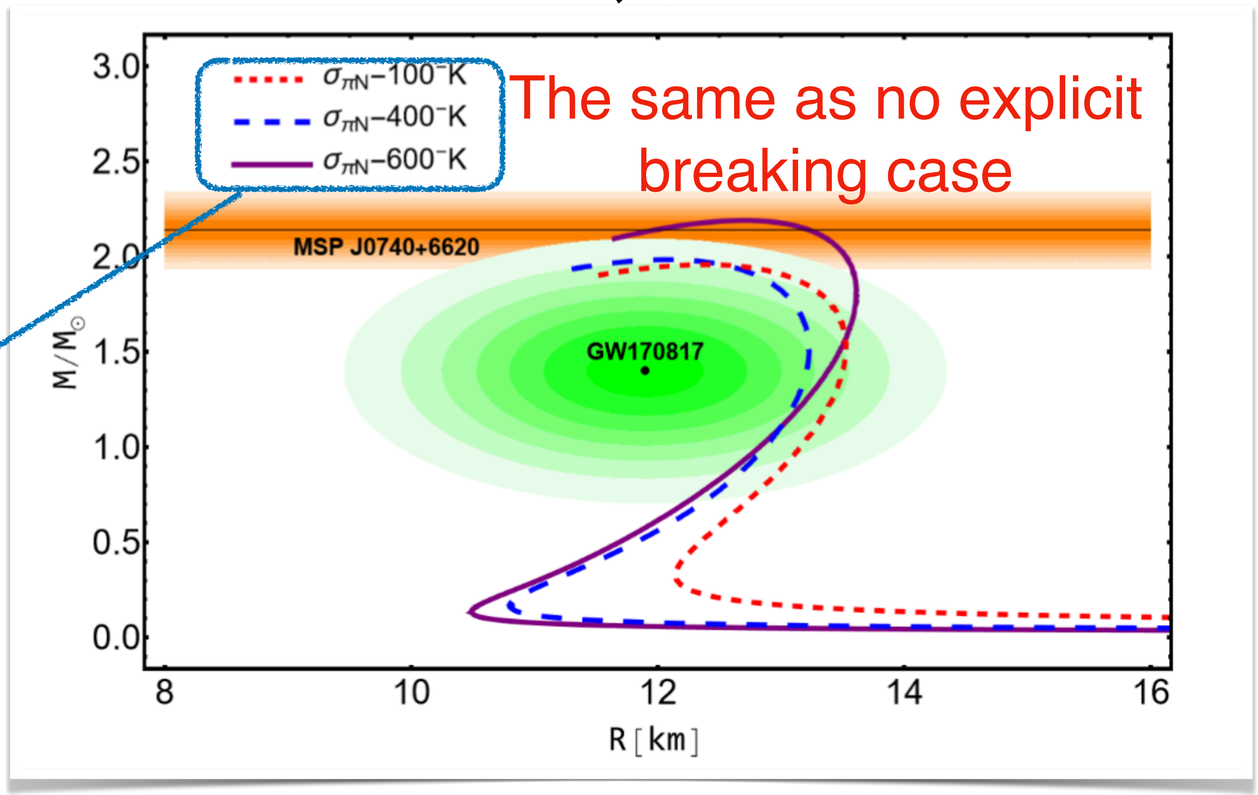
# Ignorance of exchange terms of nucleon interaction



