Recent developments of Parton Showers

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lavour hysics ecture 味物理讲座

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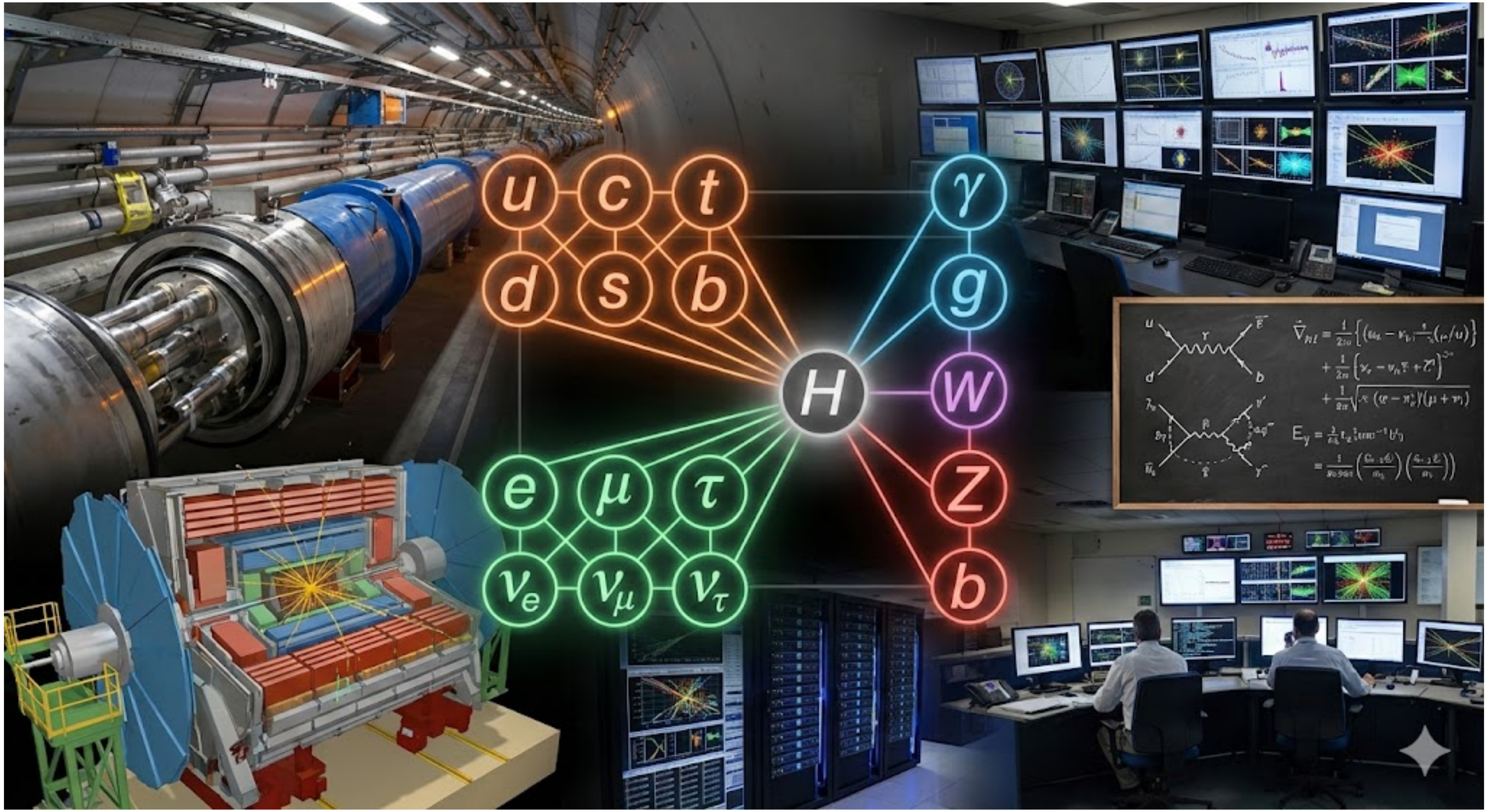
Outline

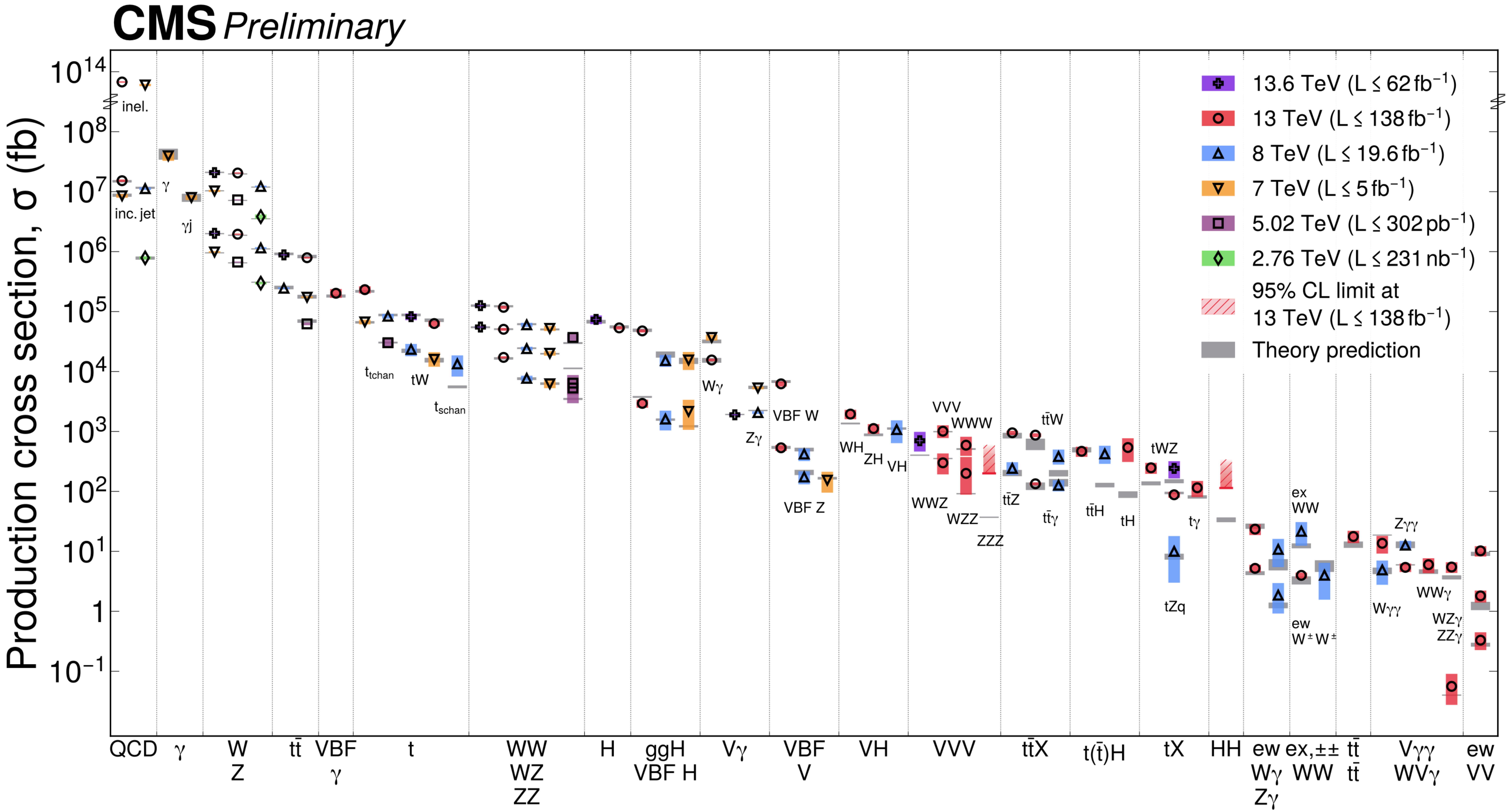
1. Introduction



generated using gemini

Particles and parameters in and beyond SM

1. Introduction

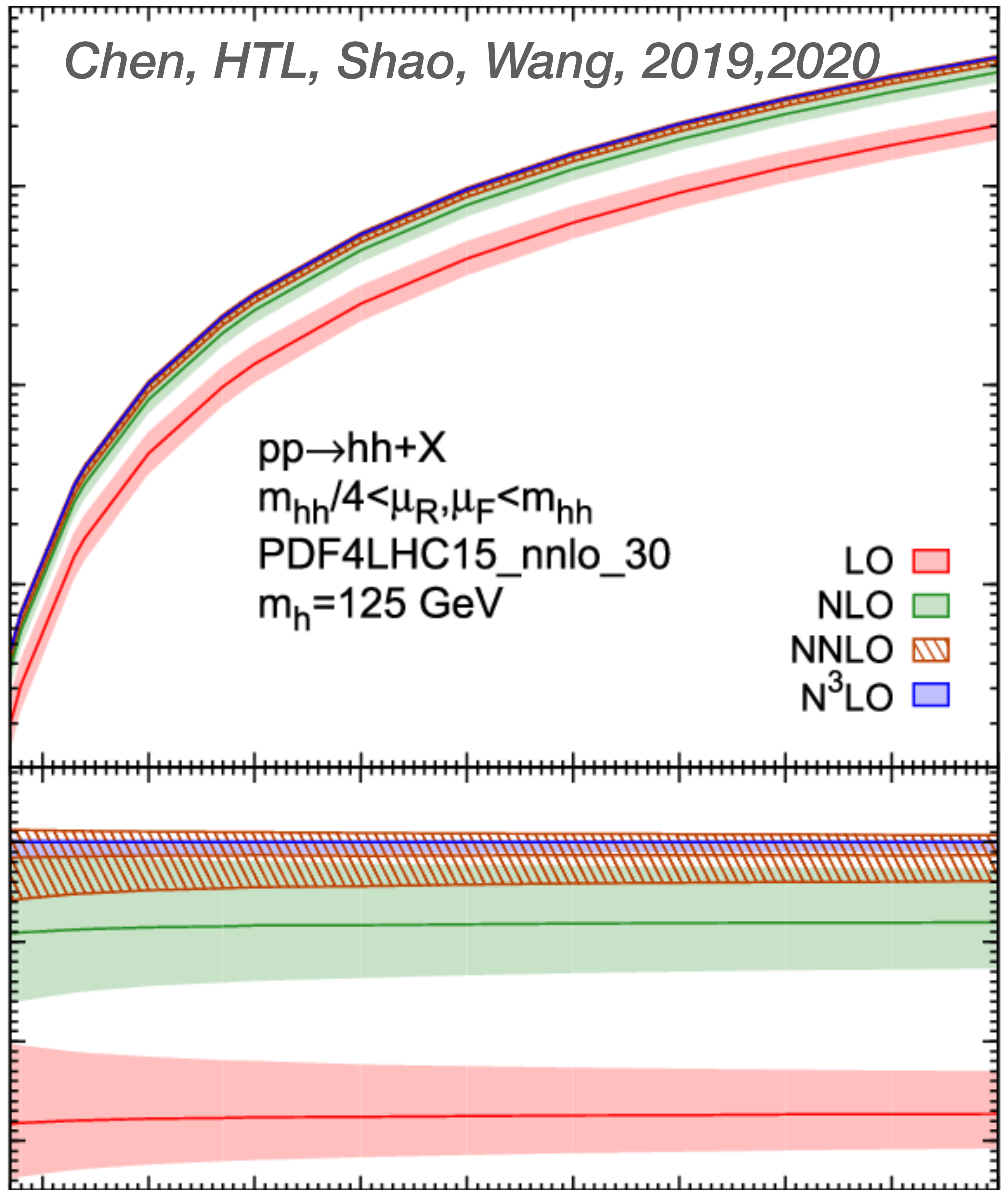


1. Introduction

- perturbative calculation for the hard cross section

$$d\hat{\sigma} = d\hat{\sigma}^{(0,0)} + \alpha_s\, d\hat{\sigma}^{(1,0)} + \alpha_s^2\, d\hat{\sigma}^{(2,0)} + \alpha_s^3\, d\hat{\sigma}^{(3,0)} + \alpha\, d\hat{\sigma}^{(0,1)} + \alpha\alpha_s\, d\hat{\sigma}^{(1,1)} + \dots$$

Leading Order (LO)	NLO QCD	NNLO QCD	N3LO QCD	NLO EW	Mixed QCD-EW
	~ 10%	~ 1%		~ 1%	



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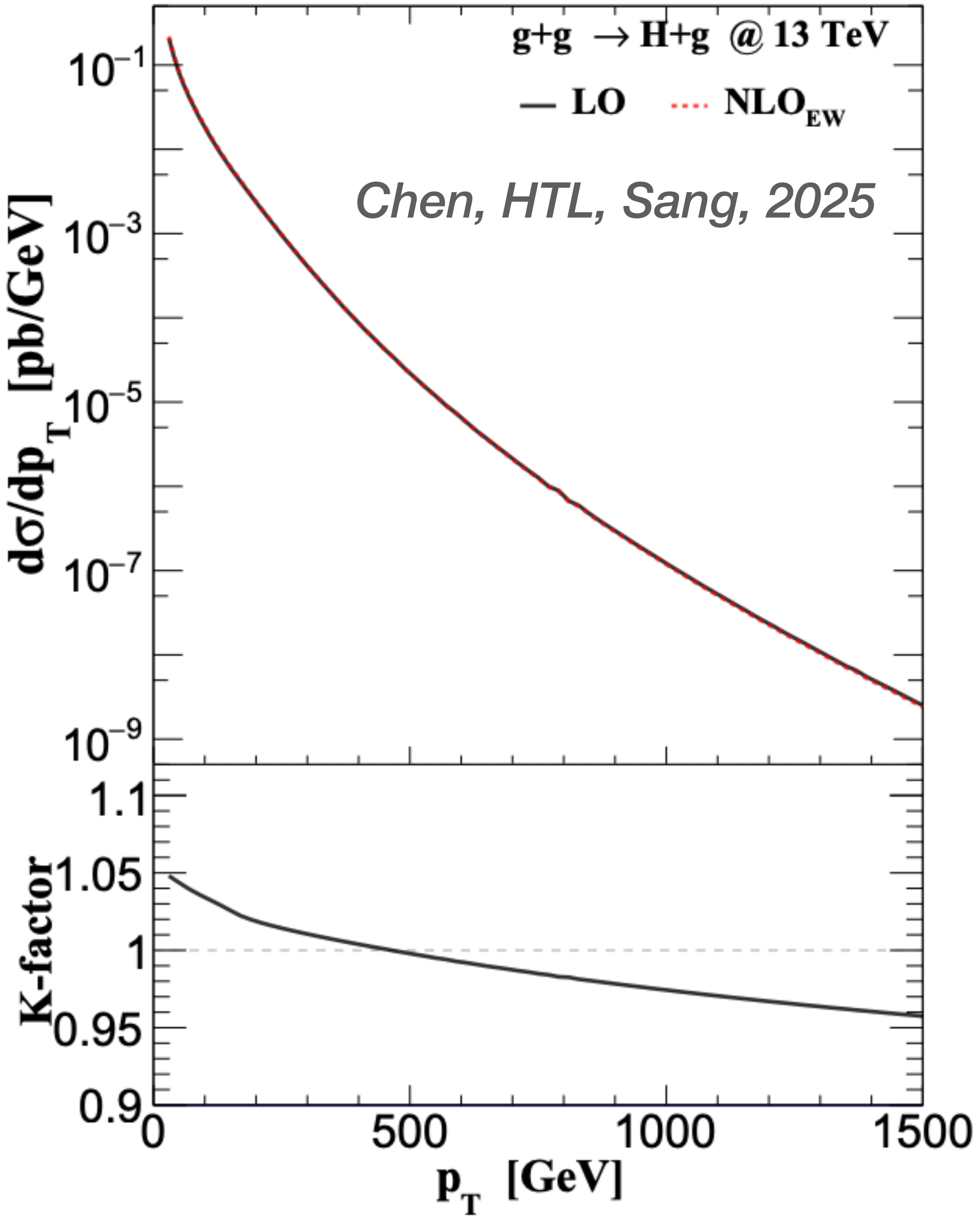

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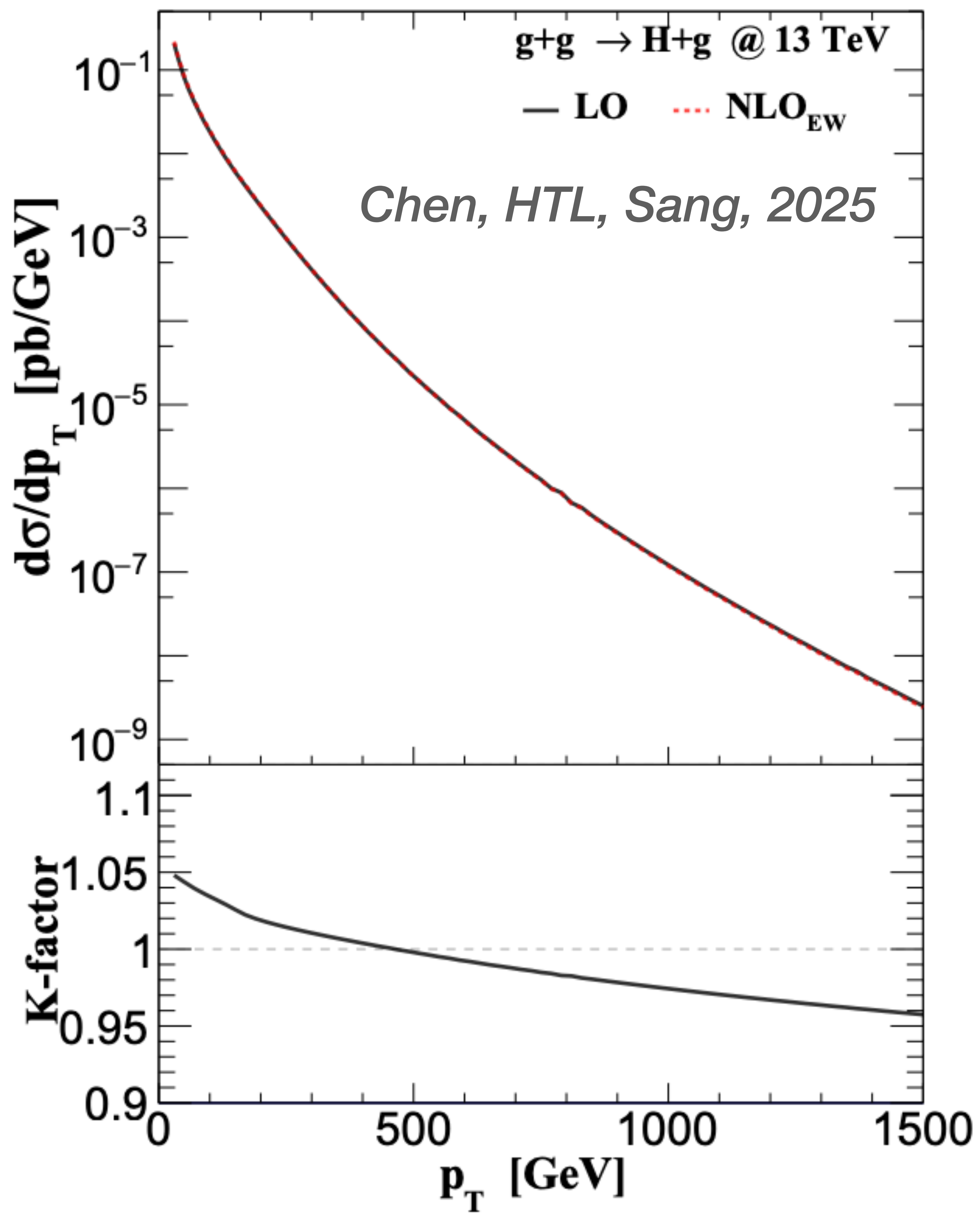

Frontiers:

- N3LO QCD for 2to2 processes
- NNLO QCD for 2to3 or 2to4 processes
- EW correction for loop induced processes
- Mixed QCD-EW corrections to 2to2 processes

1. Introduction

Resummation is essential for many collider observables.

固定阶计算

NLO, NNLO, NNNLO

$$
\begin{array}{lllll}
z\frac{1}{\sigma_0}\frac{d\sigma}{dz} \propto z + \frac{\alpha_s}{4\pi}a_{1,2}\ln z + & \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,4}\ln^3 z + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,6}\ln^5 z & + \cdots & \text{LL} \\
+ \frac{\alpha_s}{4\pi}a_{1,1} + & \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,3}\ln^2 z + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,5}\ln^4 z & + \cdots & \text{NLL} \\
+ \frac{\alpha_s}{4\pi}a_{1,0}z + & \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,2}\ln z + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,4}\ln^3 z & + \cdots & \text{NNLL} \\
& + \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,1} + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,3}\ln^2 z & + \cdots & \text{NNNLL} \\
& + \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,0}z + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2}\ln z & + \cdots & \text{NNNNLL} \\
& & + \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2} & + \cdots & \\
& & + \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2}z & + \cdots &
\end{array}
$$

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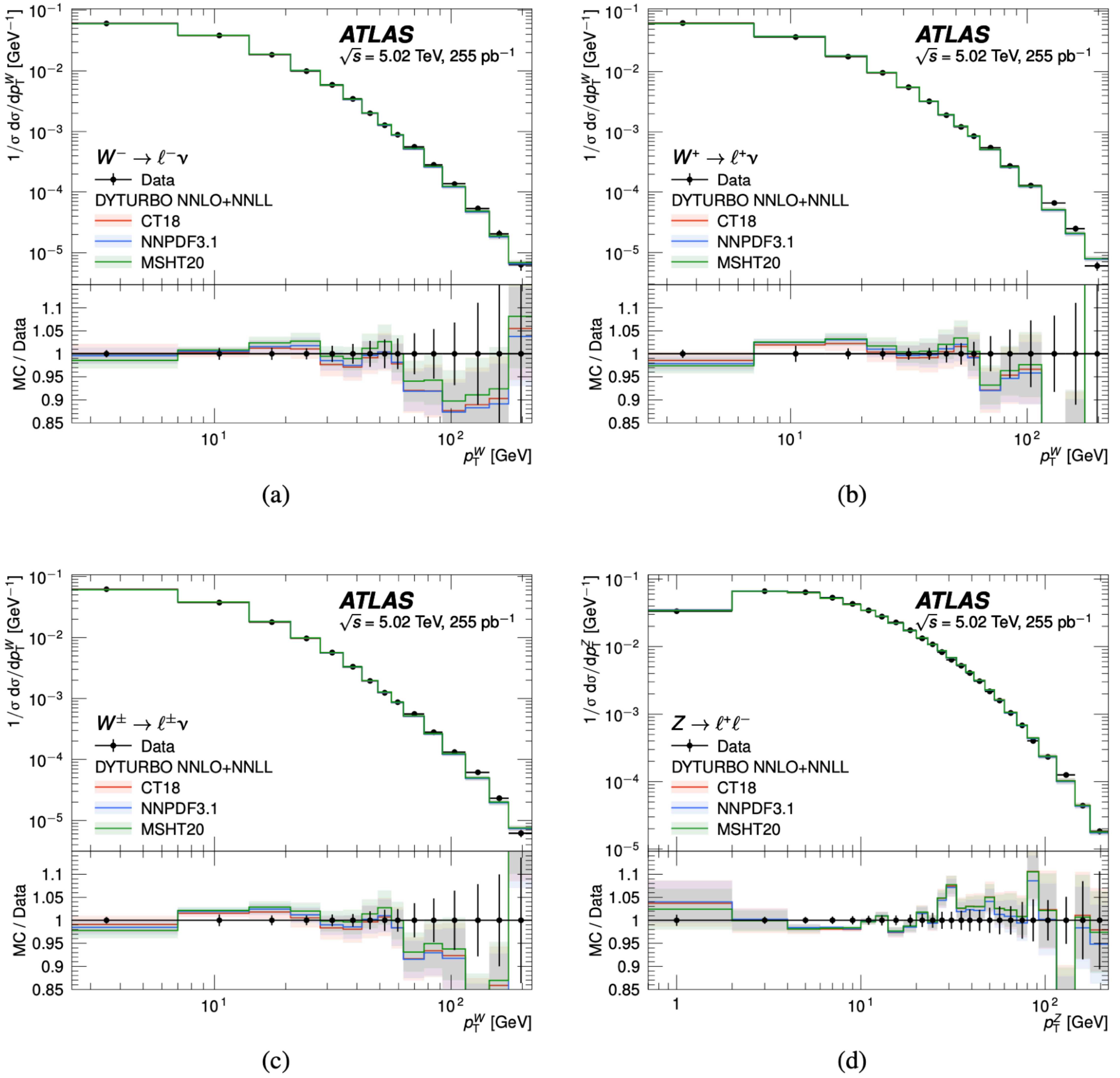
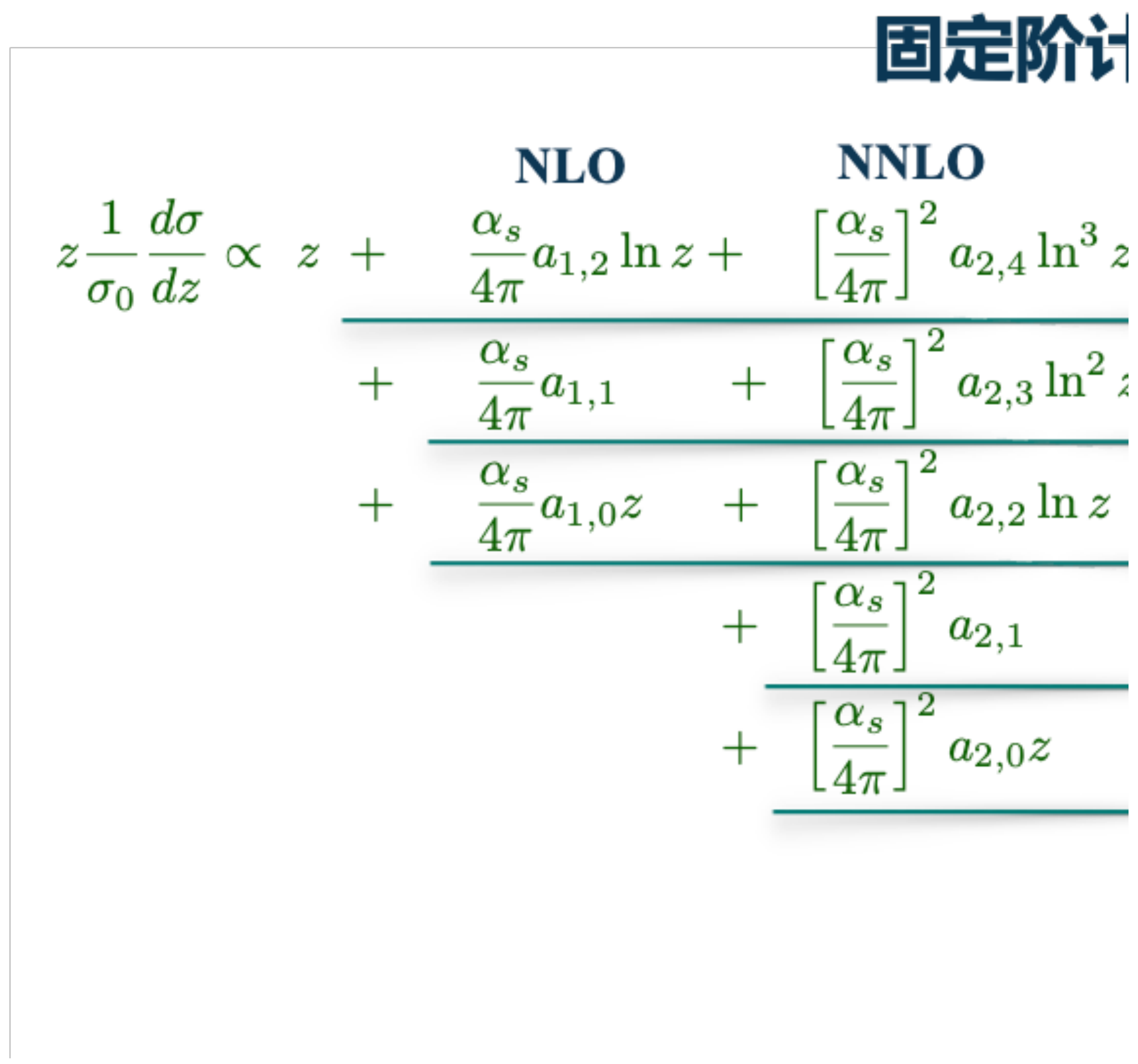
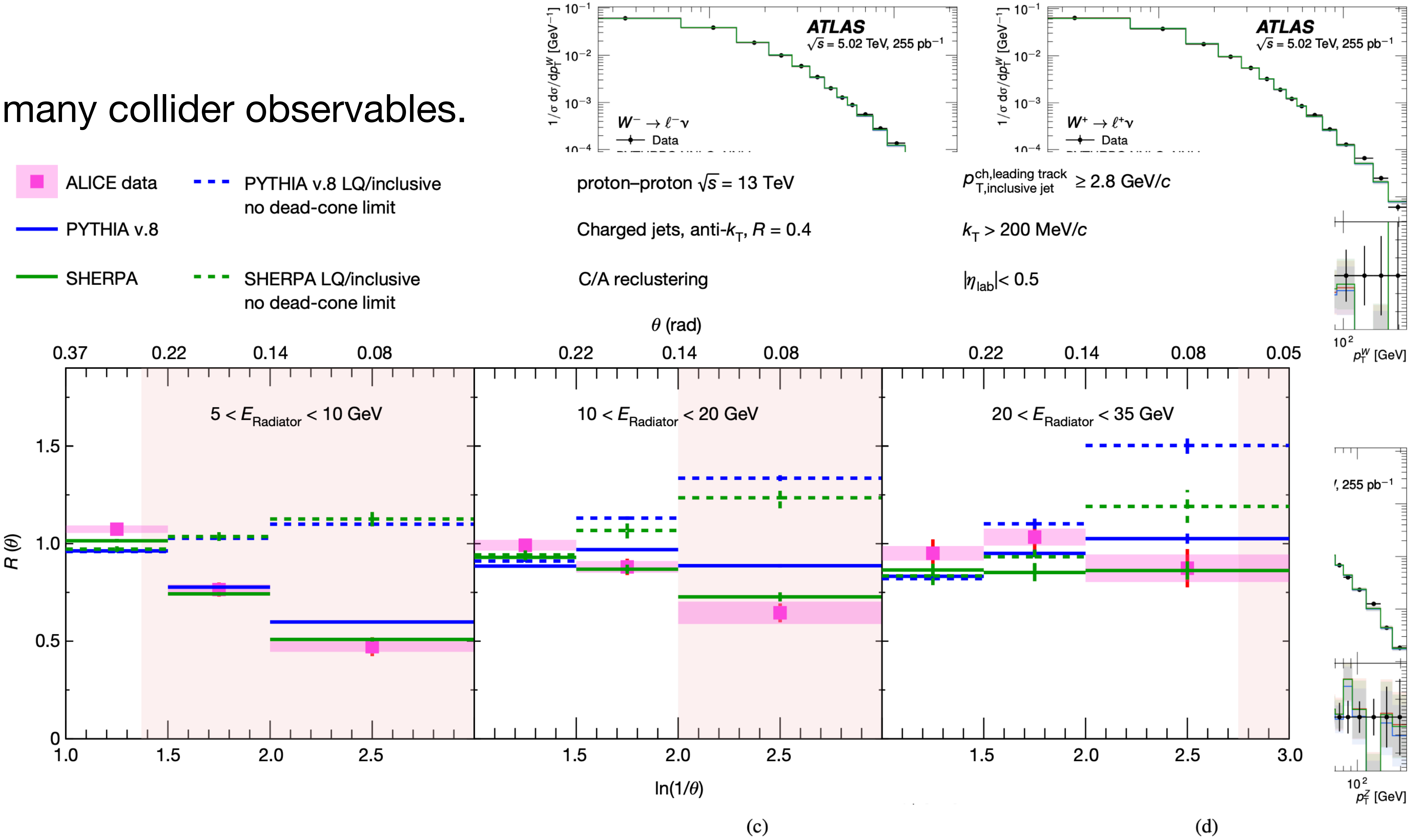


