Flavour Physics Lecture

味物理讲座

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在BESIII实验上精确测量中性D介子衰变的强相位差

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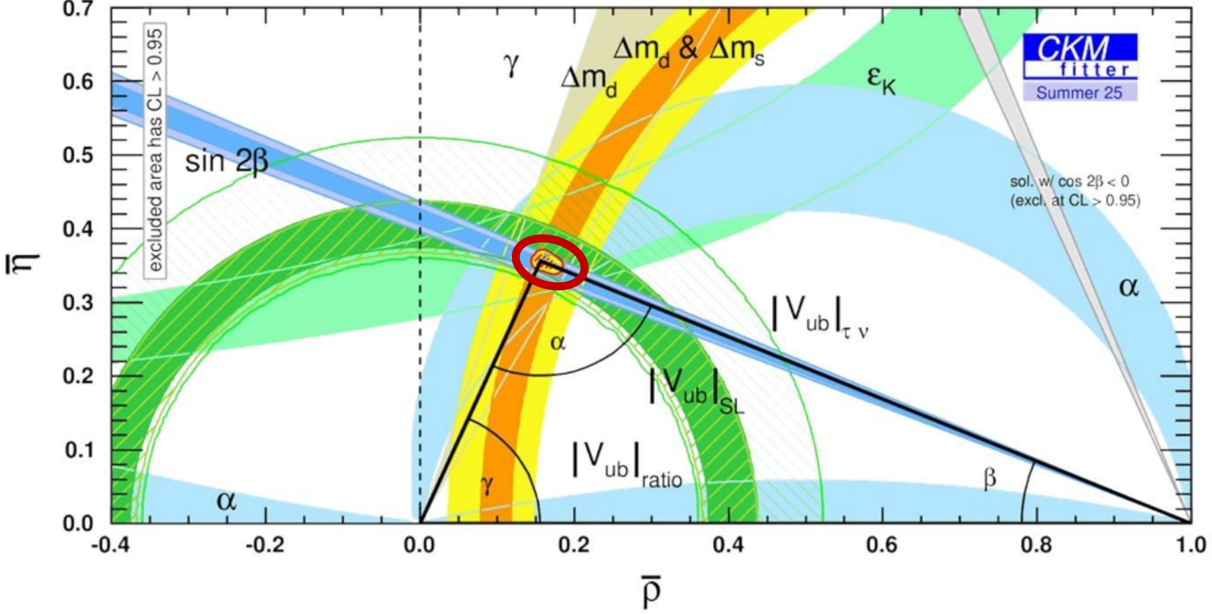
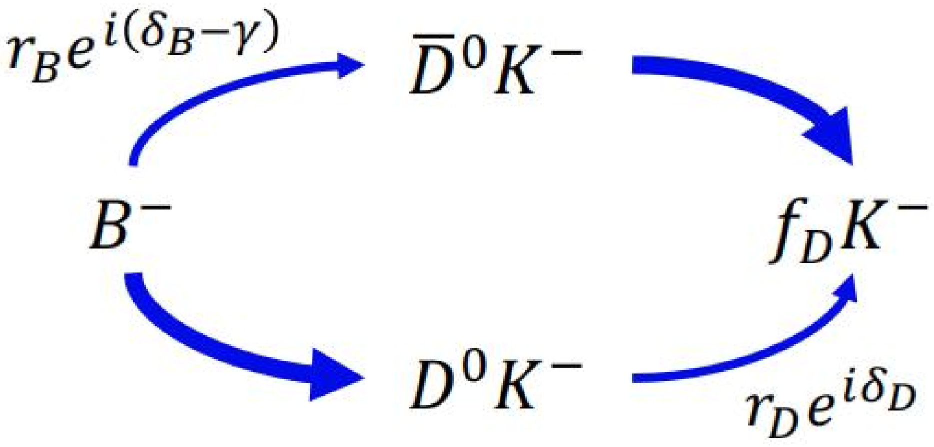
Outline

- ☐ γ角精确测量和中性粲介子的混合和间接CP破坏研究
- ☐ BESIII实验上精确测量强相位差
- ☐ 未来的相关测量
- ☐ 总结

CKM幺正三角形

[CKM2025, Olivier Deschamps]



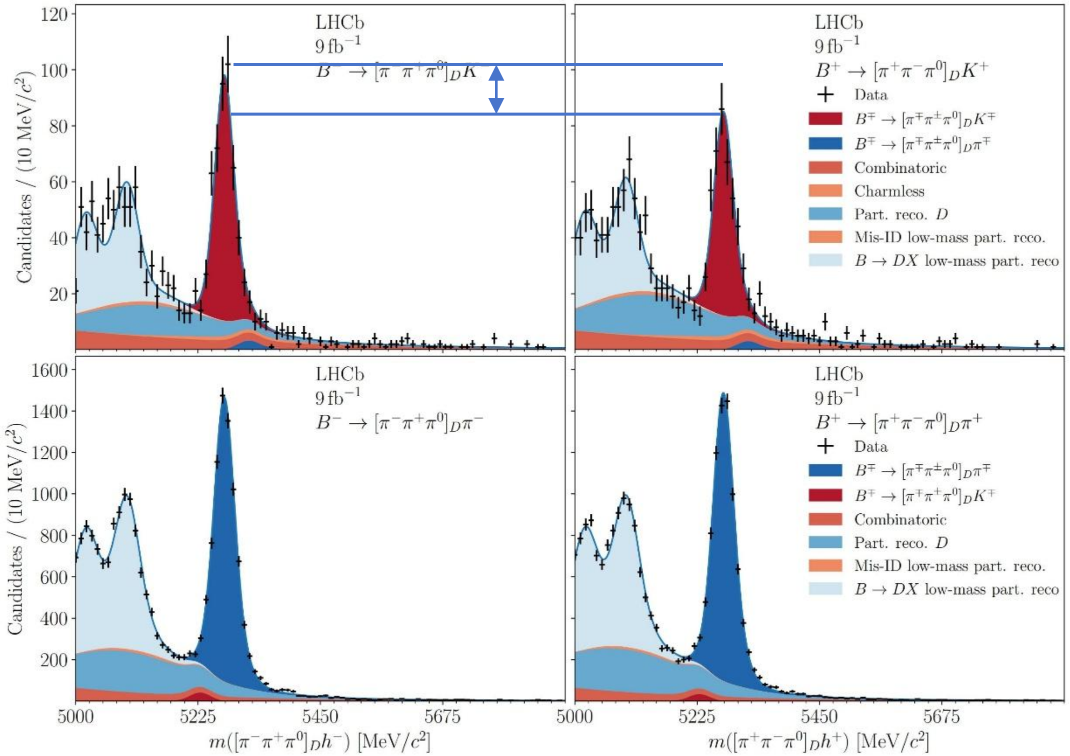


- γ/ϕ_3角主要通过树图过程$B^\pm \to f_D K^\pm$测量：$(66.7^{+2.7}_{-2.8})^o$
  - D介子为味道本征态D^0、$\bar{D}^0$的叠加态
  - 没有来自新物理的贡献[JHEP 01 (2014) 051]
- 由CKM三角形的幺正性和其他观测量约束：$(66.23^{+1.86}_{-0.72})^o$
- 验证CKM矩阵幺正性，寻找新的CP破坏来源

GLW方法

D介子衰变末态为（准）CP本征态K^+K^-、$\pi^+\pi^-$、$\pi^+\pi^-\pi^0$、$K^+K^-\pi^0$、$K_S^0\pi^0$ etc
[Phys. Lett. B253 (1991) 483; Phys. Lett. B265 (1991) 172]

$$N^\pm = h^\pm[1 + r_B^2 + 2(2F_+ - 1)r_B\cos(\delta_B \pm \gamma)]$$



$$F_+ \equiv \frac{\int |\langle f(\mathrm{x})|D_{CP+}\rangle|^2 dx}{\int |\langle f(\mathrm{x})|D_{CP+}\rangle|^2 + |\langle f(\mathrm{x})|D_{CP-}\rangle|^2 dx}$$

$$= \frac{\int a_{\mathrm{x}}^2 + a_{\bar{\mathrm{x}}}^2 + 2a_{\mathrm{x}}a_{\bar{\mathrm{x}}}\cos\delta_D^f(\mathrm{x})dx}{\int 2(a_{\mathrm{x}}^2 + a_{\bar{\mathrm{x}}}^2)dx}$$

对CP正（负）的本征态，$F_+ = 1(0)$
δ_D^f为D^0和$\overline{D}^0$介子衰变振幅的强相位差
（CP变换下不变）

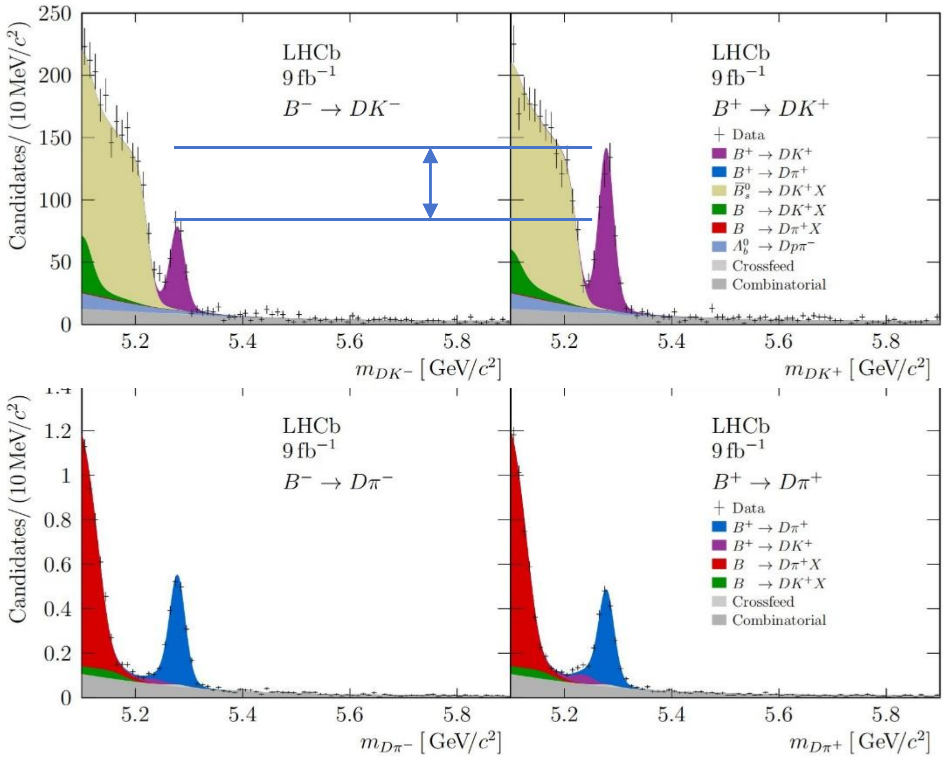
[JHEP 07 (2022) 099]; See also in [JHEP 05 (2024) 212 etc] by Belle (II)

ADS方法

D介子衰变末态为准味道本征态$K^-\pi^+$、$K^-\pi^+\pi^+\pi^-$、$K^+\pi^-\pi^0$、$K_S^0K^-\pi^+$ etc

[Phys. Rev. Lett. 78 (1997) 3257, Phys. Rev. D 63 (2001) 036005]

$$N(B^\pm \to [K^\mp n\pi]h^\pm) = h^\pm\left[\left(r_D^f\right)^2 + r_B^2 + 2r_B r_D^f R_f \cos\left(\delta_B + \delta_D^f \pm \gamma\right)\right]$$



$$r_D^f = \frac{|A_{\bar{f}}|}{|A_f|},\ R_f \exp\left(-i\delta_D^f\right) = \frac{\int dx\, A_f^*(x)\bar{A}_f(x)}{A_f\bar{A}_f}$$

δ_D^f为D^0和$\bar{D}^0$介子衰变的强相位差

r_D^f为D^0和$\bar{D}^0$介子衰变的振幅比

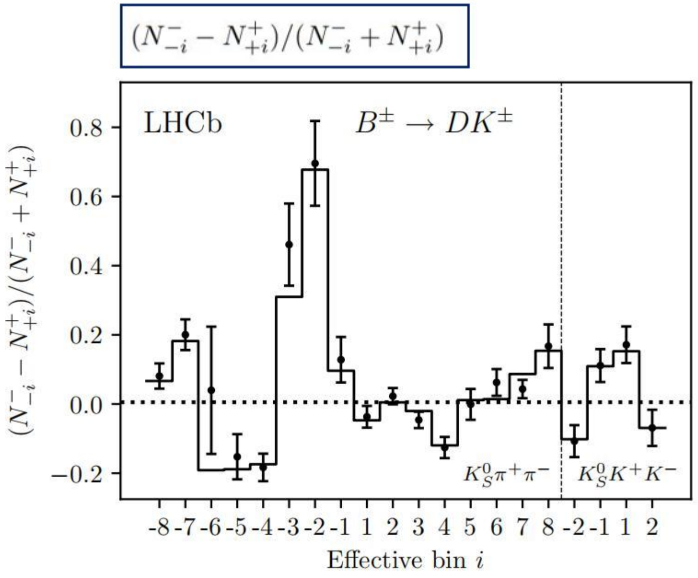
R_f为D^0和$\bar{D}^0$介子衰变振幅的“相干因子”

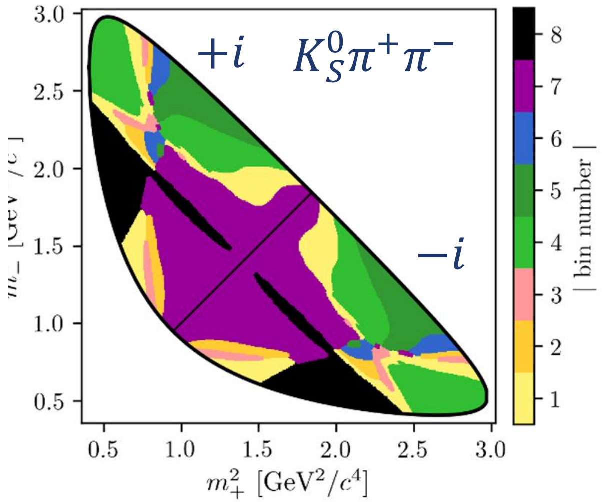
[JHEP 07 (2023) 138] by LHCb; [JHEP 09 (2023) 146 etc] by Belle (II)

BPGGSZ方法

D介子衰变为自共轭衰变$K_S^0\pi^+\pi^-$、$K_S^0K^+K^-$、$\pi^+\pi^-\pi^+\pi^-$、$K^+K^-\pi^+\pi^-$ etc
[A. Bondar, Phys. Rev. D 68 (2003) 054018, Phys. Rev. D 70 (2004) 072003]

$$N^+_{\pm i} = h_{B^+}\left[F_{\mp i} + ((x_+)^2 + (y_+)^2)F_{\pm i} + 2\sqrt{F_{\mp i}F_{\pm i}}(x_+c_i \mp y_+s_i)\right]$$
$$N^-_{\mp i} = h_{B^-}\left[F_{\mp i} + ((x_-)^2 + (y_-)^2)F_{\pm i} + 2\sqrt{F_{\mp i}F_{\pm i}}(x_-c_i \mp y_-s_i)\right]$$
$$x_\pm \equiv r_B\cos(\delta_B \pm \gamma),\ y_\pm \equiv r_B\sin(\delta_B \pm \gamma)$$





[JHEP 02 (2021) 0169]

$$c_i \equiv \frac{\int_i dm_-^2 dm_+^2 |A_D(m_-^2, m_+^2)||A_D(m_+^2, m_-^2)|\cos[\delta_D(m_-^2, m_+^2) - \delta_D(m_+^2, m_-^2)]}{\sqrt{\int_i dm_-^2 dm_+^2 |A_D(m_-^2, m_+^2)|^2 \int_i dm_-^2 dm_+^2 |A_D(m_+^2, m_-^2)|^2}}$$

$$s_i \equiv \frac{\int_i dm_-^2 dm_+^2 |A_D(m_-^2, m_+^2)||A_D(m_+^2, m_-^2)|\sin[\delta_D(m_-^2, m_+^2) - \delta_D(m_+^2, m_-^2)]}{\sqrt{\int_i dm_-^2 dm_+^2 |A_D(m_-^2, m_+^2)|^2 \int_i dm_-^2 dm_+^2 |A_D(m_+^2, m_-^2)|^2}}$$

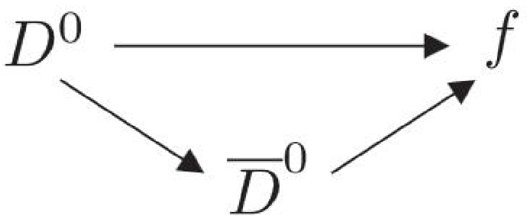
$\delta_D(m_-^2, m_+^2) - \delta_D(m_+^2, m_-^2)$为$D^0$和$\bar{D}^0$介子衰变的强相位差

$$F_i \equiv \frac{\int_i dm_-^2 dm_+^2 |A_D(m_-^2, m_+^2)|^2}{\sum_j \int_j dm_-^2 dm_+^2 |A_D(m_-^2, m_+^2)|^2}$$

F_i为D^0介子衰变率在bin i的比例

See also in [JHEP 02 (2022) 063 (Erratum ibid. 12 (2022) 034) etc] by Belle (II)

