ACAT 2013, Track 3 (Part II)

Andrej Arbuzov (JINR Dubna)

I. Loop integral calculations

1) **J. Bluemlein**: *Harmonic, generalized harmonic, cyclotomic and binomial sums, polylogarithms and special numbers*
2) **T. Riemann**: *Contractions of 1-loop 5-point tensor Feynman integrals*
3) **S. Badger**: *Multi-loop integrand reduction with computational algebraic geometry*
4) **N. Watanabe**: *One loop integration with hypergeometric series by using recursion relation*
5) **R. Lee**: *LiteRed: a new powerful tool for the reduction of multiloop integrals*

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II. Applications:

6) **M. Kompaniets**: *Theory of phase transitions and critical phenomena: new approach to numerical calculation of anomalous dimensions*
7) **A. Bednyakov**: *Three-loop beta-functions and anomalous dimensions in the SM*
8) **V. Khandramai**: *FAPT: a Mathematica package for calculations in QCD Fractional Analytic Perturbation Theory*
9) **R. Sadykov**: *SANC system and its application for LHC*
10) **V. Zykunov**: *Radiative (QCD and Electroweak) Corrections to Drell–Yan Process for Experiments at the LHC*
11) **F. Feng**: *Automated one-loop computation in quarkonium process within NRQCD framework*
12) **X.-G. Wu**: *Generators BCVEGPY and GENXICC for doubly heavy mesons and baryons*

I. Loop integral calculations

1) J. Bluemlein: *Harmonic, Generalized Harmonic, Cyclotomic...*

Mathematical basis of special functions and numbers appearing in loop integral calculations

Harmonic, Generalized Harmonic, Cyclotomic and Binomial Sums, Polylogarithms and Special Numbers

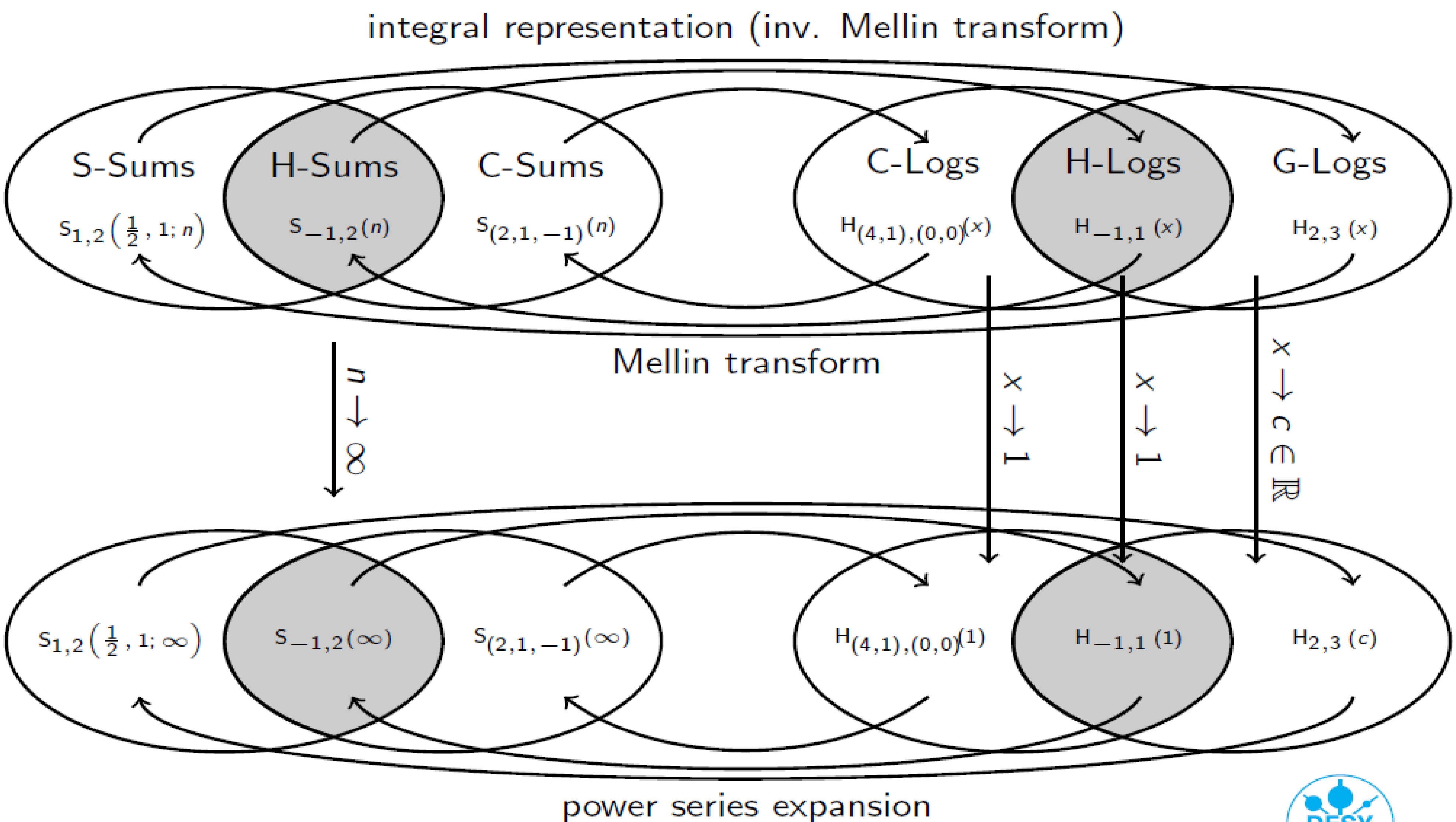
Johannes Blümlein
DESY

in collaboration with Jakob Ablinger and Carsten Schneider
RISC JKU Linz

[J. Math. Phys. 52 (2011) 102301, arXiv:1105.6063]
[arXiv:1302.0378, J. Math. Phys. to appear.]
[arXiv:1304.7071]

I. Loop integral calculations

1) J. Bluemlein: *Harmonic, Generalized Harmonic, Cyclotomic...*



I. Loop integral calculations

1) J. Bluemlein: *Harmonic, Generalized Harmonic, Cyclotomic...*

Harmonic sums: (Vermaseren; JB, Kurth, 1998)

$$S_{b,\vec{a}}(N) = \sum_{k=1}^{N} \frac{(\mathrm{sign}(b))^k}{k^{|b|}} S_{\vec{a}}(k)\ , \qquad S_{\emptyset}(N) = 1\ , \qquad b, a_i \in \mathbb{Z}\backslash\{0\}.$$

Generalized harmonic sums: (Moch, Uwer, Weinzierl, 2001; Ablinger, JB, Schneider, 2013)

$$S_{b,\vec{a}}(\zeta, \vec{\xi}; N) = \sum_{k=1}^{N} \frac{\zeta^k}{k^b} S_{\vec{a}}(\vec{\xi}; k)\ , \quad b, a_i \in \mathbb{N}_+; \zeta, \xi_i \in \mathbb{R}^* = \mathbb{R}\backslash\{0\}$$

Important:

Algebraic Relations: (JB, 2003)

All these objects form stuffle (Sums) and shuffle (Iter. Integrals) Algebras. The special numbers obey both algebras.

