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# **Hadronic enhancements of axion reactions in chiral effective field thoery**



**Zhi-Hui Guo (郭志辉)**

**Hebei Normal University (河北师范大学)**

# Introduction

## Strong CP problem

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q}(iD - m_q e^{i\theta_q})q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \theta \frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$q \rightarrow e^{i\gamma_5 \alpha} q \quad \longrightarrow \quad \theta_q \rightarrow \theta_q + 2\alpha, \quad \theta \rightarrow \theta - 2\alpha$$

implying the invariant quantity:  $\bar{\theta} = \theta + \theta_q$


$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q}(iD - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \boxed{\bar{\theta} \frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

- $G_{\mu\nu}\tilde{G}^{\mu\nu} \sim \partial_\mu K^\mu$ : this total derivative is relevant, due to the nonperturbative QCD vacuum
- Naive guess from CKM:  $\theta_{\text{CPV}} \sim \mathbf{O}(1)$
- Experimental constraints from neutron EDM:  $\bar{\theta} \leq 10^{-10}$

➤ Strong CP problem: why  $\bar{\theta}$  unnaturally tiny ?

# Peccei-Quinn mechanism to address strong CP problem

[Peccei, Quinn, PRL '77]

[Weinberg, PRL '78] [Wilczek, PRL '78]

- Promote constant  $\bar{\theta}$  as a dynamical spin-0 field  $a(x)$
- Impose new global U(1) PQ symmetry (anomalous under QCD)

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a} \frac{g_s^2}{32\pi^2} G \tilde{G}$$

$$a(x) \rightarrow a(x) + \kappa f_a \implies S \rightarrow S + \frac{g_s^2 \kappa}{32\pi^2} \int d^4x G \tilde{G} \quad (\text{cancel } \bar{\theta} \text{ term})$$

- Vafa-Witten theorem: VEV of  $\langle a \rangle = 0$  in the vector-like theory, such as QCD
- Weinberg and Wilczek: PQ mechanism indicates a pseudo-Nambu-Goldstone boson
- This pNGB strips off the unwanted strong CP phase: Wilczek names it as **Axion**

中文: 轴子 (轴矢流耦合)

$$\frac{\partial_\mu a}{2f_a} \bar{q} \gamma^\mu \gamma_5 Q_a q$$



- Original PQWW axion:  $f_a \sim v_{EW} \approx 246 \text{ GeV}$  (visible axion)  
quickly ruled out by experiments:  $K \rightarrow \pi a$ ,  $J/\psi \rightarrow \gamma a$ ,  $\Upsilon \rightarrow \gamma a$ , ... ...  
astrophysical constraints: Supernovae, Red giant, ... ... ( $NN \rightarrow NNa$ )

# Generic effective axion Lagrangian for light-flavor quarks

$$\mathcal{L}_{\text{QCD}}^{\text{axion}} = \bar{q}(iD - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_{\mu}a\partial^{\mu}a - \frac{1}{2}m_{a,0}^2a^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4}g_{a\gamma}^0aF\tilde{F} + \frac{\partial_{\mu}a}{2f_a}\bar{q}c_q^0\gamma^{\mu}\gamma_5q$$

## Diverse viable axion models

- **PQWW:**  $Y_{\text{PQ}}$  (SM fermion)  $\neq 0$ ,  $f_a \sim v_{\text{EW}}$  (ruled out)
- **KSVZ:**  $Y_{\text{PQ}}$  (SM fermion)  $= 0$ , singlet Higgs and extra BSM fermions,  $f_a \gg v_{\text{EW}}$  (invisible axion)  
model-dependent terms vanish:  $g_{a\gamma}^0 = 0, c_q^0 = 0$
- **DFSZ:**  $Y_{\text{PQ}}$  (SM fermion)  $\neq 0$ , extra singlet and doublet Higgs,  $f_a \gg v_{\text{EW}}$  (invisible)  
model-dependent terms retain:  $g_{a\gamma}^0 \neq 0, c_q^0 \neq 0$
- **QCD axion / ALP (axion-like particle):** bare axion mass term  $m_{a,0} = 0$  /  $m_{a,0} \neq 0$   
ALP case:  $m_a$  and  $f_a$  are independent

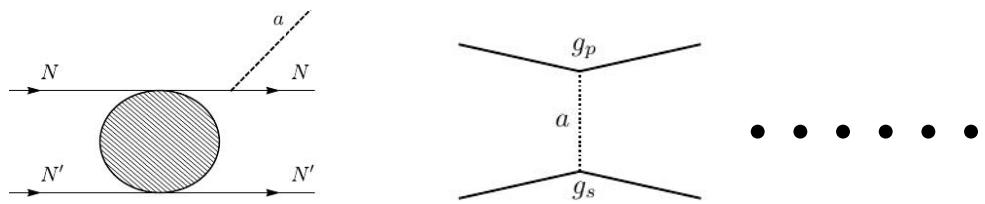
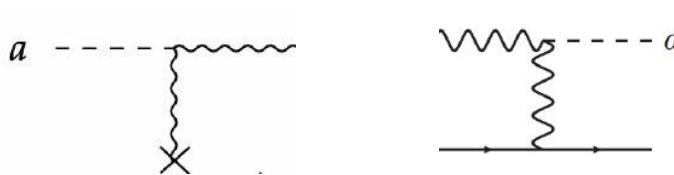
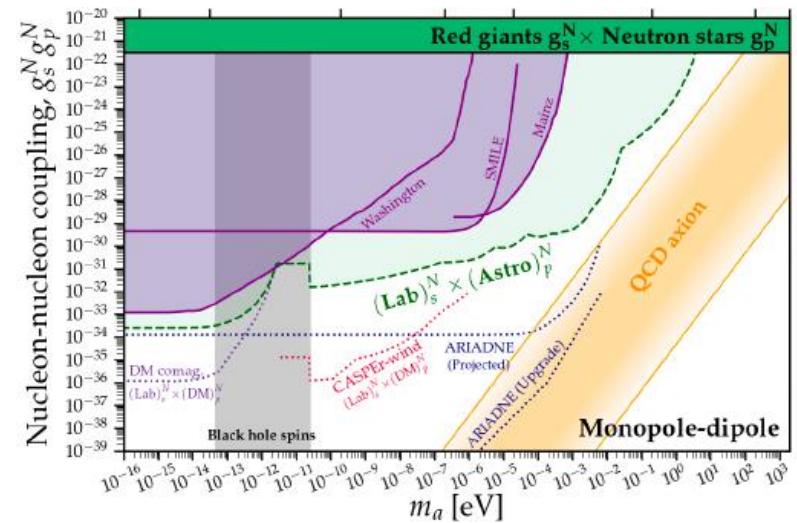
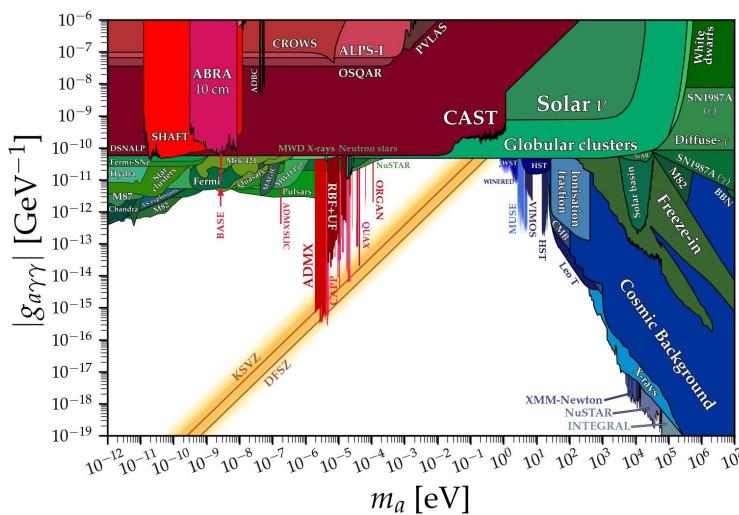
QCD axion case:  $m_a^2 \cong \frac{m_{\pi}^2 f_{\pi}^2}{4f_a^2}$  ( $f_a \gg f_{\pi}$ , axion: a very light BSM particle)

- Various constraints from rather different experiments

[Di Luzio, et al., Phy.Rep'20] [Sikivie, RMP'21] [Irastorza, Redondo,PPNP'18] ... ...

Cosmology, Astronomy, Colliders, Quantum precision measurements , Cavity Haloscope, ... ...

[O'Hare,Github, <https://cajohare.github.io/AxionLimits/>]



# Axion chiral perturbation theory

## Axion chiral perturbation theory (AxPT)

- We will focus on the QCD-like axion:  $m_{a,0} (\neq 0) \ll f_a$  with model-independent  $aG\tilde{G}$  interaction, i.e., the **MODEL INDEPENDENT QCD axion interactions**.
- Axion-hadron interactions are relevant at low energies.

$$\mathcal{L}_{\text{QCD}}^{\text{axion}} = \bar{q}(iD - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_{\mu}a\partial^{\mu}a - \frac{1}{2}m_{a,0}^2a^2 + \boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

Two ways to proceed:

(1) Remove the  $aG\tilde{G}$  term via the quark axial transformation

$$\begin{aligned} q &\rightarrow e^{i\frac{a}{2f_a}\gamma_5 Q_a} q \\ \text{Tr}(Q_a) = 1 \quad \text{curved arrow} \quad & - \frac{a\alpha_s}{8\pi f_a} G\tilde{G} - \frac{\partial_{\mu}a}{2f_a} \bar{q}\gamma^{\mu}\gamma_5 Q_a q \quad M_q \rightarrow M_q(a) = e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \end{aligned}$$

Mapping to  $\chi$ PT

$$\mathcal{L}_2 = \frac{F^2}{4} \langle D_{\mu}UD^{\mu}U^{\dagger} + \chi_a U^{\dagger} + U\chi_a^{\dagger} \rangle + \frac{\partial_{\mu}a}{2f_a} J_A^{\mu} \Big|_{\text{LO}}$$

$$\chi_a = 2B_0 e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \quad J_A^{\mu} \Big|_{\text{LO}} = -i\frac{F^2}{2} \langle Q_a(\partial^{\mu}UU^{\dagger} + U^{\dagger}\partial^{\mu}U) \rangle$$

- $Q_a = M_q^{-1}/\text{Tr}(M_q^{-1})$  [Georgi,Kaplan,Randall, PLB'86]
- $J_A^{\mu} \partial_{\mu}a$  [Bauer, et al., PRL'21]

## (2) Explicitly keep the $aG\tilde{G}$ term and match it to $\chi$ PT

Reminiscent:

QCD  $U(1)_A$  anomaly that is caused by topological charge density  $\omega(x) = \alpha_s G_{\mu\nu} \tilde{G}^{\mu\nu} / (8\pi)$  is responsible for the massive singlet  $\eta_0$ .

Axion could be similarly included as the  $\eta_0$  via the  $U(3)$   $\chi$ PT:

$$\mathcal{L}^{\text{LO}} = \frac{F^2}{4} \langle u_\mu u^\mu \rangle + \frac{F^2}{4} \langle \chi_+ \rangle + \frac{F^2}{12} M_0^2 X^2$$

$$\mathcal{L}^{\text{NLO}} = L_5 \langle u^\mu u_\mu \chi_+ \rangle + \frac{L_8}{2} \langle \chi_+ \chi_+ + \chi_- \chi_- \rangle - \frac{F^2 \Lambda_1}{12} D^\mu X D_\mu X - \frac{F^2 \Lambda_2}{12} X \langle \chi_- \rangle ,$$

$$U = u^2 = e^{i \frac{\sqrt{2}\Phi}{F}} , \quad \chi = 2B(s + ip) , \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$

$$u_\mu = iu^\dagger D_\mu U u^\dagger , \quad D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$$

$$X = \log(\det U) - i \frac{a}{f_a} \quad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

- $Q_a$  is not needed in  $U(3)$   $\chi$ PT.
- $M_0^2 = 6\tau/F^2$ , with  $\tau$  the topological susceptibility. Note that  $M_0^2 \sim O(1/N_c)$ .
- $\delta$  expansion scheme:  $\delta \sim O(p^2) \sim O(m_q) \sim O(1/N_c)$ .
- Axion interactions enter via the axion-meson mixing terms.