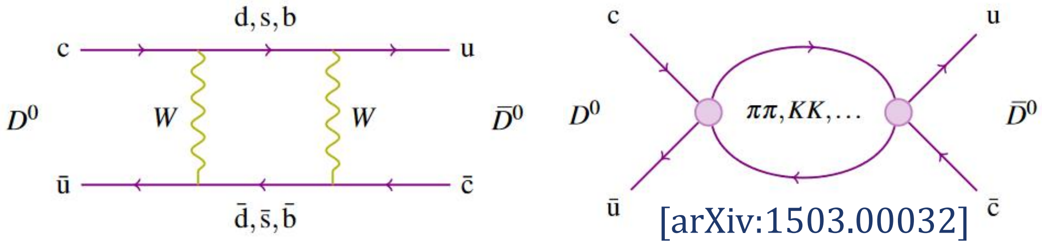
中性粲介子的混合和间接*CP*破坏研究



$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$$

$$x \equiv \frac{2(m_2 - m_1)}{\Gamma_1 + \Gamma_2},\ y \equiv \frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2}$$

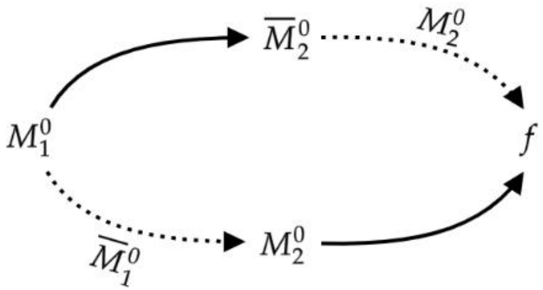
Loop level short-distance distributions are at the level $(\boldsymbol{x}, \boldsymbol{y}) \sim \mathbf{10^{-7}}$



[arXiv:1503.00032]

Long-distance contributions are at the level $(\boldsymbol{x}, \boldsymbol{y}) \sim \mathbf{10^{-3}}$
[Phys. Lett. B 810 (2020) 135802;
[Chin. Phys. C 42 (2018) 063101;
[Phys. Rev. D 81 (2010) 114020;
[Phys. Rev. D 65 (2002) 054034 etc]

- Type 1. 混合中的CP破坏$|p| \neq |q|$
    - 质量本征态不是CP本征态
- Type 2. 衰变振幅和混合之间的干涉：$\mathrm{Im}\left(\frac{q}{p}\frac{A(\overline{D}^0 \to f)}{A(D^0 \to f)}\right) \neq \mathrm{Im}\left(\frac{p}{q}\frac{A(D^0 \to \bar{f})}{A(\overline{D}^0 \to \bar{f})}\right)$
    - $\frac{q}{p}$具有非零相角 ϕ
- Type 3. 粲介子混合和衰变末态中的$K^0 - \overline{K}^0$混合之间的干涉



[PRD 110 (2024) L031301]

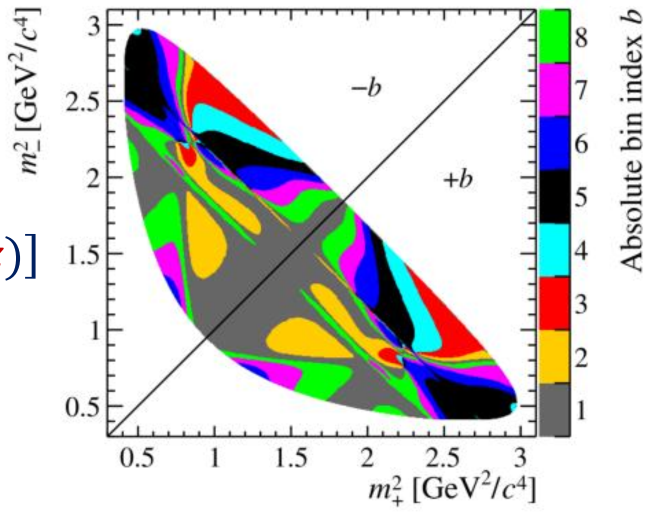
自共轭衰变

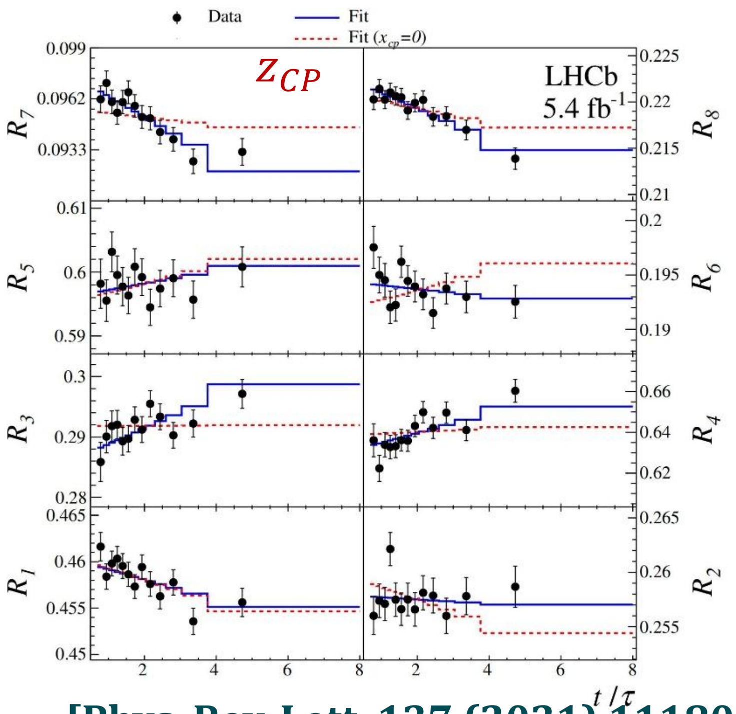
- 产生时为味道本征态的D介子数据($D^{*+} \to D^0(K_S^0\pi^+\pi^-)\pi^+$)
- 时间依赖的衰变率

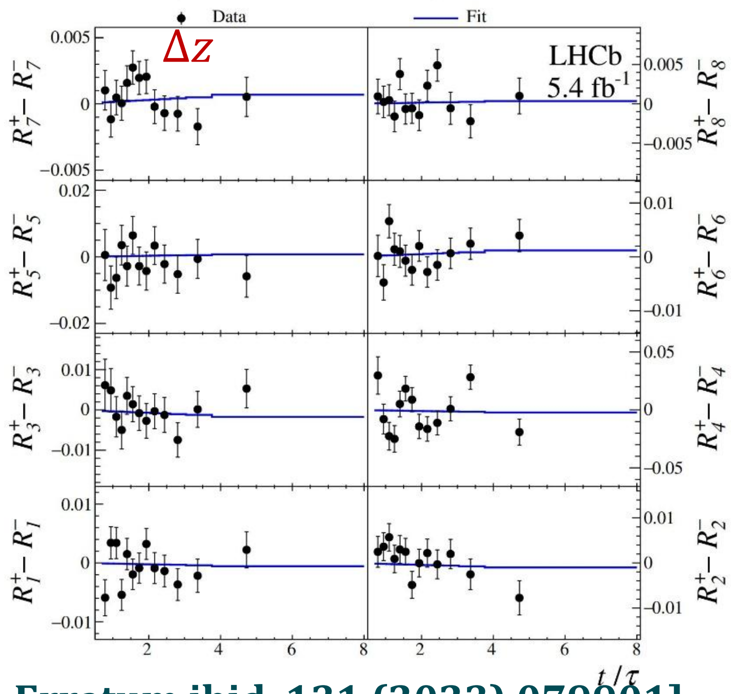
混合和间接CP破坏参数

$$p^{\pm}_{bj} \propto r_b + r_b \frac{\langle t^2\rangle_j}{4}\mathrm{Re}(z_{CP}^2 - \Delta z^2) + \frac{\langle t^2\rangle_j}{4}|z_{CP} \pm \Delta z|^2 + \sqrt{r_b}\langle t\rangle_j \mathrm{Re}[X_{\pm b}(z_{CP} \pm \Delta z)]$$

强相位差参数







首次观测到中性D介子的质量差

$$x = \left(3.98^{+0.56}_{-0.54}\right) \times 10^{-3}$$

$$y = \left(4.6^{+1.5}_{-1.4}\right) \times 10^{-3}$$

$$|q/p| = 0.996 \pm 0.052$$

$$\phi = -0.056^{+0.047}_{-0.051}$$

[Phys. Rev. Lett. 127 (2021) 111801, Erratum ibid. 131 (2023) 079901];
See also in [Phys. Rev. D 111 (2025) 11, 112011] by Belle (II)

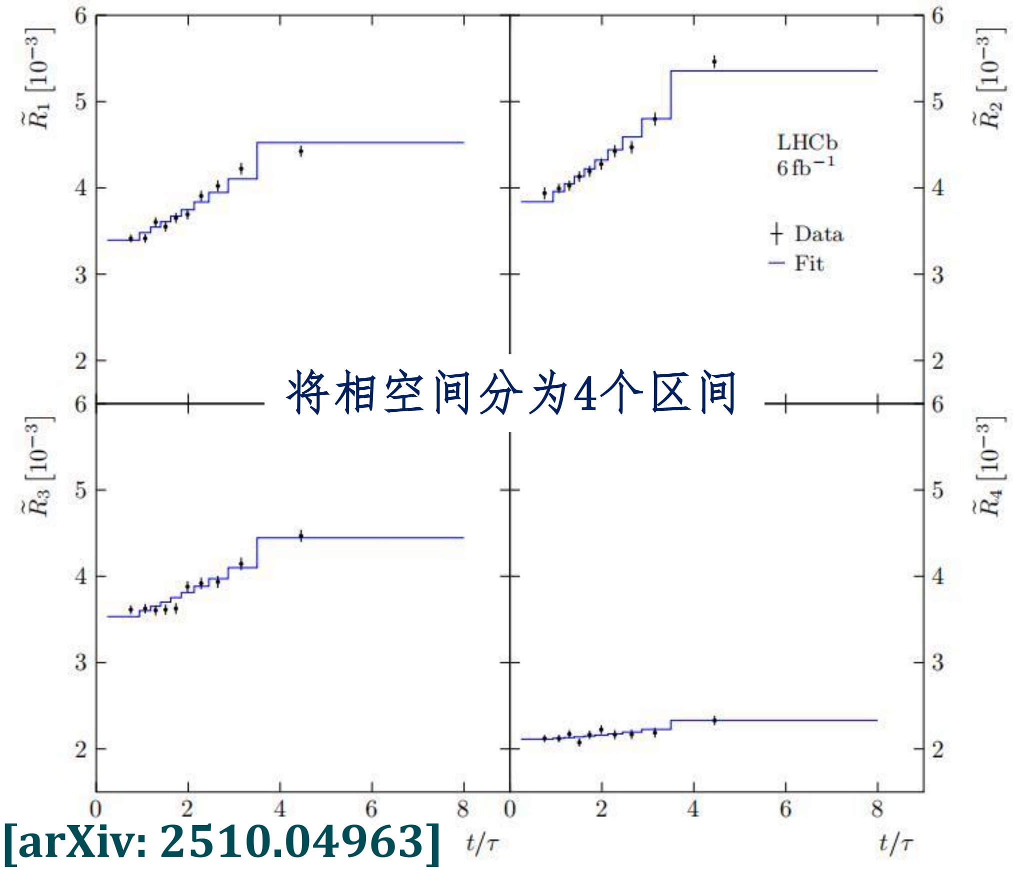
准味道本征态

- 产生时为味道本征态的D介子数据($D^{*+} \to D^0(K^{\pm}n\pi)\pi^+$)
- 时间依赖的衰变率

$$R(t) \equiv \frac{\Gamma[D^0 \to K^+3\pi](t)}{\Gamma[\overline{D}^0 \to K^-3\pi](t)} \approx \left(r_D^f\right)^2 - r_D^f R_f \frac{t}{\tau}\left(y\cos\delta_D^f - x\sin\delta_D^f\right) + \frac{x^2+y^2}{4}\left(\frac{t}{\tau}\right)^2$$

混合参数

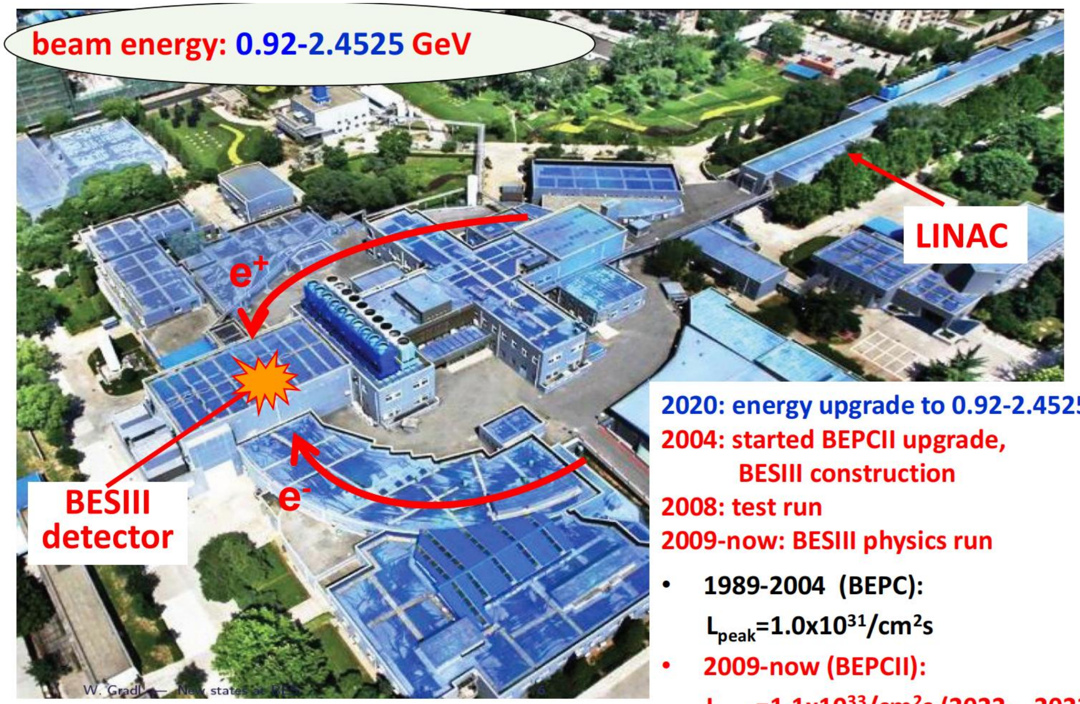
强相位差参数



[arXiv: 2510.04963]

$$\boldsymbol{x = \left(0.85^{+0.15}_{-0.24}\right)\%}$$
$$\boldsymbol{y = \left(0.21^{+0.29}_{-0.27}\right)\%}$$

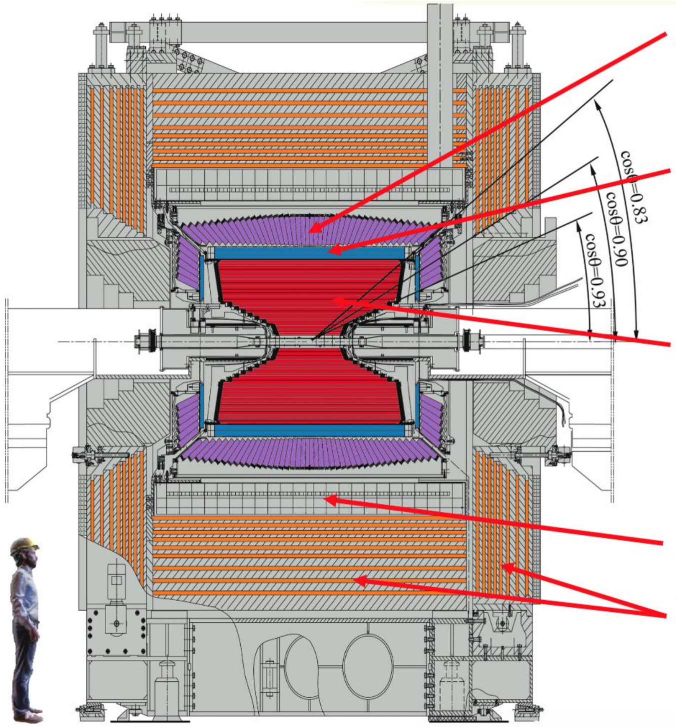
BEPCII和BESIII探测器



Credit to 马海龙

2020: energy upgrade to 0.92-2.4525 GeV
2004: started BEPCII upgrade,
BESIII construction
2008: test run
2009-now: BESIII physics run

- 1989-2004 (BEPC):
  L_{peak}=1.0x10^{31}/cm^{2}s
- 2009-now (BEPCII):
  L_{peak}=1.1x10^{33}/cm^{2}s (2022、2023)



EMC: CsI crystals
$\Delta E/E$ = 2.5% @ 1 GeV - Barrel
$\Delta E/E$ = 5.0% @ 1 GeV - Endcaps

