Relativity in statistical mechanics and black holes

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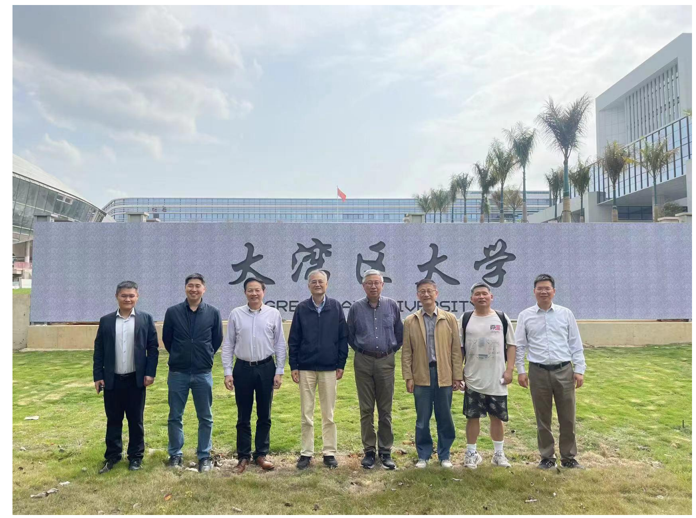
什么是**时间**？ 时间就是生命
The most fundamental problem in the Nature



什么是**温度**? 温度就是热情
It is a very deep problem, but simpler than the first one

Two questions are closely related

Complex Time: $$t \rightarrow z = \tau + it$$

生命在于运动　　温度决定杂乱无章的运动: 内能

Internal Energy

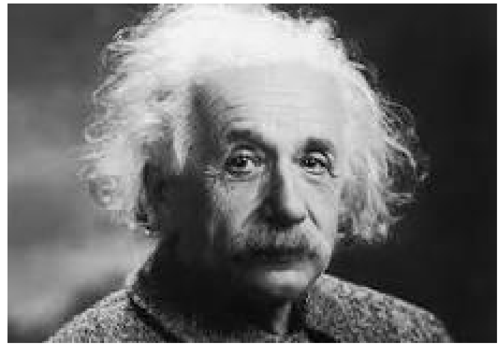
1/温度 就是**虚**时间　$$\frac{\hbar}{k_B T} = it$$

外能？

External Energy

Special theory of relativity

Poincare



Einstein

Lorentz

Unified Space and Time

In the low velocity limit, it recovers the Galileo relativity

Experiments

A moving stick becomes *shorter*

A running clock gets *slower*

A moving mass gets heavier

…………….

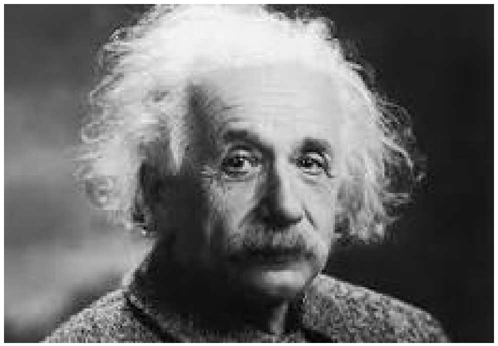
Energy-Mass Relation

$$E = mc^2$$

Atomic bomb in WWII!

General theory of relativity

Differential geometry in Pure math



Einstein Equation:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

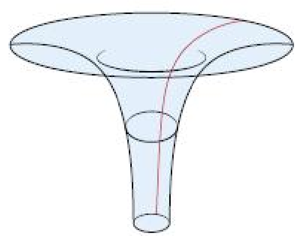
Space-time ⇄ Matter

$\Lambda < 0, = 0, > 0$

AdS, flat, dS

Experiments

Gravitational redshift
Perihelion Procession
Bending of light
Timed delay
……….
Gravitational wave
Black holes *due to star collapse*



Nobel prize in physics, 2017

In the weak field limit, it recovers the Newton's gravitation's law

$$\Delta\Phi = 4\pi G\rho$$

$$\Phi = -GM / r$$

Singularity → **Quantum effects**

Einstein's contribution to Quantum Mechanics,
Entanglement and Quantum Statistical Mechanics

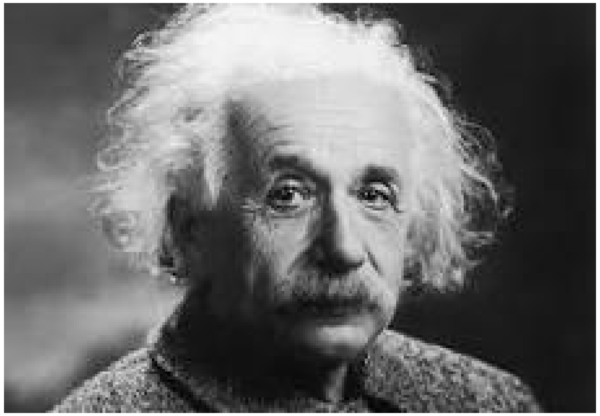


Photo-Emission, Photons

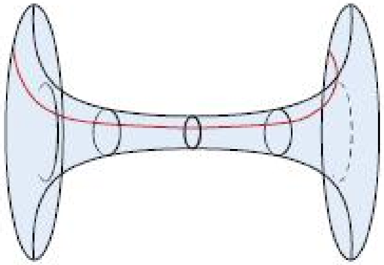
Bose-Einstein Condensation

Superfluid He4

Brownian motion

....

Traversable wormhole



Einstein, Podolsky, and Rosen
Entanglement pair

ER= EPR

$$|EPR\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Einstein-Rosen (ER) bridge

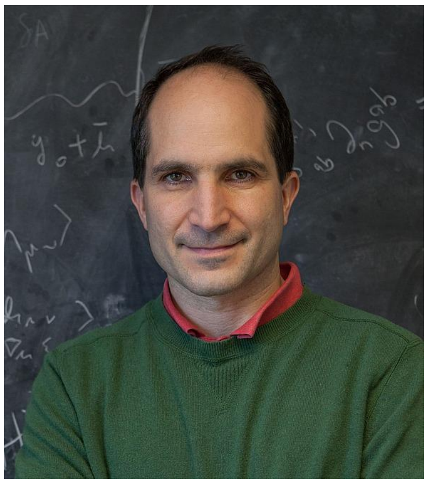
No experiments yet, but see the SYK model

Gravity/Gauge (AdS_{d+1} / CFT_d) duality

Electricity/Magnetism duality

Gauge Theory

CFT



J. Maldacena, 1998

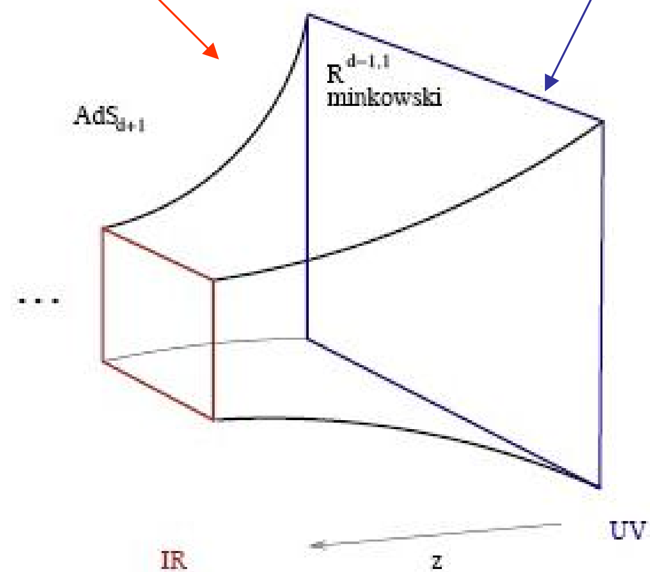
d $\quad$ $d+1$

Gauge Theory = Gravity

In Anti-De Sitter Geometry $\Lambda < 0$

RG=GR

Poincare coordinate



$\mathcal{N} = 4, SU(N)$ $\quad$ AdS_5 / CFT_4

*Supersymmetric Yang-Mills (**SYM**)*

Type IIB string theory on $AdS_5 \times S^5$

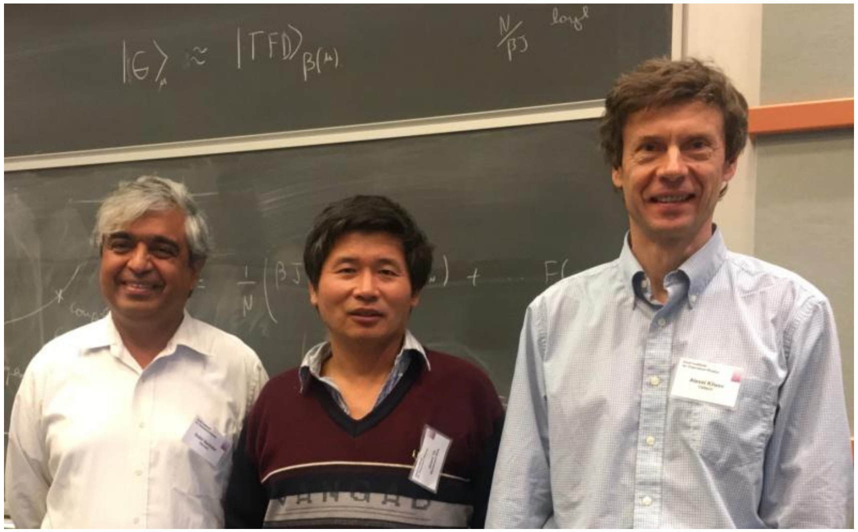
Self-dual

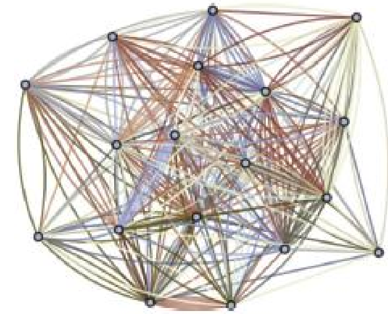
N D3 brane

No experiments yet !

Non-Supersymmetry

$$H_R = \frac{1}{\sqrt{M}} \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j \qquad P[J_{ij}] \sim e^{-NJ_{ij}^2/2J^2}$$





SYK $NAdS_2 / NCFT_1$

Materials on the Earth → SYK → black holes in the SKY

建立了天和地之间的联系

Sachdev-Ye-Kitaev model establish possible deep connections between **emergent quantum Phenomenon in materials** **and** **black holes in AdS geometry** $\Lambda < 0$

Experiments: IBM, Google, and Microsoft….are using two Coupled SYK models to test **ER= EPR** $T > 0$

Albert Einstein made outstanding contributions to the special theory of relativity, the statistical mechanics, the general theory of relativity and the quantum mechanics.

Near to the end of his life, Albert Einstein has **two goals** to achieve:

1. incorporating the special theory of relativity into the statistical mechanics

2. incorporating the general theory of relativity into the quantum mechanics.

Despite the second called quantum gravity remains elusive, Here we will show how we achieved the first goal. *It also puts some hints on the 2nd goal.*

The Poincare-Einstein-Lorentz special theory of relativity is based on two postulates:

I. *The law of any physical phenomena take the same form in all the inertial frames.*

II. *The velocity of the light in the vacuum is a constant in all the inertial frames (Causality)*

One may call the postulate I the Lorentz Invariance (LI)

The postulate II can be used to derive the Lorentz transformation (LT). So one may call the postulate II as LT.

The Maxwell equations in the vacuum satisfy the LI, but those in a media do not. The media sets up a **preferred reference frame** where the Maxwell equations hold and breaks the LI explicitly.

When going to a different inertial frame, the Maxwell equations in a media change their forms.
However, the postulate II remains true in any inertial frames.

$$\Box A^{\mu} = j^{\mu} \qquad\qquad j = j_{\mu}dx^{\mu} = \rho dt + j_i dx^i$$

The speed of light in the **medium**

$$c_r = 1/\sqrt{{}_{0}\,{}_{r}\mu_0\mu_r} = c/\sqrt{{}_{r}\mu_r} < c$$

This fact suggests that LI and LT are two different things, ***the LT is more robust than LI.***

As a comparison, Parity conservation and Parity transformation are two different and separate things (See **Lee and Yang, Nobel prize symposium in 1957**).

P.W. Anderson: More is different,

Emergent quantum Phenomena !

In a quantum matter, the *Lorentz invariance* (LI) is explicitly broken at very beginning even at

So there is an absolute frame even at $T = 0$

However, when getting to different inertial frames, one still need to apply *Lorentz transformation* (LT).

So LT is more **general** than LI.

Similarly, Parity conservation (PC) and Parity transformation (PT) are two different things. PT is more **general** than PC (See *Lee and Yang, Nobel price 1957 on Parity Violation*).

Special Relativity in Emergent quantum Phenomena !

How to apply Special Relativity to (equilibrium) condensed matter system Or quantum statistical mechanics system Which is always at a finite temperature ?

Special Theory of Relativity is well known at $T = 0$

What is Special Theory of Relativity at **a finite temperature ?** $T > 0$

Poincare (or Lorentz) group is just a subgroup of the conformal group, what is its impact on the black hole on the Gravity side or AdS/CFT at a finite $T > 0$?

庞加莱的狭义相对伦，
金晓峰，2022-2023，物理

1. Introduction on the history:

Poincare and Einstein's special theory of relativity
There is no preferred inertial frame.

Different inertial frames are related by Lorentz transformation (LT): The **Michelson–Morley experiment** (*1907 Nobel prize in physics*)

The speed of light c is the relevant velocity scale.

The Hamiltonian or Lagrangian takes identical form and owns the same set of symmetries in all inertial frames.

A moving stick becomes *shorter* $l = l_0\sqrt{1-(\frac{v}{c})^2}$

A running clock gets *slower* $t = t_0 / \sqrt{1-(\frac{v}{c})^2}$

mass

$$m = m_0 / \sqrt{1-(\frac{v}{c})^2}$$

How does the temperature of a thermal equilibrium system change in different initial frames ?

Previous works from phenomenological thermodynamics point of view initiated by *Planck, Einstein, Pauli, Laue*……..

Incorporating the special relativity into the Thermodynamics by making various ***assumptions***:

Einstein (1) first found: $$\frac{T}{T_0} = \sqrt{1-(\frac{v}{c})^2} < 1$$

a moving body gets cooler

(2) second found: $$\frac{T}{T_0} = 1/\sqrt{1-(\frac{v}{c_l})^2} > 1$$

a moving body gets hotter

then dismissed both results and *became* hesitated !

Similar phenomenological thermodynamics approach has been used by many people to get two additional results:

(3) The Temperature does not change:

$$\frac{T}{T_0} = 1$$

Einstein also said this !

(4) The Temperature is well defined only in a co-moving frame, but not in any other frames.

****

In this work, we take a completely different approach which starts from the fundamental quantum statistical mechanics point of view:

Complex time $\tau = \beta + it$

This is also the limitation of *Planck, Einstein, Pauli, Laue……..*

The Kampen-Israel covariant formalism **in statistical mechanics**

$$\rho = 1/Ze^{-\beta E} \rightarrow \rho = 1/Ze^{-\beta P_\mu u^\mu},$$

$$Z = Tre^{-\beta P_\mu u^\mu}$$

The biggest advantage of this Kampen-Israel covariant formalism is that the **second** law of thermodynamics can be expressed just in terms of all the four vectors such as the 4-temperature and the 4-momentum.

However, one severe problem suffered in this covariant formalism is that it is impossible to make Energy - momentum in a FINITE volume V co-variant. Namely, the **first** law

$$dE = TdS - PdV$$

can ***not be*** formulated in any co-variant form, From the extended symmetry point of view:

Supersymmetry (SUSY) transformations by themselves do not form a group, but form an extension of the Poincare group, the combination leads to the *Super-Poincare algebra* (also called Graded Lie algebra), a sub-algebra of **Superconformal**

Super-charge $Q, \bar{Q}$, *Super-space* $(x, \theta, \bar{\theta})$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^{\mu}_{\alpha\dot{\beta}} P_\mu$$

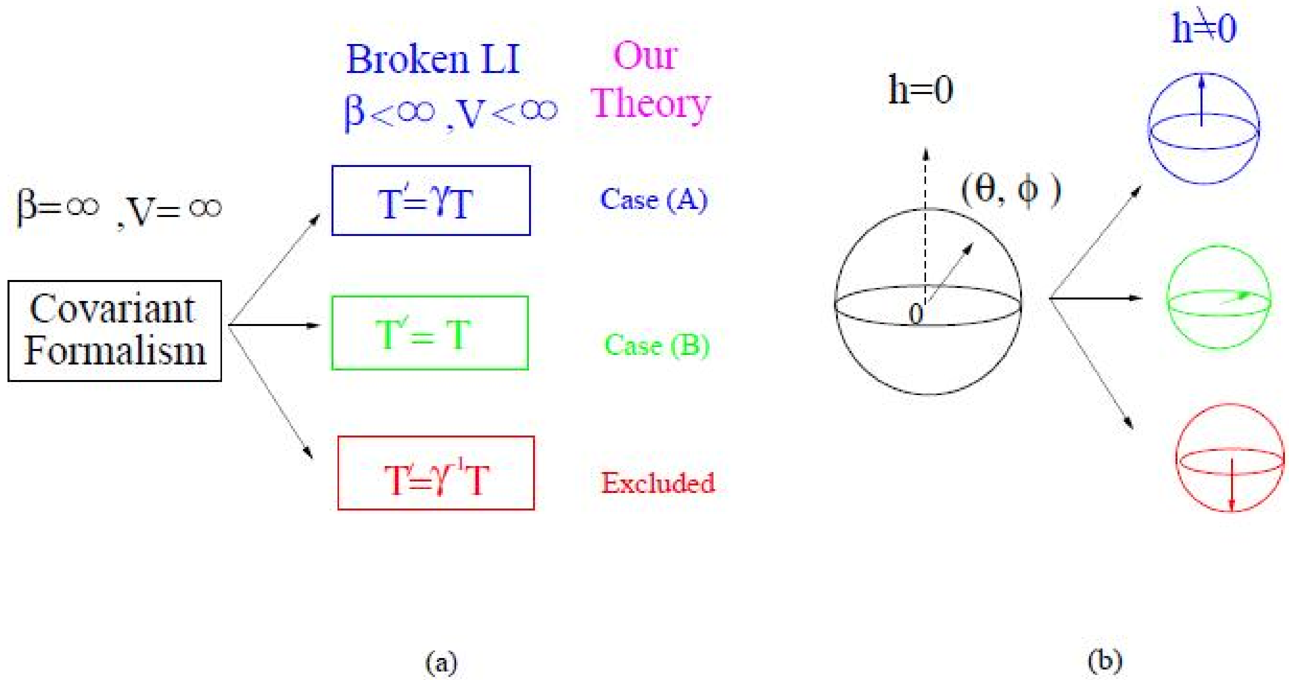
It is known that any SUSY will be explicitly broken by any $T > 0$, so does the Lorentz group symmetry.