LO

(mass mixing only)

$$\begin{pmatrix} \bar{\pi}^0 \\ \bar{\eta} \\ \bar{\eta}' \\ \bar{a} \end{pmatrix} = \begin{pmatrix} 1 + v_{11} & -v_{12} & -v_{13} & -v_{14} \\ v_{12} & 1 + v_{22} & -v_{23} & -v_{24} \\ v_{13} & v_{23} & 1 + v_{33} & -v_{34} \\ v_{41} & v_{42} & v_{43} & 1 + v_{44} \end{pmatrix} \begin{pmatrix} \pi^0 \\ \overset{\circ}{\eta} \\ \overset{\circ}{\eta}' \\ a \end{pmatrix}$$

$$v_{12} = -\frac{\epsilon}{\sqrt{3}} \frac{c_\theta - \sqrt{2}s_\theta}{m_\pi^2 - m_{\overset{\circ}{\eta}}^2}, \quad v_{13} = -\frac{\epsilon}{\sqrt{3}} \frac{\sqrt{2}c_\theta + s_\theta}{m_\pi^2 - m_{\overset{\circ}{\eta}'}^2}, \quad v_{23} = \frac{\sqrt{2}s_\theta^2 + c_\theta s_\theta - \sqrt{2}c_\theta^2}{3(m_{\overset{\circ}{\eta}'}^2 - m_{\overset{\circ}{\eta}}^2)} \epsilon, \quad v_{41} = -\frac{M_0^2 \epsilon}{6(m_a^2 - m_\pi^2)} \frac{F}{f_a} \left[ -\frac{(\sqrt{2}c_\theta - 2s_\theta)s_\theta}{m_a^2 - m_{\overset{\circ}{\eta}}^2} + \frac{c_\theta(2c_\theta + \sqrt{2}c_\theta s_\theta + s_\theta^2)}{m_a^2 - m_{\overset{\circ}{\eta}'}^2} \right]$$

$$v_{42} = \frac{M_0^2 s_\theta}{\sqrt{6}(m_a^2 - m_{\overset{\circ}{\eta}}^2)} \frac{F}{f_a} - \frac{M_0^2 \epsilon}{3\sqrt{6}(m_a^2 - m_{\overset{\circ}{\eta}'}^2)} \frac{F}{f_a} \left[ \frac{c_\theta(-\sqrt{2}c_\theta^2 + c_\theta s_\theta + \sqrt{2}s_\theta^2)}{m_a^2 - m_{\overset{\circ}{\eta}'}^2} - \frac{s_\theta(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2)}{m_a^2 - m_{\overset{\circ}{\eta}}^2} \right]$$

$$v_{43} = -\frac{M_0^2 c_\theta}{\sqrt{6}(m_a^2 - m_{\overset{\circ}{\eta}'}^2)} \frac{F}{f_a} - \frac{M_0^2 \epsilon}{3\sqrt{6}(m_a^2 - m_{\overset{\circ}{\eta}}^2)} \frac{F}{f_a} \left[ \frac{c_\theta(c_\theta^2 - 2\sqrt{2}c_\theta s_\theta + 2s_\theta^2)}{m_a^2 - m_{\overset{\circ}{\eta}'}^2} - \frac{s_\theta(-\sqrt{2}c_\theta^2 + c_\theta s_\theta + \sqrt{2}s_\theta^2)}{m_a^2 - m_{\overset{\circ}{\eta}}^2} \right]$$

... ...

with

$$m_{\overset{\circ}{\eta}}^2 = \frac{M_0^2}{2} + m_K^2 - \frac{\sqrt{M_0^4 - \frac{4M_0^2\Delta^2}{3} + 4\Delta^4}}{2}, \quad m_{\overset{\circ}{\eta}'}^2 = \frac{M_0^2}{2} + m_K^2 + \frac{\sqrt{M_0^4 - \frac{4M_0^2\Delta^2}{3} + 4\Delta^4}}{2}, \quad \sin \theta = -\left( \sqrt{1 + \frac{(3M_0^2 - 2\Delta^2 + \sqrt{9M_0^4 - 12M_0^2\Delta^2 + 36\Delta^4})^2}{32\Delta^4}} \right)^{-1}$$

## Physical masses after diagonalization

$$m_{\overset{\circ}{\eta}}^2 = m_{\overset{\circ}{\eta}}^2 + \frac{\epsilon}{3}(\sqrt{2}c_\theta + s_\theta)^2 + O(\epsilon^2)$$

$$m_{\overset{\circ}{\eta}'}^2 = m_{\overset{\circ}{\eta}'}^2 + \frac{\epsilon}{3}(c_\theta - \sqrt{2}s_\theta)^2 + O(\epsilon^2)$$

$$\begin{aligned} m_a^2 &= m_{a,0}^2 + \frac{M_0^2 F^2}{6f_a^2} \left[ 1 + \frac{c_\theta^2 M_0^2}{m_{a,0}^2 - m_{\overset{\circ}{\eta}}^2} + \frac{s_\theta^2 M_0^2}{m_{a,0}^2 - m_{\overset{\circ}{\eta}'}^2} \right] \\ &+ \frac{M_0^4 F^2 \epsilon}{9f_a^2} \left[ \frac{s_\theta^2 (\sqrt{2}c_\theta + s_\theta)^2}{2(m_{a,0}^2 - m_{\overset{\circ}{\eta}}^2)^2} + \frac{c_\theta^2 (c_\theta - \sqrt{2}s_\theta)^2}{2(m_{a,0}^2 - m_{\overset{\circ}{\eta}'}^2)^2} \right. \\ &\left. + \frac{c_\theta s_\theta (\sqrt{2}c_\theta^2 - c_\theta s_\theta - \sqrt{2}s_\theta^2)}{(m_{a,0}^2 - m_{\overset{\circ}{\eta}}^2)(m_{a,0}^2 - m_{\overset{\circ}{\eta}'}^2)} \right] + O(\epsilon^2), \end{aligned}$$



$$m_a^2 = \frac{m_\pi^2 F^2}{4f_a^2}$$

[Weinberg, PRL'78]

(keep LO terms in  $m_\pi/m_K$  &  $m_\pi/M_0$  &  $\epsilon$  expansions)

## NLO: (kinetic & mass mixing)

$$\begin{aligned}
\mathcal{L} = & \frac{1 + \delta_k^\eta}{2} \partial_\mu \bar{\eta} \partial^\mu \bar{\eta} + \frac{1 + \delta_k^{\eta'}}{2} \partial_\mu \bar{\eta}' \partial^\mu \bar{\eta}' + \delta_k^{\eta\eta'} \partial_\mu \bar{\eta} \partial^\mu \bar{\eta}' - \frac{m_\eta^2 + \delta_{m_\eta^2}}{2} \bar{\eta} \bar{\eta} - \frac{m_{\eta'}^2 + \delta_{m_{\eta'}^2}}{2} \bar{\eta}' \bar{\eta}' - \delta_{m^2}^{\eta\eta'} \bar{\eta} \bar{\eta}' \\
& + \frac{1 + \delta_k^\pi}{2} \partial_\mu \bar{\pi}^0 \partial^\mu \bar{\pi}^0 + \delta_k^{\pi\eta} \partial_\mu \bar{\pi}^0 \partial^\mu \bar{\eta} + \delta_k^{\pi\eta'} \partial_\mu \bar{\pi}^0 \partial^\mu \bar{\eta}' - \frac{m_\pi^2 + \delta_{m_\pi^2}}{2} \bar{\pi}^0 \bar{\pi}^0 - \delta_{m^2}^{\pi\eta} \bar{\pi}^0 \bar{\eta} - \delta_{m^2}^{\pi\eta'} \bar{\pi}^0 \bar{\eta}' \\
& + \frac{1 + \delta_k^a}{2} \partial_\mu \bar{a} \partial^\mu \bar{a} + \delta_k^{a\pi} \partial_\mu \bar{a} \partial^\mu \bar{\pi}^0 + \delta_k^{a\eta} \partial_\mu \bar{a} \partial^\mu \bar{\eta} + \delta_k^{a\eta'} \partial_\mu \bar{a} \partial^\mu \bar{\eta}' - \frac{m_a^2 + \delta_{m_a^2}}{2} \bar{a} \bar{a} - \delta_{m^2}^{a\pi} \bar{a} \bar{\pi}^0 \\
& - \delta_{m^2}^{a\eta} \bar{a} \bar{\eta} - \delta_{m^2}^{a\eta'} \bar{a} \bar{\eta}' 
\end{aligned}$$

Separately handle the kinetic ( $x_{ij}$ ) and mass ( $y_{ij}$ ) mixing terms

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = \begin{pmatrix} 1 & -y_{12} & -y_{13} & -y_{14} \\ y_{12} & 1 & -y_{23} & -y_{24} \\ y_{13} & y_{23} & 1 & -y_{34} \\ y_{14} & y_{24} & y_{34} & 1 \end{pmatrix} \times \begin{pmatrix} 1 - x_{11} & -x_{12} & -x_{13} & -x_{14} \\ -x_{12} & 1 - x_{22} & -x_{23} & -x_{24} \\ -x_{13} & -x_{23} & 1 - x_{33} & -x_{34} \\ -x_{14} & -x_{24} & -x_{34} & 1 - x_{44} \end{pmatrix} \begin{pmatrix} \bar{\pi}^0 \\ \bar{\eta} \\ \bar{\eta}' \\ \bar{a} \end{pmatrix}$$

$$\begin{aligned}
x_{11} &= -\frac{\delta_k^\pi}{2}, & x_{12} &= -\frac{\delta_k^{\pi\eta}}{2}, & x_{13} &= -\frac{\delta_k^{\pi\eta'}}{2}, & x_{14} &= -\frac{\delta_k^{a\pi}}{2}, & x_{22} &= -\frac{\delta_k^\eta}{2}, \\
x_{23} &= -\frac{\delta_k^{\eta\eta'}}{2}, & x_{24} &= -\frac{\delta_k^{a\eta}}{2}, & x_{33} &= -\frac{\delta_k^{\eta'}}{2}, & x_{34} &= -\frac{\delta_k^{a\eta'}}{2}, & x_{44} &= -\frac{\delta_k^a}{2}, 
\end{aligned}$$