TOF:
σ_T = 80 ps Barrel
σ_T = 110 (60) ps Endcap

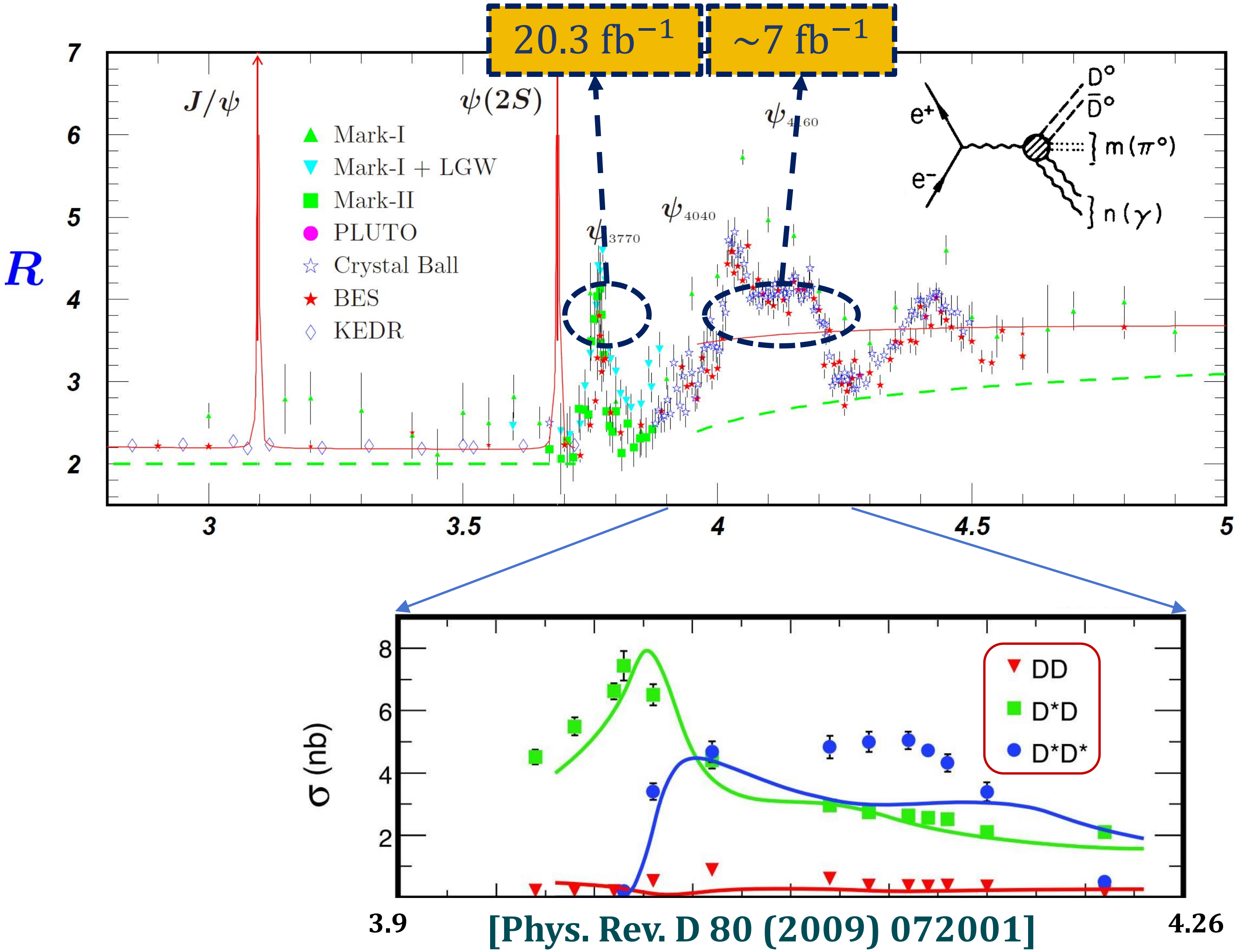
MDC: small cell & He gas
σ_{xy}= 130 μm
σ_p/p = 0.5% @1GeV
dE/dx = 6%

Magnet: 1T Super conducting

Muon ID: 9 layer RPC

Trigger: Tracks & Showers

BESIII实验采集的中性*D*介子数据样本



[Phys. Rev. D 80 (2009) 072001]

2009:	106M $\psi(3686)$
	225M J/ψ
2010:	0.98 fb^{-1} $\psi(3770)$
2011:	2.93 fb^{-1} $\psi(3770)$ ($D^{0(+)}$, total)
	0.48 fb^{-1} @4.01 GeV
2012:	0.45B $\psi(3686)$ (total)
	1.30B J/ψ (total)
2013:	1.09 fb^{-1} @4.23 GeV (XYZ&D_s^+)
	0.83 fb^{-1} @4.26 GeV
	0.54 fb^{-1} @4.36 GeV
	10×0.05 fb^{-1} XYZ scan@3.81-4.42 GeV
2014:	1.03 fb^{-1} @4.42 GeV
	0.11 fb^{-1} @4.47 GeV
	0.11 fb^{-1} @4.53 GeV
	0.05 fb^{-1} @4.575 GeV
	0.57 fb^{-1} @4.60 GeV (XYZ&Λ_c^+)
	0.80 fb^{-1} R scan @3.85-4.59 GeV
2009:	106M $\psi(3686)$
2015:	R-scan 2-3 GeV+2.175 GeV

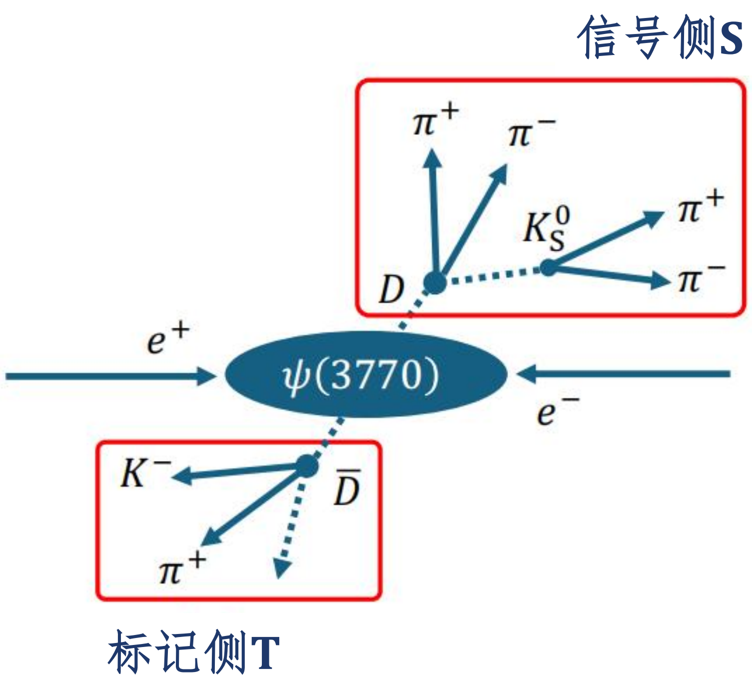
2016:	3.20 fb^{-1} @4.178 GeV (XYZ&D_s^+)
2017:	7×0.50 fb^{-1} @4.19-4.22 GeV (XYZ&D_s^+),
	@4.24-4.27 (XYZ)
2018:	More J/ψ+tuning new RF cavity
2019:	10B J/ψ (total)
	8×0.50 fb^{-1} XYZ scan@4.13, 4.16 (XYZ&D_s^+),
	4.29-4.44 GeV
2020:	3.8 fb^{-1} @ 4.61-4.7 GeV (XYZ&Λ_c^+)
2021:	2.0 fb^{-1} @ 4.74-4.946 GeV
	2.7B $\psi(3686)$ (total)
2022:	0.4 fb^{-1} @3.650 GeV
	0.4 fb^{-1} @3.682 GeV
	2.9→7.9 fb^{-1} $\psi(3770)$ ($D^{0(+)}$, total)
2023-2024:	7.9→20.3 fb^{-1} $\psi(3770)$ (for $D^{0(+)}$, total)
	2×0.42 fb^{-1} $\psi(3770)$ scan
	0.14 fb^{-1} @3.800-3.885 GeV
	0.13 fb^{-1} @3.554 GeV
	0.025 fb^{-1} @1.84-2.00 GeV

>50 fb^{-1} at E_{cm} between 1.84 and 4.95 GeV in 15 year running

Credit to 马海龙

BESIII实验测量强相位差

- $(e^+e^-)_{\psi(3770)} \rightarrow [S]_D[T]_{\bar{D}}$, C宇称负关联
  - $|\psi(3770)\rangle = 1/\sqrt{2}(|D^0\rangle|\bar{D}^0\rangle - |\bar{D}^0\rangle|D^0\rangle)$
  - $\Gamma_{D\bar{D}} \propto A_S^2 A_T^2[(r_D^S)^2 + (r_D^T)^2 - 2R_S R_T r_D^S r_D^T \cos(\delta_D^S - \delta_D^T)]$
  - 模型无关的测量
  - 忽略CP破坏，$o(x^2, y^2)$



- 双标记方法，通过不同衰变标记重建信号衰变

		标记侧	分支比	强相位差
味道本征态		$K^-e^+\nu_e$、$K^-\pi^+$、$K^-\pi^+\pi^0$、$K^-\pi^+\pi^+\pi^-$	10^{-2}	实验测量
CP本征态	正	K^+K^-、$\pi^+\pi^-$、$\pi^0\pi^0$、$K_L^0\pi^0$、$K_L^0\omega$、$K_S^0\pi^0\pi^0$、$K_S^0\pi^0\eta$、$\pi^+\pi^-\pi^0$	10^{-3}~10^{-2}	0
	负	$K_S^0\pi^0$、$K_S^0\omega$、$K_S^0\eta$、$K_S^0\eta'$、$K_L^0\pi^0\pi^0$	10^{-3}~10^{-2}	π
自共轭		$K_{S,L}^0\pi^+\pi^-(\pi^0)$、$\pi^+\pi^-\pi^+\pi^-$	10^{-3}~10^{-2}	实验测量

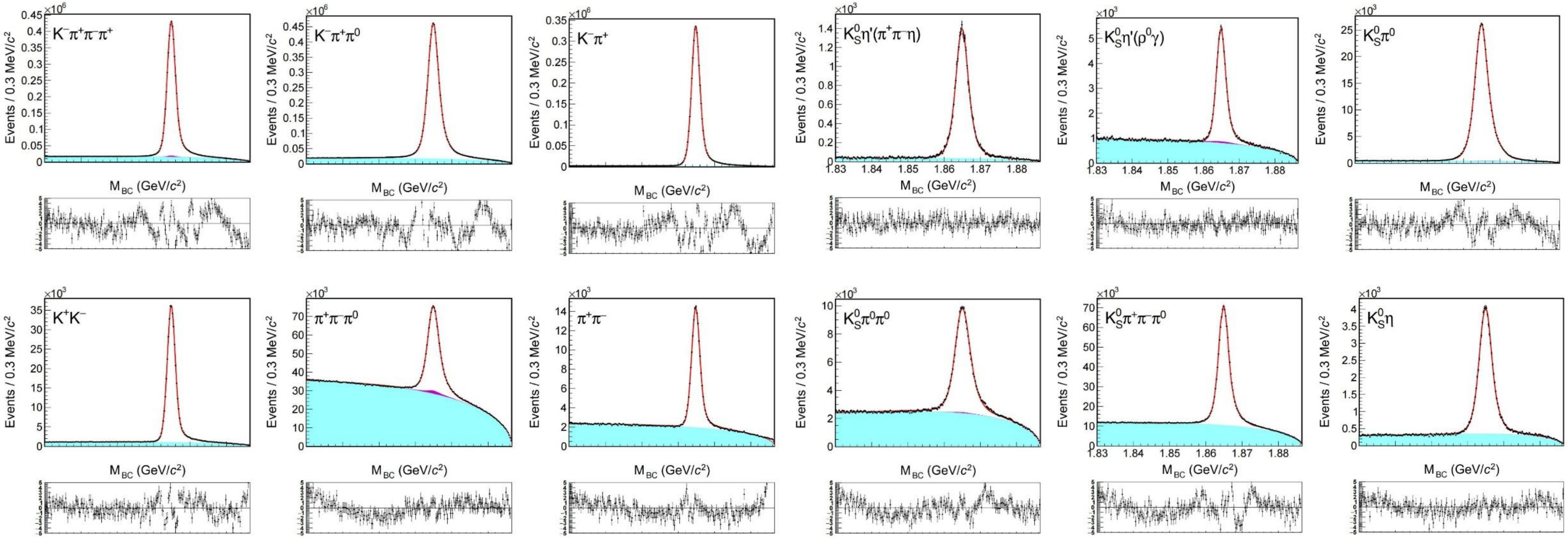
单标记

- 对双标记事例归一，消除标记侧重建的系统误差
  - $\Gamma_{D\bar{D}} \propto A_S^2 A_T^2 [(r_D^S)^2 + (r_D^T)^2 - 2R_S R_T r_D^S r_D^T \cos(\delta_D^S - \delta_D^T)]$
  - $\Gamma_D \propto A_T^2 [1 - R_T r_D^T (y\cos\delta_D^T + x\sin\delta_D^T) + (y^2 - x^2)/2]$

$K^-\pi^+$	3817700 ± 2040
$K^-\pi^+\pi^0$	7252630 ± 9050
$K^-\pi^+\pi^-\pi^+$	4847930 ± 2710
$K^-e^+\nu_e$	3030700 ± 41200
K^+K^-	387176 ± 677
$\pi^+\pi^-$	142731 ± 475
$\pi^+\pi^-\pi^0$	757190 ± 1500
$K_S^0\pi^0\pi^0$	159212 ± 616
$K_L^0\pi^0$	438500 ± 15500
$K_S^0\pi^0$	482803 ± 745
$K_S^0\eta$	65382 ± 304
$K_S^0\eta'_{\pi\pi\eta}$	24059 ± 169
$K_S^0\eta'_{\rho\gamma}$	59428 ± 310
$K_S^0\pi^+\pi^-\pi^0$	920140 ± 1760

味道标记道产额

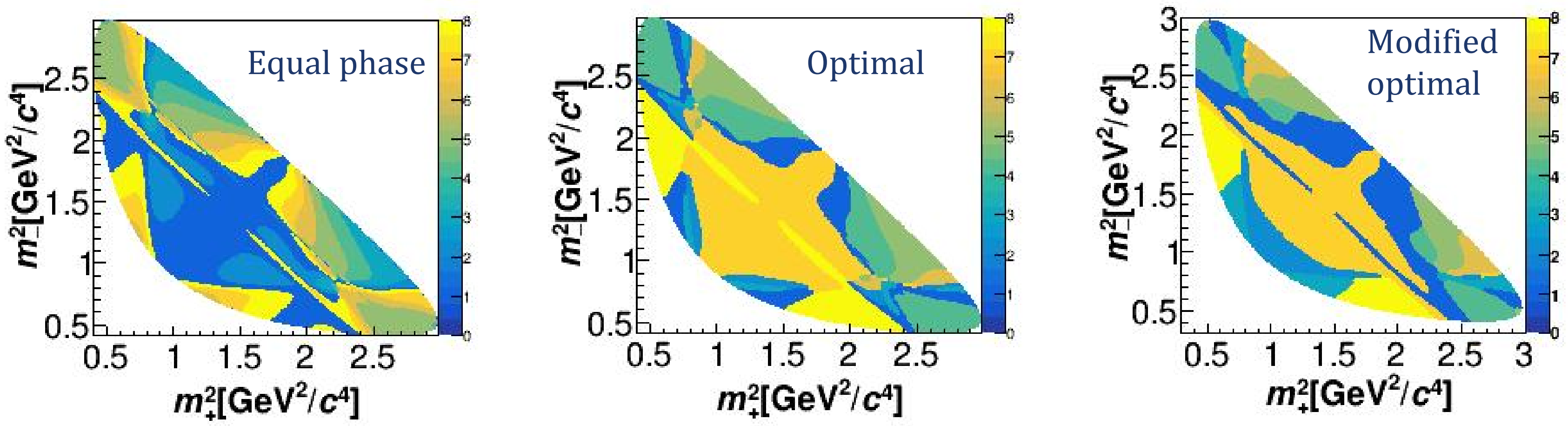
*CP*标记道产额



20.3 fb^{-1} $\psi(3770)$ @BESIII **[Phys. Rev. D 112 (2025) 012015]**

自共轭衰变

$D^0/\bar{D}^0 \to K_S^0\pi^+\pi^-$的强相位差



[Developed in Phys. Rev. D 82 (2010) 112006 by CLEO-c]

Equal phase: 粲介子混合和间接CP破坏 [Phys. Rev. Lett. 127 (2021) 11, 111801; Phys. Rev. D 111 (2025) 11, 112011]

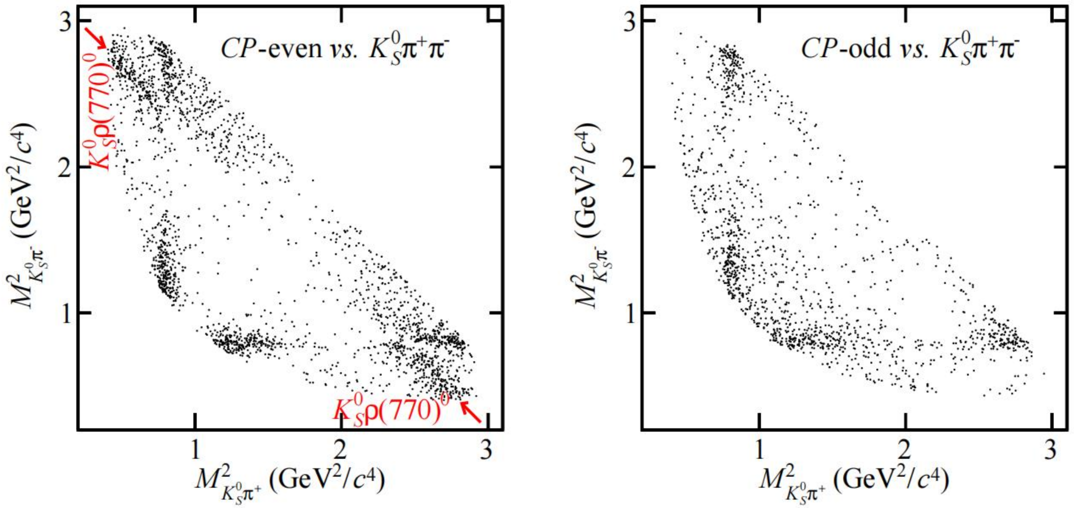
Optimal: γ角测量，达到模型依赖测量精度的85% [JHEP 02 (2021) 169; JHEP 02 (2022) 063]

