Spin density matrix for ρ^0 mesons in a pion gas

Yi-Liang Yin

University of Science and Technology of China

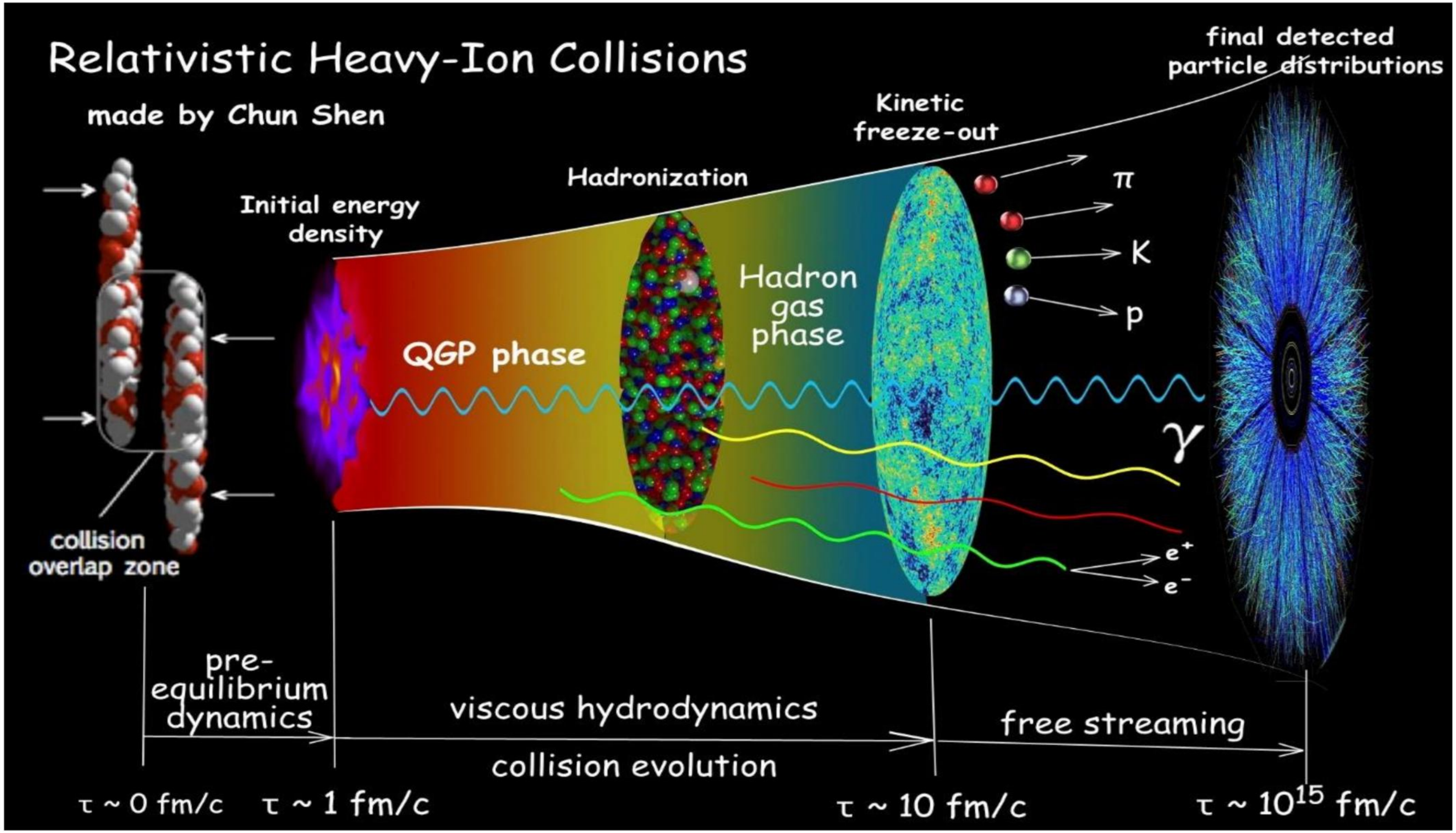
PhysRevC.110.024905 (2024)

Sci.China Phys.Mech.Astron. 69 (2026) 5, 251011

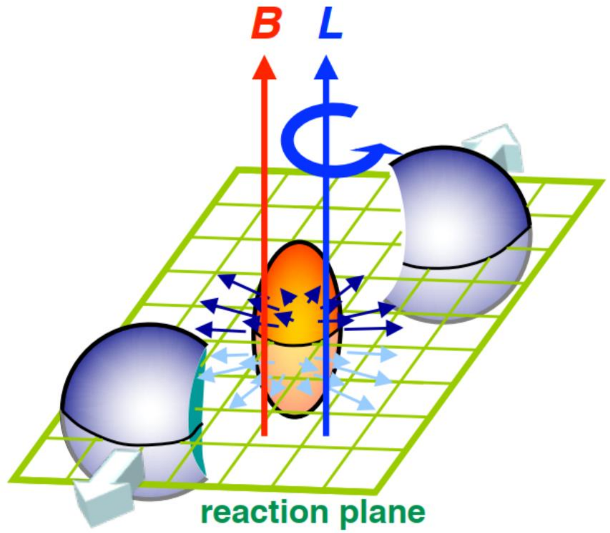
Outline

- **Background**
- Evolution of the spin alignment for ρ meson in a pion gas (Local effect)
- Shear induced spin alignment for ρ meson (Nonlocal effect)

Relativistic heavy ion collision



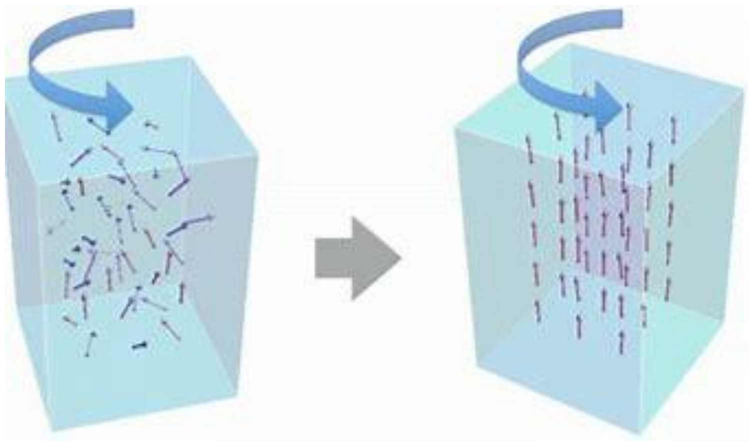
Spin polarization



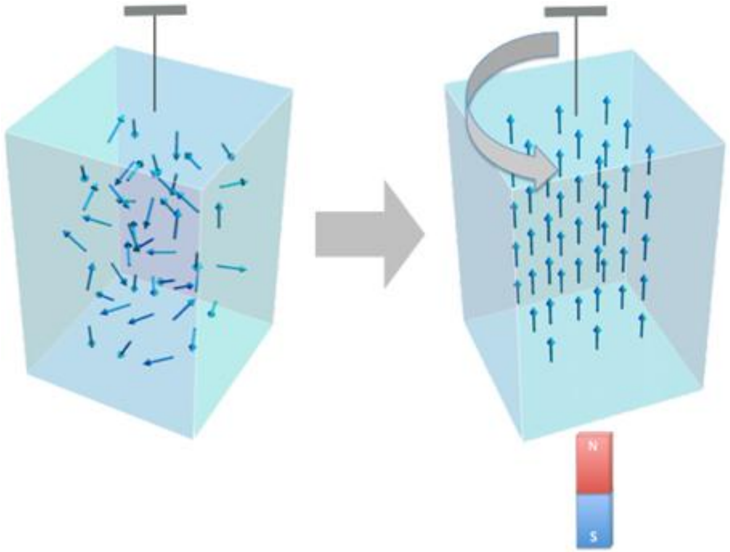
L~$10^5\hbar$
B~10^{18}G

Works on spin polarization

· *Z.-T. Liang et al. Phys. Rev. Lett. 94, 102301 (2005)*
· *Z.-T. Liang et al. Phys. Lett. B 629, 20 (2005)*
· *J.-H. Gao et al. Phys. Rev. C 77, 044902 (2008)*
· *F. Becattini et al. Phys. Rev. C 77, 024906 (2008)*
· *I. Karpenko et al. Eur. Phys. J. C 77, 213 (2017)*
· *H. Li et al. Phys. Rev. C 96, 054908 (2017)*
· *Y. Xie et al. Phys. Rev. C 95, 031901(R) (2017)*
· *S. Shi et al. Phys. Lett. B 788, 409 (2019)*
....

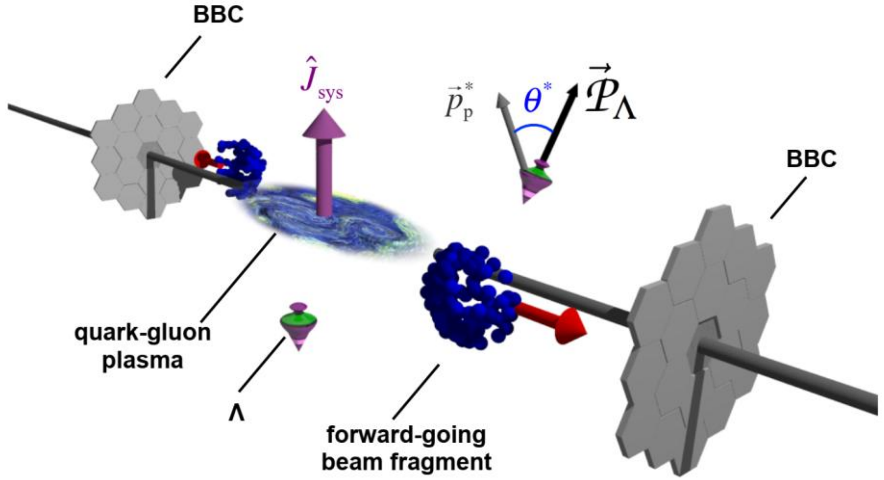


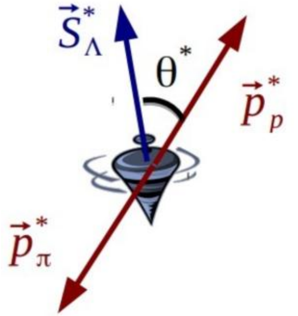
Bernett effect
S. J. Barnett,
Rev. Mod. Phys. 7, 129 (1935)



Einstein-de Haas effect
A. Einstein and W. de Haas,
Deuts. Physik. Gesells.
Verhandlun. 17, 152 (1915)

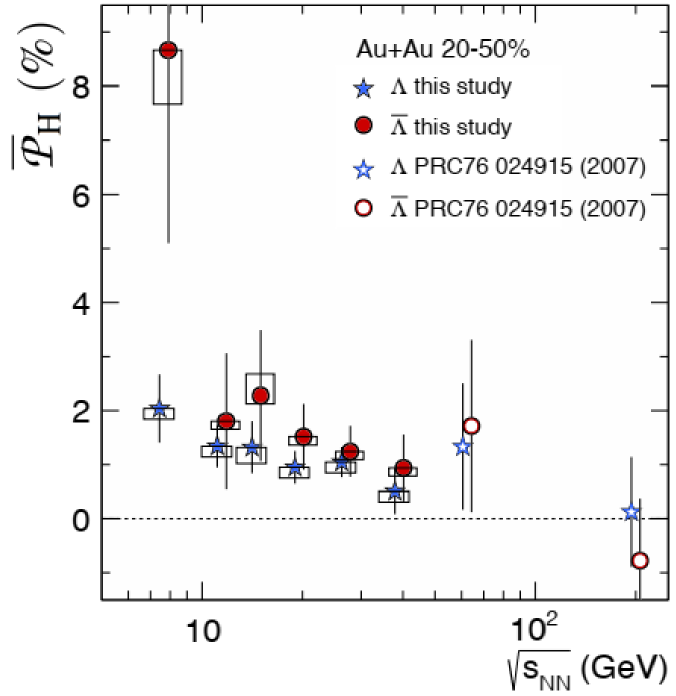
Spin polarization





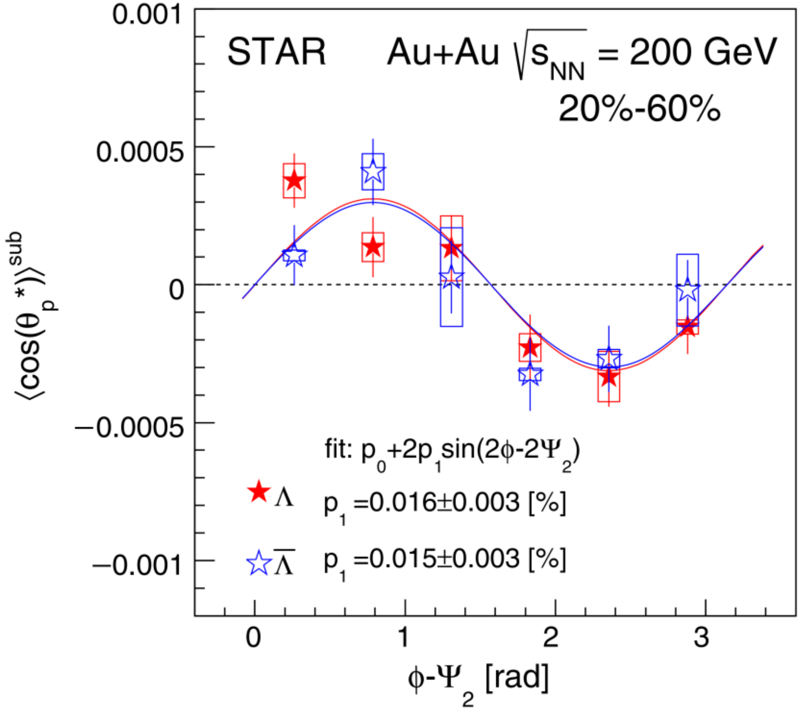
$$\Lambda \to p + \pi^-$$

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P}_\Lambda \cdot \mathbf{p}_\mathbf{p}^*)$$



