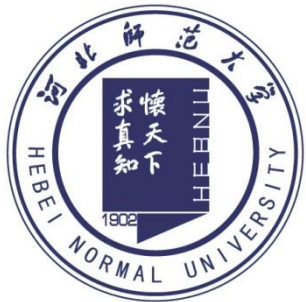


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Hadronic enhancements of axion reactions in chiral effective field theory



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Introduction

Strong CP problem

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q}(i\not{D} - m_q e^{i\theta_q})q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \theta \frac{\alpha_s}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$q \rightarrow e^{i\gamma_5\alpha} q \quad \longrightarrow \quad \theta_q \rightarrow \theta_q + 2\alpha, \quad \theta \rightarrow \theta - 2\alpha$$

implying the invariant quantity: $\bar{\theta} = \theta + \theta_q$

$$\longrightarrow \mathcal{L}_{\text{QCD}} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \boxed{\bar{\theta} \frac{\alpha_s}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

- $G_{\mu\nu}\tilde{G}^{\mu\nu} \sim \partial_\mu K^\mu$: this total derivative is relevant, due to the nonperturbative QCD vacuum
- Naive guess from CKM: $\theta_{\text{CPV}} \sim \text{O}(1)$
- Experimental constraints from neutron EDM: $\bar{\theta} \leq 10^{-10}$

➤ Strong CP problem: why $\bar{\theta}$ unnaturally tiny ?

Peccei-Quinn mechanism to address strong CP problem

[Peccei,Quinn,PRL'77]

[Weinberg,PRL'78] [Wilzeck,PRL'78]

- Promote constant $\bar{\theta}$ as a dynamical spin-0 field $a(x)$
- Impose new global U(1) PQ symmetry (anomalous under QCD)

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a} \frac{g_s^2}{32\pi^2} G\tilde{G}$$

$$a(x) \rightarrow a(x) + \kappa f_a \implies S \rightarrow S + \frac{g_s^2 \kappa}{32\pi^2} \int d^4x G\tilde{G} \quad (\text{cancel } \bar{\theta} \text{ term})$$

- Vafa-Witten theorem: VEV of $\langle a \rangle = 0$ in the vector-like theory, such as QCD
- Weinberg and Wilczek: PQ mechanism indicates a pseudo-Nambu-Goldstone boson
- This pNGB strips off the unwanted strong CP phase: Wilczek names it as **Axion**

中文：轴子 (轴矢流耦合)

$$\frac{\partial_\mu a}{2f_a} \bar{q} \gamma^\mu \gamma_5 Q_a q$$



- Original PQWW axion: $f_a \sim v_{EW} \approx 246 \text{ GeV}$ (visible axion)
quickly ruled out by experiments: $K \rightarrow \pi a$, $J/\psi \rightarrow \gamma a$, $\Upsilon \rightarrow \gamma a$,
astrophysical constraints: Supernovae, Red giant, ($NN \rightarrow NN a$)

Generic effective axion Lagrangian for light-flavor quarks

$$\begin{aligned}\mathcal{L}_{\text{QCD}}^{\text{axion}} = & \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \frac{a}{f_a} \frac{\alpha_s}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu} \\ & + \frac{1}{4}g_{a\gamma}^0 a F\tilde{F} + \frac{\partial_\mu a}{2f_a} \bar{q}c_q^0 \gamma^\mu \gamma_5 q\end{aligned}$$

Diverse viable axion models

- **PQWW**: Y_{PQ} (SM fermion) $\neq 0$, $f_a \sim v_{\text{EW}}$ (ruled out)
- **KSVZ**: Y_{PQ} (SM fermion) $= 0$, singlet Higgs and extra BSM fermions,
 $f_a \gg v_{\text{EW}}$ (invisible axion)
 model-dependent terms vanish: $g_{a\gamma}^0 = 0$, $c_q^0 = 0$
- **DFSZ**: Y_{PQ} (SM fermion) $\neq 0$, extra singlet and doublet Higgs, $f_a \gg v_{\text{EW}}$ (invisible)
 model-dependent terms retain: $g_{a\gamma}^0 \neq 0$, $c_q^0 \neq 0$
- **QCD axion / ALP (axion-like particle)**: bare axion mass term $m_{a,0} = 0$ / $m_{a,0} \neq 0$
 ALP case: m_a and f_a are independent

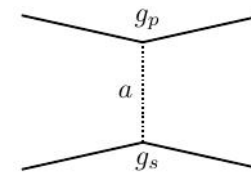
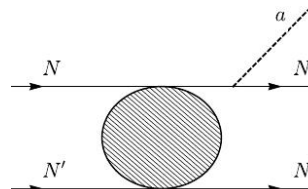
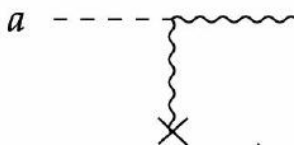
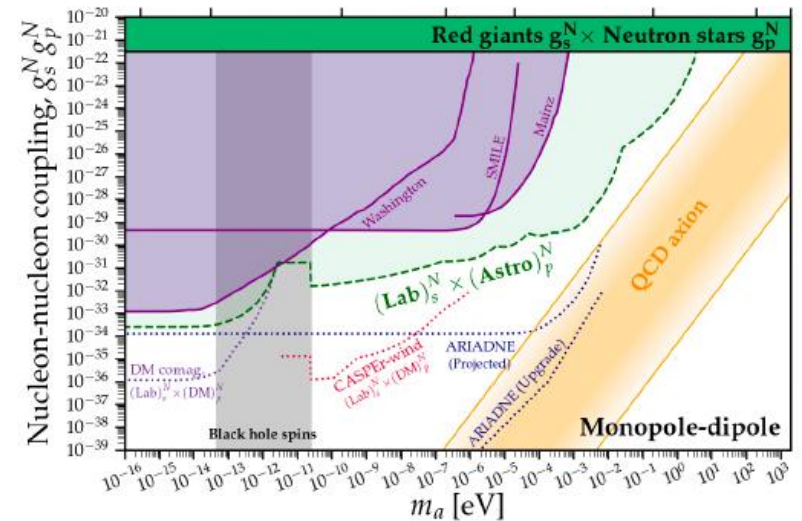
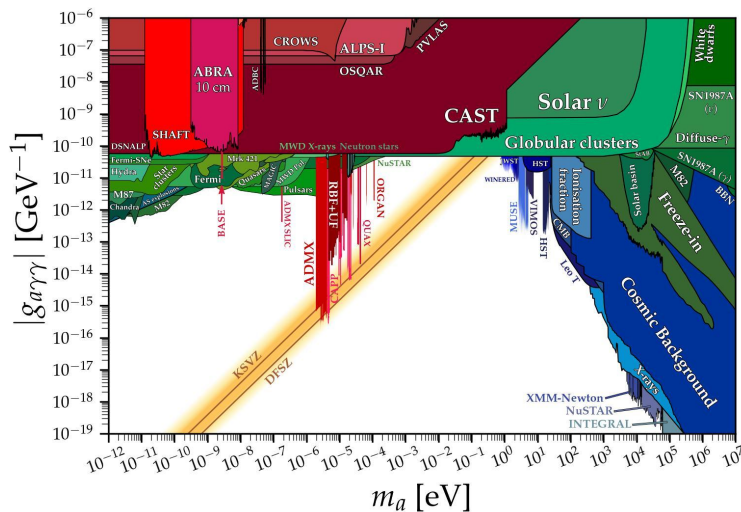
QCD axion case: $m_a^2 \cong \frac{m_\pi^2 f_\pi^2}{4f_a^2}$ ($f_a \gg f_\pi$, axion: **a very light BSM particle**)

- Various constraints from rather different experiments

[Di Luzio, et al., Phy.Rep'20] [Sikivie, RMP'21] [Irastorza, Redondo, PPNP'18]

Cosmology, Astronomy, Colliders, Quantum precision measurements , Cavity Haloscope,

[O'Hare, Github, <https://cajohare.github.io/AxionLimits/>]



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Axion chiral perturbation theory

Axion chiral perturbation theory ($\Lambda\chi$ PT)

- We will focus on the QCD-like axion: $m_{a,0}(\neq 0) \ll f_a$ with model-independent $aG\tilde{G}$ interaction, i.e., the **MODEL INDEPENDENT QCD axion** interactions.
- Axion-hadron interactions are relevant at low energies.

$$\mathcal{L}_{\text{QCD}}^{\text{axion}} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \boxed{\frac{a}{f_a} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}}$$

Two ways to proceed:

(1) Remove the $aG\tilde{G}$ term via the quark axial transformation

$$\begin{array}{l} \text{Tr}(Q_a) = 1 \\ \begin{array}{l} \text{curved arrow} \\ \text{curved arrow} \end{array} \end{array} \quad \begin{array}{l} q \rightarrow e^{i\frac{a}{2f_a}\gamma_5 Q_a} q \\ -\frac{a\alpha_s}{8\pi f_a} G\tilde{G} - \frac{\partial_\mu a}{2f_a} \bar{q}\gamma^\mu \gamma_5 Q_a q \end{array} \quad M_q \rightarrow M_q(a) = e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a}$$

Mapping to χ PT

$$\mathcal{L}_2 = \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger + \chi_a U^\dagger + U \chi_a^\dagger \rangle + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\text{LO}}$$

$$\chi_a = 2B_0 e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \quad J_A^\mu|_{\text{LO}} = -i\frac{F^2}{2} \langle Q_a (\partial^\mu U U^\dagger + U^\dagger \partial^\mu U) \rangle$$

- $Q_a = M_q^{-1} / \text{Tr}(M_q^{-1})$ [Georgi, Kaplan, Randall, PLB'86]
- $J_A^\mu \partial_\mu a$ [Bauer, et al., PRL'21]

(2) Explicitly keep the $aG\tilde{G}$ term and match it to χ PT

Reminiscent:

QCD $U(1)_A$ anomaly that is caused by topological charge density $\omega(x) = \alpha_s G_{\mu\nu} \tilde{G}^{\mu\nu} / (8\pi)$ is responsible for the massive singlet η_0 .

Axion could be similarly included as the η_0 via the $U(3)$ χ PT:

$$\mathcal{L}^{\text{LO}} = \frac{F^2}{4} \langle u_\mu u^\mu \rangle + \frac{F^2}{4} \langle \chi_+ \rangle + \frac{F^2}{12} M_0^2 X^2$$

$$\mathcal{L}^{\text{NLO}} = L_5 \langle u^\mu u_\mu \chi_+ \rangle + \frac{L_8}{2} \langle \chi_+ \chi_+ + \chi_- \chi_- \rangle - \frac{F^2 \Lambda_1}{12} D^\mu X D_\mu X - \frac{F^2 \Lambda_2}{12} X \langle \chi_- \rangle,$$

$$U = u^2 = e^{i\frac{\sqrt{2}\Phi}{F}}, \quad \chi = 2B(s + ip), \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$

$$u_\mu = iu^\dagger D_\mu U u^\dagger, \quad D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$$

$$X = \log(\det U) - i \frac{a}{f_a} \quad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

- Q_a is not needed in $U(3)$ χ PT.
- $M_0^2 = 6\tau/F^2$, with τ the topological susceptibility. Note that $M_0^2 \sim \mathcal{O}(1/N_c)$.
- δ expansion scheme: $\delta \sim \mathcal{O}(p^2) \sim \mathcal{O}(m_q) \sim \mathcal{O}(1/N_c)$.
- Axion interactions enter via the axion-meson mixing terms.