Modified optimal: 根据B衰变的事例数和本底优化

测量方法

- CP标记
  - $D \to K_S^0\pi^+\pi^-$:$M_i \propto K_i + K_{-i} \mp 2c_i\sqrt{K_iK_{-i}}$
  - $D \to K_L^0\pi^+\pi^-$:$M_i' \propto K_i' + K_{-i}' \pm 2c_i'\sqrt{K_i'K_{-i}'}$
- 自共轭标记($D \to K_{S,L}^0\pi^+\pi^-$)
  - $M_{ij} \propto K_iK_{-j} + K_{-i}K_j - 2\sqrt{K_iK_{-i}K_jK_{-j}}(c_ic_j + s_is_j)$
  - $M_{ij} \propto K_iK_{-j}' + K_{-i}K_j' + 2\sqrt{K_iK_{-i}K_j'K_{-j}'}(c_ic_j' + s_is_j')$
- 味道标记
  - $D \to K_S^0\pi^+\pi^-$: $M_i \propto K_i + (r_D^F)^2K_{-i} - 2R_Dr_D^F\sqrt{K_iK_{-i}}(\cos\delta_D^F c_i - \sin\delta_D^F s_i)$
  - $D \to K_L^0\pi^+\pi^-$: $M_i' \propto K_i' + (r_D^F)^2K_{-i}' + 2R_Dr_D^F\sqrt{K_i'K_{-i}'}(\cos\delta_D^F c_i' - \sin\delta_D^F s_i')$
- CP正/负标记的事例分布有明显差异——量子关联效应
  - MC模拟中根据关联的振幅模型抽样产生量子关联效应

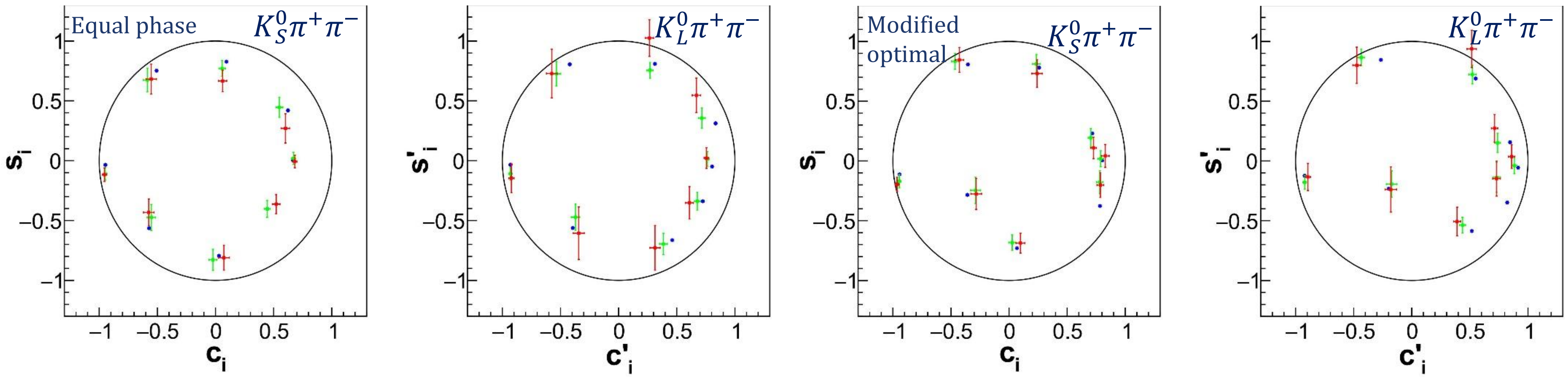
2.93 fb^{-1} $\psi(3770)$ @BESIII

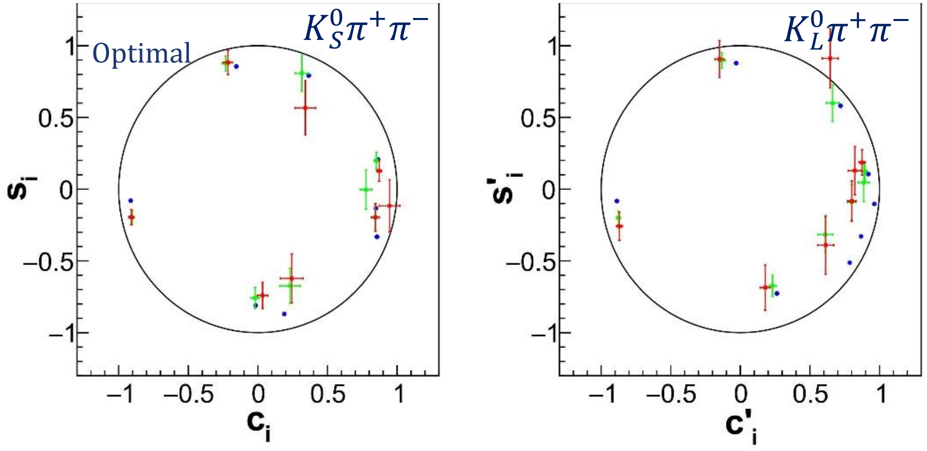


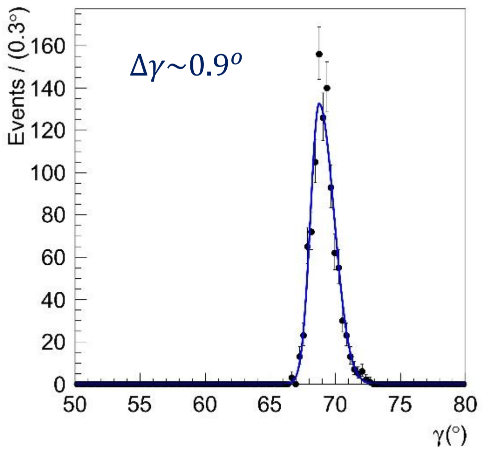
[Phys. Rev. Lett. 124 (2020) 24, 241802]
[Phys. Rev. D 101 (2020) 11, 112002]

$D^0/\bar{D}^0 \to K^0_{S,L}\pi^+\pi^-$的强相位差 [JHEP 06 (2025) 086]

7.93 fb^{-1} $\psi(3770)$ @BESIII



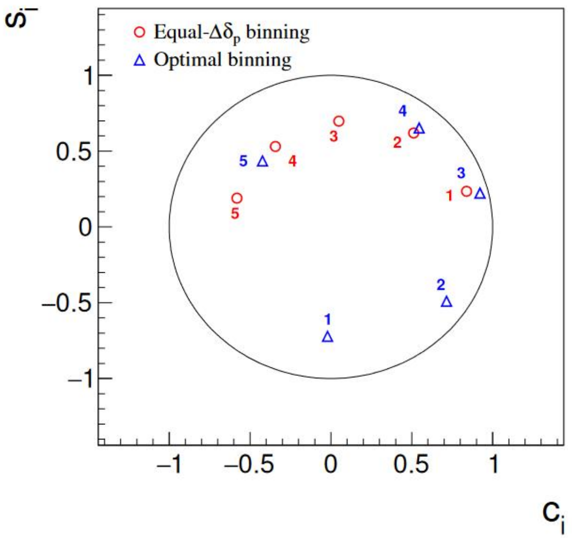


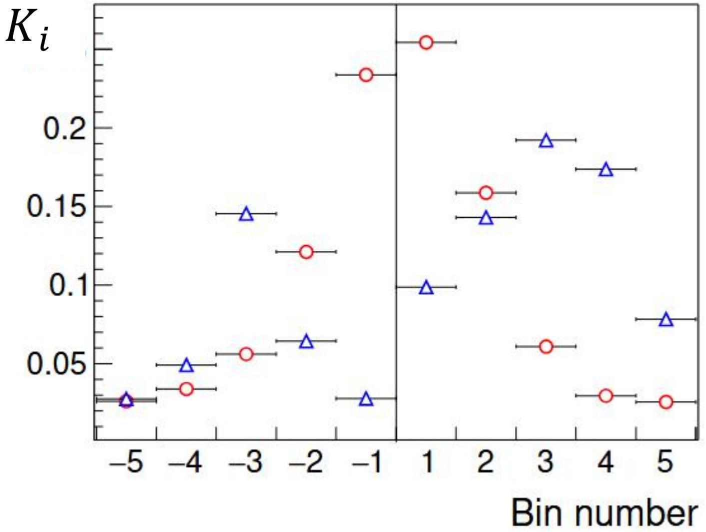


- 加入$D \to K^0_{S,L}\pi^+\pi^-$振幅相位差的约束
- 对γ系统误差贡献约为0.9度
  - 无模型约束误差约为1.5度
  - γ角中心值变化可忽略

$D \to \pi^+\pi^-\pi^+\pi^-$的强相位差

- 分bin方式
  - 5维相空间m_+, m_-,$\cos\theta_+$,$\cos\theta_-$, ϕ
  - BESIII振幅模型[Chin. Phys. C 48, 083001 (2024)]
  - 分bin方式：Equal phase & optimal
    [Phys. Rev. D 110 (2024) 11, 112008]
  - 达到模型依赖测量精度的80~85%

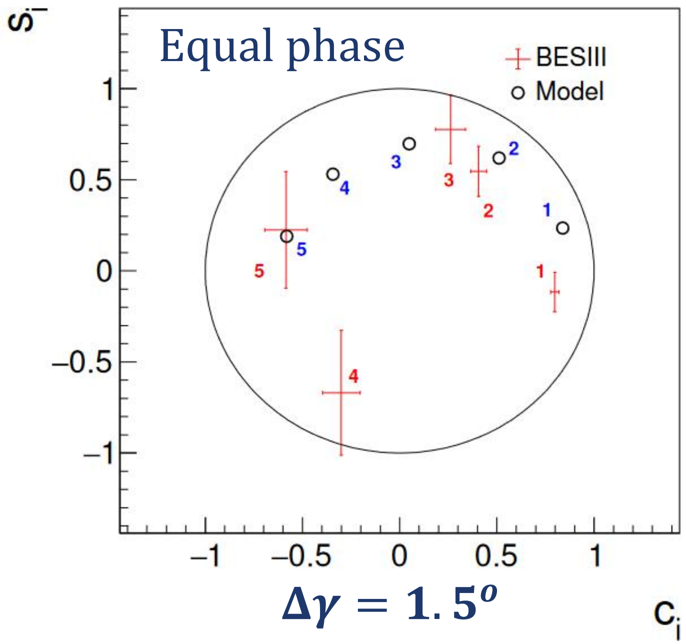




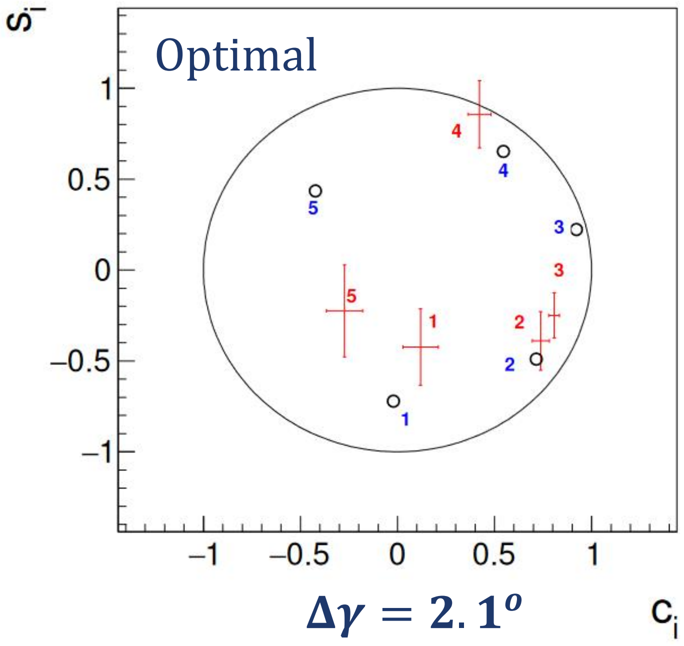
- CP标记：$M_i \propto K_i + K_{-i} \mp 2c_i\sqrt{K_iK_{-i}}$
- 自共轭标记：$M_{ij} \propto K_iK'_{-j} + K_{-i}K'_j + 2\sqrt{K_iK_{-i}K'_jK'_{-j}}(c_ic'_j + s_is'_j)$
- 味道标记：$M_i \propto K_i + (r_D^F)^2K_{-i} - 2R_Dr_D^F\sqrt{K_iK_{-i}}(\cos\delta_D^F c_i - \sin\delta_D^F s_i)$

$D \to \pi^+\pi^-\pi^+\pi^-$的强相位差 [Phys. Rev. D 110 (2024) 11, 112008]

2.93 fb^{-1} $\psi(3770)$ @BESIII



$\Delta\gamma = 1.5^o$



$\Delta\gamma = 2.1^o$

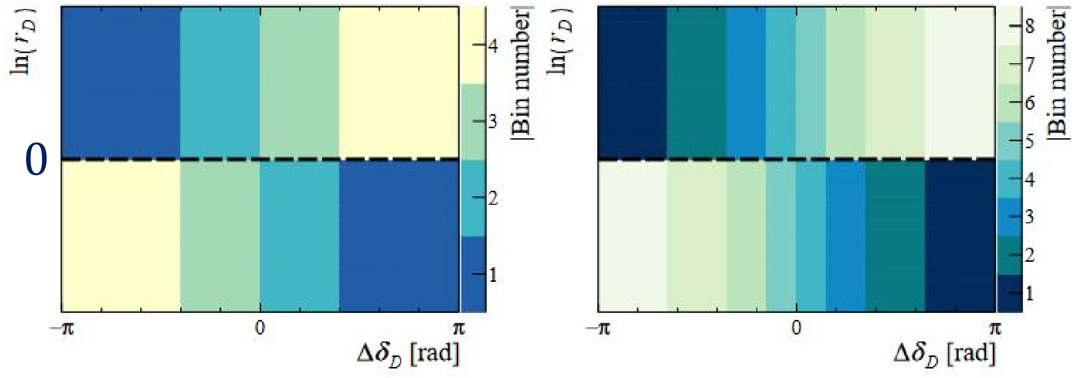
- $F_+ = 0.746 \pm 0.010 \pm 0.004$
- $F_+ = 0.735 \pm 0.015 \pm 0.005$

  [Phys. Rev. D 106 (2022) 9, 092004]

$D \to K^+K^-\pi^+\pi^-$的强相位差

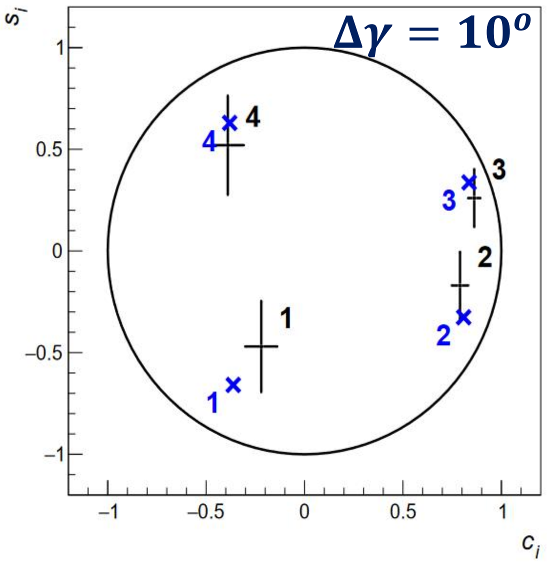
根据$D^0 \to K^+K^-\pi^+\pi^-$振幅模型得到的相位差确定分bin方式

达到模型依赖测量精度的85（90）%



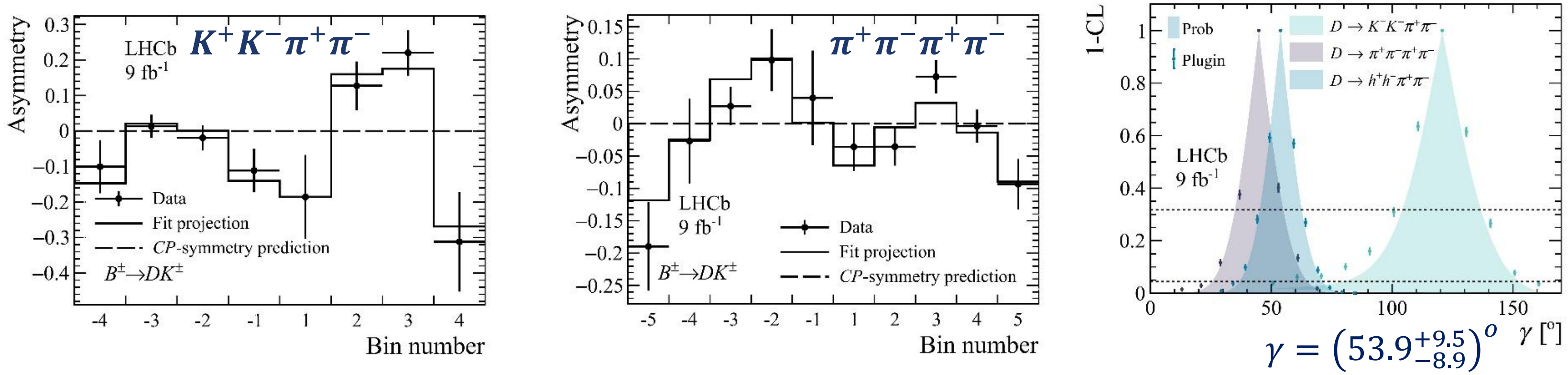
[Eur. Phys. J. C 83,1099 547 (2023), 10.5281/zenodo.6940031]