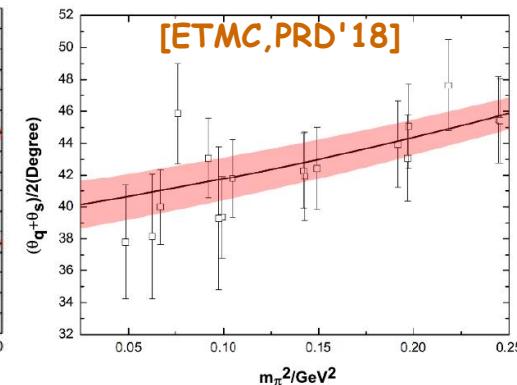
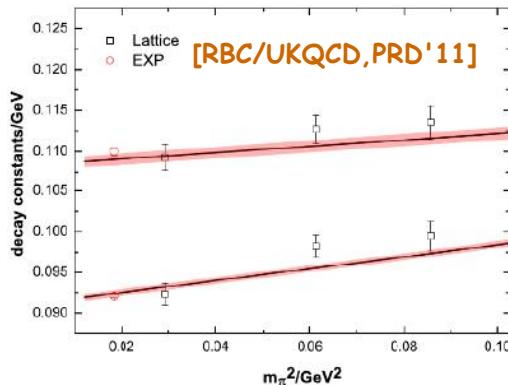
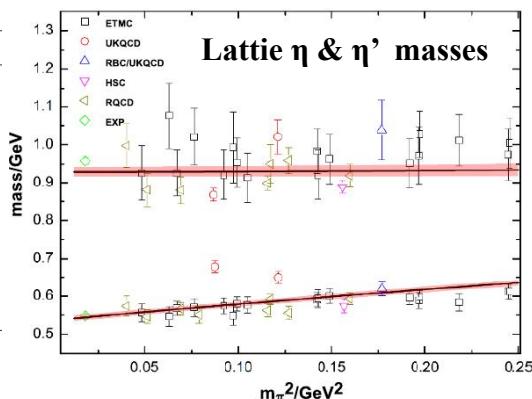
$$\delta_X \sim \mathbf{L}_5, \mathbf{L}_8, \mathbf{\Lambda}_1, \mathbf{\Lambda}_2$$

$$\begin{aligned}
y_{12} &= \frac{\delta_{m^2}^{\pi\eta} + x_{12}(m_\eta^2 + m_\pi^2)}{m_\eta^2 - m_\pi^2}, & y_{13} &= \frac{\delta_{m^2}^{\pi\eta'} + x_{13}(m_{\eta'}^2 + m_\pi^2)}{m_{\eta'}^2 - m_\pi^2}, & y_{14} &= \frac{\delta_{m^2}^{a\pi} + x_{14}(m_{a,0}^2 + m_\pi^2)}{m_{a,0}^2 - m_\pi^2}, \\
y_{23} &= \frac{\delta_{m^2}^{\eta\eta'} + x_{23}(m_\eta^2 + m_{\eta'}^2)}{m_\eta^2 - m_{\eta'}^2}, & y_{24} &= \frac{\delta_{m^2}^{a\eta} + x_{24}(m_\eta^2 + m_{a,0}^2)}{m_{a,0}^2 - m_\eta^2}, & y_{34} &= \frac{\delta_{m^2}^{a\eta'} + x_{34}(m_{\eta'}^2 + m_{a,0}^2)}{m_{a,0}^2 - m_{\eta'}^2}.
\end{aligned}$$

# Fit to lattice data

[Gao, ZHG, Oller, Zhou, JHEP'23] [Gao, Hao, ZHG, et al., EPJC'25]

Parameters	NLO Fit
$F(\text{MeV})$	$91.05^{+0.42}_{-0.44}$
$10^3 \times L_5$	$1.68^{+0.05}_{-0.06}$
$10^3 \times L_8$	$0.88^{+0.04}_{-0.04}$
$\Lambda_1$	$-0.17^{+0.05}_{-0.05}$
$\Lambda_2$	$0.06^{+0.08}_{-0.09}$
$\chi^2/(\text{d.o.f.})$	$219.9/(111-5)$



## Mixing pattern@NLO

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = M^{\text{LO+NLO}} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix}$$

$$M^{\text{LO+NLO}} = \begin{pmatrix} 1 + (0.015 \pm 0.001) & 0.017 + (-0.007 \pm 0.001) & 0.009 + (-0.011 \pm 0.001) \\ -0.019 + (0.005 \pm 0.001) & 0.94 + (0.21 \pm 0.01) & 0.33 + (-0.21 \pm 0.03) \\ -0.003 + (-0.001 \pm 0.000) & -0.33 + (-0.18 \pm 0.02) & 0.94 + (0.13^{+0.01}_{-0.02}) \\ \frac{12.1 + (0.5 \pm 0.1)}{f_a} & \frac{23.8 + (1.0^{+0.2}_{-0.1})}{f_a} & \frac{35.7 + (1.7^{+0.2}_{-0.1})}{f_a} \end{pmatrix}$$

$$1 + \frac{-12.8 + (-0.13 \pm 0.02)}{f_a} \\ \frac{-34.3 + (1.7^{+0.8}_{-0.7})}{f_a} \\ \frac{-25.9 + (0.2^{+0.4}_{-0.3})}{f_a} \\ 1 + \frac{-921.5 + (-56.6^{+7.9}_{-9.6})}{f_a^2}$$

## Mass decomposition@NLO

$$\begin{aligned} m_{\hat{\pi}} &= [134.9 + (0.1 \pm 0.07)] \text{ MeV}, \\ m_{\hat{K}} &= [492.1 + (5.1^{+3.4}_{-3.3})] \text{ MeV}, \\ m_{\hat{\eta}} &= [490.4 + (61.1^{+10.0}_{-8.7})] \text{ MeV}, \\ m_{\hat{\eta}'} &= [954.5 + (-28.5^{+11.9}_{-10.9})] \text{ MeV}, \\ m_{\hat{a}} &= [5.96 + (0.12 \pm 0.02)] \mu \text{ eV} \frac{10^{12} \text{ GeV}}{f_a}, \end{aligned}$$

## Two-photon couplings

$$\mathcal{L}_{WZW}^{\text{LO}} = -\frac{3\sqrt{2}}{8\pi^2 F} \varepsilon_{\mu\nu\rho\sigma} \partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2 \Phi \rangle, \quad Q = \text{Diag}(\frac{2e}{3}, -\frac{e}{3}, -\frac{e}{3})$$

$$\mathcal{L}_{WZW}^{\text{NLO}} = t_1 \frac{32\sqrt{2}B}{F} \varepsilon_{\mu\nu\rho\sigma} \partial^\mu A^\nu \partial^\rho A^\sigma \langle (M_q \Phi + \Phi M_q) Q^2 \rangle + 16k_3 \varepsilon_{\mu\nu\rho\sigma} \partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2 \rangle \left( \frac{\sqrt{2}}{F} \langle \Phi \rangle - \frac{a}{f_a} \right)$$

\* Note: one needs the  $\pi$ - $\eta$ - $\eta'$ - $a$  mixing as input to calculate  $g_{a\gamma\gamma}$

$$F_{\pi^0\gamma\gamma}^{\text{Exp}} = 0.274 \pm 0.002 \text{ GeV}^{-1},$$

$$F_{\eta\gamma\gamma}^{\text{Exp}} = 0.274 \pm 0.006 \text{ GeV}^{-1},$$

$$F_{\eta'\gamma\gamma}^{\text{Exp}} = 0.344 \pm 0.008 \text{ GeV}^{-1},$$



$$t_1 = -(3.8 \pm 2.4) \times 10^{-4} \text{ GeV}^{-2},$$

$$k_3 = (1.21 \pm 0.23) \times 10^{-4}$$

isospin limit(LO)    isospin breaking(LO)    NLO

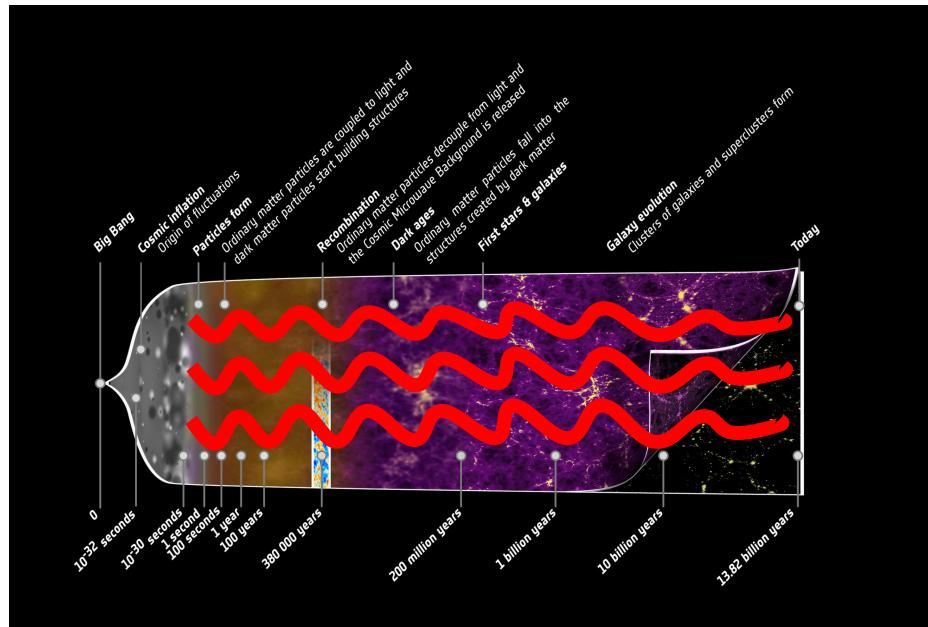
$$F_{a\gamma\gamma} = \frac{20.1 + 3.4 + (0.5 \pm 0.2)}{f_a} \times 10^{-3},$$

( IB corrections amount to be around 15%!)

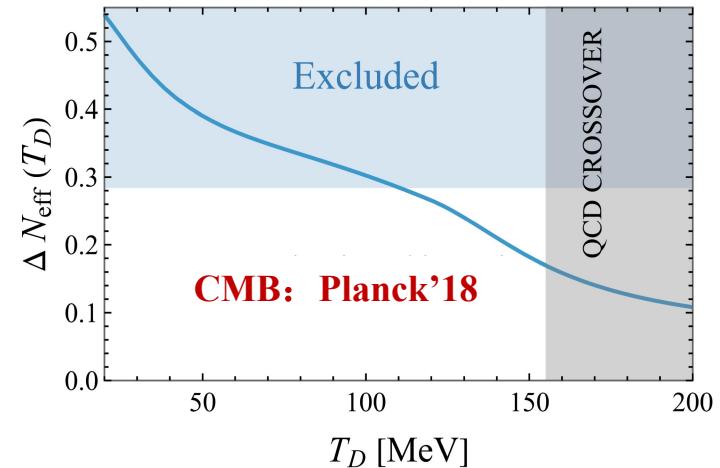
$$g_{a\gamma\gamma} = 4\pi\alpha_{em} F_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a} (1.89 \pm 0.02).$$

which can be compared to:  $1.92 \pm 0.04$  [Grilli de Cortona, et al., JHEP'16] and  $2.05 \pm 0.03$  [Lu, et al., JHEP'20]

# Cosmology constraints on axion thermalization rate



- **Axions can be copiously produced from thermal bath in the early Universe.**
- **After decoupling, its thermal relics will leave imprints today.**



- Axion thermalization rate from reaction:  $\underbrace{a + \dots}_{i} \leftrightarrow \underbrace{1 + 2 + 3 + \dots}_{j}$

$$\Gamma_a(T) = \frac{1}{n_a^{eq}} \int \left[ \prod_i \frac{d^3 p_i}{(2\pi)^3 2E_i} \right] \left[ \prod_j \frac{d^3 p_j}{(2\pi)^3 2E_j} \right] (2\pi)^4 \delta^4 \left( \sum_i p_i - \sum_j p_j \right) |\mathcal{M}_{\text{reaction}}|^2 \prod_i f_i(x_i) \prod_j [1 \pm f_j(x_j)]$$

$$f_i(x_i) = \begin{cases} \frac{1}{e^{x_i} - 1}, & \text{bosonic,} \\ \frac{1}{e^{x_i} + 1}, & \text{fermionic,} \end{cases} \quad x_i = E_i/T$$