So it is ***in-consistent*** to still impose the Lorentz symmetry at any $T > 0$ for a SUSY !

Taking $c \longrightarrow \infty$ LT reduces to GT

The SUSY decouples from the GT and behaves just like an ordinary internal symmetry

The space-time symmetry breaking in the Kampen-Israel co-variant formalism versus internal symmetry breaking in a magnet.



Incorporate the Kampen-Israel co-variant formalism into our case (B)

Comments on the co-variant formalism **in Hot QCD to enforce** $(T > 0, \xi \neq 0)$ **to be LI.**

There is a so-called co-variant formalism which **enforces** the partition function to remain co-variant even at $T > 0$

$$Z = Trexp[-\beta \int_{\Sigma(\tau)} d\Sigma n_\mu (T^{\mu\nu} u_\nu - \xi j^\mu)]$$

$u^\nu = \gamma(1, -\vec{v})$ 4-velocity of an observer

τ is the proper time

$T^{\mu\nu}$ is the energy-momentum tensor

j^μ is the conserved 4-current due to the internal $U(1)$ symmetry. $\partial_\mu j_\mu = 0$

For a static observer, $\tau = t$ it reduces to the original partition function $Z = Tre^{-\beta(H-\mu N)}$ of the thermal QFT which, then can be written in the imaginary time, The energy momentum tensor in for a perfect fluid takes the covariant form

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + p\eta_{\mu\nu}$$

It remains impossible to extend this covariant $T_{\mu\nu}$ to a finite volume V .

The original Thermodynamic first law

$$dE = TdS - PdV + \mu dN$$

is simplified all in terms of the intensive quantity only.

$$d\epsilon = Tds + \mu dn$$

but not able to write down $-PdV$

The equation of state (EOS) can not be made covariant:

EOS of the QGP is not known

The Van der Waals EOS is a modification of the ideal gas law

$$(P + an^2)(1 - nb) = nRT$$

it would be very difficult to write the Van der Waals EOS as a covariant form.

Even so, it still has only mathematical sense, no physical meanings.

Incorporate the co-variant formalism into our case (B)

In the **new relativity principle** at $T > 0$ or $\mu \neq 0$,
the LT was kept, but LI was discarded and replaced by the following new rules:

There is an absolute frame set-up by the external reservoir in a canonical ensemble or the rest frame of the medium in a micro-canonical ensemble.

The temperature T *and the real time* t *are combined into one* ***complex time*** $z = \tau + it$ $0 < \tau < \beta, -\infty < t < \infty$

LT the generalized LT

$$SO(3,1) \rightarrow SO(4) \rtimes SO(3,1)$$

$3+1$ dimension $3+2$ dimension

or S^1 bundle on $M^{3,1}$

The statistical mechanics depends on both the velocity of the sample and that of the observer in the absolute frame, ***not*** *just their relative one*

The new basic rule leads to several dramatic new predictions *on moving sample or a moving observer or both in quantum materials, particle physics experiments or AdS black holes.*

(1) It naturally incorporates the old ***Kampen-Israel*** *covariant formalism into the case (B), also fixed all the ambiguities suffered in the covariant formalism.*

(2) The known experimental results in the non-relativistic limit put strong constraints on any theory in $c \to \infty$

$c < \infty$ *and pick up our theory uniquely.*

New sector: *The internal energy is conjugate to* T

Old sector: *the external energy is conjugate to real time* t

In the gravity side, It has equivalent implications in a black hole in the AdS geometry.

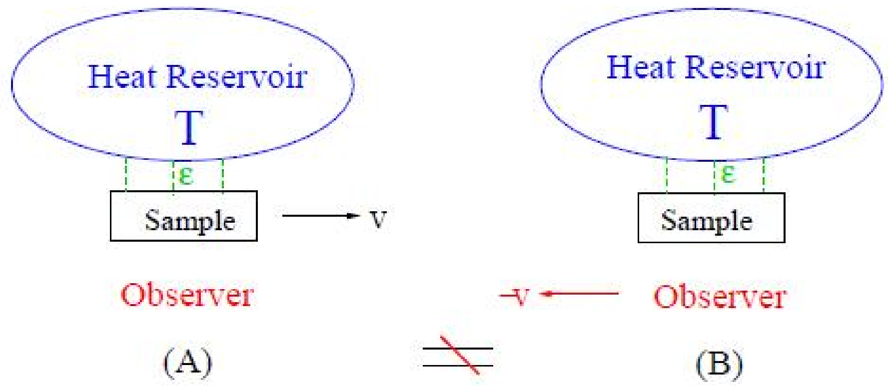
If there is a conserved number, then one must introduce a chemical potential μ *to fix the total number of particles in all the inertial frames.*

In the gravity side, it corresponds to extending charge neutral black holes to charged black holes.

The most disruptive change from the temperature $T = 0$ to $T > 0$ is the existence of a **Reservoir** which sets up the temperature.

It plays the role of Ether and breaks the LI !

$\varepsilon \to 0$



Most condensed matter experiments

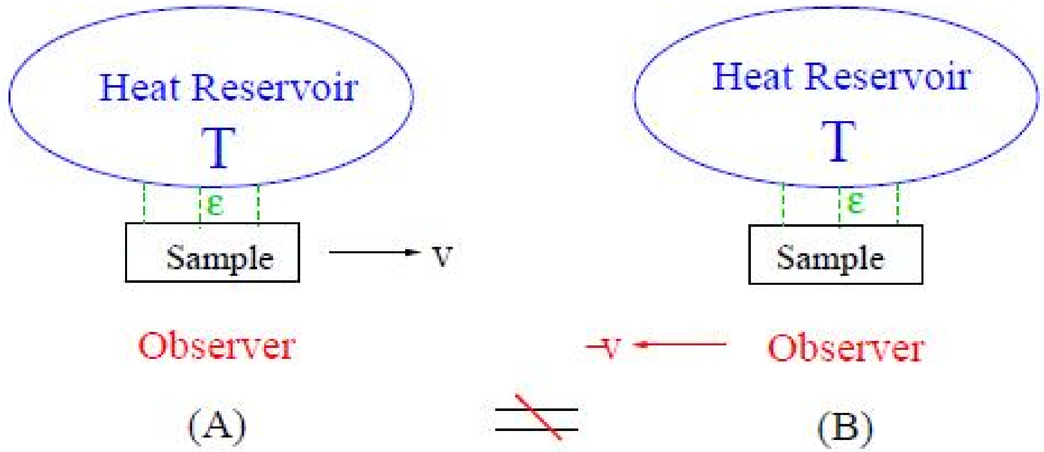
All particle physics Experiments and **Black holes in AdS**

There is a (no) relative motion between the sample and the reservoir in case A (B).

We will present the case (A) first, then Case (B), then the combined case of (A)+(B)

$\varepsilon \rightarrow 0$

$\vec{F} \sim \epsilon^2 T \vec{v}$



$$\frac{T'}{T} = 1 / \sqrt{1 - (\frac{v}{c})^2} \quad + O(\epsilon^2)$$

Internal Energy=
Doppler shifted energy

Most condensed matter experiments

$$\frac{T'}{T} = 1$$

Setting $v = 0$

Internal Energy≠
Doppler shifted energy

All particle physics Experiments and **black holes in AdS (but not in dS)**

The case (A) $\frac{T'}{T} = 1/\sqrt{1-(\frac{v}{c})^2} + O(\varepsilon^2)$

External energy

Internal Energy(内能)= Doppler shifted energy(外能)

$$\omega_I' = \omega_D' = \gamma(\omega - \vec{v}\cdot\vec{k})$$

The case (B) $\frac{T'}{T} = 1$

Internal Energy≠ Doppler shifted energy

|| 里外不一 ||

$$\omega_I' = \omega \neq \omega_D' = \gamma(\omega - \vec{v}\cdot\vec{k})$$

Only Internal Energy appears in the Fermi/Bose distribution functions. (内能)

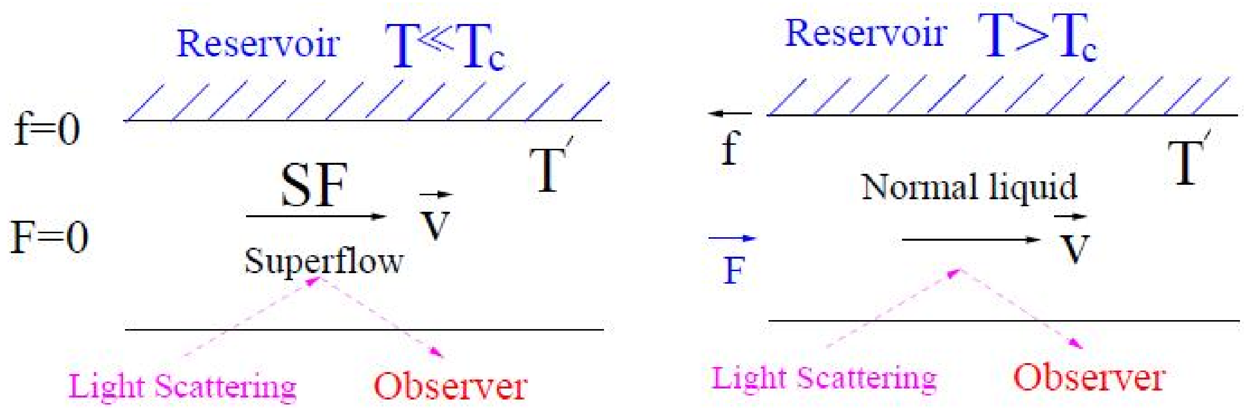
The Doppler shifted energy can be measured by an observer

外能 External energy

A flowing Superfluid He4

$$T \ll T_c \sim 2.17K, \quad v < v_c \sim 60m/s$$

$\varepsilon \to 0$



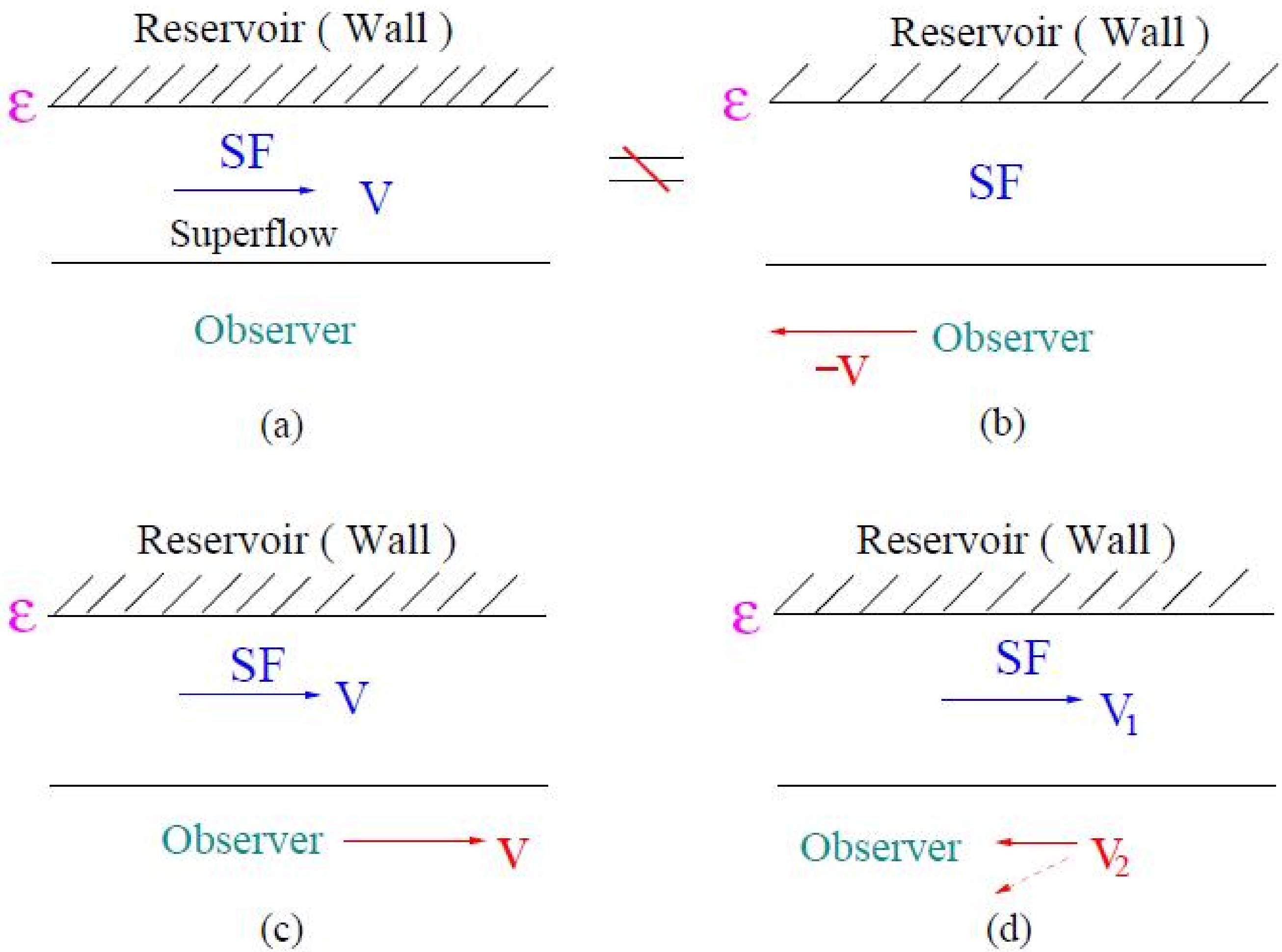
(a) (b)

$\varepsilon \to 1$

$$\frac{T'}{T} = 1/\sqrt{1-(\frac{v}{c})^2} > 1 \quad + O(\epsilon^2)$$

Due to the internal $U(1)$ symmetry breaking in the SF, the number of particles is not conserved, then a finite chemical potential need to be introduced to enforce an **average number** of He4 atoms in a grand **canonical ensemble**.

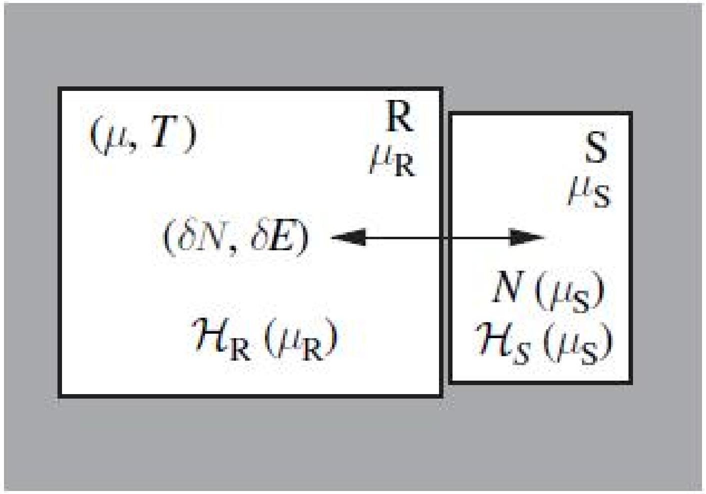
$$\varepsilon \rightarrow 0$$



The effects of a Reservoir

The free Fermi gas at a temperature T .