20.3 fb^{-1} $\psi(3770)$ @BESIII



- $F_+ = 0.754 \pm 0.010 \pm 0.008$
- $F_+ = 0.730 \pm 0.037 \pm 0.021$
  2.93 fb^{-1} $\psi(3770)$ @BESIII
  [Phys. Rev. D 107, 032009 (2023)]

- $D \to \pi^+\pi^-\pi^+\pi^-$、$K^+K^-\pi^+\pi^-$在LHCb最新的测量中使用[arXiv: 2509.15139]



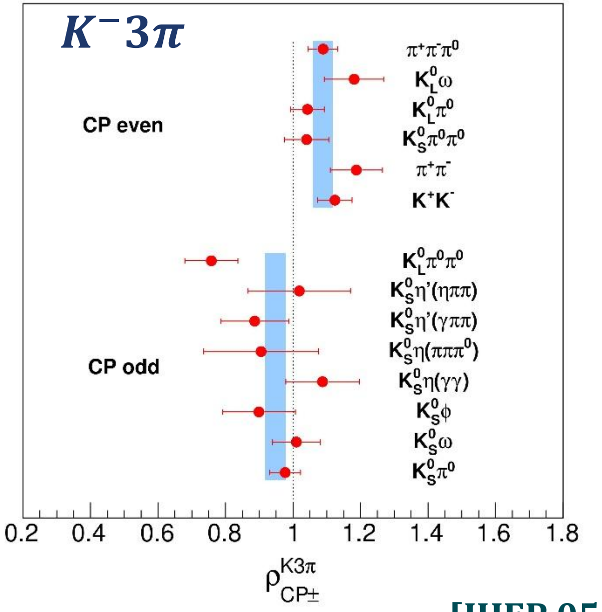
$\gamma = (53.9^{+9.5}_{-8.9})^o$

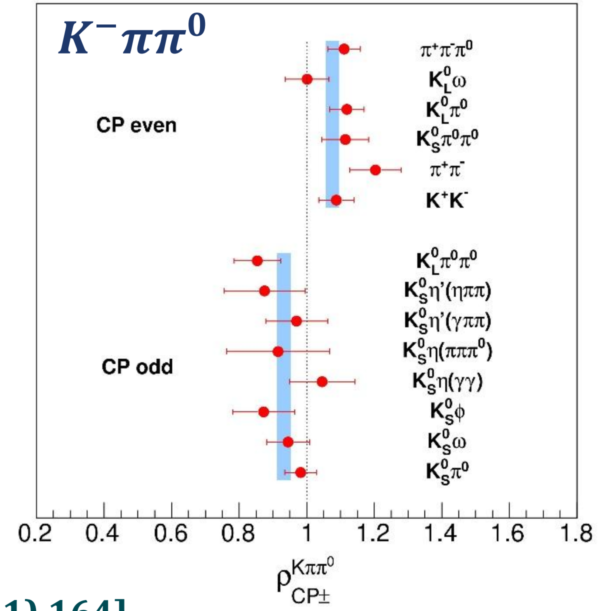
准味道本征态

$D^0/\bar{D}^0 \to K^- n\pi$的强相位差

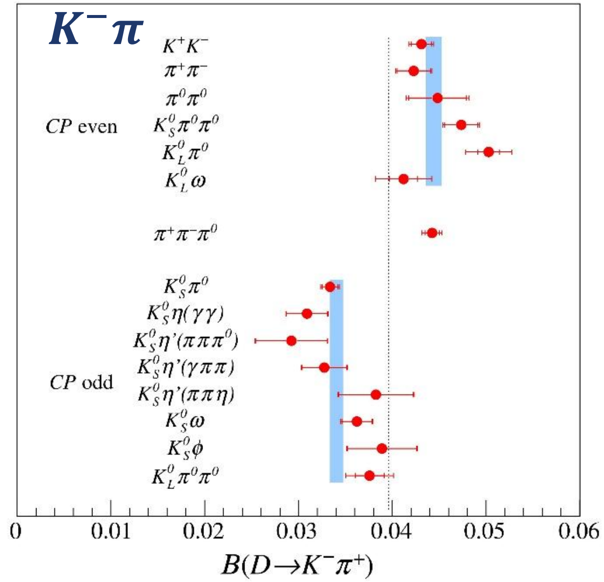
CP标记: $\Gamma \propto \left[1 + (r_D^S)^2 \mp R_F r_D^F \cos(\delta_D^F)\right]$

2.93 fb^{-1} $\psi(3770)$ @BESIII





[JHEP 05 (2021) 164]



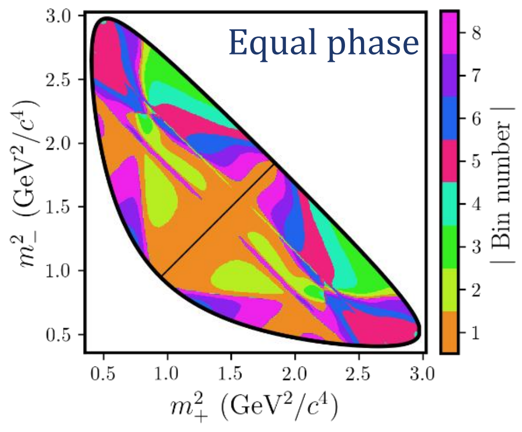
[Eur. Phys. J. C 82 (2022) 1009]

$D^0/\bar{D}^0 \to K^-n\pi$的强相位差

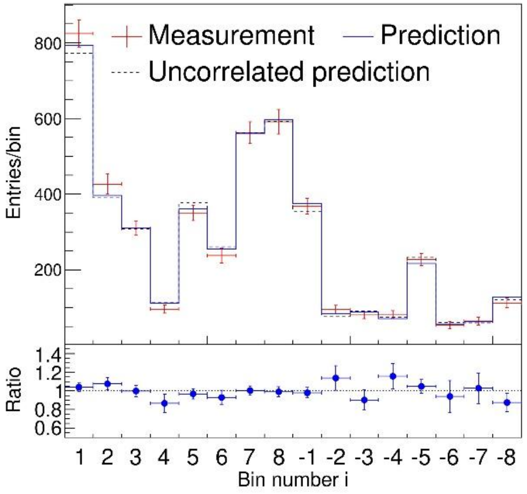
自共轭标记

$$Y_i^{K_S^0\pi\pi} = H_F\left(K_i + \left(r_D^S\right)^2 K_{-i} - 2R_F r_D^F \sqrt{K_i K_{-i}}\left[c_i \cos\delta_D^F - s_i \sin\delta_D^F\right]\right)$$

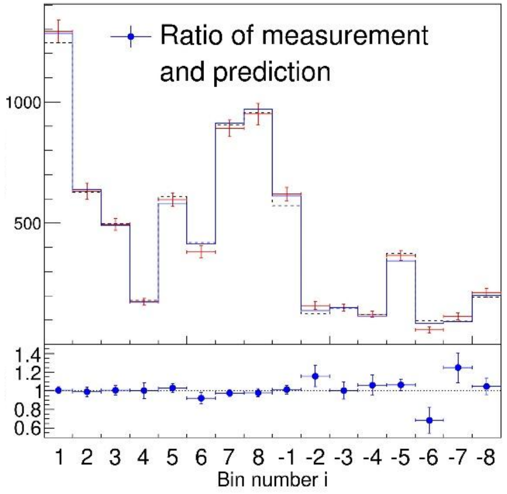
$$Y_i^{K_L^0\pi\pi} = H_F'\left(K_i' + \left(r_D^S\right)^2 K_{-i}' + 2R_F r_D^F \sqrt{K_i' K_{-i}'}\left[c_i' \cos\delta_D^F - s_i' \sin\delta_D^F\right]\right)$$



$K^-3\pi$ vs $K_S^0\pi\pi$

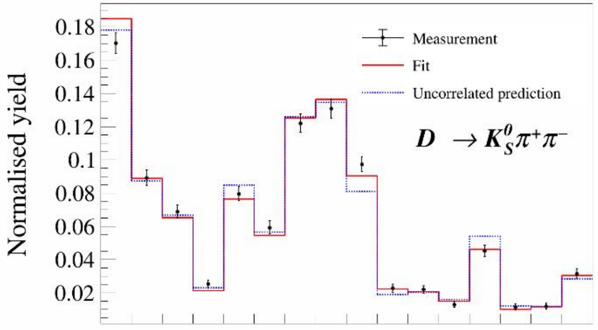


$K^-\pi\pi^0$ vs $K_S^0\pi\pi$

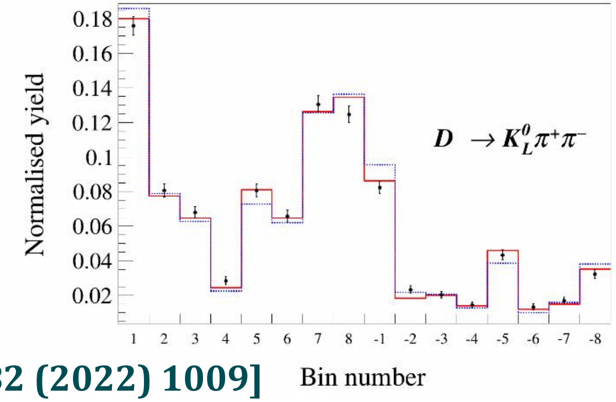


[JHEP 05 (2021) 164]

$K^-\pi$ vs $K_S^0\pi\pi$



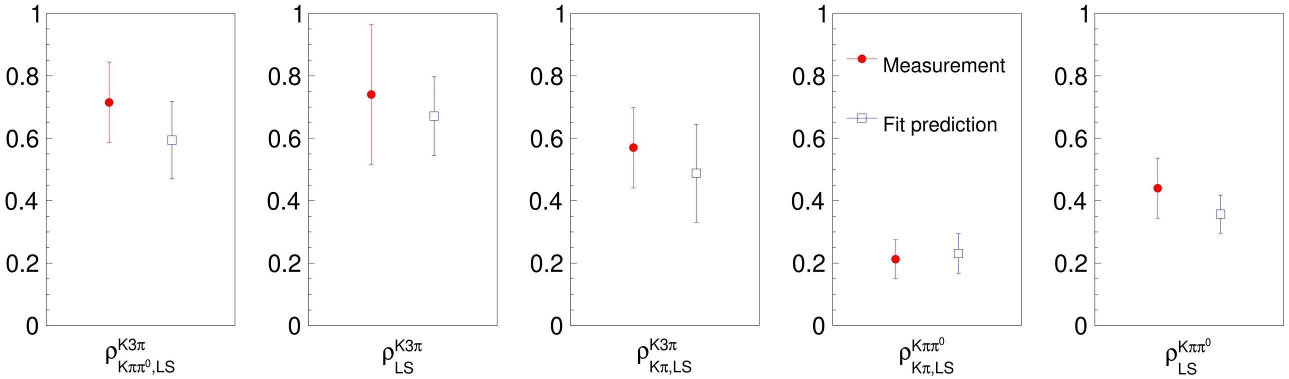
$K^-\pi$ vs $K_L^0\pi\pi$

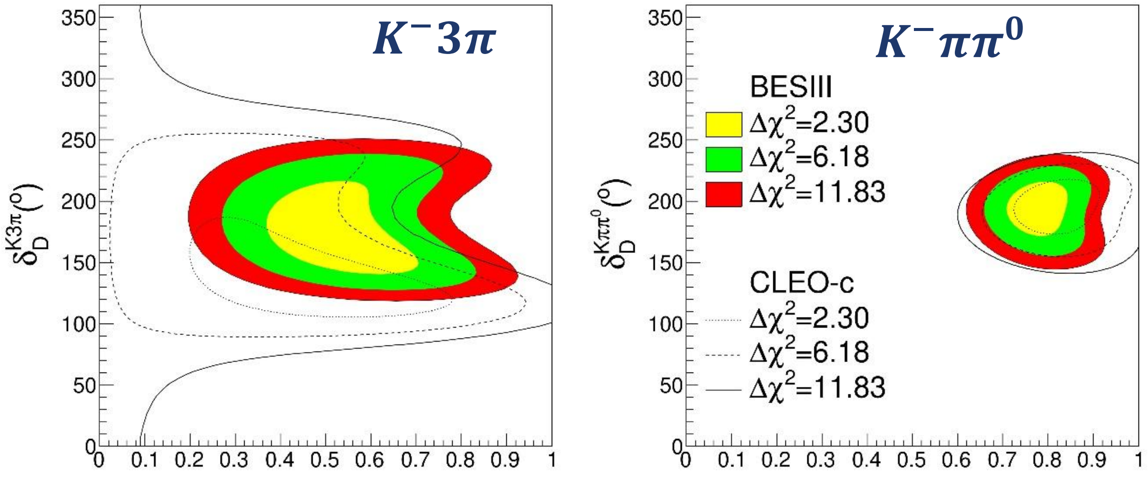


[Eur. Phys. J. C 82 (2022) 1009]

$D^0/\bar{D}^0 \to K^- n\pi$的强相位差

同号味道标记：$\Gamma = \Gamma_0 |A_S|^2 |A_T|^2 \left[\left(r_D^S\right)^2 + \left(r_D^T\right)^2 - 2R_S R_T r_D^S r_D^T \cos\left(\delta_D^T - \delta_D^S\right) \right]$





[JHEP 05 (2021) 164]

2.93 fb^{-1} $\psi(3770)$ @BESIII

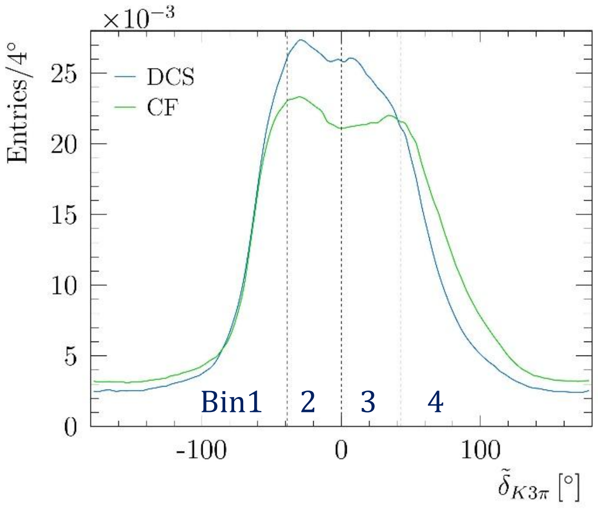
Decay	δ_D^f	R_f
$K^-3\pi$	167^{+31}_{-19}	$0.52^{+0.12}_{-0.10}$
$K^-\pi\pi^0$	196^{+14}_{-15}	0.78 ± 0.04
$K^-\pi$	$187.6^{+8.9+5.4}_{-9.7-6.4}$	–