Global spin polarization of $\Lambda/\bar{\Lambda}$

STAR, Nature 548 (2017) 62-65



Local spin polarization of $\Lambda/\bar{\Lambda}$

STAR, Phys. Rev. Lett. 123 (2019) 13, 132301

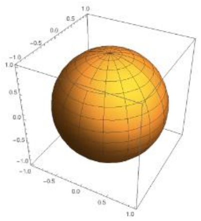
Spin alignment of vector mesons

Spin density matrix:

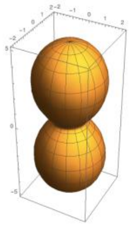
$$\rho_V = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1,-1} \\ \rho_{01} & \rho_{00} & \rho_{0,-1} \\ \rho_{-1,1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix}$$

$\rho_{11} - \rho_{-1,-1}$: Spin polarization → Not measurable

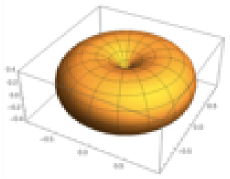
ρ_{00}: Spin alignment



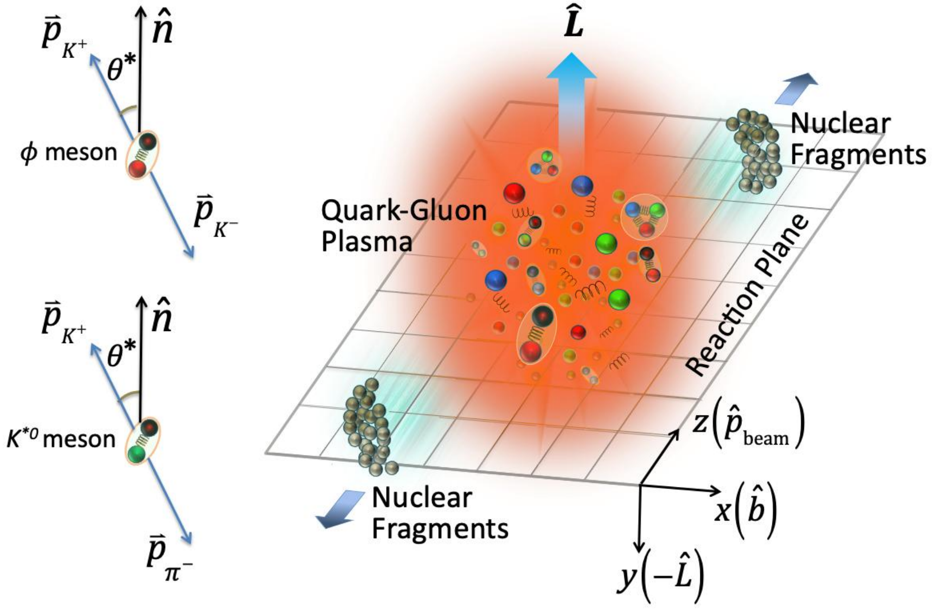
$\rho_{00} = \frac{1}{3}$



$\rho_{00} > \frac{1}{3}$

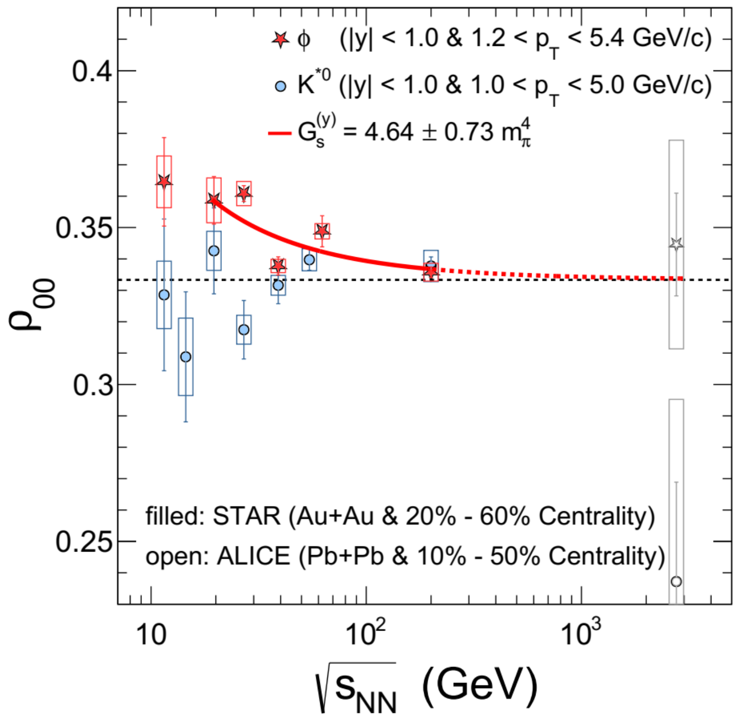


$\rho_{00} < \frac{1}{3}$

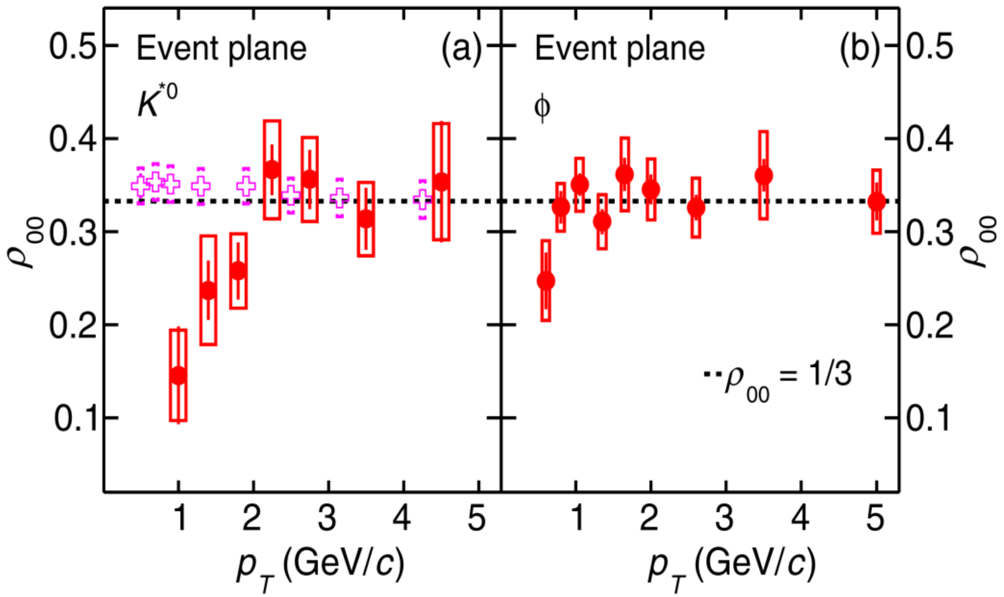


$$\frac{dN}{d(\cos\theta^*)} \propto (1-\rho_{00}) + (3\rho_{00}-1)\cos^2\theta^*$$

Spin alignment of ϕ and K^{*0} mesons



STAR, Nature 614, 244–248 (2023)



$\sqrt{S_{NN}} = 2.76\text{TeV}$

ALICE, Phys. Rev. Lett. 125, 012301 (2020)

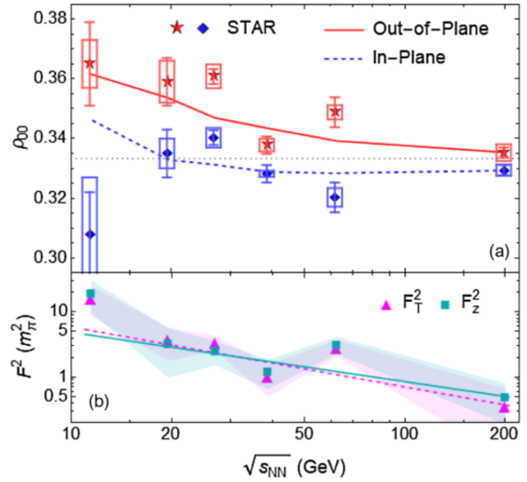
The spin alignment of ϕ meson is of $\rho_{00} - \frac{1}{3} \sim 10^{-2}$.

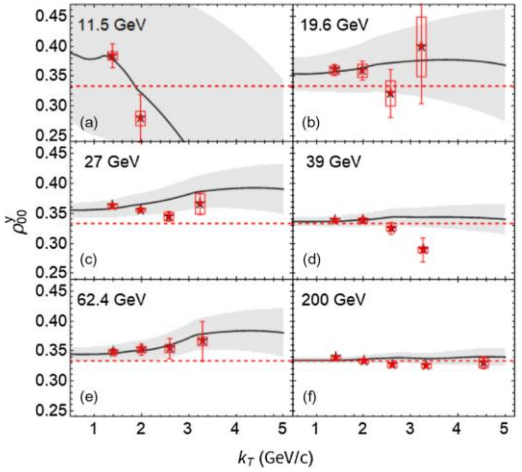
Spin alignment of ϕ mesons

Physics Mechanisms	(ρ_{00})
c_Λ: Quark coalescence vorticity & magnetic field[1]	< 1/3 (Negative ~ 10^{-5})
c_ε: E-comp. of Vorticity tensor[1]	< 1/3 (Negative ~ 10^{-4})
c_E: Electric field[2]	> 1/3 (Positive ~ 10^{-5})
c_F: Fragmentation[3]	> or, < 1/3 (~ 10^{-5})
c_L: Local spin alignments[4]	< 1/3
c_A: Turbulent color field[5]	< 1/3
c_φ: Vector meson strong force field[6]	> 1/3 (Can accommodate large positive signal)
c_g: Glasma fields + effective potential	could be significant

$$\rho_{00}^{\phi} \approx \frac{1}{3} + c_\Lambda + c_\varepsilon + c_E + c_\phi + \cdots$$

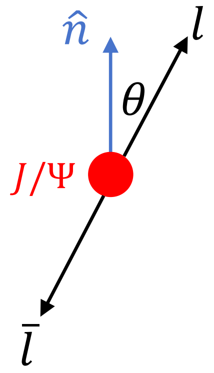
[1]. Liang et., al., Phys. Lett. B 629, (2005);
Yang et., al., Phys. Rev. C 97, 034917 (2018);
Xia et., al., Phys. Lett. B 817, 136325 (2021);
Beccattini et., al., Phys. Rev. C 88, 034905 (2013)
[2]. Sheng et., al., Phys. Rev. D 101, 096005 (2020);
Yang et., al., Phys. Rev. C 97, 034917 (2018)
[3]. Liang et., al., Phys. Lett. B 629, (2005)
[4]. Xia et., al., Phys. Lett. B 817, 136325 (2021);
Gao, Phys. Rev. D 104, 076016 (2021)
[5]. Muller et., al., Phys. Rev. D 105, L011901 (2022)
[6]. Sheng et., al., Phys. Rev. D 101, 096005 (2020);
Phys. Rev. D 102, 056013 (2020); Phys Rev. Lett. 131 042304 (2023);
…………