# Effects of sigma term and incompressibility



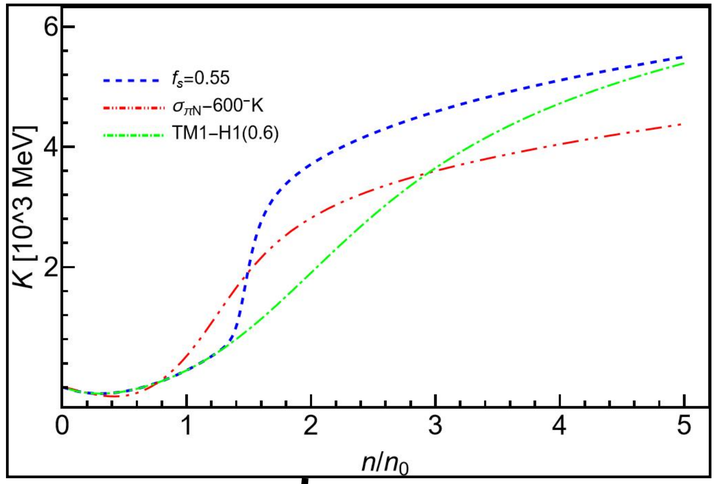

| | Empirical | $\sigma_{\pi N}$-100$^-K$ | $\sigma_{\pi N}$-400$^-K$ | $\sigma_{\pi N}$-600$^-K$ |
|---|---|---|---|---|
| $n_0$(fm$^{-3}$) | $0.1550 \pm 0.0500$ | 0.1592 | 0.1592 | 0.1592 |
| $e_0$(MeV) | $-15.00 \pm 1.00$ | $-16.00$ | $-15.99$ | $-16.00$ |
| $K(n_0)$(MeV) | — | 521.9 | 520.3 | 518.8 |
| $E_{\rm sym}(n_0)$(MeV) | $30.90 \pm 1.90$ | 30.00 | 30.00 | 30.01 |
| $L(n_0)$(MeV) | $52.50 \pm 17.50$ | 80.15 | 93.28 | 76.64 |