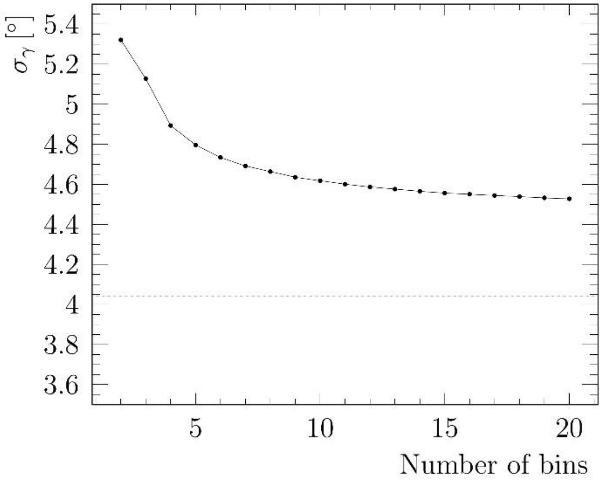
$D^0/\bar{D}^0 \to K^- n\pi$的强相位差

对γ系统误差贡献约为6.0^o

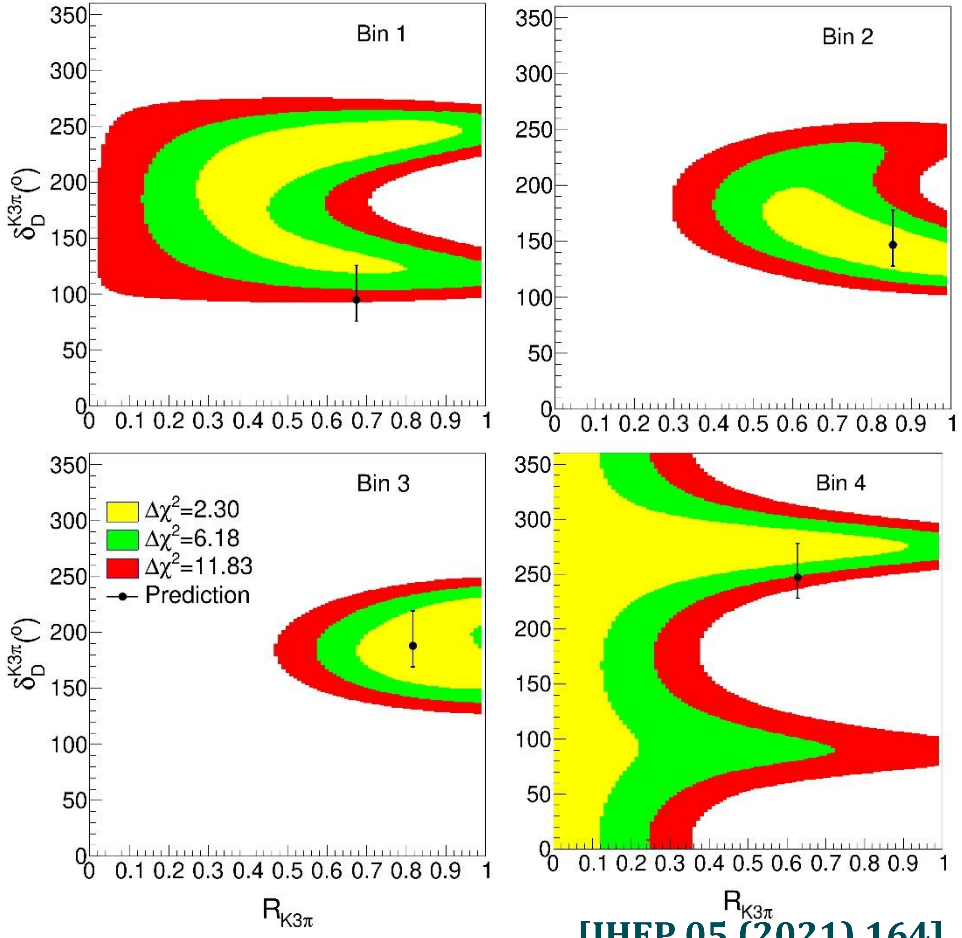
根据$D^0/\bar{D}^0 \to K^+\pi^-\pi^+\pi^-$振幅模型得到的相位差确定分bin方式

[Eur. Phys. J. C 78 (2018) 443 (模型), Phys. Lett. B 802 (2020) 135188 (分bin) by CLEO-c]





2.93 fb^{-1} $\psi(3770)$ @BESIII



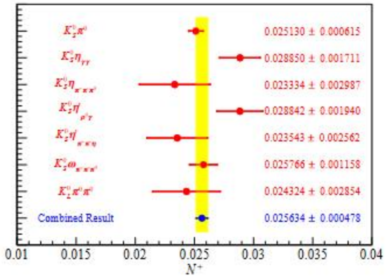
[JHEP 05 (2021) 164]

准$\boldsymbol{CP}$本征态

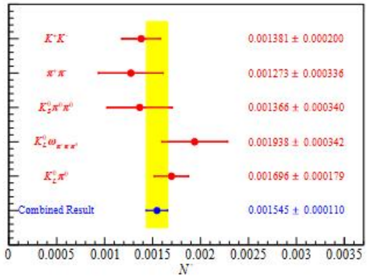
$D^0 \to \pi^+\pi^-\pi^0$, $K^+K^-\pi^0$中的偶CP比分F_+

[Phys. Rev. D 111 (2025) 1, 012007]

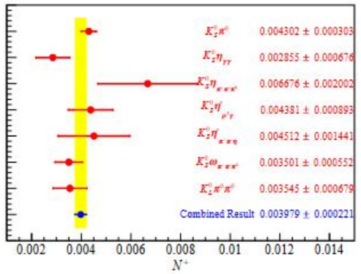
- CP标记
  - $\Gamma = A_S^2 A_T^2 [1 - (2F_+ - 1)\cos\delta_D^{CP\pm}]$



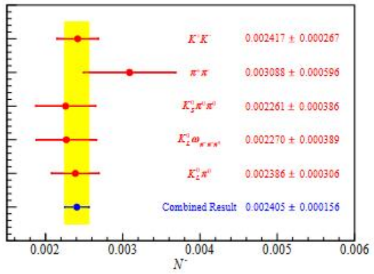
(a) N^+ for $\pi^+\pi^-\pi^0$



(b) N^- for $\pi^+\pi^-\pi^0$



(c) N^+ for $K^+K^-\pi^0$



(d) N^- for $K^+K^-\pi^0$

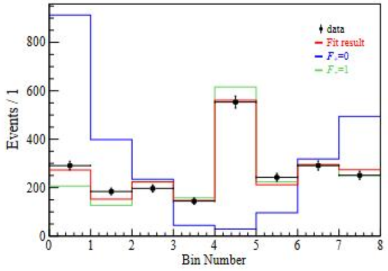
7.93 fb^{-1} $\psi(3770)$ @BESIII

Tag mode	$F_+^{\pi^+\pi^-\pi^0}$	$F_+^{K^+K^-\pi^0}$
Pure CP	$0.9432 \pm 0.0040 \pm 0.0022$	$0.623 \pm 0.020 \pm 0.008$
$K^+K^-\pi^0$	$1.0060 \pm 0.0675 \pm 0.0379$	$0.649 \pm 0.125 \pm 0.025$
$\pi^+\pi^-\pi^0$	$0.9472 \pm 0.0139 \pm 0.0081$	$0.631 \pm 0.030 \pm 0.020$
$2(\pi^+\pi^-)$	$0.9948 \pm 0.0230 \pm 0.0267$	$0.667 \pm 0.058 \pm 0.033$
$K_{S,L}^0\pi^+\pi^-$	$0.9065 \pm 0.0116 \pm 0.0073$	$0.652 \pm 0.033 \pm 0.023$
Combined	$0.9406 \pm 0.0036 \pm 0.0021$	$0.631 \pm 0.014 \pm 0.011$
PDG Scale	$0.9406 \pm 0.0055 \pm 0.0033$	—

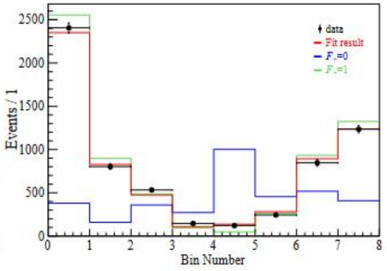
- 自共轭标记
  - $\Gamma_i = A_S^2 A_T^2 [K_i + K_{-i} - 2\sqrt{K_i K_{-i}} c_i (2F_+ - 1)]$
  - $\Gamma_i' = A_S^2 A_T^2 [K_i' + K_{-i}' + 2\sqrt{K_i' K_{-i}'} c_i' (2F_+ - 1)]$

精度比CLEO-c提高约3倍

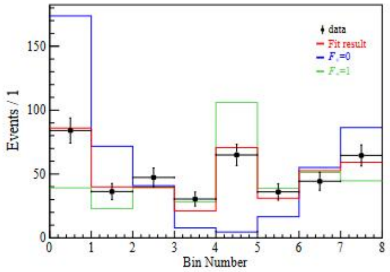
[Phys. Lett. B 747 (2015) 9-17]



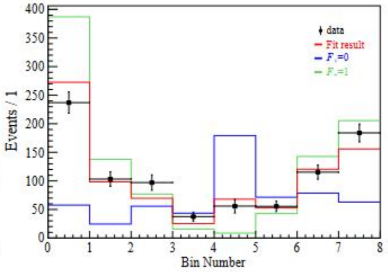
(a) $K_S^0\pi^+\pi^-$ versus $\pi^+\pi^-\pi^0$



(b) $K_L^0\pi^+\pi^-$ versus $\pi^+\pi^-\pi^0$

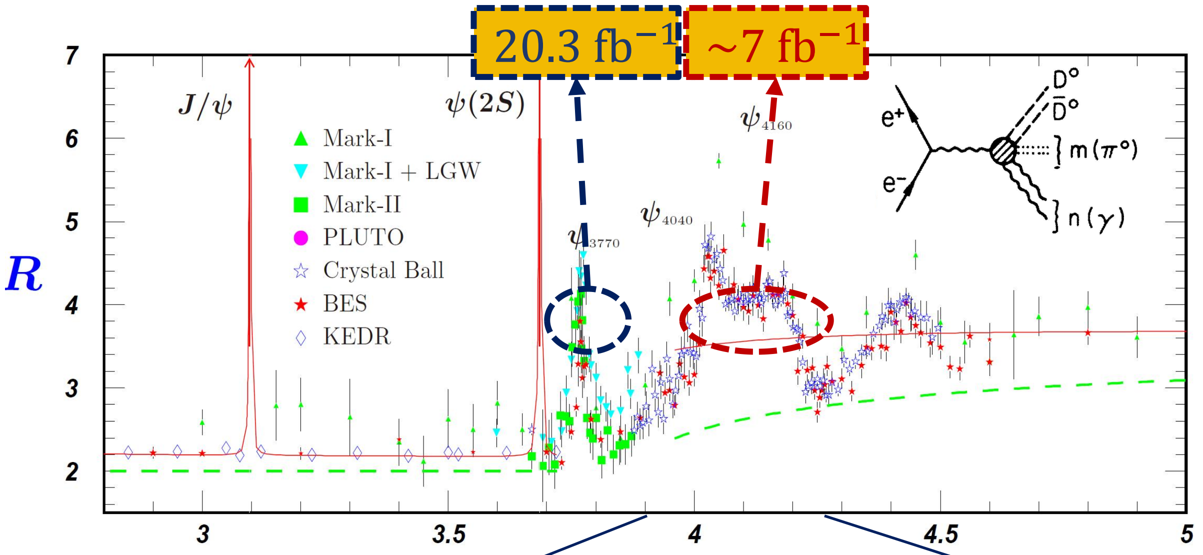


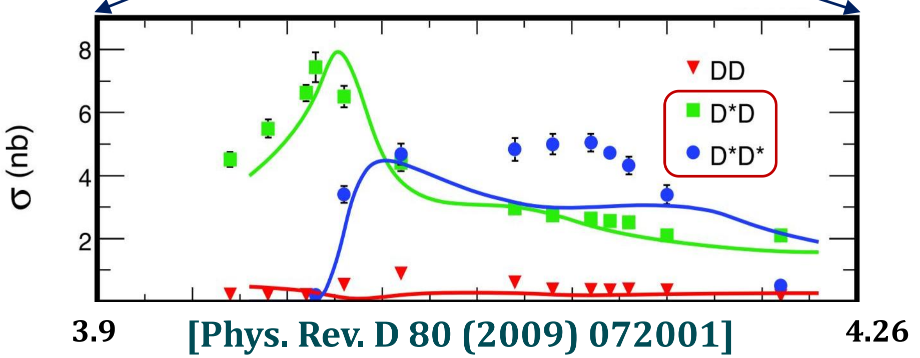
(c) $K_S^0\pi^+\pi^-$ versus $K^+K^-\pi^0$



(d) $K_L^0\pi^+\pi^-$ versus $K^+K^-\pi^0$

高能量点数据测量强相位差





[Phys. Rev. D 80 (2009) 072001]

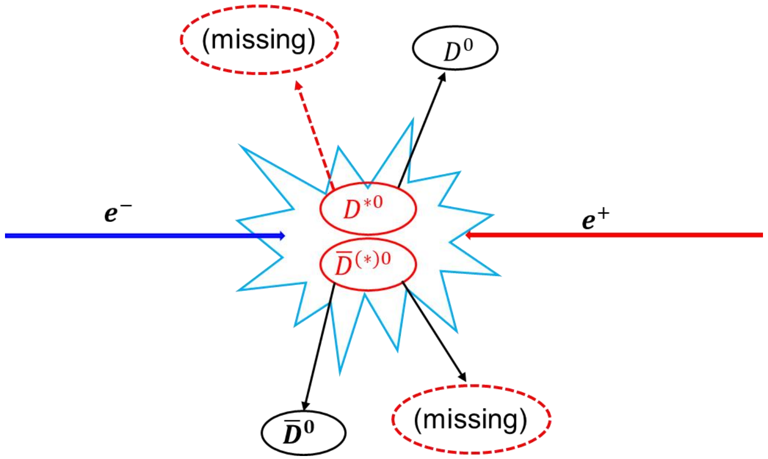
产生	末态	$C(D\bar{D})$
D^*D	$\gamma D\bar{D}$	Even
	$\pi^0 D\bar{D}$	Odd
$D^*\bar{D}^*$	$\gamma\gamma D\bar{D}$	Odd
	$\gamma\pi^0 D\bar{D}$	Even
	$\pi^0\pi^0 D\bar{D}$	Odd

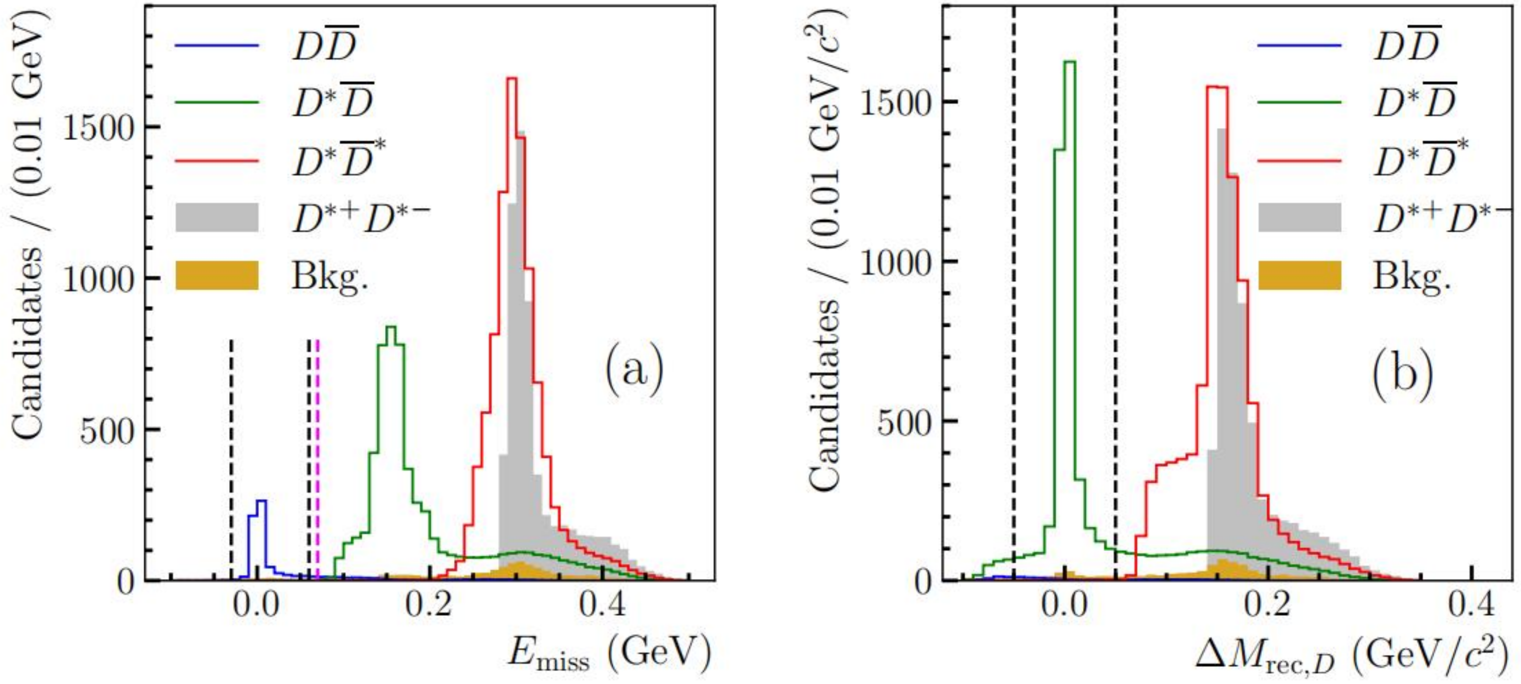
[Phys. Rev. D 15 (1977) 1254]

$$\frac{\Gamma_C(S;T)}{|A_S|^2|A_T|^2} \propto (2+C)(x^2+y^2)\left[\left|\frac{p}{q}\right|^2 + (r_D^S)^2(r_D^T)^2\left|\frac{q}{p}\right|^2 + 2CR_SR_Tr_D^Sr_D^T\cos(\delta_D^S+\delta_D^T-2\phi)\right]$$
$$+\left[2-(2+C)(x^2-y^2)\right]\left[(r_D^S)^2+(r_D^T)^2+2CR_SR_Tr_D^Sr_D^T\cos(\delta_D^S-\delta_D^T)\right]$$
$$-2(1+C)y\left[R_Tr_D^T\left(\left|\frac{p}{q}\right|+\left|\frac{q}{p}\right|(r_D^S)^2\right)\cos(\delta_D^T-\phi)+CR_Sr_D^S\left(\left|\frac{p}{q}\right|+\left|\frac{q}{p}\right|(r_D^T)^2\right)\cos(\delta_D^S-\phi)\right]$$
$$+2(1+C)x\left[R_Tr_D^T\left(\left|\frac{p}{q}\right|-\left|\frac{q}{p}\right|(r_D^S)^2\right)\sin(\delta_D^T-\phi)+CR_Sr_D^S\left(\left|\frac{p}{q}\right|-\left|\frac{q}{p}\right|(r_D^T)^2\right)\sin(\delta_D^S-\phi)\right].$$

分离不同类型的$D\bar{D}$产生过程

[Phys. Rev. Lett. 135, 171901 (2025)]
[Phys. Rev. D 112, 072006 (2025)]