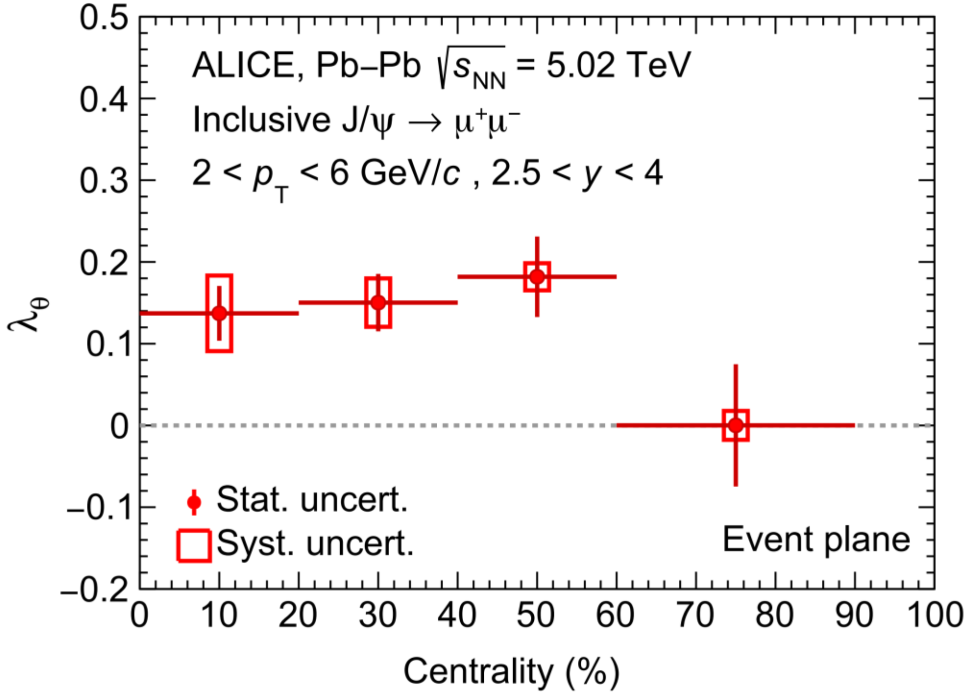


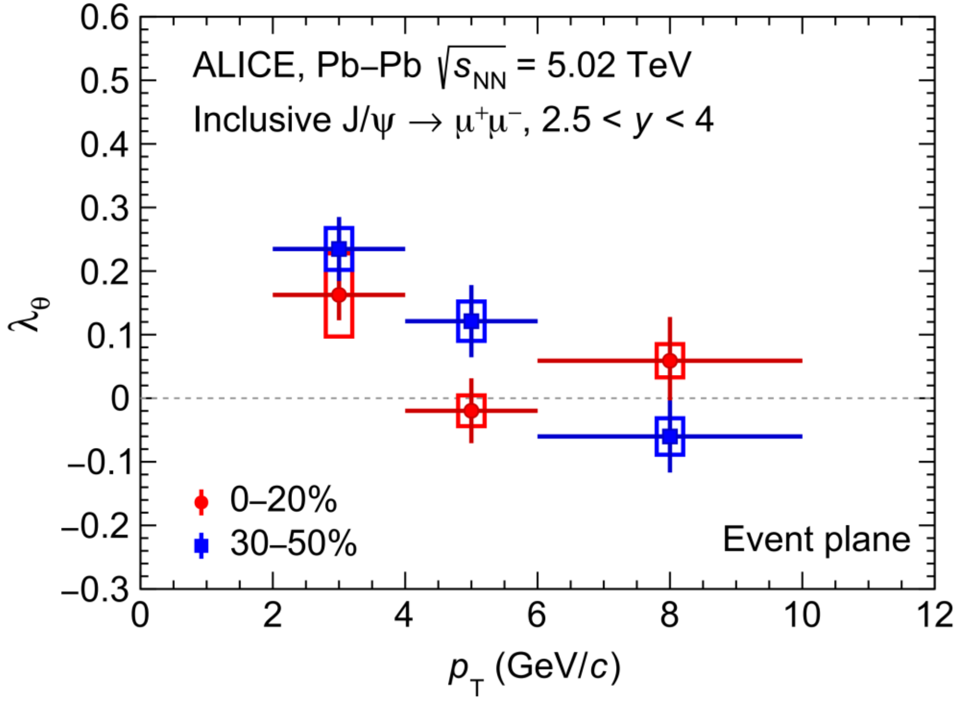


Phys Rev. Lett. 131 042304 (2023)

Spin alignment of J/Ψ mesons





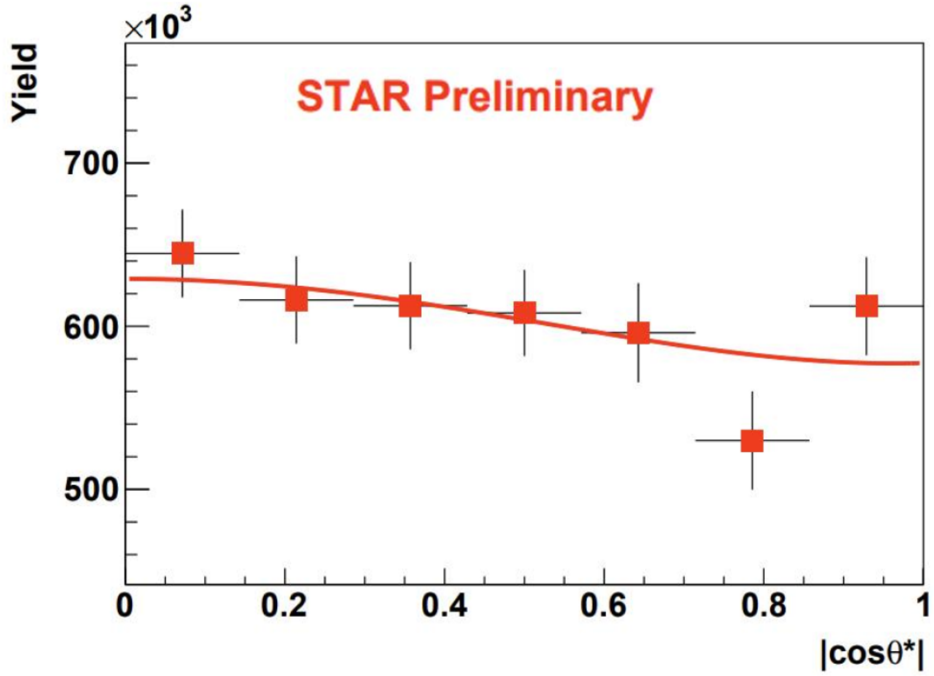


$$W(\theta) \propto \frac{1}{3+\lambda_\theta}(1+\lambda_\theta \cos^2\theta)$$

$$\lambda_\theta \propto (1-3\rho_{00})/(1+\rho_{00})$$

ALICE, Phys.Rev.Lett. 131 (2023) 4, 042303

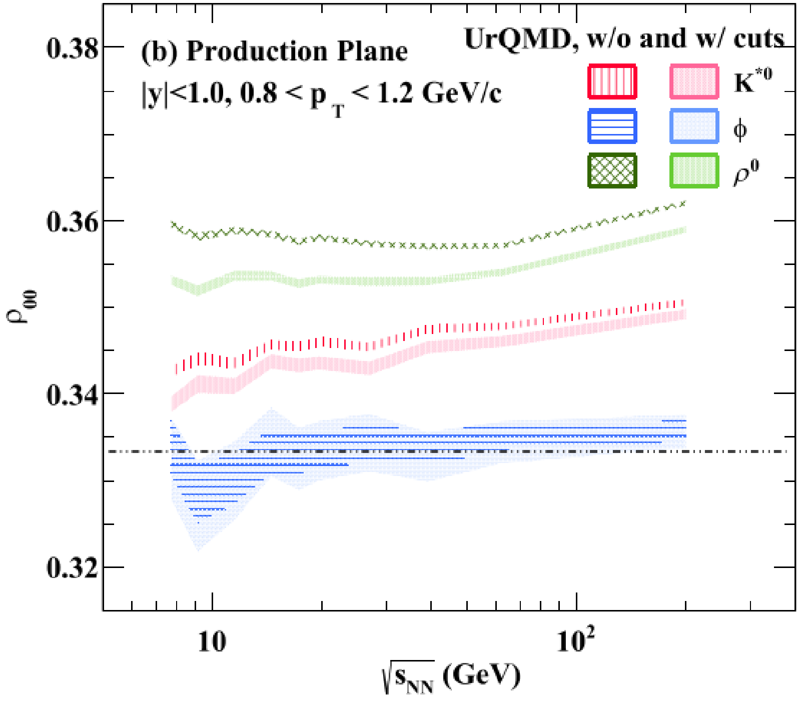
Spin alignment of ρ mesons



AuAu for run 2011 at 200 GeV,
Centrality: 60-80%, pT: 1.8-2.4 GeV/c
Taken from Baoshan Xi - Quark Matter 2023

$$\rho_{00} < \frac{1}{3} \ ?$$

$$\rho_{00} > \frac{1}{3} \ ?$$

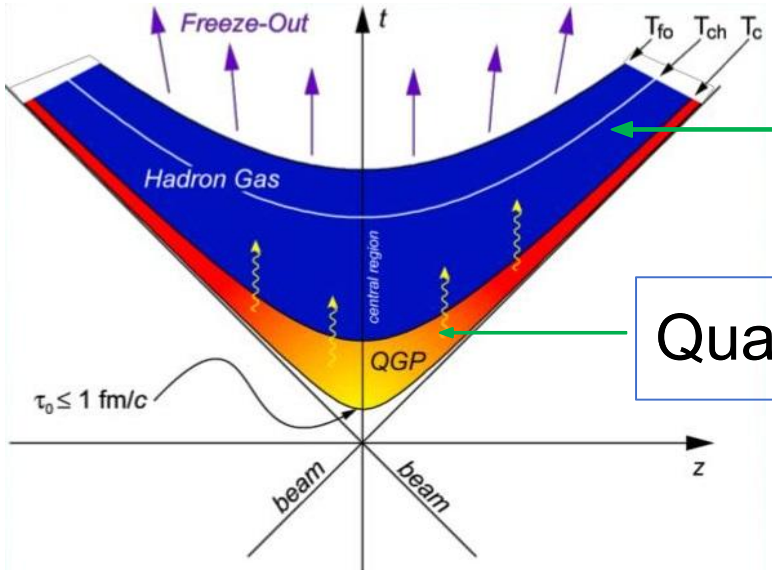


UrQMD Model
Z. Liu, Z. Li, W. Zha, Z. Tang, PLB 873, 140161 (2026)

Outline

- Background
- **Evolution of the spin alignment for ρ meson in a pion gas (Local effect)**
- Shear induced spin alignment for ρ meson (Nonlocal effect)

Motivation



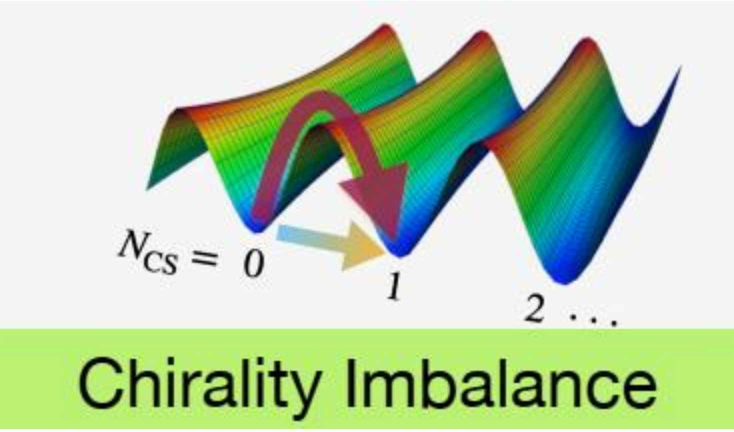
Hadron interaction

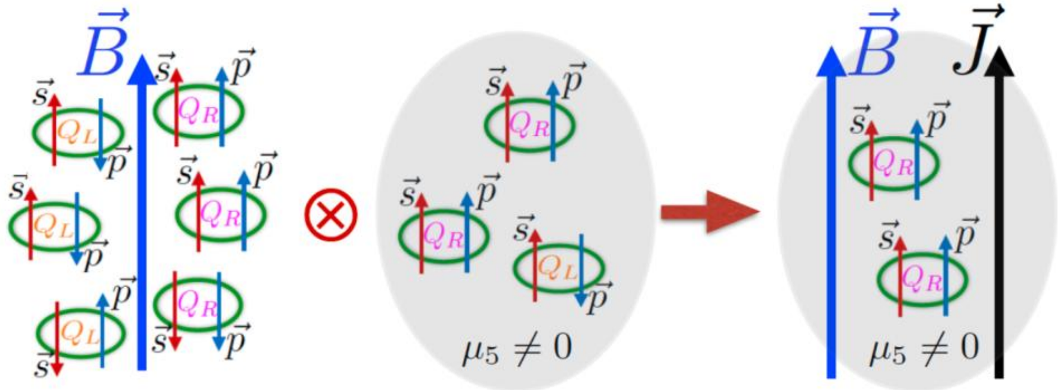
Quark-gluon interaction, $\rho_{00} - \frac{1}{3} \approx c_\Lambda + c_\epsilon + c_E + c_\phi + \cdots$

The width of ρ^0 meson is much larger than ϕ meson, so the coupling between ρ^0 mesons and hadron gas must be considered.

	ρ^0	ϕ
Mass	$m \approx 770$MeV	$m \approx 1020$MeV
Width	$\Gamma \approx$147.4MeV	$\Gamma \approx 4.249$MeV
Main decay channel	$\rho^0 \rightarrow \pi^+\pi^-$	$\phi \rightarrow K^+K^-$ $\phi \rightarrow K_L^0 K_S^0$ …
Quark constitution	$u\bar{u}, d\bar{d}$	$s\bar{s}$

Chiral magnetic effect (CME)





Chirality imbalance
$(\mu_5 \neq 0)$

External magnetic field → Charge separation

$$\overrightarrow{J_e} = \frac{e^2}{2\pi^2}\mu_5 \overrightarrow{B}$$

Odd parity ($\overrightarrow{J_e}$) Even parity ($\overrightarrow{B}$)

D. Kharzeev, Phys. Lett. B 633, 260 (2006)
D. E. Kharzeev et al. Nucl. Phys. A 803, 227 (2008)
K. Fukushima et al. Phys. Rev. D 78, 074033 (2008)

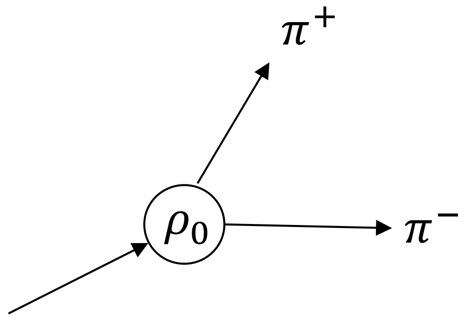
γ correlator and tensor polarization

The γ correlator for the chiral magnetic effect (CME) is defined as

$$\gamma_{112} \equiv \langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle$$

Azimuthal angle

Reaction plane



The difference between γ correlators for the opposite-sign (OS) and same-sign (SS) pairs can be regarded as a CME signal

$$\Delta\gamma_{112} \equiv \gamma_{112}^{\mathrm{OS}} - \gamma_{112}^{\mathrm{SS}}$$

The decay products of ρ meson contribute to the γ correlator.