1. Introduction

Resummation is essential for many collider observables.



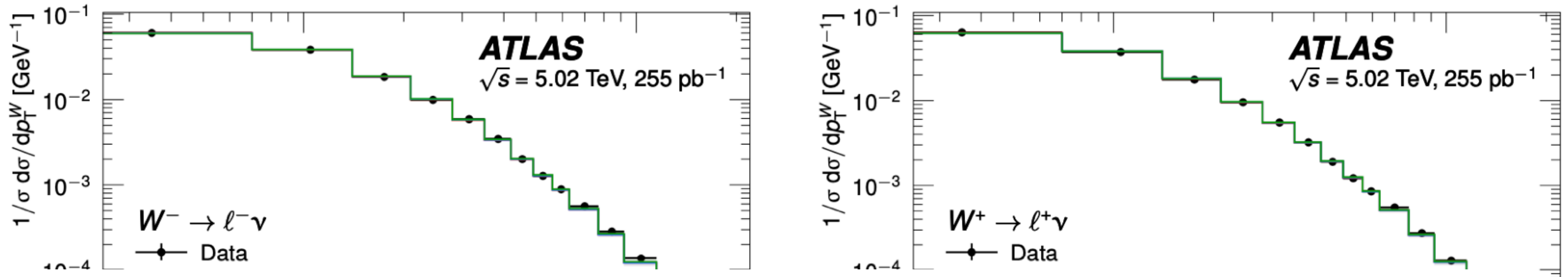


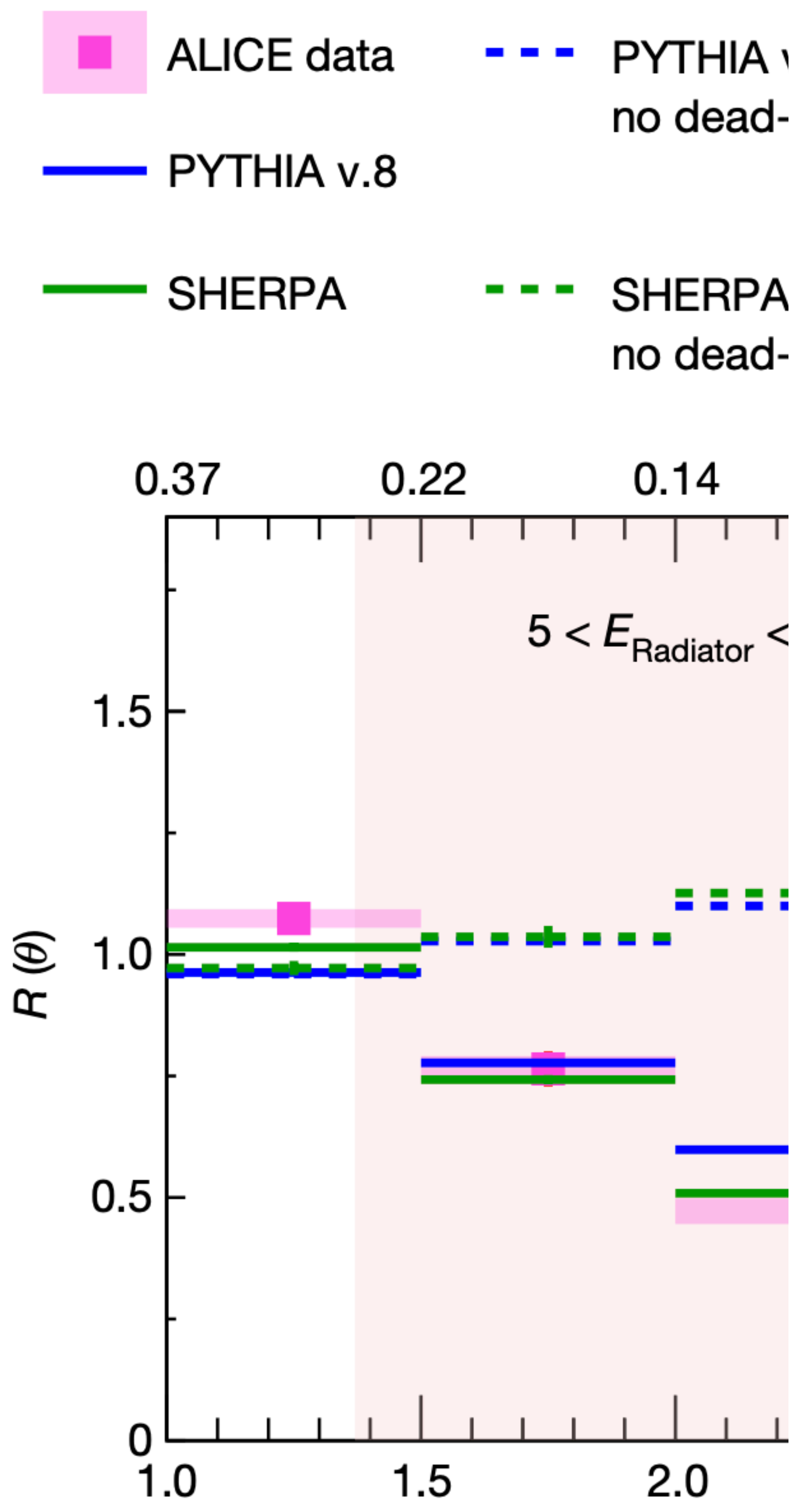
1. Introduction

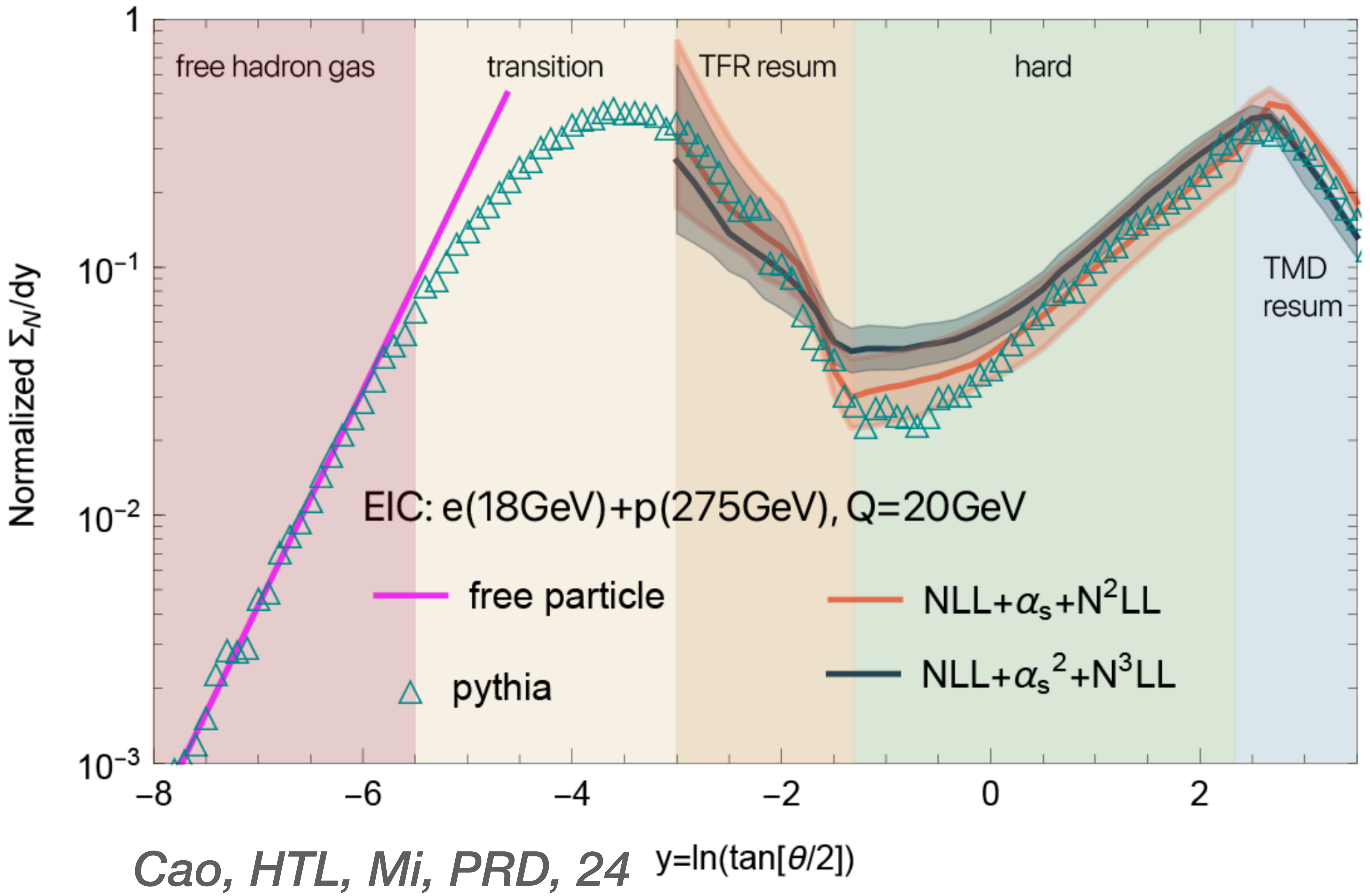
Resummation is essential for many collider observables.

固定阶计

	NLO	NNLO
$z\frac{1}{\sigma_0}\frac{d\sigma}{dz} \propto z +$	$\frac{\alpha_s}{4\pi}a_{1,2}\ln z +$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,4}\ln^3 z$
$+$	$\frac{\alpha_s}{4\pi}a_{1,1}$	$+ \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,3}\ln^2 z$
$+$	$\frac{\alpha_s}{4\pi}a_{1,0}z$	$+ \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,2}\ln z$
		$+ \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,1}$
		$+ \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,0}z$

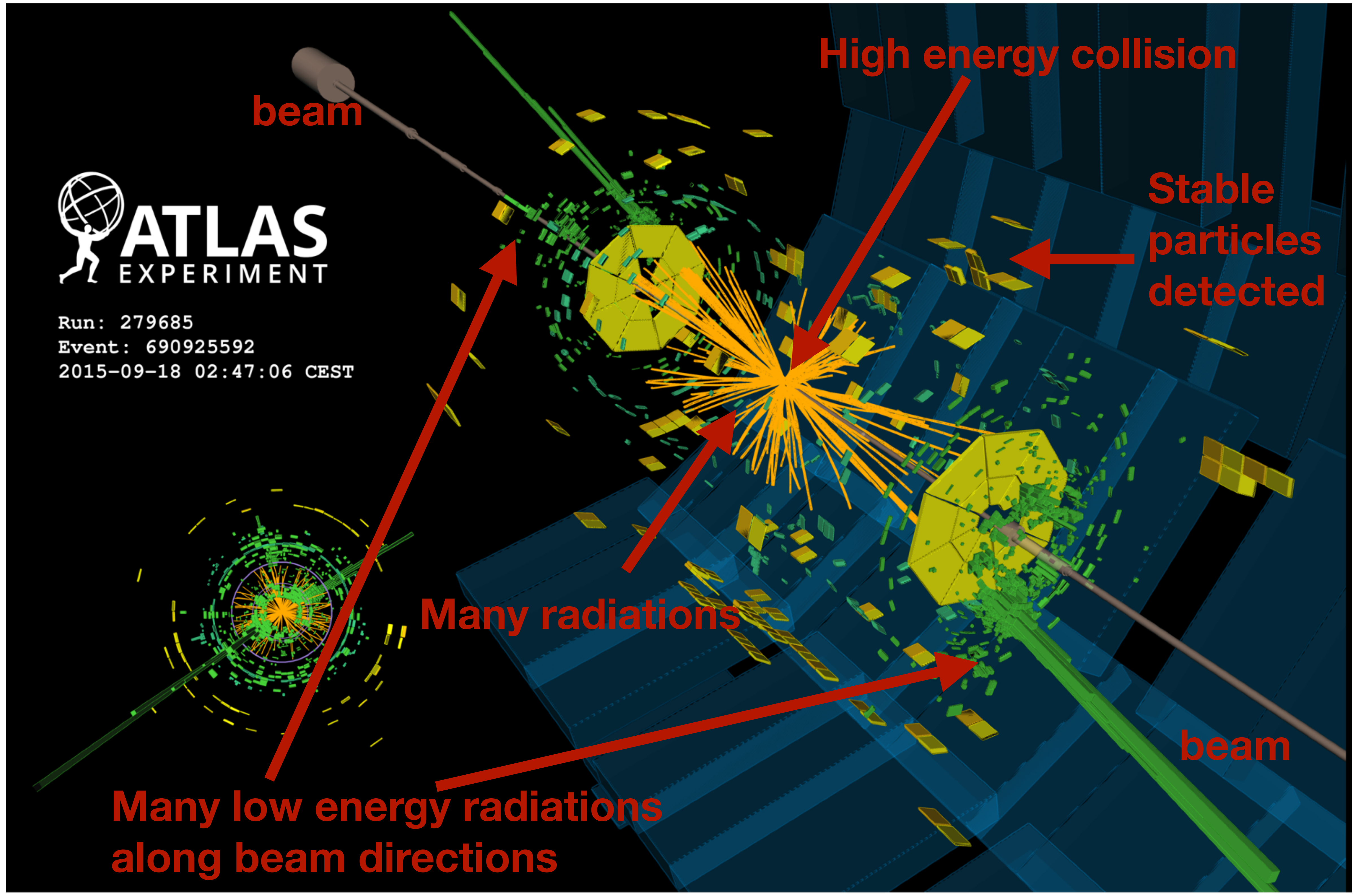






Cao, HTL, Mi, PRD, 24

1. Introduction



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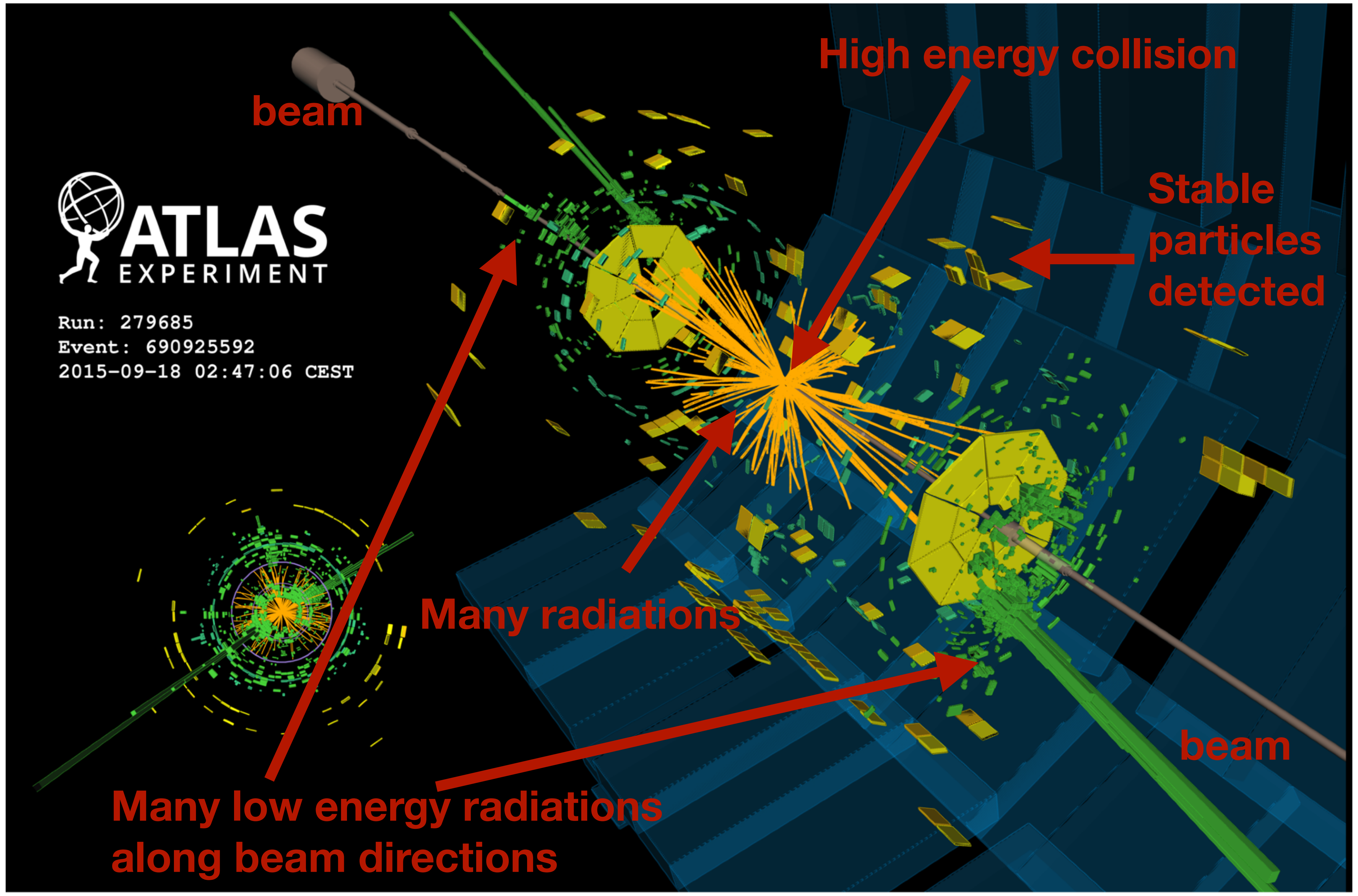
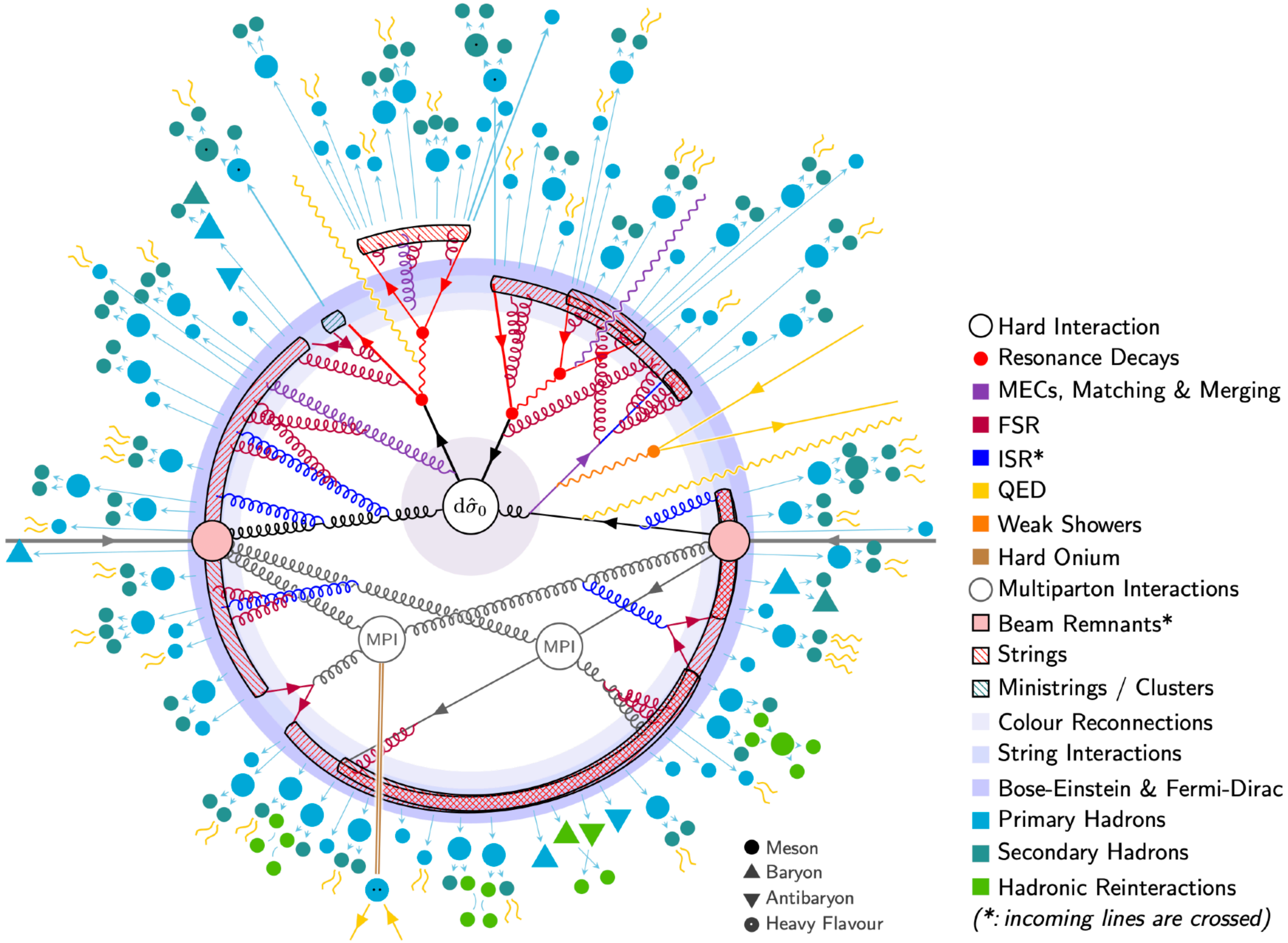




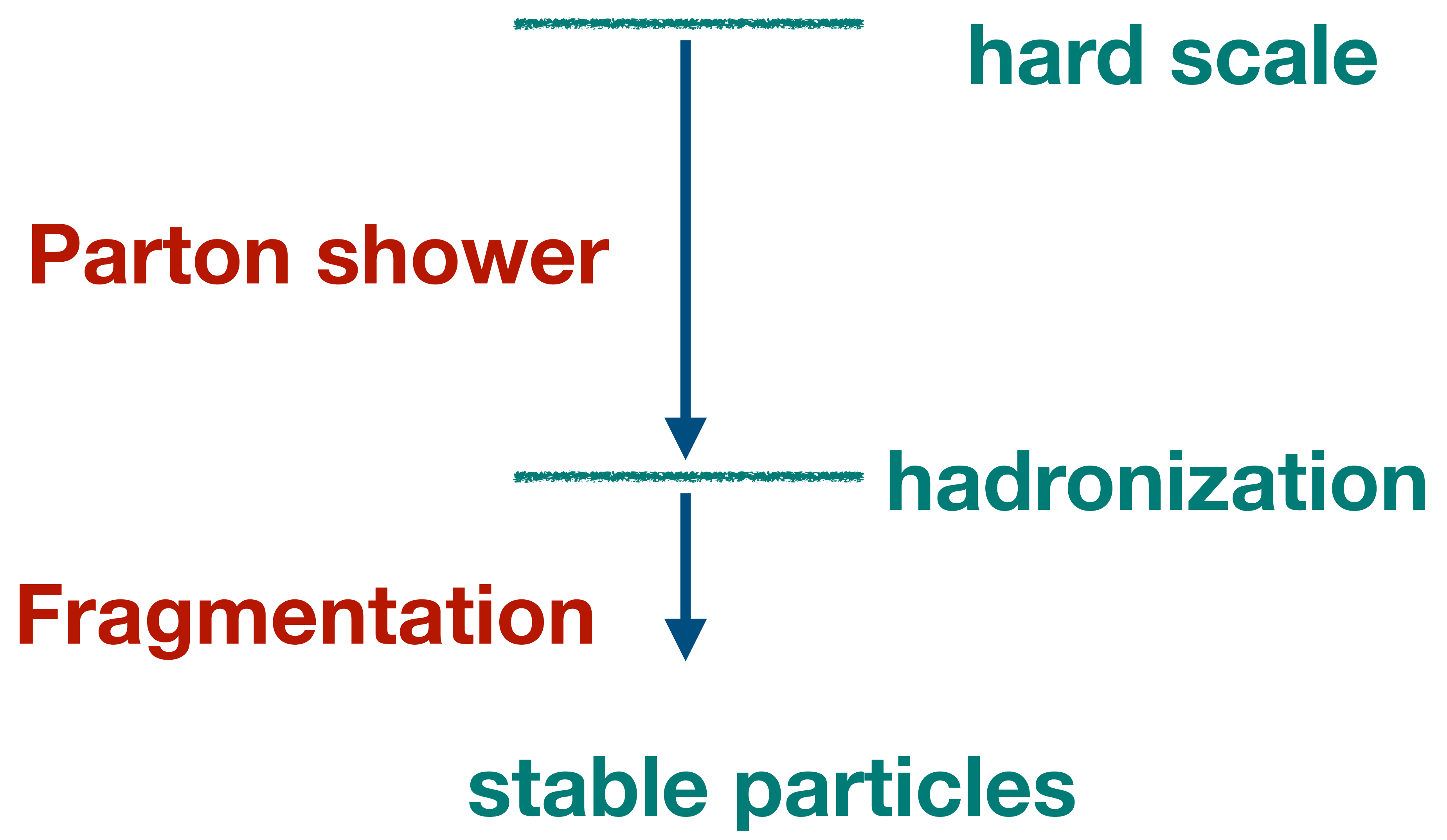
From PYTHIA 8.3

1. Introduction

The purpose of Monte Carlo event generators is to generate events in as much details as nature (generate average and fluctuation right)

$$\mathscr{P}_{\text{event}} = \mathscr{P}_{\text{Hard}} \otimes \mathscr{P}_{\text{Decay}} \otimes \mathscr{P}_{\text{ISR}} \otimes \mathscr{P}_{\text{FSR}} \otimes \mathscr{P}_{\text{MPI}} \otimes \mathscr{P}_{\text{Had}} \cdots$$

