Quark Butter

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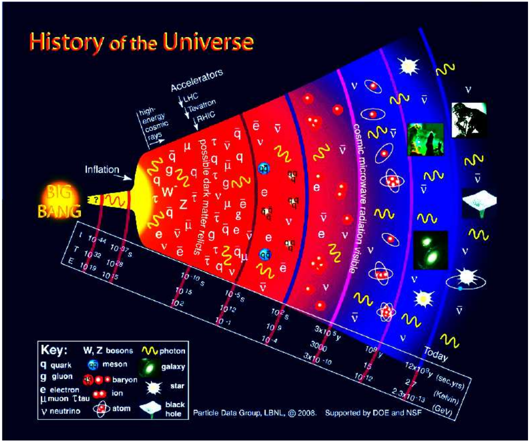
Introduction

Forest and tiger

Along with gravity, QCD is the theory which gives structure to the universe: strong interactions dominate the pressure of the universe at the electroweak phase transition, baryons get mass at the QCD cross over, baryon acoustic oscillations give a scale to the universe, nuclear short range repulsion shapes stars, supernovae, planets.

But strong interactions make it difficult to catch this tiger and study it.

Focus: the microsecond universe



Relevant magnitudes: 1 μs, 0.1 GeV, 10^{12} K

What is a better model of QCD: water or butter?

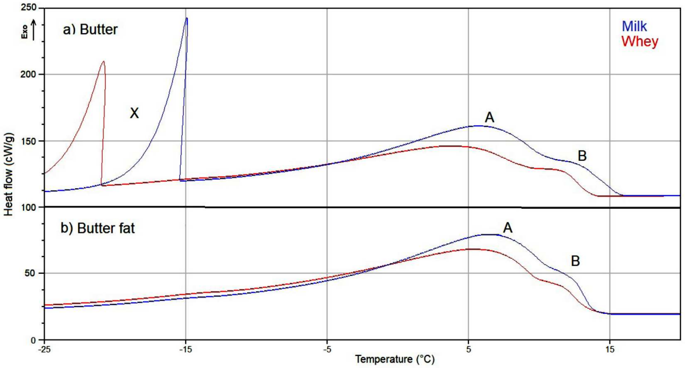


FIGURE 2. Exothermic peaks of fat crystallisation (A, B) and water freezing (X) in milk butter and whey butter (a) and the corresponding fats (b).

Brozek et al, 10.31883/pjfns/155838

Phase transition or cross over? Simple or multi-component matter?

Phase diagram

QCD or the full SM?

The EW sector is a **chiral theory**: left-handed Fermions have a different status than right-handed Fermions. Hard to simulate on the lattice.

Standard model at $T \ll M_W$ is in the **Higgsed state**, at $T \gg M_W$ expected to be in the **symmetric state**. Is there a phase transition? Need to understand thermodynamics of a gauge theory coupled to a scalar and left-handed Fermions.

The strong CP problem does not seem to have a solution at any scale relevant to QCD.

No extra symmetry

Conserved quantities of full standard model are B, Q, and lepton number L. The section with $L = 0$ has no extra symmetries beyond those of QCD. So, the phase diagram of physical QCD may be understood correctly even if we neglect the EW sector.

Quark flavour and symmetry breaking in QCD

Three flavours of light quarks in QCD ($N_f = 3$): up and down related by isospin I, and strange (related by Gell-Mann Nishijima flavour symmetry). Conserved quantities B and Q imply two chemical potentials μ_B and μ_Q.

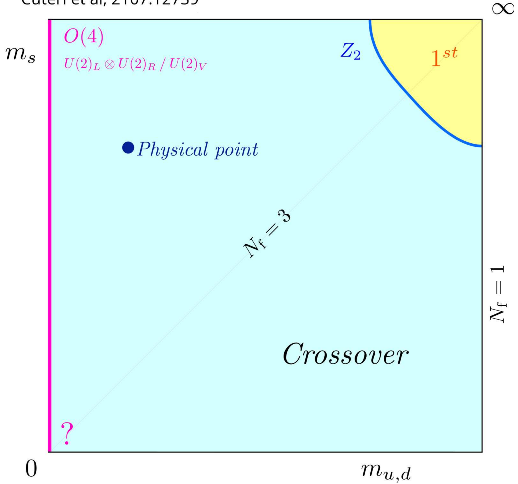
Gell-Mann Nishijima relation $I_3 = Q - (B + S)/2$ can be used to relate $dI_3 = dQ$. This implies equality of charge and isospin chemical potential $\mu_Q = \mu_I$.