The contribution to $\Delta\gamma_{112}$ from pions in the decay $\boldsymbol{\rho_0 \to \pi^+\pi^-}$:

$$\Delta\gamma_{112}^{\rho} = \frac{N_\rho}{N_+N_-}\Delta\overline{\gamma}_{112}^{\rho}$$

$$\begin{aligned}\Delta\overline{\gamma}_{112}^{\rho} \equiv & \, \mathrm{Cov}(\cos\Delta\phi_+, \cos\Delta\phi_-) \\ & - \mathrm{Cov}(\sin\Delta\phi_+, \sin\Delta\phi_-),\end{aligned}$$

Diyu Shen et al.
Phys. Lett. B 839 137777 (2023)

γ correlator and tensor polarization

$$\Delta\overline{\gamma}^{\rho}_{112} = \frac{1}{n_\rho}\int \frac{d^3\mathbf{p}_\rho}{(2\pi\hbar)^3} f(\mathbf{p}_\rho)$$
$$\times\Big[\int d\Omega^* \frac{dN}{d\Omega^*}(\cos\phi_+\cos\phi_- - \sin\phi_+\sin\phi_-)$$
$$-\int d\Omega^* \frac{dN}{d\Omega^*}\cos\phi_+ \int d\Omega^* \frac{dN}{d\Omega^*}\cos\phi_-$$
$$+\int d\Omega^* \frac{dN}{d\Omega^*}\sin\phi_+ \int d\Omega^* \frac{dN}{d\Omega^*}\sin\phi_-\Big],$$

$$\rho_{\lambda_1\lambda_2} = \left(\tfrac{1}{3} + \tfrac{1}{2}P_i\Sigma_i + T_{ij}\Sigma_{ij}\right)_{\lambda_1\lambda_2},$$

$$\frac{dN}{d\Omega^*} = \frac{3}{8\pi}[(1-\rho_{00}) + (3\rho_{00}-1)\sin^2\theta^*\sin^2\phi^*$$
$$-(T_{11}-T_{22})(\cos^2\theta^* - \sin^2\theta^*\cos^2\phi^*)$$
$$-2T_{12}\sin(2\theta^*)\cos\phi^* - 2T_{31}\sin(2\theta^*)\sin\phi^*$$
$$-2T_{23}\sin^2\theta^*\sin(2\phi^*)].$$

$$\Delta\overline{\gamma}^{\rho}_{112} = \sum_{i=1}^{5} A_i t_i + \sum_{i,j=1}^{5} B_{ij} t_i t_j$$

Linear | Quadratic

$\delta\rho_{00}$ and $Re\rho_{1,-1}$ $(T_{11}-T_{22})$ have dominant effects!

Linear and quadratic coefficients

t_i	$\rho_{00}-1/3$	$T_{11}-T_{22}$	T_{12}	T_{31}	T_{23}
A_i	0.5215	−0.1738	0	0	0

B_{ij}	$\rho_{00}-1/3$	$T_{11}-T_{22}$	T_{12}	T_{31}	T_{23}
$\rho_{00}-1/3$	0.03885	0.01295	0	0	0
$T_{11}-T_{22}$	0.01295	−0.01295	0	0	0
T_{12}	0	0	-6.089×10^{-4}	0	0
T_{31}	0	0	0	-6.089×10^{-4}	0
T_{23}	0	0	0	0	0

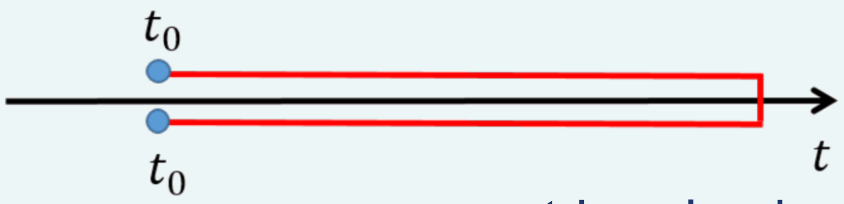
Spin Boltzmann equation

1. Two-point Green's function in CTP formalism

Xin-Li Sheng et al. Phys. Rev. D 109, 036004 (2024)

Two-point Green's functions in closed-time-path (CTP) formalism for vector mesons:

$$G^{\mu\nu}_{CTP}(x_1, x_2) = \langle T_C A^\mu(x_1) A^\nu(x_2) \rangle$$



$$\begin{aligned} G^{<}_{\mu\nu}(x,p) = 2\pi\hbar \sum_{\lambda_1,\lambda_2} \delta\left(p^2 - m_\rho^2\right) \{ & \theta(p^0)\epsilon_\mu(\lambda_1,\mathbf{p})\,\epsilon^*_\nu(\lambda_2,\mathbf{p})\,\boxed{f_{\lambda_1\lambda_2}(x,\mathbf{p})} \\ & + \theta(-p^0)\epsilon^*_\mu(\lambda_1,-\mathbf{p})\,\epsilon_\nu(\lambda_2,-\mathbf{p})\left[\delta_{\lambda_2\lambda_1} + f_{\lambda_2\lambda_1}(x,-\mathbf{p})\right] \}, \end{aligned}$$

→ matrix valued spin dependent distribution (**MVSD**)

$$\begin{aligned} G^{>}_{\mu\nu}(x,p) = 2\pi\hbar \sum_{\lambda_1,\lambda_2} \delta\left(p^2 - m_\rho^2\right) \{ & \theta(p^0)\epsilon_\mu(\lambda_1,\mathbf{p})\,\epsilon^*_\nu(\lambda_2,\mathbf{p})\left[\delta_{\lambda_1\lambda_2} + f_{\lambda_1\lambda_2}(x,\mathbf{p})\right] \\ & + \theta(-p^0)\epsilon^*_\mu(\lambda_1,-\mathbf{p})\,\epsilon_\nu(\lambda_2,-\mathbf{p})\, f_{\lambda_2\lambda_1}(x,-\mathbf{p}) \}, \end{aligned}$$

$$\rho_{00} \equiv \frac{f_{00}}{\mathrm{tr} f}$$

Spin Boltzmann equation

2. Kadanoff-Baym (KB) equation

$$p\cdot\partial_x G^{<,\mu\nu}(x,p)-\frac{1}{4}\left[p^\mu\partial^x_\eta G^{<,\eta\nu}(x,p)+p^\nu\partial^x_\eta G^{<,\mu\eta}(x,p)\right]$$
$$=\frac{1}{4}[\Sigma^{<,\mu}{}_\alpha(x,p)G^{>,\alpha\nu}(x,p)-\Sigma^{>,\mu}{}_\alpha(x,p)G^{<,\alpha\nu}(x,p)]$$
$$+\frac{1}{4}[G^{>,\mu}{}_\alpha(x,p)\boxed{\Sigma^{<,\alpha\nu}(x,p)}-G^{<,\mu}{}_\alpha(x,p)\Sigma^{>,\alpha\nu}(x,p)]$$

Self energy

We have neglected the Poisson bracket terms (Nonlocal effect).

- The self energy should be calculated with a explicit Lagrangian;
- Since ρ meson is a real vector meson, we can only consider the positive-energy part of the equation.

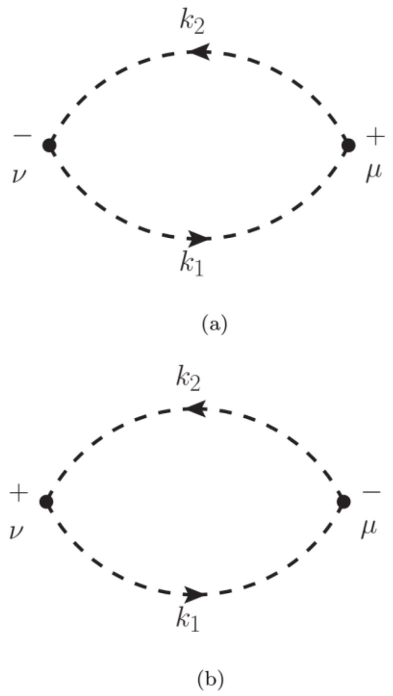
Spin Boltzmann equation

3. Collision terms

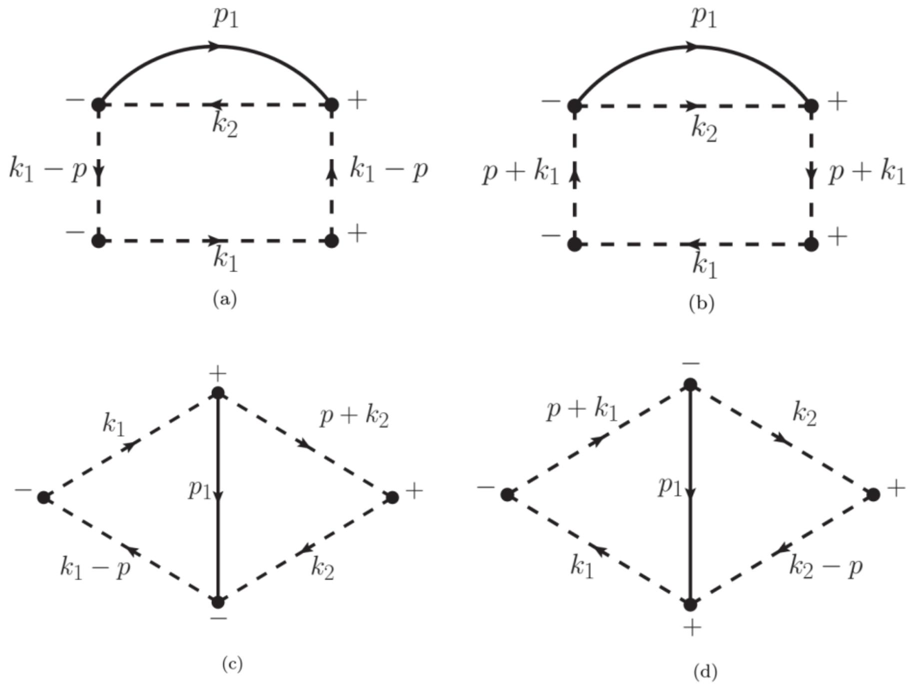
We decompose the collision terms into coalescence-dissociation part and scattering part, and assumed that the system is homogeneous in space.

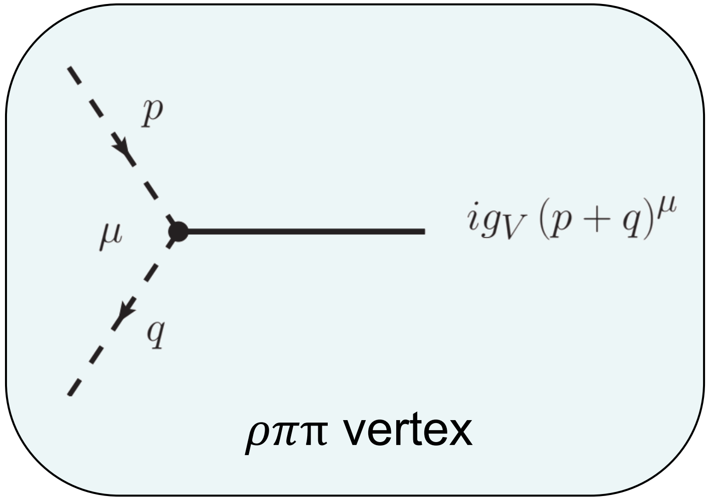
$$\partial_t f_{\lambda_1\lambda_2}(x, \mathbf{p}) \;=\; C_{\text{coal/diss}} + C_{\text{scat}}$$

Leading order



Next-to-leading order





T. Fujiwara et al., Prog. Theor. Phys. ***74****, 128 (1985)*

Spin Boltzmann equation

4. Regulation of pion propagators

$$S^F(k) = \frac{i\hbar}{k^2 - m_\pi^2},$$

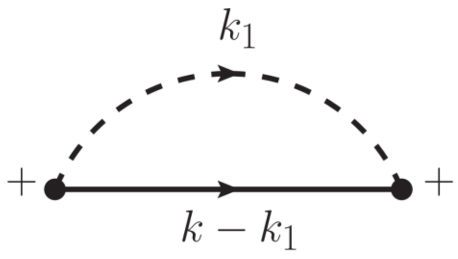
$$S^{\overline{F}}(k) = \frac{-i\hbar}{k^2 - m_\pi^2}.$$

Divergent

Introduce self-energy corrections with medium effects:

$$S^F(k) = \frac{i\hbar}{k^2 - m_\pi^2 - \hbar\Sigma^F(k)}$$

$$S^{\overline{F}}(k) = \frac{-i\hbar}{k^2 - m_\pi^2 + \hbar\Sigma^{\overline{F}}(k)},$$



$$\begin{aligned}\Gamma(k) \equiv \mathrm{Im}\Sigma^F(k) &= 2g_V^2\theta(k^0)\int\frac{d^3k_1}{(2\pi\hbar)^3 2E^\pi_{k_1}}\int\frac{d^3p}{(2\pi\hbar)^3 2E^\rho_p}\\ &\times(2\pi\hbar)^4\delta^{(4)}(k+k_1-p)\,f_{\pi^-}(\mathbf{k}_1)\left[m_\pi^2 - \frac{(k_1\cdot p)^2}{m_\rho^2}\right]\\ &+2g_V^2\theta(-k^0)\int\frac{d^3k_1}{(2\pi\hbar)^3 2E^\pi_{k_1}}\int\frac{d^3p}{(2\pi\hbar)^3 2E^\rho_p}\\ &\times(2\pi\hbar)^4\delta^{(4)}(k-k_1+p)\,f_{\pi^+}(\mathbf{k}_1)\left[m_\pi^2 - \frac{(k_1\cdot p)^2}{m_\rho^2}\right]\end{aligned}$$

we only consider the imaginary part of the self-energy since the mass correction from the real part is much smaller.