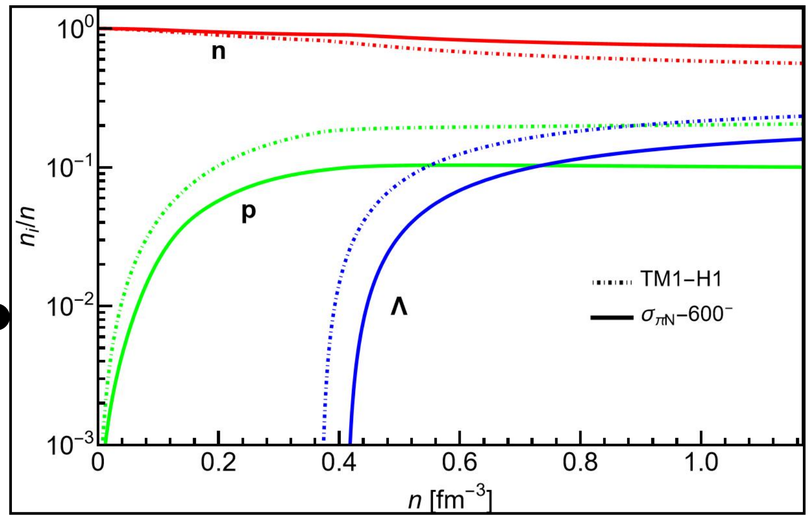

Still negative sigma term

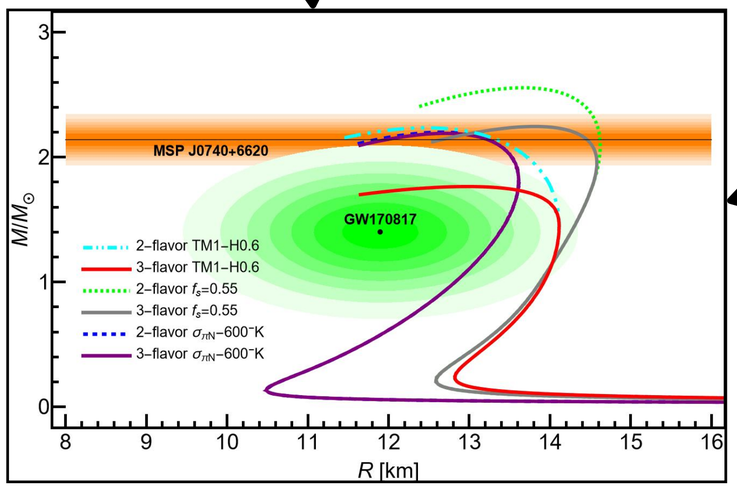
# Comparison with Walecka models

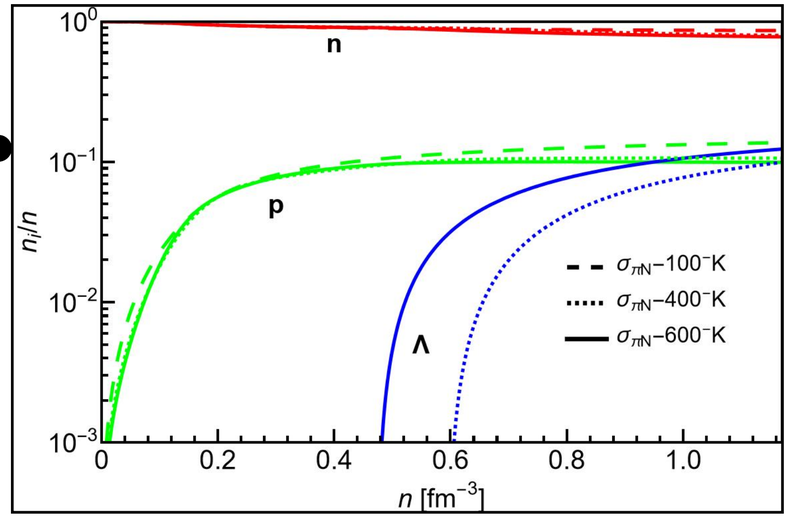
Y. Zhang, J. Hu, and P. Liu, Phys. Rev. C 97, 015805 (2018).
K. A. Maslov, E. E. Kolomeitsev, and D. N. Voskresensky, Phys. Rev. C 92, 052801 (2015).




Hyperon Ratio



# What could be learned?

I. The low energy EFTs/models of QCD can reproduce the vacuum spectra, NM properties and NS structures with appropriate parameter choice but different from past studies based on Walecka models;

II. A scaling scheme for parameters to cover different physical regions may be crucial to connect the QCD symmetry patterns and NS observations;

III. At high densities, Fock terms (quantum exchange effects) ignored in RMF may play an important role.

# Thank you!

# Backup


# Outline

嚴謹求實
勤篤創新

- Motivation (How to connect the microscopic symmetries to the rich macroscopic phenomenons)
- Theoretical framework and phenomenological analysis (Nuclear matter properties and neutron star structures with an extended linear sigma model (chiral symmetry))

# The lowest order Lagrangians (The system only with nucleons)


The scalar meson masses by spontaneous chiral symmetry breaking

$$\mathscr{L}_{\mathrm{M}} = \frac{1}{2}\mathrm{Tr}\left(\partial_\mu \Phi' \partial^\mu \Phi'^\dagger\right) + \frac{1}{2}\mathrm{Tr}\left(\partial_\mu \hat{\Phi}' \partial^\mu \hat{\Phi}'^\dagger\right) + c_2\,\mathrm{Tr}\left(\Phi'\Phi'^\dagger\right) - c_4\,\mathrm{Tr}\left(\Phi'\Phi'^\dagger\Phi'\Phi'^\dagger\right) - d_2\,\mathrm{Tr}\left(\hat{\Phi}'\hat{\Phi}'^\dagger\right) -$$

$$e_3\left(\epsilon_{abc}\epsilon^{def}\Phi'^a_d\Phi'^b_e\hat{\Phi}'^c_f + \text{ h.c. }\right) - c_3\left[\gamma_1 \ln\left(\frac{\det\Phi'}{\det\hat{\Phi}'^\dagger}\right) + (1-\gamma_1)\ln\left(\frac{\mathrm{Tr}\left(\Phi'\hat{\Phi}'^\dagger\right)}{\mathrm{Tr}\left(\hat{\Phi}'\Phi'^\dagger\right)}\right)\right]^2$$