How do the *non-interacting* fermions reach equilibrium at a temperature T satisfying *Fermi-Dirac* distribution ?



$$H_M = \psi^{\dagger}(\vec{k})[\frac{\hbar^2 k^2}{2m} - \mu]\psi(\vec{k}),$$

$$\mu = \frac{\hbar^2 k_{0F}^2}{2m}$$

Lyapunov exponent $\lambda_L = 0$

RMT: Possion distribution

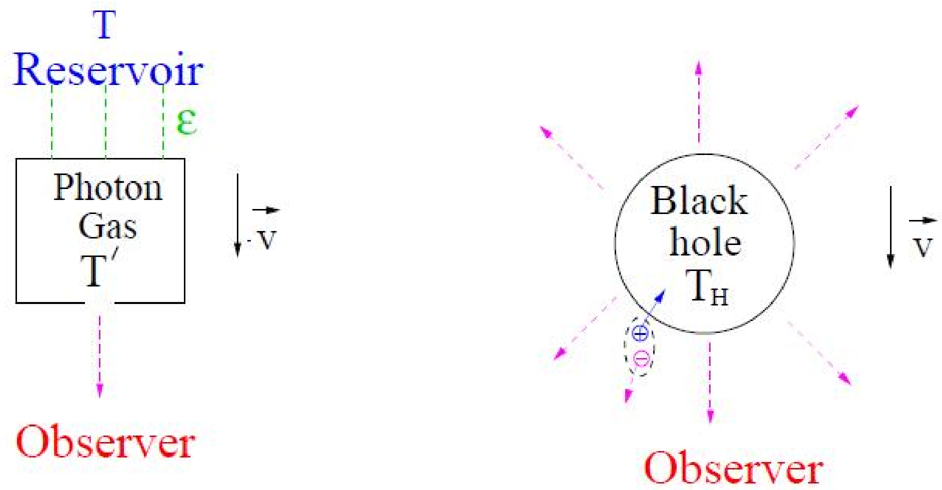
Idea Bose gas such as Photon gas also belongs to this class

Any integrable systems belong to this class

Black body radiations of a moving box
versus
Hawking radiations of a moving black hole

$\varepsilon \to 0$

Integrable



(a) (b)

Isolated system

Quantum chaotic

Because the photons are non-interacting, can not thermalize on its own. So the temperature must be given by **an external reservoir**

While the blackhole is an isolated system and a quantum chaotic system before it starts to radiates and evaporates.

2. Imaginary time in the thermal QFT. $T > 0$

In a canonical ensemble, the partition function in a static sample:

given by the Hamiltonian

$$Z = Tre^{-\beta H}$$

$\beta = 1/k_B T \quad T > 0$

is given by the reservoir

The Z of any (non-) relativistic quantum field theory in the Euclidean signature at a finite temperature is:

$$\tau = it$$

$$Z = \int D\phi e^{-S/\hbar}, \quad S[\beta] = \int_0^{\hbar\beta} d\tau \int d^d x L[\phi, \partial_\mu \phi]$$

An effective Lagrangian in a material, the space-time is an emergent space-time.

The free energy

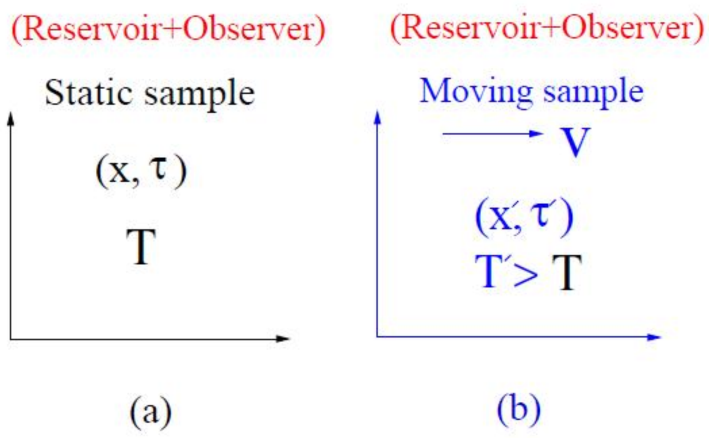
$$F = -\ln Z / \beta$$

$$dF = -SdT - PdV$$

At any finite temperature, it is always more convenient to get to the Euclidean signature by going to the imaginary time $\boxed{\tau = i\,t}$: $x' = \Lambda x, \quad \Lambda \in SO(4)$

There is a relative motion between the sample and the reservoir in ***the case A***

Co-moving*
Reservoir+Observer
are moving together with *the sample*
(not experimental realizable)



$$\boxed{\begin{aligned} x' &= \gamma(x + iv\tau) \\ \tau' &= \gamma(\tau - i\frac{v}{c^2}x) \end{aligned}}$$

$$\gamma = 1/\sqrt{1-(v/c)^2}$$

(τ, x) and (τ', x') are the space-time in the **Co-moving*** frame and lab frame with the velocity v respectively.

Assuming the temperature in the static sample is T .
What is the corresponding temperature T' in the moving one ?

The quickest way to identify the temperature of a system is to identify the (anti-) periodicity along the imaginary time direction for bosons or fermions respectively.

Inertial frame: Special Relativity

Non-inertial frame: acceleration a **General** Relativity

Similar technique (but in metric, instead of action) was used to determine the **Unruh** Temperature

$$k_B T_U = \frac{\hbar a}{2\pi c}$$

Quantum Entanglement Temperature

and also the **black hole Hawking temperature** in a curved space-time metric

$$k_B T_H = \frac{\hbar \kappa}{2\pi c}$$

3. Temperature transformation law:

The two events in the imaginary time $(\tau_1 = 0, x_1), (\tau_2 = \beta, x_2 \neq x_1)$ in the **Co-moving*** frame corresponds to

$$0 = \gamma(\tau_1' + i\frac{v}{c^2}x')$$

$$\beta = \gamma(\tau_2' + i\frac{v}{c^2}x')$$

Due to assuming the same position $i\,x'$ in the lab frame, one finds

$$\Delta\tau' = \gamma^{-1}\beta$$

which leads to

$$\frac{T'}{T} = 1/\sqrt{1-(\frac{v}{c})^2} > 1$$

$i\,x'$ must drop out in the final answer

a moving body gets hotter

In the non-relativistic limit $c \to \infty$, $T' = T$

the effect is completely **relativistic** !

When $v/c \to 1^-$, $T' \to \infty$.

However, any QFT *breaks down* at sufficiently high temp.

Due to assuming the same position x' in the lab frame, one can also see the difference in the co-moving* frame:

$$i\Delta x = i(x_2 - x_1) = \gamma v \Delta\tau') = v\beta$$

It correspond to two different positions in the co-moving* frame which do not cause any concern

The i in the LT contains **deep** physics, it is due to the imaginary time $\tau = it$ in the Euclidean signature.

Complex Time: $\tau = \hbar\beta + it$

*Same techniques has been used to find the out of time ordered correlation (**OTOC**) functions and spectral form factors in the **Sachdev-Ye-Kitaev** models*

$$\tau_1 = 3\beta/4, \tau_2 = \beta/4, \tau_3 = \beta/2 + it, \tau_4 = it$$

When there is no relative motion between the body and the reservoir, the body's temperature is the same as that dictated by the reservoir.

When there is a relative motion, then the body's temperature measured (or detected) by **any** observer is **higher** than that dictated by the reservoir.

One can take this new effect as the **generalization** of the thermodynamic **zeroth law** in the *static* case to the *moving* case.

The third law remains to apply: $T = 0 \rightarrow T' = 0$

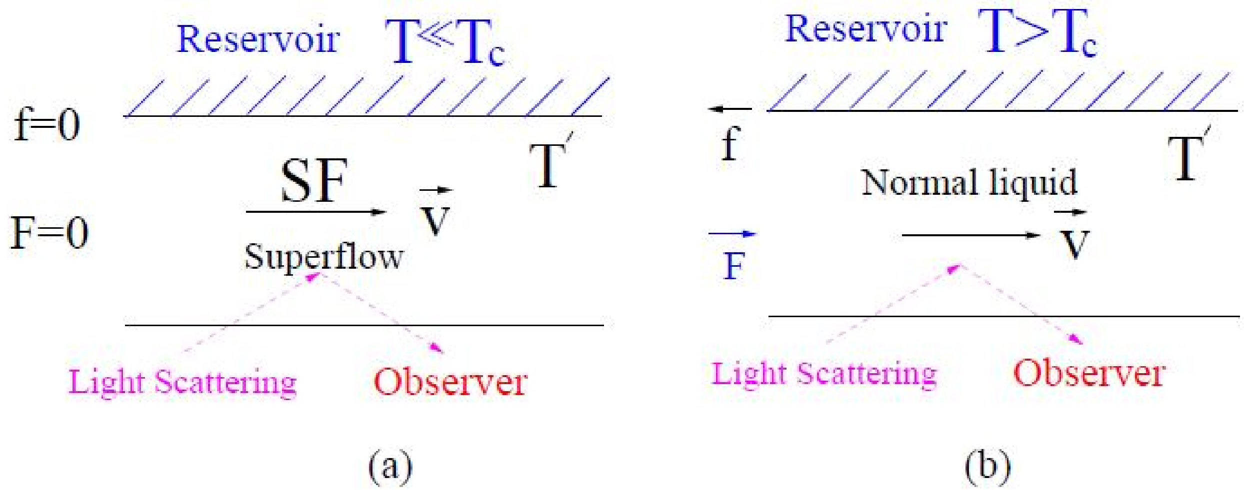
Pure state to Pure state

Ground state $\longrightarrow$ Ground state

A flowing Superfluid He4

$$T \ll T_c \sim 2.17K, \quad v < v_c \sim 60m/s$$

$\varepsilon \to 0$



$\varepsilon \to 1$

$$\frac{T'}{T} = 1/\sqrt{1-(\frac{v}{c})^2} > 1 \quad +O(\epsilon^2)$$

Due to the internal $U(1)$ symmetry breaking in the SF, the number of particles is not conserved, then a finite chemical potential need to be introduced to enforce an **average number** of He4 atoms in a grand **canonical ensemble**.

The opposite Temperature transformation law: *A wrong result*

The imaginary time $(\tau_1 = 0, x), (\tau_2 = \beta, x)$ corresponds to

$$\tau_1' = \gamma(-i\frac{v}{c^2}x)$$

$$\tau_2' = \gamma(\beta - i\frac{v}{c^2}x)$$

(Co-moving* frame)

Due to assuming the same position x in the static sample, one finds

$$\Delta\tau' = \tau_2' - \tau_1' = \gamma\beta$$

which leads to $\frac{T'}{T} = \sqrt{1-(\frac{v}{c})^2} < 1$ a moving body gets cooler

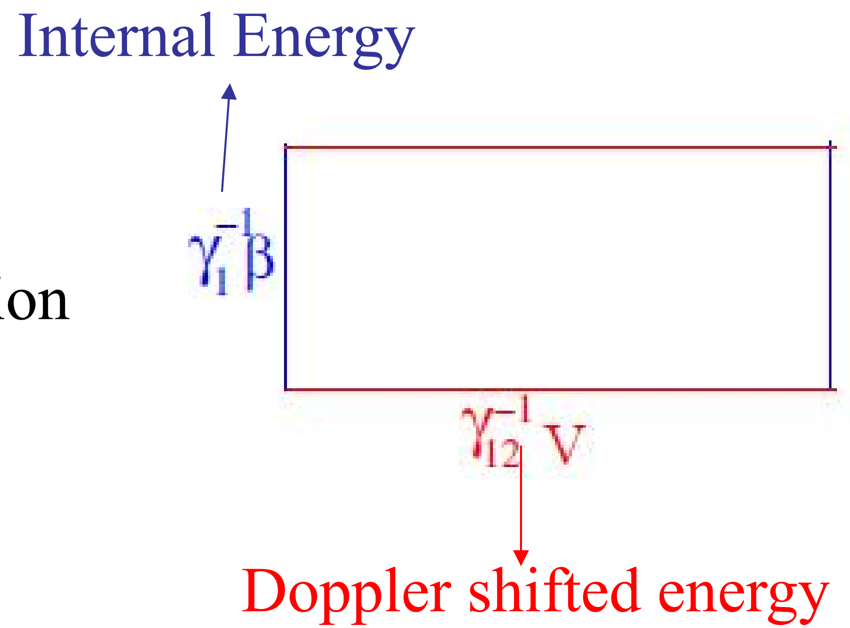
When $v/c \to 1^-, \quad T \to 0$ looks *quite funny*

The difference in the lab frame:

must be wrong !

$$\Delta x' = x_2' - x_1' = \gamma(iv\beta)$$

The unified geometric interpretation



Temperature and Volume can be viewed as **the finite size** along the imaginary time direction and the real space direction respectively.

How do their corresponding 2 *conjugate variables*

$$S,\ P$$

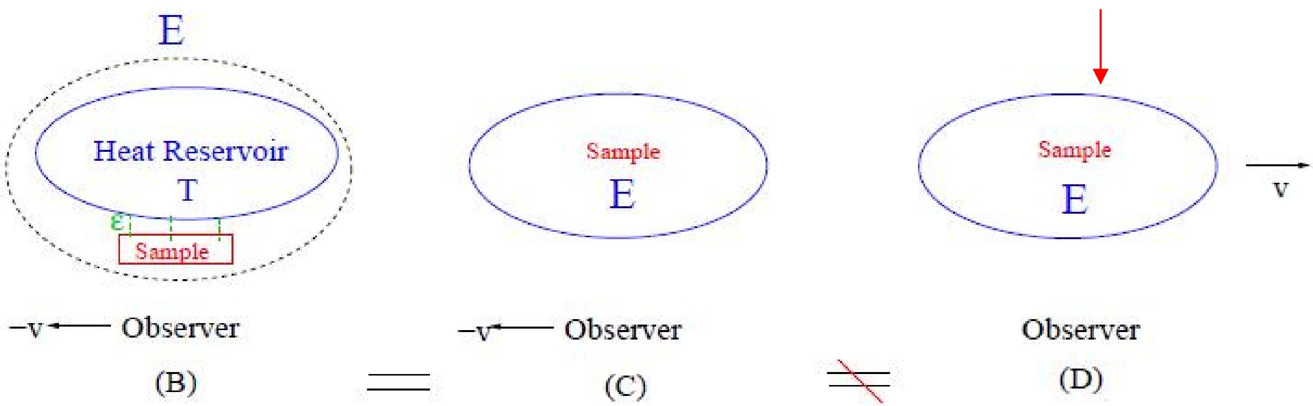
transform are not universal, depend on the specific systems. also both $\vec{v}_1$ and $\vec{v}_{12}$

Micro-canonical ensemble

(Isolated system)

Quantum Materials $N \sim 10^{23}$

Planck, Einstein, Pauli, Laue……..

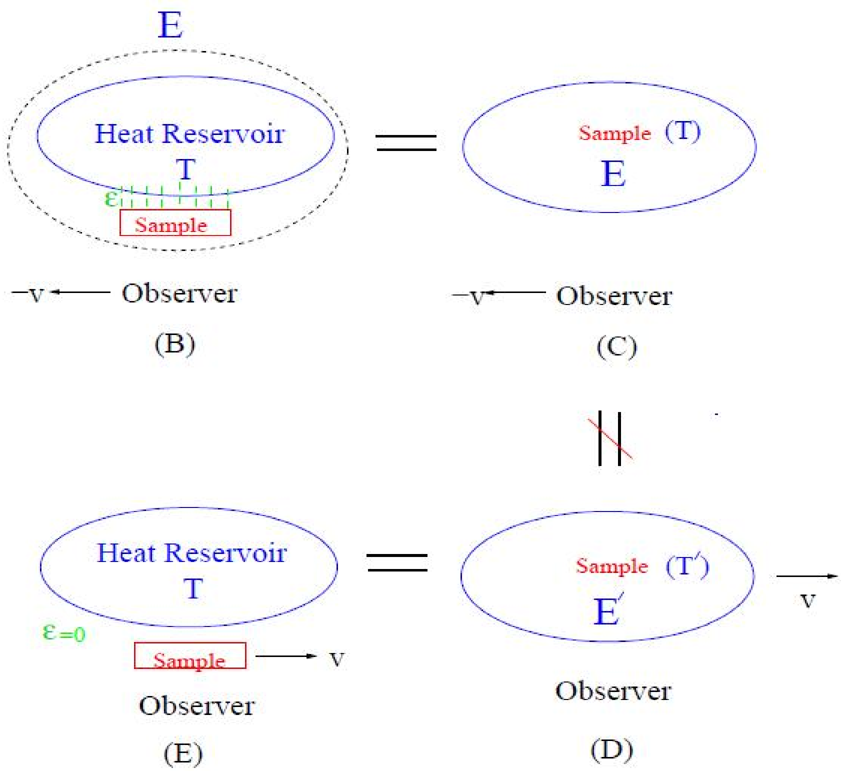


All particle physics Experiments $N \sim 10^3$

Some cold atom system inside a trap $N \sim 10^3 - 10^6$

Black holes in AdS Quantum Entanglement Temperature

$$k_B T_H = \frac{\hbar \kappa}{2\pi c}$$



Case (D)

Planck, Einstein, Pauli, Laue……..

$$\frac{T'}{T} = 1/\sqrt{1-(\frac{v}{c})^2}$$

Micro-canonical ensemble

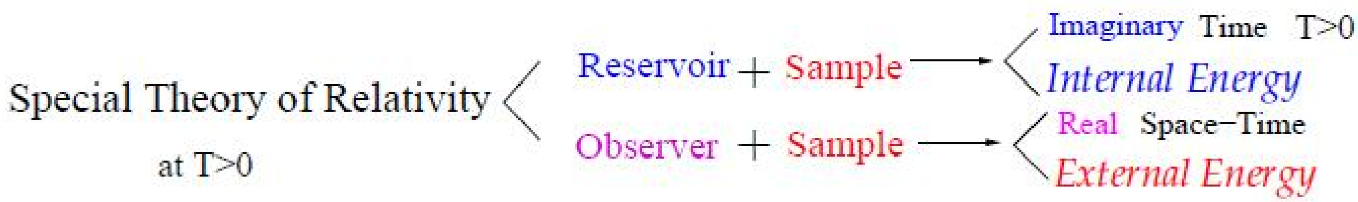
No Reservoir or $\varepsilon \to 0$

Only Intrinsic part

$$\boxed{\tau = \hbar\beta + i\,t}$$

Temperature effects *Internal Energy*

Independent of the observer



External energy (*Doppler effects*) $\omega' = \gamma(\omega - \vec{v}\cdot\vec{k})$

Independent of the reservoir

$\boxed{SO(4) \rtimes SO(3,1)}$ in 2+3=5 dimension

or S^1 bundle on $M^{3,1}$

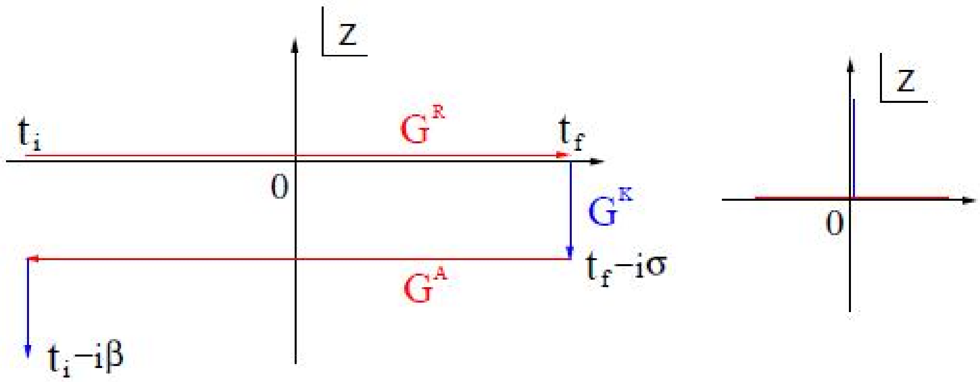


FIG. 9. The Keldysh contour in the whole complex time plane $z = t + i\tau$ in the static frame. The forward and backward path along the real (red) axis gives the Retarded G^R and Advanced G^A Green function respectively which lead to the external energy. While the path along the pure imaginary (green) axis gives the distribution function G^K which leads to internal energy. $0 < \sigma < \beta$ can be chosen for the convenience, but the physics is independent of such a choice. The case (B) in Eq.F3 is just performing a conventional LT along the real (red) time (Inset) which leads to the external energy. While the co-moving case Eq.F7 is performing a new LT along the pure imaginary (green) time (Inset) which leads to the internal energy. Because nothing happens in the co-moving frame in the conventional special theory of relativity, so it is new and not contained in the conventional special theory of relativity (Fig.7). For the generic case, it involves a generalized LT in the whole complex plane, namely in the $3 + 2 = 5$ dimensional space-time (or a S^1 fiber bundle over $\mathcal{M}^{3,1}$).