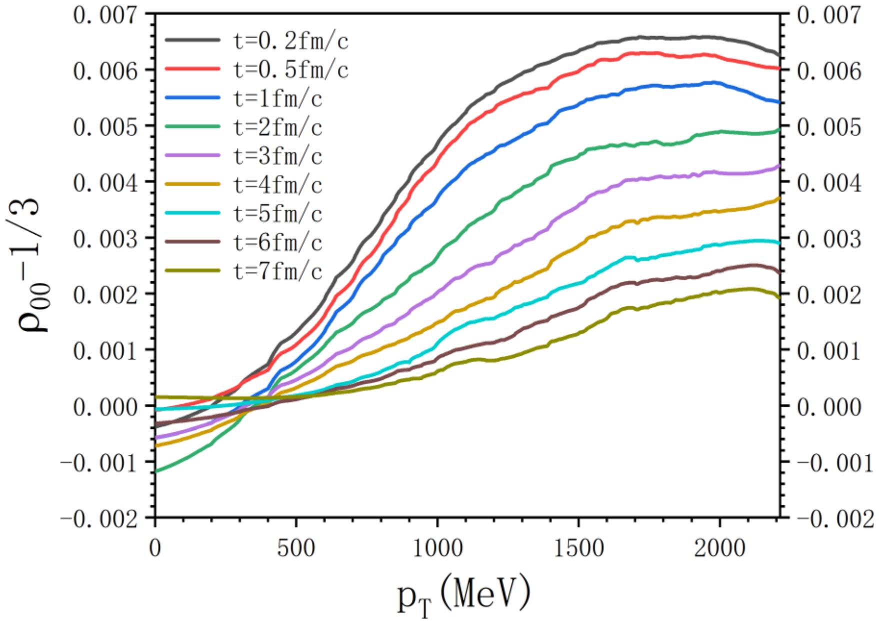
Numerical results

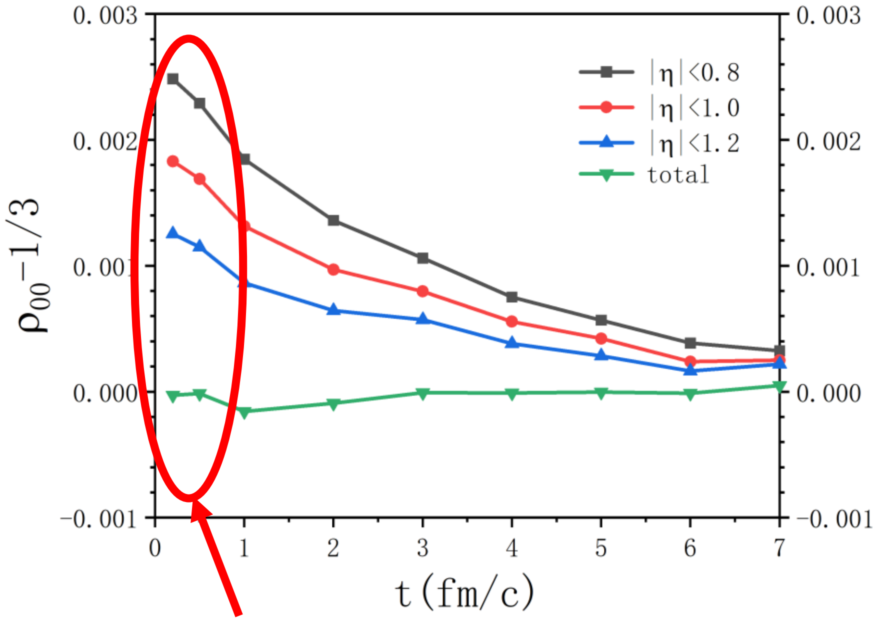
We assume that $\pi^{\pm}$ are in global thermal equilibrium, so they obey the Bose-Einstein distribution

$$f_{\pi^{\pm}}(x,\mathbf{p}) = f_{\pi^{\pm}}(\mathbf{p}) = \frac{1}{\exp\left[\beta\left(E_p \mp \mu_\pi\right)\right]-1},$$

and we choose $\mu_\pi = 0$ and $T = 156.5$MeV.

The ρ^0 spin alignment with initial condition $f_{\lambda_1\lambda_2}(t=0)=0$.



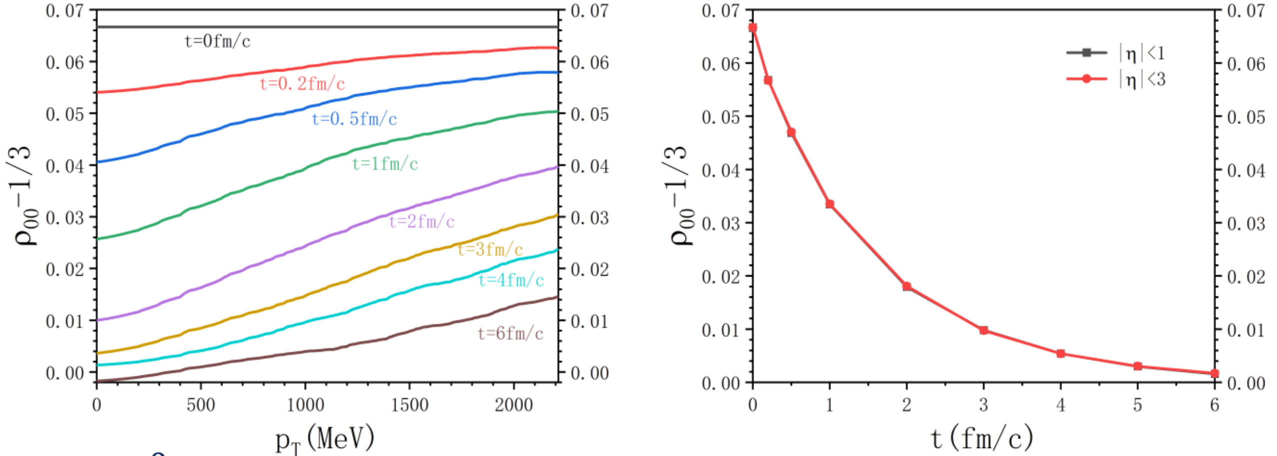


Non-zero spin alignment comes from the choice of rapidity range.

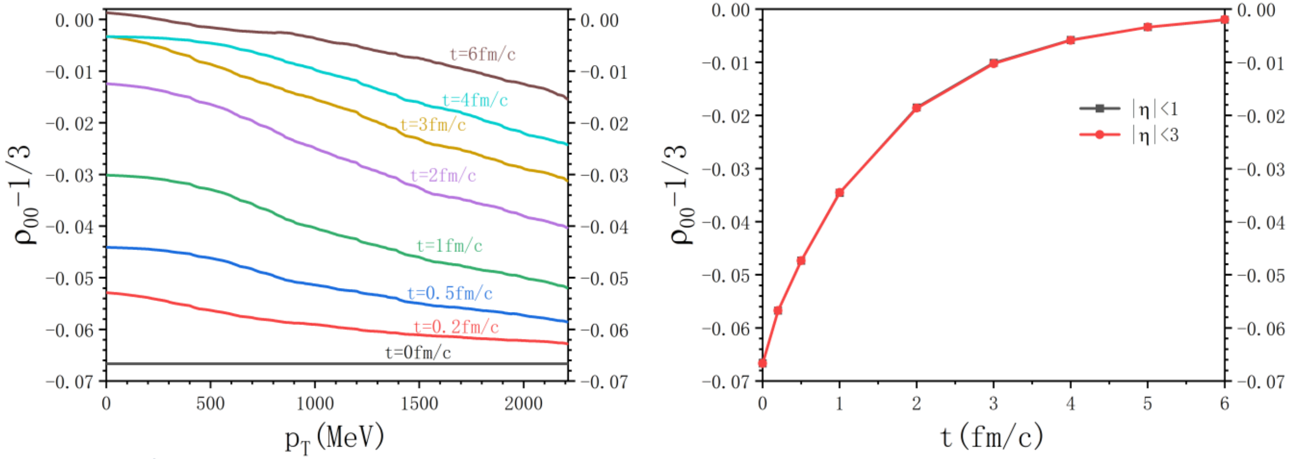
Monte Carlo method:
Momentum range: -2.5 ~ 2.5 GeV
Lattice: 100×100×100 MeV^3
Time step: 10^{-3} fm/c

Does the choice of ρ meson's **initial state** affect the final spin alignment?

Numerical results



The ρ^0 spin alignment with initial condition $\rho_{00} = 0.4 > 1/3$.



The ρ^0 spin alignment with initial condition $\rho_{00} = 0.27 < 1/3$.

Initial state:

$$f_{\lambda_1\lambda_2} = \text{diag}(0.9, 1.2, 0.9) \times f_{\text{BE}},$$

The spin alignment for ρ^0 mesons will **decay rapidly toward zero** no matter what the initial state is.

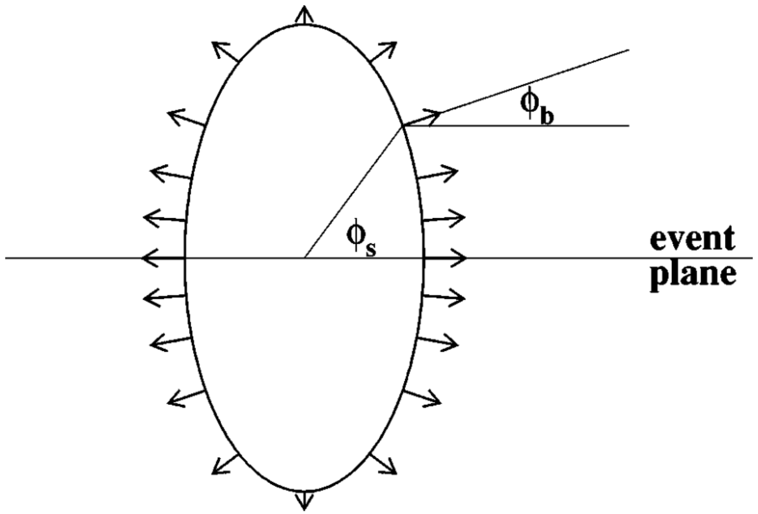
YLY et al., PhysRevC.110.024905 (2024)

Initial state:

$$f_{\lambda_1\lambda_2} = \text{diag}(1.1, 0.8, 1.1) \times f_{\text{BE}}.$$

Numerical results

Blast wave model with elliptic flow



$$\rho_{00} = \frac{\int_{|\eta|<1} d^3p \int_0^R r dr d\phi_s f_{00}(u,p)}{\int_{|\eta|<1} d^3p \int_0^R r dr d\phi_s \mathrm{tr} f(u,p)},$$

Fabrice Retiere et al.
Phys. Rev. C 70, 044907 (2004)

$$f_\pi = \frac{1}{\mathrm{e}^{u_\mu \mathrm{p}^\mu / \mathrm{T}} - 1}$$

$$u^\mu(r,\phi_s) = (\cosh\rho(r,\phi_s),\ \sinh\rho(r,\phi_s)\cos\phi_s,\ \sinh\rho(r,\phi_s)\sin\phi_s, 0),$$

$$\rho(r,\phi_s) = \frac{r}{R}[\rho_0 + \rho_2\cos(2\phi_s)].$$

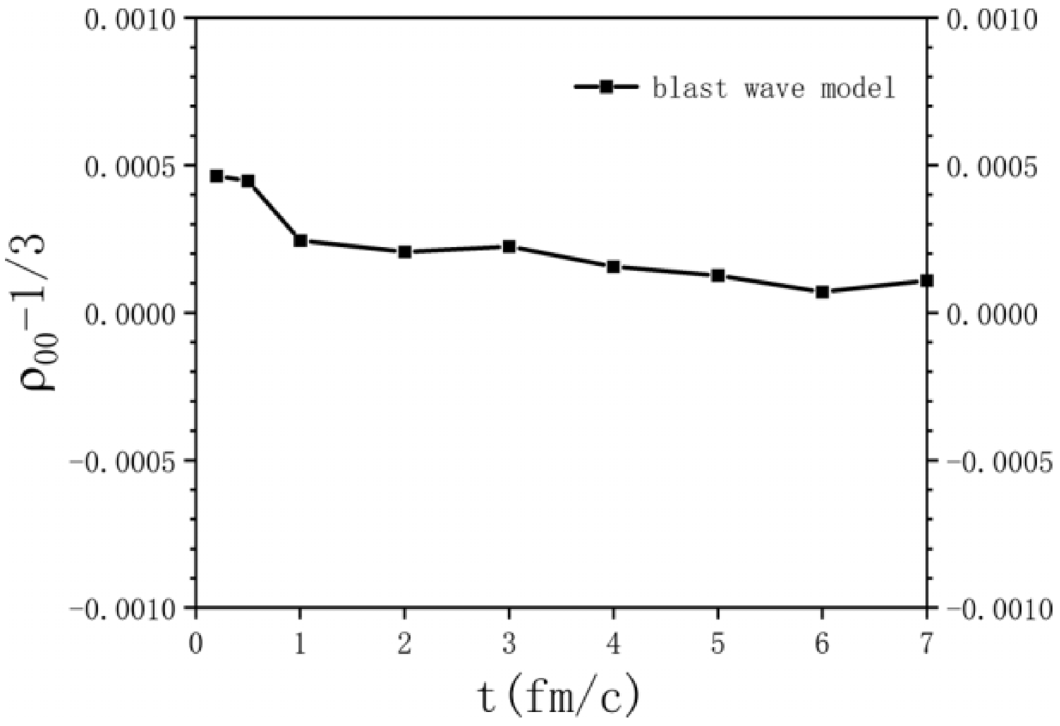
The parameters are chosen as:

$$R = 13\ \mathrm{fm},\ \rho_0 = 0.89,\ \rho_2 = 0.06$$

elliptic flow

Numerical results

The ρ^0 spin alignment in blast wave model



The spin alignment of the ρ^0 meson at z = 0 for $|\eta| < 1$ in the blast wave model with the elliptic flow.