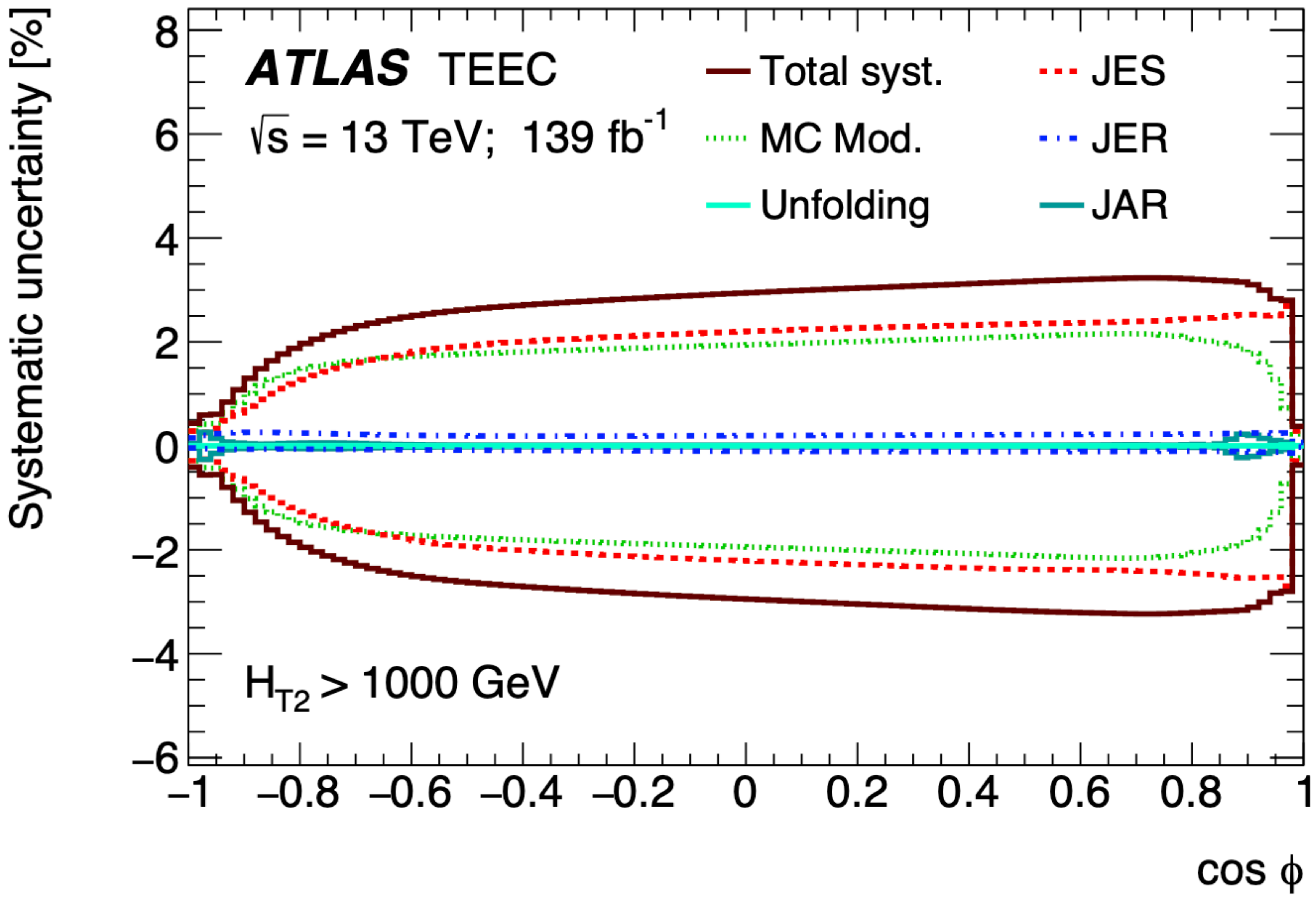
- ❑ **Hard process in high energy**
- ❑ **Transition from high energy to low energy**
  —parton shower
- ❑ **Low energy soft regime**
  —fragmentation

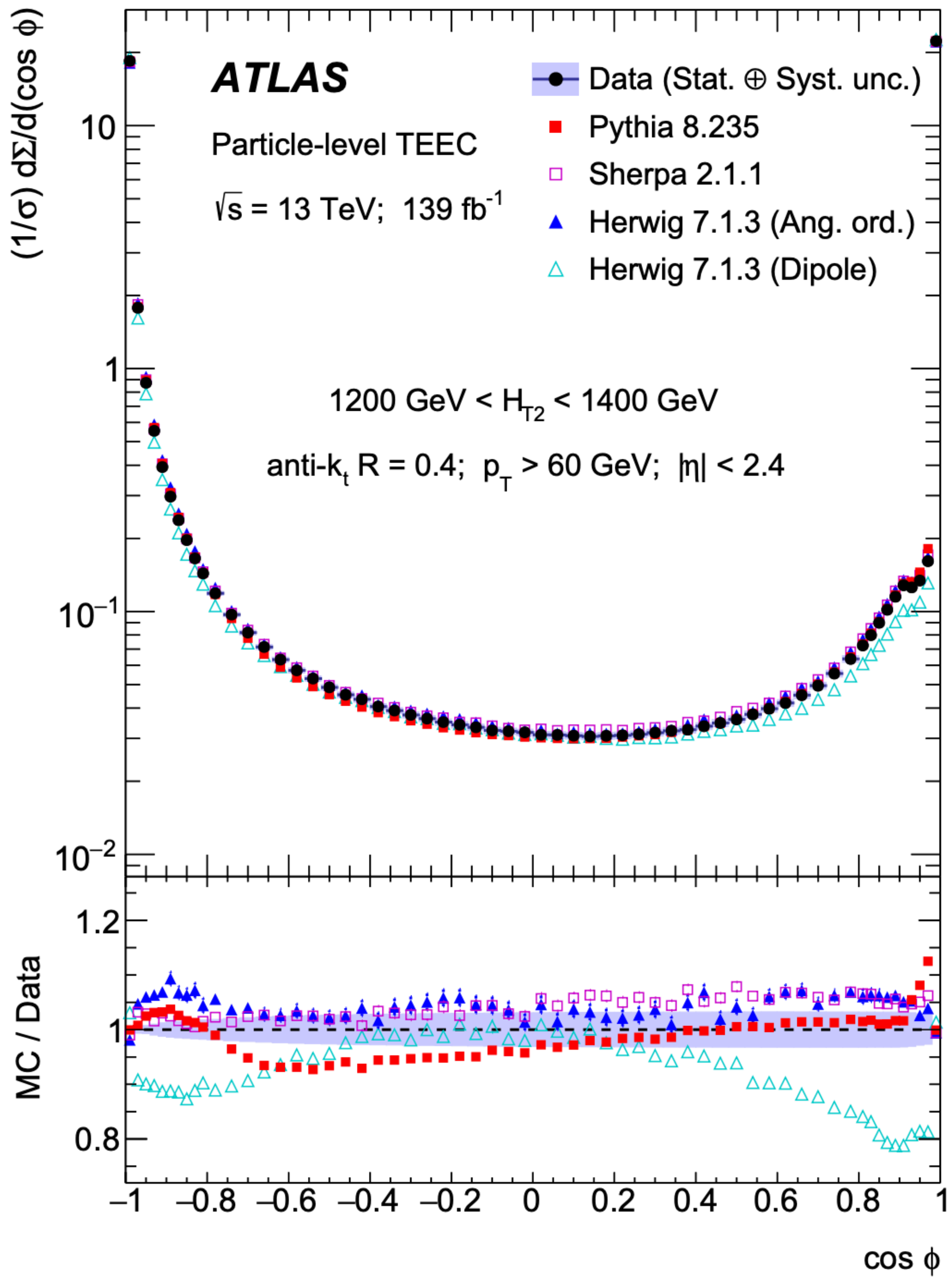


Parton shower: a model for the evolution from high scale to hadronization scale

1. Introduction

Uncertainties

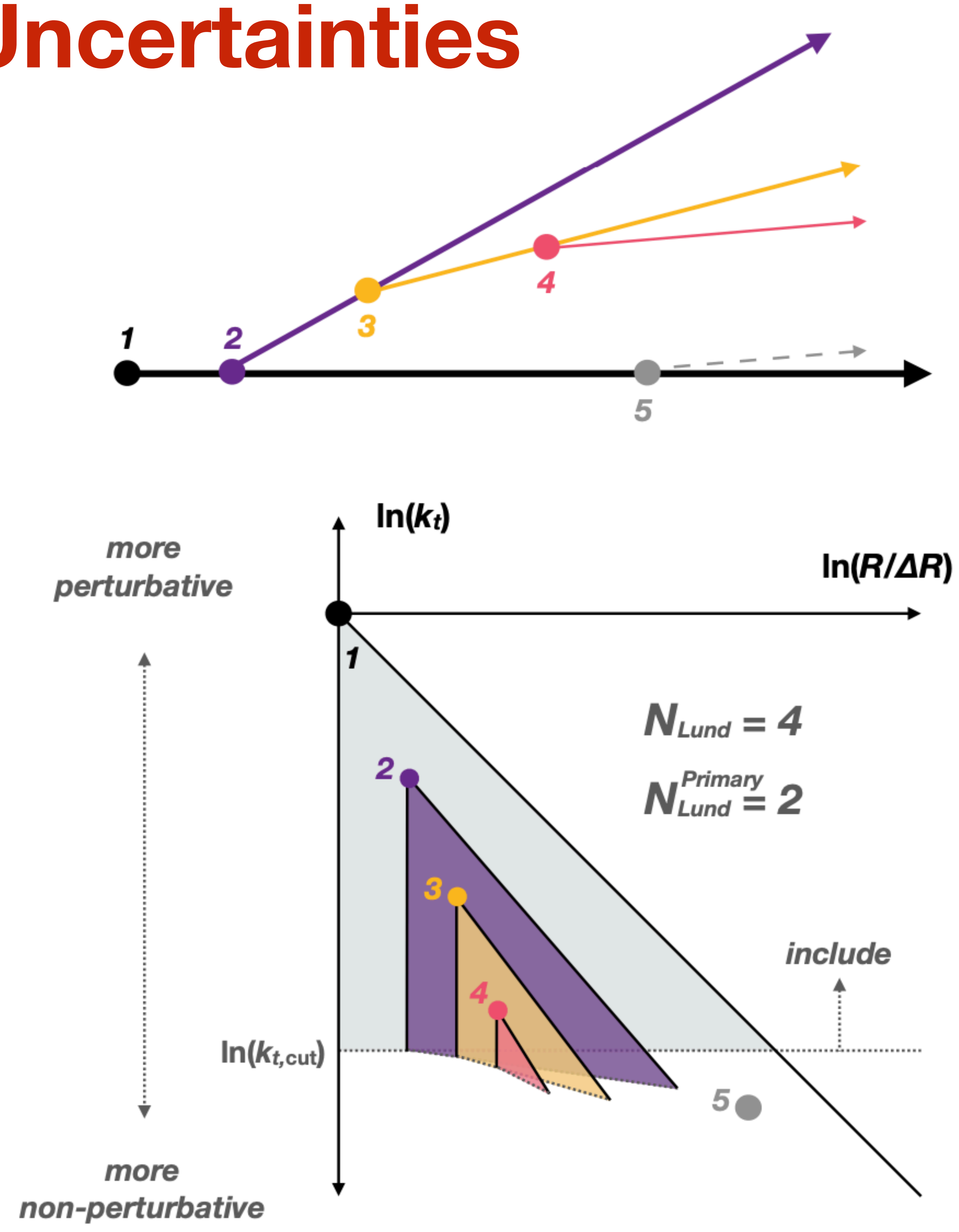


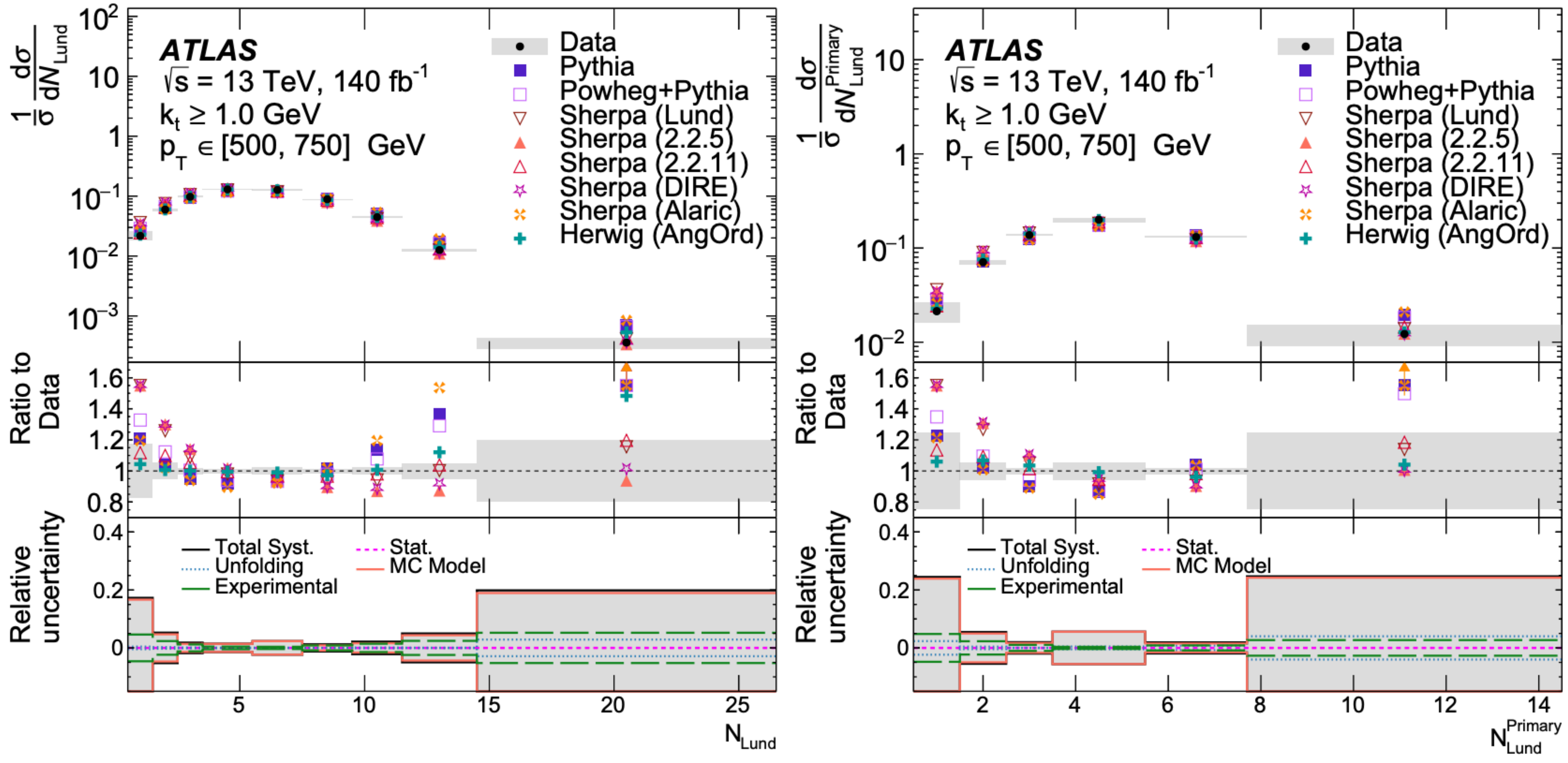


ATLAS, 2301.09351

1. Introduction

Uncertainties





ATLAS, 2402.13052

1. Introduction

Uncertainties arise from

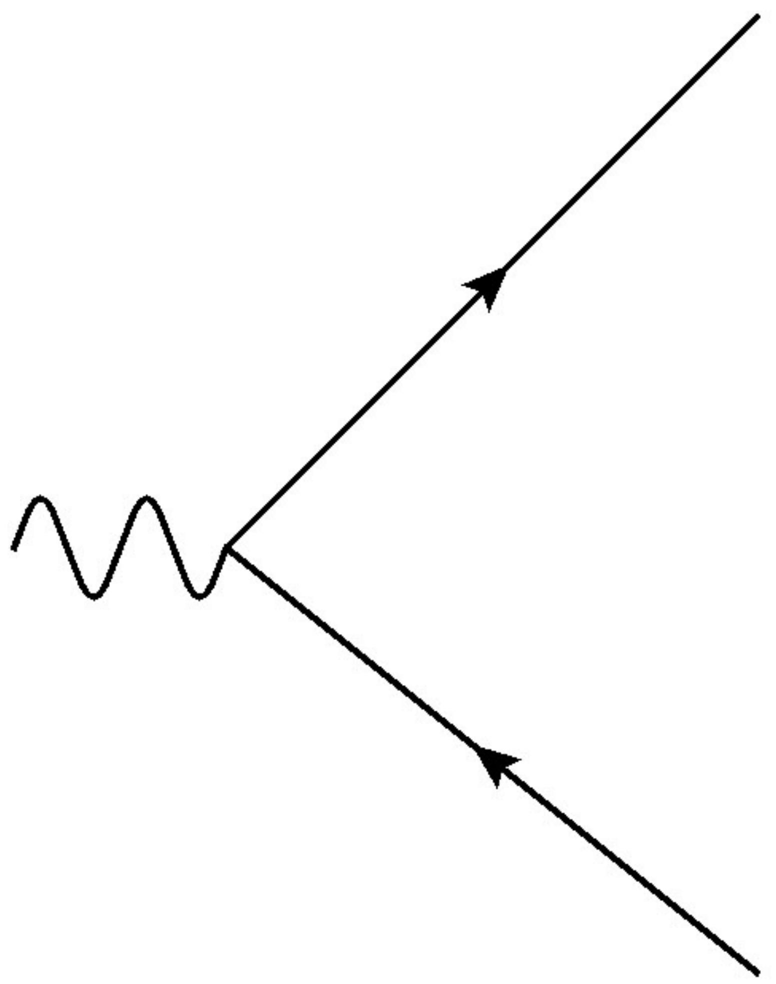
Perturbative
- Hard scattering matrix elements
- Parton shower

Non-perturbative
- Hadronization models
- Color reconnection
- MPI

2. Parton Showers

Parton showers approximate higher-order real-emission corrections to the hard scattering process

- Generate cascades of radiation automatically
- Locally conserved four momentum
- Locally conserved flavor
- Unitarity by construction



Parton showers

- sample infrared configurations
- simulate the evolution of parton (resummation)

Parton shower indispensable tools for particle physics phenomenology

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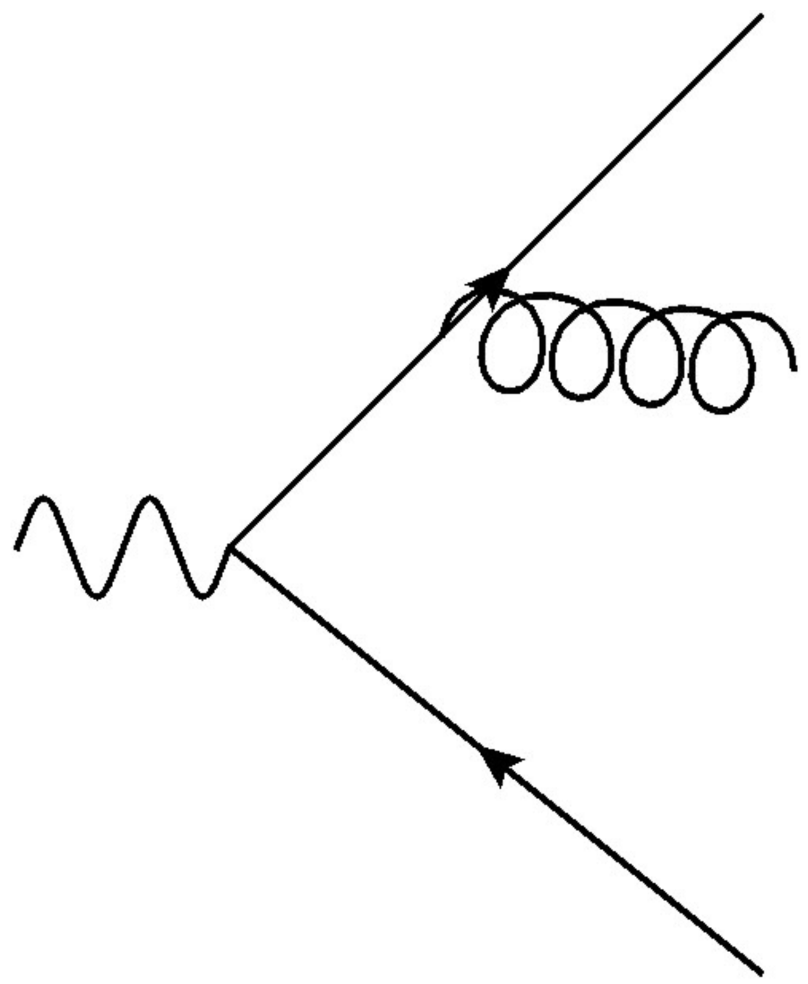

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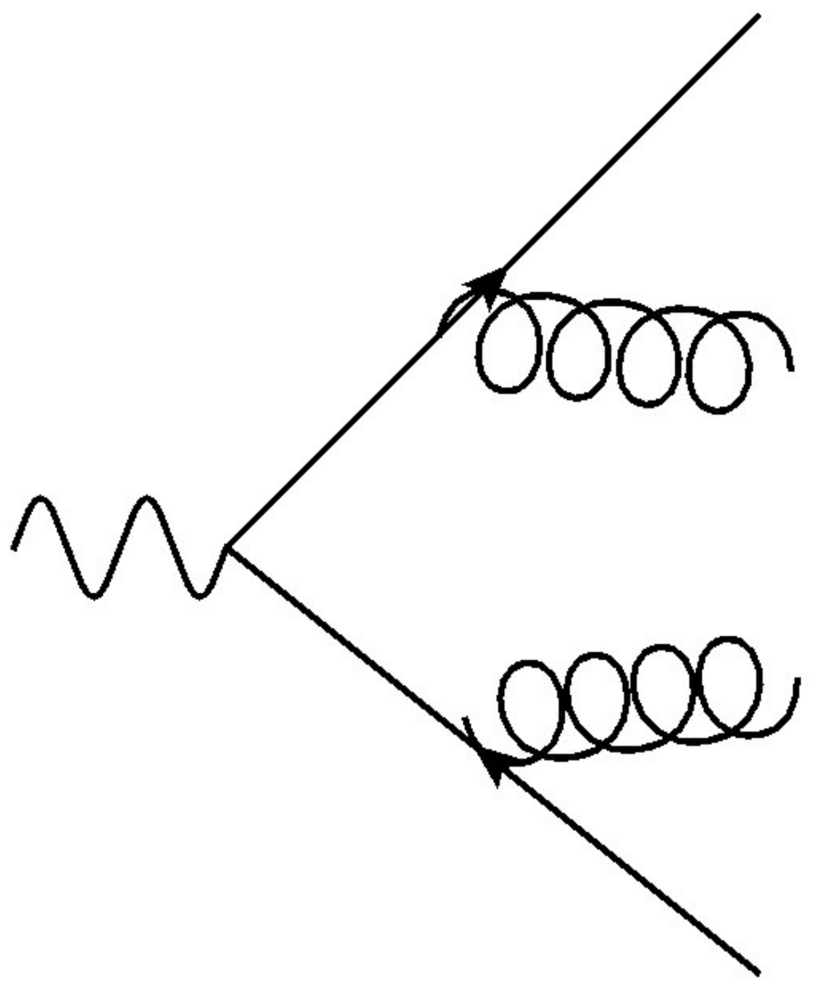

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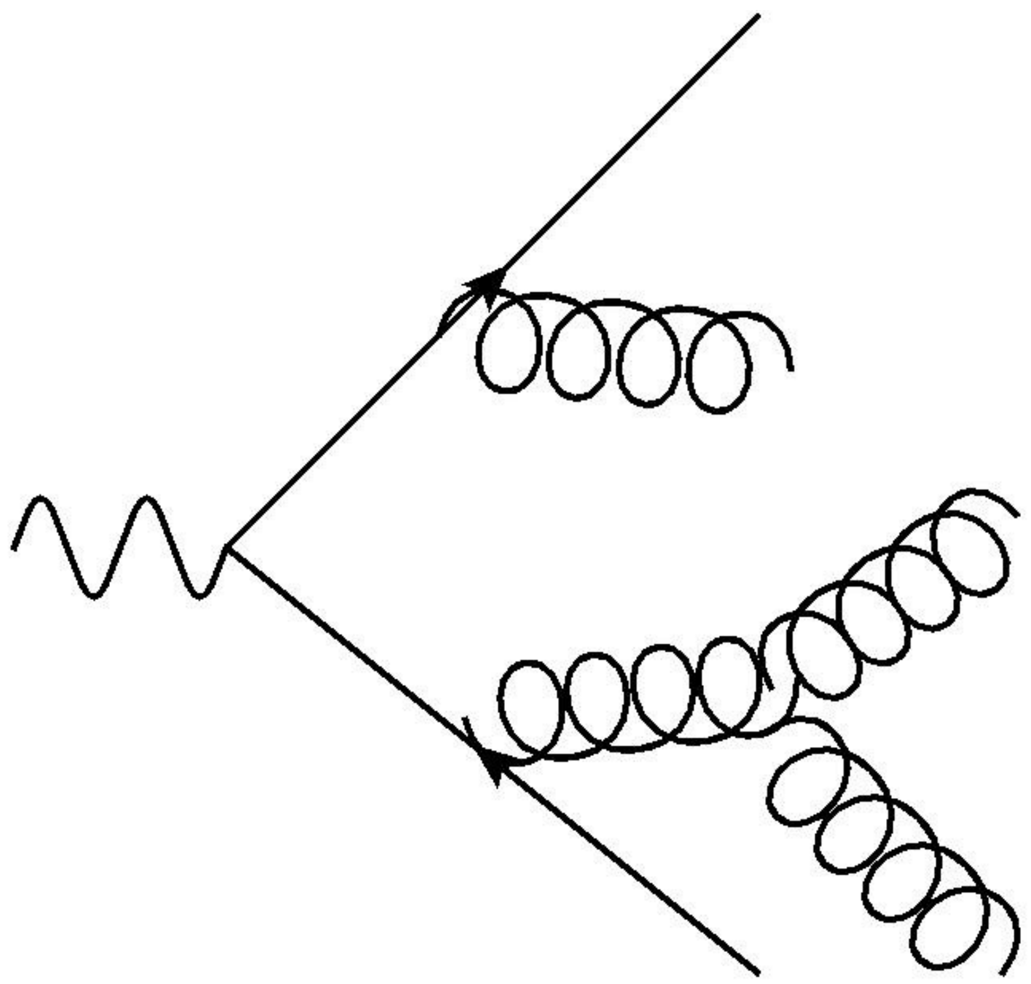