Structural Relations: (JB, 2009)

Specific relations between these quantities

I. Loop integral calculations

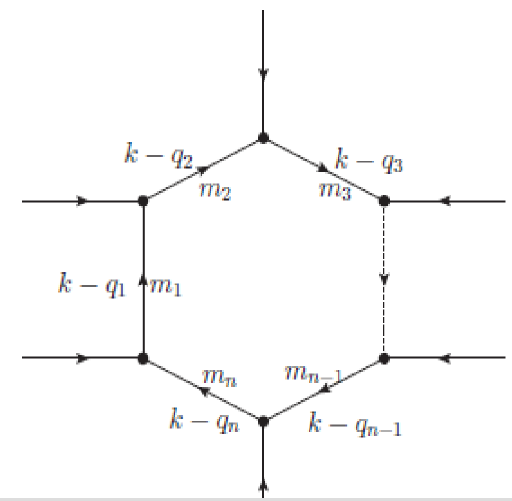
2) T. Riemann: *Contractions of 1-loop 5-point tensor Feynman integrals*

Tord Riemann DESY, Zeuthen, Germany

in cooperation with **J. Fleischer** (1937-2013), **J. Gluza, A. Almasy**

n-point tensor integrals of rank *R*: (n,R)-integrals

$$I_n^{\mu_1\cdots\mu_R} = \int \frac{d^dk}{i\pi^{d/2}} \frac{\prod_{r=1}^R k^{\mu_r}}{\prod_{j=1}^n c_j^{\nu_j}},$$



~~Open source~~ Source-open programs for 5,6-point reductions:

- LoopTools/FF ($n \leq 5$, $rank \leq 4$), T. Hahn [2, 3] 1998,1990.
- Golem95 T. Binoth et al. [4] 2008
- PJFry ($n \leq 5$, $rank \leq 5$), V. Yundin et al. [5, 6] July 2011

I. Loop integral calculations

2) T. Riemann: *Contractions of 1-loop 5-point tensor Feynman integrals*

This talk: Efficient reduction formulae in the algebraic Davydychev-Tarasov-Fleischer-Jegerlehner-TR approach

- Get $n > 4$ tensor reduction with $\cdots$:
- $\cdots$ arbitrary masses
- $\cdots$ inverse pentagon Gram determinants killed
- $\cdots$ full kinematics treated, also with small inverse sub-diagram Gram determinants

- **new:** $\cdots$ multiple sums over tensor coefficients made efficient by **contracting with external momenta**

  Fleischer, TR [10] PLB 701(2011)646 + further simplifications

  Programs:

  OLEC (C++), J. Fleischer, J. Gluza, M. Gluza, TR [11]

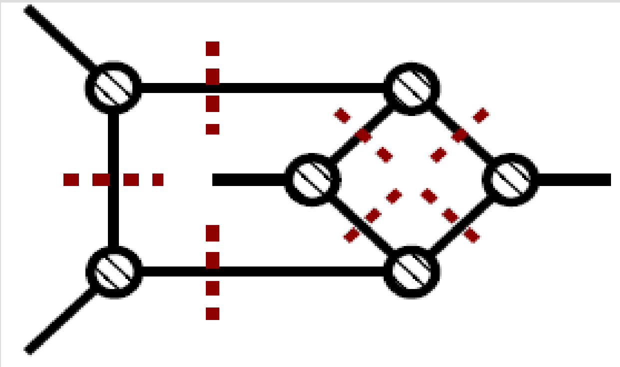
  CONTRACTIONS (F95), Andrea Almasy, J. Fleischer, TR [12]

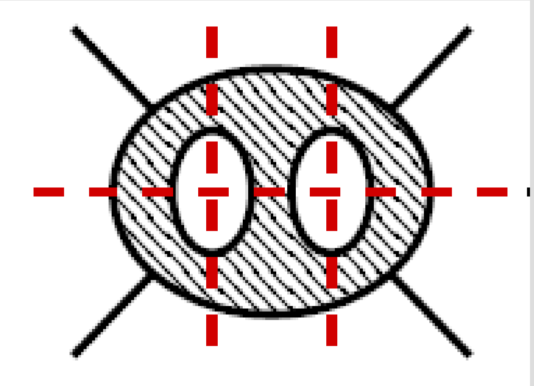
- $\cdots$ higher n point functions, $n \geq 7$ Fleischer, TR [13] PLB 707(2012)375

  Programs: to be done

I. Loop integral calculations

3) S. Badger: *Multi-Loop Integrand Reduction with Computational Algebraic Geometry*





Simon Badger (NBIA & Discovery Center)