LO

(mass mixing only)

$$\begin{pmatrix} \pi^0 \\ \bar{\eta} \\ \bar{\eta}' \\ \bar{a} \end{pmatrix} = \begin{pmatrix} 1 + v_{11} & -v_{12} & -v_{13} & -v_{14} \\ v_{12} & 1 + v_{22} & -v_{23} & -v_{24} \\ v_{13} & v_{23} & 1 + v_{33} & -v_{34} \\ v_{41} & v_{42} & v_{43} & 1 + v_{44} \end{pmatrix} \begin{pmatrix} \pi^0 \\ \bar{\eta} \\ \bar{\eta}' \\ a \end{pmatrix}$$

$$v_{12} = -\frac{\epsilon}{\sqrt{3}} \frac{c_\theta - \sqrt{2}s_\theta}{m_\pi^2 - m_{\bar{\eta}}^2}, \quad v_{13} = -\frac{\epsilon}{\sqrt{3}} \frac{\sqrt{2}c_\theta + s_\theta}{m_\pi^2 - m_{\bar{\eta}'}^2}, \quad v_{23} = \frac{\sqrt{2}s_\theta^2 + c_\theta s_\theta - \sqrt{2}c_\theta^2}{3(m_{\bar{\eta}'}^2 - m_{\bar{\eta}}^2)} \epsilon, \quad v_{41} = -\frac{M_0^2 \epsilon}{6(m_a^2 - m_\pi^2)} \frac{F}{f_a} \left[-\frac{(\sqrt{2}c_\theta - 2s_\theta)s_\theta}{m_a^2 - m_{\bar{\eta}}^2} + \frac{c_\theta(2c_\theta + \sqrt{2}s_\theta)}{m_a^2 - m_{\bar{\eta}'}^2} \right]$$

$$v_{42} = \frac{M_0^2 s_\theta}{\sqrt{6}(m_a^2 - m_{\bar{\eta}}^2)} \frac{F}{f_a} - \frac{M_0^2 \epsilon}{3\sqrt{6}(m_a^2 - m_{\bar{\eta}}^2)} \frac{F}{f_a} \left[\frac{c_\theta(-\sqrt{2}c_\theta^2 + c_\theta s_\theta + \sqrt{2}s_\theta^2)}{m_a^2 - m_{\bar{\eta}'}^2} - \frac{s_\theta(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2)}{m_a^2 - m_{\bar{\eta}}^2} \right]$$

$$v_{43} = -\frac{M_0^2 c_\theta}{\sqrt{6}(m_a^2 - m_{\bar{\eta}'}^2)} \frac{F}{f_a} - \frac{M_0^2 \epsilon}{3\sqrt{6}(m_a^2 - m_{\bar{\eta}'}^2)} \frac{F}{f_a} \left[\frac{c_\theta(c_\theta^2 - 2\sqrt{2}c_\theta s_\theta + 2s_\theta^2)}{m_a^2 - m_{\bar{\eta}}^2} - \frac{s_\theta(-\sqrt{2}c_\theta^2 + c_\theta s_\theta + \sqrt{2}s_\theta^2)}{m_a^2 - m_{\bar{\eta}}^2} \right] \quad \dots \dots$$

with $m_{\bar{\eta}}^2 = \frac{M_0^2}{2} + m_K^2 - \frac{\sqrt{M_0^4 - \frac{4M_0^2 \Delta^2}{3} + 4\Delta^4}}{2}, \quad m_{\bar{\eta}'}^2 = \frac{M_0^2}{2} + m_K^2 + \frac{\sqrt{M_0^4 - \frac{4M_0^2 \Delta^2}{3} + 4\Delta^4}}{2}, \quad \sin \theta = -\left(\sqrt{1 + \frac{(3M_0^2 - 2\Delta^2 + \sqrt{9M_0^4 - 12M_0^2 \Delta^2 + 36\Delta^4})^2}{32\Delta^4}} \right)^{-1}$

Physical masses after diagonalization

$$m_{\bar{\eta}}^2 = m_{\bar{\eta}}^2 + \frac{\epsilon}{3} (\sqrt{2}c_\theta + s_\theta)^2 + O(\epsilon^2)$$

$$m_{\bar{\eta}'}^2 = m_{\bar{\eta}'}^2 + \frac{\epsilon}{3} (c_\theta - \sqrt{2}s_\theta)^2 + O(\epsilon^2)$$

$$m_a^2 = m_{a,0}^2 + \frac{M_0^2 F^2}{6f_a^2} \left[1 + \frac{c_\theta^2 M_0^2}{m_{a,0}^2 - m_{\bar{\eta}'}^2} + \frac{s_\theta^2 M_0^2}{m_{a,0}^2 - m_{\bar{\eta}}^2} \right] + \frac{M_0^4 F^2 \epsilon}{9f_a^2} \left[\frac{s_\theta^2 (\sqrt{2}c_\theta + s_\theta)^2}{2(m_{a,0}^2 - m_{\bar{\eta}}^2)^2} + \frac{c_\theta^2 (c_\theta - \sqrt{2}s_\theta)^2}{2(m_{a,0}^2 - m_{\bar{\eta}'}^2)^2} + \frac{c_\theta s_\theta (\sqrt{2}c_\theta^2 - c_\theta s_\theta - \sqrt{2}s_\theta^2)}{(m_{a,0}^2 - m_{\bar{\eta}}^2)(m_{a,0}^2 - m_{\bar{\eta}'}^2)} \right] + O(\epsilon^2),$$



$$m_a^2 = \frac{m_\pi^2 F^2}{4f_a^2}$$

[Weinberg,PRL'78]

(keep LO terms in m_π/m_K & m_π/M_0 & ϵ expansions)

NLO: (kinetic & mass mixing)

$$\begin{aligned}
\mathcal{L} = & \frac{1 + \delta_k^\eta}{2} \partial_\mu \bar{\eta} \partial^\mu \bar{\eta} + \frac{1 + \delta_k^{\eta'}}{2} \partial_\mu \bar{\eta}' \partial^\mu \bar{\eta}' + \delta_k^{\eta\eta'} \partial_\mu \bar{\eta} \partial^\mu \bar{\eta}' - \frac{m_\eta^2 + \delta_{m_\eta^2}}{2} \bar{\eta} \bar{\eta} - \frac{m_{\eta'}^2 + \delta_{m_{\eta'}^2}}{2} \bar{\eta}' \bar{\eta}' - \delta_{m^2}^{\eta\eta'} \bar{\eta} \bar{\eta}' \\
& + \frac{1 + \delta_k^\pi}{2} \partial_\mu \bar{\pi}^0 \partial^\mu \bar{\pi}^0 + \delta_k^{\pi\eta} \partial_\mu \bar{\pi}^0 \partial^\mu \bar{\eta} + \delta_k^{\pi\eta'} \partial_\mu \bar{\pi}^0 \partial^\mu \bar{\eta}' - \frac{m_\pi^2 + \delta_{m_\pi^2}}{2} \bar{\pi}^0 \bar{\pi}^0 - \delta_{m^2}^{\pi\eta} \bar{\pi}^0 \bar{\eta} - \delta_{m^2}^{\pi\eta'} \bar{\pi}^0 \bar{\eta}' \\
& + \frac{1 + \delta_k^a}{2} \partial_\mu \bar{a} \partial^\mu \bar{a} + \delta_k^{a\pi} \partial_\mu \bar{a} \partial^\mu \bar{\pi}^0 + \delta_k^{a\eta} \partial_\mu \bar{a} \partial^\mu \bar{\eta} + \delta_k^{a\eta'} \partial_\mu \bar{a} \partial^\mu \bar{\eta}' - \frac{m_a^2 + \delta_{m_a^2}}{2} \bar{a} \bar{a} - \delta_{m^2}^{a\pi} \bar{a} \bar{\pi}^0 \\
& - \delta_{m^2}^{a\eta} \bar{a} \bar{\eta} - \delta_{m^2}^{a\eta'} \bar{a} \bar{\eta}'
\end{aligned}$$

Separately handle the kinetic (x_{ij}) and mass (y_{ij}) mixing terms

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = \begin{pmatrix} 1 & -y_{12} & -y_{13} & -y_{14} \\ y_{12} & 1 & -y_{23} & -y_{24} \\ y_{13} & y_{23} & 1 & -y_{34} \\ y_{14} & y_{24} & y_{34} & 1 \end{pmatrix} \times \begin{pmatrix} 1 - x_{11} & -x_{12} & -x_{13} & -x_{14} \\ -x_{12} & 1 - x_{22} & -x_{23} & -x_{24} \\ -x_{13} & -x_{23} & 1 - x_{33} & -x_{34} \\ -x_{14} & -x_{24} & -x_{34} & 1 - x_{44} \end{pmatrix} \begin{pmatrix} \bar{\pi}^0 \\ \bar{\eta} \\ \bar{\eta}' \\ \bar{a} \end{pmatrix}$$

$$\begin{aligned}
x_{11} &= -\frac{\delta_k^\pi}{2}, & x_{12} &= -\frac{\delta_k^{\pi\eta}}{2}, & x_{13} &= -\frac{\delta_k^{\pi\eta'}}{2}, & x_{14} &= -\frac{\delta_k^{a\pi}}{2}, & x_{22} &= -\frac{\delta_k^\eta}{2}, \\
x_{23} &= -\frac{\delta_k^{\eta\eta'}}{2}, & x_{24} &= -\frac{\delta_k^{a\eta}}{2}, & x_{33} &= -\frac{\delta_k^{\eta'}}{2}, & x_{34} &= -\frac{\delta_k^{a\eta'}}{2}, & x_{44} &= -\frac{\delta_k^a}{2},
\end{aligned}$$