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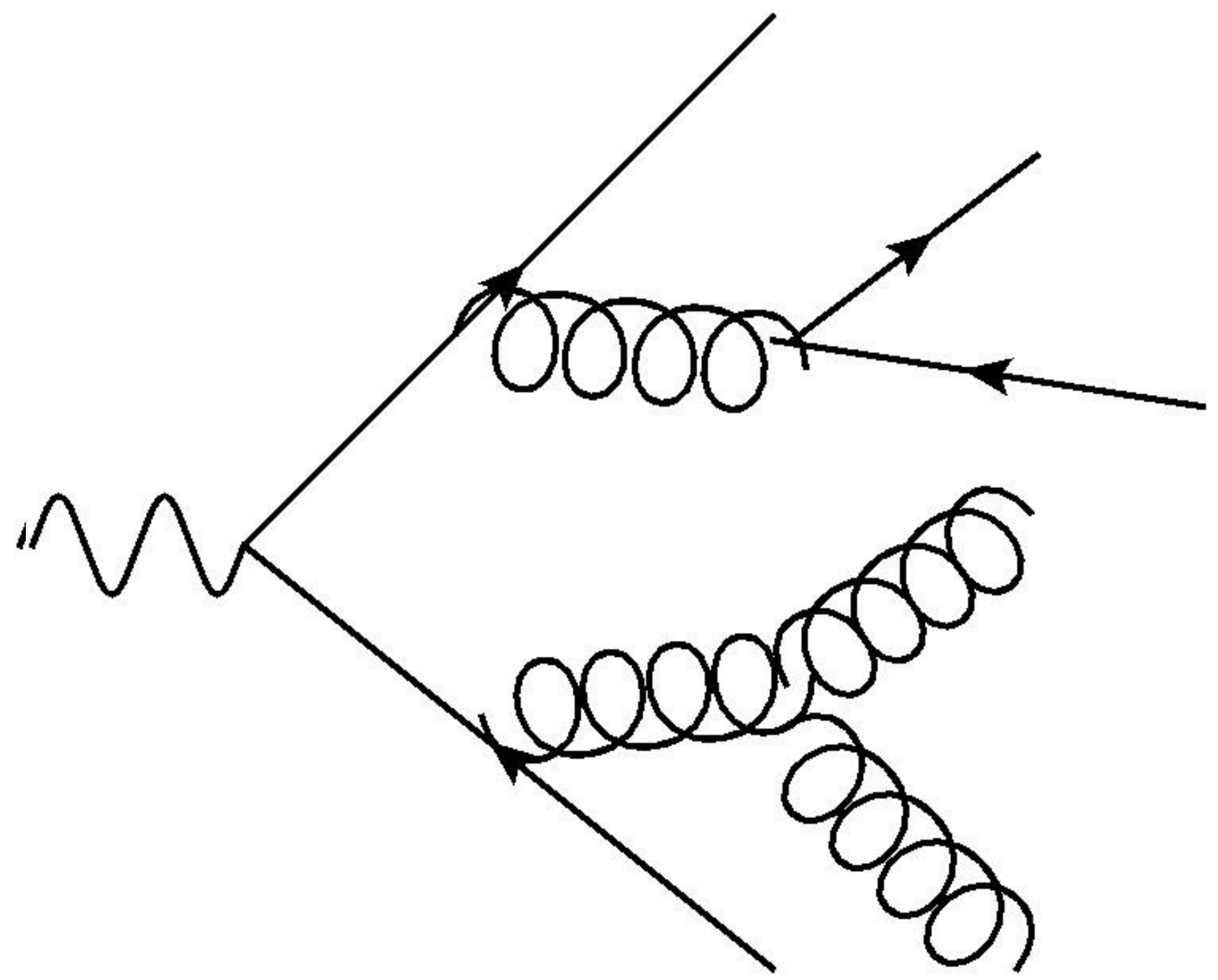

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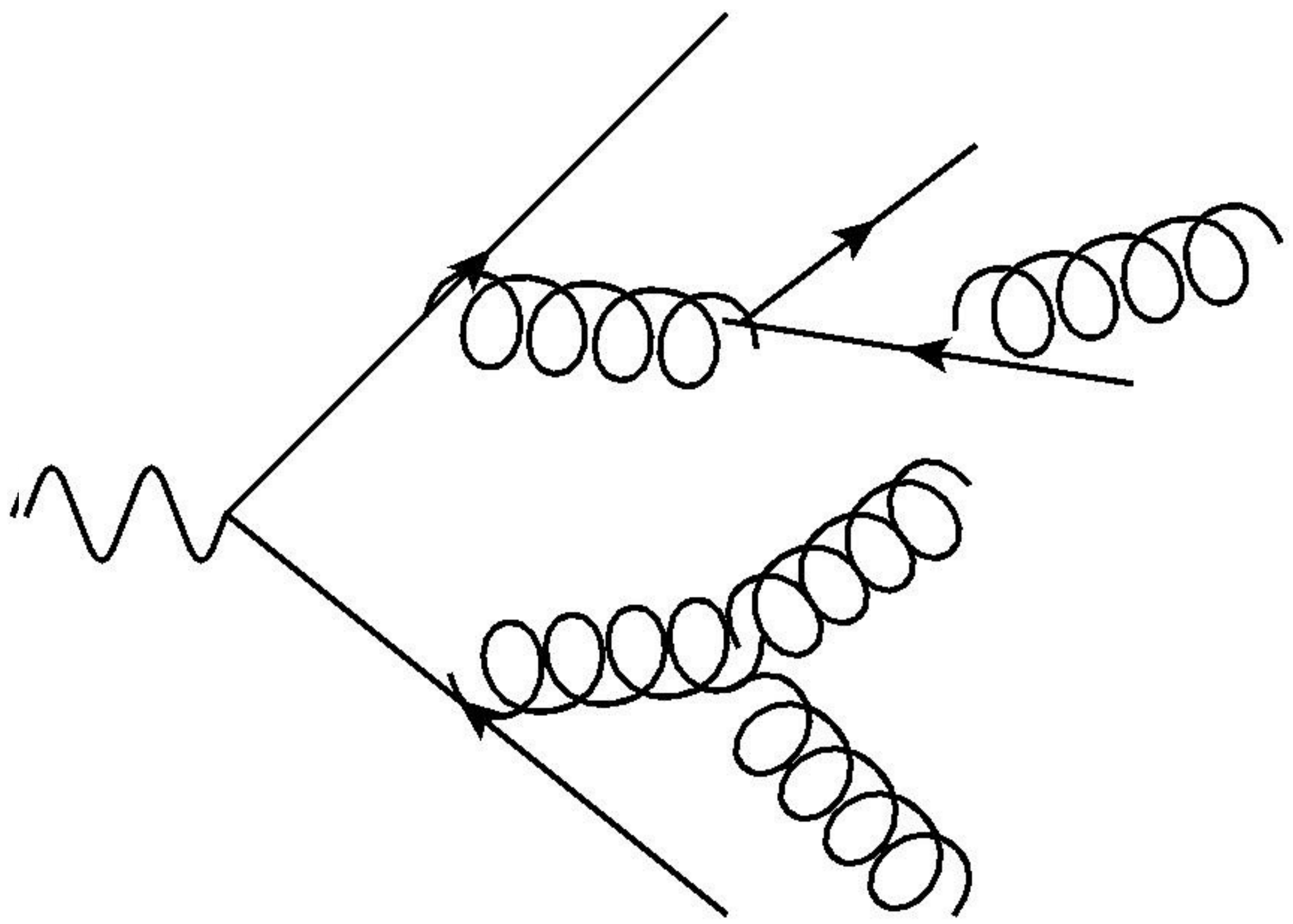

Parton showers

- sample infrared configurations
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Parton shower indispensable tools for particle physics phenomenology

2. Parton Showers

In the collinear or soft limit, the matrix element can be factorized as

$$|M(\cdots, p_i, p_j, \cdots)|^2 \xrightarrow{i||j} g_s^2 \mathscr{C} \frac{P(z)}{s_{ij}} |M(\cdots, p_i + p_j, \cdots)|^2$$

$$|M(\cdots, p_i, q, p_j, \cdots)|^2 \xrightarrow{q \to 0} g_s^2 \mathscr{C} \frac{p_i \cdot p_j}{p_i \cdot q \; p_j \cdot q} |M(\cdots, p_i + p_j, \cdots)|^2$$

n+1 external legs **n external legs**

Together with phase space integration, the cross section is

$$d\sigma_{n+1} = \frac{1}{2s} \int d\phi_{n+1} |M_{n+1}|^2 = d\sigma_n \otimes d\phi_{n \to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}$$

If we want to get the single unresolved limit correct, $\frac{|M_{n+1}|^2}{|M_n|^2}$ can be written as universal functions.

higher multiplicities can be obtained recursively

2. Parton Showers

$$d\sigma_{n+1} = \frac{1}{2s}\int d\phi_{n+1}|M_{n+1}|^2 = d\sigma_n \otimes d\phi_{n\to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}$$

In the exact single-unresolved limit

$s_{ij} = 0$ or $E_q = 0$

$$d\sigma_{n+2} = \frac{1}{2s}\int d\phi_{n+2}|M_{n+2}|^2 = d\sigma_n \times \frac{1}{2}\left(\int d\phi_{n\to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}\right)^2$$

$$d\sigma_{n+m} = \frac{1}{2s}\int d\phi_{n+1}|M_{n+1}|^2 = d\sigma_n \times \frac{1}{m!}\left(\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right)^m$$

2. Parton Showers

$$d\sigma_{n+1} = \frac{1}{2s}\int d\phi_{n+1}|M_{n+1}|^2 = d\sigma_n \otimes d\phi_{n\to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}$$

In the exact single-unresolved limit

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$$d\sigma_{n+m} = \frac{1}{2s}\int d\phi_{n+1}|M_{n+1}|^2 = d\sigma_n \times \frac{1}{m!}\left(\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right)^m$$

$$\sum_{m=0}^{\infty} \sigma_{n+m}$$

$$d\sigma_n \times \exp\left[\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right]$$

2. Parton Showers

$$d\sigma_{n+1} = \frac{1}{2s}\int d\phi_{n+1}|M_{n+1}|^2 = d\sigma_n \otimes d\phi_{n\to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}$$

$$d\sigma_{n+2} = \frac{1}{2s}\int d\phi_{n+2}|M_{n+2}|^2 = d\sigma_n \times \frac{1}{2}\left(\int d\phi_{n\to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}\right)^2$$

In the exact single-unresolved limit

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$$d\sigma_{n+m} = \frac{1}{2s}\int d\phi_{n+1}|M_{n+1}|^2 = d\sigma_n \times \frac{1}{m!}\left(\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right)^m$$

$$\sum_{m=0}^{\infty}\sigma_{n+m}$$

$$d\sigma_n \times \exp\left[\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right]$$

no additional radiation observed

with the probability function $\exp\left[\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right]$

2. Parton Showers

Sudakov form factor: Non-branching probability $\exp\left[\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right]$

Probability that there is no branching from Q to q is $\Delta_i\left(Q^2, q^2\right)$

choose kinematic variable as the evolution scale

$$\Delta\left(Q^2, q^2\right) = \exp\left\{\int_{Q^2}^{q^2} d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right\}$$

Probability for one observed branching $1 - \Delta\left(Q^2, q^2\right)$

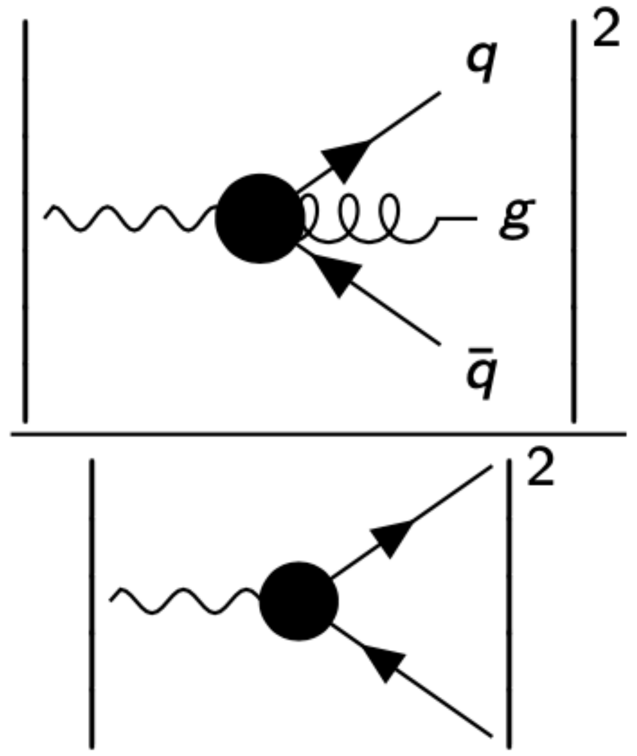
Probability one branching between the scale q^2 to $q^2 + dq^2$

$$\frac{d}{dq^2}\Delta\left(Q^2, q^2\right) = \Delta\left(Q^2, q^2\right) \times d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}$$

Additional radiations can be added according to the function $\Delta\left(Q^2, q^2\right)$

2. Parton Showers

Infrared structure for single unresolved limit is well known



$$\rightarrow \begin{cases} \underbrace{\frac{P_{q\mapsto qg}(z_q)}{s_{qg}}}_{q\text{-collinear}} + \underbrace{\frac{P_{\bar{q}\mapsto \bar{q}g}(z_{\bar{q}})}{s_{g\bar{q}}}}_{\bar{q}\text{-collinear}} & \text{DGLAP} \\ \underbrace{\frac{K_{q\mapsto qg;\bar{q}}(z_q)}{s_{qg}}}_{q\text{-(soft-)collinear}} + \underbrace{\frac{K_{\bar{q}\mapsto \bar{q}g;q}(z_{\bar{q}})}{s_{g\bar{q}}}}_{\bar{q}\text{-(soft-)collinear}} & \text{CS Dipoles} \\ \underbrace{\frac{2s_{q\bar{q}}}{s_{qg}s_{g\bar{q}}}}_{\text{soft}} + \underbrace{\frac{1}{s}\left(\frac{s_{g\bar{q}}}{s_{qg}} + \frac{s_{qg}}{s_{g\bar{q}}}\right)}_{\text{collinear}} & \text{Antennae} \end{cases}$$

DGLAP splitting functions used

applied widely used CS dipole subtraction terms

antenna function obtained directly from matrix element square

Phase space mapping $\int d\phi_{n\to n+1} \frac{|M_{n+1}|^2}{|M_n|^2} = \int_{q^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2}\frac{\alpha_s}{2\pi}\int_{Q_0^2/k^2}^{1-Q_0^2/k^2} \mathrm{d}z P_{ji}(z)$

$$\frac{d\theta^2}{\theta^2} = \frac{dq^2}{q^2} = \frac{dk_\perp^2}{k_\perp^2}$$

many choices for the evolution variables

2. Parton Showers

Phase space mapping $\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2} = \int_{q^2}^{Q^2}\frac{\mathrm{d}k^2}{k^2}\frac{\alpha_s}{2\pi}\int_{Q_0^2/k^2}^{1-Q_0^2/k^2}\mathrm{d}z P_{ji}(z)$

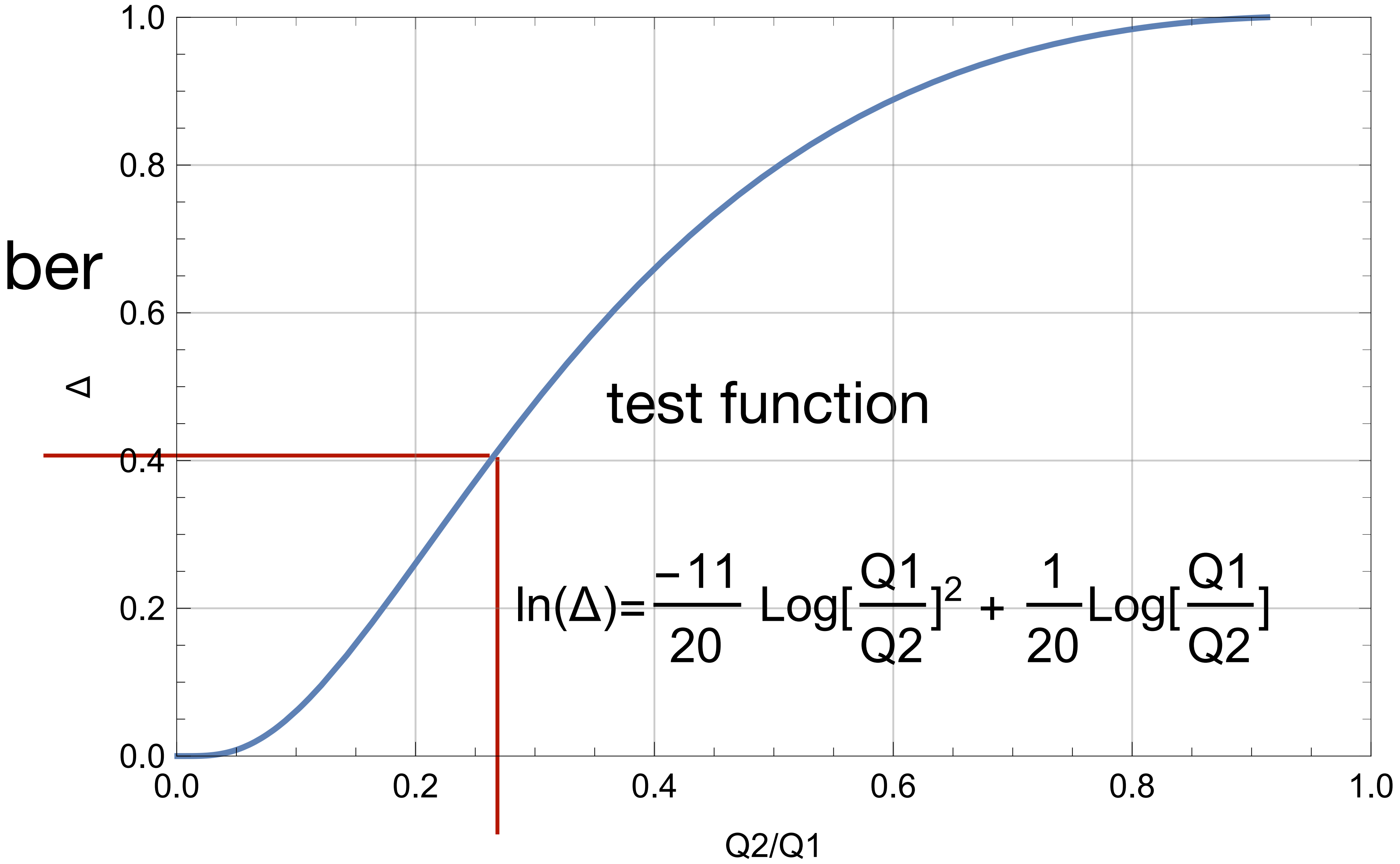
$$\frac{d\theta^2}{\theta^2} = \frac{dq^2}{q^2} = \frac{dk_\perp^2}{k_\perp^2}$$

many choices for the evolution variables

Monte-Carlo Technique and resummation

Sudakov factor $\Delta\left(Q_1^2, Q_2^2\right)$

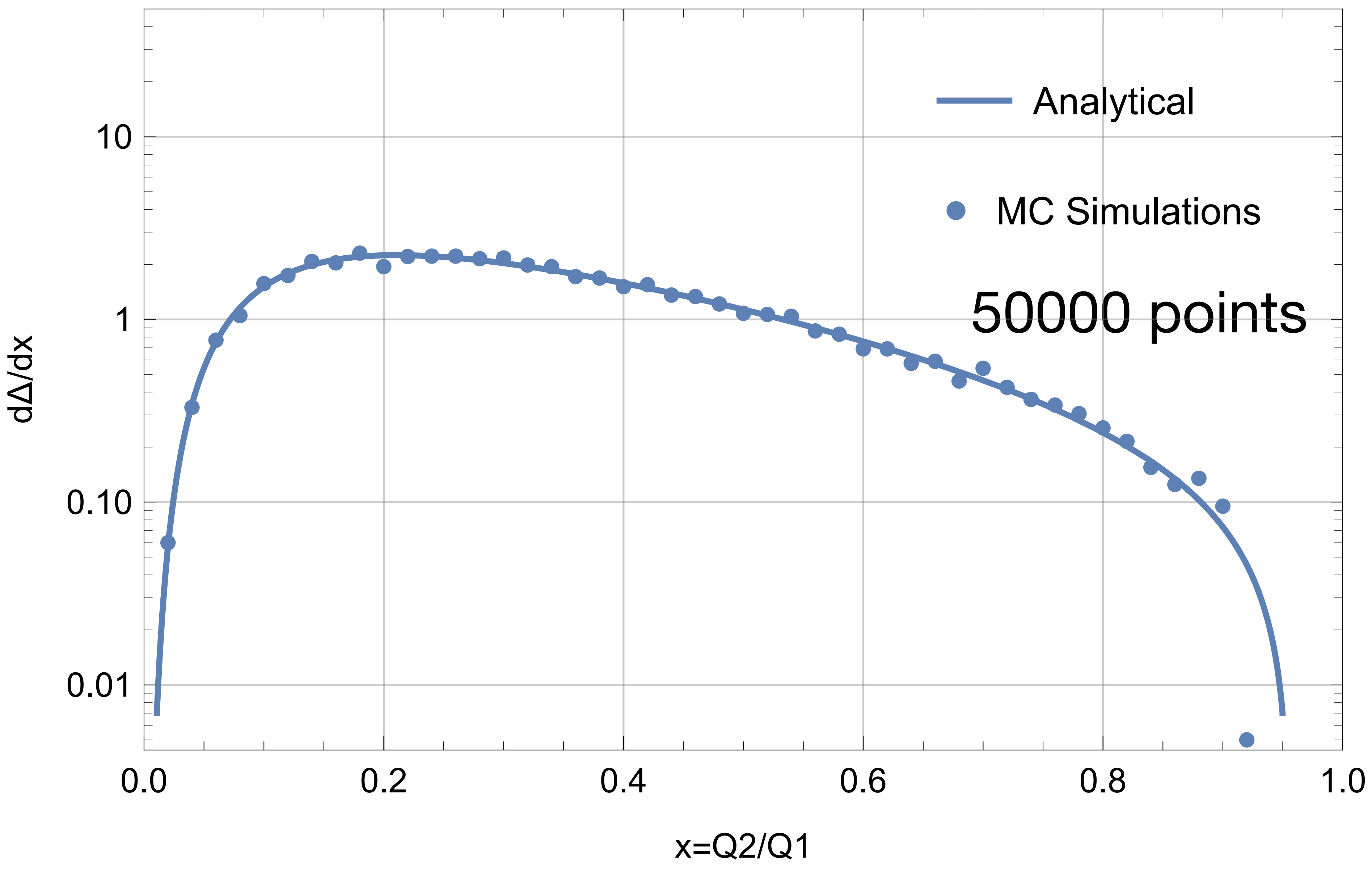
Generate random number $R \in (0,1)$



Solve $R = \Delta$ for Q1/Q2

Q_2/Q_1 **distribution generated by**

$$\frac{d}{dq^2}\Delta\left(Q^2,q^2\right) = \Delta\left(Q^2,q^2\right)\times d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}$$



new phase space point generated according to the new scales

2. Parton Showers

For multi-scale problem

固定阶计算

	NLO	NNLO	NNNLO	
$z\frac{1}{\sigma_0}\frac{d\sigma}{dz} \propto z +$	$\frac{\alpha_s}{4\pi}a_{1,2}\ln z +$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,4}\ln^3 z +$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,6}\ln^5 z$	$+\cdots$ **LL**
$+$	$\frac{\alpha_s}{4\pi}a_{1,1} +$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,3}\ln^2 z +$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,5}\ln^4 z$	$+\cdots$ **NLL**
$+$	$\frac{\alpha_s}{4\pi}a_{1,0}z +$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,2}\ln z +$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,4}\ln^3 z$	$+\cdots$ **NNLL**
	$+$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,1} +$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,3}\ln^2 z$	$+\cdots$ **NNNLL**
	$+$	$\left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,0}z +$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2}\ln z$	$+\cdots$ **NNNNLL**
		$+$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2}$	$+\cdots$
		$+$	$\left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2}z$	$+\cdots$

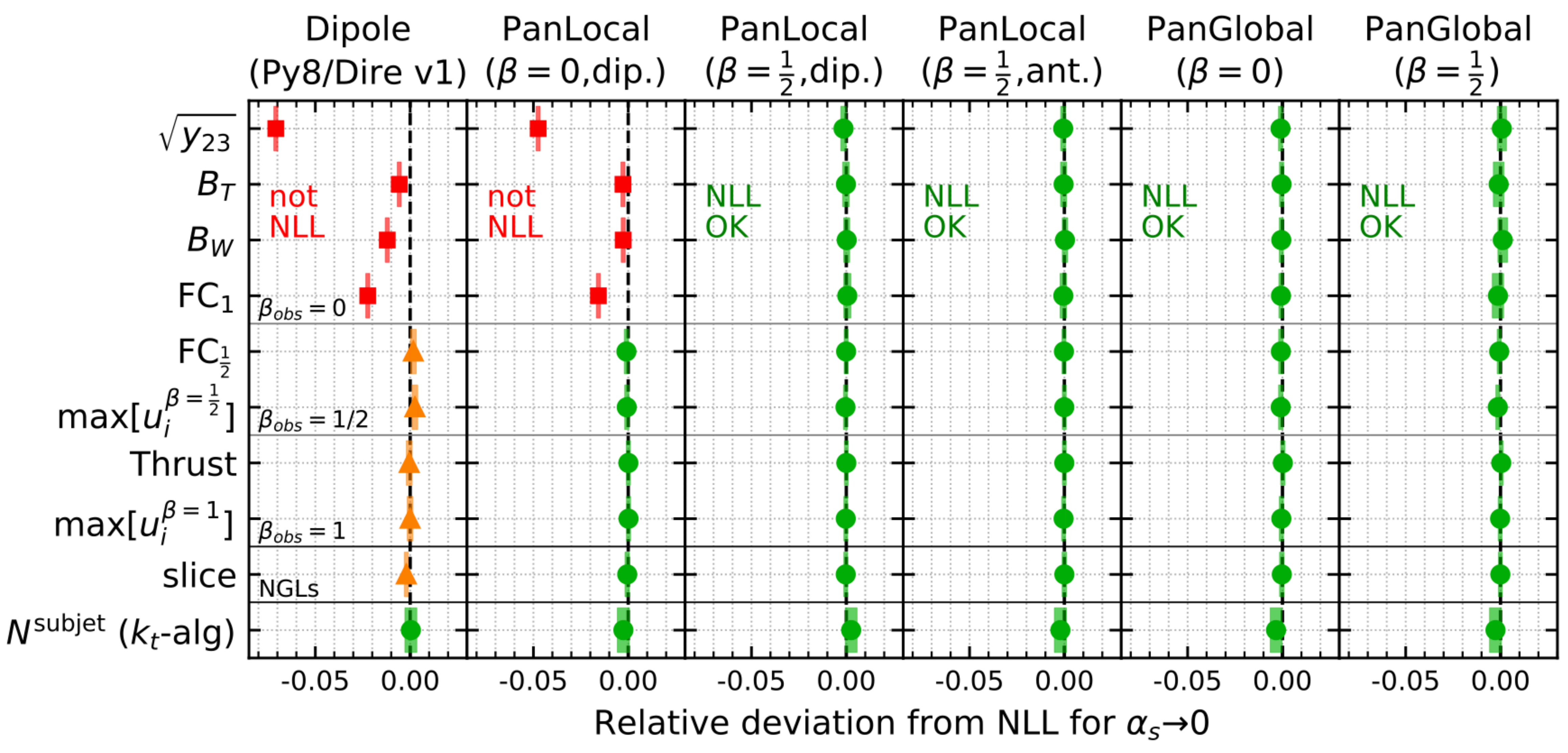
重求和

For observables that involve scale hierarchies resummation is required

NLL: PanScales, Alaric, Herwig et al

with higher order effects: Vincia, DIRE et al

PanScales:



arXiv:2002.11114

2. Parton Showers

NLL accuracy is the becoming the new standard

Logarithmic accuracy of parton showers: a fixed-order study
Dasgupta, Dreyer, Hamilton, Monni, Salam [1805.093927]