Generalized Unitarity Cuts

Algorithm to fit a generic integrand

- Parametrize the full set of on-shell solutions, $l_i^{(s)}(\tau_1, \ldots, \tau_p)$
- Identify the ISPs on each solution:

$$k_i \cdot p_j = f_{ij}(\tau_1, \ldots, \tau_p)$$

- Construct and solve the resulting linear system:

$$\Delta^{(s)}(\tau_1, \ldots, \tau_p) = \sum d_a \tau_1 \ldots \tau_p$$

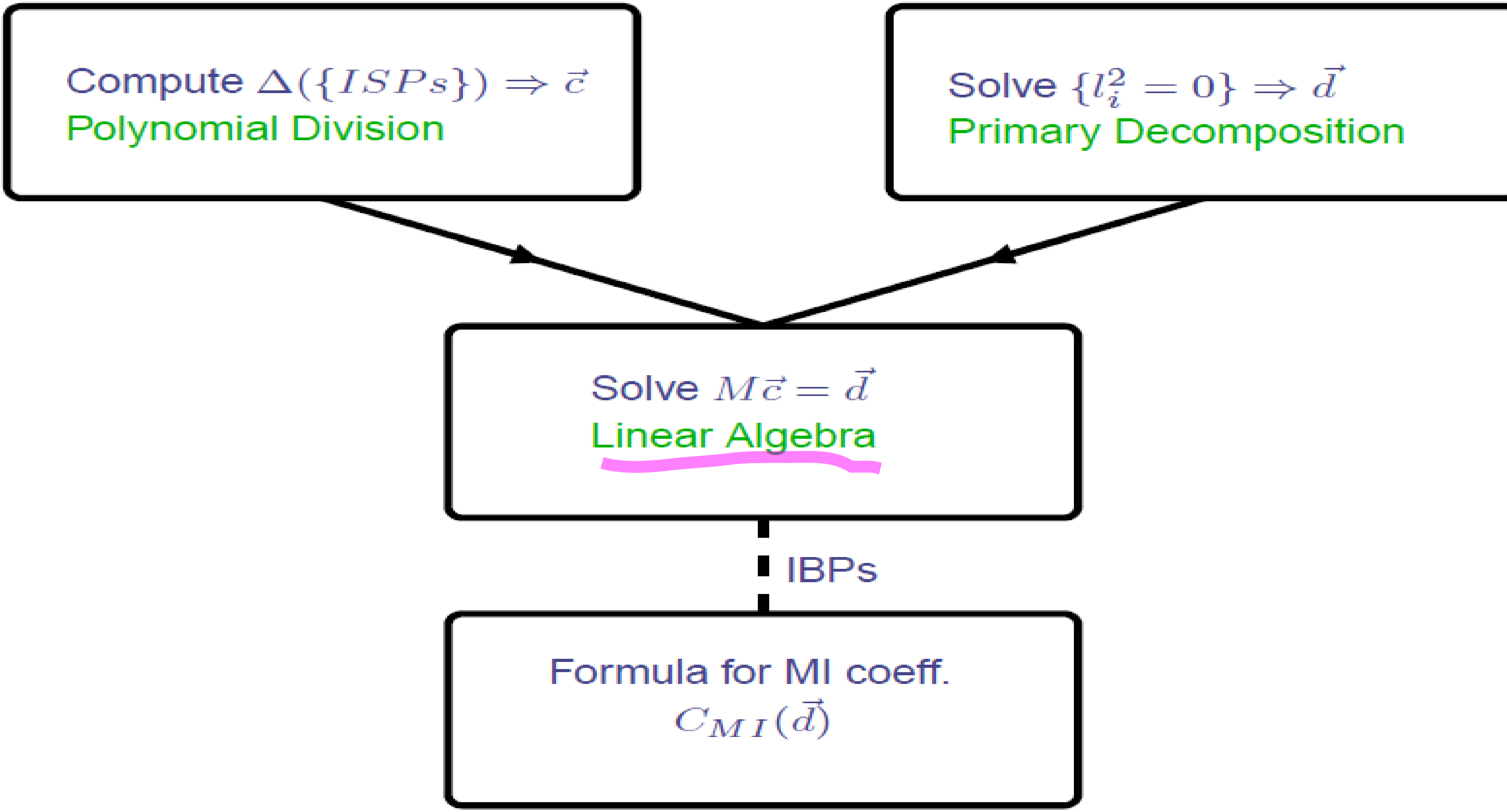
Public Mathematica code BasisDet

[http://www.nbi.dk/~zhang/BasisDet.html]

I. Loop integral calculations

3) S. Badger: *Multi-Loop Integrand Reduction with Computational Algebraic Geometry*

Integrand Reduction Procedure



I. Loop integral calculations

4) N. Watanabe: *One loop integration with hypergeometric series by using recursion relation* Geometry

Norihisa Watanabe(KEK)

Collaborated with T.Kaneko(KEK)

The one loop n-point functions are exactly expressed in terms of some set of hypergeometric functions.

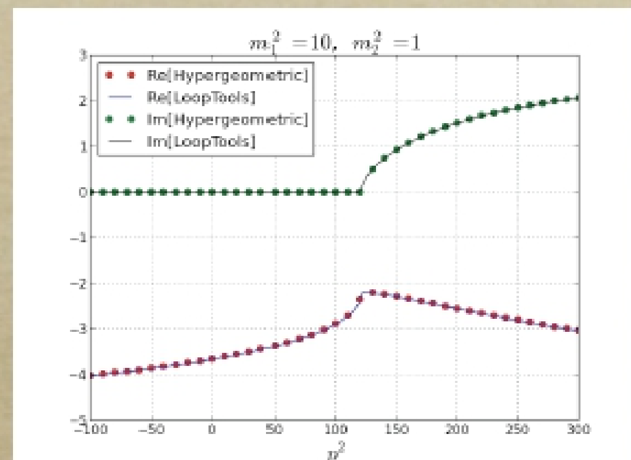
- Representation with hypergeometric functions
  - Regge(1969,a class of generalized hypergeometric equations)
  - Tarasov et al., Davydychev, Kalmykov,...(1-, 2-loop, ...)
  - Duplančić and Nižić, Kurihara, (1-loop, for massless QCD)

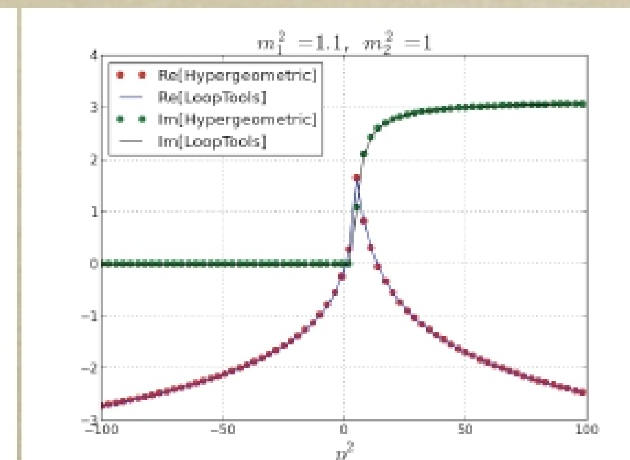
I. Loop integral calculations

4) N. Watanabe: *One loop integration with hypergeometric series by using recursion relation Geometry*