$N_f = 3$ SU(3) flavour symmetry explicitly broken to $N_f = 2 + 1$ when quark masses are not equal: $m_\ell < m_s$. Lattice QCD tunes two quark masses to match m_π and m_K (isospin averaged mass for each S).

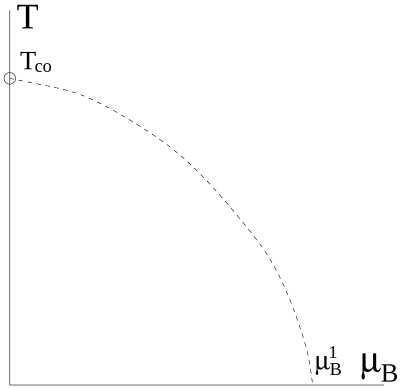
$N_f = 2 + 1$ SU(2) flavour symmetry broken to $N_f = 1 + 1 + 1$ when $\Delta m = m_d - m_u$ is non-zero. Remnant symmetry is U(1) generated by τ_3. Same symmetry also broken by α_{EM}. Lattice QCD tunes Δm so that $m_{\pi^0}/m_{\pi^\pm}$ is close to physical.

The Columbia plot on the lattice: tuning theory parameters

Cuteri et al, 2107.12739



What we know from lattice QCD and what we guess



When $m_\pi \simeq 140$ MeV there is a cross over at $T_{co} = 156$ MeV and $\mu_B = 0$ with a width $\Delta T \simeq 15$ MeV. No sign of 1st order phase transition for $T > 100$ MeV and $\mu_B < 450$ MeV. Line of crossover (or first order transitions) meets $T = 0$ axis at $\mu_B^1 \simeq 1250$ MeV.

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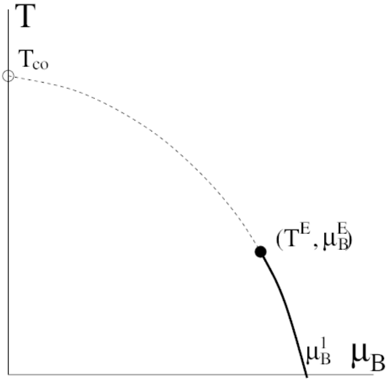

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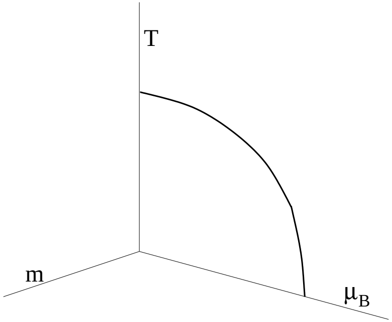
Phase diagram as a function of light quark mass



At this time cannot decide whether there is a surface of first order transitions, and a QCD critical point at finite chemical potential. Experiments and improvements in lattice computations are needed.

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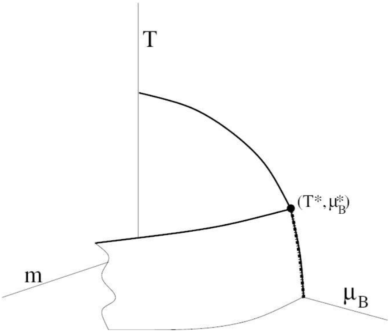

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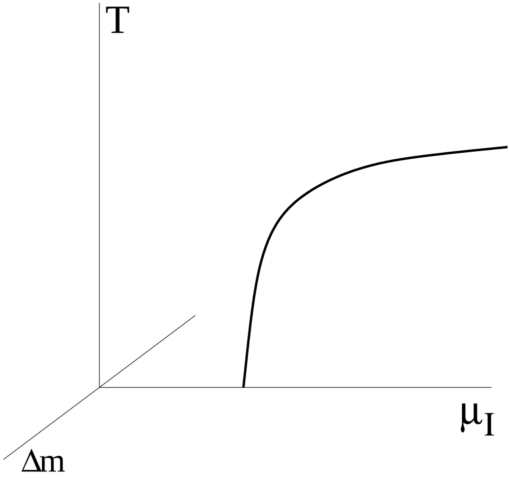
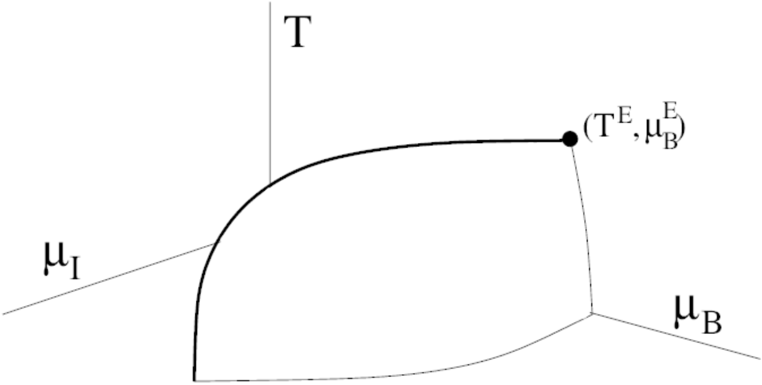
Phase diagram also has isospin