“+” for Bose enhancement  
“-” for Pauli blocking

# Cosmology constraints on axion thermalization rate

Axion thermal production in the early Universe : Extra radiation ( $\Delta N_{\text{eff}}$ )

Extra effective number of relativistic d.o.f :

$$\Delta N_{\text{eff}} \simeq \frac{4}{7} \left( \frac{43}{4g_{\star s}(T_D)} \right)^{\frac{4}{3}}$$

$g_{\star s}(T)$  : effective number of entropy d.o.f at temperature  $T$

$T_D$ : axion decoupling temperature from the thermal medium

- CMB constraint (Plank'18) [Aghanim et al., 2020] :  $\Delta N_{\text{eff}} \leq 0.28$
- $T_D$  : Instantaneous decoupling approximation

$$\Gamma_a(T_D) = H(T_D)$$

Axion thermalization rate

Hubble expansion parameter

$$\Gamma_a(T) = \frac{1}{n_a^{\text{eq}}} \int d\tilde{\Gamma} |\mathcal{M}_{a-\text{SM}}|^2 \prod_i f_i(x_i) \prod_j [1 \pm f_j(x_j)] \quad H(T) = T^2 \sqrt{4\pi^3 g_*(T)/45} / m_{\text{Pl}}$$

Axion-SM particle scattering amplitudes

# Key thermal channels of axion-SM scatterings at different temperatures

- 👉  $T_D \gtrsim 1$  GeV:  $ag \leftrightarrow gg$ .  
[Masso et al., 2002, Graf and Steffen, 2011]
- 👉  $T_D \lesssim 1$  GeV: Hadrons need to be included.
- 👉  $T_D \lesssim 200$  MeV:  $a\pi \leftrightarrow \pi\pi$ .  
[Chang and Choi, 1993, Hannestad et al., 2005,  
Giusarma et al., 2014, D'Eramo et al., 2022]

- ◻ **Reliable  $a\pi$  interaction is crucial to determine  $\Gamma_a$  for  $T_a < T_c \approx 155$  MeV**
  - For a long time, only the LO  $a\pi \leftrightarrow \pi\pi$  amplitude is employed to calculate  $\Gamma_a$ , e.g.,  
[Chang, Choi, PLB'93] [Hannestad, et al., JCAP'05] [Hannestad, et al., JCAP'05] [D'Eramo, et al., PRL'22] ... ...
  - Recent NLO calculation of  $\Gamma_a$ :  $\chi$ PT invalid for  $T_\chi > 70$  MeV [Di Luzio, et al., PRL'21]
  - Chiral unitarization approach for  $a\pi \leftrightarrow \pi\pi$ : [Di Luzio, et al., PRD'23]
  - **However, all the previous works have ignored thermal corrections to the  $a\pi \leftrightarrow \pi\pi$  amplitudes. The first estimation of such effect is given: [Wang, ZHG, Zhou, PRD'24]**

$$\Gamma_a(T) = \frac{1}{n_a^{\text{eq}}} \int d\tilde{\Gamma} \boxed{|\mathcal{M}_{a\pi;\pi\pi}|^2} \prod_i f_i(x_i) \prod_j [1 + f_j(x_j)]$$

- **First realistic calculation of  $aK \leftrightarrow \pi K$  shows significant contribution to axion thermalization rate: [Wang, ZHG, Zhou, PRD'25]**

# Calculation of **THERMAL** $a\pi \leftrightarrow \pi\pi$ amplitudes at one-loop level

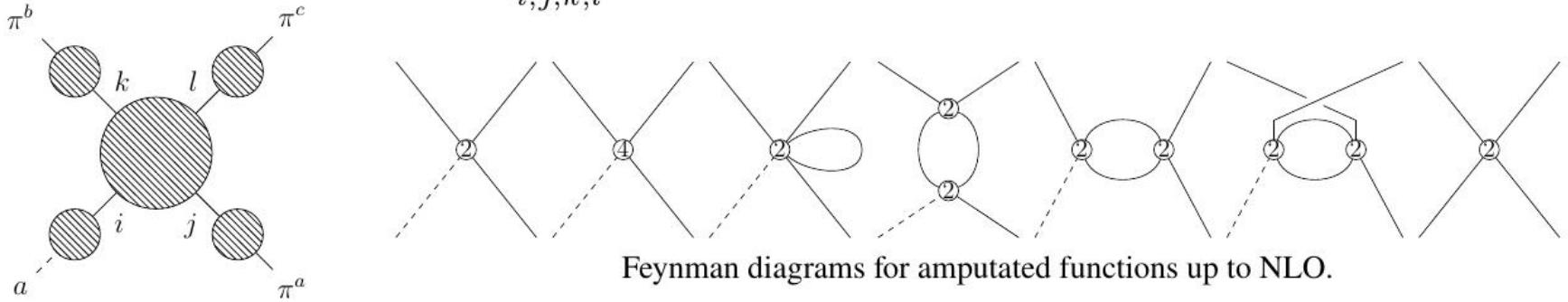
- Finite-temperature effects are included by imaginary time formalism (ITF), where  
[Kapusta and Gale, 2011, Bellac, 2011, Laine and Vuorinen, 2016]

$$p^0 \rightarrow i\omega_n, \quad \text{with } \omega_n = 2\pi n T, n \in \mathbb{Z},$$

$$-i \int \frac{d^d q}{(2\pi)^d} \rightarrow -i \int_{\beta} \frac{d^d q}{(2\pi)^d} \equiv T \sum_n \int \frac{d^{d-1} q}{(2\pi)^{d-1}}.$$

- Compute the thermal Green functions in ITF

$$G_{a\pi^a; \pi^b \pi^c}^T(p_1, p_2; p_3, p_4) = \sum_{i,j,k,l} G_{ai}(p_1^2) G_{\pi^a j}(p_2^2) G_{k\pi^b}(p_3^2) G_{l\pi^c}(p_4^2) A_{ij;kl}(p_1, p_2; p_3, p_4).$$



- The effective Lagrangian at  $\mathcal{O}(p^4)$

$$\begin{aligned} \mathcal{L}_4 &\supset \frac{l_3 + l_4}{16} \left\langle \chi_a U^\dagger + U \chi_a^\dagger \right\rangle \left\langle \chi_a U^\dagger + U \chi_a^\dagger \right\rangle + \frac{l_4}{8} \left\langle \partial_\mu U \partial^\mu U^\dagger \right\rangle \left\langle \chi_a U^\dagger + U \chi_a^\dagger \right\rangle & J_A^\mu|_{\text{NLO}} &\supset -il_1 \left\langle Q_a \left\{ \partial^\mu U, U^\dagger \right\} \right\rangle \left\langle \partial_\nu U \partial^\nu U^\dagger \right\rangle \\ &- \frac{l_7}{16} \left\langle \chi_a U^\dagger - U \chi_a^\dagger \right\rangle \left\langle \chi_a U^\dagger - U \chi_a^\dagger \right\rangle + \frac{h_1 - h_3 - l_4}{16} \left[ \left( \left\langle \chi_a U^\dagger + U \chi_a^\dagger \right\rangle \right)^2 \right. & - i \frac{l_2}{2} \left\langle Q_a \left\{ \partial_\nu U, U^\dagger \right\} \right\rangle \left\langle \partial^\mu U \partial^\nu U^\dagger + \partial^\nu U \partial^\mu U^\dagger \right\rangle \\ &\left. + \left( \left\langle \chi_a U^\dagger - U \chi_a^\dagger \right\rangle \right)^2 - 2 \left\langle \chi_a U^\dagger \chi_a U^\dagger + U \chi_a^\dagger U \chi_a^\dagger \right\rangle \right] + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\text{NLO}}, & - i \frac{l_4}{4} \left\langle Q_a \left\{ \partial^\mu U, U^\dagger \right\} \right\rangle \left\langle \chi_a U^\dagger + U \chi_a^\dagger \right\rangle. \end{aligned}$$

# Unitarization of the partial-wave $a\pi \leftrightarrow \pi\pi$ amplitude

## Inverse amplitude method (IAM)

$$\mathcal{M}_{a\pi;IJ}^{\text{IAM}} = \frac{\left(\mathcal{M}_{a\pi;IJ}^{(2)}\right)^2}{\mathcal{M}_{a\pi;IJ}^{(2)} - \mathcal{M}_{a\pi;IJ}^{(4)}}$$

$$\mathcal{M}_{a\pi;IJ}(E_{cm}) = \frac{1}{2} \int_{-1}^{+1} d\cos\theta \mathcal{M}_{a\pi;I}(E_{cm}, \cos\theta) P_J(\cos\theta)$$

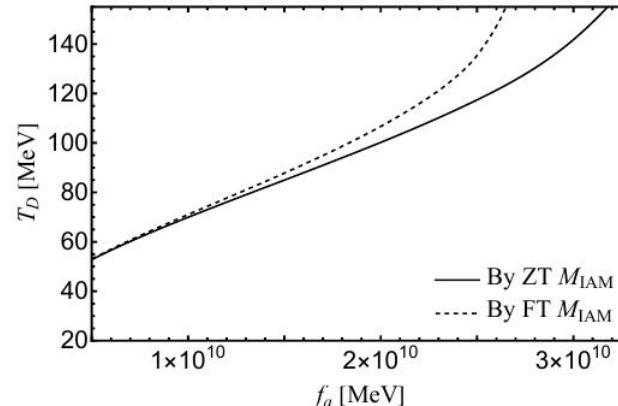
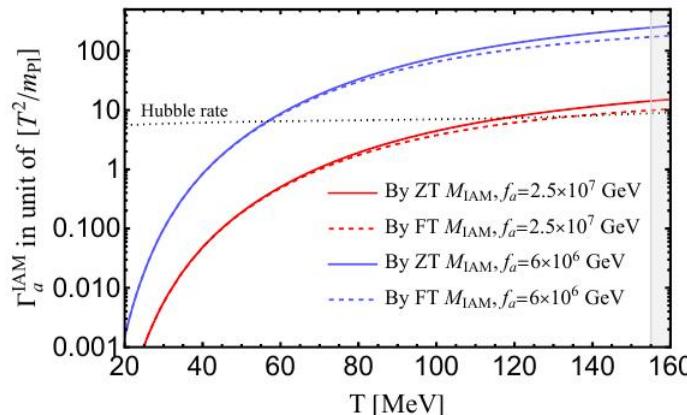
$$\text{Im}\mathcal{M}_{a\pi;IJ}(E_{cm}) \stackrel{s}{=} \frac{1}{2} \rho_{\pi\pi}^T(E_{cm}) \mathcal{M}_{\pi\pi;\pi\pi}^{IJ*} \mathcal{M}_{a\pi;IJ}, \quad (E_{cm} > 2m_\pi)$$

$$\rho_{\pi\pi}^T(E_{cm}) = \frac{\sigma_\pi(E_{cm}^2)}{16\pi} \left[ 1 + 2n_B\left(\frac{E_{cm}}{2}\right) \right], \quad \sigma_\pi(s) = \sqrt{1 - \frac{4m_\pi^2}{s}}, \quad n_B(E) = \frac{1}{e^{E/T} - 1}$$