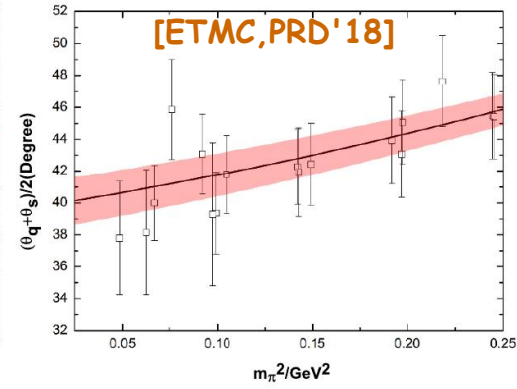
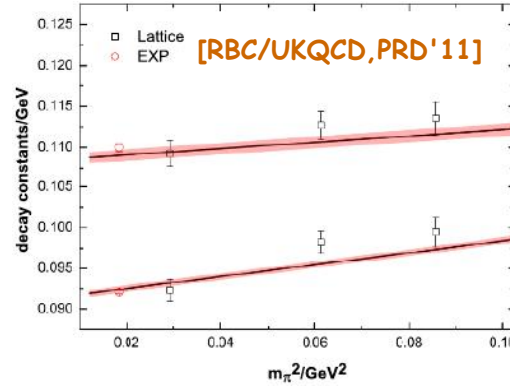
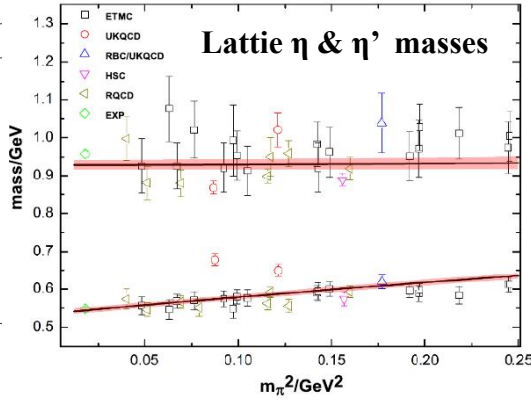
$$\delta_X \sim \mathbf{L}_5, \mathbf{L}_8, \Lambda_1, \Lambda_2$$

$$\begin{aligned}
y_{12} &= \frac{\delta_{m^2}^{\pi\eta} + x_{12}(m_\eta^2 + m_\pi^2)}{m_\eta^2 - m_\pi^2}, & y_{13} &= \frac{\delta_{m^2}^{\pi\eta'} + x_{13}(m_{\eta'}^2 + m_\pi^2)}{m_{\eta'}^2 - m_\pi^2}, & y_{14} &= \frac{\delta_{m^2}^{a\pi} + x_{14}(m_{a,0}^2 + m_\pi^2)}{m_{a,0}^2 - m_\pi^2}, \\
y_{23} &= \frac{\delta_{m^2}^{\eta\eta'} + x_{23}(m_\eta^2 + m_{\eta'}^2)}{m_{\eta'}^2 - m_\eta^2}, & y_{24} &= \frac{\delta_{m^2}^{a\eta} + x_{24}(m_\eta^2 + m_{a,0}^2)}{m_{a,0}^2 - m_\eta^2}, & y_{34} &= \frac{\delta_{m^2}^{a\eta'} + x_{34}(m_{\eta'}^2 + m_{a,0}^2)}{m_{a,0}^2 - m_{\eta'}^2}.
\end{aligned}$$

Fit to lattice data

[Gao,ZHG,Oller,Zhou,JHEP'23] [Gao,Hao,ZHG,et al.,EPJC'25]

Parameters	NLO Fit
$F(\text{MeV})$	$91.05^{+0.42}_{-0.44}$
$10^3 \times L_5$	$1.68^{+0.05}_{-0.06}$
$10^3 \times L_8$	$0.88^{+0.04}_{-0.04}$
Λ_1	$-0.17^{+0.05}_{-0.05}$
Λ_2	$0.06^{+0.08}_{-0.09}$
$\chi^2/(\text{d.o.f.})$	$219.9/(111-5)$



Mixing pattern@NLO

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = M^{\text{LO+NLO}} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix} \quad M^{\text{LO+NLO}} = \begin{pmatrix} 1 + (0.015 \pm 0.001) & 0.017 + (-0.007 \pm 0.001) & 0.009 + (-0.011 \pm 0.001) & \frac{-12.8 + (-0.13 \pm 0.02)}{f_a} \\ -0.019 + (0.005 \pm 0.001) & 0.94 + (0.21 \pm 0.01) & 0.33 + (-0.21 \pm 0.03) & \frac{-34.3 + (1.7^{+0.8}_{-0.7})}{f_a} \\ -0.003 + (-0.001 \pm 0.000) & -0.33 + (-0.18 \pm 0.02) & 0.94 + (0.13^{+0.01}_{-0.02}) & \frac{-25.9 + (0.2^{+0.4}_{-0.3})}{f_a} \\ \frac{12.1 + (0.5 \pm 0.1)}{f_a} & \frac{23.8 + (1.0^{+0.2}_{-0.1})}{f_a} & \frac{35.7 + (1.7^{+0.2}_{-0.1})}{f_a} & 1 + \frac{-921.5 + (-56.6^{+7.9}_{-9.6})}{f_a^2} \end{pmatrix}$$

Mass decomposition@NLO

$$\begin{aligned} m_{\hat{\pi}} &= [134.9 + (0.1 \pm 0.07)] \text{ MeV}, \\ m_{\hat{K}} &= [492.1 + (5.1^{+3.4}_{-3.3})] \text{ MeV}, \\ m_{\hat{\eta}} &= [490.4 + (61.1^{+10.0}_{-8.7})] \text{ MeV}, \\ m_{\hat{\eta}'} &= [954.5 + (-28.5^{+11.9}_{-10.9})] \text{ MeV}, \\ m_{\hat{a}} &= [5.96 + (0.12 \pm 0.02)] \mu\text{eV} \frac{10^{12} \text{ GeV}}{f_a}, \end{aligned}$$

Two-photon couplings

$$\mathcal{L}_{WZW}^{\text{LO}} = -\frac{3\sqrt{2}}{8\pi^2 F} \varepsilon_{\mu\nu\rho\sigma} \partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2 \Phi \rangle, \qquad Q = \text{Diag}(\frac{2e}{3}, -\frac{e}{3}, -\frac{e}{3})$$

$$\mathcal{L}_{WZW}^{\text{NLO}} = t_1 \frac{32\sqrt{2}B}{F} \varepsilon_{\mu\nu\rho\sigma} \partial^\mu A^\nu \partial^\rho A^\sigma \langle (M_q \Phi + \Phi M_q) Q^2 \rangle + 16k_3 \varepsilon_{\mu\nu\rho\sigma} \partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2 \rangle \left(\frac{\sqrt{2}}{F} \langle \Phi \rangle - \frac{a}{f_a} \right)$$

★ **Note: one needs the π - η - η' - a mixing as input to calculate $g_{a\gamma\gamma}$**

$F_{\pi^0\gamma\gamma}^{\text{Exp}} = 0.274 \pm 0.002 \text{GeV}^{-1},$
 $F_{\eta\gamma\gamma}^{\text{Exp}} = 0.274 \pm 0.006 \text{GeV}^{-1},$
 $F_{\eta'\gamma\gamma}^{\text{Exp}} = 0.344 \pm 0.008 \text{GeV}^{-1},$

\longrightarrow

$t_1 = -(3.8 \pm 2.4) \times 10^{-4} \text{GeV}^{-2},$
 $k_3 = (1.21 \pm 0.23) \times 10^{-4}$

\longrightarrow

isospin limit(LO)
isospin breaking(LO)
NLO

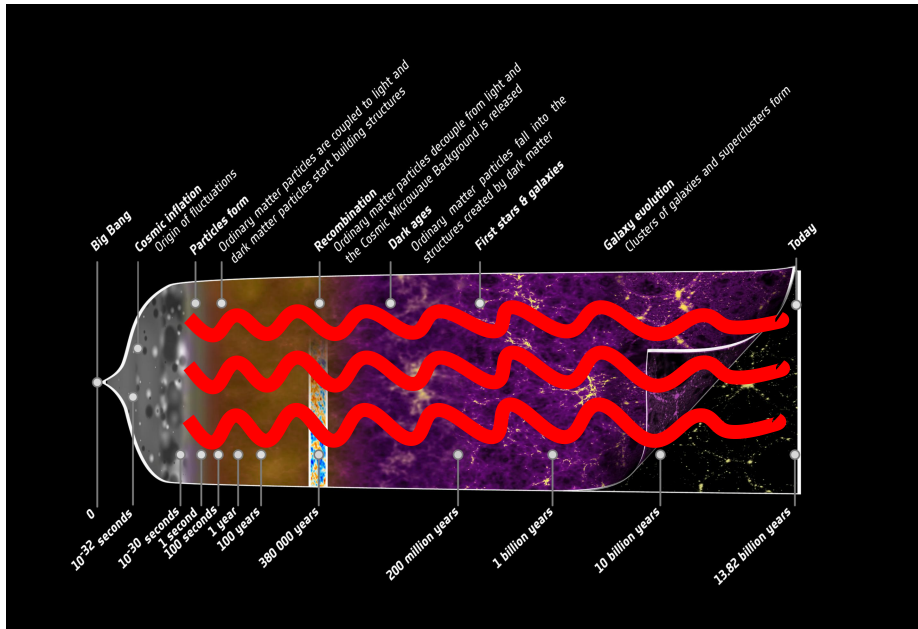
$$F_{a\gamma\gamma} = \frac{20.1 + 3.4 + (0.5 \pm 0.2)}{f_a} \times 10^{-3},$$

(IB corrections amount to be around 15%!)

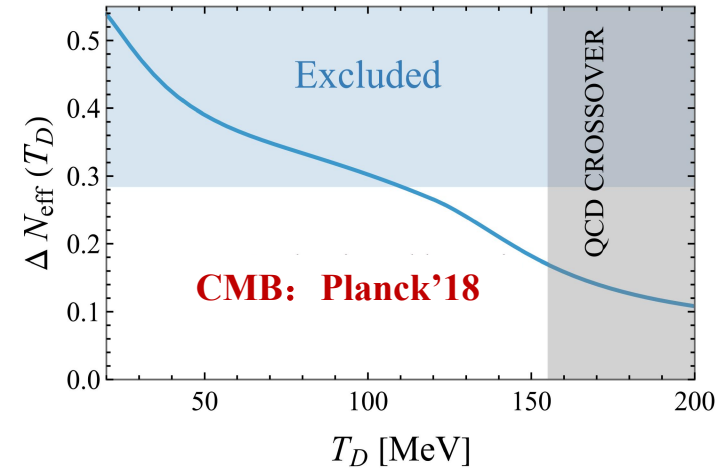
$$g_{a\gamma\gamma} = 4\pi \alpha_{em} F_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a} (1.89 \pm 0.02).$$

which can be compared to: 1.92 ± 0.04 [Grilli de Cortona, et al., JHEP'16] and 2.05 ± 0.03 [Lu, et al., JHEP'20]

Cosmology constraints on axion thermalization rate



- Axions can be copiously produced from thermal bath in the early Universe.
- After decoupling, its thermal relics will leave imprints today.



- Axion thermalization rate from reaction: $\underbrace{a + \dots}_i \leftrightarrow \underbrace{1 + 2 + 3 + \dots}_j$

$$\Gamma_a(T) = \frac{1}{n_a^{eq}} \int \left[\prod_i \frac{d^3 p_i}{(2\pi)^3 2E_i} \right] \left[\prod_j \frac{d^3 p_j}{(2\pi)^3 2E_j} \right] (2\pi)^4 \delta^4 \left(\sum_i p_i - \sum_j p_j \right) |\mathcal{M}_{\text{reaction}}|^2 \prod_i f_i(x_i) \prod_j [1 \pm f_j(x_j)]$$

$$f_i(x_i) = \begin{cases} \frac{1}{e^{x_i} - 1}, & \text{bosonic,} \\ \frac{1}{e^{x_i} + 1}, & \text{fermionic,} \end{cases} \quad x_i = E_i/T$$

“+” for Bose enhancement
“−” for Pauli blocking

Cosmology constraints on axion thermalization rate

Axion thermal production in the early Universe : Extra radiation (ΔN_{eff})

Extra effective number of relativistic d.o.f :

$$\Delta N_{\text{eff}} \simeq \frac{4}{7} \left(\frac{43}{4g_{\star s}(T_D)} \right)^{\frac{4}{3}}$$

$g_{\star s}(T)$: effective number of entropy d.o.f at temperature T

T_D : axion decoupling temperature from the thermal medium

➤ CMB constraint (Planck'18) [Aghanim et al., 2020] : $\Delta N_{\text{eff}} \leq 0.28$

➤ T_D : Instantaneous decoupling approximation

$$\Gamma_a(T_D) = H(T_D)$$

Axion thermalization rate

Hubble expansion parameter

$$\Gamma_a(T) = \frac{1}{n_a^{\text{eq}}} \int d\tilde{\Gamma} |\mathcal{M}_{a\text{-SM}}|^2 \prod_i f_i(x_i) \prod_j [1 \pm f_j(x_j)]$$

$$H(T) = T^2 \sqrt{4\pi^3 g_*(T)/45} / m_{\text{Pl}}$$

Axion-SM particle scattering amplitudes

Key thermal channels of axion-SM scatterings at different temperatures

☞ $T_D \gtrsim 1 \text{ GeV}$: $ag \leftrightarrow gg$.