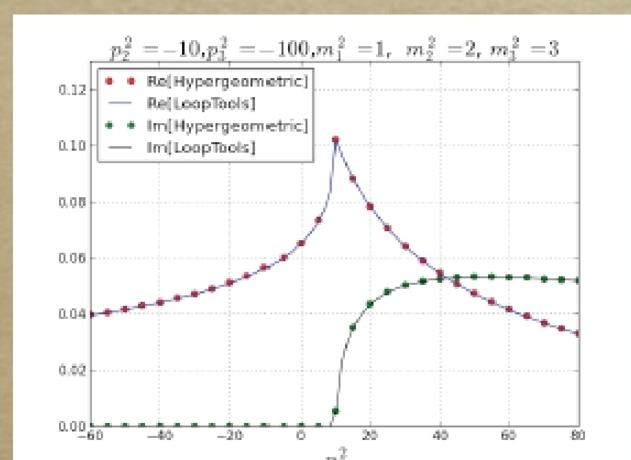
- Results are compared with LoopTools T. Hahn-M.Perez-Victoria

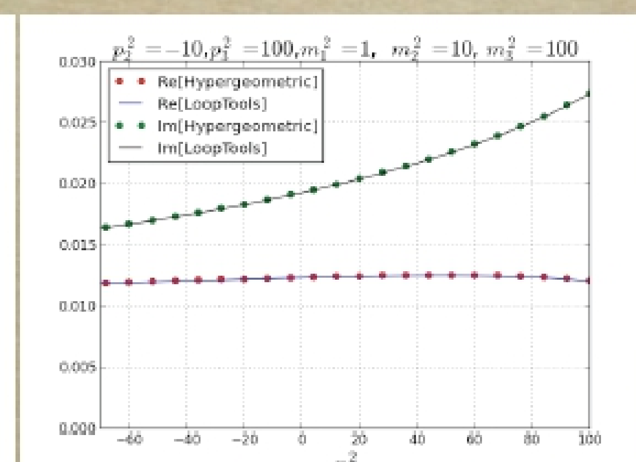
2-pt func.





3-pt func.





4-pt func. is not compared yet.

I. Loop integral calculations

4) N. Watanabe: *One loop integration with hypergeometric series by using recursion relation* Geometry

- The general one loop integral with any physical parameter are written in a linear combination of Aomoto-Gelfand hypergeometric functions

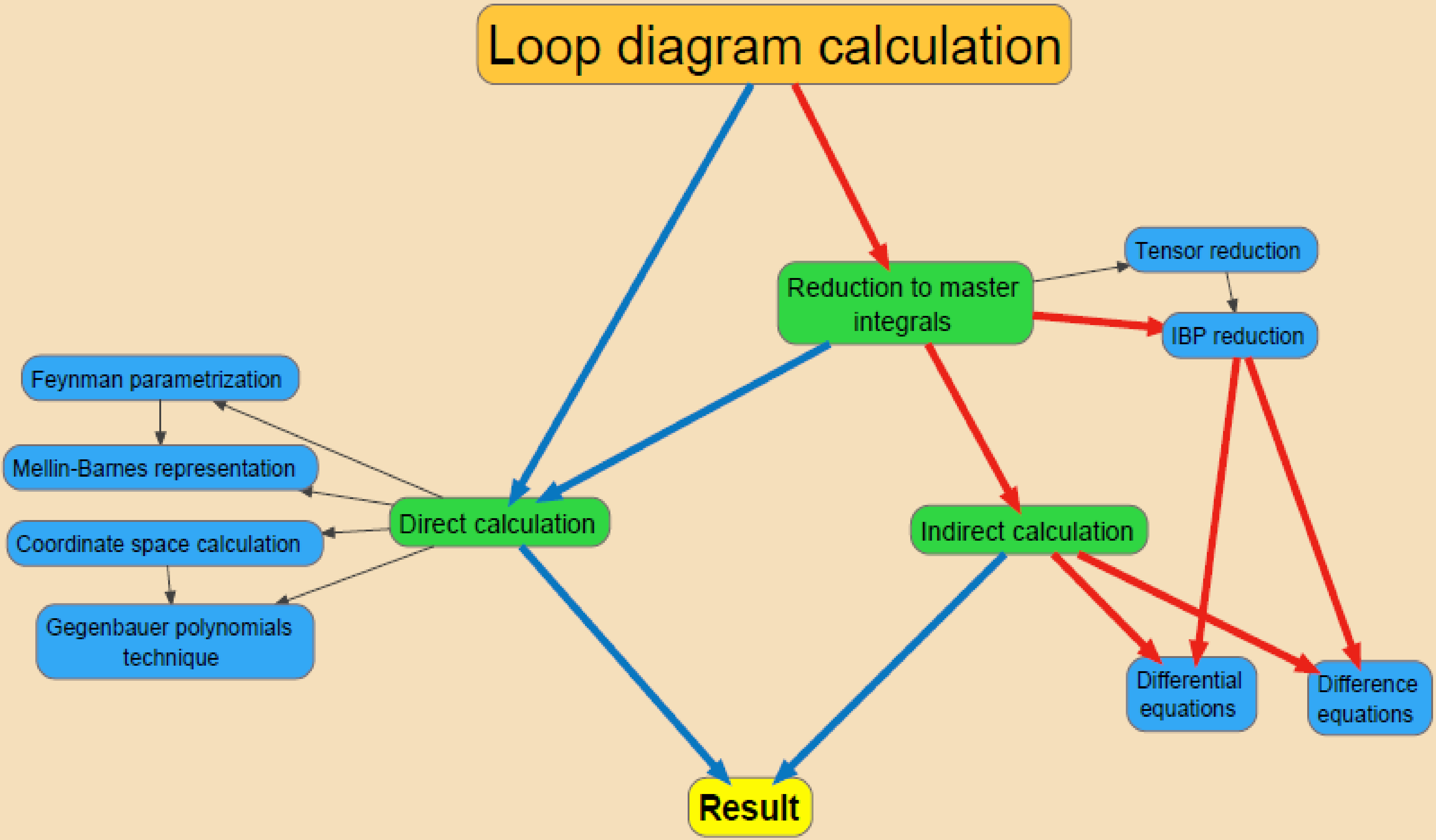
- Many identities of hypergeometric funcs. are developed
- Our method is consistent with well-known results.