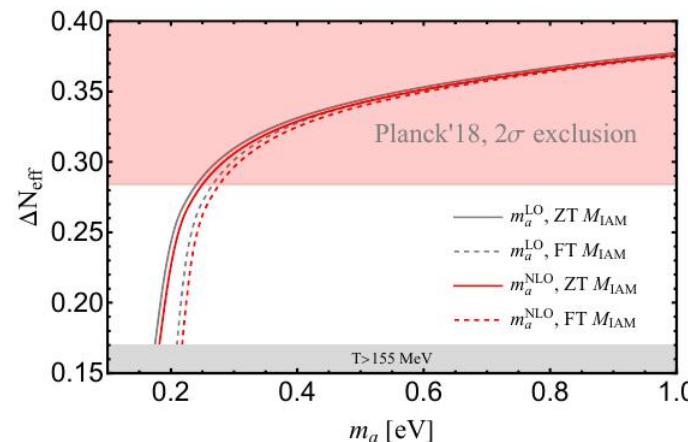
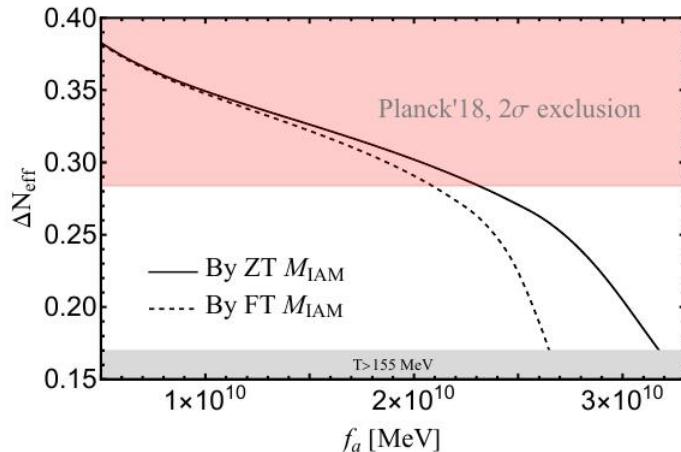
- Resonances poles on the second Riemann sheet

$f_0(500)/\sigma$		$\rho(770)$			
		$M_\sigma \pm i\frac{\Gamma_\sigma}{2}$	$ f_a g_{\sigma a\pi} $	$M_\rho \pm i\frac{\Gamma_\rho}{2}$	$ f_a g_{\rho a\pi} $
$T = 0 \text{ MeV}$	$422 \pm i240 \text{ MeV}$	$0.032 \text{ GeV}^2$		$739 \pm i72 \text{ MeV}$	$0.035 \text{ GeV}^2$
$T = 100 \text{ MeV}^*$	$368 \pm i310 \text{ MeV}$	$0.037 \text{ GeV}^2$		$744 \pm i77 \text{ MeV}$	$0.036 \text{ GeV}^2$

\*Only include  $s$ -channel unitary thermal correction.



## Updated bounds on the axion parameters



□ The constrians

**10% corrections are observed**

	lower limit of $f_a$	upper limit of $m_a$ by $m_a^{\text{LO}}$	upper limit of $m_a$ by $m_a^{\text{NLO}}$
ZT	$2.3 \times 10^7$ GeV	0.24 eV	0.25 eV
FT	$2.1 \times 10^7$ GeV	0.27 eV	0.28 eV

# Combined analyses with $a\pi \leftrightarrow \pi\pi$ & $aK \leftrightarrow \pi K$ channels

SU(3) Axion ChPT@LO

[Wang, ZHG, Zhou, PRD'25]

$$\mathcal{L}_2 = \frac{F_\pi^2}{4} \langle \partial_\mu U \partial^\mu U^\dagger + \chi(a) U^\dagger + U \chi^\dagger(a) \rangle - \frac{\partial_\mu a}{2f_a} \sum_{i=1}^8 C_i J_{A,i}^\mu$$

$$\chi(a) = 2B_0 e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \quad Q_a = M_q^{-1} / \langle M_q^{-1} \rangle \quad J_{A,i}^\mu = i \frac{F_\pi^2}{4} \langle \lambda_i \{ \partial^\mu U, U^\dagger \} \rangle \quad (\text{singlet component of axial currents neglected})$$

$$C_3 = \frac{z(1-r^2)}{2r+z(1+r)^2}, \quad C_8 = \frac{z(1+r)^2 - 4r}{\sqrt{3}[2r+z(1+r)^2]} \quad z = \frac{m_s}{\hat{m}}, r = \frac{m_u}{m_d}, \hat{m} = \frac{m_u+m_d}{2}$$

## Unitarized partial-wave axion-meson/meson-meson amplitudes

Unitarized meson-meson Amp:  $T_{IJ}^{\text{uni}} = T_{IJ}^{(2)} \cdot \left[ T_{IJ}^{(2)} - T_{IJ}^{(4)\text{LECs}} - T_{IJ}^{(2)} \cdot \mathcal{G} \cdot T_{IJ}^{(2)} \right]^{-1} \cdot T_{IJ}^{(2)}$

Unitarized axion-meson Amp:  $\vec{M}_{IJ}^{\text{uni}} = T_{IJ}^{(2)} \cdot \left[ T_{IJ}^{(2)} - T_{IJ}^{(4)\text{LECs}} - T_{IJ}^{(2)} \cdot \mathcal{G} \cdot T_{IJ}^{(2)} \right]^{-1} \cdot \vec{M}_{IJ}^{(2)}$

$$\mathcal{G} = \text{diag}(G_n, G_m, \dots) \quad G_n(s) = G(a_{\text{sc}}^n, s, m_{n_1}, m_{n_2}) = -\frac{1}{(4\pi)^2} \left[ a_{\text{sc}}^n - 1 + \log \frac{m_{n_2}^2}{\mu^2} + \frac{m_{n_2}^2 \sqrt{\lambda(s, m_{n_1}^2, m_{n_2}^2)}}{s} \log \frac{m_{n_1}^2 + m_{n_2}^2 - s + \sqrt{\lambda(s, m_{n_1}^2, m_{n_2}^2)}}{2m_{n_1}m_{n_2}} \right]$$

Example:  $\begin{pmatrix} M_{00, \pi\pi}^{\text{uni}} \\ M_{00, KK}^{\text{uni}} \end{pmatrix} = \begin{pmatrix} \widehat{T}_{00}^{\pi\pi \rightarrow \pi\pi} & \widehat{T}_{00}^{\pi\pi \rightarrow KK} \\ \widehat{T}_{00}^{\pi\pi \rightarrow KK} & \widehat{T}_{00}^{KK \rightarrow KK} \end{pmatrix} \begin{pmatrix} M_{00, \pi\pi}^{(2)} \\ M_{00, KK}^{(2)} \end{pmatrix}$

## Relevant channels: $S + P$ waves

(1)  $a \pi^0 \rightarrow \pi^+ \pi^- , \pi^0 \pi^0$

$IJ=00$ :  $f_0(500), f_0(980)$  [K-Kbar coupled-channel included]

$IJ=20$ : nonresonant case [single  $\pi\pi$  channel]

(2)  $a \pi^+ \rightarrow \pi^+ \pi^0$

$IJ=11$ :  $\rho(770)$  [K-Kbar coupled-channel included]

$IJ=20$ : nonresonant case [single  $\pi\pi$  channel, same as  $a\pi^0$  case]

(3)  $a K^+ \rightarrow \pi^+ K^0 , \pi^0 K^+$

$IJ=1/2 1$ :  $K^*(892)$  [K $\eta$  coupled-channel included]

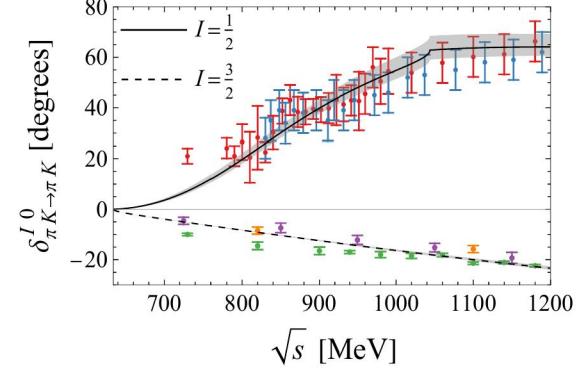
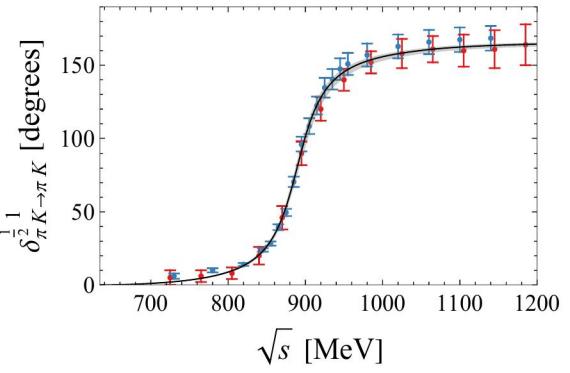
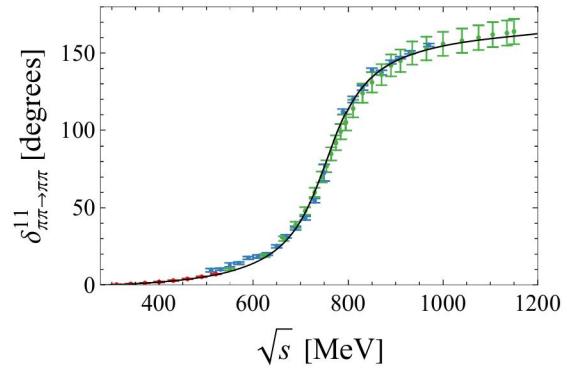
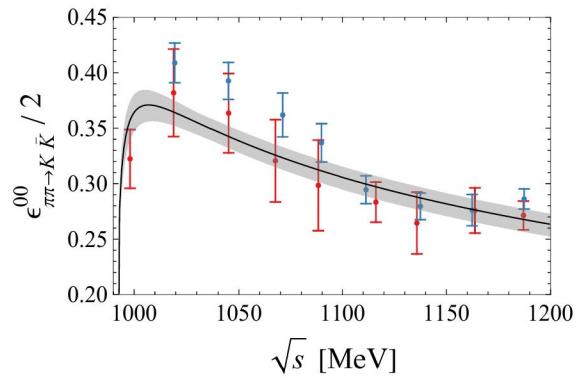
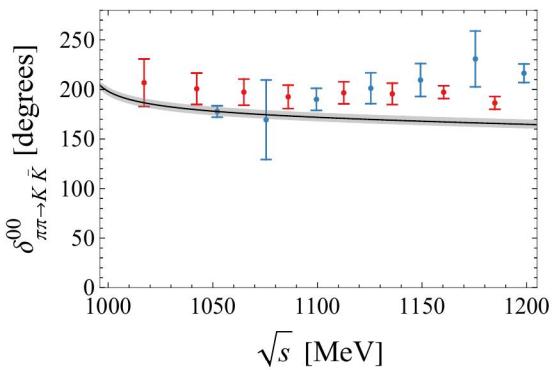
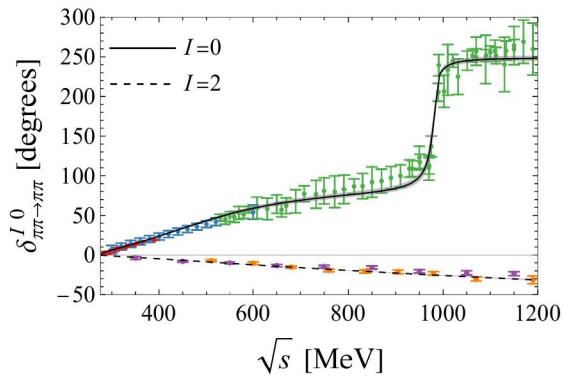
$IJ=1/2 0$ :  $K^*_0(700)$  [K $\eta$  coupled-channel included]

$IJ=3/2 0$ : nonresonant case [single  $K\pi$  channel]

$IJ=3/2 1$ : nonresonant case [neglected]

(4)  $a K^0 \rightarrow \pi^- K^+ , \pi^0 K^0$  [similar as  $aK^+$  case]

(5) Other channels can be obtained via the charge-conjugation symmetry.