Particle physics experiments like LHC, RHIC, FAIR……
belong to **micro-canonical ensemble**

Bombarding and splitting atoms

Very hot Temp is $T \sim 10^{12} K$

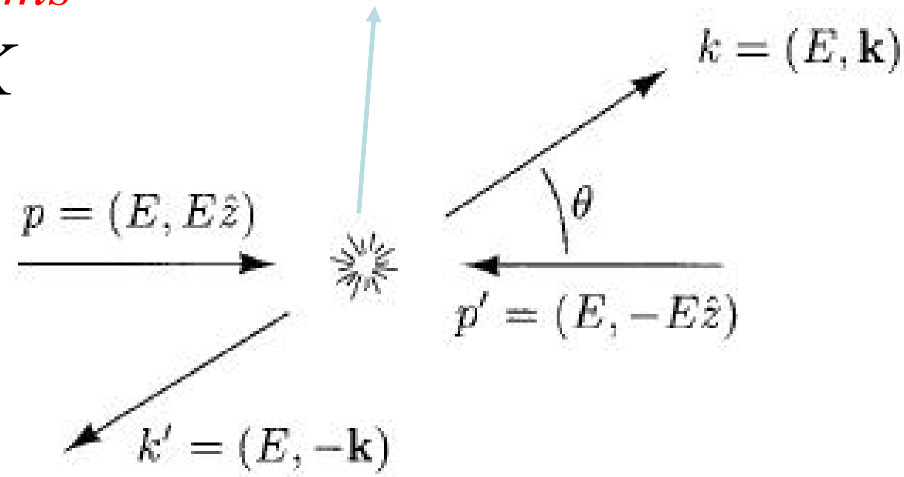
self-generated

No reservoir

Not really in the usual Statistical Mechanics !

NOT a truly equilibrium System !

Hot clouds expands and Cool down $N \sim 10^3$



Mimic the **early universe** which expands slowly up to now

AdS Quantum black hole is also isolated !

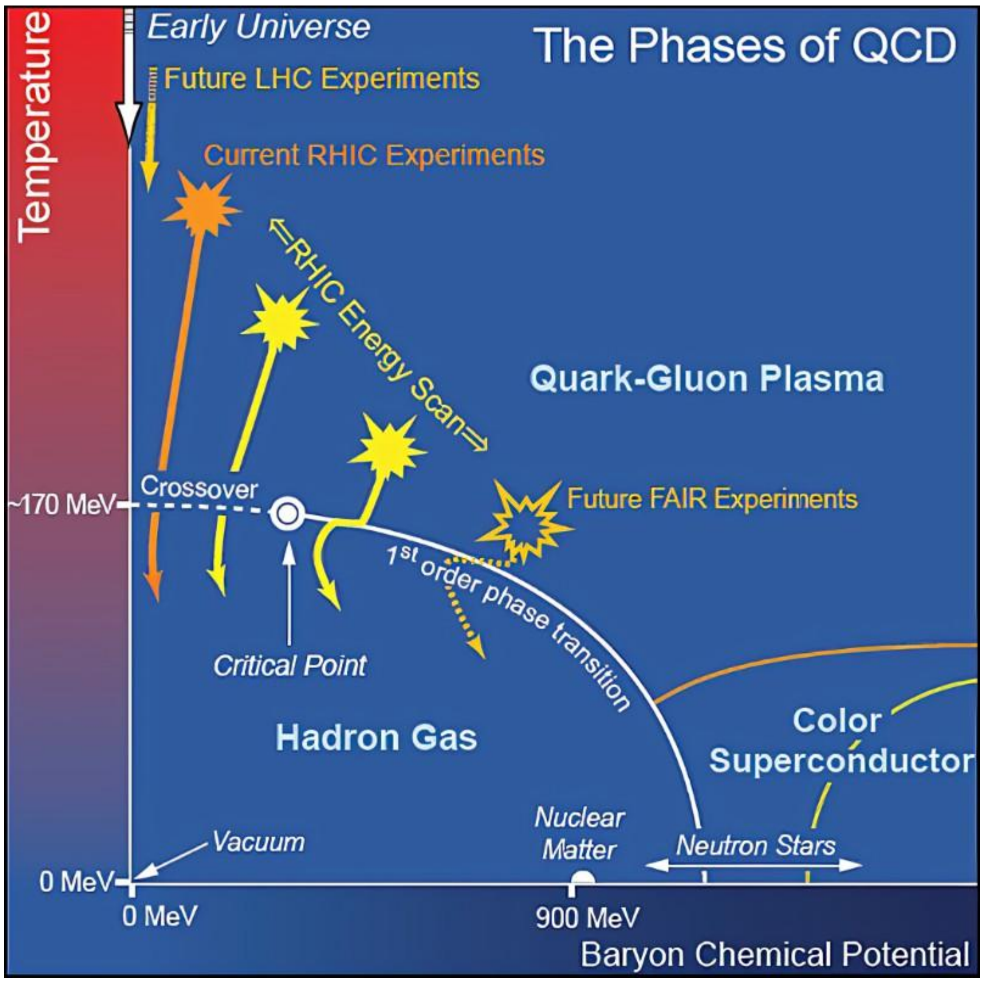
Condensed matter experiments belong to **Canonical Ensemble**: there is always a reservoir

Cold atom experiments maybe isolated systems (micro-canonical ensemble) or both

Looks like (fictitious) grand canonical ensemble (T, μ)

In Reality, Micro-canonical ensemble **fixed** (E, N)

T



μ

A moving detector

$$T_o^{'} / T = 1$$

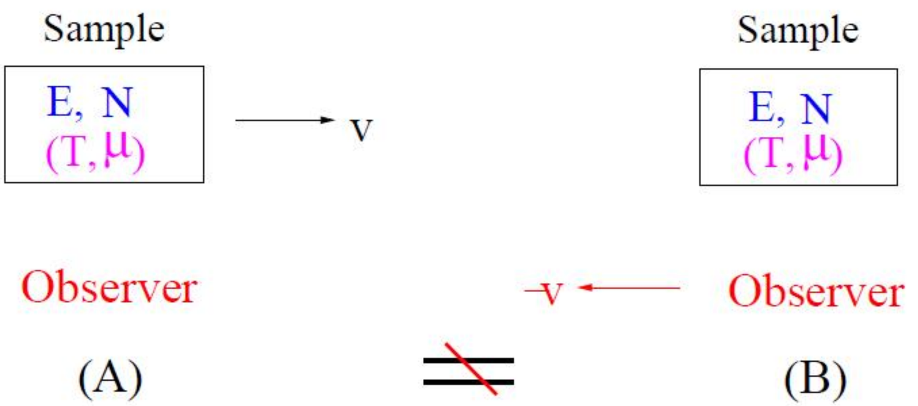
hard in Experiment

A moving hot cloud

$$\frac{T^{'}}{T} = 1 / \sqrt{1 - (\frac{v}{c})^2}$$

easy in Experiment

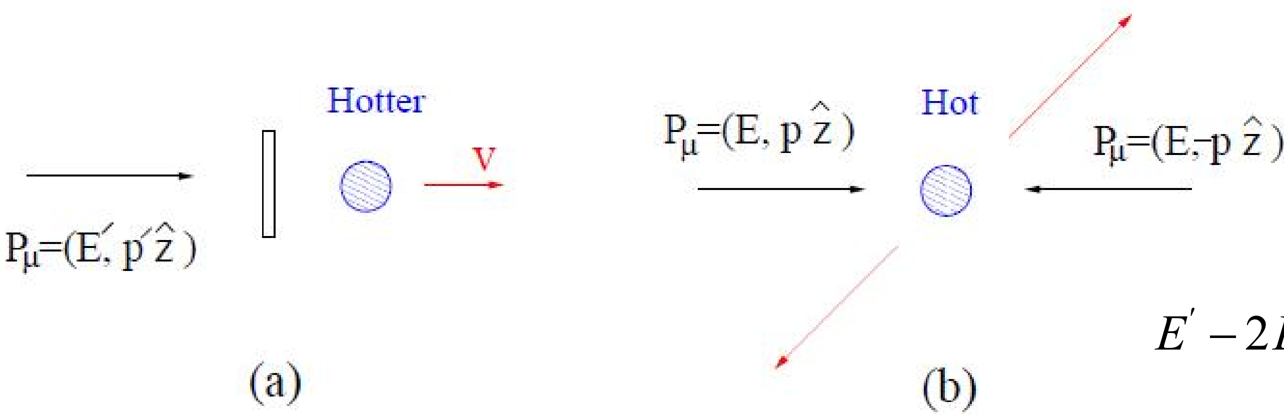
Planck, Einstein, Pauli, Laue……..



Lorentz boost the static cloud (b) into the moving cloud (a).

the LI mass $M^2 = (p_{1\mu} + p_{2\mu})^2$

$$\gamma_i = E / m_0 c^2$$

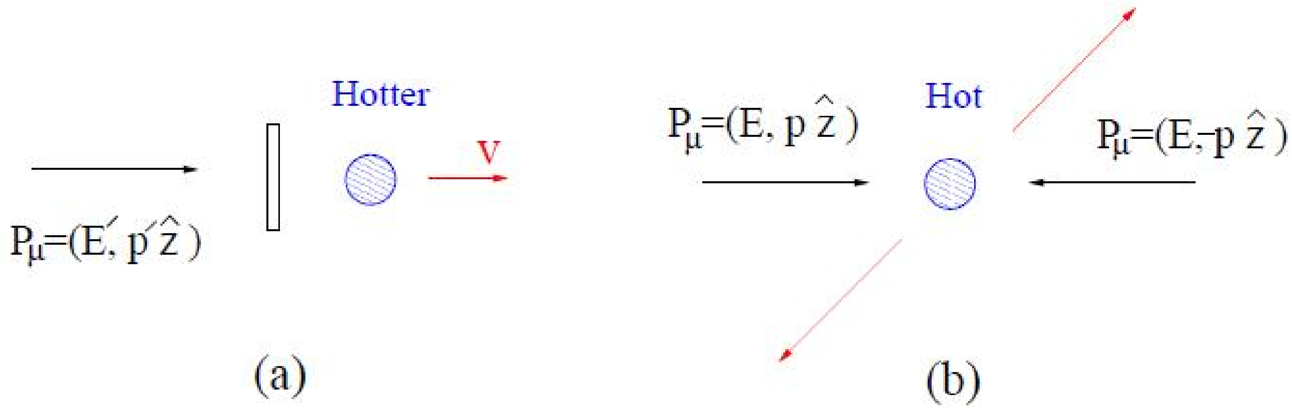


$$E' - 2E \sim 2E(\frac{\gamma_i^2}{1+\gamma_i})$$

in the ultra-relativistic limit.

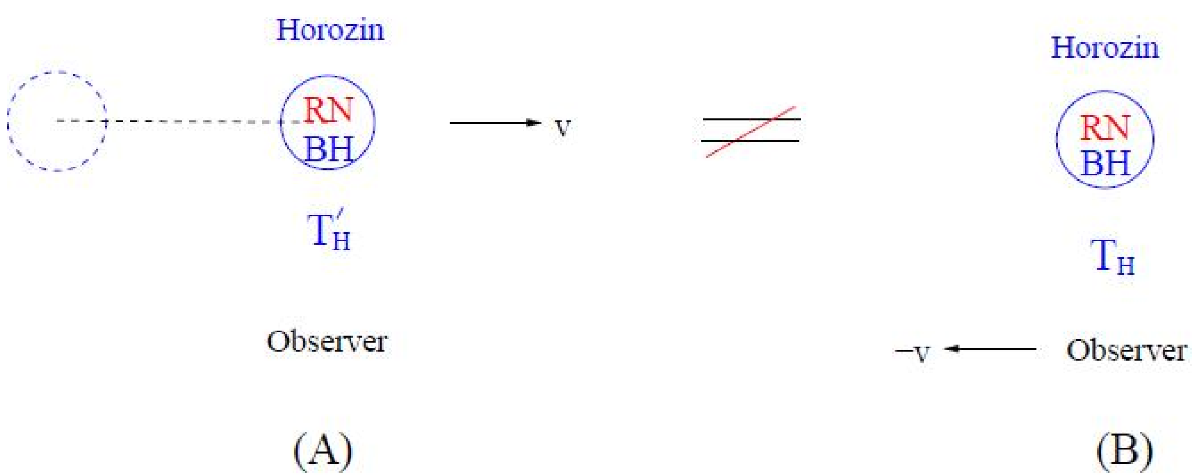
$$E \gg m_0 c^2$$

$$\frac{v}{c} \sim 1 - \frac{1}{8}(\frac{m_0 c^2}{E})^4 + \cdots$$



Hotter
V
P_μ=(E′, p′ẑ)
(a)
Hot
P_μ=(E, p ẑ)
P_μ=(E,-p ẑ)
(b)

is dual to



Horozin
RN
BH
T′_H
v
Observer
(A)
Horozin
RN
BH
T_H
−v
Observer
(B)

Case (A) for a moving cloud

the momentum distribution curve (MDC) number of quarks **in the lab frame** satisfies

$$n_F(\omega'(\vec{k})) = \frac{1}{e^{\frac{\hbar\omega'(\vec{k})-\mu'}{k_B T'}}+1}$$

the measure $V' dk'_{E,x} dk'_{E,y} dk'_{E,z}$

the density of states (DOS):

$$V' D'(\omega') = V' \int \frac{d^d k'}{(2\pi)^d} \delta[\omega' - \omega'(\vec{k}')]$$

the **energy** distribution curve (**EDC**):

$$n'_F(\omega') = \frac{V' D'(\omega')}{e^{\frac{\hbar\omega'-\mu'}{k_B T'}}+1}$$

μ' need to be determined by the total number of fermions

$$N_F \sim 10^3$$

the total number of fermions is

$$N_F = N_F' = \int d\omega' n_F'(\omega')$$

MDC and EDC can be mapped out by various neutron, light or X-ray scattering on a moving cloud with a velocity $\vec{v}$

Unfortunately, in contrast to the condensed matter systems, there is not an external reservoir to compare the temperature, one needs to compare the *two Lorentz boost related clouds* in (a) and (b)

In terms of the static coordinates

Now transferring back to the static frame,

$$\omega'(\vec{k}') = \gamma(\omega(\vec{k}) - \vec{v}\cdot\vec{k}) \qquad T' = \gamma T$$

So the MDC can be re-written in the static coordinate:

$$n'_F(\vec{k}) = \frac{1}{e^{\frac{\hbar(\omega(\vec{k})-\vec{v}\cdot\vec{k})-\mu'/\gamma}{k_B T}}+1}$$

with the corresponding measure:

$$V' dk'_{E,x} dk'_{E,y} dk'_{E,z} = V(1 - \frac{v}{c^2}\frac{\partial\omega(\vec{k})}{\partial k_x}) dk_x dk_y dk_z$$

$V' = \gamma^{-1} V$ *Lorentz contraction* of the spacial volume

In terms of the static coordinates

The DOS:

$$V'D'(\omega') = V\int \frac{d^d k}{(2\pi)^d}\left(1-\frac{v}{c^2}\frac{\partial\omega}{\partial k_x}\right)\delta[\omega' - \gamma(\omega(\vec{k}) - \vec{v}\cdot\vec{k})]$$

for any relativistic spectrum $\omega(\vec{k})$, the DOS ***per volume*** takes the same form in any inertial frame .

$$D'(\omega') = D(\omega')$$

The EDC

$$n_F'(\omega') = \frac{V'D'(\omega')}{e^{\frac{\hbar\omega' - \mu'}{k_B T'}} + 1} = \left(\frac{1}{\gamma}\right)\frac{VD(\omega')}{e^{\frac{\hbar\omega' - \mu'}{k_B(\gamma T)}} + 1}$$

Both the MDC and the EDC can be contrasted to those in the static cloud:

$$n_F(\omega(\vec{k})) = \frac{1}{e^{\frac{\hbar\omega(\vec{k})-\mu}{k_B T}} + 1}$$

with the measure $V dk_x dk_y dk_z$

$$n_F(\omega') = \frac{VD(\omega')}{e^{\frac{\hbar\omega' - \mu}{k_B T}} + 1}$$

$$N_F = \int d\omega' n_F(\omega') = N_F' = \int d\omega' n_F'(\omega')$$

The ***relation*** between μ' and μ can be determined from this constraint.