Outlook

- The general expressions for n≧4-point functions are constructed
- Extend this method for massless QCD loop with IR singularity.

I. Loop integral calculations

5) R. Lee: *LiteRed: a new powerful tool for the reduction of multiloop integrals*



I. Loop integral calculations

5) R. Lee: *LiteRed: a new powerful tool for the reduction of multiloop integrals*

Relations between the integrals

The **symmetry relations** arise from the shifts of loop momenta which map denominators to denominators.

The **integration-by-part identities** arise due to the fact, that, in dimensional regularization the integral of the total derivative is zero (Tkachov 1981, Chetyrkin and Tkachov 1981)

The **Lorentz-invariance identities** arise due to the fact that loop integrals are scalar functions of the external momenta (Gehrmann and Remiddi 2000).

Important fact!

Reduction not only reduces the number of integrals to be calculated. It also allows one to obtain for the master integrals the closed systems of equations: differential and/or difference. Solving these equations is often simpler then the direct integration.

I. Loop integral calculations

5) R. Lee: *LiteRed: a new powerful tool for the reduction of multiloop integrals*

LiteRed

One more reduction package?

Many reduction packages on the market: FIRE, Reduze, etc., why creating another?

- LiteRed is publicly available
- LiteRed implements a new approach to the reduction
- The results obtain in LiteRed can be used in other programs

II. Applications

6) M. Kompaniets: *Theory of phase transitions and critical phenomena: new approach to numerical calculation os anomalous dimensions*

L.Ts. Adzhemyan, D.V. Batkovich, M.V. Kompaniets, S.V. Novikov

Department of Theoretical Physics, Saint Petersburg State University

Critical dynamics

- speed of sound
- viscosity
- thermal conductivity coefficient
- diffusion coefficient

Models of critical dynamics are constructed on basis of

φ^4 model.

II. Applications

6) M. Kompaniets: *Theory of phase transitions and critical phenomena: new approach to numerical calculation os anomalous dimensions*

Renormalization group

Model φ^4

- $d = 4 - 2\epsilon \rightarrow$ 5-loop order (analytical)[2]
- $d = 2 \rightarrow$ 5-loop order (numerical)[3]
- $d = 3 \rightarrow$ 6-loop order (numerical)[4]

more loops are needed

Normalization point scheme

This scheme is in some sense intermediate between minimal subtraction(MS) scheme and subtraction at zero momenta (ZM)

II. Applications

6) M. Kompaniets: *Theory of phase transitions and critical phenomena: new approach to numerical calculation os anomalous dimensions*

We developed **fully automated** software that allows to calculate anomalous dimensions in wide range of models (e.g. ϕ^3, ϕ^4 models and models of critical dynamics)

(NEW!) Using this approach we calculated RG-functions for ϕ^3 model up to 4-loop order [5]

- $O(N)$-symmetric ϕ^4 model ($d = 4 - \epsilon$) in 5-loop order
  Results agree within 10^{-6} with analitical results[6]
- $O(N)$-symmetric ϕ^4 model ($d = 2$) in 5-loop order
  Results agree within 10^{-6} with results obtained by Orlov and Sokolov.[7]
- NEW! Preliminary results for $O(N)$-symmetric ϕ^4 model ($d = 2$) in 6-loop order
- NEW! Preliminary result for model A of critical dynamics in 4-loop order

II. Applications

7) A. Bednyakov: *Three-loop beta-functions and anomalous dimensions in the SM*

A.V. Bednyakov[1], A.F. Pikelner[1] and V.N. Velizhanin[2]

- The discovery of the Higgs boson "finalizes" the SM
- No clear experimental hints of New Physics

ArXiv: 1210.6873
ArXiv: 1212.6829
ArXiv: 1303.4364

More precise studies of the SM is required

- Find Beta-functions and anomalous dimensions

SM (running $\overline{\text{MS}}$) parameters

$$a_i = \left(\frac{5}{3}\frac{g_1^2}{16\pi^2}, \frac{g_2^2}{16\pi^2}, \frac{g_s^2}{16\pi^2}, \frac{y_t^2}{16\pi^2}, \frac{y_b^2}{16\pi^2}, \frac{y_\tau^2}{16\pi^2}, \frac{\lambda}{16\pi^2}\right)$$

$\frac{\lambda}{16\pi^2} \equiv \hat{\lambda}$

II. Applications

7) A. Bednyakov: *Three-loop beta-functions and anomalous dimensions in the SM*

II. Applications

7) A. Bednyakov: *Three-loop beta-functions and anomalous dimensions in the SM*

- 3-loop beta-functions for all the fundamental parameters of the SM are obtained and a full agreement is found with

  [Mihaila, Salomon, Steinhauser,'12]
  [Chetyrkin, Zoller,'12-'13]

  (3-loop Yukawa beta-functions - new result :)

- A framework is established for calculation of three-loop RGEs within "arbitrary" QFT model
  (with the help of LanHEP/FeynRules)

II. Applications

8) V. Khandramai: *FAPT: a Mathematica package for calculations in QCD Fractional Analytic Perturbation Theory*

Analytic Perturbation Theory **(APT)** [Shirkov, Solovtsov (1996,1997)]

Fractional Analytic Perturbation Theory **(FAPT)** [Bakulev, Mikhailov, Stefanis (2005-2010)], [Bakulev, Karanikas, Stefanis (2007)]

- Power PT set in $\overline{MS}$-scheme $\{\bar{\alpha}_s^k(Q^2)\}$ $\Rightarrow$ a non-power APT expansion set $\bar{\mathcal{A}}_k(Q^2)$ **in Euclidian domain** and $\bar{\mathfrak{A}}_k(s)$ **in Minkowskian domain** with both $\bar{\mathcal{A}}_k(Q^2)$ and $\bar{\mathfrak{A}}_k(s)$ regular in the IR region.

$$\begin{array}{ccll} \bar{\alpha}_s^k & \to & \bar{\mathcal{A}}_k\,, & \bar{\mathfrak{A}}_k \\ \sum d_k \bar{\alpha}_s^k & \to & \sum d_k \bar{\mathcal{A}}_k\,, & \sum d_k \bar{\mathfrak{A}}_k \end{array}$$