The initial condition is chosen as $f_{\lambda_1\lambda_2}(t=0)=0$.

$\delta\rho_{00} \sim 10^{-4}$
in blast wave model

Conclusion 1

- Because of the strong $\rho - \pi$ interaction in hadron gas, the spin alignment of ρ mesons **decreases rapidly.** The initial value of the spin alignment can be easily washed out **in several fm/c**.
- In this work we only consider the **local effect** in collision terms, which proves that ρ mesons will reach local equilibrium rapidly. The spin alignment of ρ mesons may come from **nonlocal effects**.

Outline

- Shear induced spin alignment for Φ meson
- Evolution of the spin alignment for ρ meson in a pion gas (Local effect)
- **Shear induced spin alignment for ρ meson (Nonlocal effect)**

Spectral function

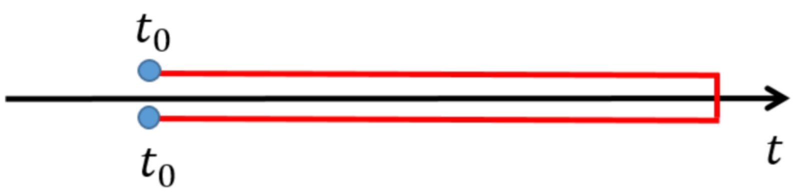
Interaction from SU(3) chiral perturbation theory

$$\mathcal{L}_{\text{int}} = i g_V A^\mu \left(\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger\right) + g_V^2 A_\mu A^\mu \phi^\dagger \phi,$$

A_μ: Vector field
ϕ: Scalar field

Dyson Schwinger equation in closed-time-path (CTP) formalism:

$$-i\left[g^\mu_{\ \rho}(\partial^2_{x_1} + m_V^2) - \partial^\mu_{x_1}\partial^{x_1}_\rho\right]\begin{pmatrix} 0 & G_A^{\rho\nu} \\ G_R^{\rho\nu} & G_C^{\rho\nu} \end{pmatrix}(x_1, x_2)$$
$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} g^{\mu\nu}\delta^{(4)}(x_1 - x_2)$$
$$+ \int dx' \begin{pmatrix} \Sigma^\mu_{A,\rho} & 0 \\ \Sigma^\mu_{C,\rho} & \Sigma^\mu_{R,\rho} \end{pmatrix}(x_1, x')\begin{pmatrix} 0 & G_A^{\rho\nu} \\ G_R^{\rho\nu} & G_C^{\rho\nu} \end{pmatrix}(x', x_2)$$



Kuang-chao Chou et al. Phys. Rept. 118, 1 (1985)

$$i\left[g^\mu_{\ \rho}(p^2 - m_V^2) - p^\mu p_\rho\right] G_R^{\rho\nu}(p) = g^{\mu\nu} + \Sigma^\mu_{R,\rho}(p) G_R^{\rho\nu}(p)$$

Self energy

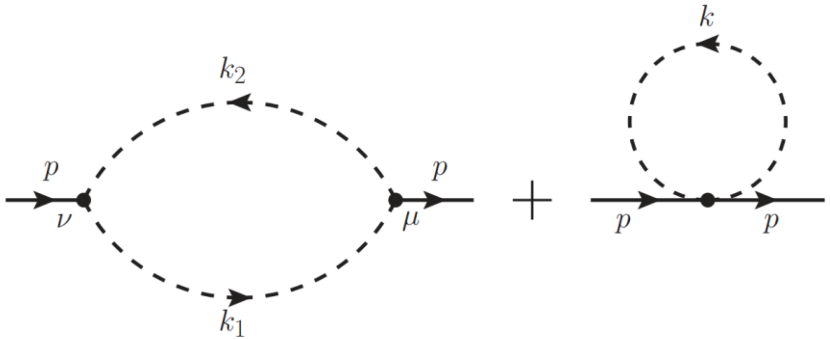
The retarded Green's function is related to the spectral function.

Spectral function

Self energy:

C. Gale and J. I. Kapusta, Nucl. Phys. B 357, 65 (1991)

$$
\begin{aligned}
\Sigma_R^{\mu\nu}(p) &= \Sigma_F^{\mu\nu}(p) - \Sigma_<^{\mu\nu}(p) \\
&= -g_V^2 \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(p-k_1+k_2)(k_1^\mu + k_2^\mu)(k_1^\nu + k_2^\nu) S^F(k_1) S^F(k_2) \\
&\quad + g_V^2 \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(p-k_1+k_2)(k_1^\mu + k_2^\mu)(k_1^\nu + k_2^\nu) S^<(k_1) S^>(k_2) \\
&\quad + 2g_V^2 i g^{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m_\pi^2 + i\epsilon}
\end{aligned}
$$



$$
\rho_L(p) = -\mathrm{Im} \frac{1}{p^2 - m_V^2 - \Pi_L}
$$

$$
\rho_T(p) = -\mathrm{Im} \frac{1}{p^2 - m_V^2 - \Pi_T}
$$

The difference between longitudinal (L) and transverse (T) modes is originated from **medium effect.**

The leading order of Green's function:

$$
G_{<,\mathrm{LO}}^{\mu\nu}(p) = -2n_B(p^0)\left[\Delta_L^{\mu\nu}\rho_L(p) + \Delta_T^{\mu\nu}\rho_T(p)\right]
$$

Linear response theory

Local Equilibrium Density Operator:

$$\widehat{\rho}_{\mathrm{LE}} = \frac{1}{Z_{\mathrm{LE}}} \exp\left[- \int_{\Sigma(\tau)} \mathrm{d}\Sigma \, n_\mu \left(\widehat{T}^{\mu\nu}(x)\beta_\nu(x) - \zeta(x)\widehat{j}^\mu(x) \right) \right]$$

Vanishes for ρ^0 meson

F. Becattini et al. Particles 2, 197 (2019)

The linear response of $\hat{O}(x)$ to the perturbation $\partial_\mu \beta_\nu$:

$$\left\langle \hat{O}(x) \right\rangle - \left\langle \hat{O}(x) \right\rangle_{\mathrm{LE}} = \partial_\mu \beta_\nu(x) \lim_{K^\mu \to 0} \frac{\partial}{\partial K_0} \times \mathrm{Im}\left[iT(x) \int_{-\infty}^{t} d^4x' \left\langle \left[\hat{O}(x), T^{\mu\nu}(x') \right] \right\rangle_{\mathrm{LE}} e^{-iK\cdot(x'-x)} \right]$$

$$T^{\mu\nu} = F^\mu_{\ \rho} F^{\rho\nu} + m_V^2 A^\mu A^\nu - g^{\mu\nu}\left(-\frac{1}{4} F_{\rho\sigma}F^{\rho\sigma} + \frac{1}{2} m_V^2 A_\rho A^\rho \right)$$

Linear response theory

The next-to-leading order of Green's function:

$$G^{\mu\nu}_{<,\mathrm{NLO}}(x,p) = 2T\xi_{\gamma\lambda}\frac{\partial n_B(p_0)}{\partial p_0} I^{\mu\nu\gamma\lambda}(p)$$

Thermal shear: $\xi_{\gamma\lambda} = \partial_{(\gamma}\beta_{\lambda)}$

$$\begin{aligned} I^{\mu\nu\gamma\lambda}(p) \equiv & -\left[g^{\lambda\gamma}\left(p^2-m^2\right)-2p^\lambda p^\gamma\right]\left(\Delta_L^{\mu\nu}\rho_L^2+\Delta_T^{\mu\nu}\rho_T^2\right) \\ & +2\left(p^2-m^2\right)\left(\Delta_L^{\mu\lambda}\rho_L+\Delta_T^{\mu\lambda}\rho_T\right)\left(\Delta_L^{\nu\gamma}\rho_L+\Delta_T^{\nu\gamma}\rho_T\right) \end{aligned}$$

Longitudinal and transverse projectors:

$$\Delta_L^{\mu\nu}(p) \equiv \frac{\Delta^{\mu\rho}u_\rho\Delta^{\nu\sigma}u_\sigma}{\Delta^{\rho\sigma}u_\rho u_\sigma},$$

$$\Delta_T^{\mu\nu}(p) \equiv \Delta^{\mu\nu} - \Delta_L^{\mu\nu}.$$

Feng Li et al. 2206.11890

The total Green's function:

$$G^{\mu\nu}_{<}(x,p) = G^{\mu\nu}_{<,\mathrm{LO}}(x,p) + G^{\mu\nu}_{<,\mathrm{NLO}}(x,p)$$

$G^{\mu\nu}_{<,\mathrm{LO}}(x,p)$ → Off-shell effect

$G^{\mu\nu}_{<,\mathrm{NLO}}(x,p)$ → Off-shell effect; Thermal shear effect (nonlocal)

Spin density matrix

Corrections for spin density matrix:

Off-shell projector: $L_{\mu\nu}(\lambda_1,\lambda_2,p) \equiv \epsilon^*_\mu(\lambda_1,p)\epsilon_\nu(\lambda_2,p) + \frac{1}{3}\Delta_{\mu\nu}(p)\delta_{\lambda_1\lambda_2}$

$$\delta\rho_{\lambda_1\lambda_2}(\mathbf{p}) = \frac{\int_0^\infty dp_0 \int d\Sigma^\mu p_\mu L_{\mu\nu}(\lambda_1,\lambda_2,p)\left[G^{\mu\nu}_{<,\mathrm{LO}}(x,p)+G^{\mu\nu}_{<,\mathrm{NLO}}(x,p)\right]}{-\int_0^\infty dp_0 \int d\Sigma^\mu p_\mu \Delta_{\mu\nu}(p)\left[G^{\mu\nu}_{<,\mathrm{LO}}(x,p)+G^{\mu\nu}_{<,\mathrm{NLO}}(x,p)\right]}$$

Hypersurface

$\epsilon_\mu(\lambda, p_{on}) \longrightarrow \epsilon_\mu(\lambda, p)$

More general case with off-shell effect

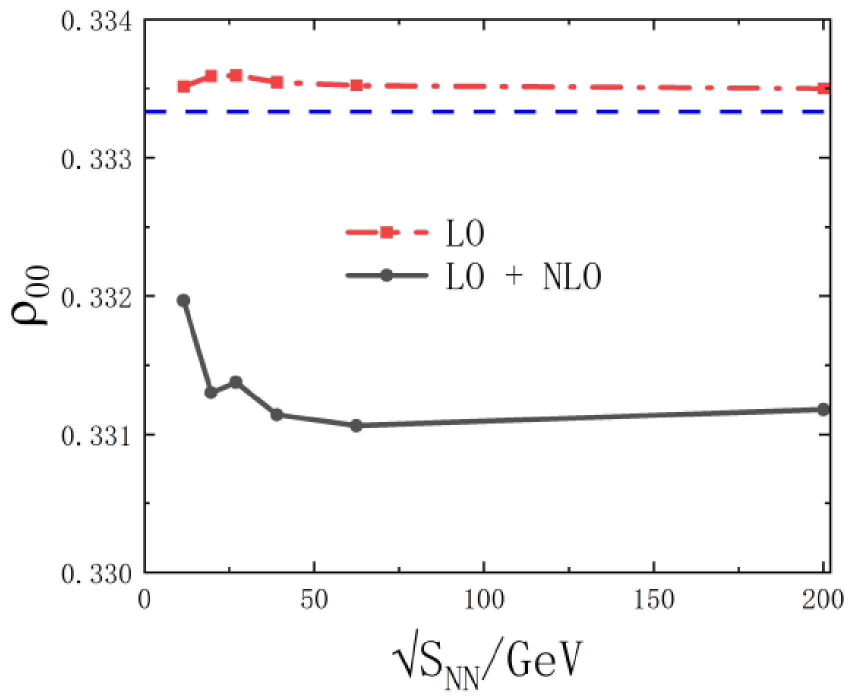
The thermal shear contribution to **ϕ** meson is calculated in *Wen-Bo Dong et al. Phys.Rev.C 110 (2024) 2, 024905*