For the lightest **up** quark , $m_u c^2 = 2.2 MeV$
taking $T \to 0$ limit leads to

$$\mu'^2 - (mc^2)^2 = \gamma^{2/3}[\mu^2 - (mc^2)^2]$$

$$\mu'(T=0) > \mu(T=0)$$

$\mu'(T=0) \sim \gamma^{1/3}\mu(T=0)$ in the $\mu \gg mc^2$ limit.

At any $T > 0$, scaling out $k_B T$ in the unit of $100 MeV$ scale

$$N_F = 10^2 (k_B T)^3 \int_a^\infty dx \frac{x\sqrt{x^2 - a^2}}{e^{x-y} + 1}$$

$$= 10^2 (k_B T)^3 \gamma^2 \int_{a/\gamma}^\infty dx \frac{x\sqrt{x^2 - (a/\gamma)^2}}{e^{x-y'} + 1}$$

$a = mc^2 / k_B T, y = \mu / k_B T$

$y' = \mu' / \gamma k_B T$

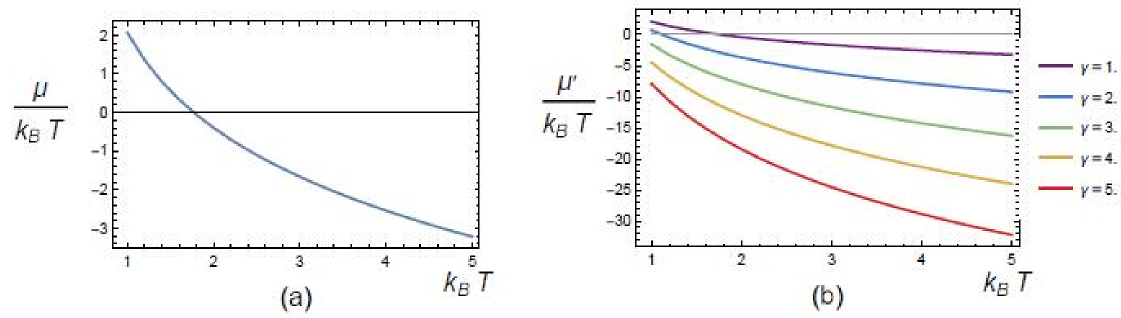
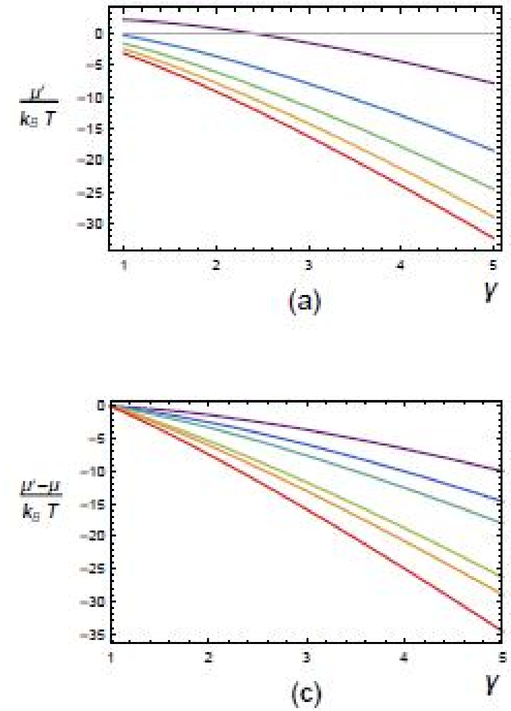


FIG. 4. The temperature dependence of the chemical potential for the static cloud and the moving cloud in (A). $k_B T$ is using the limit of $100MeV$. (a) In a static cloud, $\mu/k_B T$ decreases from positive to negative value. (b) For a moving cloud, $\mu'/k_B T$ decreases from positive to negative value more rapidly as its velocity increases. $\gamma = 1$ recovers the static cloud. Of course, as $k_B T \to 0$, both diverges to infinity (not shown).

$k_B T = 1, 2, 3, 4, 5$



the transformation law for μ of a moving cloud is much less **universal** than T

It fails in the strong coupling limit where the theory is intractable, but the experimental result shows $\mu \to 0^+$ in the hight temperature limit.

Case (B) for a moving observer

the MDC of the number of quarks in the running frame satisfies

$$n_F(\omega(\vec{k})) = \frac{1}{e^{\frac{\hbar\omega(\vec{k})-\mu'}{k_B T}}+1}$$

with the measure of distribution function

$$V' dk'_{E,x} dk'_{E,y} dk'_{E,z} = V(1-\frac{v}{c^2}\frac{\partial\omega(\vec{k})}{\partial k_x}) dk_x dk_y dk_z$$

$V' = \gamma^{-1} V$ *Lorentz contraction* of the spacial volume

μ' need to be determined by the total number of fermions

$$N_F \sim 10^3$$

EDC: $$n'_{SF}(\omega') = \frac{V'D'(\omega')}{e^{\frac{\hbar\omega' - \mu'}{k_B T}} + 1} = \frac{VD(\omega')}{e^{\frac{\hbar\omega' - \mu'}{k_B T}} + 1}$$

the density of states (DOS)

$$V'D'(\omega') = V\int \frac{d^d k}{(2\pi)^d}\left(1 - \frac{v}{c^2}\frac{\partial\omega}{\partial k_x}\right)\delta[\omega' - \omega(\vec{k})]$$

$$= V\int \frac{d^d k}{(2\pi)^d}\delta[\omega' - \omega(\vec{k})] = VD(\omega')$$

which can be contrasted to the EDC in the **static** observer:

$$n_F(\omega') = \frac{VD(\omega')}{e^{\frac{\hbar\omega' - \mu}{k_B T}} + 1} \longrightarrow \mu' = \mu$$

ıtrast the Case (A) and the Case (B) with the static sample in various physical quantities. P
table also applies to a charged RN black hole through AdS_5/CFT_4.

$(A) \neq (B)$	Static	Case (A)	Case (B)
Temp.	T	γT	T
Chemical potential	μ	$\mu' \neq \mu$	μ
Critical Temp.	T_c	$\gamma^{-1} T_c$	T_c
EDC	$n_F(\omega') = \frac{VD(\omega')}{e^{\frac{\hbar\omega'-\mu}{k_B T}}+1}$	$n'_F(\omega') = (\frac{1}{\gamma})\frac{VD(\omega')}{e^{\frac{\hbar\omega'-\mu'}{k_B(\gamma T)}}+1}$	$n'_F(\omega') = n_F(\omega')$
MDC	Eq.9	Eq.5,6	Eq.11,12
Sound velocity	c_s	$v > c_s$ induces a PT	$v > c_s$ induces no PT
Shear viscosity	η	$\gamma^3 \eta$	η
Bulk viscosity	ξ	$\gamma^3 \xi$	ξ
QGP lifetime	τ	$\gamma^2 \tau$	$\gamma \tau$
QGP size	L	$\gamma^{-1} L$	$\gamma^{-1} L$

The QGP lifetime τ is a many body one instead of for a single particle.

So the MDC differs from that in the static observer by an extra piece **anti-symmetric** in $\vec{k}$

the EDC is **identical** as that in the static case

can be mapped out by various neutron, light or X-ray scattering on a moving detector with a velocity $-\vec{v}$

the case (B) is essentially the same as the previously proposed co-variant approach $T^{'} = T, \quad \mu^{'} = \mu$

If the observer is accelerated, then it becomes the celebrated Unruh effect .
For a fermion system with a finite chemical potential, it remains interesting to study what is the μ viewed by the Rindler observer.

The QGP is a de-confined phase confined in a finite volume $L \sim 10\, fm$ $1\, fm = 10^{-15}\, m$
which is about 10 times of a nucleon. Its lifetime is $\tau \sim 10^{-22}\, s$ in the COMM frame.

Due to the expansion of the hot clouds, its putative temperature may decrease during such an expansion.

In testing the relativistic temperature effects, the quantum matter and the QGP are *complementary* to each other.
The former is stable, but the relativistic effects is small $(v/c)^2 \sim 10^{-5}$.

The latter is in the ultra-relativistic limit, but its lifetime is short $\tau \sim 10^{-22}\, s$

The QGP phase in the strong coupling: the collective (sound) mode

The (slihtly off-equilibrium) sound mode in the QGP phase breaks the LI with a sound velocity $c_s < c$

$$\nabla^2 u^i(\vec{x},t) - \frac{1}{c_s^2}\frac{\partial^2 u^i(\vec{x},t)}{\partial t^2} = 0 \qquad c_s = \sqrt{dp/d\epsilon}$$

u^i is the displacement along one of the three spacial dimensions,

For a perfect fluid $c_s = 1/\sqrt{3} \sim 0.57$

Experimentally $c_s \sim 0.27$ at $T \sim 222 MeV \sim 10^{13} K$

It should also be ***sensitive*** to T .

the sound mode breaks LI. In fact, it is precisely due to the breaking of the LI in the medium of QGP.

For the case (A), when the hot cloud moves faster than c_s in addition to its temperature increases, the QGP gets into a different phase through a phase transition.

The nature of the phase transition and the new phases may be derived from the relativistic hydrodynamics of QGP and will be presented elsewhere.

For the case (B), the moving observer can only see a Doppler shifted energy of the sound wave, but no such new phases and the phase transition can be observed.

The decrease of the de-confinement transition $T_c \sim 10^{12} K$ in the case (A).

The LT at $T = 0, \mu = 0$ dictates

$$Z_A = Z'_A[\gamma_1^{-1}\beta, V] = Z_A[\gamma_1^{-1}\beta, V]$$

If there is a quantum or thermal phase transition, it only happens in the thermodynamic $V \to \infty$ limit

There is a singularity at $Z(\beta_c)$ indicating a critical temperature T_c in the static sample, then in the moving sample, the singularity shifts to:

$$\frac{T'_c}{T_c} = \sqrt{1 - (\frac{v}{c})^2} < 1$$

For $\gamma \sim 10$, the drop by one order of magnitude of T_c is quite appreciable.

The case (A) In terms of the static coordinates

The DOS:

$$V^{'}D^{'}(\omega^{'})=V\int\frac{d^dk}{(2\pi)^d}(1-\frac{v}{c^2}\frac{\partial\omega}{\partial k_x})\delta[\omega^{'}-\gamma(\omega(\vec{k})-\vec{v}\cdot\vec{k})]$$

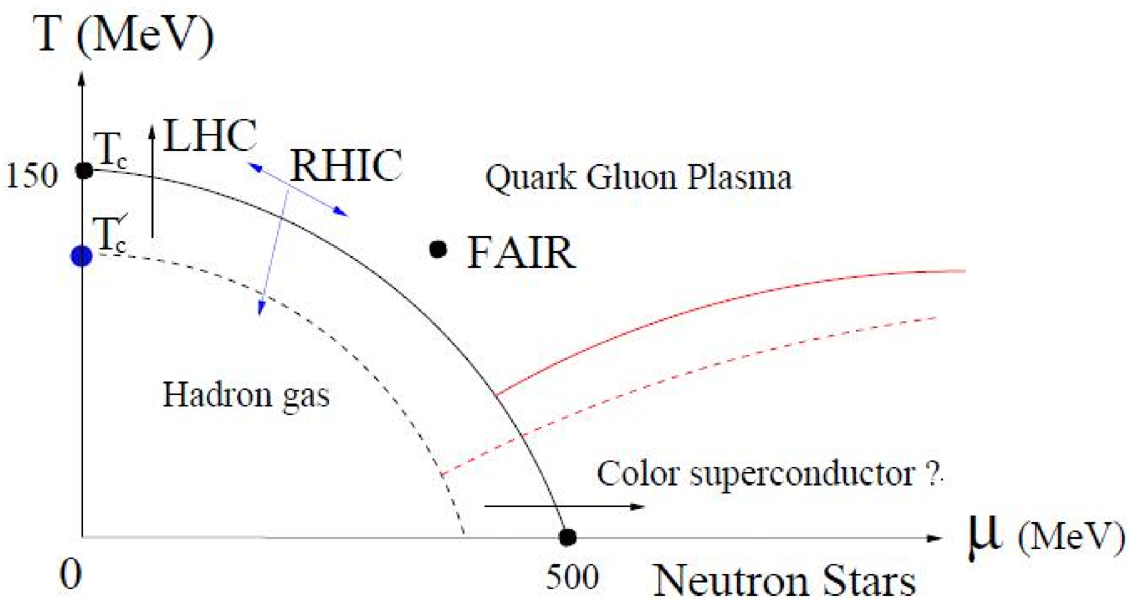
for any relativistic spectrum $\omega(\vec{k})$, the DOS ***per volume*** takes the same form in any inertial frame .

$$D^{'}(\omega^{'})=D(\omega^{'})$$

The EDC $$n_F^{'}(\omega^{'})=\frac{V^{'}D^{'}(\omega^{'})}{e^{\frac{\hbar\omega^{'}-\mu^{'}}{k_BT^{'}}}+1}=(\frac{1}{\gamma})\frac{VD(\omega^{'})}{e^{\frac{\hbar\omega^{'}-\mu^{'}}{k_B(\gamma T)}}+1}$$

If setting , then $\mu=\mu^{'}=0$ the only difference is the scale of the temperature which has important application on $T_c\sim10^{12}K$ in the moving cloud the finite chemical potential $\mu \neq 0$ will change the result only quantitatively, but not quantitatively.

Namely, the de-confinement transition of a moving QGP cloud drops. Moving favors the formation of the QGP



There is no change of $T_c \sim 10^{12} K$ in the case (B).

ntrast the Case (A) and the Case (B) with the static sample in various physical quantities. P
e table also applies to a charged RN black hole through AdS_5/CFT_4.

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Bulk viscosity	ξ	$\gamma^3 \xi$	ξ
QGP lifetime	τ	$\gamma^2 \tau$	$\gamma \tau$
QGP size	L	$\gamma^{-1} L$	$\gamma^{-1} L$

The QGP lifetime τ is a many body one instead of for a single particle.

Shear viscosity of QGP $\mu = 0$ and AdS black holes:

The de-confinement phase transition from the QLP phase to the Hadron gas transition should be similar to the Hawking-Page phase transition in the bulk . By using the well known $\mathcal{N} = 4\, SUSY,\, SU(N)$ Yang-Mills

dual to

Type IIB string theory on $AdS_5 \times S^5$

One can study the **shear viscosity** of the QLP phase from both the gauge theory and the gravity side.

G. Policastro, D. T. Son, A. O. Starinets, Phys.Rev.Lett.87:081601,2001.

P. Kovtun, D. T. Son, A. O. Starinets, Phys.Rev.Lett. 94 (2005) 111601.

In the strong t' Hooft coupling limit

$$\lambda = N^2 g_{YM} \to \infty$$

the shear viscosity $\eta = \frac{\pi}{8} N^2 T^3$

It also satisfies a universal bound

$$\eta / s \geq \hbar / 4\pi k_B$$

In the large $\lambda \to \infty$ limit:

$$\eta / s = \hbar / 4\pi k_B (1 + \lambda^{-3/2} + \cdots)$$

In the case (A) for a moving hot cloud $\eta \sim \gamma^3$

so is the entropy density $s \sim \gamma^3$

their ratio remains the same in the moving cloud.

In the case (B) for a moving observer
both stay the same, so its their ratio.

So the differences between case (A) and case (B) can be easily distinguished in the **shear viscosity**.

The $U(1)$ global symmetry in the boundary leads to the conserved current j_μ , which is dual to the $U(1)$ gauge field A_μ in the AdS_5 bulk geometry.

The time component of the gauge field at the boundary corresponds to the chemical potential .

$$A_\tau(r=\infty)=i\mu$$

At a finite T , there is a charged RN black hole with a horizon area A and a Hawking temperature T_H .

The bulk calculation in the presence of such a charged RN black hole leads to

$$\eta = \frac{a}{16\pi G} \sim T_H^3$$

where $a = \frac{A}{V}$ with V as the spatial volume of S^5

While $S = \frac{A}{4G}$ so the entropy density $s = \frac{a}{4G} \sim T_H^3$

It also leads to the universal bound

$$\eta / s \geq \hbar / 4\pi k_B$$

In the case (A) for a moving RN black hole in the bulk, $T_{Hm} = \gamma T_H$ implies $\eta \sim \gamma^3$

so is the entropy density $s \sim \gamma^3$

However, their ratio remains the same in the moving black hole.

In the case (B) for a moving observer along the boundary, $T_{Ho} = T_H$ implies

both stay the same, so its their ratio.