Parton showers beyond leading logarithmic accuracy
Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez [2002.11114]

Colour and logarithmic accuracy in final-state parton showers
Hamilton, Medves, Salam, Scyboz, Soyez [2011.10054]

Spin correlations in final-state parton showers and jet observables
Karlberg, Salam, Scyboz, Verheyen [2103.16526]

Soft spin correlations in final-sta[te] parton showers
Hamilton, Karlberg, Salam, Scyboz, Verhe[yen] [2111.01161]

PanScales parton showers for hadron collisions: formulation and fixed-order studies
van Beekveld, Ferrario Ravasio, Salam, Soto Ontoso, Soyez, Verheyen [2205.02237]

PanScales

PanScales parton showers for hadron collisions: all-order validation
van Beekveld, Ferrario Ravasio, Hamilton, Salam, Soto Ontoso, Soyez, Verheyen [2207.09467]

Next-to-leading-logarithmic PanScales showers for deep inelastic scattering and vector boson fusion
van Beekveld, Ferrario Ravasio [2305.08645]

Introduction to the PanScales framework, version 0.1
van Beekveld, Dasgupta, El-Menoufi, Ferrario Ravasio, Hamilton, Helliwell, Karlberg, Medves, Monnim Salam, Scyboz, Soto Ontoso, Soyez, Verheyen [2312.13275]

Building a consistent parton shower
Forshaw, Holguin, Plätzer [2003.06400]

Improvements on dipole shower colour
Forshaw, Holguin, Plätzer [2011.15087]

Logarithmic accuracy of angular-ordered parton showers
Bewick, Ferrario Ravasio, Richar[dson], Seymour [1904.11866]

Herwig

Initial state radiation in the Herwig 7 angular-ordered parton shower
Bewick, Ferrario Ravasio, Richardson, Seymour [2107.04051]

A new approach to color-coherent parton evolution
Herren, Höche, Krauss, Reichelt, Schönherr [2208.06057]

Alaric

New approach to QCD final-state evolution i[n] processes with massive partons
Assi, Höche [2307.00728]

The Alaric parton shower for hadron colliders
Höche, Krauss, Reichelt [2404.14360]

A partitioned dipole-antenna shower with improved transverse recoil
Preuss [2403.19452]

Apollo

Summation of large logarithms by parton showers
Nagy, Soper [2011.04773]

Deductor

Summation by parton showers of large logarithms in electron-positron annihilation
Nagy, Soper [2011.04777]

2. Parton Showers

固定阶计算

NLO | NNLO | NNNLO

$$z\frac{1}{\sigma_0}\frac{d\sigma}{dz} \propto z + \frac{\alpha_s}{4\pi}a_{1,2}\ln z + \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,4}\ln^3 z + \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,6}\ln^5 z + \cdots \quad \text{LL}$$

$$+ \frac{\alpha_s}{4\pi}a_{1,1} + \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,3}\ln^2 z + \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,5}\ln^4 z + \cdots \quad \text{NLL}$$

$$+ \frac{\alpha_s}{4\pi}a_{1,0}z + \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,2}\ln z + \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,4}\ln^3 z + \cdots \quad \text{NNLL}$$

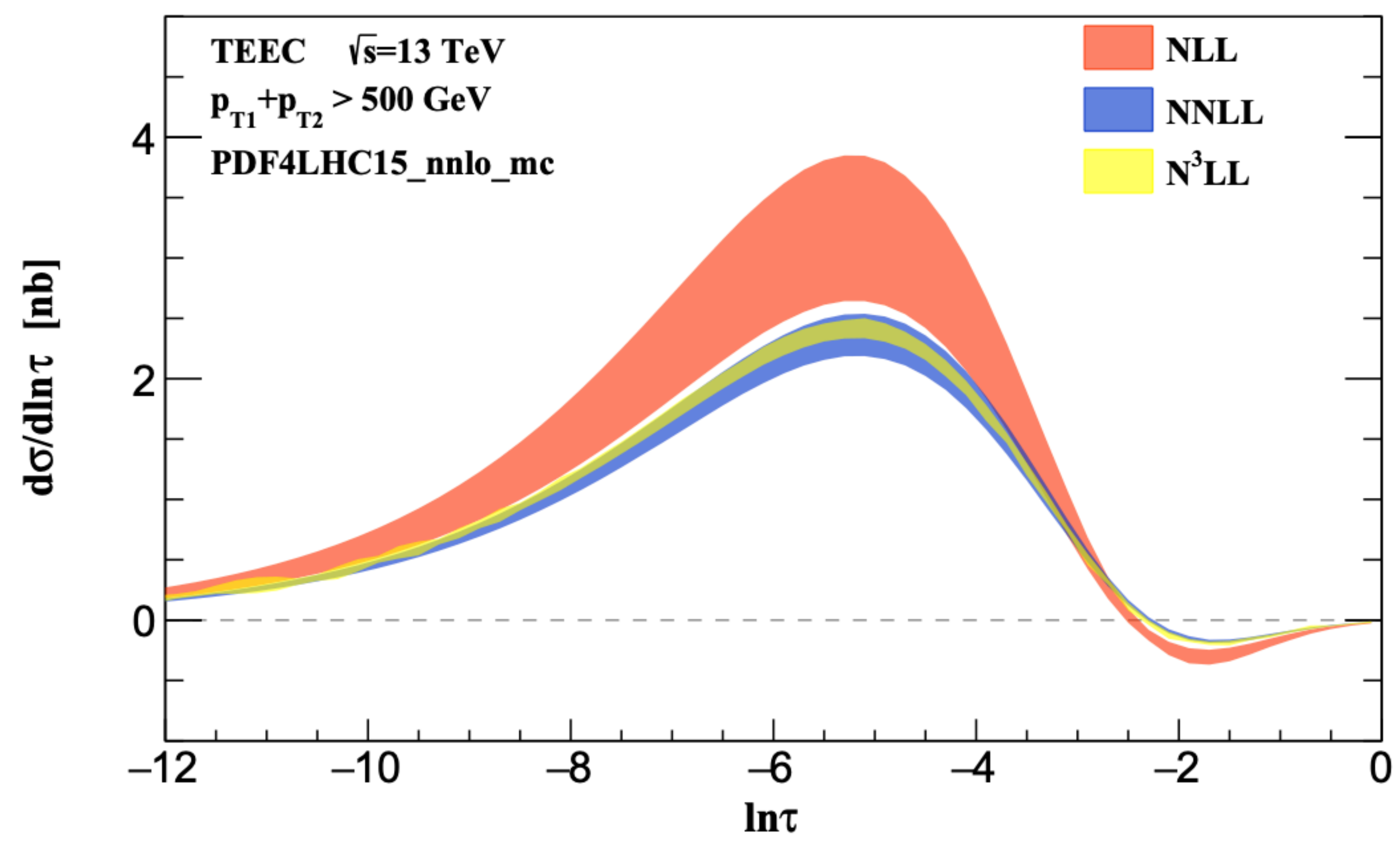
$$+ \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,1} + \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,3}\ln^2 z + \cdots \quad \text{NNNLL}$$

$$+ \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,0}z + \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2}\ln z + \cdots \quad \text{NNNNLL}$$

$$+ \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2} + \cdots$$

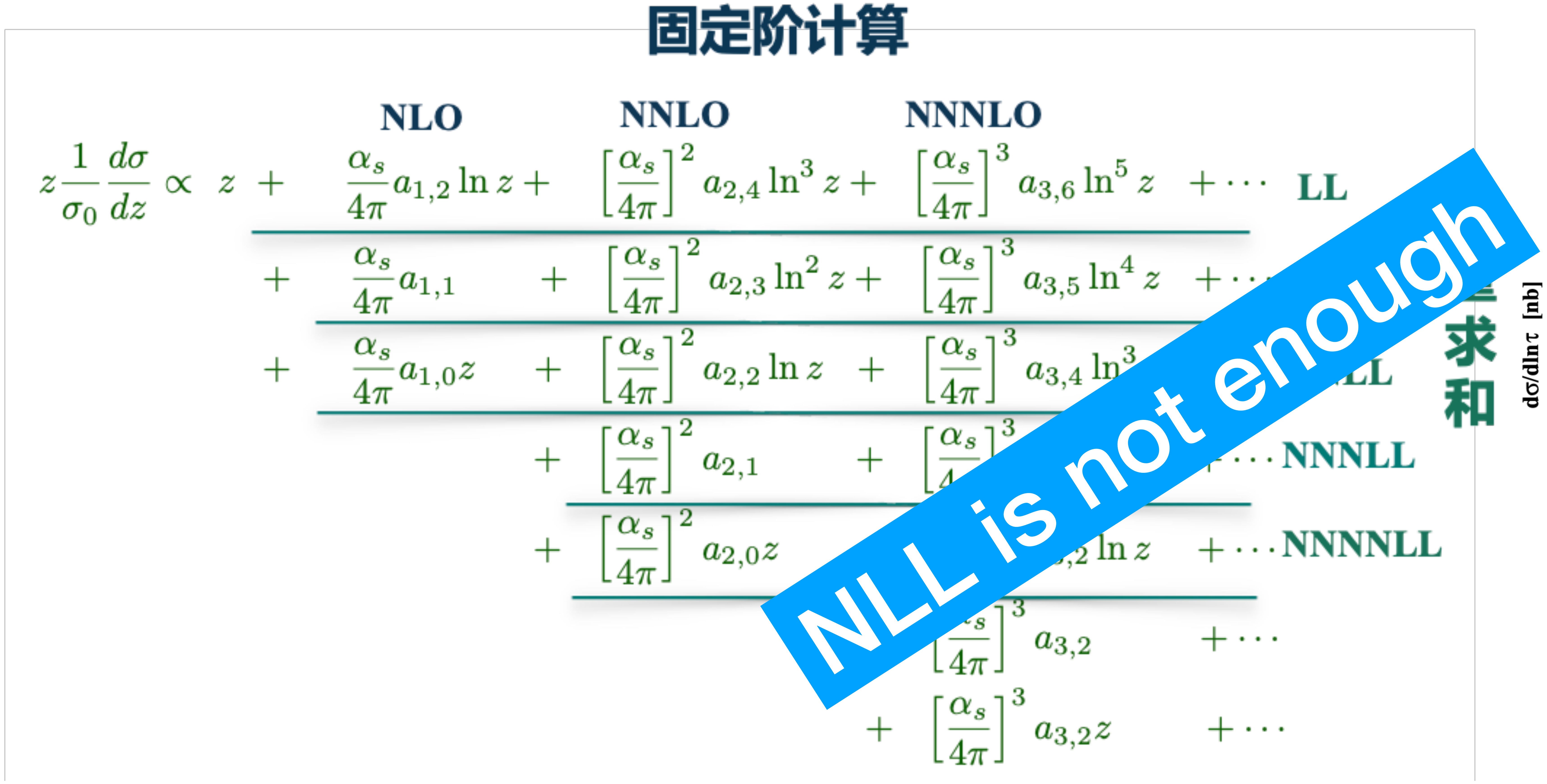
$$+ \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2}z + \cdots$$

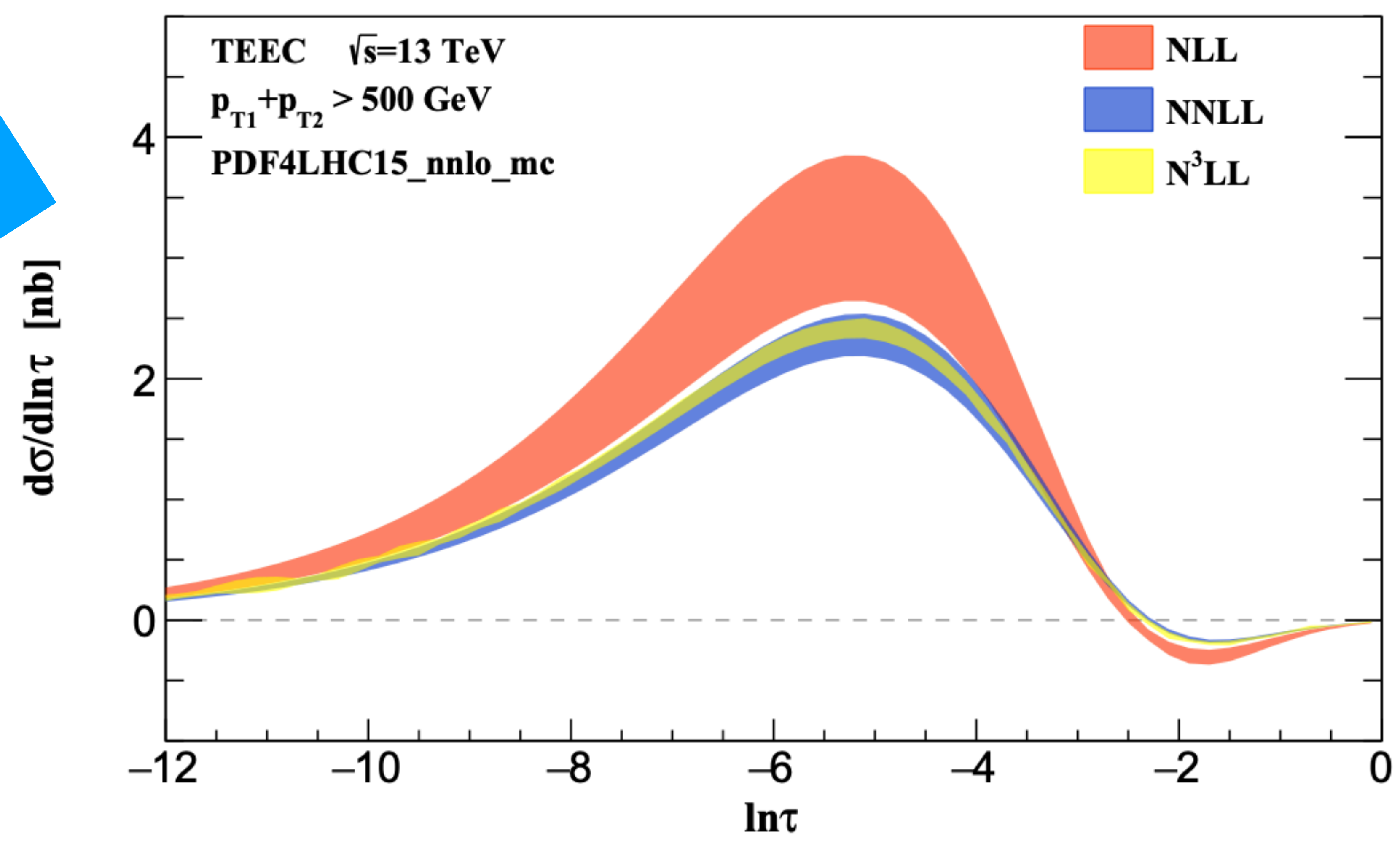
重求和



Gao, HTL, Moult, Zhu, JHEP 2024

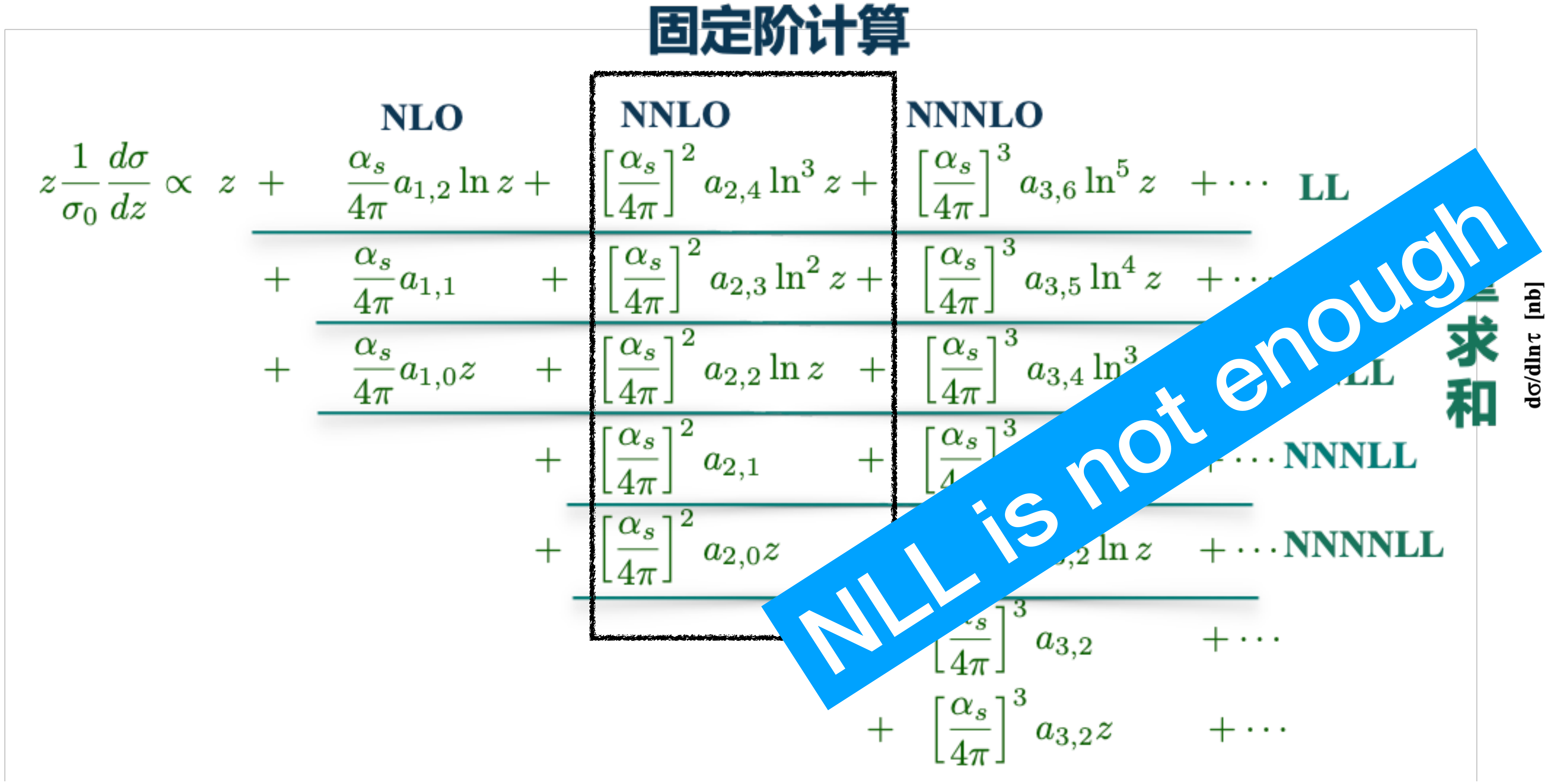
2. Parton Showers

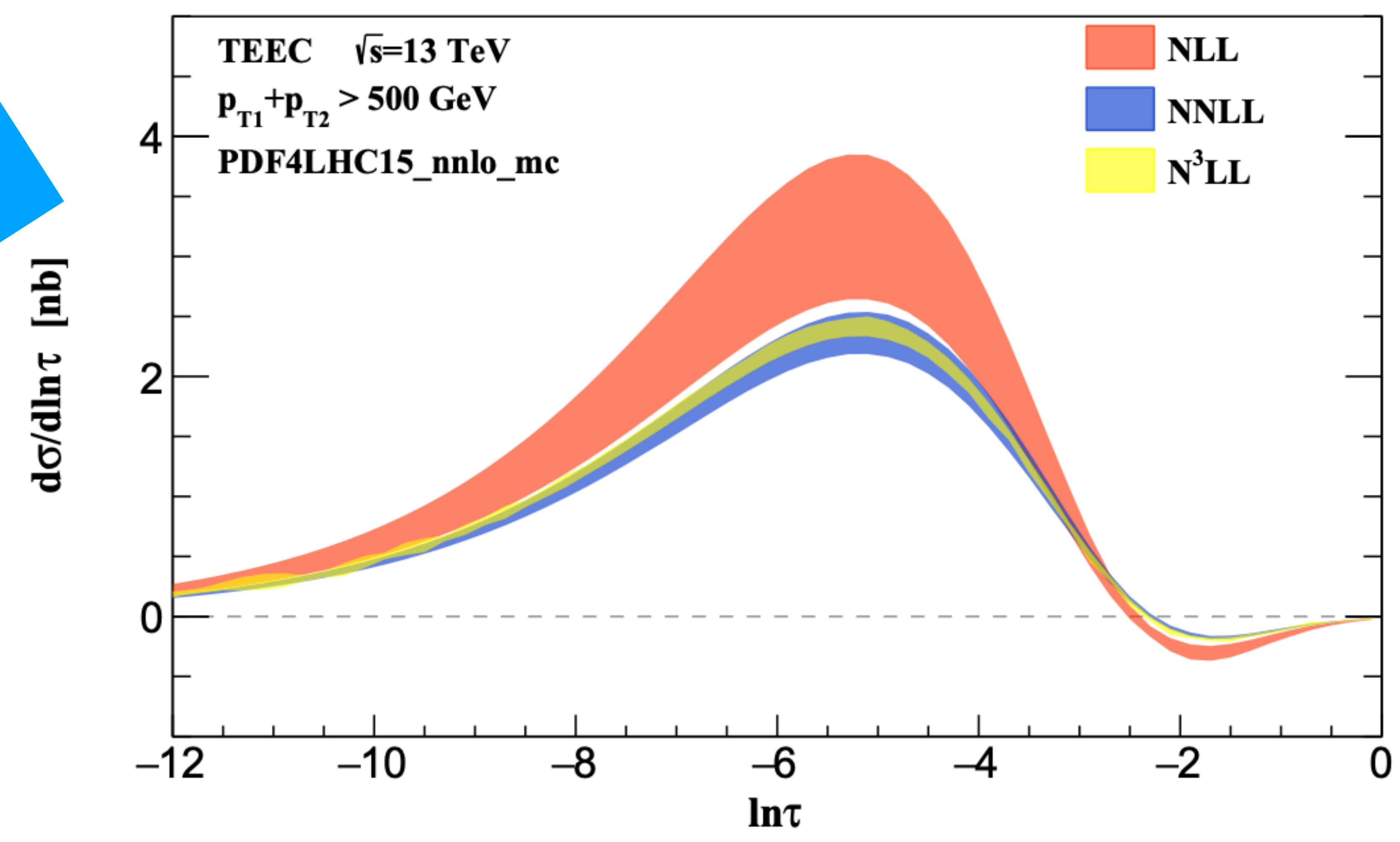




Gao, HTL, Moult, Zhu, JHEP 2024

2. Parton Showers





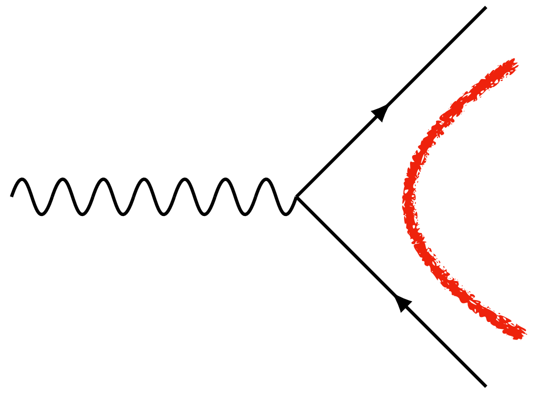
Gao, HTL, Moult, Zhu, JHEP 2024

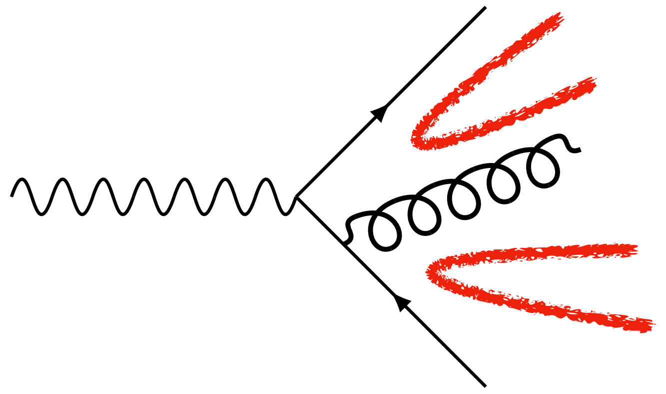
NNLL would include the full NNLO pole structures

2. Parton Showers

LO parton shower

From parton shower





$1 - \Delta_i(Q^2, Q_0^2)$

$$\sigma_{\rm NLO}^{\rm PS} = \sigma_0 \Pi_i \left(\Delta_i(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \Delta_i(Q^2, q^2) \int dz P_{ji}(z) \right)$$

0-radiation **1-radiation (Sudakov suppressed)**

From the definition of Sudakov factor, we have

$$\mathscr{P}(\text{unresolved}) + \mathscr{P}(\text{resolved}) = 1$$

probability conservation from the definition of Δ

From NLO calculations

$$\sigma_{\rm NLO} = \sigma_0 + \left(\int d\Phi_n V + \int d\Phi_{n+1} S \right) \mathcal{O}_n + \int d\Phi_{n+1} (R \mathcal{O}_{n+1} - S \mathcal{O}_n)$$

virtual **integrated subtraction** **subtracted real**

$$\sigma_{\rm NLO} = \sigma_0^n + \int_0^{t_n} d\sigma_{(1)}^n + \int_{t_n} d\sigma_{(1)}^{n+1}$$

t_n as the resolution scale for 1-radiation

LO parton showers reproduce the NLO singular behavior of the underlying hard process with unitarity assumption

$$V + \int R = 0.$$

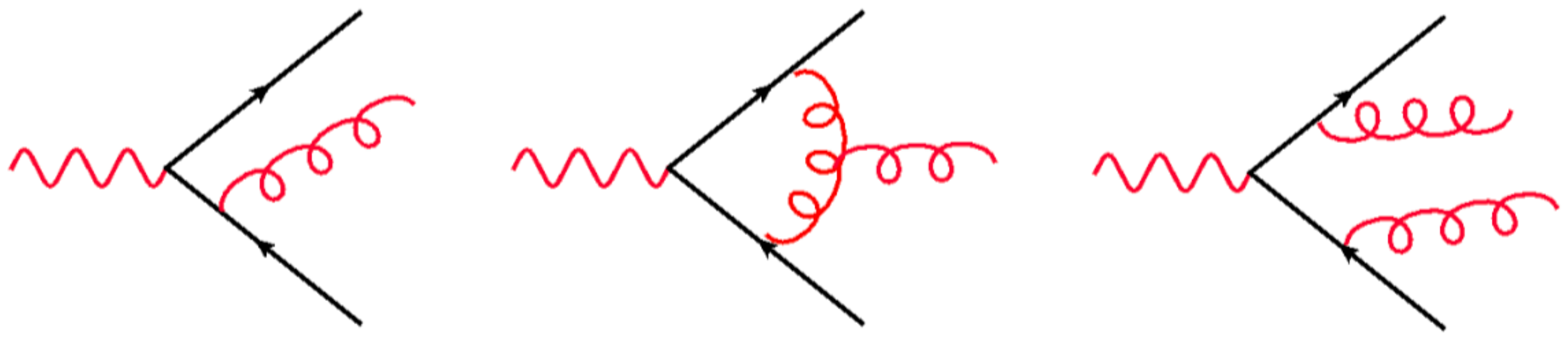
Resummation from Showers + Hard emissions From fixed orders

2. Parton Showers

To which order can Parton Showers do?