Two quark configurations mixing

A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 72, 034001 (2005)
A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 77, 034006 (2008)
A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 79, 074014 (2009)

$$\mathscr{L}_{\mathrm{V}} = -\frac{1}{8}\mathrm{Tr}\left(L_{\mu\nu}L^{\mu\nu} + R_{\mu\nu}R^{\mu\nu}\right) + g_1\left[\mathrm{Tr}\left(\partial_\nu R_\mu R^\mu R^\nu\right) + \mathrm{Tr}\left(\partial_\nu L_\mu L^\mu L^\nu\right)\right] +$$

$$g_2\left[\mathrm{Tr}\left(\partial_\nu R_\mu R^\nu R^\mu\right) + \mathrm{Tr}\left(\partial_\nu L_\mu L^\nu L^\mu\right)\right] + g_3\left[\mathrm{Tr}\left(R_\mu R^\mu R_\nu R^\nu\right) + \mathrm{Tr}\left(L_\mu L^\mu L_\nu L^\nu\right)\right] +$$

$$g_4\left[\mathrm{Tr}\left(R_\mu R^\nu R_\mu R^\nu\right) + \mathrm{Tr}\left(L_\mu L^\nu L_\mu L^\nu\right)\right] + h_1\left[\mathrm{Tr}\left(\Phi'^{\dagger}\Phi' R_\mu R^\mu\right) + \mathrm{Tr}\left(\Phi'\Phi'^{\dagger} L_\mu L^\mu\right)\right] +$$

$$h_2\left[\mathrm{Tr}\left(L^\mu \Phi' R_\mu \Phi'^{\dagger}\right)\right] + h_3\left[\mathrm{Tr}\left(\Phi'^{\dagger}\partial_\mu \Phi' R^\mu\right) + \mathrm{Tr}\left(\Phi'\partial_\mu \Phi'^{\dagger} L^\mu\right)\right] +$$

$$\frac{a_1}{2}\epsilon_{abc}\epsilon^{def}\left[\left(R_\mu\right)_{ad}\left(R_\nu\right)_{be}\left(\Phi'^{\dagger}\Phi\right)_{cf} + \left(L_\mu\right)_{ad}\left(L_\nu\right)_{be}\left(\Phi\Phi'^{\dagger}\right)_{cf}\right] +$$

$$\frac{a_2}{2}\epsilon_{abc}\epsilon^{\mathrm{def}}\left[\left(R_\mu\right)_{ad}\left(R_\nu\right)_{be}\left(R^\mu R^\nu\right)_{cf} + \left(L_\mu\right)_{ad}\left(L_\nu\right)_{be}\left(L^\mu L^\nu\right)_{cf}\right] +$$

$$\frac{a_3}{2}\epsilon_{abc}\epsilon^{def}\left[\left(R_\mu\right)_{ad}\left(R^\mu\right)_{be}\left(R^\nu R_\nu\right)_{cf} + \left(L_\mu\right)_{ad}\left(L^\mu\right)_{be}\left(L^\nu L_\nu\right)_{cf}\right] +$$

$$\frac{a_4}{2}\epsilon_{abc}\epsilon^{def}\left[\left(R_\mu\right)_{ad}\left(R^\nu\right)_{be}\left(\partial^\mu R_\nu\right)_{cf} + \left(L_\mu\right)_{ad}\left(L^\nu\right)_{be}\left(\partial^\mu L_\nu\right)_{cf}\right]$$

The vector meson masses by spontaneous chiral symmetry breaking

# Di-quark approximation

L. Olbrich, Master's thesis, Goethe U., Frankfurt (main) (2015).
L. Olbrich, M. Zétényi, F. Giacosa, and D. H. Rischke, Phys. Rev. D 93, 034021 (2016).

$$N^{(\mathrm{RR})}_{\mathrm{R,\,L}} = \frac{1}{\sqrt{2}}\frac{1\pm\gamma_5}{2}B, \quad N^{(\mathrm{LL})}_{\mathrm{R,\,L}} = -\frac{1}{\sqrt{2}}\frac{1\pm\gamma_5}{2}B$$

$$\mathscr{L}_{\mathrm{B}} = \mathrm{Tr}(\bar{B}i \not{\partial} B) + c\,\mathrm{Tr}\left(\bar{B}\gamma_\mu V^\mu B + \bar{B}\gamma_\mu\gamma_5 A^\mu B\right) + c'\,\mathrm{Tr}\left(\bar{B}\gamma_\mu \bar{B} V^\mu + \bar{B}\gamma_\mu\gamma_5 \bar{B} A^\mu\right) +$$

$$h\epsilon_{abc}\epsilon^{def}\left[(\bar{B})_{ad}\gamma_\mu (B)_{be}(V^\mu)_{cf} + (\bar{B})_{ad}\gamma_\mu\gamma_5 (B)_{be}(A^\mu)_{cf}\right] -$$

$$\frac{g}{2}\,\mathrm{Tr}\left[\bar{B}\left(\Phi' + \Phi'^{\dagger}\right)B + \bar{B}\gamma_5\left(\Phi' - \Phi'^{\dagger}\right)B\right] -$$