- We provide here easy-to-use `Mathematica` system procedures collected in the package **"FAPT"** A.P. Bakulev and V.L. Khandramai *Comp. Phys. Comm.* (2013)
- Our package is organized as well-known package **RunDec** [Chetyrkin, Kühn, Steinhauser (2000)]
- This task has been partially realized for APT as the `Maple` package **QCDMAPT** and as the `Fortran` package **QCDMAPT_F** [Nesterenko, Simolo (2010)]

II. Applications

8) V. Khandramai: *FAPT: a Mathematica package for calculations in QCD Fractional Analytic Perturbation Theory*

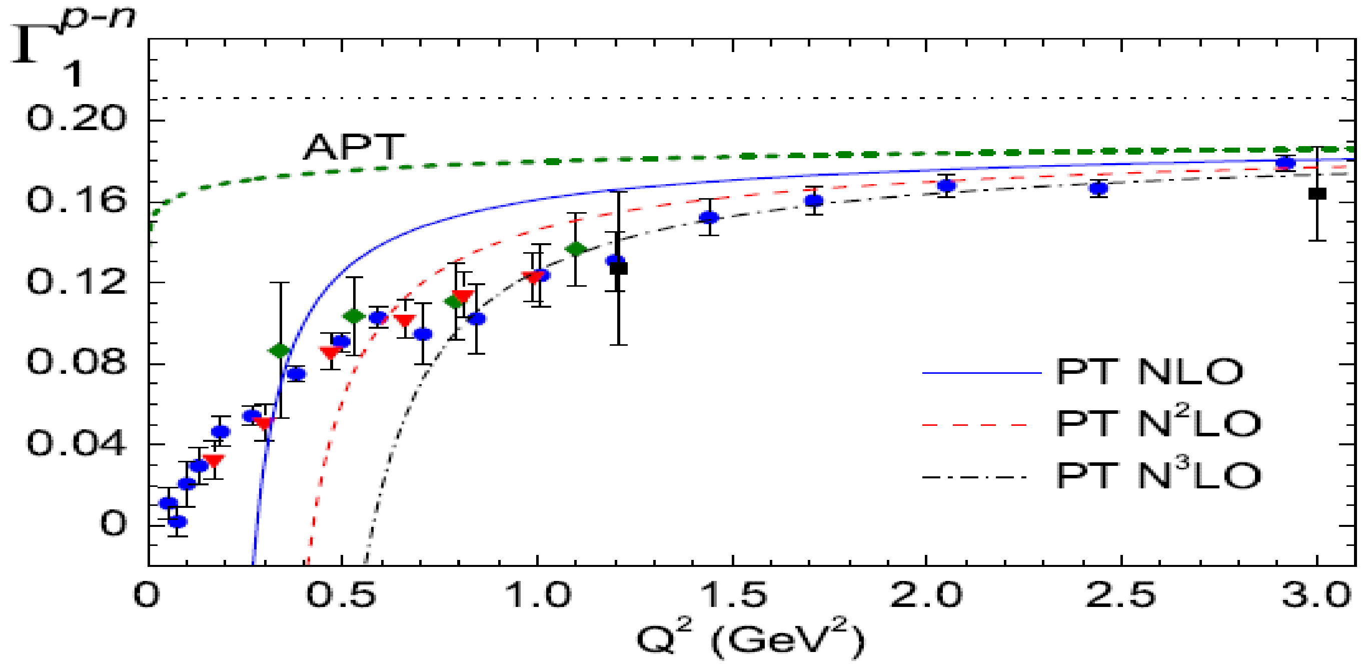
The polarized Bjorken Sum Rule

Perturbative power-correction:

[Baikov, Chetyrkin, Kühn (2010)]

$$\Delta_{\rm Bj}^{\rm PT}(Q^2) = 0.318\,\bar{\alpha}_s(Q^2) + 0.363\,\bar{\alpha}_s^2(Q^2) + 0.652\,\bar{\alpha}_s^3(Q^2) + 1.804\,\bar{\alpha}_s^4(Q^2) + \ldots$$

$$\Delta_{\rm Bj}^{\rm APT}(Q^2) = 0.318\bar{\mathcal{A}}_1(Q^2) + 0.363\bar{\mathcal{A}}_2(Q^2) + 0.652\bar{\mathcal{A}}_3(Q^2) + 1.804\bar{\mathcal{A}}_4(Q^2) + \ldots$$

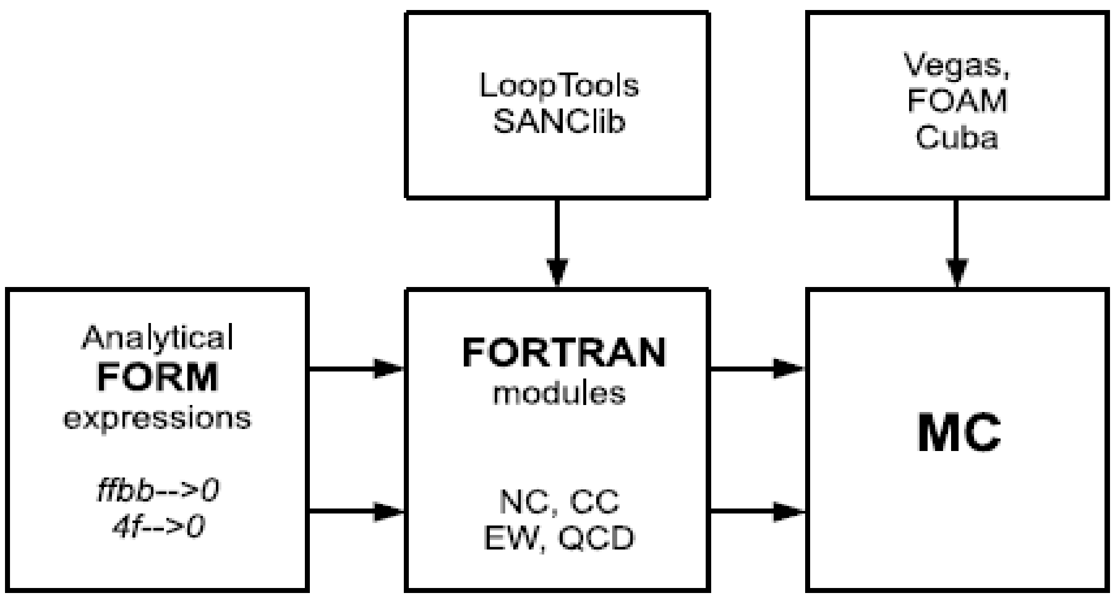


II. Applications

9) R. Sadykov: *SANC system and its application for LHC*

Renat Sadykov (JINR, Dubna)
on behalf of SANC group

- SANC: project to Support of Analytic and Numeric calculations for experiments at Colliders
- SANC web-page: http://sanc.jinr.ru. Stand-alone modules for differential cross-sections as well as Monte Carlo programs (integrators and generators) are available to download