$\Delta m = m_d - m_u$ and $\mu_Q = \mu_I$ for fixed B. Normal hadron phase to pion condensed phase, eventually to colour superconducting phase.

Phase diagram for physical QCD



Physical QCD: $N_f = 1+1+1$. $m_{\pi^\pm} \neq m_{\pi^0}$ and both about 135 MeV. m_K, m_η physical.

Effective (Quantum) Field Theory

Effective Field Theories for hadrons at $T = 0$

Chiral perturbation theory (χPT)

Very widely used tool for understanding low-energy physics of strong interactions. Roy equations give properties of $\pi\pi$ scattering, extended to πN scattering; captured well in χPT.

Heavy quark effective theory HQET

The systematics of form factors and decay rates of mesons and baryons with one heavy quark: b or c.

Non-relativistic QED and QCD (NRQED, NRQCD)

The Schrödinger's equation for the Hydrogen atom is used as the start of an expansion which exactly reproduces the bound state in QED. Now used in chemistry. Similar approach for QCD bound states with two heavy quarks.

Construction of EFTs in matter

Use a UV cutoff Λ: only describe physics at momentum scales $k \ll \Lambda$.

Start with symmetries: in the presence of matter ($T > 0$ or $\mu > 0$) all Lorentz frames are not equivalent. Theory is Lorentz covariant but not Lorentz invariant. Easiest treatment: work in the rest frame of matter.

$$\text{Lorentz group} \longrightarrow \text{Rotation group} \times CPT$$

Effect: new LECs appear, thermal physics emerges automatically.

$$\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) \to \frac{1}{2}(\dot{\phi})^2 - \frac{1}{2}c_4(\nabla\phi)^2,$$

New LEC c_4. Effect: temporal propagator has pole at m, static propagator has pole at $m/\sqrt{c_4}$; difference between pole mass and screening mass.

Next choose the fields. Quark fields or meson fields? Cannot use both—double counting degrees of freedom. χPT uses mesons, NJL uses quarks. Quark EFT allows easy incorporation of μ_B. Lagrangian organized by mass dimension D.

Thermal EFT setup

Quark EFT for $T > 0$ with chiral symmetry, UV cutoff $\Lambda <$ proton mass

$$
\begin{aligned}
D &= 3 \qquad L_3 = \text{mass terms, break chiral symmetry} \\
D &= 4 \qquad L_4 = \text{thermal kinetic terms}: \quad \overline{\psi}\partial\!\!\!/_0\psi + d_4\overline{\psi}\nabla\!\!\!\!/\,\psi \\
D &= 6 \qquad L_6 = \text{many Nambu JonaLasinio} - \text{like terms} \\
&\qquad\qquad + \text{derivative corrections to propagator} \\
D &= 8 \qquad L_8 = \text{NJL} - \text{like terms with two derivatives} \\
&\qquad\qquad + \text{derivative corrections to propagator} \\
D &= 9 \qquad L_9 = {}'\text{t Hooft} - \text{Schaeffer terms}
\end{aligned}
$$

Use all terms constrained only by symmetry: EFT is not a model but a tool to describe QCD accurately. Accuracy: $(T/\Lambda)^{D-4}$

SG+RS doi:10.1103/PhysRevD.97.036025 and SG+RS+PS arxiv:2511.00409

How to do a tractable computation

1: Introduce a chiral condensate, and do a Hartree-Fock computation. Find the chiral condensate self-consistently: gap equation. This gives the vacuum: spontaneous symmetry breaking. Only 3 free LECs; tune one to pion mass, then 2 LECs remain to be determined.

2: Examine the effects of fluctuations in the quark fields. The most important fluctuations are the ones that cost least energy: the pseudo-Goldstone bosons. Write the EFT of these fluctuations (thermal χPT). This is a coupled theory of pions, Kaons, η and η' mesons.