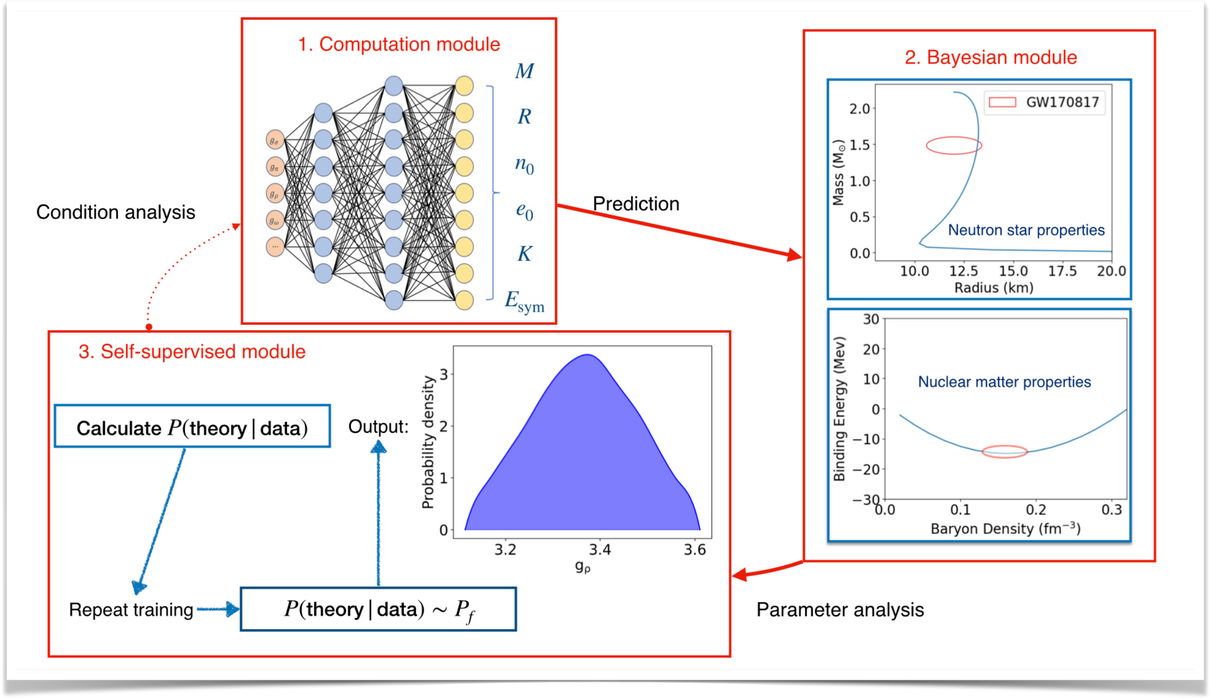
$$\frac{e}{2}\epsilon_{abc}\epsilon^{def}\left[(\bar{B})_{ad}\left(\Phi' + \Phi'^{\dagger}\right)_{be}(B)_{cf} + (\bar{B})_{ad}\gamma_5\left(\Phi' - \Phi'^{\dagger}\right)_{be}(B)_{cf}\right]$$

The baryon masses by spontaneous chiral symmetry breaking

# Massive data processing (ML methods)

L.-J. Guo, J.-Y. Xiong, **YM**, and Y.-L. Ma, Astrophys. J. 965, 47 (2024)

- The parameter space searching of an EFT with data across broad scale is a rather labor- and time-consuming work
- AI techs based on machine learning have advantage to analysis massive data and statistical correlations