II. Applications

9) R. Sadykov: *SANC system and its application for LHC*

Tuned comparison of different codes

Processes	Codes	Ref.
CC DY (NLO EW)	SANC, HORACE, WGRAD2	arXiv:0705.3251
NC DY (NLO EW)	SANC, HORACE, ZGRAD2	arXiv:0803.0678
CC & NC DY (NLO QCD & EW)	SANC, HORACE, WZGRAD, RADY, FEWZ, DYNNLO, POWHEG-w, POWHEG-z	W-mass workshop (in progress)

SANC and PHOTOS: tuned comparison

FSR QED in W and Z production: arXiv:1212.6783

II. Applications

9) R. Sadykov: *SANC system and its application for LHC*

mcsanc integrator: supported processes

pid	$f + f \to$	SANC ref.
001:003	$\ell^+ + \ell^- (\ell = e, \mu, \tau)$	arXiv:0711.0625, 0901.2785
004	$Z^0 + H$	arXiv:hep-ph/0506120, 0812.4207
± 101:103	$\ell^\pm + \nu_\ell$	arXiv:hep-ph/0506110
± 104	$W^\pm + H$	-
105	$t + \bar{b}$ (s-channel)	arXiv:1110.3622,1207.4400
106	$t + q$ (t-channel)	-//-
-105	$\bar{t} + b$ (s-channel)	-//-
-106	$\bar{t} + q$ (t-channel)	-//-

parallelization thanks to Cuba library

«Missed» corrections: EW NLO — FSR were used by ATLAS

II. Applications

10) V. Zykunov: *Radiative (QCD and Electroweak) Corrections to Drell–Yan Process for Experiments at the LHC*

The possible traces of NP can be production of high-mass dilepton resonances, extra spatial dimensions and so on.

$$pp \to \gamma, Z \to l^+ l^- X$$

NLO EW+QCD

at large invariant mass of lepton-pair ($M \geq 1$ TeV)

FORTRAN program READY: (Radiative corrEctions to lArge invariant mass Drell-Yan process).

- Our motto is "ALL FOR USERS!"
- All Tables were developed as working tool for RDMS CMS (JINR, Dubna) NP group (A.Lanev, S.Shmatov et al.) and concentrates on extra large M (and $\sqrt{S}$) region;

II. Applications

10) V. Zykunov: *Radiative (QCD and Electroweak) Corrections to Drell–Yan Process for Experiments at the LHC*

Relative corrections to fully differential cross sections: EWK RC, $l = \mu$ and $\sqrt{S} = 14$ TeV, MRST2004QED

M	y	ψ										
		-0.985	-0.800	-0.600	-0.400	-0.200	0.000	0.200	0.400	0.600	0.800	0.985
0.5	0.0	0.007	0.018	0.011	0.012	0.016	0.018	0.016	0.012	0.011	0.018	0.007
	0.6		0.038	0.034	0.026	0.021	0.019	0.020	0.021	0.019	0.009	0.028
	1.2						0.024	0.035	0.033	0.031	0.026	0.004
	1.8											-0.006
1.0	0.0	-0.027	-0.024	-0.033	-0.033	-0.029	-0.026	-0.029	-0.033	-0.033	-0.024	-0.027
	0.6		0.018	0.006	-0.005	-0.012	-0.015	-0.017	-0.019	-0.027	-0.042	-0.030
	1.2						0.010	0.018	0.014	0.007	-0.016	-0.068
	1.8											-0.083
2.0	0.0	-0.109	-0.104	-0.110	-0.107	-0.102	-0.099	-0.102	-0.107	-0.110	-0.104	-0.109
	0.5		-0.055	-0.071	-0.079	-0.082	-0.083	-0.085	-0.092	-0.108	-0.128	-0.126
	1.0				-0.046	-0.035	-0.038	-0.041	-0.049	-0.069	-0.112	-0.174
	1.5									-0.013	-0.069	-0.213
	1.9											-0.309
3.0	0.0	-0.196	-0.175	-0.173	-0.164	-0.155	-0.151	-0.155	-0.164	-0.173	-0.175	-0.196
	0.5		-0.132	-0.138	-0.139	-0.136	-0.134	-0.136	-0.148	-0.169	-0.196	-0.212
	1.0				-0.058	-0.055	-0.060	-0.068	-0.088	-0.127	-0.190	-0.271
	1.5									-0.135	-0.218	-0.408
4.0	0.0	-0.286	-0.241	-0.229	-0.213	-0.199	-0.193	-0.199	-0.213	-0.229	-0.241	-0.286
	0.4		-0.207	-0.204	-0.196	-0.188	-0.182	-0.186	-0.202	-0.227	-0.255	-0.280
	0.8			-0.093	-0.098	-0.103	-0.111	-0.129	-0.161	-0.209	-0.265	-0.333
	1.2						-0.092	-0.107	-0.133	-0.185	-0.286	-0.427
5.0	0.0	-0.370	-0.303	-0.280	-0.257	-0.238	-0.231	-0.238	-0.257	-0.280	-0.303	-0.370
	0.3		-0.271	-0.259	-0.244	-0.230	-0.224	-0.231	-0.251	-0.279	-0.311	-0.352
	0.6		-0.153	-0.158	-0.160	-0.163	-0.174	-0.197	-0.234	-0.277	-0.323	-0.386
	0.9			-0.124	-0.129	-0.149	-0.175	-0.213	-0.270	-0.344	-0.424	-0.480

II. Applications

11) Feng Feng: *Automated one-loop computation in quarkonium process within NRQCD framework*