[Masso et al., 2002, Graf and Steffen, 2011]

☞ $T_D \lesssim 1 \text{ GeV}$: Hadrons need to be included.

☞ $T_D \lesssim 200 \text{ MeV}$: $a\pi \leftrightarrow \pi\pi$.

[Chang and Choi, 1993, Hannestad et al., 2005,
Giusarma et al., 2014, D'Eramo et al., 2022]

❑ **Reliable $a\pi$ interaction is crucial to determine Γ_a for $T_D < T_c \approx 155 \text{ MeV}$**

➤ For a long time, only the LO $a\pi \leftrightarrow \pi\pi$ amplitude is employed to calculate Γ_a , e.g.,

[Chang, Choi, PLB'93] [Hannestad, et al., JCAP'05] [Hannestad, et al., JCAP'05] [D'Eramo, et al., PRL'22]

➤ Recent NLO calculation of Γ_a : χ PT invalid for $T_\chi > 70 \text{ MeV}$ [Di Luzio, et al., PRL'21]

➤ Chiral unitarization approach for $a\pi \leftrightarrow \pi\pi$: [Di Luzio, et al., PRD'23]

➤ **However, all the previous works have ignored thermal corrections to the $a\pi \leftrightarrow \pi\pi$ amplitudes. The first estimation of such effect is given: [Wang, ZHG, Zhou, PRD'24]**

$$\Gamma_a(T) = \frac{1}{n_a^{\text{eq}}} \int d\tilde{\Gamma} \boxed{|\mathcal{M}_{a\pi;\pi\pi}|^2} \prod_i f_i(x_i) \prod_j [1 + f_j(x_j)]$$

➤ **First realistic calculation of $aK \leftrightarrow \pi K$ shows significant contribution to axion thermalization rate: [Wang, ZHG, Zhou, PRD'25]**

Calculation of **THERMAL** $a\pi \leftrightarrow \pi\pi$ amplitudes at one-loop level

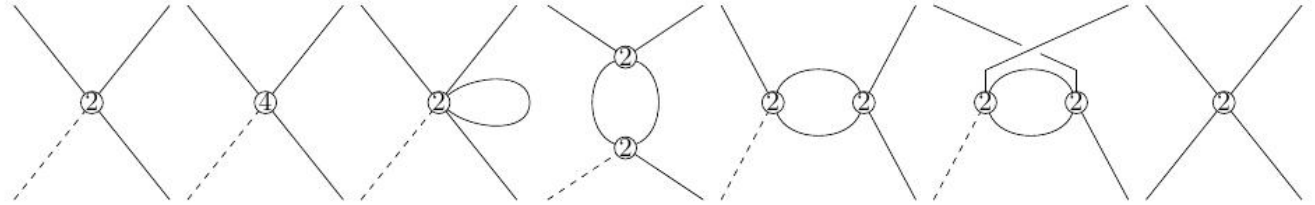
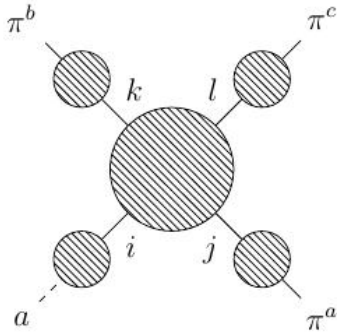
- Finite-temperature effects are included by imaginary time formalism (ITF), where [Kapusta and Gale, 2011, Bellac, 2011, Laine and Vuorinen, 2016]

$$p^0 \rightarrow i\omega_n, \quad \text{with } \omega_n = 2\pi nT, n \in \mathbb{Z},$$

$$-i \int \frac{d^d q}{(2\pi)^d} \rightarrow -i \int_{\beta} \frac{d^d q}{(2\pi)^d} \equiv T \sum_n \int \frac{d^{d-1} q}{(2\pi)^{d-1}}.$$

- Compute the thermal Green functions in ITF

$$G_{a\pi^a; \pi^b \pi^c}^T(p_1, p_2; p_3, p_4) = \sum_{i,j,k,l} G_{ai}(p_1^2) G_{\pi^a j}(p_2^2) G_{k\pi^b}(p_3^2) G_{l\pi^c}(p_4^2) A_{ij;kl}(p_1, p_2; p_3, p_4).$$



Feynman diagrams for amputated functions up to NLO.

- The effective Lagrangian at $\mathcal{O}(p^4)$

$$\begin{aligned} \mathcal{L}_4 \supset & \frac{l_3 + l_4}{16} \langle \chi_a U^\dagger + U \chi_a^\dagger \rangle \langle \chi_a U^\dagger + U \chi_a^\dagger \rangle + \frac{l_4}{8} \langle \partial_\mu U \partial^\mu U^\dagger \rangle \langle \chi_a U^\dagger + U \chi_a^\dagger \rangle \\ & - \frac{l_7}{16} \langle \chi_a U^\dagger - U \chi_a^\dagger \rangle \langle \chi_a U^\dagger - U \chi_a^\dagger \rangle + \frac{h_1 - h_3 - l_4}{16} \left[\left(\langle \chi_a U^\dagger + U \chi_a^\dagger \rangle \right)^2 \right. \\ & \left. + \left(\langle \chi_a U^\dagger - U \chi_a^\dagger \rangle \right)^2 - 2 \langle \chi_a U^\dagger \chi_a U^\dagger + U \chi_a^\dagger U \chi_a^\dagger \rangle \right] + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\text{NLO}}, \end{aligned}$$

$$\begin{aligned} J_A^\mu|_{\text{NLO}} \supset & -il_1 \langle Q_a \{ \partial^\mu U, U^\dagger \} \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle \\ & - i \frac{l_2}{2} \langle Q_a \{ \partial_\nu U, U^\dagger \} \rangle \langle \partial^\mu U \partial^\nu U^\dagger + \partial^\nu U \partial^\mu U^\dagger \rangle \\ & - i \frac{l_4}{4} \langle Q_a \{ \partial^\mu U, U^\dagger \} \rangle \langle \chi_a U^\dagger + U \chi_a^\dagger \rangle. \end{aligned}$$

Unitarization of the partial-wave $a\pi \leftrightarrow \pi\pi$ amplitude

Inverse amplitude method (IAM)

$$\mathcal{M}_{a\pi;IJ}^{\text{IAM}} = \frac{\left(\mathcal{M}_{a\pi;IJ}^{(2)}\right)^2}{\mathcal{M}_{a\pi;IJ}^{(2)} - \mathcal{M}_{a\pi;IJ}^{(4)}}$$

$$\mathcal{M}_{a\pi;IJ}(E_{cm}) = \frac{1}{2} \int_{-1}^{+1} d\cos\theta \mathcal{M}_{a\pi;I}(E_{cm}, \cos\theta) P_J(\cos\theta)$$

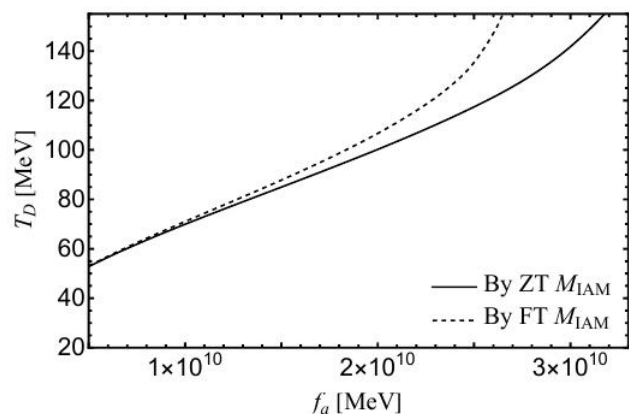
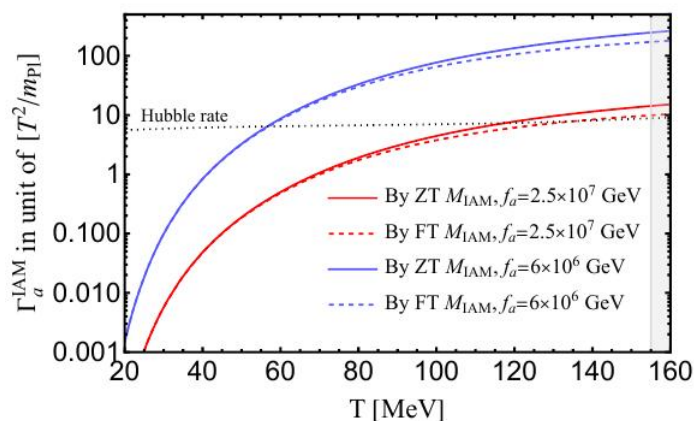
$$\text{Im}\mathcal{M}_{a\pi;IJ}(E_{cm}) \stackrel{s}{=} \frac{1}{2} \rho_{\pi\pi}^T(E_{cm}) \mathcal{M}_{\pi\pi;\pi\pi}^{IJ*} \mathcal{M}_{a\pi;IJ}, \quad (E_{cm} > 2m_\pi)$$

$$\rho_{\pi\pi}^T(E_{cm}) = \frac{\sigma_\pi(E_{cm}^2)}{16\pi} \left[1 + 2n_B\left(\frac{E_{cm}}{2}\right) \right], \quad \sigma_\pi(s) = \sqrt{1 - \frac{4m_\pi^2}{s}}, \quad n_B(E) = \frac{1}{e^{E/T} - 1}$$