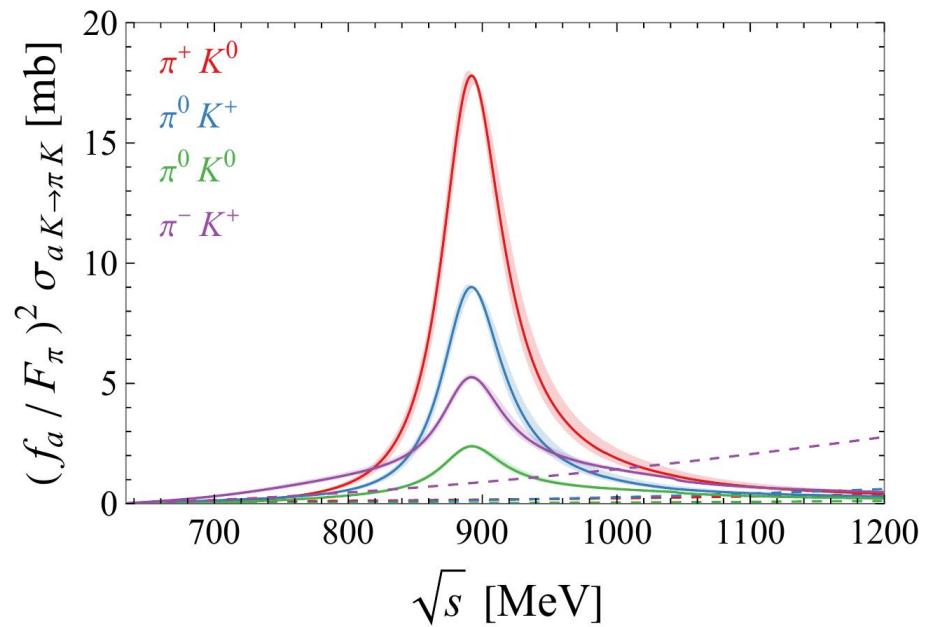
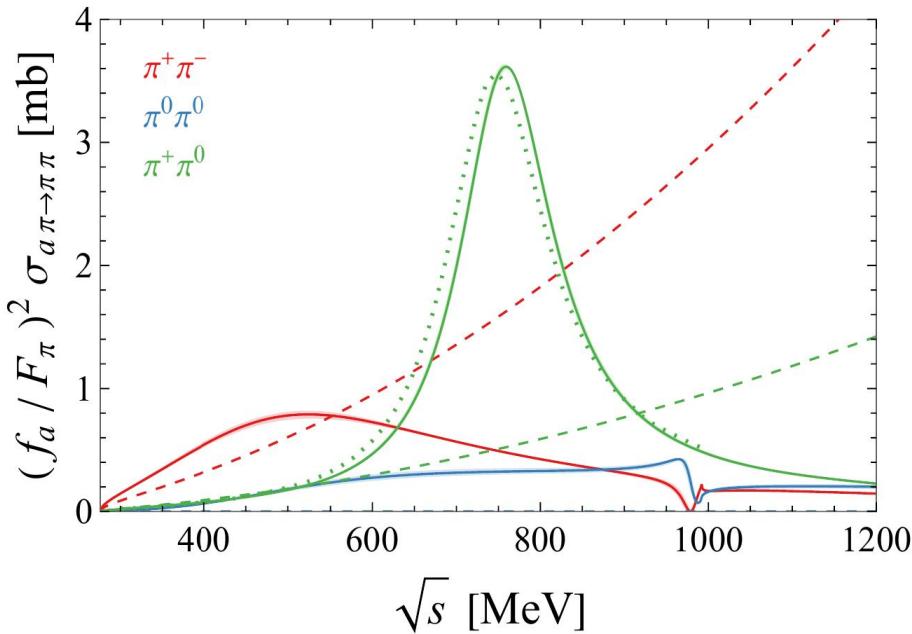


### Subst. const.

$a_{\text{sc}}^{\pi\pi,00}$	$-0.49^{+0.24}_{-0.23}$
$a_{\text{sc}}^{K\bar{K},00}$	$-1.51^{+0.20}_{-0.19}$
$a_{\text{sc}}^{11}$	$-1.38^{+0.33}_{-0.26}$
$a_{\text{sc}}^{\frac{1}{2}0}$	$0.15^{+0.18}_{-0.21}$
$a_{\text{sc}}^{\frac{1}{2}1}$	$1.53^{+0.76}_{-0.80}$

### Low energy constants

$\hat{L}_1 \times 10^3$	$0.33^{+0.02}_{-0.02}$
$\hat{L}_2 \times 10^3$	$0.97^{+0.05}_{-0.05}$
$\hat{L}_3 \times 10^3$	$-2.71^{+0.10}_{-0.11}$
$\hat{L}_4 \times 10^3$	$-0.77^{+0.09}_{-0.11}$
$\hat{L}_5 \times 10^3$	$3.51^{+1.39}_{-1.62}$
$\hat{L}_6 \times 10^3$	$-1.47^{+0.20}_{-0.24}$
$\hat{L}_7 \times 10^3$	$-0.77^{+0.24}_{-0.18}$
$\hat{L}_8 \times 10^3$	$4.05^{+0.37}_{-0.45}$

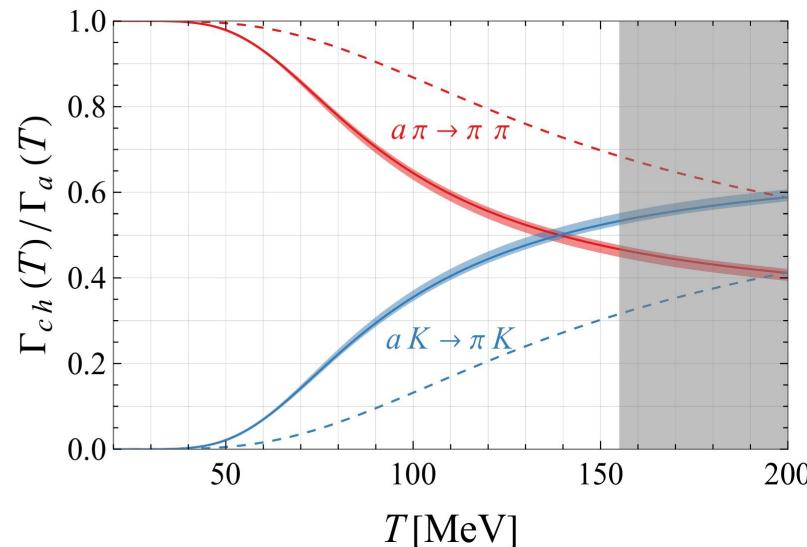
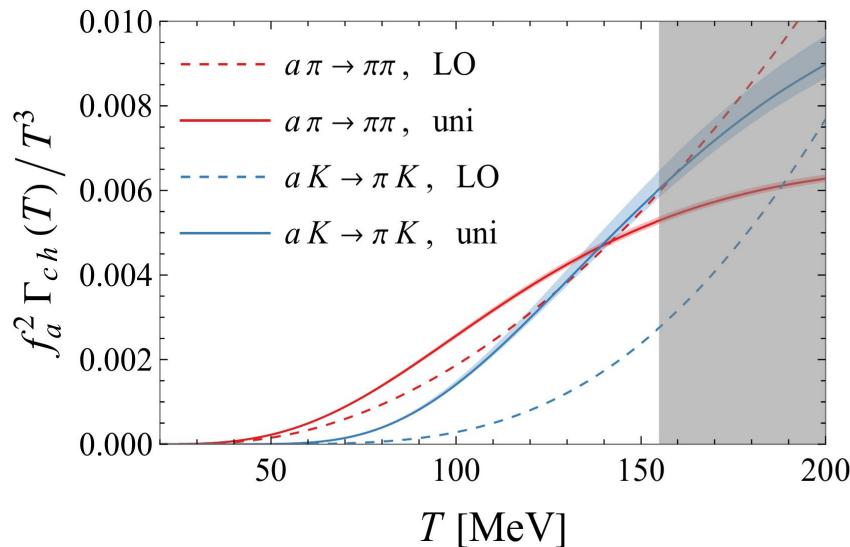


*Resonance poles*

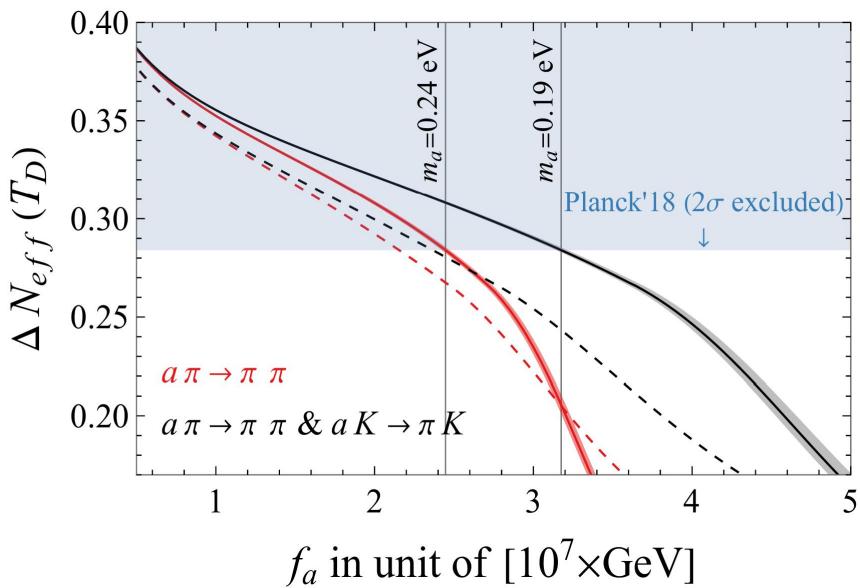
$\rho$ :  $(754.3 - i67.9)$  MeV ;  $K^*$ :  $(889.5 - i28.0)$  MeV;

$f_0(500)$ :  $435.4 - i238.0$ ;  $f_0(980)$ :  $981.1 - i11.4$ ;  $K^*_0(700)$ :  $801.9 - i195.2$ ;

- **Clear enhancement from the unitarized amplitudes (solid lines)**
- **$\rho(770)$  &  $K^*(892)$  lead to the most prominent effects**
- **Scalar resonances mostly give mild contributions**
- **$\alpha\eta$  related processes are much less important than the  $aK$  ones. (working in progress)**
- **axion-baryon is expected to be much suppressed, due to the heavy thresholds.**



[Wang, ZHG, Zhou, PRD'25]



$$f_a \geq 2.45^{+0.03}_{-0.02} \times 10^7 \text{ GeV} \quad (\mathbf{a\pi \rightarrow \pi\pi})$$

$$f_a \geq 3.18^{+0.04}_{-0.03} \times 10^7 \text{ GeV} \quad (\mathbf{a\pi \rightarrow \pi\pi + aK \rightarrow \pi K})$$

- Enhancement in  $\tau \rightarrow \nu_\tau Ka$  is also seen. However this belongs to a Cabibbo suppressed reaction. [Hao, Duan, ZHG, 2507.00383]

# Axion production in $\eta \rightarrow \pi\pi a$ decay

# Axion production from $\eta \rightarrow \pi\pi a$ decay in SU(3) $\chi$ PT

## Why focus on axion in $\eta$ decay:

- ✓ Valuable channel to search axion @colliders: many available experiments with large data samples of  $\eta/\eta'$  [BESIII, STCF, JLab, REDTOP, ... ...]
- ✓  $\eta \rightarrow \pi\pi\pi$  (IB suppressed),  $\eta \rightarrow \pi\pi a$  (no IB suppression)
- ✓  $\eta \rightarrow \pi\pi a$ : theoretically easier to handle than  $\eta' \rightarrow \pi\pi a$  (next step)

## Previous works:

- ❖ Most of them rely on leading-order  $\chi$ PT
- ❖ Possible issue: bulk contributions @LO  $\chi$ PT are constant terms, and potential large corrections from higher orders may result.
- ❖ Hadron resonance effects may lead to enhancements.