Bayesian statistical analyses of experimental data from RHIC are used to determine the effective QGP shear viscosity as a function of the net baryon chemical potential.

Of course, $SU(3)$ QCD with no SUSY is different than the $\mathcal{N}=4,\ SU(N)$ Yang-Mills gauge field.

Of course, at any finite $T>0$, there is no SUSY anymore

G. Policastro, D. T. Son, A. O. Starinets, Phys.Rev.Lett.87:081601,2001.

P. Kovtun, D. T. Son, A. O. Starinets, Phys.Rev.Lett. 94 (2005) 111601.

did all the caculations at $\mu=0$ which maybe valid at $T\gg\mu$, but the QGP phase happens at $T\sim\mu$

Bulk viscosity of QGP $\mu > 0$ and AdS black holes:

Note that a finite $\mu > 0$ also breaks the conformal symmetry (in fact, also the space-time LI) at $\mu = 0$, so may lead to a non-vanishing bulk viscosity ζ .

In the strong coupling limit, the calculations from the gravity side also found a universal bound

$$\xi / \eta \geq 2(\frac{1}{D-1} - c_s^2)$$

D space-time dim. c_s sound wave velocity which is *sensitive* to T

The perfect fluid satisfies the lower bound $\xi / \eta = 0$

In fact, in the weak coupling limit, when solving the kinetic Boltzmann equation to the linear (or including the leading Log or beyond) order in the moving velocity v , several authors always used MDC for the case (B).

This is because in the covariant approach, the LI dictates the equivalence between the case (A) and the case (B). However, in our theory, the LI was explicitly broken, so Case (B) is different than case (A).

In the case (A) which is the experimental situation considered, one needs to use the correct MDC,

Furthermore, the transformation law of the chemical potential $\mu^{'} / \mu$ in the boundary are non-universal in the case (A) and depends on T and γ sensitively as shown in a previous slide, but does not change in the case (B).

It remains interesting to use the correct MDC to ***re-do all the calculations*** in the weak coupling limit.
The differences already show up in the linear order of the velocity v .

6. Experimental detections in materials.

The Temperature transformation law also holds for any non-relativistic system,

$$\frac{T'}{T} = 1/\sqrt{1-(\frac{v}{c})^2} > 1$$

$$\varepsilon \to 0$$

This makes it experimental detection in non-relativistic systems such as materials or cold atom systems feasible.

Taking the lowest non-trivial limit $v/c \to 0$, then it reduces to

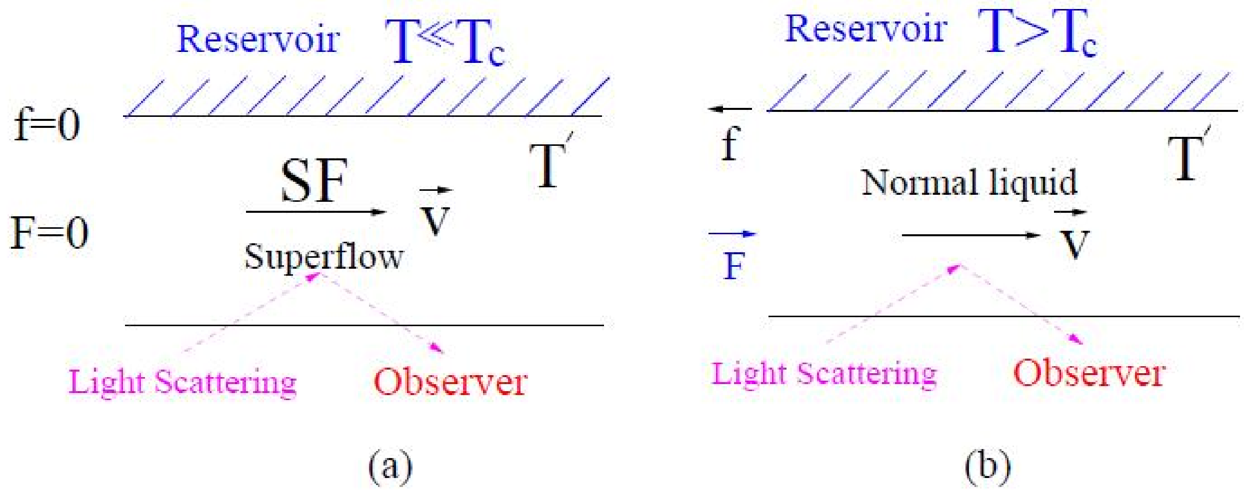
$$T'/T \sim 1 + \frac{1}{2}(v/c)^2$$

the relativistic effect show up at the order of $(v/c)^2$

A flowing Superfluid He4

$$T \ll T_c \sim 2.17K, \quad v < v_c \sim 60m/s$$

$\varepsilon \to 0$



$\varepsilon \to 1$

$$\frac{T'}{T} = 1/\sqrt{1-(\frac{v}{c})^2} > 1 \quad + O(\epsilon^2)$$

Due to the internal $U(1)$ symmetry breaking in the SF, the number of particles is not conserved, then a finite chemical potential need to be introduced to enforce an **average number** of He4 atoms in a grand **canonical ensemble**.

The free energy and the number of quasi-particle in a static sample are given by :

$$F(T,V) = k_B T V \int \frac{d^d k}{(2\pi)^d} \ln(1 - e^{-\omega(\vec{k})/k_B T}) = k_B T V \int d\omega D(\omega) \ln(1 - e^{-\beta\omega})$$

$$N_{qp}(T,V) = V \int \frac{d^d k}{(2\pi)^d} (e^{\omega(\vec{k})/k_B T} - 1)^{-1} = V \int d\omega D(\omega) (e^{\beta\omega} - 1)^{-1}$$

Density of states (DOS)

$$VD(\omega) = V \int \frac{d^d k}{(2\pi)^d} \delta(\omega - \omega(\vec{k}))$$

For the case (A)

Internal Energy= Doppler shifted energy

$$(k,\omega) \to (k', \omega') \qquad \omega' = \gamma(\omega - \vec{v} \cdot \vec{k}), \; k_1' = \gamma(k_1 - \frac{v}{c^2}\omega)$$

$$T' = \gamma T, \quad V' = \gamma^{-1} V$$

The free energy and the number of quasi-particle in a static sample are given by :

$$F(T,V) = k_B T V \int \frac{d^d k}{(2\pi)^d} \ln(1 - e^{-\omega(\vec{k})/k_B T}) = k_B T V \int d\omega D(\omega) \ln(1 - e^{-\beta\omega})$$

$$N_{qp}(T,V) = V \int \frac{d^d k}{(2\pi)^d} (e^{\omega(\vec{k})/k_B T} - 1)^{-1} = V \int d\omega D(\omega) (e^{\beta\omega} - 1)^{-1}$$

Density of states (DOS)

$$VD(\omega) = V \int \frac{d^d k}{(2\pi)^d} \delta(\omega - \omega(\vec{k}))$$

For the case (A) Internal Energy= Doppler shifted energy

The **MDC** of the quasi-particle excitations in the lab frame:

$$n'_{SF}(\omega') = \frac{1}{e^{\frac{\hbar\omega'(\vec{k}')}{k_B T'}} - 1}$$

The measure is

$$V' d^d k' / (2\pi)^d$$

$T' = \gamma T$ is slightly higher than that of the reservoir (the wall).

the energy distribution curve (**EDC**): $$n'_{SF}(\omega') = \frac{V' D'(\omega')}{e^{\frac{\hbar\omega'}{k_B T'}} - 1}$$

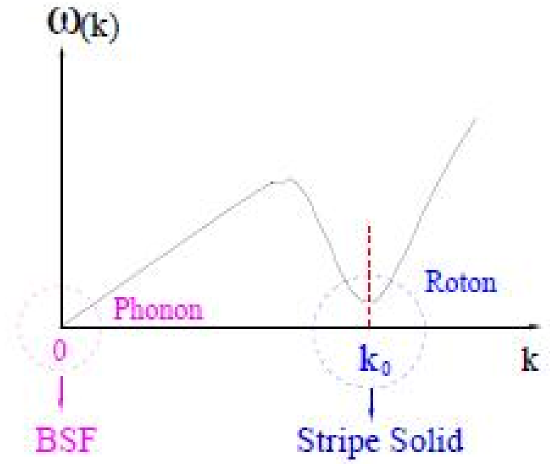
The DOS $$V' D'(\omega') = V' \int \frac{d^d k'}{(2\pi)^d} \delta[\omega' - \omega'(\vec{k}')]$$

Both MDC and EDC can be mapped out by various , light, neutron or X-ray scattering on a moving sample with a velocity $\vec{v}$.

In terms of the coordinates in the static frame:

$$\omega' = \gamma(\omega - \vec{v}\cdot\vec{k}) \qquad T' = \gamma T$$

$$\frac{\omega'}{T'} = \frac{\omega - \vec{v}\cdot\vec{k}}{T}$$



the relativistic factor γ drops out in the MDC

$$n'_{SF}(\vec{k}) = \frac{1}{e^{\frac{\hbar(\omega(\vec{k}) - \vec{v}\cdot\vec{k})}{k_B T}} - 1}$$

with the measure

$$V' \frac{dk'_{E,x} dk'_{E,y} dk'_{E,z}}{(2\pi)^d} = V\left(1 - \frac{v}{c^2}\frac{\partial \omega(\vec{k})}{\partial k_x}\right)\frac{dk_x dk_y dk_z}{(2\pi)^d}$$

$\omega(\vec{k})$ is the excitation spectrum in a SF containing the phonon and the roton part in a static sample.

The EDC

$$n'_{SF}(\omega') = \frac{V' D'(\omega')}{e^{\frac{\hbar\omega'}{k_B T'}} - 1} \qquad T' = \gamma T$$

The density of states (DOS)

$$V' D'(\omega') = V \int \frac{d^d k}{(2\pi)^d} \left(1 - \frac{v}{c^2}\frac{\partial\omega}{\partial k_x}\right) \delta[\omega' - \gamma(\omega(\vec{k}) - \vec{v}\cdot\vec{k})]$$

satisfies the sum rule:

$$V' \int d\omega' D'(\omega') = V \int \frac{d^d k}{(2\pi)^d} \left(1 - \frac{v}{c^2}\frac{\partial\omega}{\partial k_x}\right) = V \int \frac{d^d k}{(2\pi)^d}$$

the size of the Hilbert space is the same in any inertial frame. But not necessarily for the total number of particles N'_{SF} which is not a conserved quantity in any given reference frame.

$$(v/c)^2 \sim 10^{-15}$$

The recent experimental detection of the red shift in a gravitational field to reach the un-precedent accuracy $10^{-20}\, mm^{-1}$ by the atomic clocks, we expect it is within the current experimental precision measurement.

*Tobias Bothwell, et.al, **Jun Ye**, Resolving the gravitational redshift across a millimetre-scale atomic sample, Nature volume 602, pages420–424 (2022).*

May also be used measure the transverse Doppler shift

it is still much larger than the frequency shift

$$\Delta f / f \sim 10^{-21}$$

caused by **the gravitational wave** which was detected by the Laser interferometry.

Unruh Temperature

$$k_B T_U = \frac{\hbar a}{2\pi c}$$

$a = 1m/s^2$ $\quad T_U \sim 4 \times 10^{-21} K$ Beyond current Tech

The T' of the Surface super-current versus T of the bulk in a type-I superconductor

The flowing velocity is $(v/c)^2 \sim 10^{-5}$, which is about 10 orders of magnitude larger than that of a flowing SF

$$T'/T \sim 1 + \frac{1}{2}(v/c)^2$$

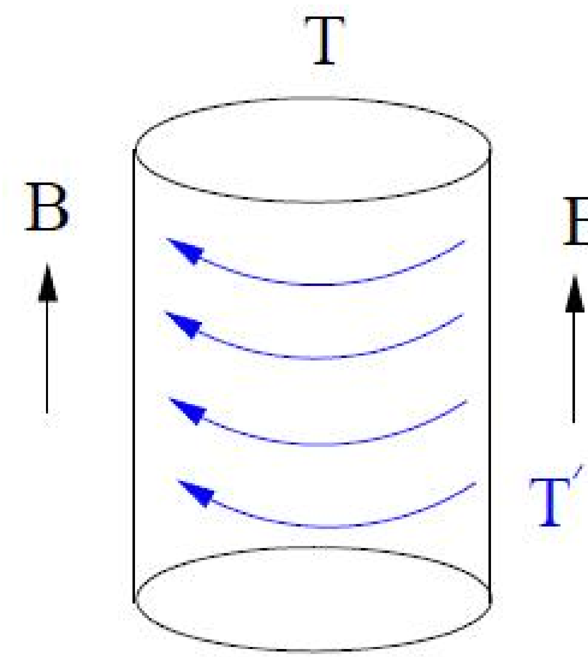
The London penetration length into the bulk at $T = 0$

$$\lambda_L = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{1/2} \sim 100\ \AA$$

diverges near $T \sim T_c^-$

perform the measurement at

$$T \ll T_c$$



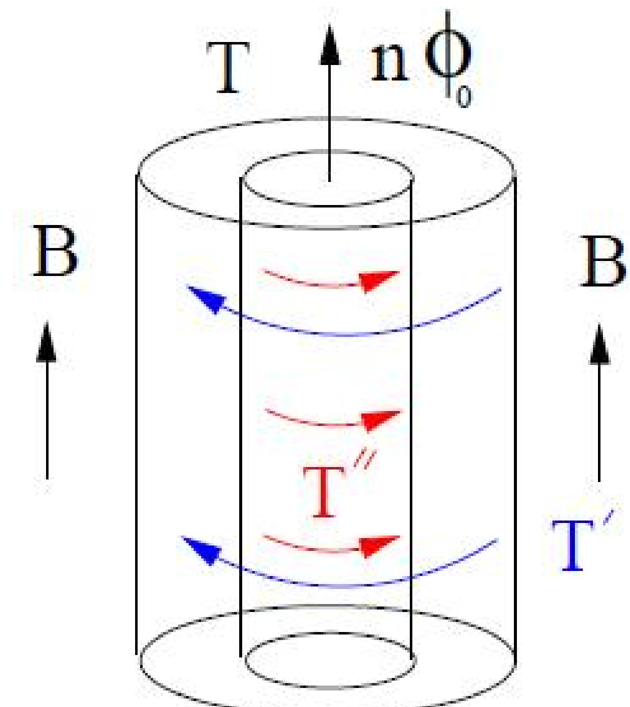
With a hole in the center may act more like a type-II superconductor with the hole behaves as the vortex core carrying $n\phi_0$ vortex quanta.

The supercurrent flowing around the inner surface resembles the superflow around the vortex.

The temperature sequence

$$T^{'} > T^{''} > T$$

should be measured by local temperature probes.

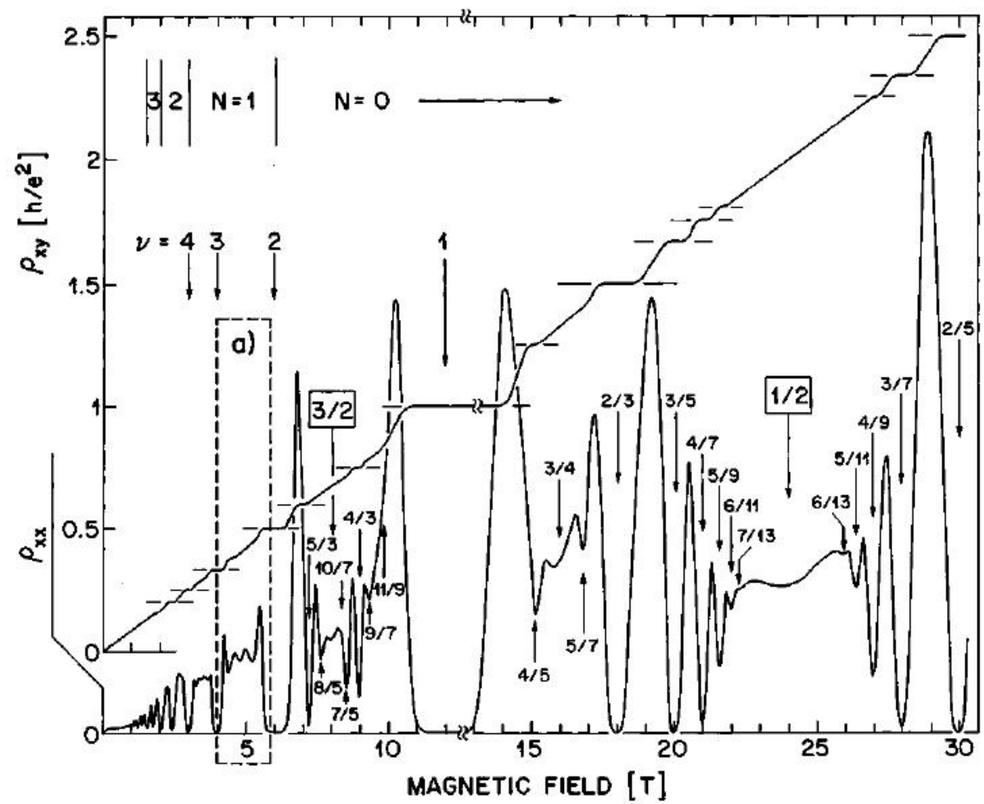


Integer or fractional quantum Hall State:

$$\rho_{xx} = 0, \rho_{xy} = h / e^2, \qquad T = 0$$

$$\rho_{xx} \sim e^{-\Delta/T}, \rho_{xy} \sim h / e^2 + e^{-\Delta/T}, \quad T > 0$$

Can one make use of the nearly Dissipation-less edge state of IQH or FQH to detect the temperature effects?