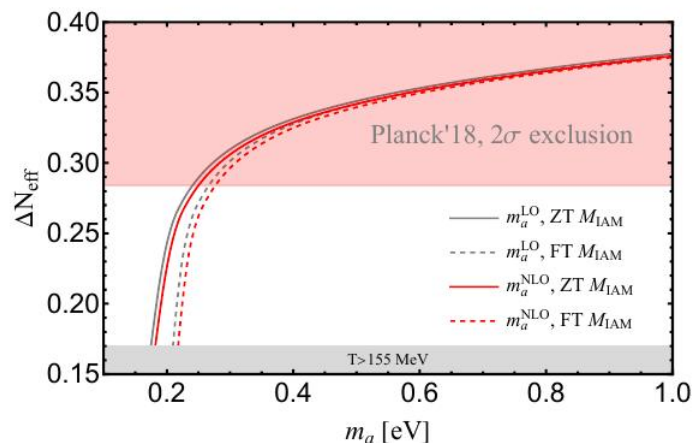
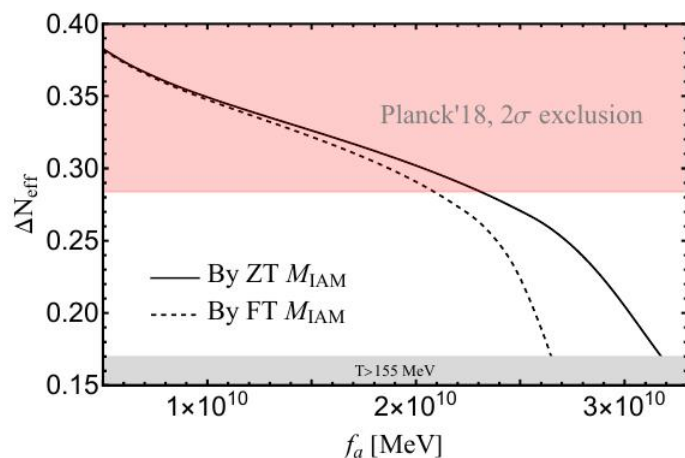
● Resonances poles on the second Riemann sheet

	$f_0(500)/\sigma$		$\rho(770)$	
	$M_\sigma \pm i\frac{\Gamma_\sigma}{2}$	$ f_a g_{\sigma a \pi} $	$M_\rho \pm i\frac{\Gamma_\rho}{2}$	$ f_a g_{\rho a \pi} $
$T = 0 \text{ MeV}$	$422 \pm i240 \text{ MeV}$	0.032 GeV^2	$739 \pm i72 \text{ MeV}$	0.035 GeV^2
$T = 100 \text{ MeV}^*$	$368 \pm i310 \text{ MeV}$	0.037 GeV^2	$744 \pm i77 \text{ MeV}$	0.036 GeV^2

*Only include s -channel unitary thermal correction.



Updated bounds on the axion parameters



□ The constraints

10% corrections are observed

	lower limit of f_a	upper limit of m_a by m_a^{LO}	upper limit of m_a by m_a^{NLO}
ZT	2.3×10^7 GeV	0.24 eV	0.25 eV
FT	2.1×10^7 GeV	0.27 eV	0.28 eV

Combined analyses with $a\pi \leftrightarrow \pi\pi$ & $aK \leftrightarrow \pi K$ channels

SU(3) Axion ChPT@LO

[Wang, ZHG, Zhou, PRD'25]

$$\mathcal{L}_2 = \frac{F_\pi^2}{4} \langle \partial_\mu U \partial^\mu U^\dagger + \chi(a) U^\dagger + U \chi^\dagger(a) \rangle - \frac{\partial_\mu a}{2f_a} \sum_{i=1}^8 C_i J_{A,i}^\mu$$

$$\chi(a) = 2B_0 e^{-i\frac{a}{2f_a} Q_a} M_q e^{-i\frac{a}{2f_a} Q_a} \quad Q_a = M_q^{-1} / \langle M_q^{-1} \rangle \quad J_{A,i}^\mu = i\frac{F_\pi^2}{4} \langle \lambda_i \{ \partial^\mu U, U^\dagger \} \rangle \quad (\text{singlet component of axial currents neglected})$$

$$C_3 = \frac{z(1-r^2)}{2r+z(1+r)^2}, \quad C_8 = \frac{z(1+r)^2-4r}{\sqrt{3}[2r+z(1+r)^2]} \quad z = \frac{m_s}{\hat{m}}, r = \frac{m_u}{m_d}, \hat{m} = \frac{m_u+m_d}{2}$$

Unitarized partial-wave axion-meson/meson-meson amplitudes

Unitarized meson-meson Amp:

$$T_{IJ}^{\text{uni}} = T_{IJ}^{(2)} \cdot \left[T_{IJ}^{(2)} - T_{IJ}^{(4)\text{LECs}} - T_{IJ}^{(2)} \cdot \mathcal{G} \cdot T_{IJ}^{(2)} \right]^{-1} \cdot T_{IJ}^{(2)}$$

$$\text{Im} T = T^\dagger \cdot q / (8\pi\sqrt{s}) \cdot T$$

Unitarized axion-meson Amp:

$$\vec{M}_{IJ}^{\text{uni}} = T_{IJ}^{(2)} \cdot \left[T_{IJ}^{(2)} - T_{IJ}^{(4)\text{LECs}} - T_{IJ}^{(2)} \cdot \mathcal{G} \cdot T_{IJ}^{(2)} \right]^{-1} \cdot \vec{M}_{IJ}^{(2)}$$

$$\text{Im} \vec{M} = T^\dagger \cdot q / (8\pi\sqrt{s}) \cdot \vec{M}$$

$$\mathcal{G} = \text{diag}(G_n, G_m, \dots) \quad G_n(s) = G(a_{\text{sc}}^n, s, m_{n_1}, m_{n_2}) = -\frac{1}{(4\pi)^2} \left[a_{\text{sc}}^n - 1 + \log \frac{m_{n_2}^2}{\mu^2} + \frac{m_{n_1}^2 - m_{n_2}^2}{s} - \frac{\sqrt{\lambda(s, m_{n_1}^2, m_{n_2}^2)}}{s} \log \frac{m_{n_1}^2 + m_{n_2}^2 - s + \sqrt{\lambda(s, m_{n_1}^2, m_{n_2}^2)}}{2m_{n_1}m_{n_2}} \right]$$

Example:

$$\begin{pmatrix} M_{00,\pi\pi}^{\text{uni}} \\ M_{00,KK}^{\text{uni}} \end{pmatrix} = \begin{pmatrix} \hat{T}_{00}^{\pi\pi \rightarrow \pi\pi} & \hat{T}_{00}^{\pi\pi \rightarrow KK} \\ \hat{T}_{00}^{\pi\pi \rightarrow KK} & \hat{T}_{00}^{KK \rightarrow KK} \end{pmatrix} \begin{pmatrix} M_{00,\pi\pi}^{(2)} \\ M_{00,KK}^{(2)} \end{pmatrix}$$

Relevant channels: $S + P$ waves

(1) $a \pi^0 \rightarrow \pi^+ \pi^- , \pi^0 \pi^0$

$IJ=00$: $f_0(500), f_0(980)$ [K-Kbar coupled-channel included]

$IJ=20$: nonresonant case [single $\pi\pi$ channel]

(2) $a \pi^+ \rightarrow \pi^+ \pi^0$

$IJ=11$: $\rho(770)$ [K-Kbar coupled-channel included]

$IJ=20$: nonresonant case [single $\pi\pi$ channel, same as $a\pi^0$ case]

(3) $a \mathbf{K}^+ \rightarrow \pi^+ \mathbf{K}^0 , \pi^0 \mathbf{K}^+$

$IJ=1/2 \ 1$: $K^*(892)$ [$K\eta$ coupled-channel included]

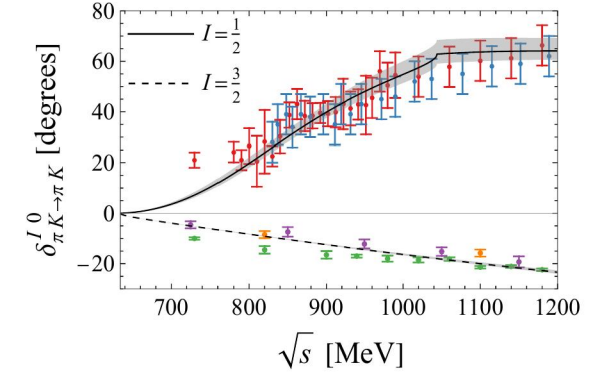
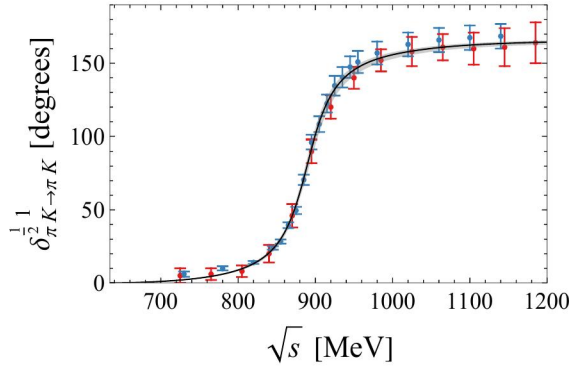
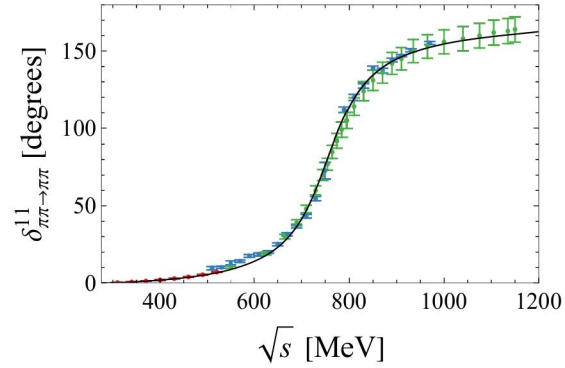
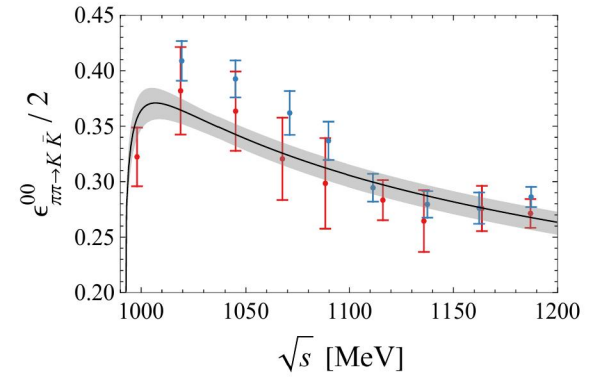
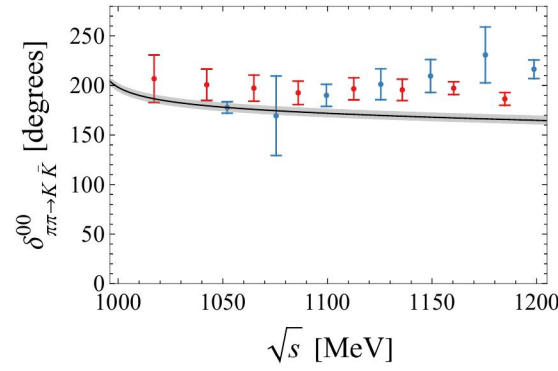
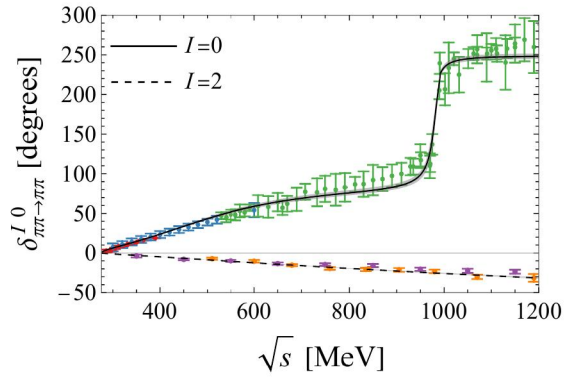
$IJ=1/2 \ 0$: $K^*_0(700)$ [$K\eta$ coupled-channel included]