## Advances in our work :

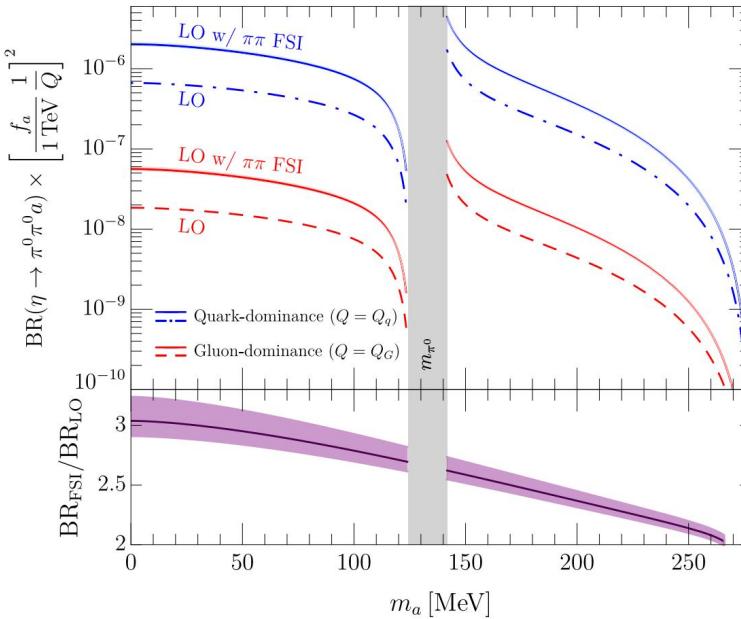
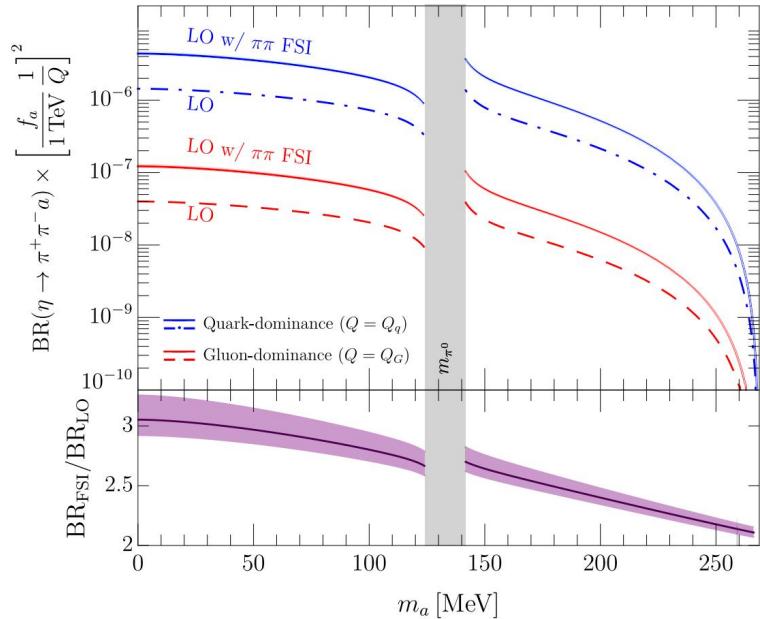
- Study of renormalization of  $\eta \rightarrow \pi\pi a$  @1-loop level in SU(3)  $\chi$ PT
- To implement unitarization to the  $\eta \rightarrow \pi\pi a$   $\chi$ PT amplitude
- Uncertainty analyses in the phenomenological discussions

$$M_0(s) = P(s)\Omega_0^0(s)$$

Red arrows pointing to the following two sections:

### $\eta \rightarrow \pi\pi a$ LO amplitude

### Omnes function: $\pi\pi$ FSI



### Our improvements:

- NLO perturbative decay amplitude include  $s$ - and  $t(u)$ -channel interactions perturbatively.
- The unitarized decay amplitude will be constructed to account for the  $s$ -channel  $\pi\pi$  final state interaction (FSI) effect that respect the chiral symmetry.
- Dalitz plots will be explored to decode the dynamics in  $\eta \rightarrow \pi\pi a$ .

**LO  $\chi$ PT Lagrangian**

$$\mathcal{L}_2 = \frac{F^2}{4} \langle \partial_\mu U \partial^\mu U^\dagger + \chi_a U^\dagger + U \chi_a^\dagger \rangle + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\text{LO}} + \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2$$

$$\chi_a = 2B_0 M(a) \quad M(a) \equiv \exp \left( -i \frac{a}{2f_a} Q_a \right) M \exp \left( -i \frac{a}{2f_a} Q_a \right) \quad J_A^\mu|_{\text{LO}} = -i \frac{F^2}{2} \langle Q_a \{ \partial^\mu U, U^\dagger \} \rangle$$

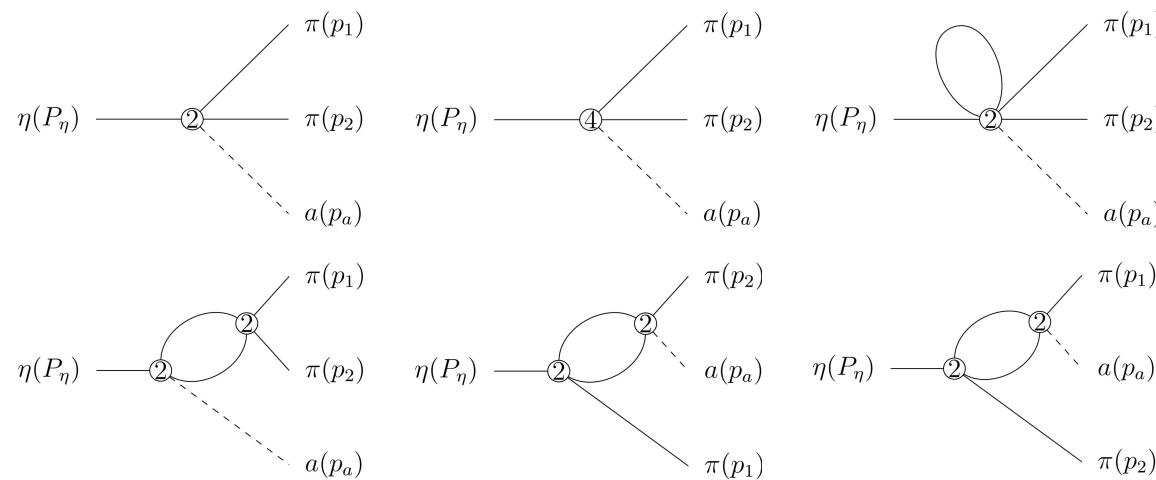
Note: we consider the octet part ( $\bar{Q}_a$ ) of  $Q_a$  in SU(3)  $\chi$ PT

**NLO  $\chi$ PT Lagrangian**

$$\mathcal{L}_4 = L_1 \langle \partial_\mu U \partial^\mu U^\dagger \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \dots + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\text{NLO}},$$

$$J_A^\mu|_{\text{NLO}} = -4iL_1 \langle \bar{Q}_a \{ U^\dagger, \partial^\mu U \} \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \dots$$

**Feynman diagrams up to NLO**



**Parameters**

Masses and $F_\pi$ [MeV]				LECs $L_i^r(\mu)$ at $\mu = 770$ MeV (in unit of $10^{-3}$ )							
$m_\pi$	$m_K$	$m_\eta$	$F_\pi$	$L_1^r$	$L_2^r$	$L_3^r$	$L_4^r$	$L_5^r$	$L_6^r$	$L_7^r$	$L_8^r$
137	496	548	92.1	1.0(1)	1.6(2)	-3.8(3)	0.0(3)	1.2(1)	0.0(4)	-0.3(2)	0.5(2)

[J. Bijnens and G. Ecker, Ann. Rev. Nucl. Part. Sci. 64, 149 (2014)]

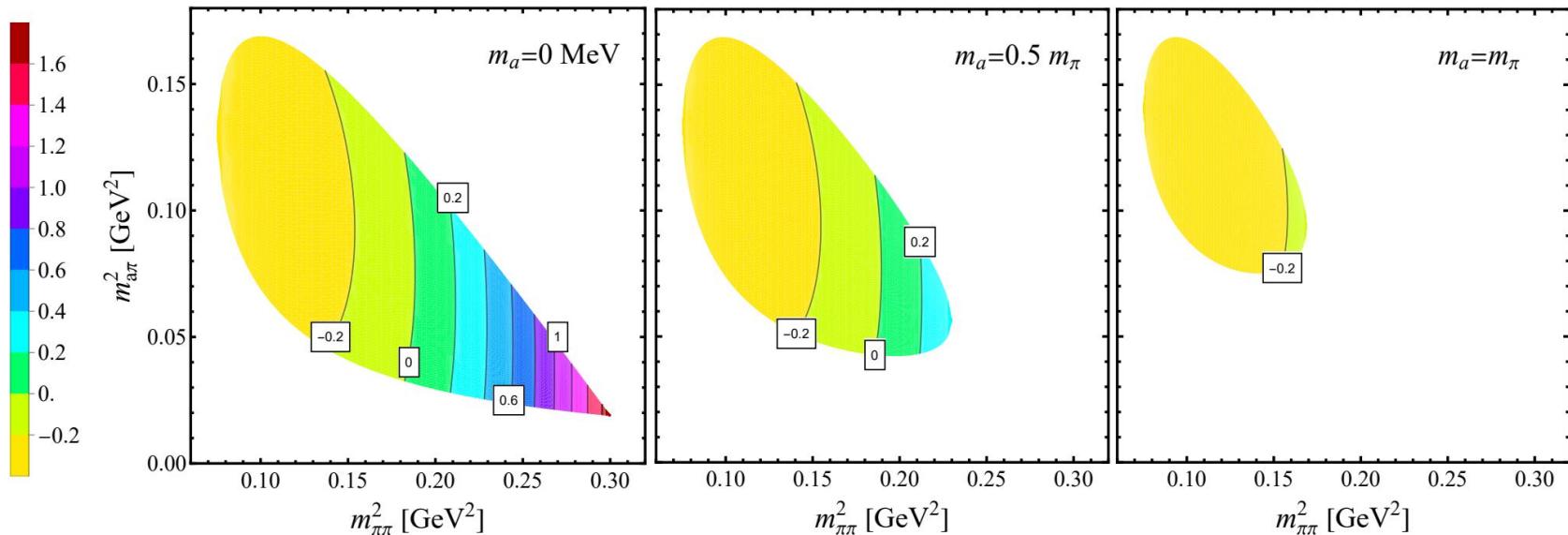
✓ Renormalization condition is verified to be consistent with conventional ChPT.