The T' of Edge channel versus T of the bulk in a quantum Hall state

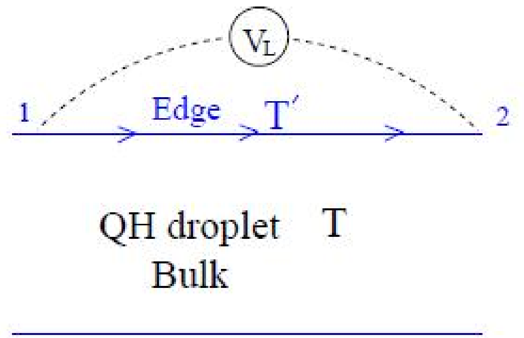
The drift velocity is given by the ratio of the local electric field and the magnetic field:

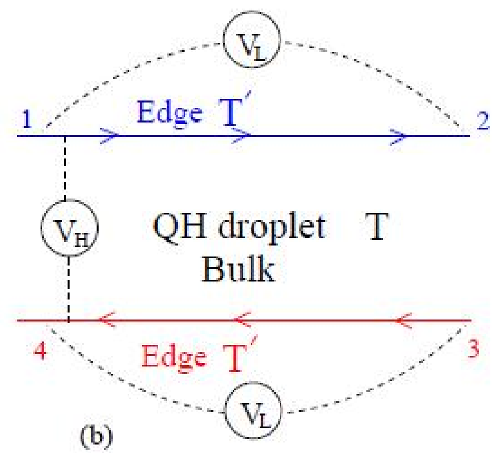
$$v_x = c\frac{E_y}{B_z} \qquad (v/c)^2 \sim 10^{-5},$$

$$\rho_{xx} = 0, \rho_{xy} = h/e^2, \qquad T = 0$$

$$\rho_{xx} \sim e^{-\Delta/T'}, \rho_{xy} \sim h/e^2 + e^{-\Delta/T'}, \quad T > 0$$

$$T'/T \sim 1 + 10^{-5} \rightarrow T/\Delta < 5$$





$$T' > T$$

For the case (B) in Fig.1

In the running frame with the velocity $\vec{v}_2 = -\vec{v}$
the **MDC** of the bosonic excitation number satisfies:

$$n'_{SF}(\omega') = \frac{1}{e^{\frac{\hbar\omega(\vec{k})}{k_B T}} - 1}$$

all the relativistic effects due to the moving of the observer are encoded in the measure of MDC

$$V' \frac{dk'_{E,x} dk'_{E,y} dk'_{E,z}}{(2\pi)^d} = V\left(1 - \frac{v}{c^2} \frac{\partial\omega(\vec{k})}{\partial k_x}\right) \frac{dk_x dk_y dk_z}{(2\pi)^d}$$

the system has the parity symmetry in the static frame, $\omega(\vec{k}) = \omega(-\vec{k})$

the first part is symmetric, the second part due to the moving of the observer is *anti-symmetric*.

the **EDC**:

$$n'_{SF}(\omega') = \frac{V' D'(\omega')}{e^{\frac{\hbar\omega'}{k_B T}} - 1}$$

the density of states (DOS)

$$V' D'(\omega') = V\int \frac{d^d k}{(2\pi)^d}\left(1 - \frac{v}{c^2}\frac{\partial\omega}{\partial k_x}\right)\delta[\omega' - \omega(\vec{k})] = V\int \frac{d^d k}{(2\pi)^d}\delta[\omega' - \omega(\vec{k})] = VD(\omega')$$

is identical to that in the static observer.

the moving observer contributes to the *anti-symmetric* part in the MDC, but does **not** affect the EDC.

Both MDC and EDC can be mapped out by various neutron, light or X-ray scattering using a moving detector with a velocity $\vec{v}_2 = -\vec{v}$.

2. The Hawking temperature viewed by an observer moving along the boundary at infinity

Applying the asymptotic Lorentz transformation leads to the metric of a Schwarzschild black hole viewed by an moving observer in the Kerr-Schild coordinate:

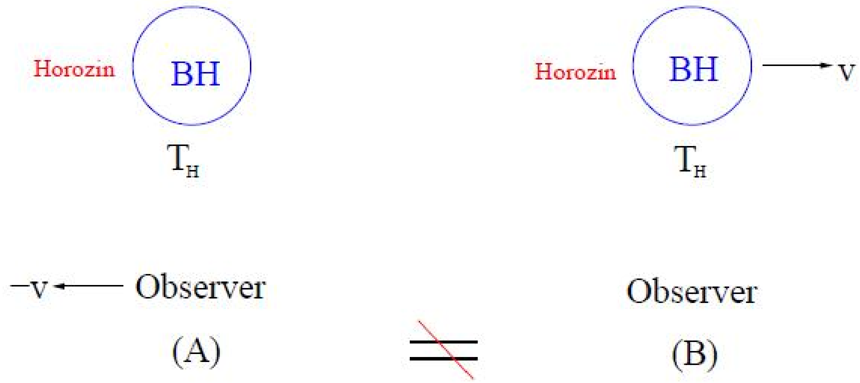
$$ds_m^2 = -(1-\frac{2GM}{r})dt^2 + (1-\frac{2GM}{r})^{-1}dr^2 + r^2\frac{d\theta^2 + \sin^2\theta d\phi^2}{\Lambda}$$

$\Lambda(\theta) = (a + b\cos\theta)^2$ depends the boost

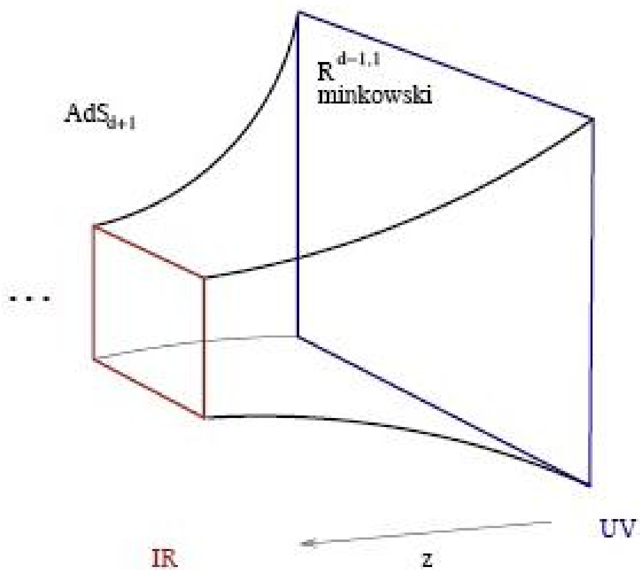
$$a = \gamma = \frac{1}{\sqrt{1-v^2}}, b = \frac{v}{\sqrt{1-v^2}}, a^2 - b^2 = 1$$

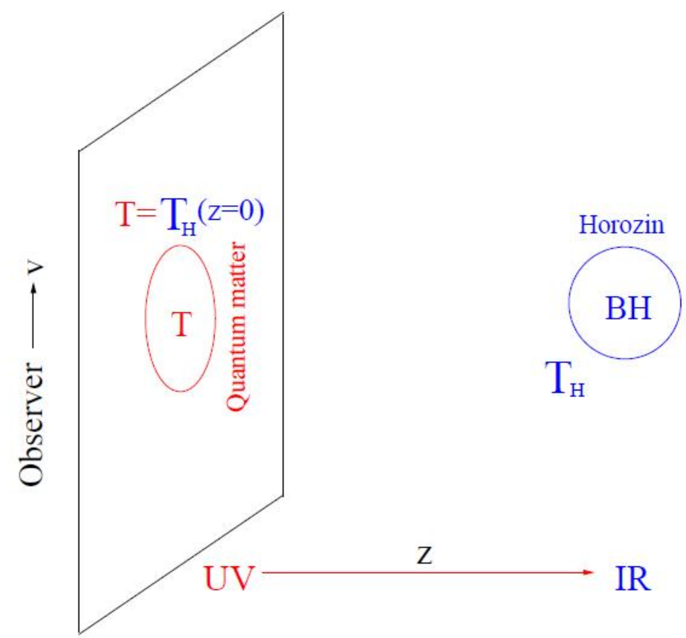
The explicit spherical symmetry breaking viewed by an moving observer is encoded in Λ

Ivano Damião Soares, PHYSICAL REVIEW D 99, 084054 (2019).See also Comment by Emanuel Gallo, Thomas Mädler (Jun 20, 2019),Phys.Rev.D 101 (2020) 2, 028501; Reply, Ivano Damião Soares (Jan 4, 2020), Phys.Rev.D 101 (2020) 2, 028502.



Poincare coordinate



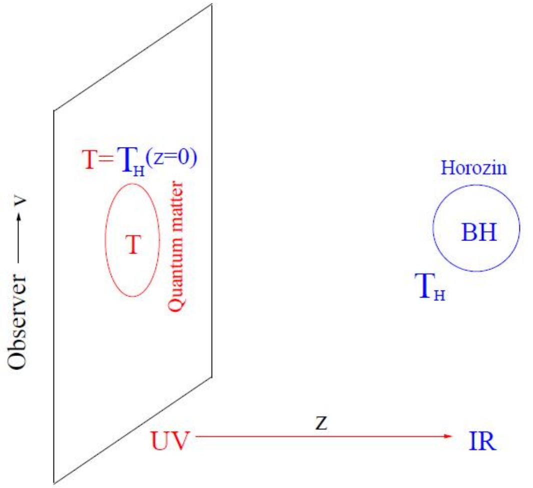


One can extract the Hawking temperature **viewed by a moving observer:**

$$\frac{T_{Ho}}{T_H} = 1 \qquad\qquad k_B T_H = \frac{\hbar c^3}{8\pi G_4 M}$$

Intuitively, a uniform moving observer should see the same surface gravity on the black hole horizon which determines the Hawking Temperature

$$k_B T_H = \frac{\hbar \kappa}{2\pi c}$$



Intuitively, in the boundary, a uniform motion should not affect the internal random motion of the system which is described by T

$$\frac{T'}{T} = 1$$

One can compute the moving (ADM) mass (or energy)

$$E_o = \frac{1}{4\pi G}\int \frac{r^2 \sin\theta d\theta d\phi}{\Lambda}\frac{GM}{r^2} = M$$

the total Bondi-Sachs linear momentum

$$\vec{P}_{BS} = -bM\hat{z}$$

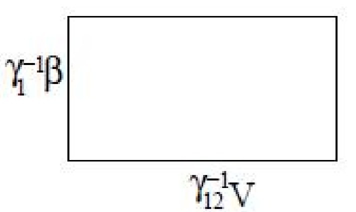
the area of the horizon

$$A_o = \int \frac{r^2 \sin\theta d\theta d\phi}{\Lambda} = 4\pi r^2 = A$$

$$\rightarrow S_o = S$$

the volume of the horizon remains as $V_o = V$

in contrast to the area shrinking
in the special theory of relativity



look at the thermodynamics of the black hole viewed by a moving observer,

the first law

$$dM_o = T_H dS_o \rightarrow dM = T_H dS$$

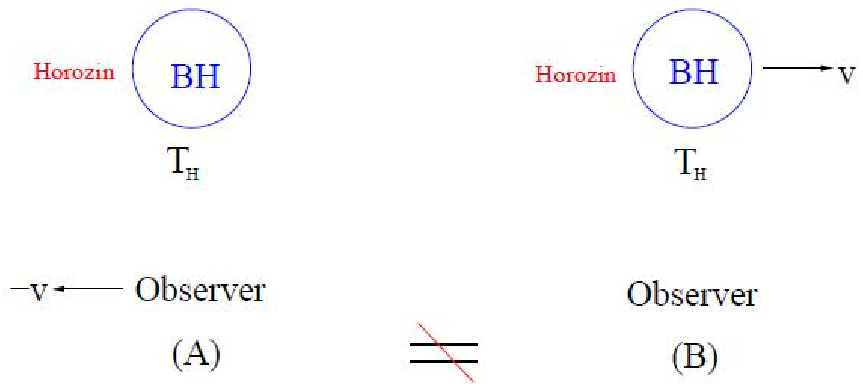
$$M_o = M, \qquad S_o = S$$

all these results may also be derived using the Einstein-Hilbert action with the proper Gibbons-Hawking boundary term at $r = r_\infty \rightarrow \infty$ and some counter terms (also called holographic renormalization) integrating over the whole space

$$I_m = \int_0^{\beta_H} d\tau \int_{r_H}^{r_\infty} dr \int \frac{r^2 \sin\theta d\theta d\phi}{\Lambda} R + \cdots \qquad R = 0$$

$I_m = I$ the action is Lorentz invariant.

A moving black hole in the bulk



Drawing the result in the boundary with a canonical ensemble, its extension to a micro-canonical ensemble and the insights gained from AdS/CFT, one obtain the Hawking Temperature for a **moving black hole**:

$$\frac{T_{Hm}}{T_H} = \gamma = 1 / \sqrt{1 - (\frac{v}{c})^2}$$

the first law

$$dM_m = T_{Hm} dS_m \rightarrow \gamma dM = \gamma T_H dS$$

$$M_m = \gamma M, T_{Hm} = \gamma T_H$$

$S_m = S$ the invariance of Horizon area under any boost

the momentum carried by the black hole

$$\vec{P}_m = \gamma M \vec{v}$$

As reported in several references

M. Cvetic, H. Lu, C. N. Pope, Spacetimes of Boosted p-branes, and CFT in Infinite-momentum Frame, Nucl.Phys.B545:309-339,1999

Rong-Gen Cai, Boosted Domain Wall and Charged Kaigorodov Space, Phys.Lett.B; 572:75-80,2003

Cristian Barrera Hinojosa, Justo López-Sarrión , Moving Schwarzschild Black Hole and Modified Dispersion Relations, Phys. Lett. B 749 (2015) 431-436.

$$\frac{T_{Hm}}{T_H} = \gamma^{-1} < 1$$

This result is not consistent with $\frac{T_{Hm}}{T_H} = \gamma$ for a moving black hole , the first law of black hole thermodynamics and AdS/CFT correspondence.

The opposite Temperature transformation law: *A wrong result*

The imaginary time $(\tau_1 = 0, x), (\tau_2 = \beta, x)$ corresponds to

$$\tau_1' = \gamma(-i\frac{v}{c^2}x)$$

$$\tau_2' = \gamma(\beta - i\frac{v}{c^2}x)$$

(Co-moving* frame)

Due to assuming the same position x in the static sample, one finds

$$\Delta\tau' = \tau_2' - \tau_1' = \gamma\beta$$

which leads to $\frac{T'}{T} = \sqrt{1-(\frac{v}{c})^2} < 1$ a moving body gets cooler

When $v/c \to 1^-, \quad T \to 0$ looks *quite funny*

The difference in the lab frame:

must be wrong !

$$\Delta x' = x_2' - x_1' = \gamma(iv\beta)$$

Recall that in the **C-metric** solution of the Einstein equation for an accelerating black hole with the acceleration a and the static Hawking temperature normalized to be 1 ,

there are also **two** kinds of solutions:

increasing $T_H^+ = \sqrt{1+a^2}$

decreasing $T_H^- = \sqrt{1-a^2}$ vanishes as $a \to 1^-$.

The physical meanings of the two C-metric solutions, especially the lower branch T_H^- , remain obscure.

So we expect the result by the others $T_{Hm} / T = \gamma^{-1} < 1$ can revert to ours $T_{Hm} / T = \gamma$ **by correcting the *similar mistake*.**

The third kind: $T_H = \sqrt{a^2 - 1}, \;\; a > 1, \quad ???$

3. The Hawking temperature viewed by an observer moving into the bulk

The Hawking temperature T_H detected by an observer far away from the horizon at the spacial infinity (boundary)

If the observer moves close to the horizon, the observed temperature increases as dictated by the Tolman relation:

$$T_H(r)\sqrt{-g_{tt}(r)} = const.$$

Plugging $-g_{tt}(r) = 1 - \frac{r_H}{r}$ $\frac{T_H(r)}{T_H} = \frac{1}{\sqrt{1 - r_H / r}}$

at the spatial infinity $r \to \infty, T_H(r) \to T_H$

approaches the horizon $r \to r_H^+, T_H(r) \to \infty$

The Tolman relation from Entanglement entropy

In the flat space at $T = 0$ on the boundary, the EE between two regions leads to a reduced density matrix

$$\rho_A = e^{-\beta_A H_A}$$

one can calculate the Von Neumann entropy

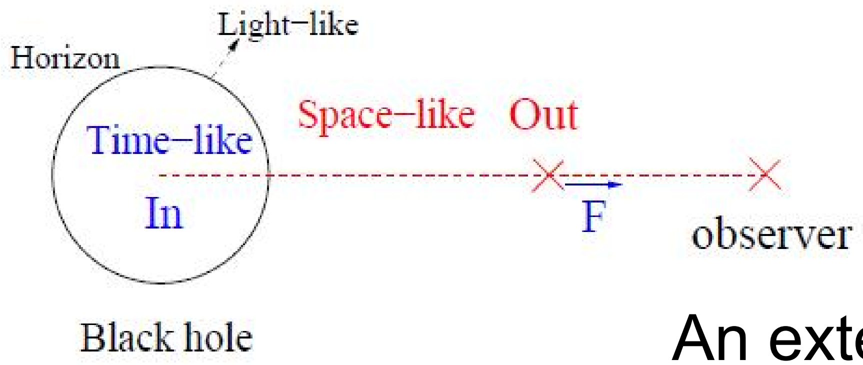
$$S_A = -Tr \rho_A \log \rho_A$$

one can only determine the reduced (also called modular) Hamiltonian H_A upto a scale β_A

Amazingly such an ambiguity can be resolved in the curved space-time

just as happened in the identification of the Hawking temperature and the Unruh temperature.