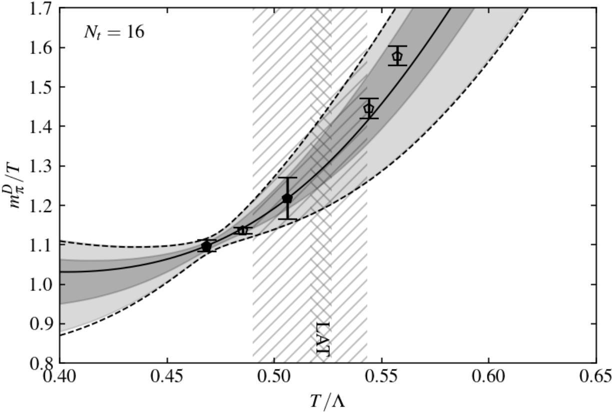
It is straightforward to remove the η' from consideration: requires tuning the 't Hooft-Schaeffer LEC.

3: Remove the K and η through a one-loop integration, giving a pion EFT

$$L = \frac{1}{2}c_2\Lambda^2\pi^2 + \frac{1}{2}\dot{\pi}^2 + \frac{1}{2}c_4(\nabla\pi)^2 + c_{41}\pi^4 + \cdots$$

The first 3 terms are LO in chiral power counting, the 4th term is one of several NLO terms.

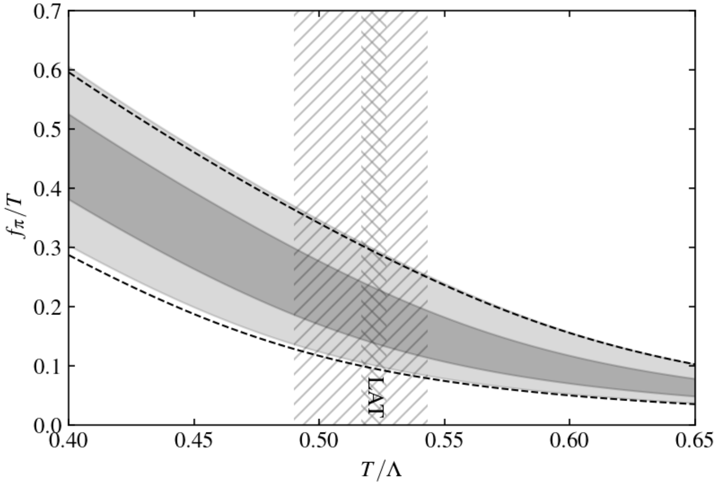
Input pion properties to fix LECs



Tune two LECs to match two pion properties at finite T — here screening mass at two Ts. Lattice errors define LEC uncertainties.

SG+RS+PS arxiv:2511.00409, using HotQCD doi:10.1103/PhysRevD.100.094510

EFT predictions for thermal pions



Looking forward to measurements from HotQCD, others

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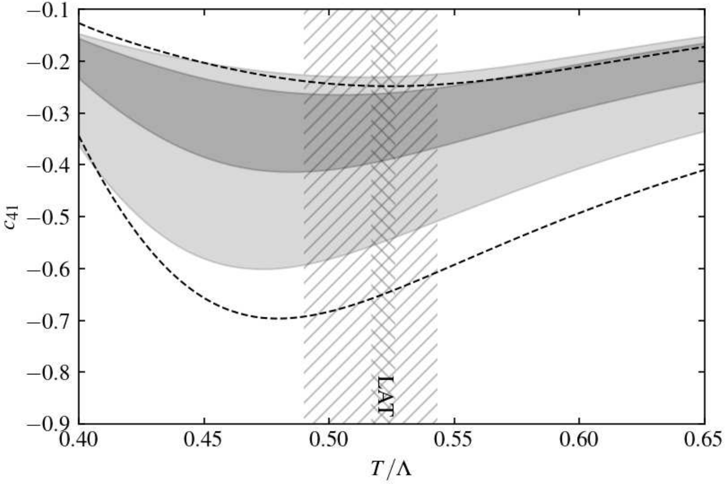