$IJ=3/2 \ 0$: nonresonant case [single $K\pi$ channel]

$IJ=3/2 \ 1$: nonresonant case [neglected]

(4) $a \mathbf{K}^0 \rightarrow \pi^- \mathbf{K}^+ , \pi^0 \mathbf{K}^0$ [similar as $a\mathbf{K}^+$ case]

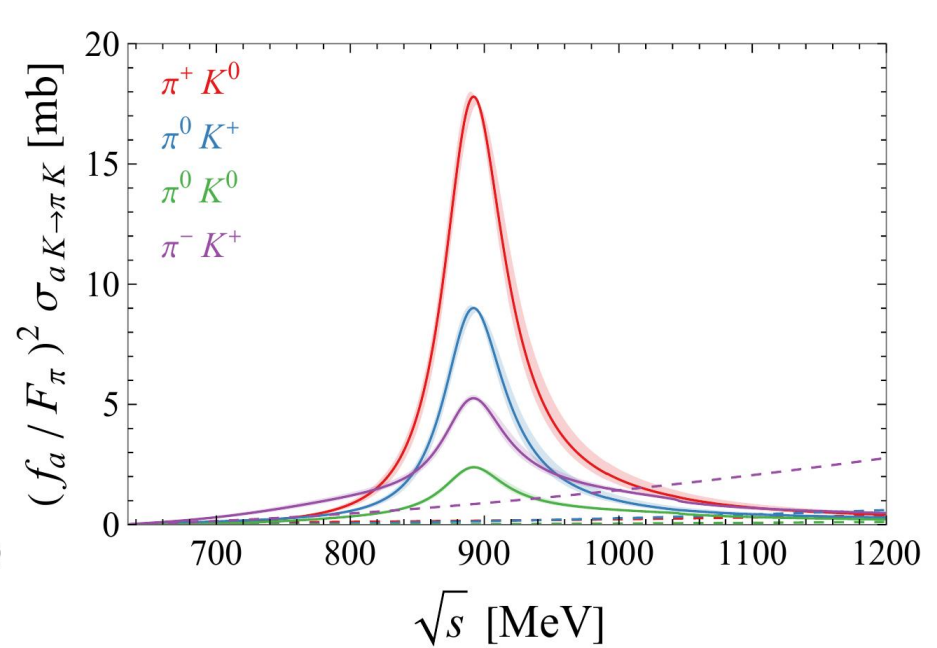
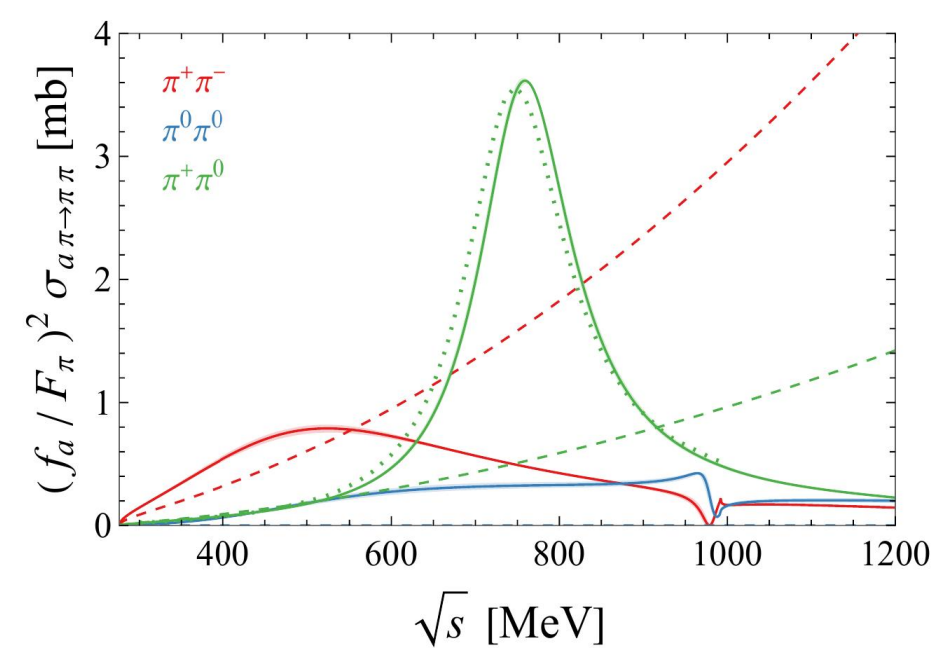
(5) Other channels can be obtained via the charge-conjugation symmetry.



Subst. const.

Low energy constants

$a_{sc}^{\pi\pi,00}$	$-0.49^{+0.24}_{-0.23}$	$\hat{L}_1 \times 10^3$	$0.33^{+0.02}_{-0.02}$
$a_{sc}^{K\bar{K},00}$	$-1.51^{+0.20}_{-0.19}$	$\hat{L}_2 \times 10^3$	$0.97^{+0.05}_{-0.05}$
a_{sc}^{11}	$-1.38^{+0.33}_{-0.26}$	$\hat{L}_3 \times 10^3$	$-2.71^{+0.10}_{-0.11}$
$a_{sc}^{\frac{1}{2}0}$	$0.15^{+0.18}_{-0.21}$	$\hat{L}_4 \times 10^3$	$-0.77^{+0.09}_{-0.11}$
$a_{sc}^{\frac{1}{2}1}$	$1.53^{+0.76}_{-0.80}$	$\hat{L}_5 \times 10^3$	$3.51^{+1.39}_{-1.62}$
		$\hat{L}_6 \times 10^3$	$-1.47^{+0.20}_{-0.24}$
		$\hat{L}_7 \times 10^3$	$-0.77^{+0.24}_{-0.18}$
		$\hat{L}_8 \times 10^3$	$4.05^{+0.37}_{-0.45}$

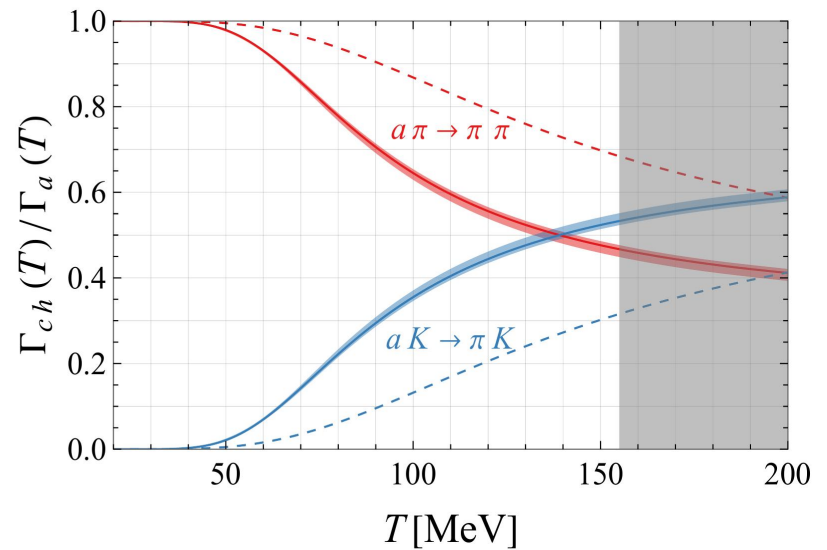
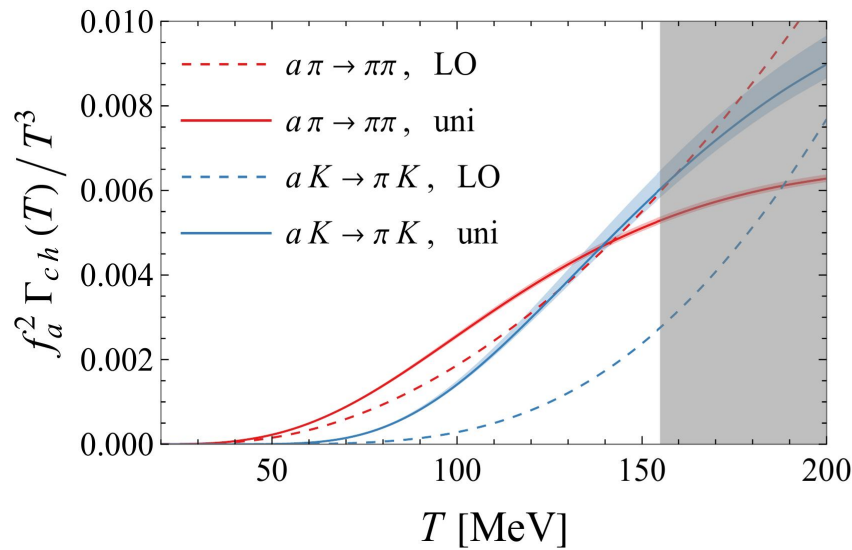


Resonance poles

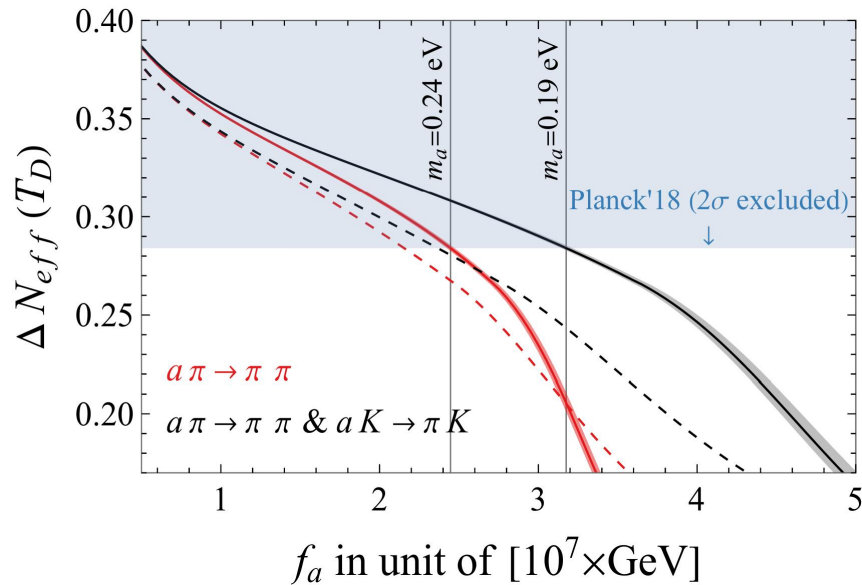
ρ : (754.3 - i 67.9) MeV ; K^* : (889.5 - i 28.0) MeV;

$f_0(500)$: 435.4- i 238.0; $f_0(980)$: 981.1- i 11.4; $K^*_0(700)$: 801.9- i 195.2;

- **Clear enhancement from the unitarized amplitudes (solid lines)**
- **$\rho(770)$ & $K^*(892)$ lead to the most prominent effects**
- **Scalar resonances mostly give mild contributions**
- **$a\eta$ related processes are much less important than the aK ones. (working in progress)**
- **a axion-baryon is expected to be much suppressed, due to the heavy thresholds.**



[Wang, ZHG, Zhou, PRD'25]



$$f_a \geq 2.45_{-0.02}^{+0.03} \times 10^7 \text{ GeV} \quad (\mathbf{a\pi \rightarrow \pi\pi})$$

$$f_a \geq 3.18_{-0.03}^{+0.04} \times 10^7 \text{ GeV} \quad (\mathbf{a\pi \rightarrow \pi\pi + aK \rightarrow \pi K})$$

➤ **Enhancement in $\tau \rightarrow \nu_\tau K a$ is also seen. However this belongs to a Cabibbo suppressed reaction.** [Hao, Duan, ZHG, 2507.00383]

Axion production in $\eta \rightarrow \pi\pi a$ decay

Axion production from $\eta \rightarrow \pi\pi a$ decay in SU(3) χ PT

Why focus on axion in η decay:

- ✓ Valuable channel to search axion @colliders: many available experiments with large data samples of η/η' [BESIII, STCF, JLab, REDTOP,]
- ✓ $\eta \rightarrow \pi\pi\pi$ (IB suppressed), $\eta \rightarrow \pi\pi a$ (no IB suppression)
- ✓ $\eta \rightarrow \pi\pi a$: theoretically easier to handle than $\eta' \rightarrow \pi\pi a$ (next step)

Previous works:

- ❖ Most of them rely on leading-order χ PT
- ❖ Possible issue: bulk contributions @LO χ PT are constant terms, and potential large corrections from higher orders may result.
- ❖ Hadron resonance effects may lead to enhancements.