NLO corrections to resummation kernel

What we expect for NLO showers

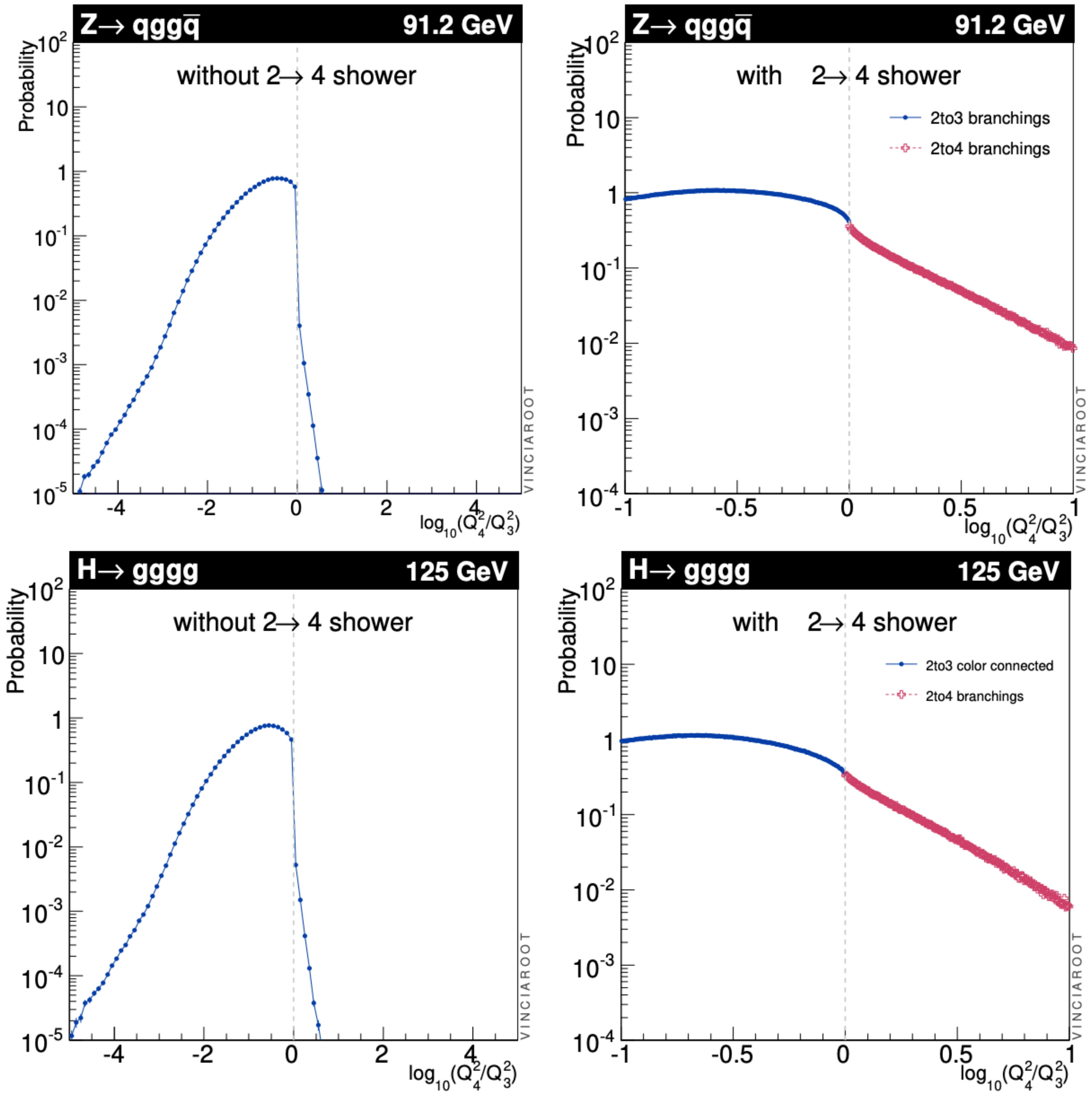


NLO parton shower

$$\frac{d}{dQ^2}\underbrace{\left(1-\Delta\left(Q_0^2,Q^2\right)\right)}_{\text{branching probability}} = -\underbrace{\int \frac{d\Phi_3}{d\Phi_2}\delta\left(Q^2-Q^2\left(\Phi_3\right)\right)\left(a_3^0+a_3^1\right)\Delta\left(Q_0^2,Q^2\right)}_{\text{born and virtual correction}}$$

$$-\underbrace{\int \frac{d\Phi_4}{d\Phi_2}\delta\left(Q^2-Q^2\left(\Phi_4\right)\right)a_4^0\,\Delta\left(Q_0^2,Q^2\right)}_{\text{real correction}}$$

HTL, Skands, arXiv:1611.00013



include correct logs and cover the full space

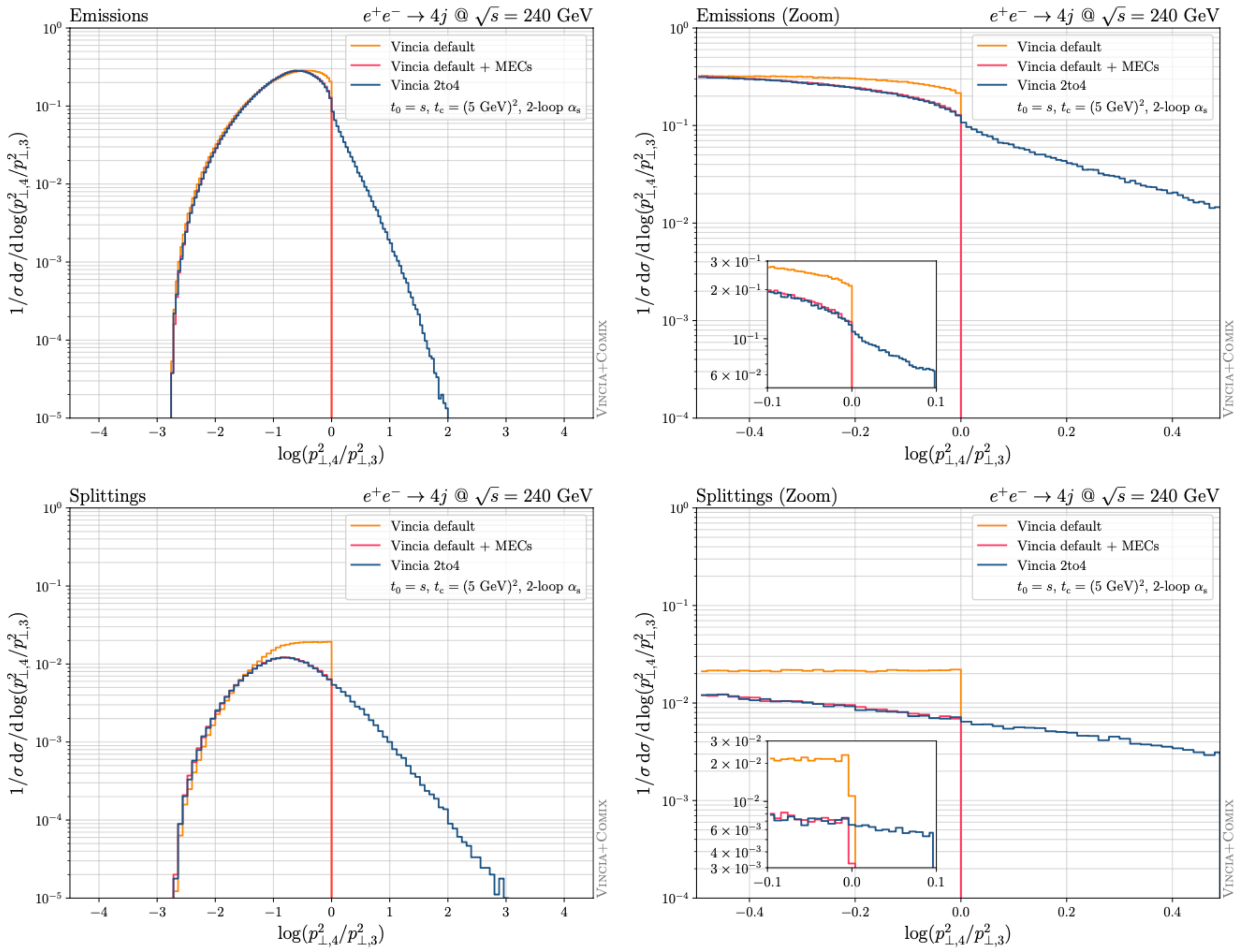
2. Parton Showers

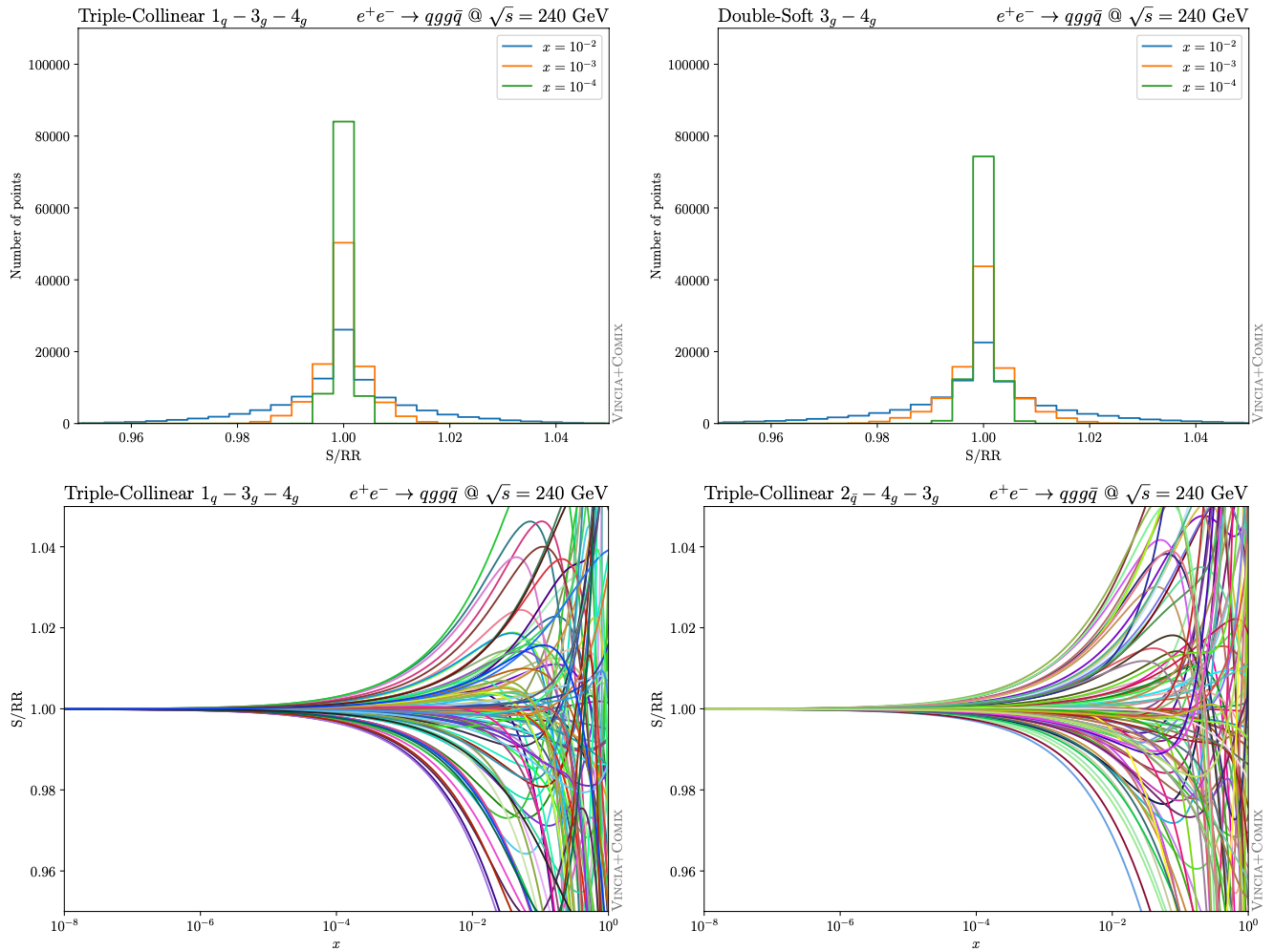
Matching using NLO antenna shower

$$\Delta_2^{\mathrm{NLO}}\left(t_0, t\right) = \exp\left\{-\int_t^{t_0} \mathrm{d}\Phi_{+1}\ \mathrm{A}_{2\mapsto 3}^{(0)}\left(\Phi_{+1}\right) w_{2\mapsto 3}^{\mathrm{NLO}}\left(\Phi_2, \Phi_{+1}\right)\right\} \times \exp\left\{-\int_t^{t_0} \mathrm{d}\Phi_{+2}^{>} \mathrm{A}_{2\mapsto 4}^{(0)}\left(\Phi_{+2}\right) w_{2\mapsto 4}^{\mathrm{LO}}\left(\Phi_2, \Phi_{+2}\right)\right\}$$

Expanding the Sudakov factor to NNLO and compare it with full NNLO corrections

First fully differentially matching





Campbella, Hoech, HTL, Preuss, Skands, 2108.07133

2. Parton Showers

Sunshine

Sudakov Nesting of Hard Integrals

Using generalized parton shower to generate fixed order corrections

Fixed order should look like

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\Phi_n} = |M_0|^2 \prod_{i=0}^{n-1} \mathrm{ant}_{i\mapsto i+1}$$

matrix element ratio $(0 \to 1) \times (1 \to 2) \times \cdots \times (n-1 \to n)$

Usually showers will give $(0 \to n)$

$$\frac{\mathrm{d}\mathcal{P}_{00\ldots0}}{\mathrm{d}\Phi_n} = |M_0|^2 \prod_{i=0}^{n-1} \mathrm{ant}_{i\mapsto i+1} \Delta_i(t_i, t_{i+1})$$

Sudakov factor from showers $\Delta_0 \times \Delta_1 \times \cdots \times \Delta_{(n-1)}$

Altmann, HTL, Scyboz, Skands, arXiv:2507.00111

2. Parton Showers

Sunshine by @vector_corp on freepik.es

Sunshine

Sudakov Nesting of Hard Integrals

Using generalized parton shower to generate fixed order corrections

Altmann, HTL, Scyboz, Skands, arXiv:2507.00111

keep the parent events after branching, and ask the event branches m times at stage $0 \to 1$, then shower them afterwards

$$\frac{\mathrm{d}\mathcal{P}_{m0\ldots0}}{\mathrm{d}\Phi_n} = \frac{\mathrm{d}\mathcal{P}_{00\ldots0}}{\mathrm{d}\Phi_n} \prod_{j=1}^{m} \int_{t_1}^{\tilde{t}_{j-1}} \mathrm{ant}_{0\mapsto1}(\tilde{t}_j)\, \mathrm{d}\tilde{t}_j$$

keep all the intermediate states and shower them m_k times from $k-1$ partons to k partons

$$\frac{\mathrm{d}\mathcal{P}_{m_1 m_2 \ldots m_n}}{\mathrm{d}\Phi_n} = \frac{\mathrm{d}\mathcal{P}_{00\ldots0}}{\mathrm{d}\Phi_n} \prod_{k=1}^{n} \prod_{j=1}^{m_k} \int_{t_k}^{\tilde{t}_{k_j-1}} \mathrm{ant}_{k-1\mapsto k}(\tilde{t}_{k_j})\, \mathrm{d}\tilde{t}_{k_j}$$

sum m_k to infinity

$$\sum_{m_k \geq 0} \frac{\mathrm{d}\mathcal{P}_{m_1 m_2 \ldots m_n}}{\mathrm{d}\Phi_n} = \frac{\mathrm{d}\mathcal{P}_{00\ldots0}}{\mathrm{d}\Phi_n} \prod_{k=1}^{n} \frac{1}{\Delta_k(t_{k-1}, t_k)}$$

$$\textsc{Sunshine}: \quad \sum_{m_k \geq 0} \frac{\mathrm{d}\mathcal{P}_{m_1 m_2 \ldots m_n}}{\mathrm{d}\Phi_n} = |M_0|^2 \prod_{i=0}^{n-1} \mathrm{ant}_{i\mapsto i+1} \,.$$

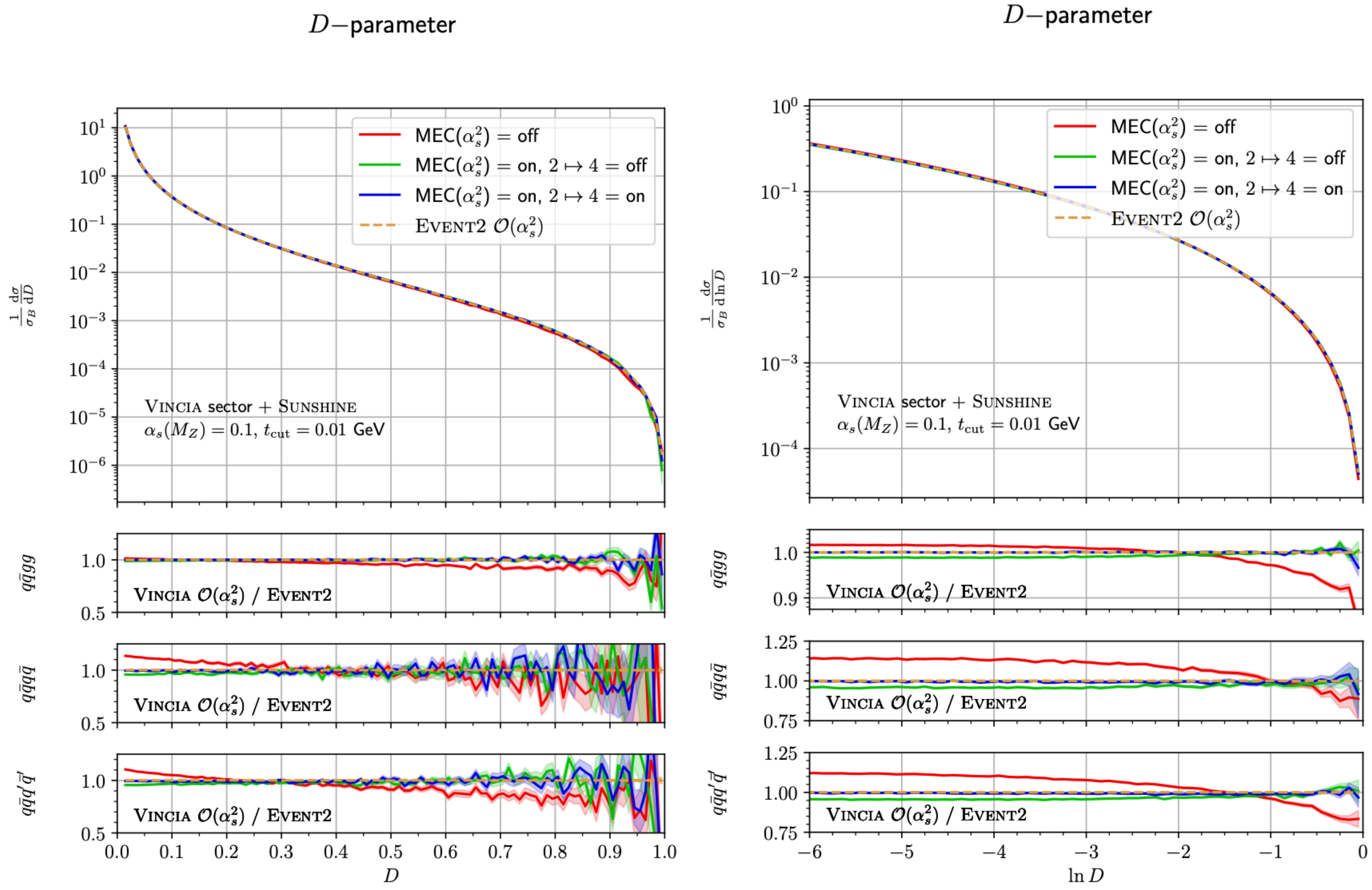
2. Parton Showers

Sunshine by @vector_corp on freepik.es

Sunshine

Sudakov Nesting of Hard Integrals

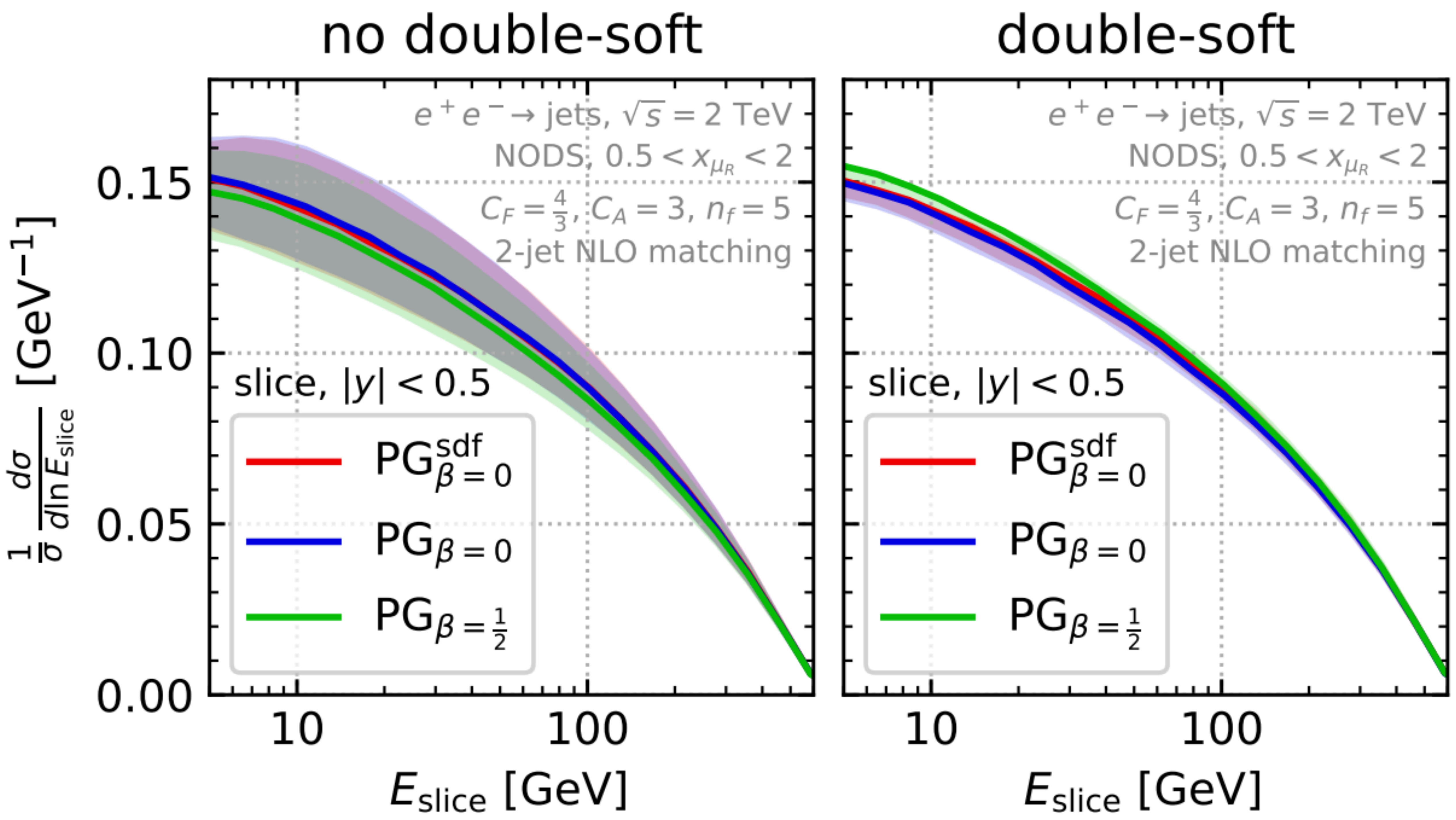
Using generalized parton shower to generate fixed order corrections



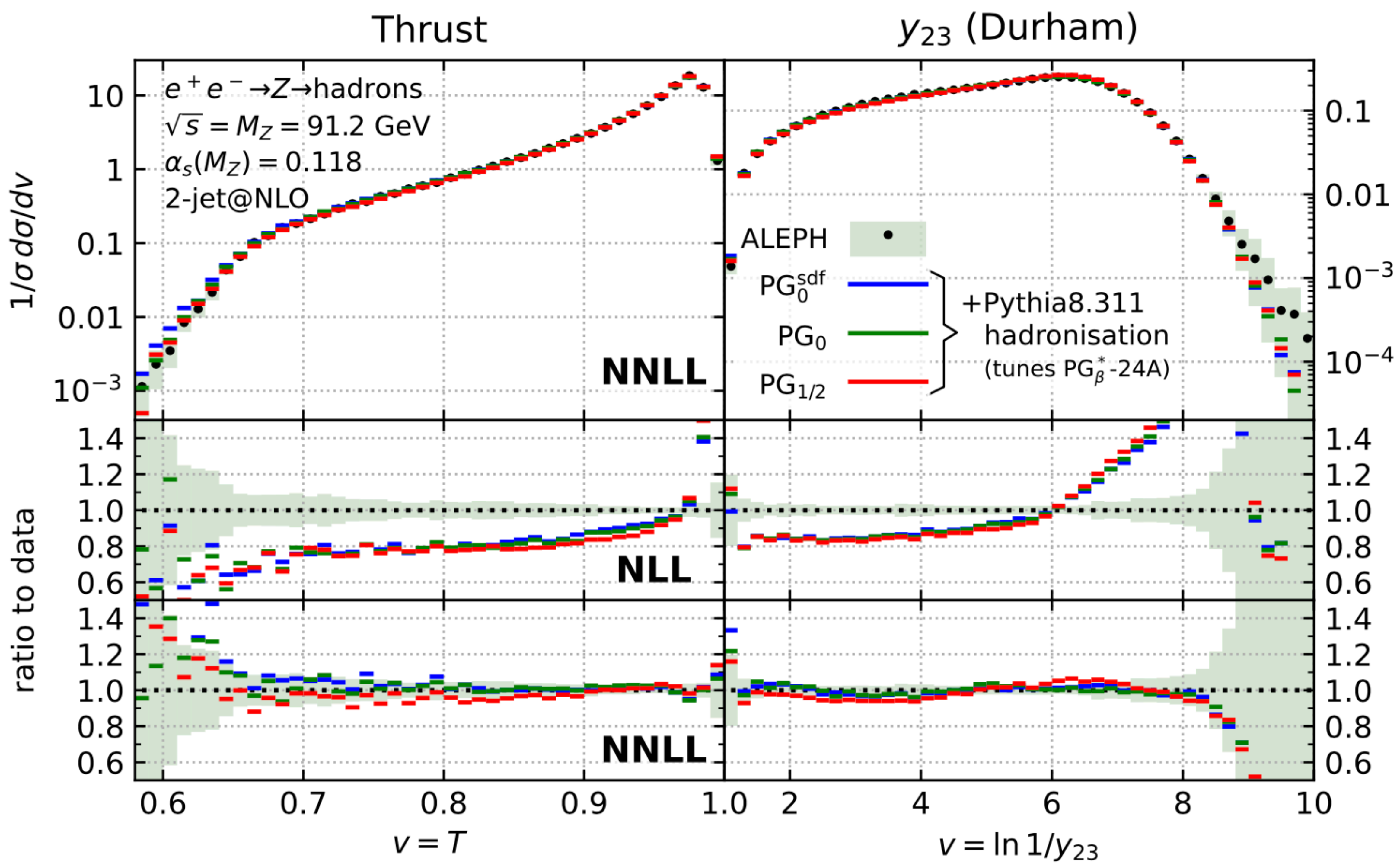
Altmann, HTL, Scyboz, Skands, arXiv:2507.00111