Looking forward to measurements from HotQCD, others

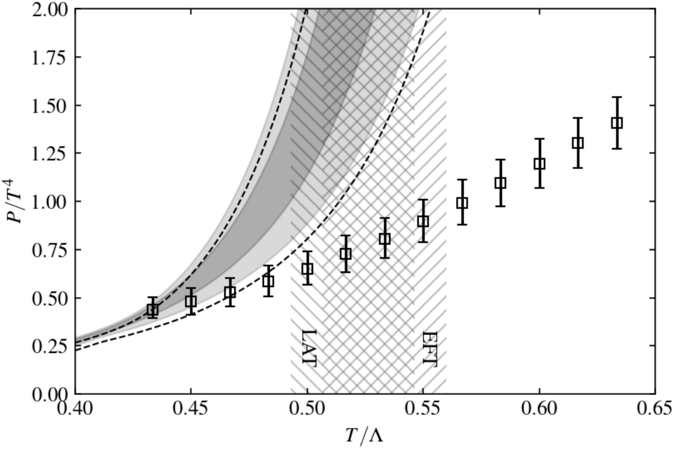
EFT predictions for the phase diagram

	T_{co} (MeV)	κ_2	κ_4
HotQCD 2017	156.5 (1.5)	0.015 (4)	-0.001 (3)
BHJW 2020	158.0 (0.6)	0.0153 (18)	0.00032 (67)
EFT	**157 (7)**	**0.0169 (20)**	**0.00014 (2)**

EFT is used to compute the phase diagram in the HF approximation.

Errors in T_c for the EFT are induced from the pion Debye mass. Improved lattice measurement of pion screening mass will reduce the errors in the EFT prediction. Needed to understand how closely pion properties are related to the phase diagram.

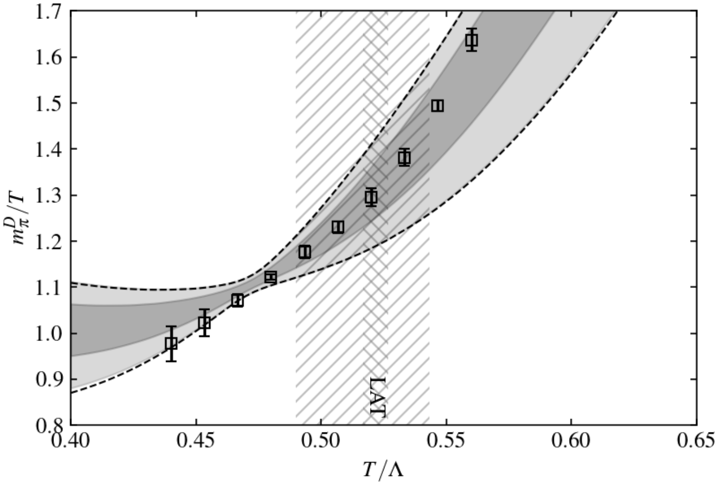
Pressure



Uses only the LO (kinetic) terms; NLO terms in TχEFT may be needed.

SG+RS+PS arxiv:2511.00409 using HotQCD doi:10.1103/PhysRevD.100.094510

Pion properties continuous across phases



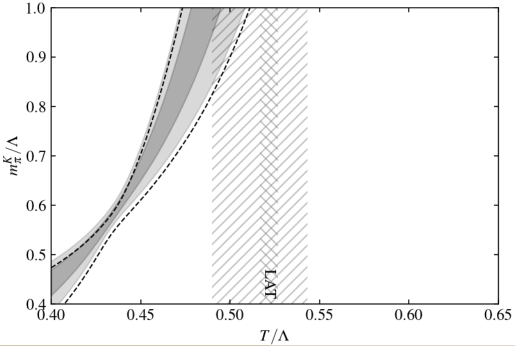
SG+RS+PS arxiv:2511.00409, using HotQCD doi:10.1103/PhysRevD.100.094510

Analytic continuation: kinetic mass

In Minkowski space

$$E = \sqrt{m_\pi^2 + c_4 p^2} \simeq m_\pi + \frac{p^2}{(2m_\pi/c_4)} + \cdots$$

Pole mass (rest mass) is m_π, but kinetic energy needs a different mass.
Kinetic mass $m_\pi^K = m_\pi/c_4$. **SG+RS doi:10.1142/S0217751X20300215**



Increasing m_π^K imply that scattering channels which are open at low T close rapidly as T increases.

Increasing m_π^K reason why the pion decouples from the dynamics above T_c

Quick outline

1. Lattice QCD gives precise predictions for parts of the QCD phase diagram: crossover from hadron phase to chiral symmetric phase at finite temperature and chemical potential. No phase transitions seen yet: future program for heavy-ion experiments and lattice computations.
2. The pion mass and other properties are continuous across the chiral crossover. Implies that pions remain in strongly interacting matter, and can be excited by appropriate probes: for example, axial currents.
3. The kinetic mass increases very rapidly, and reaches the UV cutoff of the EFT just below T_{co}. Implies that pions stop participating in the dynamics at around T_{co}. Difference between screening and kinetic mass shows complex behaviour of hadrons near QCD crossover: quark butter.
4. Axial current correlators in real time can give rise to transport coefficient: in the chiral limit axial charge diffusion constant. Can be computed in the EFT.