Hamiltonian formulation of general Relativity



An external force $\vec{F}$ needs to be applied to keep the observer static

Setting $A = out$ regime, then the Tolman relation is nothing but $\beta_{out}(r_1)H_{out}(r_1) = \beta_{out}(r_2)H_{out}(r_2)$

can be achieved from the metric just by a *coordinate transformation:*

$$\frac{T_H(r)}{T_H} = \frac{1}{\sqrt{1 - r_H / r}}$$

The reduced density matrix presents the quantum interpretation of Tolman relation from EE.

Contrast Black hole to the Unruh effect

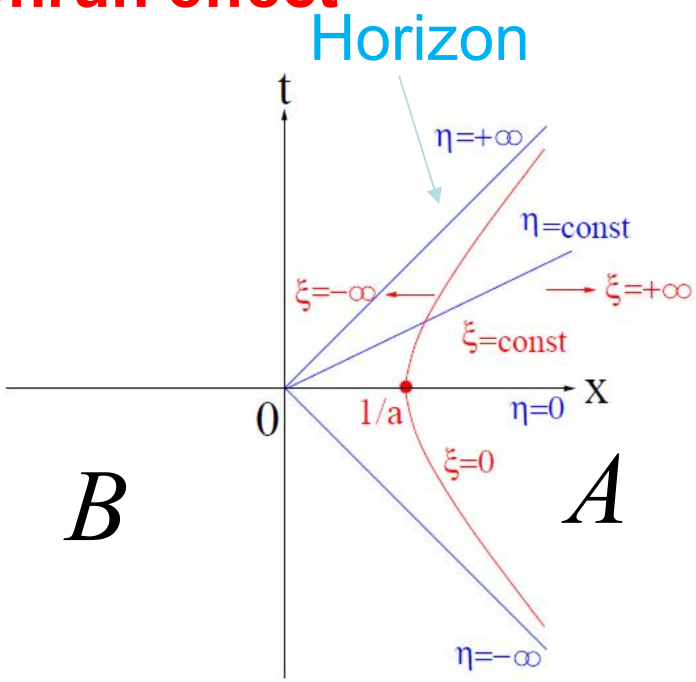
A static observer see $T = 0$

An accelerating observer see

$$k_B T_U = \frac{\hbar a}{2\pi c}$$

$$a = 1m/s^2$$

$$T_U \sim 4\times 10^{-21} K$$



The duality relation between the temperature in a black hole and that in the Unruh effect.

Observer	Black hole	Unruh effect
Static	$k_B T_H = \hbar\kappa/2\pi c$	$T = 0$
Uniform moving	$k_B T_H = \hbar\kappa/2\pi c$	$T = 0$
Accelerating	$T = 0$	$k_B T_U = \hbar a/2\pi c$

RG=GR in AdS/CFT

RG transformation in $(\vec{x},\tau)$ space

$$x^{'}=b^{-1}x,\tau^{'}=b^{-1}\tau$$

Temperature acts as the periodicity in the imaginary time

$$0<\tau<\beta$$

$$\beta^{'}=b^{-1}\beta \longrightarrow T^{'}=bT$$

going from the co-moving* (or static) frame to the lab frame,

$$l^{'}=\gamma^{-1}l \longrightarrow b=\gamma>1$$

$$T^{'}=\gamma T$$

the transformation law of the temperature !

in the Poincare coordinate: **RG=GR**

$$E \sim r = 1/z$$

the energy scale in the boundary corresponds to the radial coordinate in the bulk:

boundary at $r = \Lambda$ → UV at the lattice scale,

the horizon at $r = r_H$ → IR scale

$$ds^2 = -(1-\frac{2GM}{r})dt^2 + (1-\frac{2GM}{r})^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

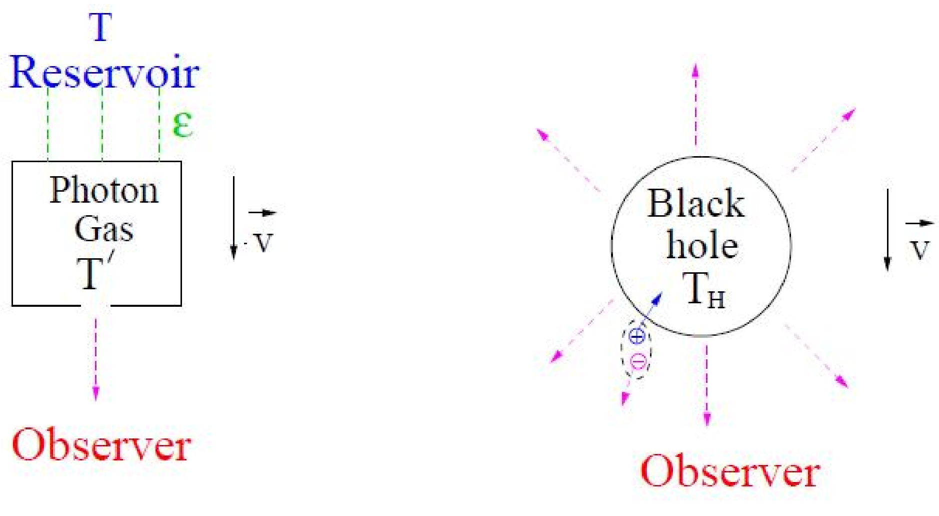
$$dt' = \sqrt{1 - r_H / r}\, dt \quad \longrightarrow \quad b = 1/\sqrt{1 - r_H / r} > 1$$

So one can unify under the same framework of RG=GR

$$\frac{T'}{T} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} \quad \Leftrightarrow \quad \frac{T_H(r)}{T_H} = \frac{1}{\sqrt{1 - r_H / r}}$$

Black body radiations of a moving box versus
Hawking radiations of a moving black hole

$$\varepsilon \to 0$$



(a) (b)

Because the photons are non-interacting, can not thermalize on its own. So the temperature must be given by **an external reservoir**

While the blackhole is an isolated system and a quantum chaotic system before it starts to radiates and evaporates.

The EDC (per volume) of the Blackbody radiation of a moving box in the lab frame satisfies

$$n'_{bb}(\omega') = \frac{\omega'^2}{2\pi^2 c^3} \frac{1}{e^{\frac{\hbar\omega'}{k_B T'}} - 1} \qquad D'(\omega') = D(\omega')$$

$T' = \gamma T$ is slightly higher than that of the reservoir

In terms of the coordinates in the static sample:

$$\omega' = \gamma(\omega - \vec{v} \cdot \vec{k}) \qquad T' = \gamma T$$

$\omega' / T' = (\omega - \vec{v} \cdot \vec{k}) / T$ the relativistic factor γ drops out.

All the relativistic effects are encoded in the DOS

$$\sim \omega'^2$$

Hawking radiation (of bosons) from a moving black hole satisfies

$$n'_{BH}(\omega') = \frac{D'_G(\omega')}{e^{\frac{\hbar\omega'}{k_B T_{Hm}(r')}} + 1} \qquad r' = r - vt$$

$D'_G(\omega')$ is the Greybody factor

Unfortunately, the black hole can thermalize on its own, so there is no an external reservoir to compare in the lab frame.

In the coordinates in the static frame: $r' = r - vt$

$$\omega' = \gamma(\omega - \vec{v}\cdot\vec{k}) \qquad \frac{T_{Hm}(r')}{T_H} = \frac{\gamma}{\sqrt{1 - r_H / r'}}$$

all the relativistic effects are encoded in the DOS

$$D'_G(\omega') = D_G(\omega')$$

7. Conclusions and perspectives.

The path integral approach in imaginary time is beyond the canonical quantization when combing quantum field theory with the statistical mechanics. It has been used to establish:

$$l / l_0 = \gamma_{12}^{-1}, \; t / t_0 = \gamma_{12}, \; m / m_0 = \gamma_{12}$$

$$\omega' = \gamma(\omega - \vec{v} \cdot \vec{k})$$

Doppler effects

$$\frac{T_o'}{T} = 1 \leftrightarrow \frac{T_{Ho}}{T_H} = 1$$

$$\frac{T'}{T} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} = \frac{t}{\tau} \leftrightarrow \frac{T_H(r)}{T_H} = \frac{1}{\sqrt{1-r_H/r}} = \frac{t}{\tau}$$

The first three depend on the relative motion between the sample and the observer. γ_{12} Doppler shifted energy

T'/T depends on the relative motion between the sample and the reservoir. γ_1 Internal Energy

Internal Energy= Doppler shifted energy Only when

$$\gamma_{12} = \gamma_1$$

From the fundamental physics point of view,

(x, t) and m are the three most fundamental elements.

they set up the three most fundamental units:

Centimeter (C), Second (S) and Gram (G).

For any equilibrium system, the temperature T can be treated as the imaginary time,

$$\tau = \beta + i\,t$$

so it is *as fundamental as the time*,

T introduces an completely **new unit**: Kelvin (K).

Talk 3: On the quantum temperature of a classical black hole the chemical potential μ is just a Lagrangian multiplier to enforce the total number of fermion numbers N_F .

It also carries the same unit as the energy, so its transformation law is *much less* universal

When there is no relative motion between the body and the reservoir, the phase transition happens in a static sample at T_c

When there is a relative motion, the body's temperature gets higher, then the phase transition happens at **a lower** reservoir's temperature in the co-moving frame:

$$\frac{T_c'}{T_c} = \sqrt{1-(\frac{v}{c})^2} < 1$$

the critical temperature T_c drops and approaches zero as $v \to c^-$.

But in condensed matter systems $v \ll c$

Its applications to 3d XY transition and 2d KT transition in a running Helium4 superfluid

The limitation of *Planck, Einstein, Pauli, Laue……..*
(also followed up by many…….)

did not start from the fundamental quantum statistical mechanics point of view:

$$\frac{T'}{T} = 1/\sqrt{1-(\frac{v}{c})^2}$$

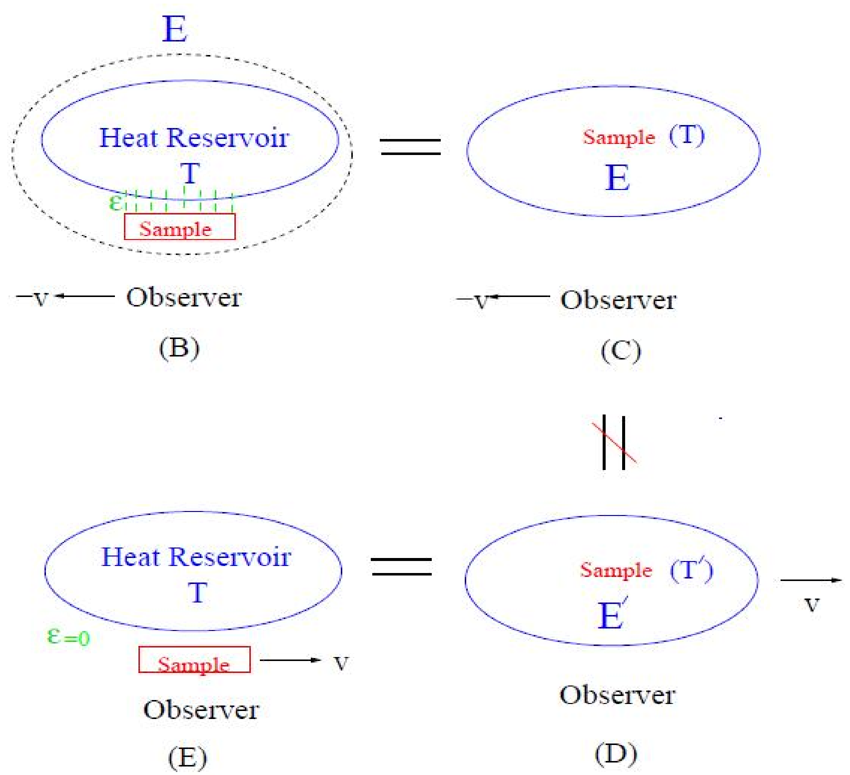
$$\frac{T'}{T} = 1$$

$$\omega_I' = \omega_D' = \gamma(\omega - \vec{v}\cdot\vec{k})$$

$$\omega_I' = \omega$$

$$\omega_D' = \gamma(\omega - \vec{v}\cdot\vec{k})$$

Did not have the concept to unify the temperature and the real time into a Complex time $\tau = \beta + it$



Case (D)

Planck, Einstein,
Pauli, Laue……..

$$\frac{T'}{T} = 1/\sqrt{1-(\frac{v}{c})^2}$$

Micro-canonical ensemble

No Reservoir or $\varepsilon \to 0$

Only Intrinsic part

The opposite Temperature transformation law: *A wrong result*

The imaginary time $(\tau_1 = 0, x), (\tau_2 = \beta, x)$ corresponds to

$$\tau_1' = \gamma(-i\frac{v}{c^2}x)$$

$$\tau_2' = \gamma(\beta - i\frac{v}{c^2}x)$$

(Co-moving* frame)

Due to assuming the same position x in the static sample, one finds

$$\Delta\tau' = \tau_2' - \tau_1' = \gamma\beta$$

which leads to $\frac{T'}{T} = \sqrt{1-(\frac{v}{c})^2} < 1$ a moving body gets cooler

When $v/c \to 1^-$, $T \to 0$ looks *quite funny*

The difference in the lab frame:

must be wrong !

$$\Delta x' = x_2' - x_1' = \gamma(iv\beta)$$

Physical interpretation in terms of Lorentizan and Galileo algebra

The Lorentz group $K_i, i=1,2,3$ in the Lorentizian signature has 3 boosts $K_i, i=1,2,3$ and 3 rotations K_i $SO(3,1)$

$$[J_i, J_j] = i \ {}_{ijk} J_k, \quad [K_i, K_j] = -i \ {}_{ijk} J_k, \quad [J_i, K_j] = i \ {}_{ijk} K_k$$

Taking $c \longrightarrow \infty$ LT reduces to GT $K_i \rightarrow C_i$

$$[J_i, J_j] = i \ {}_{ijk} J_k, \ [C_i, C_j] = 0, \ [J_i, C_j] = i \ {}_{ijk} C_k$$

This decoupling leads to the Center of Mass (COM) in the *Newtonian Mechanics*

The Noether theorem relating the symmetry to the conserved currents: **Poincare Group**

The translational symmetry $x^\mu \rightarrow x^\mu + a^\mu$

→ the conservation of the energy-momentum

$$H = \int d^3x T^{00}, P^i = \int d^3x T^{0i}$$

$$M^{\mu\nu\rho} = T^{\mu\nu}x^\rho - T^{\mu\rho}x^\nu$$

anti-symmetric in the last two indices ρ, ν

$$M^{\nu\rho} = \int d^3x M^{0\nu\rho}$$

The three rotations $J_i, i = 1,2,3$

→ the conservation of the angular momentum M^{ij}

The three boots $K_i, i = 1,2,3$

→ the conservation of the Center of Momentum (COMM) M^{0i}

$$c \to \infty$$

In the Newtonian mechanics, any motion can be decomposed into the Center of mass (COM) motion plus the motions around the COM,

The COM motion of the sample is completely decoupled from its internal motion which is controlled by the temperature.

This fact not only holds in the inertial frame, but also the non-inertial frame such as the transit process

$$\boxed{W = K} \longrightarrow T^{'} / T = 1$$

Classical Picture

However, at any finite c , there is no such **decoupling** between the Center of momentum (COMM) and the internal motion anymore, no such decoupling in the equation of motion:

$$\vec{F} = \frac{d\vec{P}}{dt}, \quad \vec{F} \cdot \vec{v} = \frac{dE_e}{dt} \qquad \vec{F} \cdot d\vec{l} = \vec{v} \cdot d\vec{P} = dE_e$$

$(iE_e / c, \vec{P})$ is just the 4-vector external energy and momentum of the sample

It is this coupling which leads to **Classical Picture**

$$\boxed{W > K} \longrightarrow T' / T > 1$$

the increase (**Work goes to Heat**)is completely a relativistic effect. It should be independent of the direction of the velocity v , so should be a function of $(v / c)^2$ only, then

seems the most likely answer.

$$\frac{T'}{T} = 1 / \sqrt{1 - (\frac{v}{c})^2} > 1$$

Intuitively: as the sample moves, the molecules inside the sample has to do two jobs:

keeping its center of momentum motion, also random motion. However, the two motions are coupled together at any finite $c < \infty$.

While the molecules inside the reservoir only need to do one job, just random motion. So in order for the random motion between the sample and the reservoir to match, the temperature of the moving sample should be higher.

Classical Picture !

For the case (A)

For bosons, the boson or excitation number in the lab frame satisfies:

$$n'(\omega')d\omega' = \frac{D(\omega')d\omega'}{e^{\frac{\hbar\omega'}{k_B T'}} - 1} \qquad \frac{\omega'}{T'} = \frac{\omega - \vec{v}\cdot\vec{k}}{T}$$

$D(\omega')$ is the density of states (DOS) in the lab frame

can be mapped out by various *light or neutron* scattering on a moving sample

Internal Energy= Doppler shifted energy

$$\omega' = \gamma(\omega - \vec{v}\cdot\vec{k})$$

Odd under $\vec{v} \rightarrow -\vec{v}$

The law of T

$$T' = \gamma T$$

Even under $\vec{v} \rightarrow -\vec{v}$

determined by the relative motion between

the sample and the observer, the sample and the reservoir.

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一起追求卓越，成就经典

一起走天涯！

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