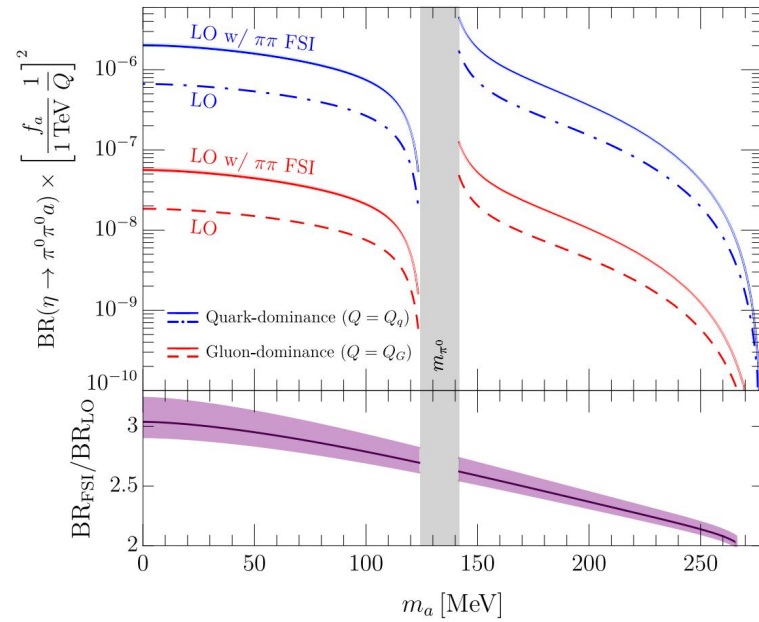
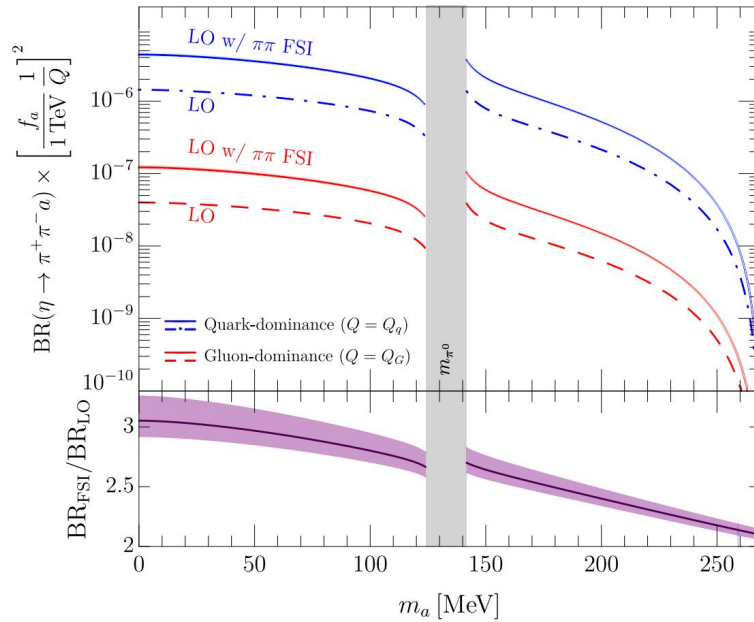
Advances in our work :

- Study of renormalization of $\eta \rightarrow \pi\pi a$ @1-loop level in SU(3) χ PT
- To implement unitarization to the $\eta \rightarrow \pi\pi a$ χ PT amplitude
- Uncertainty analyses in the phenomenological discussions

$$M_0(s) = P(s)\Omega_0^0(s)$$

$\eta \rightarrow \pi\pi a$ LO amplitude

Omnes function: $\pi\pi$ FSI



Our improvements:

- NLO perturbative decay amplitude include s - and $t(u)$ -channel interactions perturbatively.
- The unitarized decay amplitude will be constructed to account for the s -channel $\pi\pi$ final state interaction (FSI) effect that respect the chiral symmetry.
- Dalitz plots will be explored to decode the dynamics in $\eta \rightarrow \pi\pi a$.

LO χ PT Lagrangian

$$\mathcal{L}_2 = \frac{F^2}{4} \langle \partial_\mu U \partial^\mu U^\dagger + \chi_a U^\dagger + U \chi_a^\dagger \rangle + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\text{LO}} + \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2$$

$$\chi_a = 2B_0 M(a) \quad M(a) \equiv \exp\left(-i\frac{a}{2f_a} Q_a\right) M \exp\left(-i\frac{a}{2f_a} Q_a\right) \quad J_A^\mu|_{\text{LO}} = -i\frac{F^2}{2} \langle Q_a \{ \partial^\mu U, U^\dagger \} \rangle$$

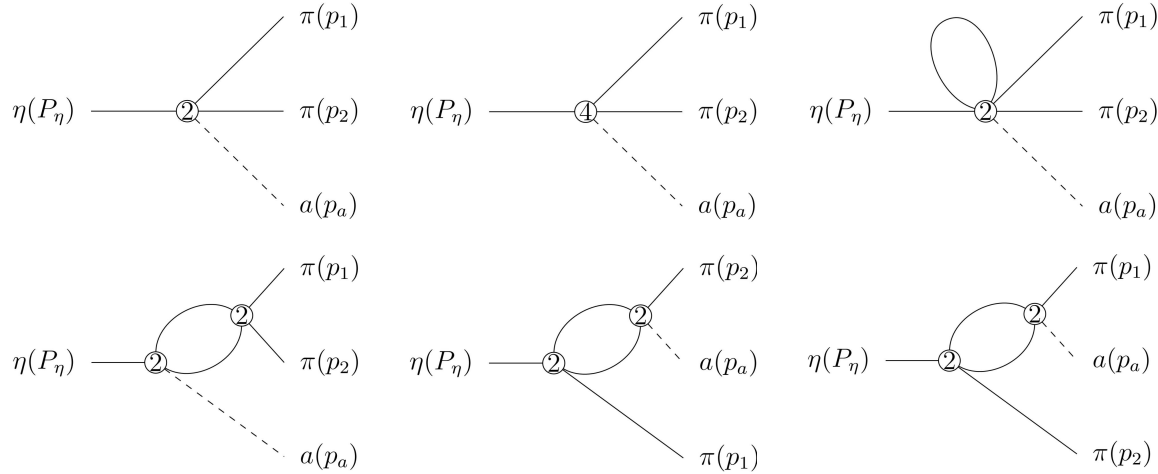
Note: we consider the octet part (\bar{Q}_a) of Q_a in SU(3) χ PT

NLO χ PT Lagrangian

$$\mathcal{L}_4 = L_1 \langle \partial_\mu U \partial^\mu U^\dagger \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \dots + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\text{NLO}},$$

$$J_A^\mu|_{\text{NLO}} = -4iL_1 \langle \bar{Q}_a \{ U^\dagger, \partial^\mu U \} \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \dots$$

Feynman diagrams up to NLO



Parameters

Masses and F_π [MeV]				LECs $L_i^r(\mu)$ at $\mu = 770$ MeV (in unit of 10^{-3})							
m_π	m_K	m_η	F_π	L_1^r	L_2^r	L_3^r	L_4^r	L_5^r	L_6^r	L_7^r	L_8^r
137	496	548	92.1	1.0(1)	1.6(2)	-3.8(3)	0.0(3)	1.2(1)	0.0(4)	-0.3(2)	0.5(2)

[J. Bijnens and G. Ecker, Ann. Rev. Nucl. Part. Sci. 64, 149 (2014)]

✓ Renormalization condition is verified to be consistent with conventional ChPT.

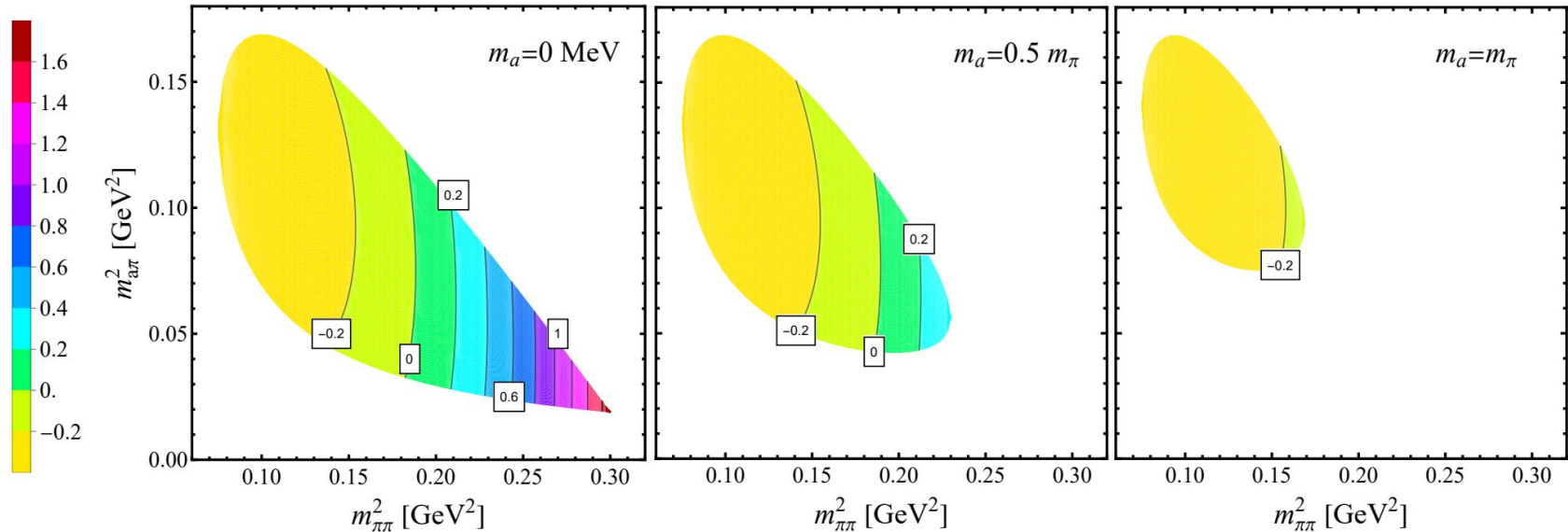
Observations:

- Strong isospin breaking effects enter the $\eta \rightarrow \pi\pi a$ amplitudes at the order of $(m_u - m_d)^2$
- In the isospin limit ($m_u = m_d$), the amplitudes with $\pi^+\pi^-$ and $\pi^0\pi^0$ in $\eta \rightarrow \pi\pi a$ processes are identical.