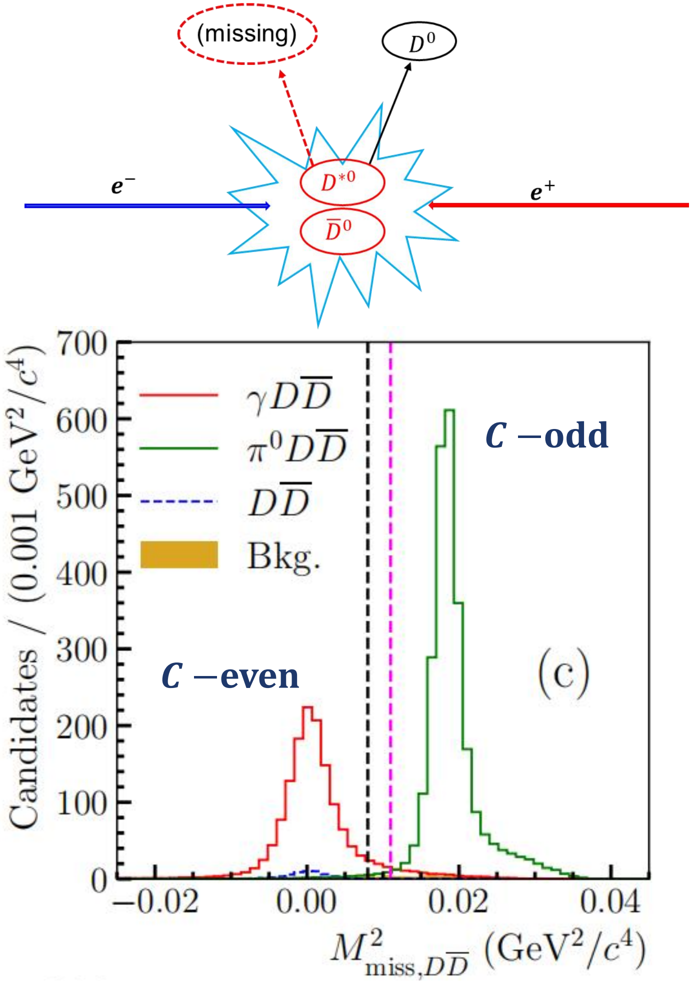


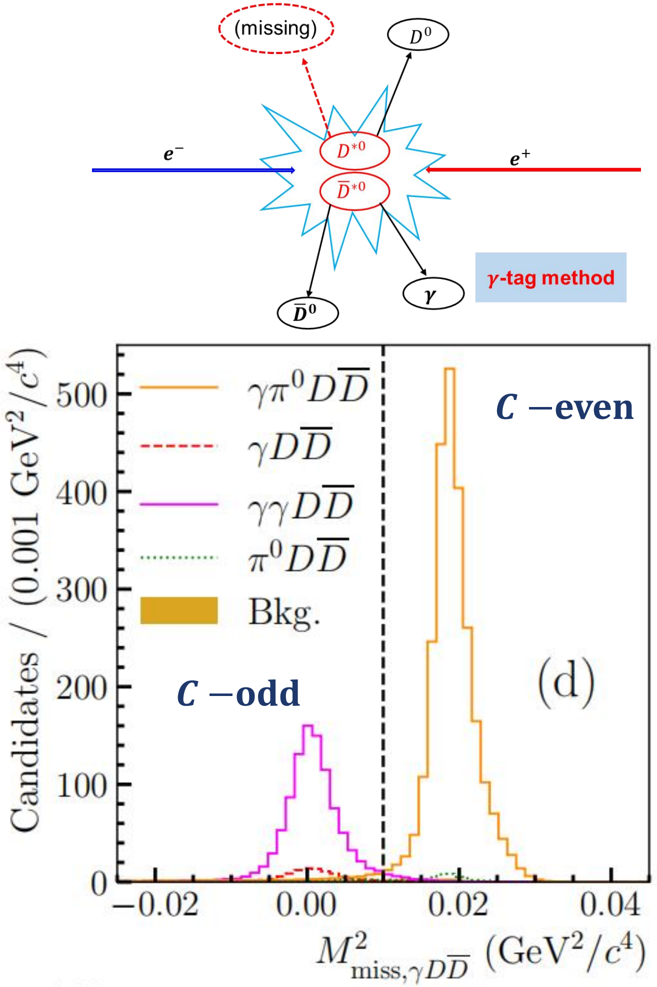
同时重建两个D介子衰变

区分不同产生过程$D\bar{D}$, $D^*\bar{D}$ and $D^*\bar{D}^*$

- 伴随γ/π^0粒子的“丢失能量”→区分$D\bar{D}$和$D^*\bar{D}^{(*)}$
- 两个D介子更接近D^*质量的“反冲”质量→区分$D^*\bar{D}$和$D^*\bar{D}^*$

分离不同类型的量子关联数据





$D^*\bar{D} \to \gamma/\pi^0 D\bar{D}$

- 通过两个D介子的“丢失质量平方”区分

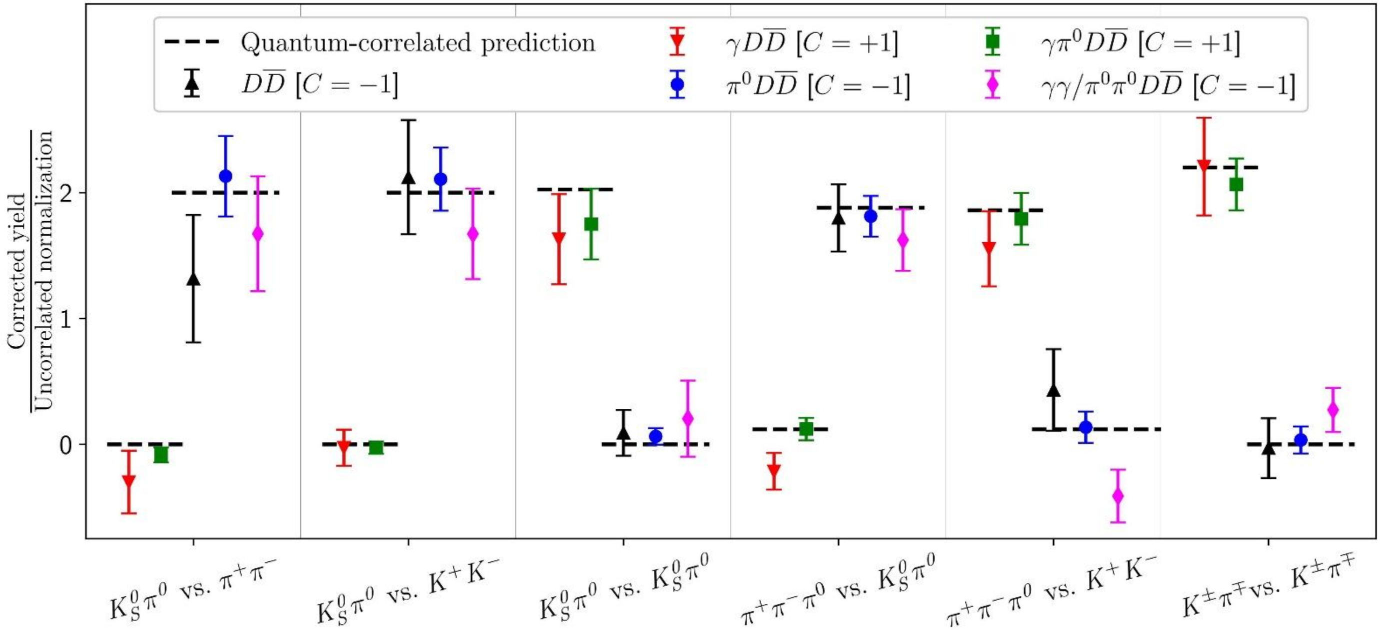
$D^*\bar{D}^* \to \gamma\gamma/\gamma\pi^0 D\bar{D}$

- 重建其中一个D^*的伴随光子
- 通过两个D介子和光子的“丢失质量平方”区分

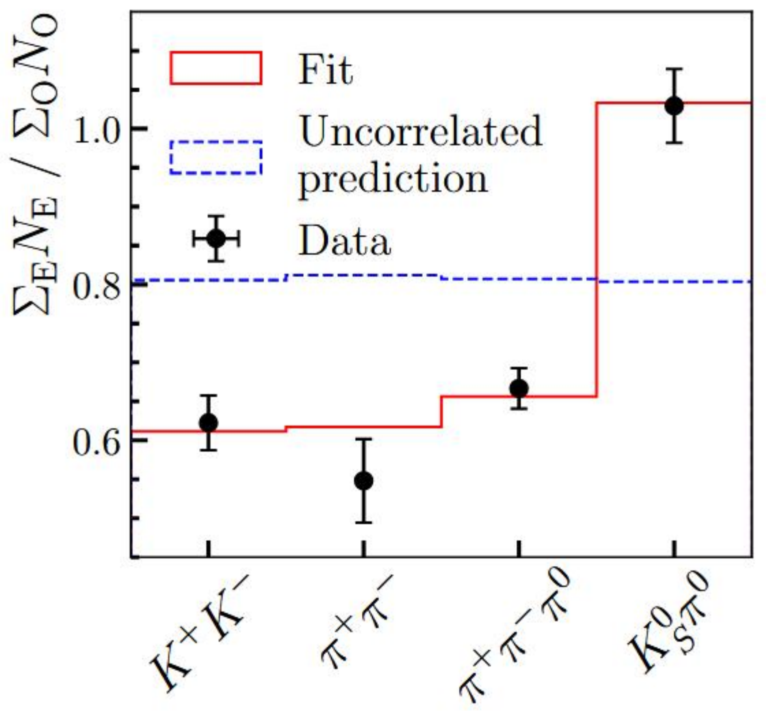
不同量子关联效应的观测

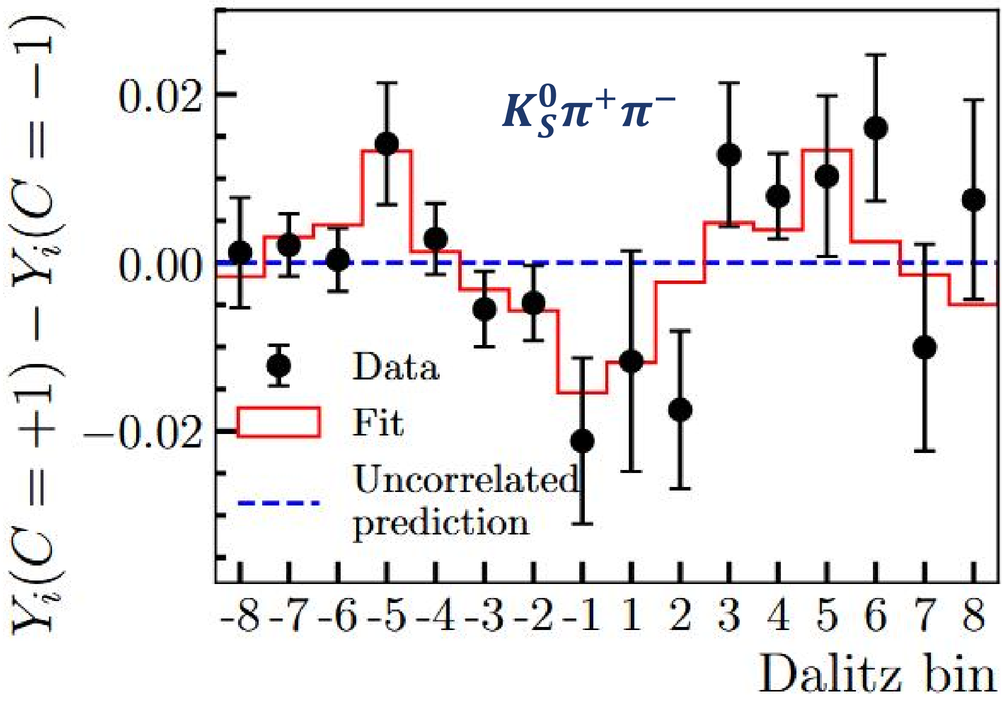
- C −odd关联的$D\bar{D}$
  - 两个D介子衰变末态具有相同CP本征值时禁戒
  - 两个D介子衰变末态具有相同CP本征值时允许
- C −even关联的$D\bar{D}$
  - 两个D介子衰变末态具有相同CP本征值时允许
  - 两个D介子衰变末态具有相同CP本征值时禁戒
- 上述重建方法可以有效区分不同量子关联效应的$D\bar{D}$

实验上首次测量C −even 量子关联效应



$\delta_D^{K\pi}$的测量





- 比较C −even和C −odd关联的双标记$D \rightarrow K\pi$产额联合得到$\delta_D^{K\pi} = (192.8^{+11.0+1.9}_{-12.4-2.4})^o$
- 与探测效率有关的系统误差可显著减小
- 通过2.93 fb^{-1} $\psi(3370)$数据得到$\delta_D^{K\pi} = (187.6^{+8.9+5.4}_{-9.7-6.4})^o$

可对强相位差提供新的约束
可用于进行粲介子混合和间接CP破坏研究

未来展望

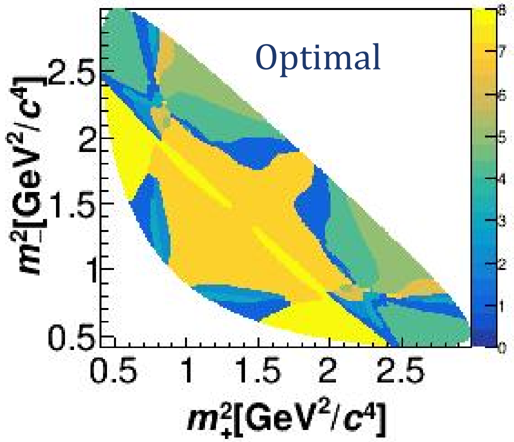
$\psi(3770)$与高能量点数据的测量

$D \to$	2.93 fb^{-1}	7.93/20.3 fb^{-1}	$D^*\bar{D}^{(*)}$	Parameter
$K^0_{S,L}\pi^+\pi^-$	✔	✔(7.93) Ongoing(20.3)	–	c_i, s_i
$K^0_{S,L}K^+K^-$	✔	Ongoing(20.3)	–	c_i, s_i
$K^-\pi^+\pi^-\pi^+$、$K^-\pi^+\pi^0$	✔	Ongoing(7.93/20.3)	Ongoing (x, y)	δ_D^f, R_f
$K^0_{S,L}\pi^+\pi^-\pi^0$	✔	Ongoing(20.3)	–	c_i, $s_i + F_+$
$\pi^+\pi^-\pi^+\pi^-$	✔(c_i, $s_i +$ F_+)	Ongoing(20.3)	–	c_i, $s_i + F_+$
$K^+K^-\pi^+\pi^-$	✔(F_+)	✔(20.3, c_i, s_i)	–	c_i, $s_i + F_+$
$\pi^+\pi^-\pi^0$、$K^+K^-\pi^0$	–	✔(7.93, F_+) Ongoing(20.3)	–	c_i, $s_i + F_+$
$K^0_S K\pi$	–	Ongoing(20.3)	–	δ_D^f, R_f
$K^-\pi^+$	✔	Ongoing(20.3)	✔	δ_D^f
$\eta\pi^+\pi^-$	–	Ongoing(20.3)	–	F_+

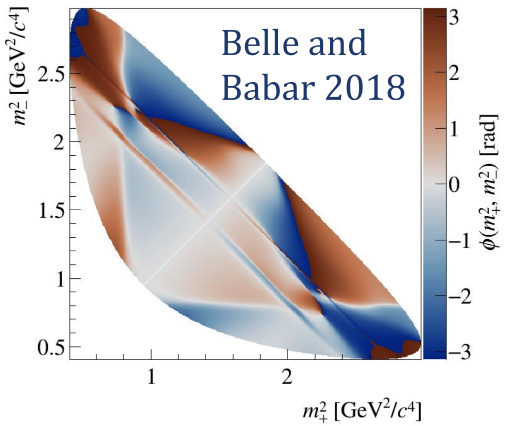
$D \to K_S^0\pi^+\pi^-$不分bin的强相位差

- 傅里叶展开方法：[Eur. Phys. J. C 78 (2018) 2, 121]
  - 对振幅模型的相位差进行傅里叶展开，采用所有Dalitz plot点上的相位信息
  - 提高γ角的统计精度
- 分bin测量方法：
  - 仅采用每个Dalitz plot区间内的平均相位（85%）

分bin方式



振幅模型的相位差



[Phys. Rev. D98 (2018) 112012]

$$p_{D^0} = \frac{a_0^D}{2} + \Sigma_{n=1}^M (a_n^D \cos(n\delta_D) + b_n^D \sin(n\delta_D))$$

$$C(z) = \mathrm{Re}[A_D^* \overline{A}_D] = \frac{a_0^C}{2} + \Sigma_{n=1}^M (a_n^C \cos(n\delta_D) + b_n^C \sin(n\delta_D))$$

$$S(z) = \mathrm{Im}[A_D^* \overline{A}_D] = \frac{a_0^S}{2} + \Sigma_{n=1}^M (a_n^S \sin(n\delta_D) + b_n^S \sin(n\delta_D))$$