26

2. Parton Showers



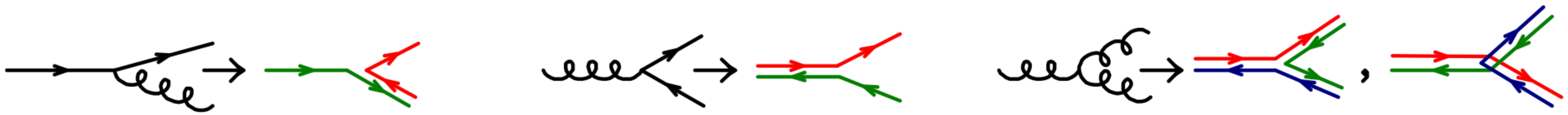
Ravasio et al arXiv:2307.11142



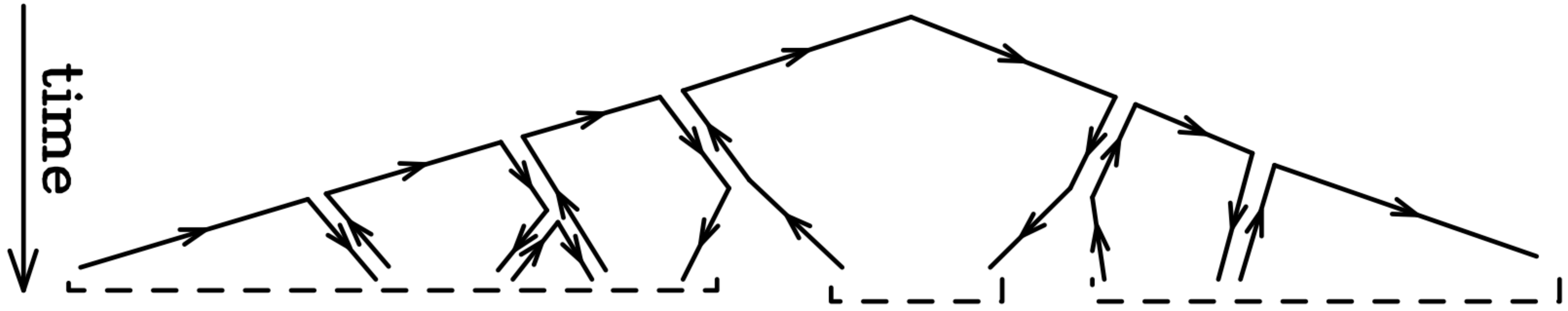
Beekveld et al arXiv:2406.02661

2. Parton Showers

Leading Color Approximation: Dipole Shower

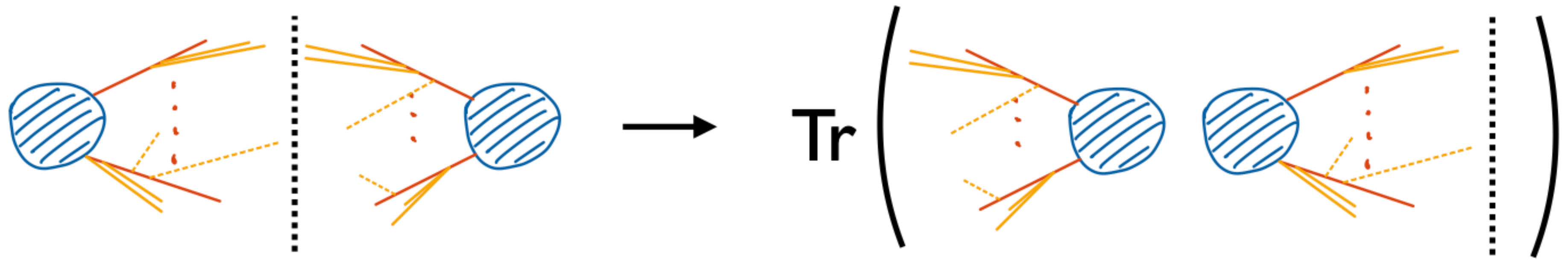


QCD radiation in this approximation is always simulated as the radiation from a single color dipole, rather than a coherent sum from a color multipole.



a color density operator   *Deductor, arXiv:1902.02105*

simulates parton showers at the amplitude level with full color information   *CVolver, arXiv:2502.12133*



$$\mathbf{A}_n(E) = \mathbf{V}_{E,E_n}\mathbf{D}_n^{\mu}\mathbf{A}_{n-1}(E_n)\mathbf{D}^{\dagger}_{n\mu}\mathbf{V}^{\dagger}_{E,E_n}\Theta(E \leq E_n),$$

3. Hadronization

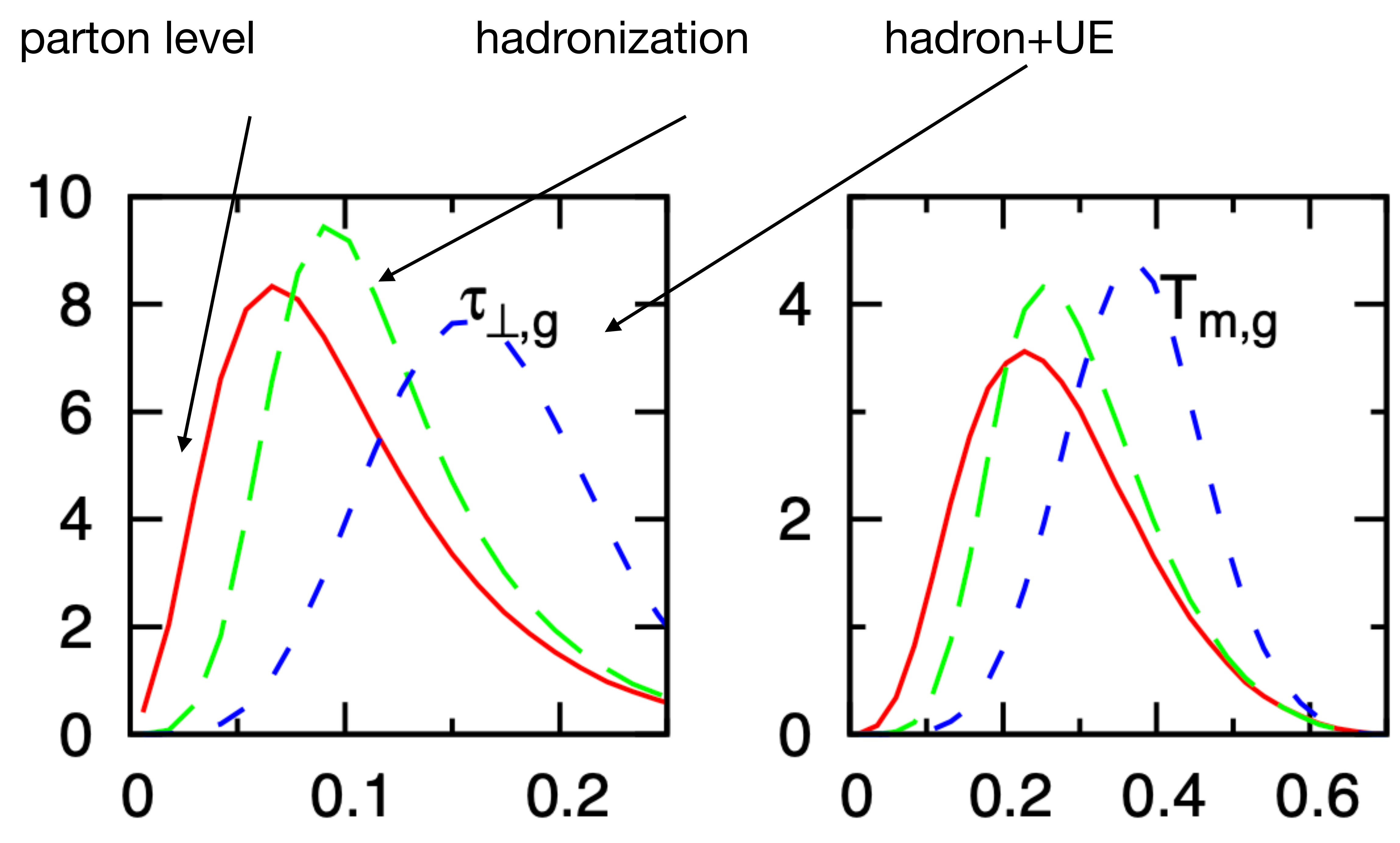
hadronization effects

27	-2	(ubar)	-71	17	17	28	29	0	103	-4.526	1.098	4.097	6.212	0.330
28	3322	(Xi0)	-83	26	27	46	47	0	0	-16.278	-0.490	18.555	24.723	1.315
29	-3222	(Sigmabar-)	-84	26	27	48	49	0	0	-7.403	1.034	7.932	10.963	1.189
30	2	(u)	-71	22	22	36	45	101	0	-5.718	1.277	7.107	9.216	0.330
31	21	(g)	-71	21	21	36	45	107	101	-0.390	0.609	0.444	0.849	0.000
32	21	(g)	-71	23	23	36	45	110	107	3.597	-0.501	-4.105	5.481	0.000
33	21	(g)	-71	24	24	36	45	106	110	1.334	0.455	-1.320	1.932	0.000
34	21	(g)	-71	25	25	36	45	105	106	7.964	-0.514	-7.933	11.253	0.000
35	-3	(sbar)	-71	11	11	36	45	0	105	16.893	-1.870	-20.679	26.772	0.500
36	111	(pi0)	-83	30	35	50	51	0	0	-3.511	0.738	4.002	5.377	0.135
37	211	pi+	83	30	35	0	0	0	0	0.002	0.218	0.085	0.273	0.140
38	-211	pi-	83	30	35	0	0	0	0	-1.767	-0.071	2.475	3.045	0.140
39	211	pi+	83	30	35	0	0	0	0	-0.182	0.285	0.651	0.747	0.140
40	-211	pi-	83	30	35	0	0	0	0	0.016	0.232	0.209	0.342	0.140
41	211	pi+	83	30	35	0	0	0	0	-0.413	0.450	-0.145	0.643	0.140
42	-211	pi-	84	30	35	0	0	0	0	2.478	-0.473	-2.622	3.642	0.140
43	2212	p+	84	30	35	0	0	0	0	6.374	-0.009	-6.640	9.252	0.938
44	111	(pi0)	-84	30	35	52	53	0	0	0.270	0.111	-0.364	0.486	0.135
45	-3122	(Lambdabar0)	-84	30	35	54	55	0	0	20.414	-2.024	-24.136	31.696	1.116
46	3122	(Lambda0)	-91	28	0	56	57	0	0	-14.222	-0.534	16.090	21.510	1.116
47	111	(pi0)	-91	28	0	58	59	0	0	-2.056	0.043	2.465	3.213	0.135
48	-2112	nbar0	91	29	0	0	0	0	0	-5.613	0.671	6.203	8.445	0.940
49	-211	pi-	91	29	0	0	0	0	0	-1.790	0.363	1.728	2.518	0.140
50	22	gamma	91	36	0	0	0	0	0	-3.222	0.667	3.613	4.887	0.000
51	22	gamma	91	36	0	0	0	0	0	-0.289	0.071	0.388	0.490	0.000
52	22	gamma	91	44	0	0	0	0	0	0.028	-0.020	-0.008	0.036	0.000
53	22	gamma	91	44	0	0	0	0	0	0.242	0.131	-0.356	0.450	0.000
54	-2212	pbar-	91	45	0	0	0	0	0	18.123	-1.732	-21.512	28.198	0.938
55	211	pi+	91	45	0	0	0	0	0	2.291	-0.292	-2.624	3.498	0.140
56	2212	p+	91	46	0	0	0	0	0	-10.893	-0.393	12.398	16.535	0.938
57	-211	pi-	91	46	0	0	0	0	0	-3.329	-0.140	3.692	4.975	0.140
58	22	gamma	91	47	0	0	0	0	0	-0.678	0.003	0.911	1.136	0.000
59	22	gamma	91	47	0	0	0	0	0	-1.378	0.040	1.554	2.077	0.000

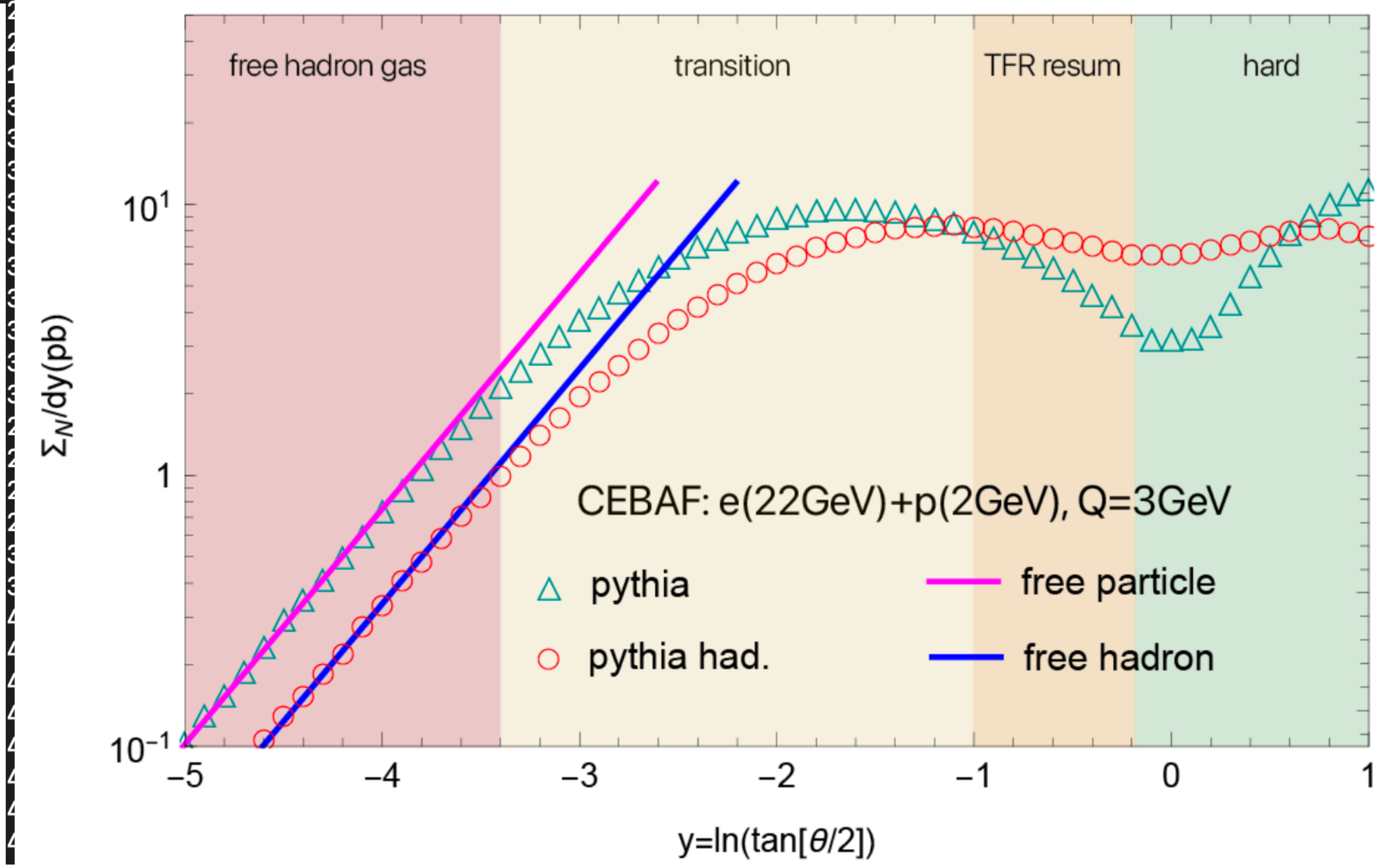
3. Hadronization

hadronization effects

```
27        -2  (ubar)         -71   17   17   28   29    0  103    -4.526    1.098    4.097    6.212    0.330
28      3322  (Xi0)          -83   26   27   46   47    0    0   -16.278   -0.490   18.555   24.723    1.315
29     -3222  (Sigmabar-)    -84   26   27   48   49    0    0    -7.403    1.034    7.932   10.963    1.189
30         2  (u)            -71   22   22   36   45  101    0    -5.718    1.277    7.107    9.216    0.330
31        21  (g)            -71   21   21   36   45  107  101    -0.390    0.609    0.444    0.849    0.000
32        21  (g)            -71   23   23   36   45  110  107     3.597   -0.501   -4.105    5.481    0.000
```

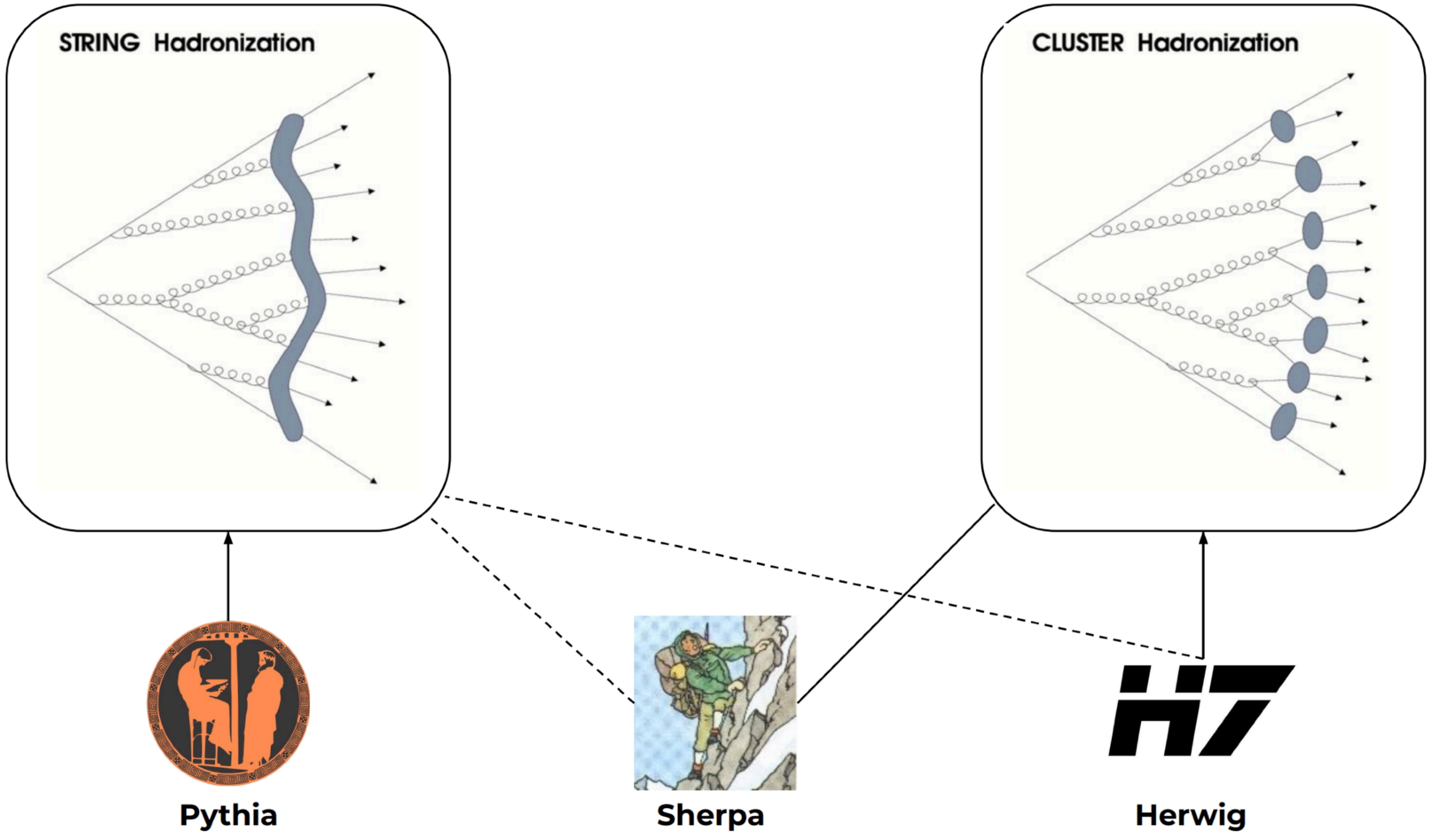


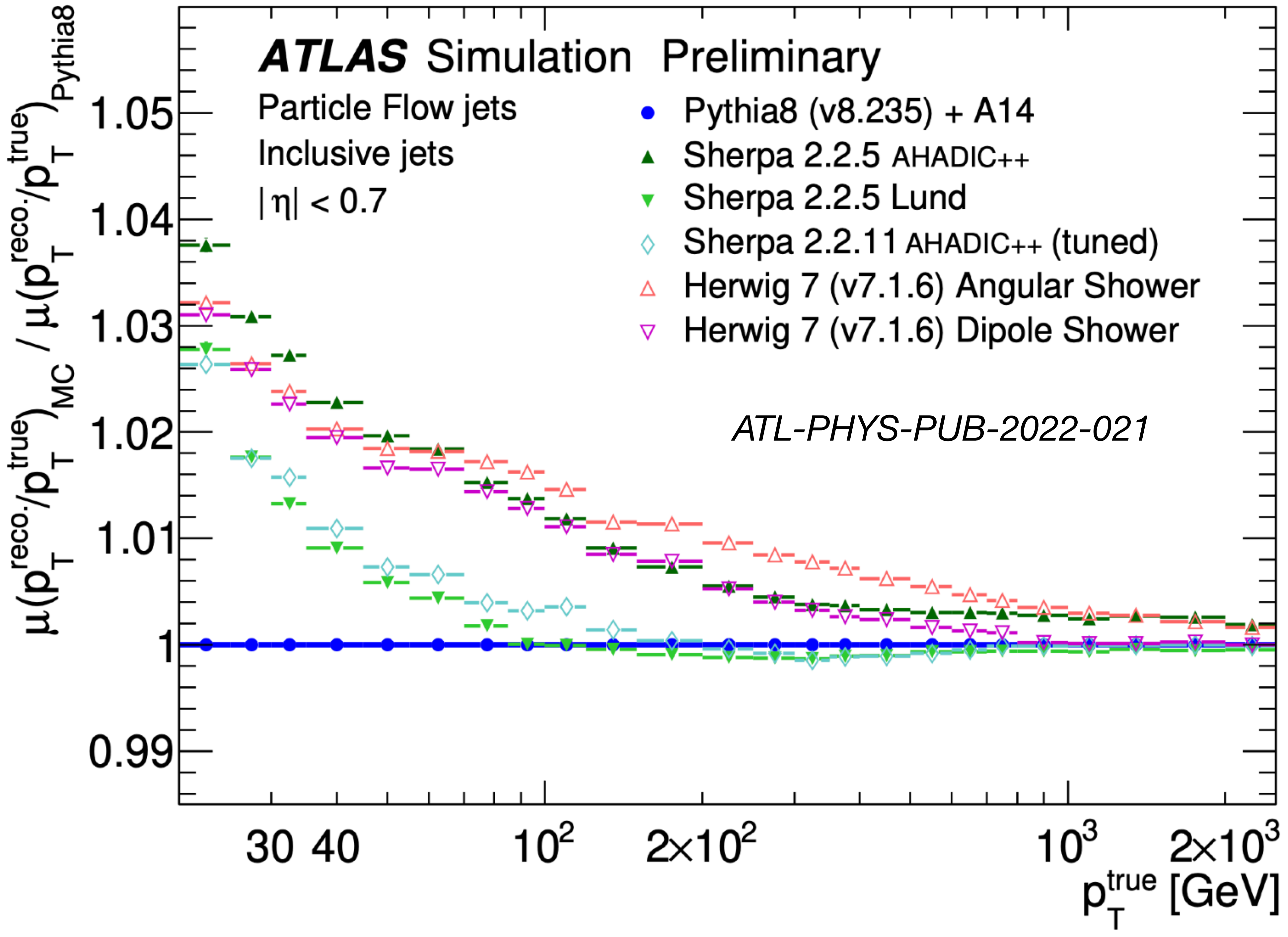
Banfi, Salam and Zanderighi arXiv:1001.4082