# An application

$$\mathscr{L}_{\mathrm{RMF}} = \bar{\psi}\left[i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu}\right]\psi$$
$$+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4$$
$$-\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2$$
$$-\frac{1}{4}R^a_{\mu\nu}R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho^a_\mu\rho^{a\mu}$$

Y. Sugahara and H. Toki, Nucl. Phys. A 579, 557 (1994)
H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi, Nucl. Phys. A 637, 435 (1998)

Bayesian analysis

TABLE I. $M$ and $R$ is the mass and radius of the primary star in GW170817. $e_0$ is the binding energy of nucleon, $E_{\mathrm{sym}}(n) = \frac{1}{2}\frac{\partial^2 E(n,\alpha)}{\partial\alpha^2}\Big|_{\alpha=0}$ is the symmetry energy and $K_0 = 9n_0^2\frac{\partial^2 E(n,0)}{\partial n^2}\Big|_{n=n_0}$ is the incompressibility coefficient at $n_0$, respectively. Normal distribution is assumed for data error bars with 95% confidence level.

| | $M$ ($M_\odot$) | $R$ (km) | $n_0$ (fm$^{-3}$) | $e_0$ (MeV) | $E_{\mathrm{sym}}$ (MeV) | $K_0$ (MeV) |
|---|---|---|---|---|---|---|
| Constraints | $1.48 \pm 0.12$ | $11.9 \pm 1.4$ | $0.155 \pm 0.05$ | $-16 \pm 1.0$ | $32.0 \pm 2.0$ | $250 \pm 50$ |
| Optimal | — | — | 0.161 | −15.7 | 28.6 | 266 |

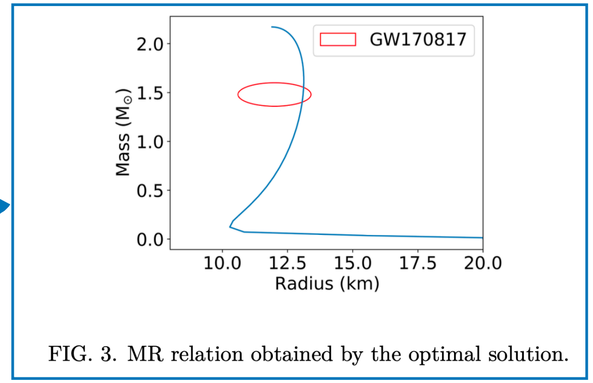



FIG. 3. MR relation obtained by the optimal solution.

TABLE II. Optimal values of the parameters calculated from NN. The emprical values $M_N = 938$ MeV, $m_\omega = 782$ MeV and $m_\rho = 765$ are physical values at vacuum taken as inputs from Ref. [68] for simplicity.

| $g_\sigma$ | $g_\omega$ | $g_\rho$ | $g_3$ | $c_3$ | $g_2$ | $m_\sigma$ |
|---|---|---|---|---|---|---|
| 9.82 | 11.8 | 3.42 | 1.26 | 72.6 | −1550 MeV | 531 MeV |

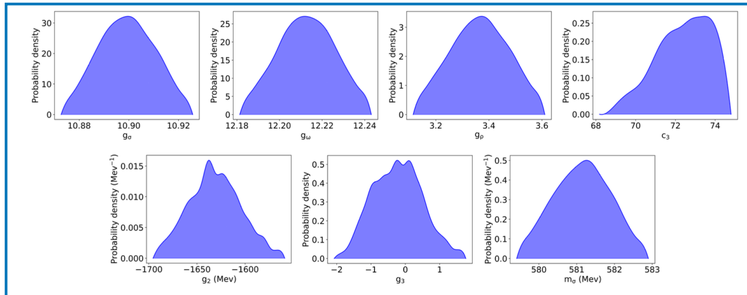
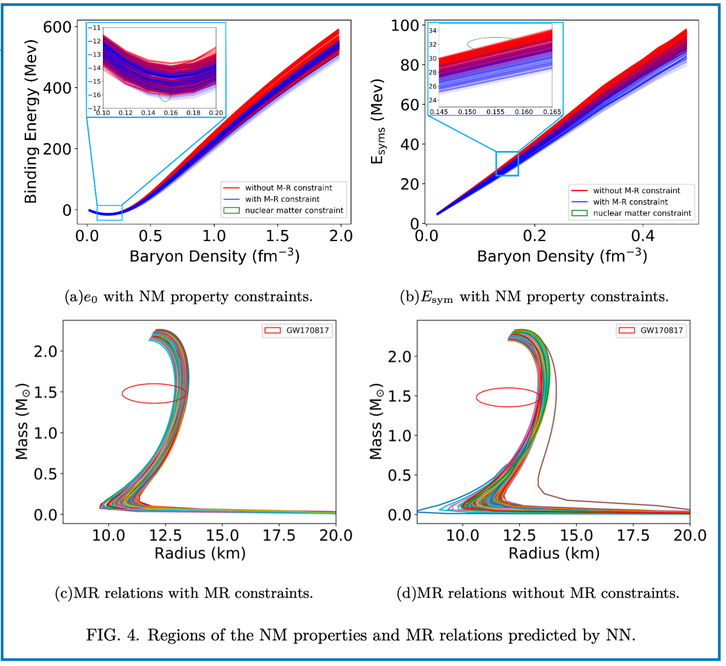