$D \to K_S^0\pi^+\pi^-$不分bin的强相位差

对衰变率p_B, p_D和强相差$\mathcal{C}$, $\mathcal{S}$在Dalitz plot上对数据点求和得到傅里叶系数

- B介子衰变

$$N_n^+ = h_{B^+}[P_n^- + ((x_+)^2 + (y_+)^2)P_n^+ + 2(-)^n(x_+C_n - y_+S_n)]$$
$$\overline{N}_n^- = h_{B^-}[P_n^- + ((x_-)^2 + (y_-)^2)P_n^+ + 2(-)^n(x_-C_n - y_-S_n)]$$

- 量子关联的$D\overline{D}$衰变
  - 味道标记

$$N_n^f = h_{flav}[P_n^+ + \left(r_D^f\right)^2 P_n^- - 2R_f r_D^f(-)^n(C_n \cos\delta_D^f - S_n \sin\delta_D^f)]$$

  - CP标记

$$N_n^{CP} = h_{CP\pm}[P_n^+ + P_n^- - (2F_+ - 1)(-)^n C_n]$$

  - 自共轭标记

$$N_{n1n2} = h_{DD}[P_{n1}^+ P_{n2}^+ + P_{n1}^- P_{n2}^- - 2(-)^{n1+n2}(C_{n1}C_{n2} + S_{n1}S_{n2})]$$

$$x_\pm = r_B\cos(\delta_B \pm \gamma),\ y_\pm = r_B\sin(\delta_B \pm \gamma)$$

开展基于LHCb和BESIII数据的联合测量

$D \to K_S^0\pi^+\pi^-$不分bin的强相位差 [JHEP 09 (2023) 007]

- 通过两个一维勒让德多项式修正振幅模型的相位
  - 提高γ角的统计精度
  - 固定振幅的大小，减小强相位差对γ角贡献的系统误差

$$\delta_D^{corr} = \sum_{i}^{i\le N}\sum_{j}^{j\le\frac{N-i}{2}} C_i C_{2j+1} p_i(z'_+) p_{2j+1}(z''_-),$$

$$\delta_D^{obs} = \delta_D^{model} + \delta_D^{corr}$$

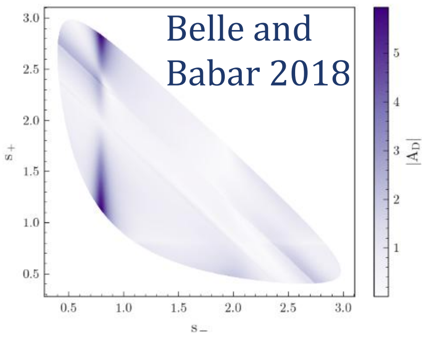
- 量子关联的$D\bar{D}$衰变进行振幅拟合
  - CP标记

$$\Gamma_{D\bar{D}} \propto \left|A_D^f(s_-,s_+)\right|^2 + \left|\bar{A}_D^f(s_-,s_+)\right|^2 \pm 2(2F_+ - 1)\left|A_D^f(s_-,s_+)\right|\left|\bar{A}_D^f(s_-,s_+)\right|\cos\delta_D^{obs}$$

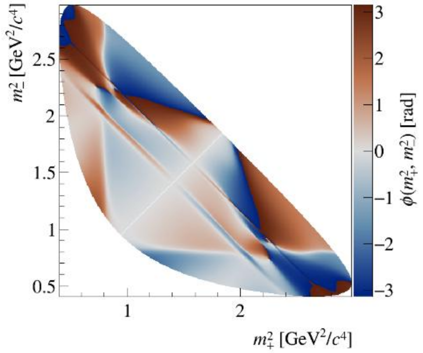
  - 自共轭标记

$$\Gamma_{D\bar{D}} \propto \left|A_D^f(s_-,s_+)\right|^2\left|\bar{A}_D^f(s'_-,s'_+)\right|^2 + \left|A_D^f(s'_-,s'_+)\right|^2\left|\bar{A}_D^f(s_-,s_+)\right|^2 - 2\left|A_D^f(s_-,s_+)\right|\left|\bar{A}_D^f(s'_-,s'_+)\right|\left|\bar{A}_D^f(s_-,s_+)\right|\left|A_D^f(s'_-,s'_+)\right|\cos\left(\cos\delta_D^{obs} - \cos\delta_D'^{obs}\right)$$

振幅模型的大小



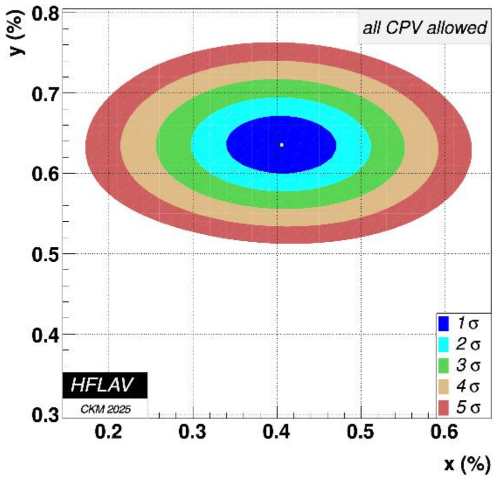
振幅模型的相位差

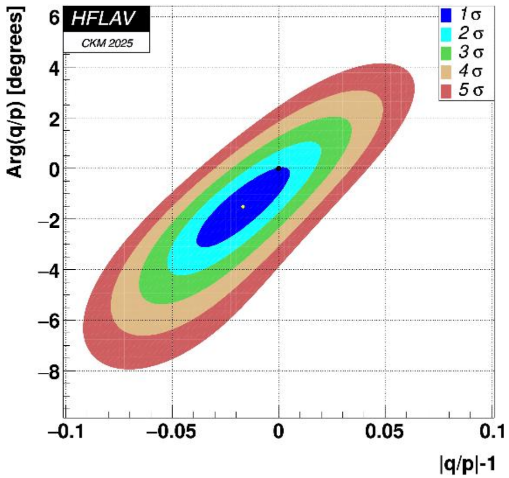


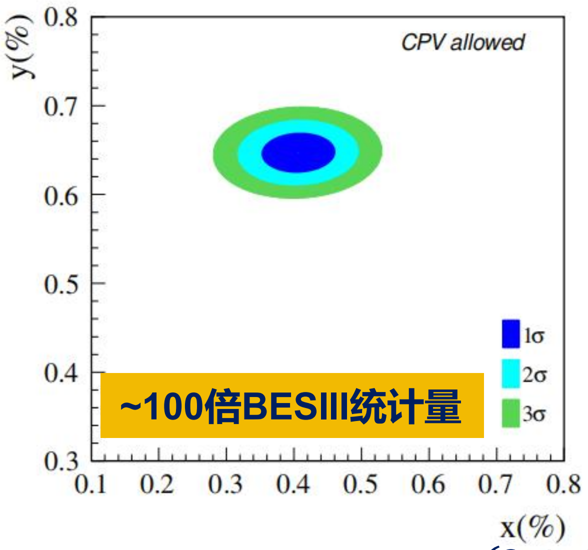
CP observable	Input value	Mean	Width
γ [°]	68.7	68.37	4.65
r_B^{DK} [$\times10^{-2}$]	9.04	9.122	0.663
δ_B^{DK} [°]	118.3	118.19	4.83
$r_B^{D\pi}$ [$\times10^{-2}$]	0.5	0.545	0.155
$\delta_B^{D\pi}$ [°]	291	290.6	20.9

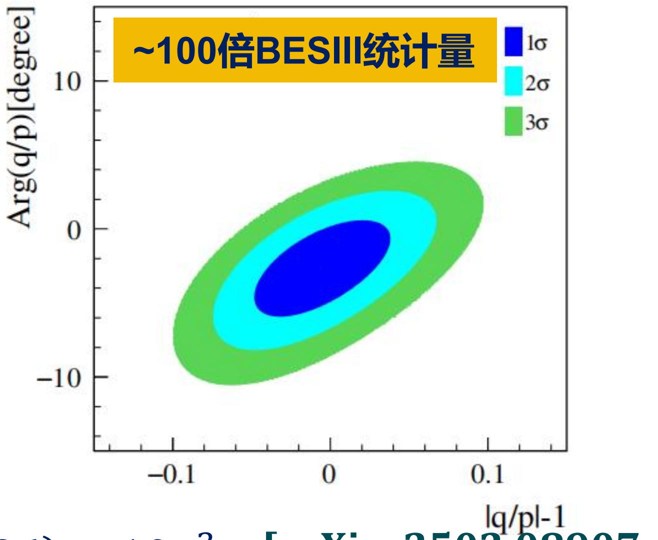
高能量点数据中的量子关联研究

双标记$K^-\pi^+\pi^0$, $K^-3\pi$, $K_S^0\pi^+\pi^-$









$$x = (4.05 \pm 0.43) \times 10^{-3}$$
$$y = (6.38 \pm 0.23) \times 10^{-3}$$
$$|q/p| = 0.983^{+0.015}_{-0.014}$$
$$\arg(q/p) = (-1.51^{+1.03}_{-1.06})^{o}$$

$$x = (? \pm 0.36) \times 10^{-3}$$
$$y = (? \pm 0.15) \times 10^{-3}$$
$$|q/p| = ? \pm 0.028$$
$$\arg(q/p) = (? \pm 2.14)^{o}$$

[arXiv: 2502.08907, 2303.15790]

- 未来超级陶粲装置可精确测量粲介子混合和间接CP破坏，由$D \to K\pi\pi^0$主导
- Belle II精度能达到$\sigma(x)$~0.028% and $\sigma(y)$~0.023% ($D \to K_S^0\pi^+\pi^-$) **[arXiv: 1808.10567]**

additional modes such as $D^0 \to K^{\mp}\pi^{\pm}\pi^+\pi^-$ [3]. With the precision on $|q/p|$ and ϕ_D reaching 0.0020 and 0.15°, respectively, with 300 fb^{-1}, LHCb Upgrade II is the only planned facility with a realistic possibility of observing CP-violating phenomena in charm mixing.

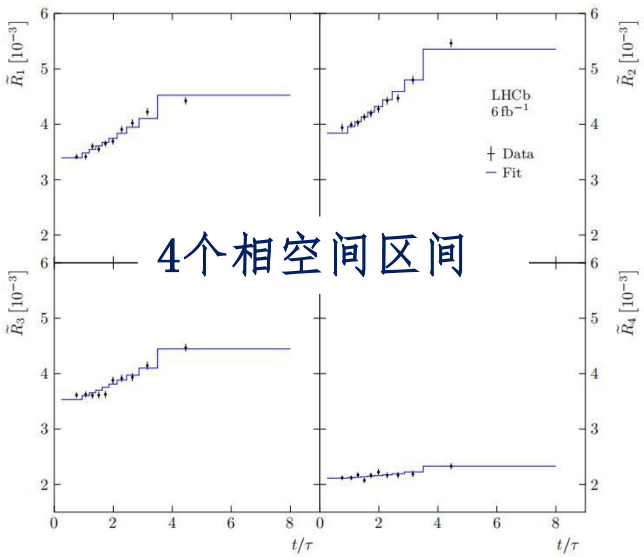
未来强相位差的精确测量

LHCb/Belle (II)可联合测量粲介子混合和强相位差

$$R(t) \equiv \frac{\Gamma[D^0 \to K^+3\pi](t)}{\Gamma[\overline{D}^0 \to K^-3\pi](t)} \approx (r_D^f)^2 - r_D^f R_f \frac{t}{\tau}(y\cos\delta_D^f - x\sin\delta_D^f) + \frac{x^2+y^2}{4}\left(\frac{t}{\tau}\right)^2$$

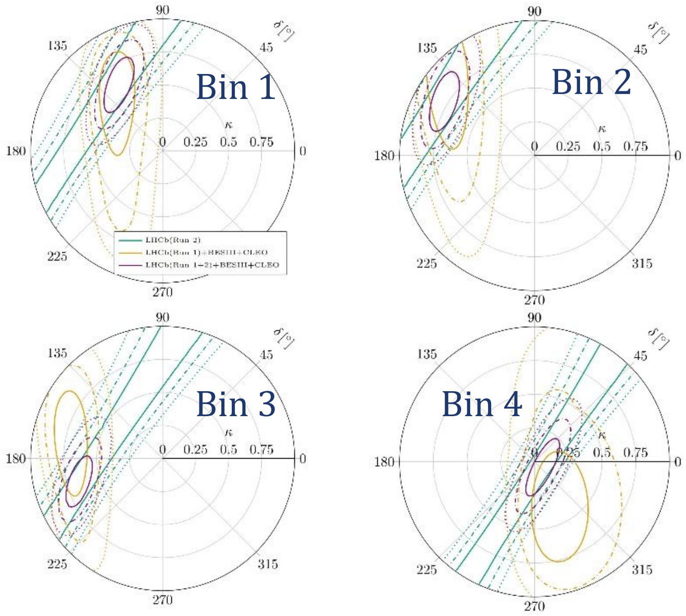
混合参数

强相位差参数



[arXiv: 2510.04963]

LHCb 6 fb^{-1} vs BESIII 2.93 fb^{-1}



$\delta_D^{K\pi}$测量

BESIII: $\left(187.6^{+8.9+5.4}_{-9.7-6.4}\right)^o$ $(2.93\ \text{fb}^{-1}\ \psi(3770))$

$\sigma\sim4^o$ with 20.3 fb^{-1} $\psi(3770)$ 数据

$\left(187.6^{+8.9+5.4}_{-9.7-6.4}\right)^o$ （高能量点）

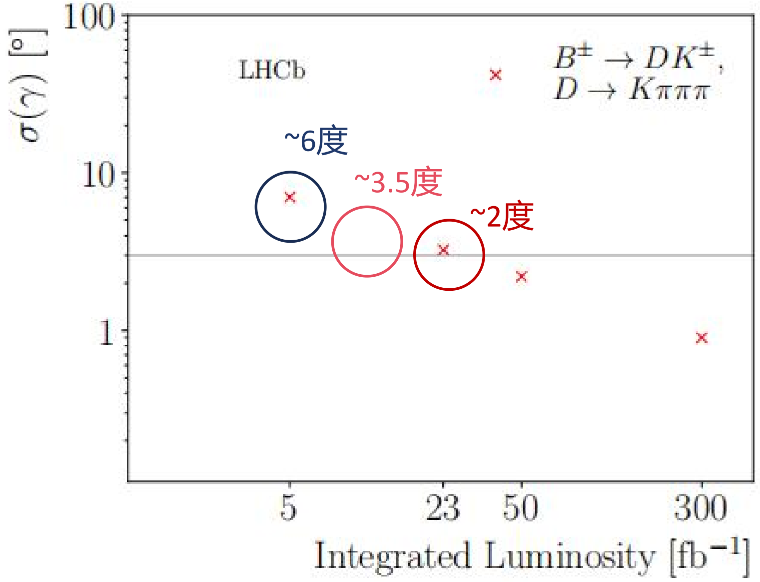
LHCb: $(191.4 \pm 2.4)^o$ **[LHCb-CONF-2024-004]**

包含BESIII、CLEO-c的测量结果

- LHCb/Belle (II)上极高统计量的粲介子数据可用于联合精确测定强相位差
- BESIII等陶粲装置需要采集更大统计量的数据来满足粲介子混合和CP破坏研究

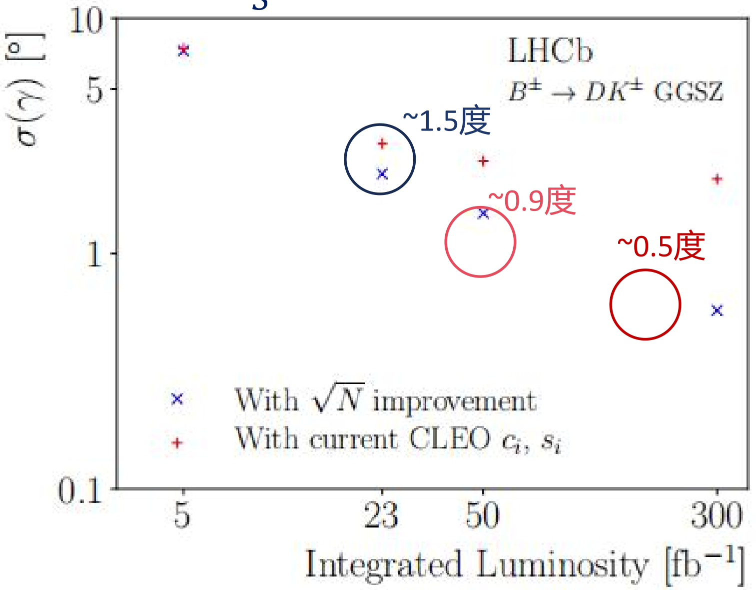
未来强相位差的精确测量

$D \to K^- 3\pi$



[arXiv: 1808.08865]

$D \to K_S^0 \pi^+ \pi^-$



BESIII强相位差对γ角的误差贡献

- 2.93 fb^{-1}
- 7.93 fb^{-1}
- 20.3 fb^{-1}

- BESIII等陶粲装置需要采集更大统计量的数据来满足γ角精确测量的需要

总结

量子关联的$D\bar{D}$数据目前是精确测量强相位差的“最佳”场所

- 正在开展基于BESIII实验全部$\psi(3770)$数据的分析
  - 开发新的测量方法
- 正在开展更多基于BESIII实验高能量点4.009 – 4.230 GeV数据的分析
  - 强相位差和混合参数测量
- 未来超级陶粲工厂上的相关研究
  - 强相位差、混合参数和间接CP破坏联合精确测量
  - 与LHCb (Upgrade)、Belle II等实验在γ角、粲介子混合和CP破坏互补