## Observations:

- Strong isospin breaking effects enter the  $\eta \rightarrow \pi\pi a$  amplitudes at the order of  $(m_u - m_d)^2$
- In the isospin limit ( $m_u = m_d$ ), the amplitudes with  $\pi^+\pi^-$  and  $\pi^0\pi^0$  in  $\eta \rightarrow \pi\pi a$  processes are identical.
- **Dalitz plots to show the NLO/LO convergence**

$$\left(2\mathcal{M}_{\eta;\pi\pi a}^{(2)} \operatorname{Re}(\mathcal{M}_{\eta;\pi\pi a}^{(4)}) + |\mathcal{M}_{\eta;\pi\pi a}^{(4)}|^2\right) / |\mathcal{M}_{\eta;\pi\pi a}^{(2)}|^2$$

[Wang, ZHG, Lu, Zhou, JHEP'24]



## Important lessons:

- Non-perturbative effect in the  $\pi\pi$  subsystem can be important.
- Perturbative treatment of the  $a\pi$  subsystem is justified.

## ● Unitarization of the partial-wave $\eta \rightarrow \pi\pi a$ amplitude

$$\begin{aligned}\mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni}}(s) &= \frac{\mathcal{M}_{\eta;\pi\pi a}^{00,\text{L}}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)} , \\ G_{\pi\pi}(s) &= -\frac{1}{(4\pi)^2} \left( \log \frac{m_\pi^2}{\mu^2} - \sigma_\pi(s) \log \frac{\sigma_\pi(s) - 1}{\sigma_\pi(s) + 1} - 1 \right) , \\ \mathcal{M}_{\eta;\pi\pi a}^{00,\text{L}}(s) &= \mathcal{M}_{\eta;\pi\pi a}^{00,(2)}(s) + \mathcal{M}_{\eta;\pi\pi a}^{00,(4)}(s) - G_{\pi\pi}(s) \mathcal{M}_{\eta;\pi\pi a}^{00,(2)}(s) T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s) .\end{aligned}$$

**The unitarized amplitude satisfies the relation**

$$\text{Im} \mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni}}(s) = \rho_{\pi\pi}(s) \mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni}}(s) (T_{\pi\pi \rightarrow \pi\pi}^{00,\text{Uni}}(s))^* , \quad (2m_\pi < \sqrt{s} < 2m_K)$$

**with the unitarized PW  $\pi\pi$  amplitude**  $T_{\pi\pi \rightarrow \pi\pi}^{00,\text{Uni}}(s) = \frac{T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)}$

## ● Unitarized PW amplitude based on LO $\eta \rightarrow \pi\pi a$ amplitude

$$\mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni-LO}}(s) = \frac{\mathcal{M}_{\eta;\pi\pi a}^{00,(2)}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)} .$$

**Resemble the method:**

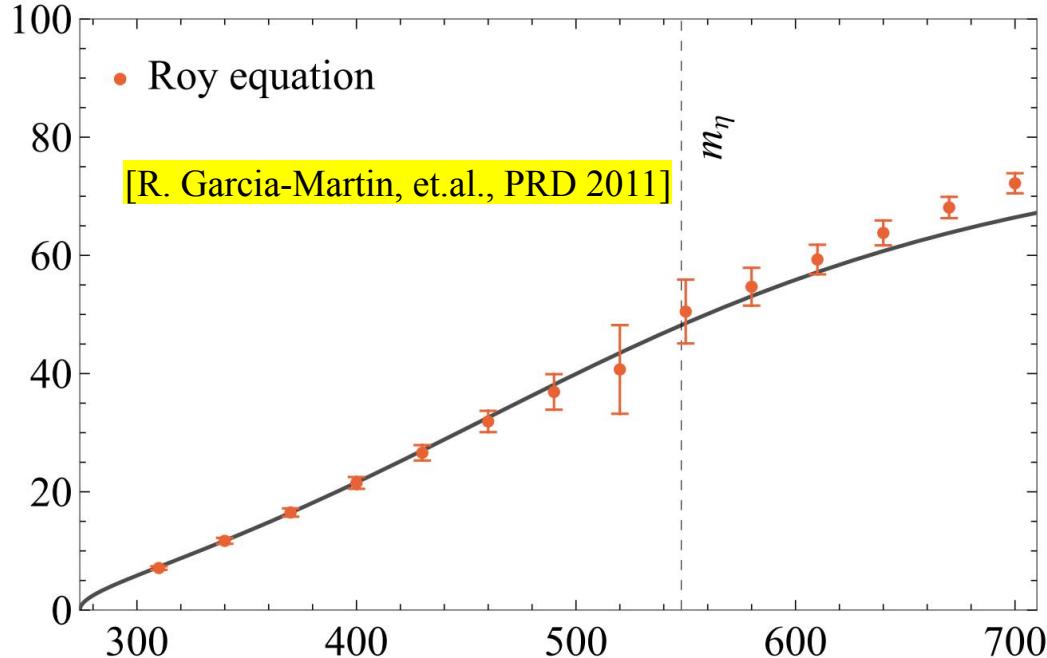
**[Alves, Gonzalez-Solis, JHEP'24]**

$$M_0(s) = P(s)\Omega_0^0(s)$$

## Phase shifts from the unitarized PW $\pi\pi$ amplitude

$$T_{\pi\pi \rightarrow \pi\pi}^{00, \text{Uni}}(s) = \frac{T_{\pi\pi \rightarrow \pi\pi}^{00, (2)}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi \rightarrow \pi\pi}^{00, (2)}(s)}$$

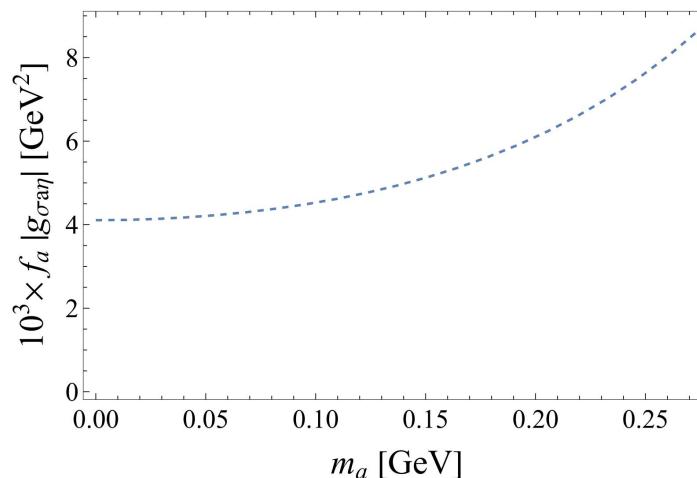
$$\delta_0^0$$



- Pole position of  $f_0(500)/\sigma$ :

$$\sqrt{s_\sigma} = 457 \pm i 251 \text{ MeV}$$

$$\mathcal{M}_{\eta; \pi\pi a}^{00, \text{Uni,II}}(s) \Big|_{s \rightarrow s_\sigma} \sim - \frac{g_{\sigma\pi\pi} g_{\sigma a\eta}}{s - s_\sigma}$$



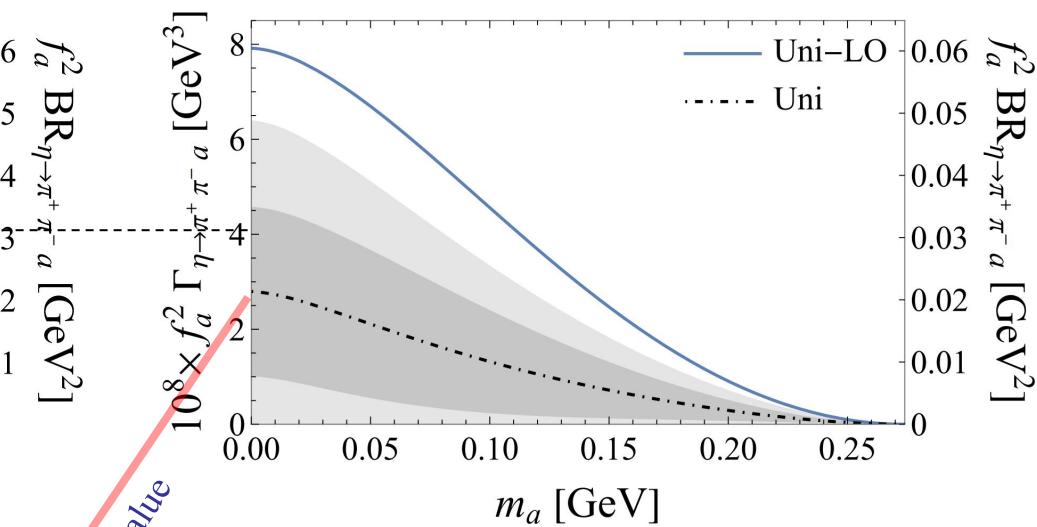
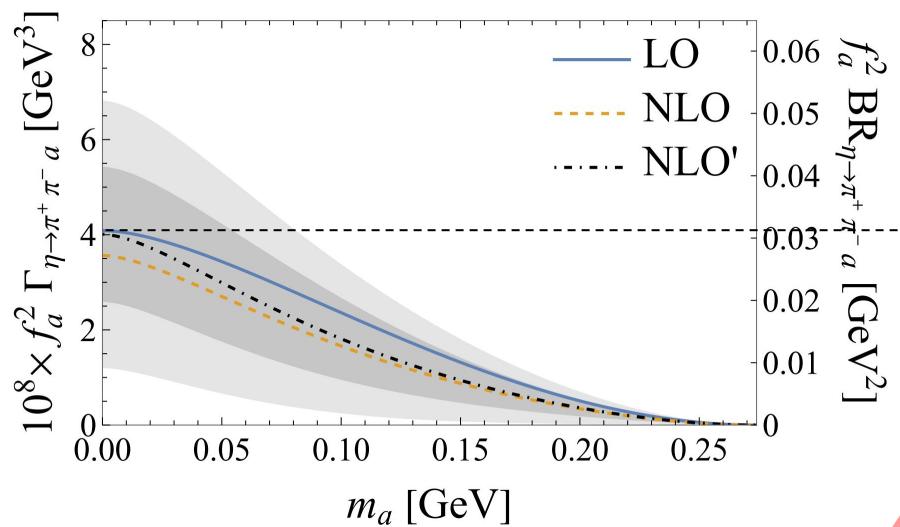
## Predictions of the $\eta \rightarrow \pi\pi a$ branching ratios by varying $m_a$

### Uncertainty bands:

➤ Lighter regions:  $L_1^r, L_2^r, L_3^r, L_5^r, L_6^r, L_8^r$   
 $1.0(1), 1.6(2), -3.8(3), 0.0(3), 1.2(1), 0.0(4), -0.3(2), 0.5(2)$

➤ Darker regions: freeze the  $1/N_c$  suppressed ones ( $L_4, L_6, L_7$ )

[Wang, ZHG, Lu, Zhou, JHEP'24]



Central value

$$\text{BR}_{\eta \rightarrow \pi^+ \pi^- a} \Big|_{m_a \rightarrow 0} = 2.1 \times 10^{-2} \left( \frac{\text{GeV}^2}{f_a^2} \right)$$

Possible detection channels:  $a \rightarrow \gamma\gamma, a \rightarrow e^+e^-, a \rightarrow \mu^+\mu^-$

# Summary

- Chiral effective field theory provides a systematical and useful framework to study the axion-hadron reactions.
- Synergies of Lattice QCD, hadron phenomenologies and chiral EFT are demonstrated to be powerful to build axion amplitudes.
- Further involvements with the experiments, cosmology, astronomy are needed to set up stronger constraints on axion parameters!

谢谢！