● Dalitz plots to show the NLO/LO convergence

$$\left(2\mathcal{M}_{\eta;\pi\pi a}^{(2)} \text{Re}(\mathcal{M}_{\eta;\pi\pi a}^{(4)}) + |\mathcal{M}_{\eta;\pi\pi a}^{(4)}|^2 \right) / |\mathcal{M}_{\eta;\pi\pi a}^{(2)}|^2$$

[Wang,ZHG,Lu,Zhou, JHEP'24]



Important lessons:

- Non-perturbative effect in the $\pi\pi$ subsystem can be important.
- Perturbative treatment of the $a\pi$ subsystem is justified.

● Unitarization of the partial-wave $\eta \rightarrow \pi\pi a$ amplitude

$$\mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni}}(s) = \frac{\mathcal{M}_{\eta;\pi\pi a}^{00,\text{L}}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)} ,$$

$$G_{\pi\pi}(s) = -\frac{1}{(4\pi)^2} \left(\log \frac{m_\pi^2}{\mu^2} - \sigma_\pi(s) \log \frac{\sigma_\pi(s) - 1}{\sigma_\pi(s) + 1} - 1 \right) ,$$

$$\mathcal{M}_{\eta;\pi\pi a}^{00,\text{L}}(s) = \mathcal{M}_{\eta;\pi\pi a}^{00,(2)}(s) + \mathcal{M}_{\eta;\pi\pi a}^{00,(4)}(s) - G_{\pi\pi}(s) \mathcal{M}_{\eta;\pi\pi a}^{00,(2)}(s) T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s) .$$

The unitarized amplitude satisfies the relation

$$\text{Im} \mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni}}(s) = \rho_{\pi\pi}(s) \mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni}}(s) \left(T_{\pi\pi \rightarrow \pi\pi}^{00,\text{Uni}}(s) \right)^* , \quad (2m_\pi < \sqrt{s} < 2m_K)$$

with the unitarized PW $\pi\pi$ amplitude $T_{\pi\pi \rightarrow \pi\pi}^{00,\text{Uni}}(s) = \frac{T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)}$

● Unitarized PW amplitude based on LO $\eta \rightarrow \pi\pi a$ amplitude

$$\mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni-LO}}(s) = \frac{\mathcal{M}_{\eta;\pi\pi a}^{00,(2)}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)} .$$

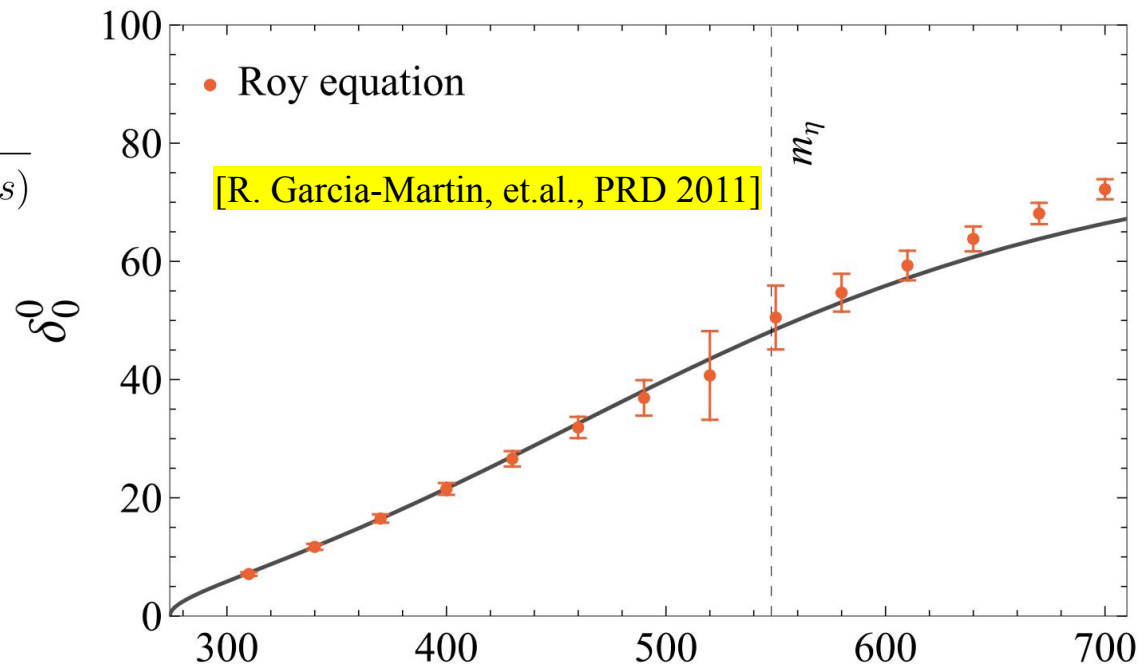
Resemble the method:

[Alves, Gonzalez-Solis, JHEP'24]

$$M_0(s) = P(s)\Omega_0^0(s)$$

Phase shifts from the unitarized PW $\pi\pi$ amplitude

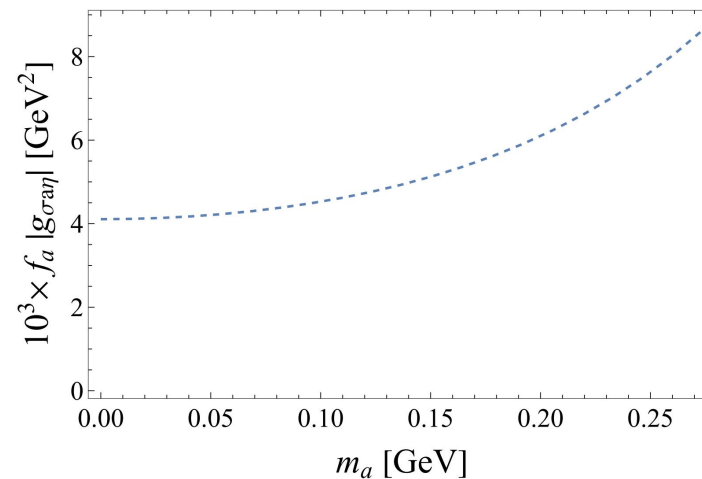
$$T_{\pi\pi\rightarrow\pi\pi}^{00,\text{Uni}}(s) = \frac{T_{\pi\pi\rightarrow\pi\pi}^{00,(2)}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi\rightarrow\pi\pi}^{00,(2)}(s)}$$



- Pole position of $f_0(500)/\sigma$:

$$\sqrt{s_\sigma} = 457 \pm i251 \text{ MeV}$$

$$\mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni},\text{II}}(s) \Big|_{s \rightarrow s_\sigma} \sim - \frac{g_{\sigma\pi\pi} g_{\sigma a \eta}}{s - s_\sigma}$$



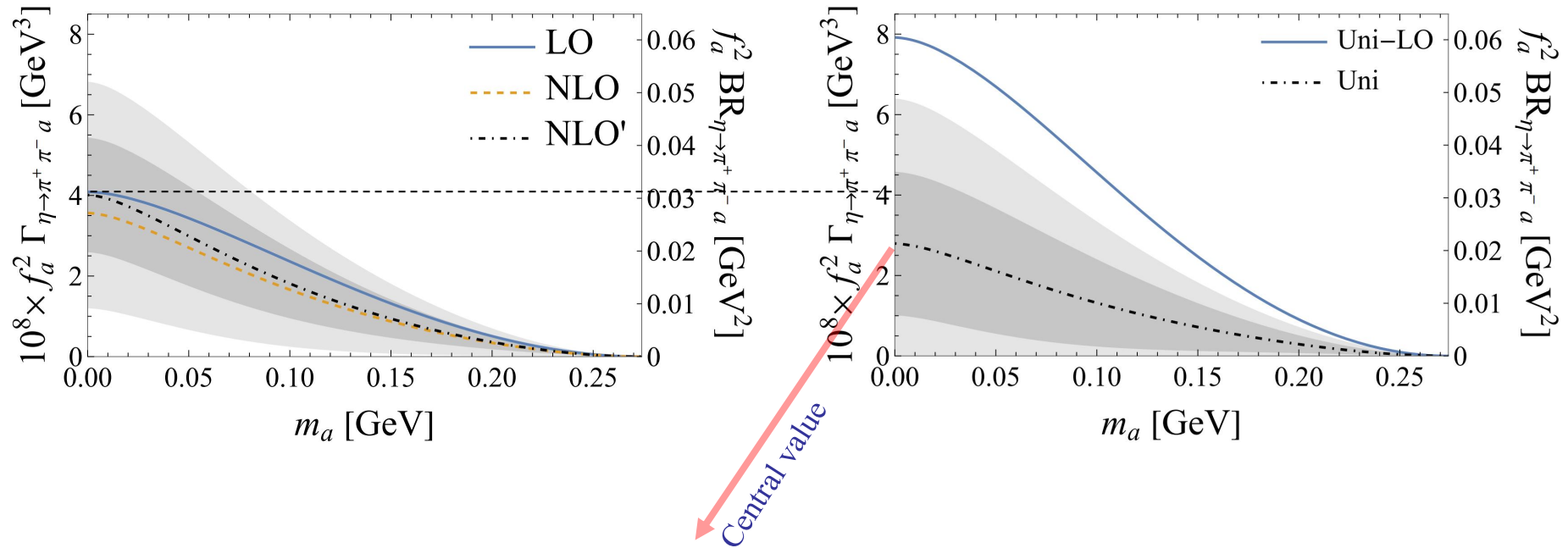
Predictions of the $\eta \rightarrow \pi\pi a$ branching ratios by varying m_a

Uncertainty bands:

- **Lighter regions:**

L_1^r	L_2^r	L_3^r	L_4^r	L_5^r	L_6^r	L_7^r	L_8^r
1.0(1)	1.6(2)	-3.8(3)	0.0(3)	1.2(1)	0.0(4)	-0.3(2)	0.5(2)
- **Darker regions:** freeze the $1/N_c$ suppressed ones (L_4, L_6, L_7)

[Wang,ZHG,Lu,Zhou, JHEP'24]



$$\text{BR}_{\eta \rightarrow \pi^+ \pi^- a} \Big|_{m_a \rightarrow 0} = 2.1 \times 10^{-2} \left(\frac{\text{GeV}^2}{f_a^2} \right)$$

Possible detection channels: $a \rightarrow \gamma\gamma$, $a \rightarrow e^+e^-$, $a \rightarrow \mu^+\mu^-$

Summary

- **Chiral effective field theory provides a systematical and useful framework to study the axion-hadron reactions.**
- **Synergies of Lattice QCD, hadron phenomenologies and chiral EFT are demonstrated to be powerful to build axion amplitudes.**
- **Futher involvements with the experiments, cosmology, astronomy are needed to set up stronger constraints on axion parameters!**

谢谢！