$\xi^{\mu\nu}, \Sigma^\mu$ ⇒ hydrodynamical model CLVisc

Xiang-Yu Wu, Phys. Rev. C 105, 064909 (2022)

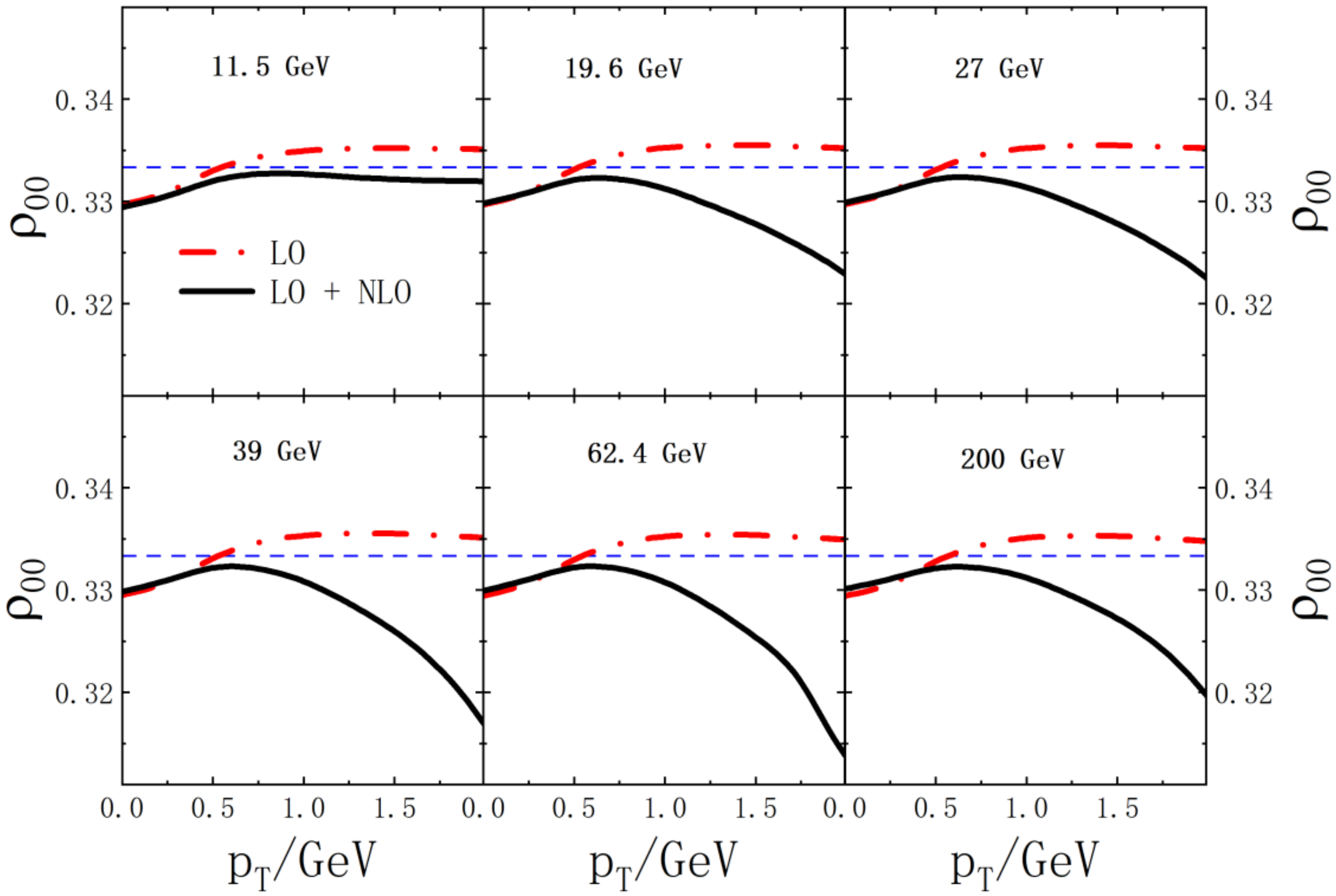
Numerical result for ρ_{00}

Global spin alignment:



- Thermal shear induces negative $\delta\rho_{00}$ at the order of $O(10^{-3})$;
- ρ_{00} decreases when p_T goes larger, when $p_T > 1.5GeV, \ \delta\rho_{00} \sim -0.01$.

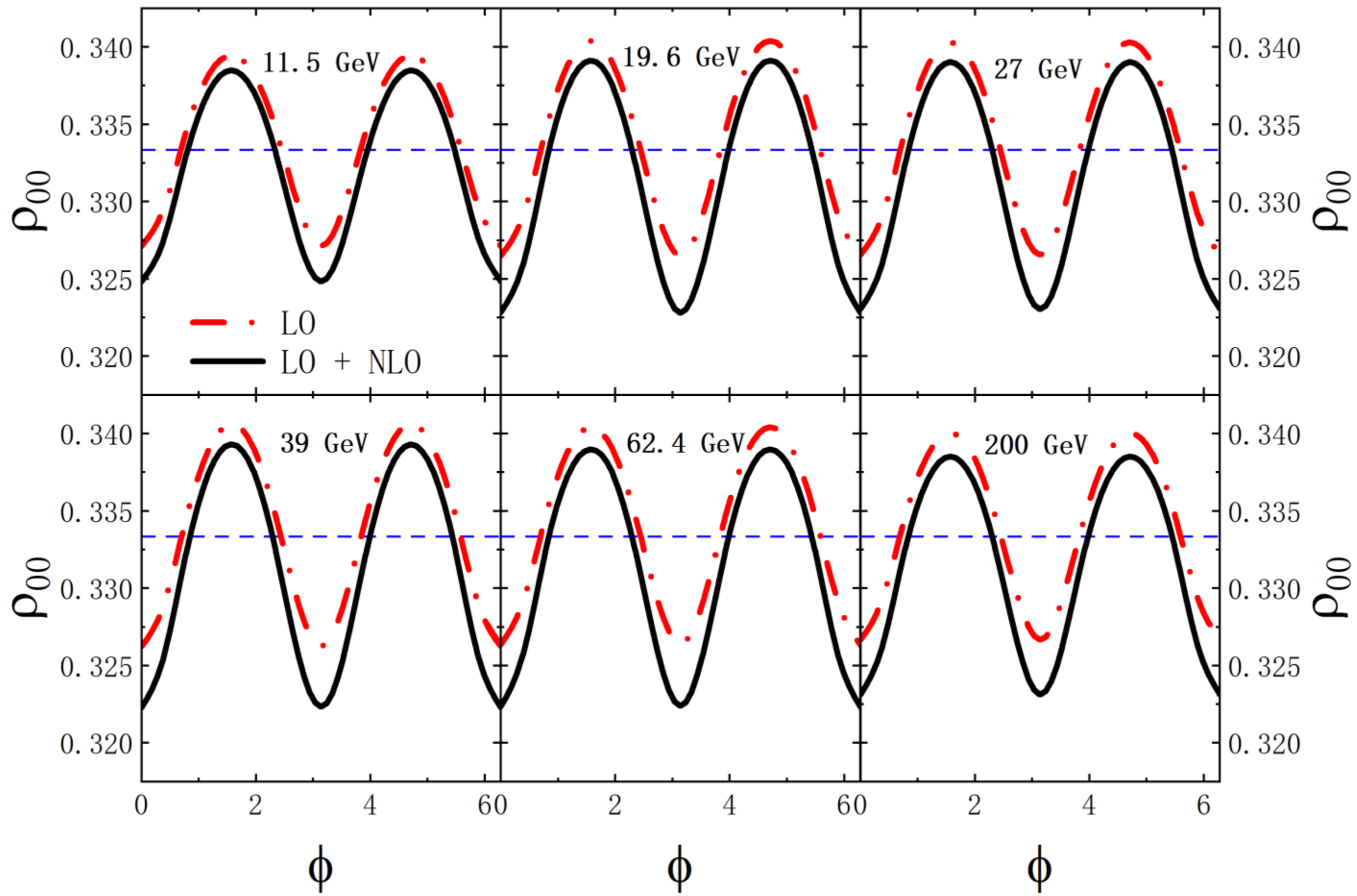
ρ_{00} as a function of p_T:



|Y|<1, 20-50%, $p_T^{cut} = 2GeV$

Numerical result for ρ_{00}

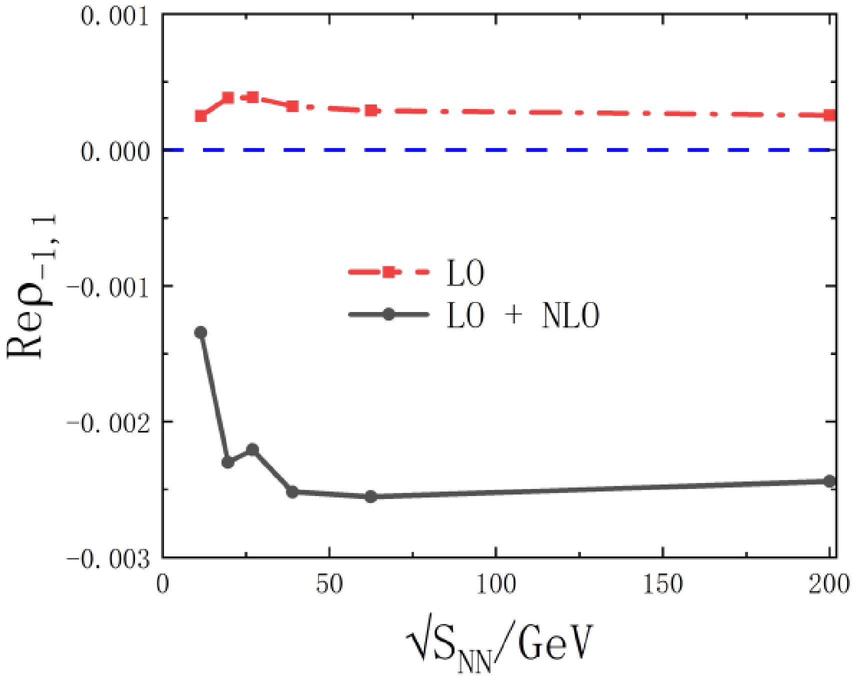
ρ_{00} as a function of ϕ:



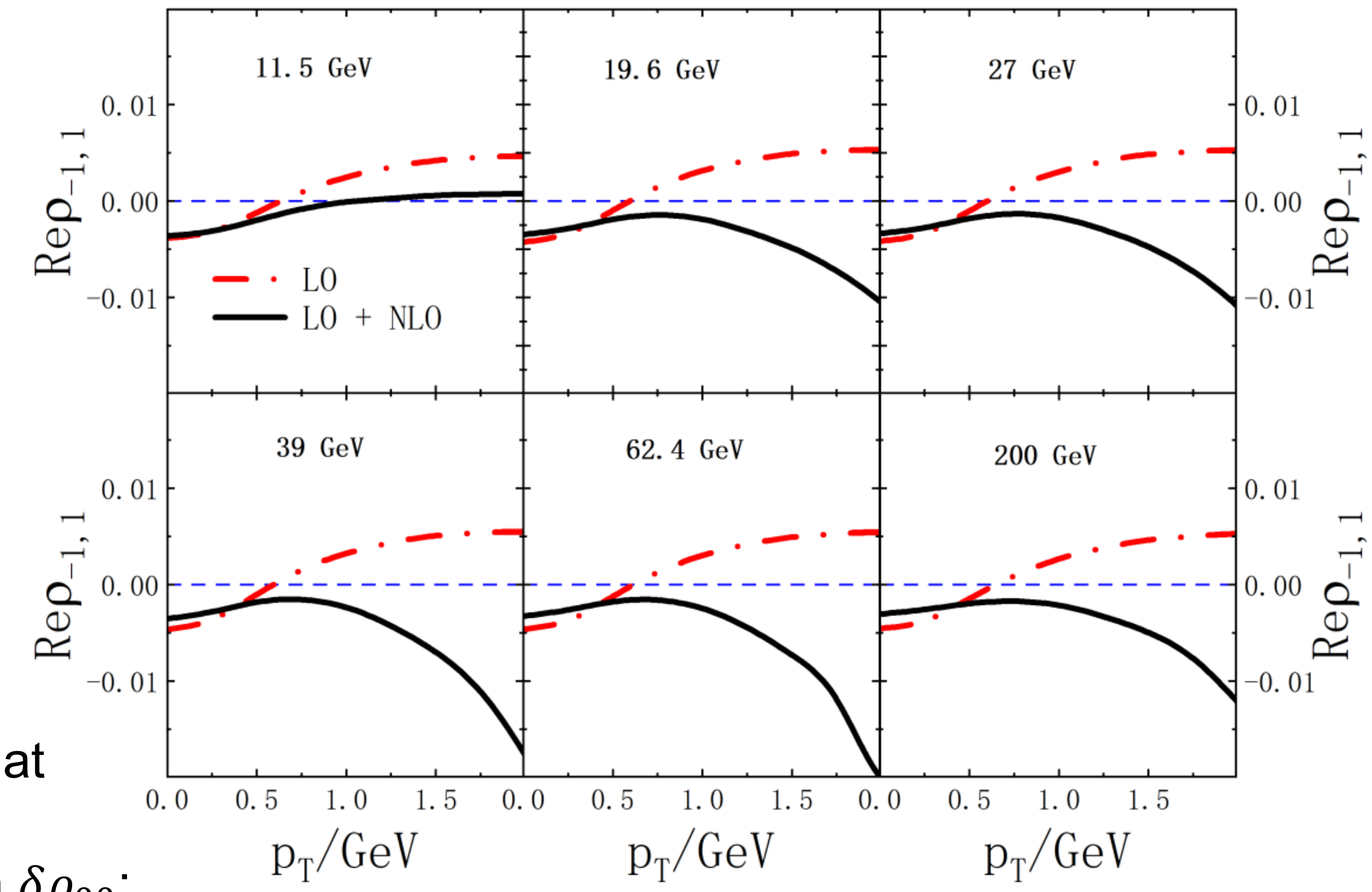
- $\delta\rho_{00}$ is positive for $\phi = \pi/2, 3\pi/2$ and negative for $\phi = 0, \pi$.
- Thermal shear has a negative contribution to ρ_{00} for all ϕ.