Cao, HTL, Mi, arXiv:2312.07655

3. Hadronization

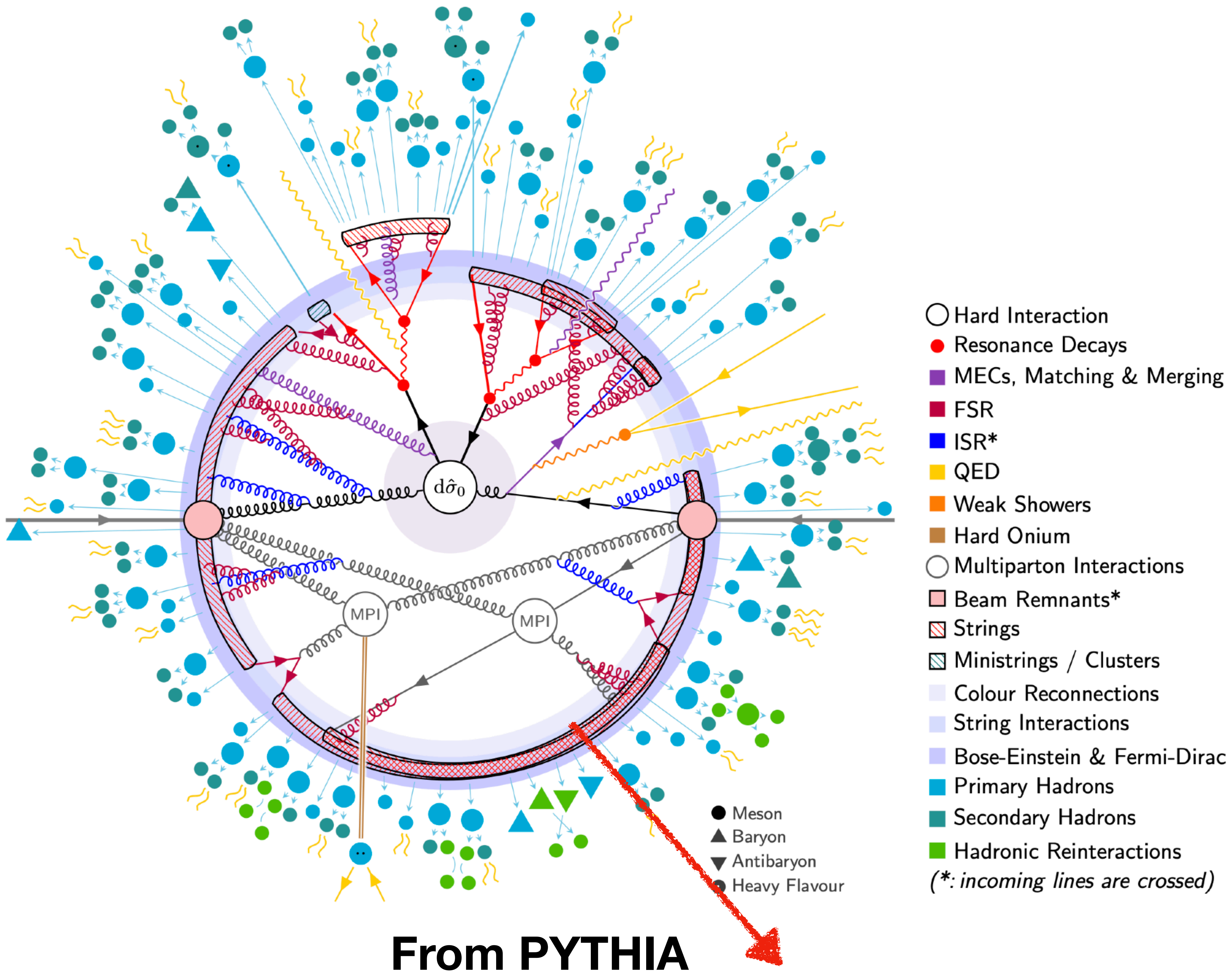




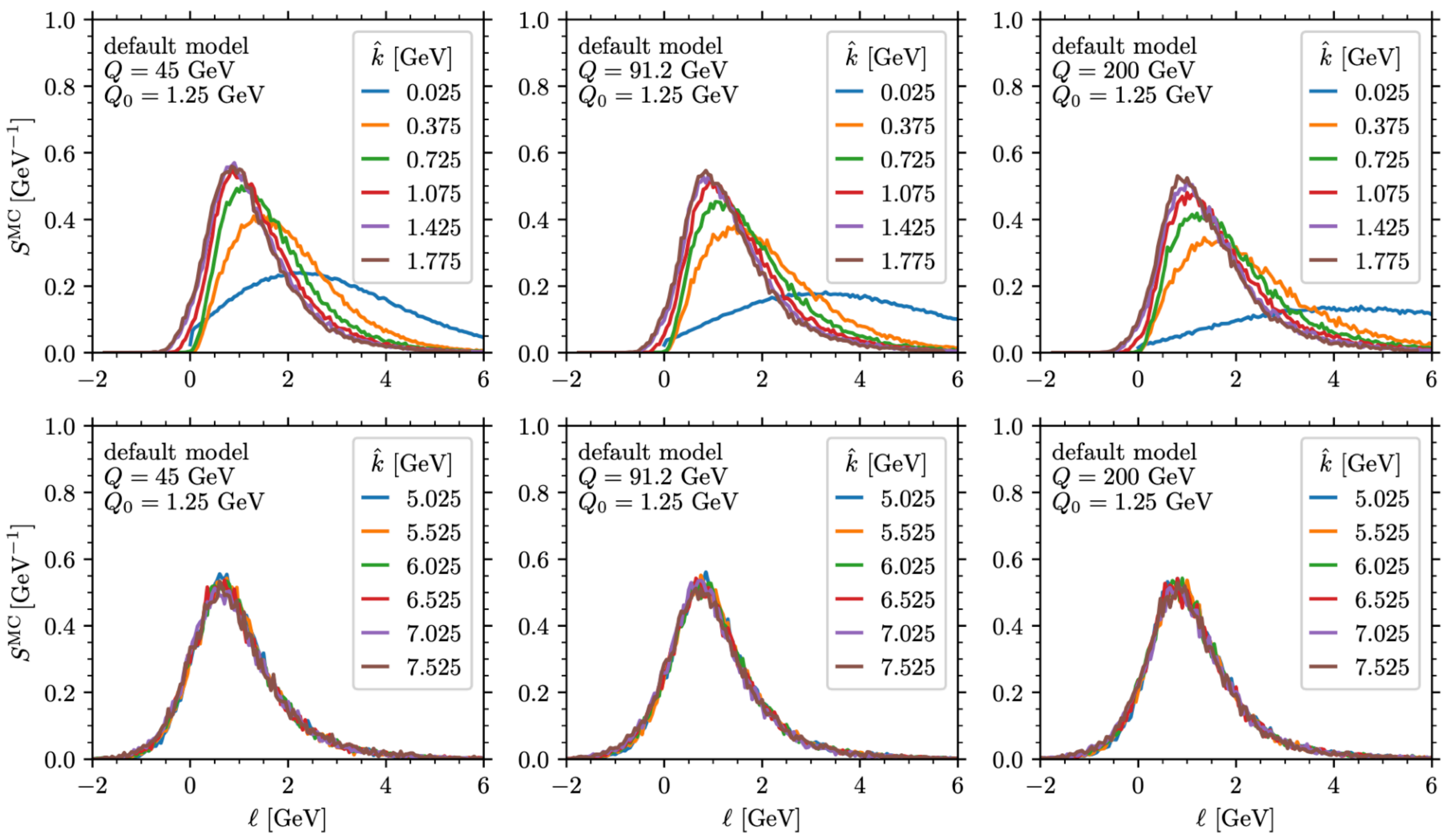
differences between

different models and tunes

3. Hadronization



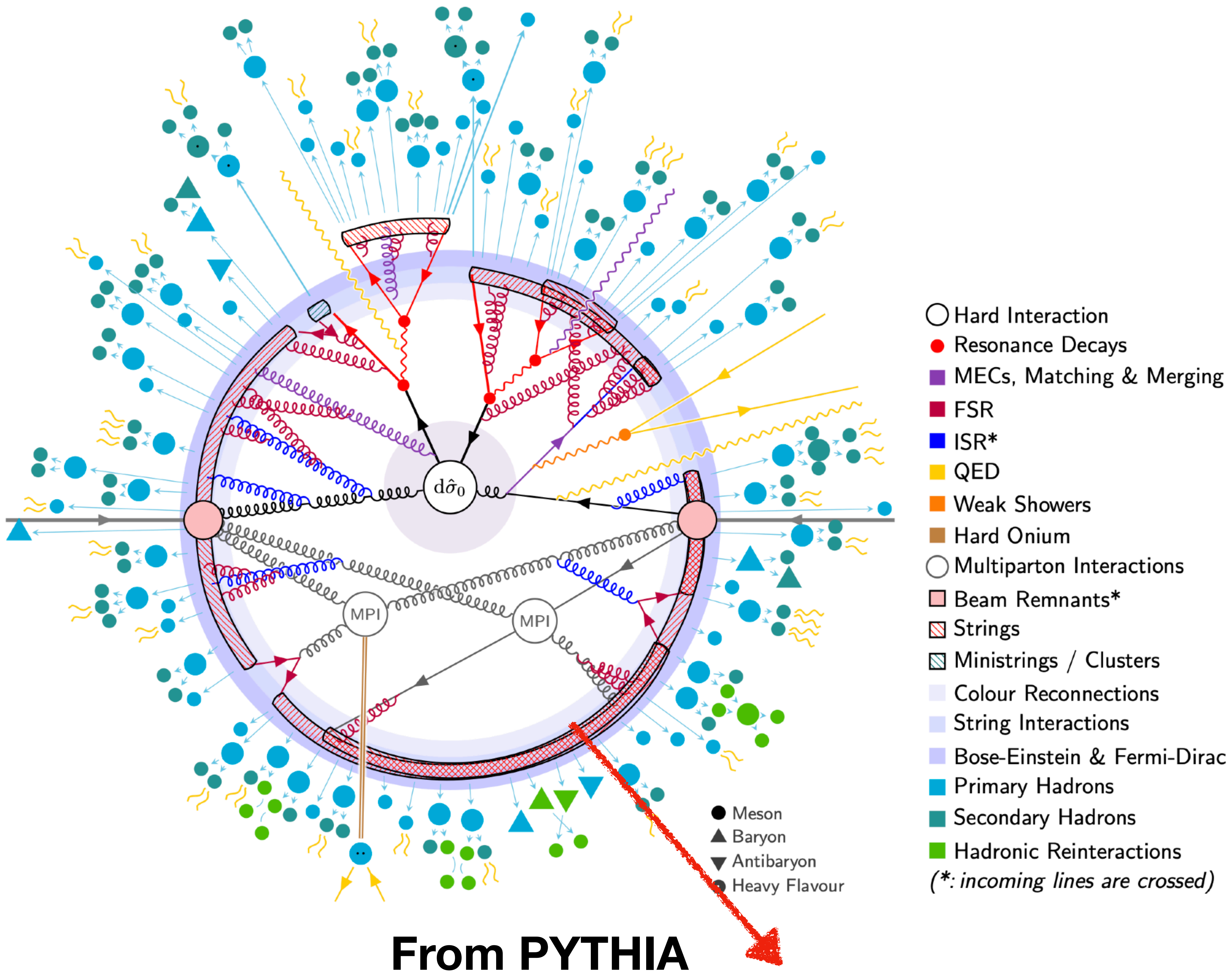
From PYTHIA



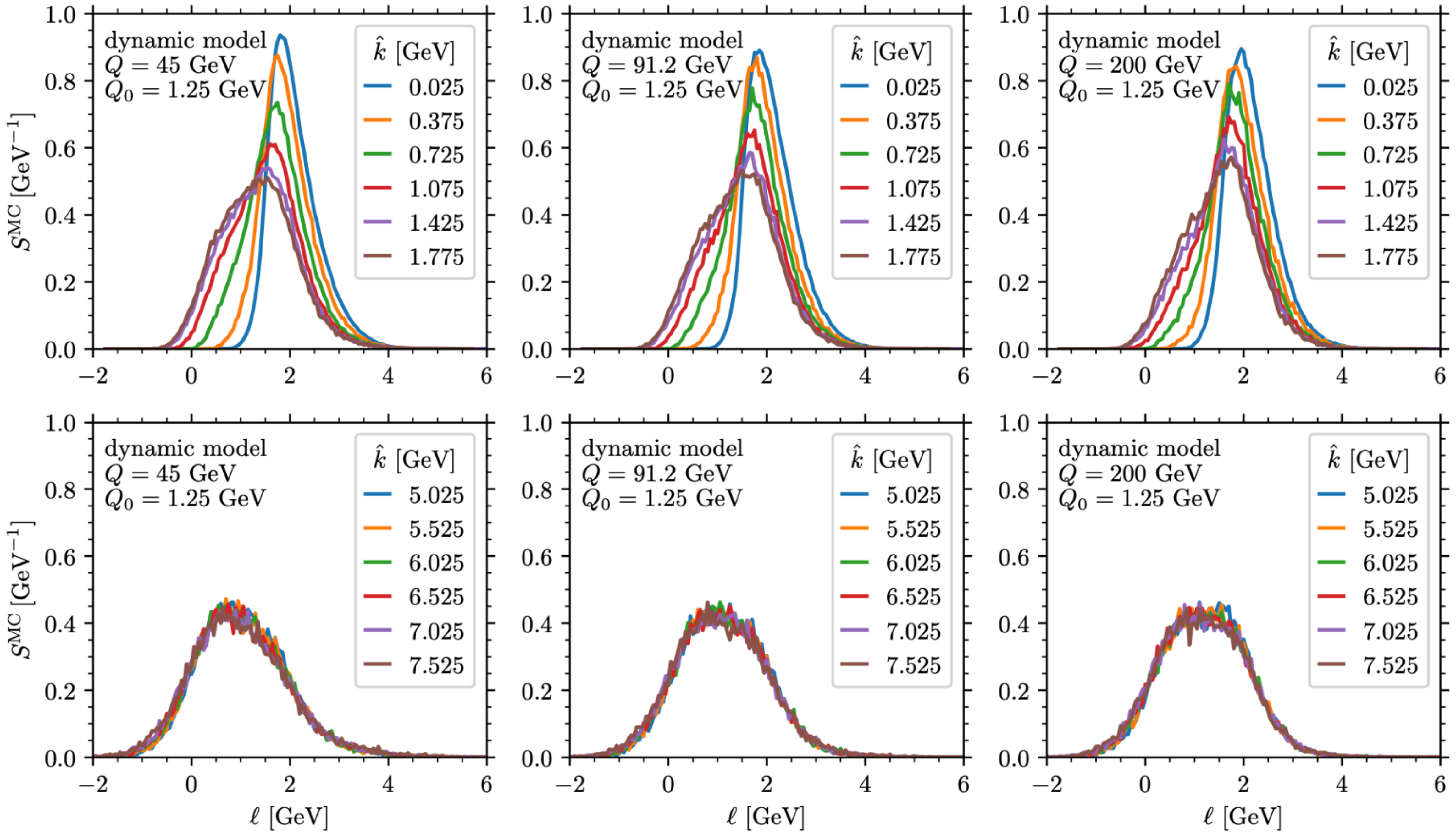
Physics should be independent on the transition scales

Matching the evolution of the perturbative evolution with hadronization arXiv:2404.09856

3. Hadronization



From PYTHIA

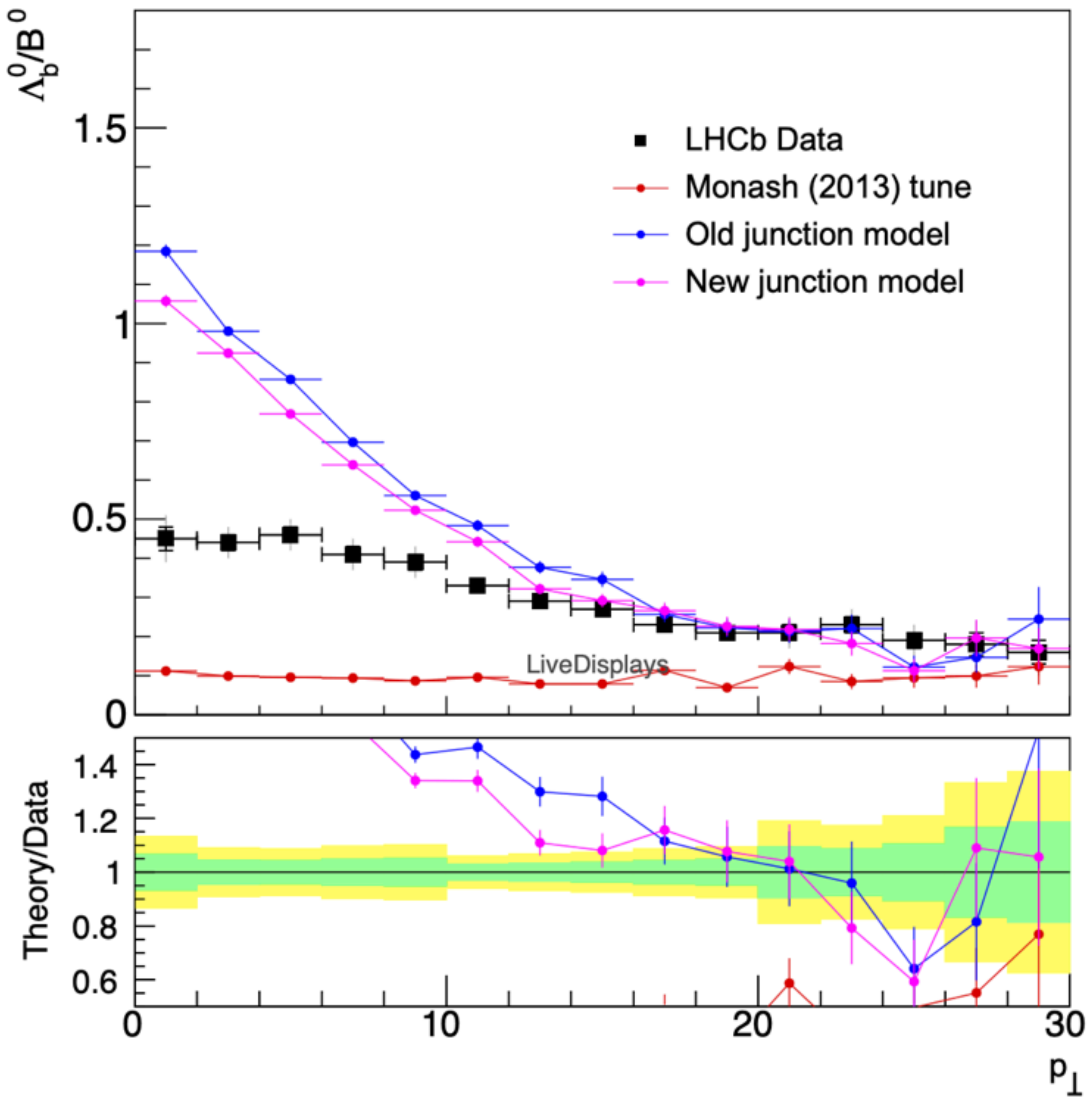


Physics should be independent on the transition scales

Matching the evolution of the perturbative evolution with hadronization arXiv:2404.09856

4. Summary

- MCEGs are essential computational tools for experimentalists and theorists
- Starting from hard processes to generate the perturbative and nonperturbative QCD radiations
- Recently, a lot progresses on improving the logarithmic resummation order of Parton Showers
- Also, subleading color effects are discussed
- Hadronization model, multiple parton interactions (MPI), and underlying event descriptions introduce uncertainties



Altmann, Skands, 2404.12040

4. Summary

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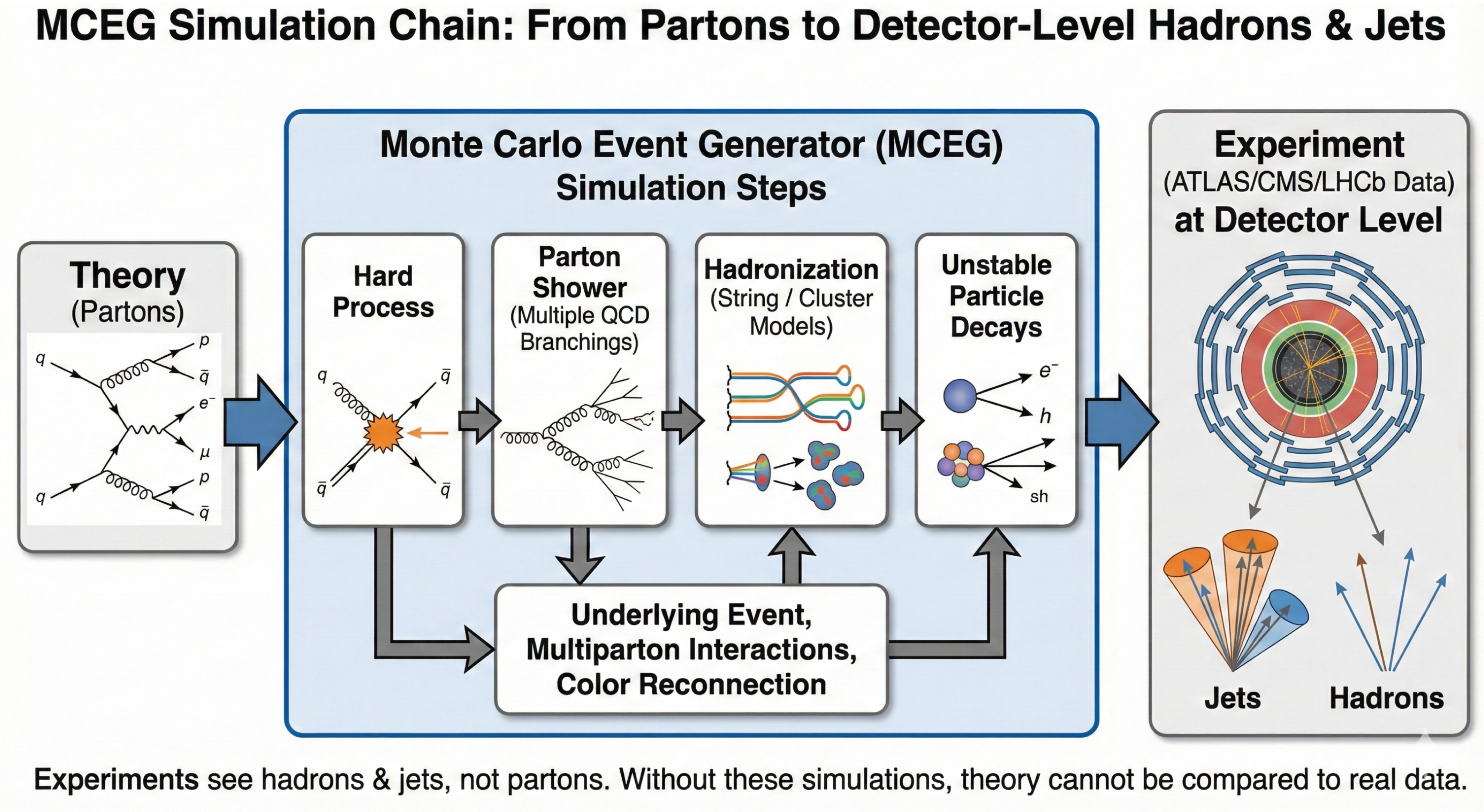

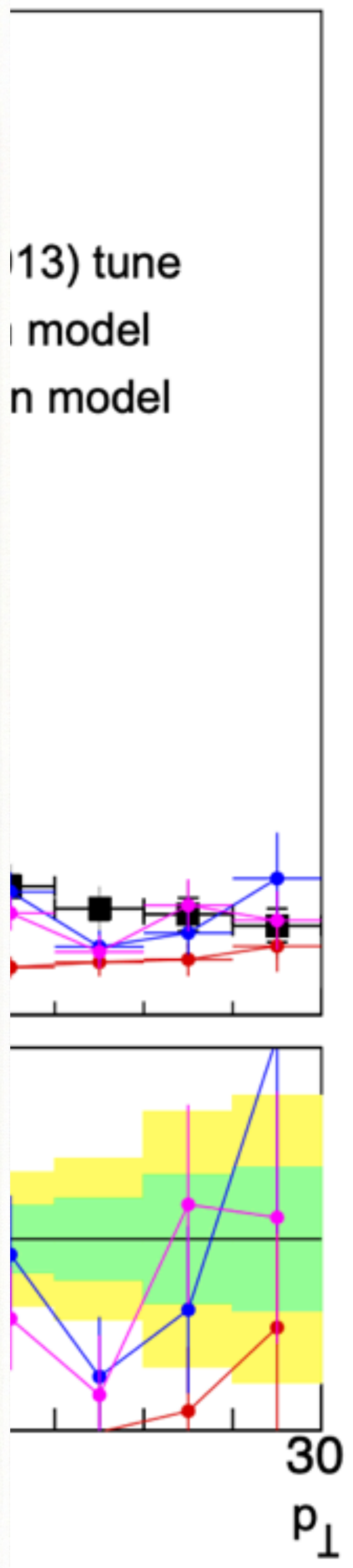

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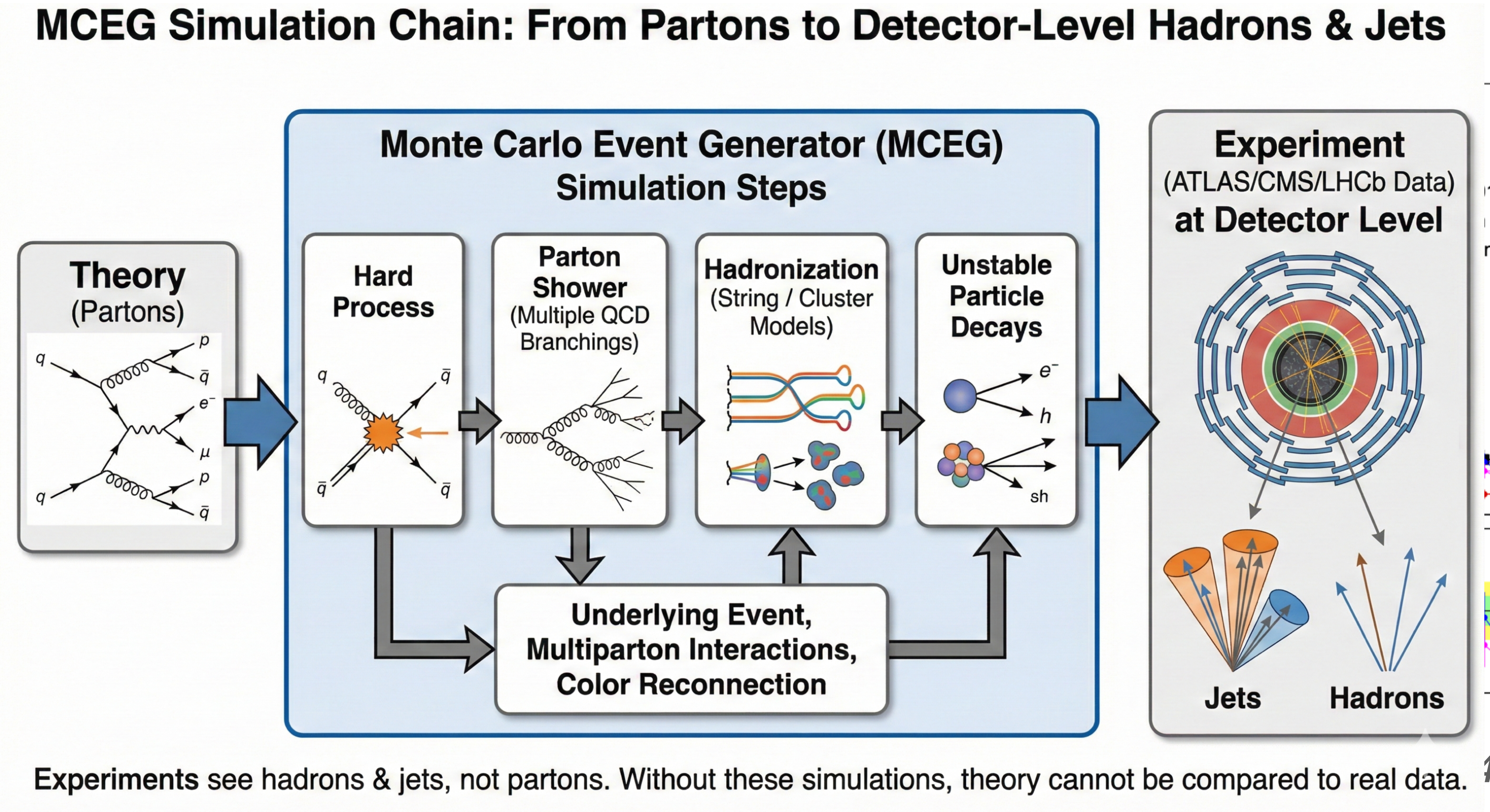

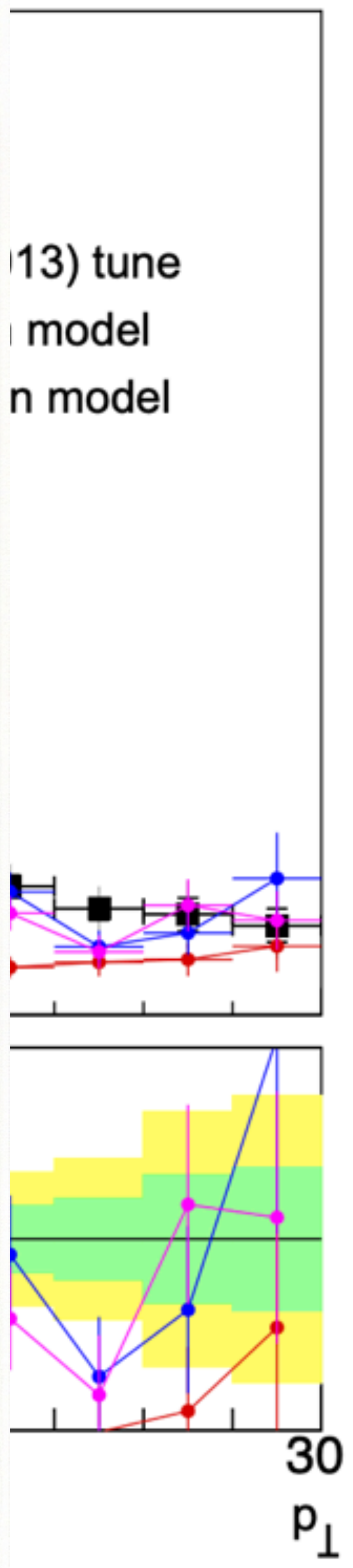

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Thank you!