FIG. 5. Distributions of the parameters estimated by NN.




(a)$e_0$ with NM property constraints. (b)$E_{\mathrm{sym}}$ with NM property constraints.

(c)MR relations with MR constraints. (d)MR relations without MR constraints.

FIG. 4. Regions of the NM properties and MR relations predicted by NN.

# An analysis on a more general Lagrangian

In collaboration with Ling-Jun Guo (郭凌君) and Jia-YIng Xiong(熊佳颖)

**Include all possible operators contributing to RMF EOS up to dimension 4**

$$\mathscr{L}_N = \bar{\Psi}\left(i\gamma^\mu\partial_\mu - m_N\right)\Psi,$$

$$\mathscr{L}_\sigma = \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - \frac{1}{2}g_4\sigma^2\omega^\mu\omega_\mu - \frac{1}{2}g_5\sigma^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu - g_6\sigma\vec{\rho}^\mu\cdot\vec{\rho}_\mu - g_7\sigma\omega^\mu\omega_\mu,$$

$$\mathscr{L}_\omega = -\frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu + \frac{1}{4}c_3\left(\omega^\mu\omega_\mu\right)^2 + \frac{1}{2}c_4\omega^\mu\omega_\mu\vec{\rho}^\mu\cdot\vec{\rho}_\mu$$

$$\mathscr{L}_\rho = -\frac{1}{4}\vec{P}^{\mu\nu}\cdot\vec{P}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu + \frac{1}{4}d_3\left(\vec{\rho}^\mu\cdot\vec{\rho}_\mu\right)^2$$

$$\mathscr{L}_{a_0} = \frac{1}{2}\partial_\mu\vec{a}_0\cdot\partial^\mu\vec{a}_0 - m_{a_0}^2\vec{a}_0\cdot\vec{a}_0 + \frac{1}{4}b_3\left(\vec{a}_0\cdot\vec{a}_0\right)^2 + \frac{1}{2}b_4\left(\vec{a}_0\cdot\vec{a}_0\right)\left(\vec{\rho}^\mu\cdot\vec{\rho}_\mu\right) + \frac{1}{2}b_5\sigma\left(\vec{a}_0\cdot\vec{a}_0\right)$$
$$+\frac{1}{2}b_6\vec{a}_0\cdot\vec{a}_0\sigma^2 + \frac{1}{2}b_7\vec{a}_0\cdot\vec{a}_0\omega^\mu\omega_\mu + b_8\vec{a}_0\cdot\vec{\rho}^\mu\omega_\mu\sigma + b_9\vec{a}_0\cdot\vec{\rho}_\mu\omega^\mu,$$

$$\mathscr{L}_I = \bar{\Psi}\left(-g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu - g_\rho\gamma^\mu\vec{\rho}_\mu - g_{a_0}\vec{a}_0\right)\Psi$$

$$\mathscr{L} = \mathscr{L}_N + \mathscr{L}_\sigma + \mathscr{L}_\omega + \mathscr{L}_\rho + \mathscr{L}_{a_0} + \mathscr{L}_I$$