Numerical result for $Re\rho_{-1,1}$

Global $Re\rho_{-1,1}$:



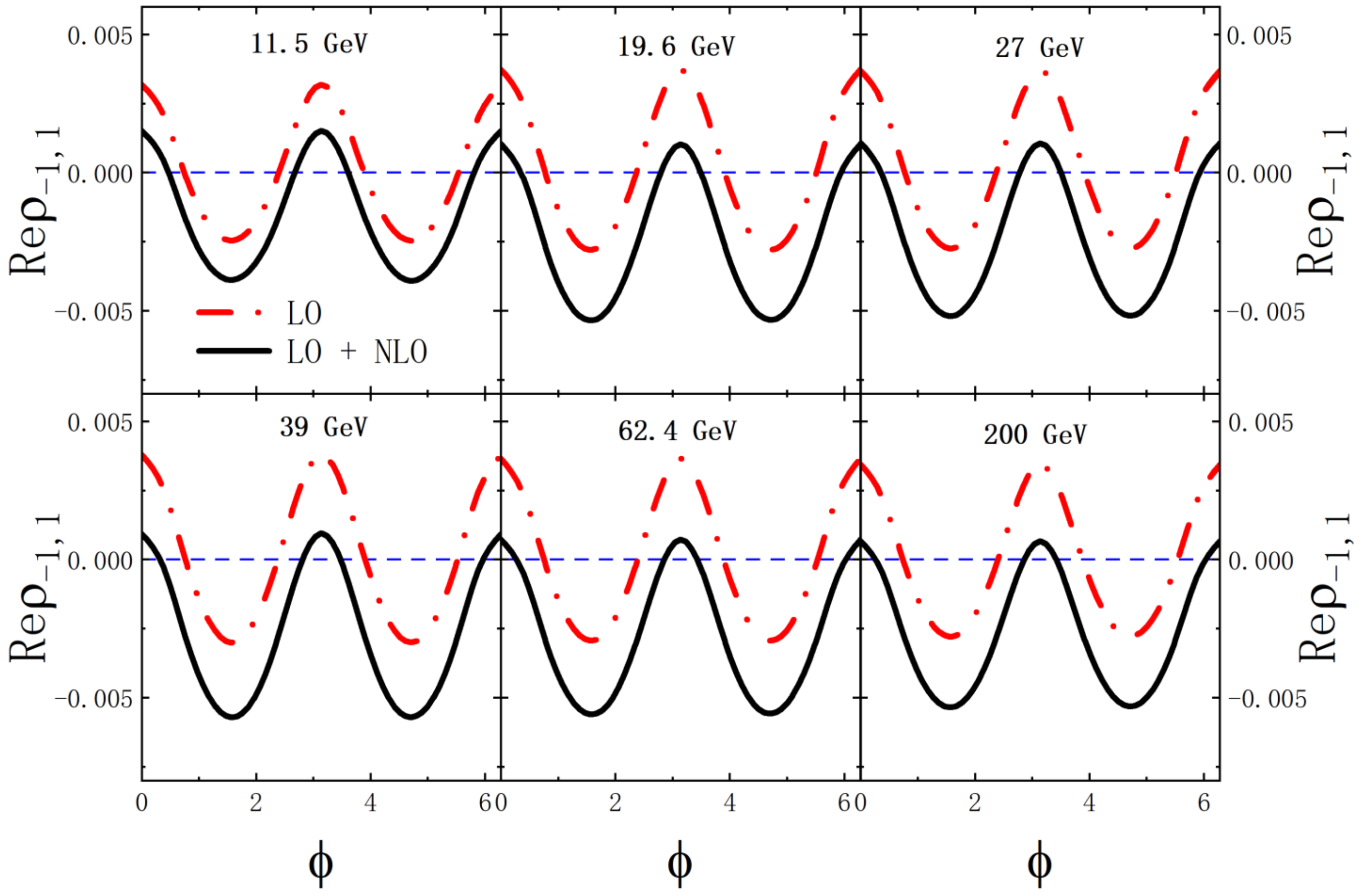
$Re\rho_{-1,1}$ as a function of p_T:



- Thermal shear induces negative $Re\rho_{-1,1}$ at the order of $O(10^{-3})$;
- The p_T behavior of $Re\rho_{-1,1}$ is similar with $\delta\rho_{00}$;

Numerical result for $Re\rho_{-1,1}$

$Re\rho_{-1,1}$ as a function of ϕ:



- $Re\rho_{-1,1}$ is positive for $\phi = 0, \pi$ and negative for $\phi = \pi/2, 3\pi/2$.
- Thermal shear has a negative contribution to $Re\rho_{-1,1}$.

YLY et al., Sci.China Phys.Mech.Astron. 69 (2026) 5, 251011

Conclusion 2

- Although the $\rho-\pi$ interaction can wash out the initial spin alignment of ρ mesons, the **spectral effect** and **thermal shear effect** can induce local and global spin alignment for ρ mesons;
- $\delta\rho_{00}$ and $Re\rho_{-1,1}$ are both negative at the order $\mathbf{10^{-3}\sim10^{-2}}$ and **decrease with $\boldsymbol{p_T}$**;
- The **global** corrections for spin density matrix are resulted from the **thermal shear effect** (NLO)(nonlocal), while the corrections as a function of **azimuthal angle** are mainly resulted from the **spectral effect** (LO).

Summary

1. The spin density matrix of ρ mesons has an influence on the **CME signal $\Delta\gamma_{112}$**, especially for the elements $\boldsymbol{\rho_{00}}$ and $\boldsymbol{Re\rho_{-1,1}}$.

2. The width of ρ meson is very large, so the evolution of ρ meson in the **hadron phase** should be considered.

3. Because of the strong $\rho-\pi$ interaction in hadron gas, the spin alignment of ρ mesons **decreases rapidly**, so the messages carried from QGP can be washed out.

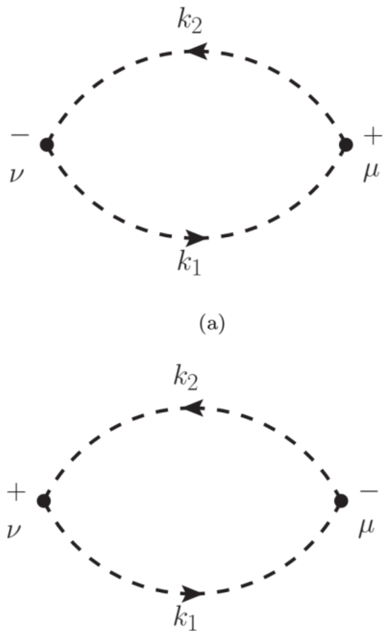
4. Thermal shear induces a **negative** global correction to $\boldsymbol{\rho_{00}}$ and $\boldsymbol{Re\rho_{-1,1}}$ at the order 10^{-3}, and for high p_T region, it will reach the order of 10^{-2}.

Thanks

Collision terms

Leading order

$$
\begin{aligned}
&C^{(0)}_{\text{coal/diss}}(\rho^0 \leftrightarrow \pi^+\pi^-) \\
&\quad = \hbar \frac{g_V^2}{E_p^\rho} \int \frac{d^3k}{(2\pi\hbar)^3 4E_k^\pi E_{p-k}^\pi} 2\pi\hbar\delta\left(E_p^\rho - E_k^\pi - E_{p-k}^\pi\right) \\
&\quad\quad \times \left[\delta_{\lambda_2\lambda_2'} k\cdot\epsilon^*(\lambda_1,\mathbf{p}) k\cdot\epsilon(\lambda_1',\mathbf{p}) \right. \\
&\quad\quad\quad \left. + \delta_{\lambda_1\lambda_1'} k\cdot\epsilon(\lambda_2,\mathbf{p}) k\cdot\epsilon^*(\lambda_2',\mathbf{p})\right] \\
&\quad\quad \times \left\{ f_{\pi^+}(x,\mathbf{k}) f_{\pi^-}(x,\mathbf{p}-\mathbf{k})\left[\delta_{\lambda_1'\lambda_2'} + f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right] \right. \\
&\quad\quad\quad \left. - [1+f_{\pi^+}(x,\mathbf{k})][1+f_{\pi^-}(x,\mathbf{p}-\mathbf{k})] f_{\lambda_1'\lambda_2'}(x,\mathbf{p}) \right\},
\end{aligned}
$$



(a)
(b)

Back up

Collision terms

Next-to-leading order

$$C_{\text{scat}}(\rho^0\pi^\pm \leftrightarrow \rho^0\pi^\pm) = \frac{4g_V^4}{E_p^\rho}\hbar^2 \int \frac{d^3k_1}{(2\pi\hbar)^3 2E_{k_1}^\pi} \int \frac{d^3k_2}{(2\pi\hbar)^3 2E_{k_2}^\pi} \int \frac{d^3p_1}{(2\pi\hbar)^3 2E_{p_1}^\rho} (2\pi\hbar)^4 \delta^{(4)}(p+k_2-p_1-k_1)$$
$$\times \left[\delta_{\lambda_2\lambda_2'} D_{(1)}(s_1,\lambda_1) D_{(1)}^*(s_2,\lambda_1') + \delta_{\lambda_1\lambda_1'} D_{(1)}(s_1,\lambda_2') D_{(1)}^*(s_2,\lambda_2)\right]$$
$$\times \left[f_{s_1s_2}(x,\mathbf{p}_1) f_{\pi^\pm}(x,\mathbf{k}_1)(1+f_{\pi^\pm}(x,\mathbf{k}_2))\left(\delta_{\lambda_1'\lambda_2'} + f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right)\right.$$
$$\left. - \left(\delta_{s_1s_2} + f_{s_1s_2}(x,\mathbf{p}_1)\right)(1+f_{\pi^\pm}(x,\mathbf{k}_1)) f_{\pi^\pm}(x,\mathbf{k}_2) f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right],$$

$$C^{(1)}_{\text{coal/diss}}(\rho^0\rho^0 \leftrightarrow \pi^+\pi^-) = \frac{4g_V^4}{E_p^\rho}\hbar^2 \int \frac{d^3k_1}{(2\pi\hbar)^3 2E_{k_1}^\pi} \int \frac{d^3k_2}{(2\pi\hbar)^3 2E_{k_2}^\pi} \int \frac{d^3p_1}{(2\pi\hbar)^3 2E_{p_1}^\rho} (2\pi\hbar)^4 \delta^{(4)}(p+p_1-k_1-k_2)$$
$$\times \left[\delta_{\lambda_2\lambda_2'} D_{(2)}(s_1,\lambda_1') D_{(2)}^*(s_2,\lambda_1) + \delta_{\lambda_1\lambda_1'} D_{(2)}(s_1,\lambda_2) D_{(2)}^*(s_2,\lambda_2')\right]$$
$$\times \left[f_{\pi^+}(x,\mathbf{k}_1) f_{\pi^-}(x,\mathbf{k}_2)\left(\delta_{s_1s_2} + f_{s_1s_2}(x,\mathbf{p}_1)\right)\left(\delta_{\lambda_1'\lambda_2'} + f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right)\right.$$
$$\left. -(1+f_{\pi^+}(x,\mathbf{k}_1))(1+f_{\pi^-}(x,\mathbf{k}_2)) f_{s_1s_2}(x,\mathbf{p}_1) f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right],$$

$$D_{\pi^+(1)}(s,\lambda) = \hbar \frac{[k_1\cdot\epsilon(s,\mathbf{p}_1)][k_2\cdot\epsilon^*(\lambda,\mathbf{p})]}{(p+k_2)^2 - m_\pi^2 + i\hbar\Gamma(p+k_2)} + \hbar \frac{[k_2\cdot\epsilon(s,\mathbf{p}_1)][k_1\cdot\epsilon^*(\lambda,\mathbf{p})]}{(p-k_1)^2 - m_\pi^2 + i\hbar\Gamma(-p+k_1)}$$

$$D_{\pi^-(1)}(s,\lambda) = \hbar \frac{[k_1\cdot\epsilon(s,\mathbf{p}_1)][k_2\cdot\epsilon^*(\lambda,\mathbf{p})]}{(p+k_2)^2 - m_\pi^2 + i\hbar\Gamma(-p-k_2)} + \hbar \frac{[k_2\cdot\epsilon(s,\mathbf{p}_1)][k_1\cdot\epsilon^*(\lambda,\mathbf{p})]}{(p-k_1)^2 - m_\pi^2 + i\hbar\Gamma(p-k_1)}$$