Feng Feng

Center for High Energy Physics, PEKING University

- Generating Feynman diagrams
  - Replacing hadrons with on-shell partonic states

    $$J/\psi \longrightarrow c(p_1)\bar{c}(p_2) \qquad J/\psi \longrightarrow c(p_1)\bar{c}(p_2)g(k)$$

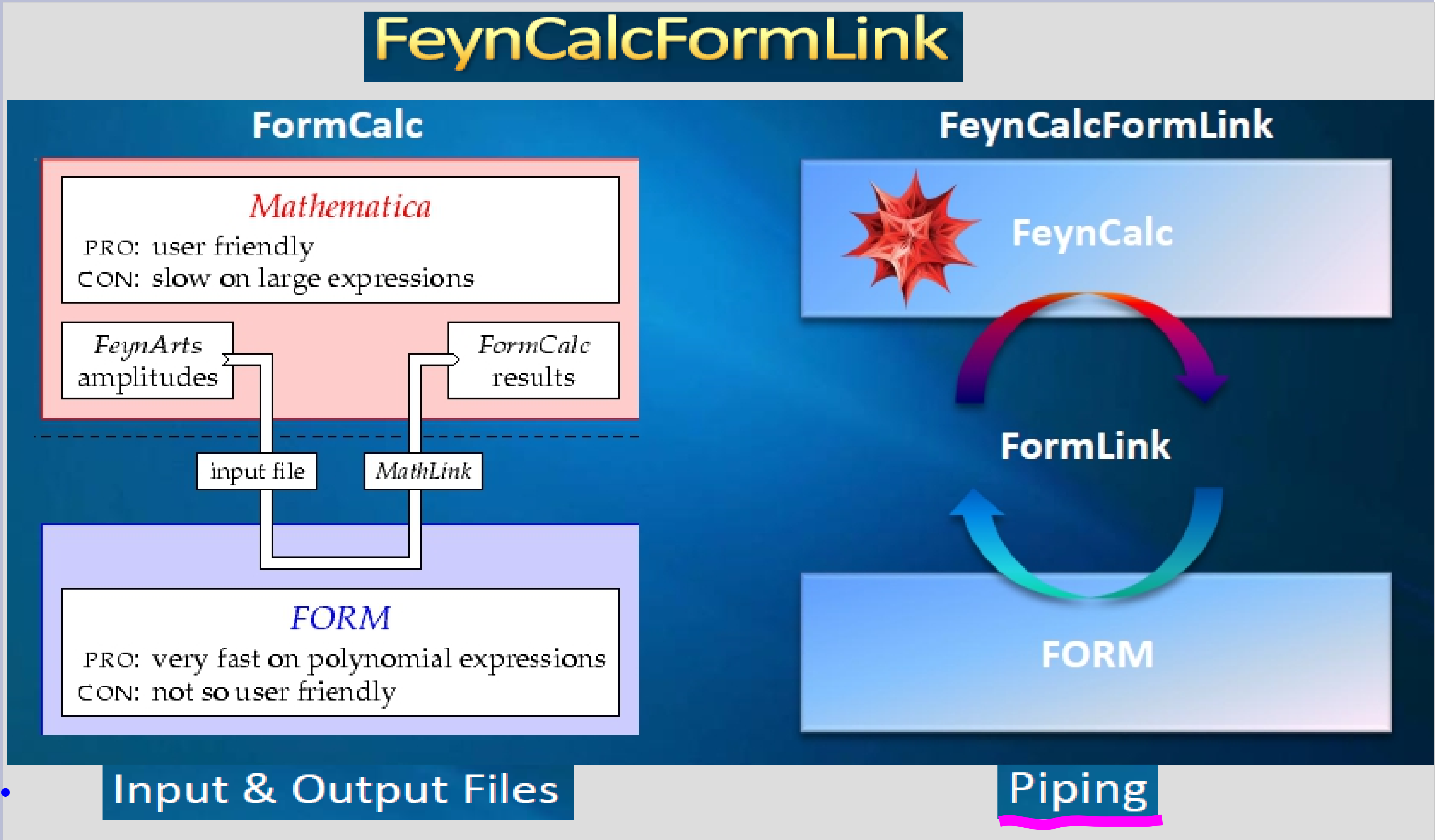
  - FeynArts – Mathematica 9

    http://www.feynarts.com
  - QGraf – Fortran 77

    http://cfif.ist.utl.pt/~paulo/qgraf.html

II. Applications

11) Feng Feng: *Automated one-loop computation in quarkonium process within NRQCD framework*



II. Applications

11) Feng Feng: *Automated one-loop computation in quarkonium process within NRQCD framework*

- Using FeynArts for Feyman diagrams and amplitudes generation.
- Using FeynCalc/FormLink for Dirac- and Color-algebra simplification.
- Using $Apart and FIRE for partial fraction and Integrate-By-Part reduction respectively.
- Solving Master Integrals by any other mean and substituting them to get the final result.
- Post-processing the final result in last previous step.

II. Applications

12) Xing-Gang Wu: *Generators BCVEGPY and GENXICC for doubly heavy mesons and baryons*

In collaboration with Profs.C.H.Chang, J.X.Wang, X.Y.Wang

Department of Physics, Chongquing University

Generators for Bc meson and Xicc baryon events

1) << BCVEGPY1.0 …>> Comput.Phys.Commun.159, 192 (2004) – S-wave
2) << BCVEGPY2.0… >> Comput.Phys.Commun.174, 241 (2006) –P-wave
3) << BCVEGPY2.1 …>> Comput.Phys.Commun.175, 624(2006) – Linux
4) <<BCVEGPY2.2…>> Comput.Phys.Commun. 183,442 (2012) - Present version

==== BCVEGPY ↑ ======= GENXICC ↓ ====

5) << GENXICC1.0 …>> Comput.Phys.Commun.177, 467 (2007)
6) << GENXICC2.0 …>> Comput.Phys.Commun.181, 1144 (2010)
7) <<GENXICC2.1 …>> Comput.Phys.Commun. 184 , 1070(2013) - Present version

Improved Helicity Amplitude Technologies

- **To improve the efficiency of the generators**

II. Applications

12) Xing-Gang Wu: *Generators BCVEGPY and GENXICC for doubly heavy mesons and baryons*

Key point: using the symmetries of the Feynman diagrams

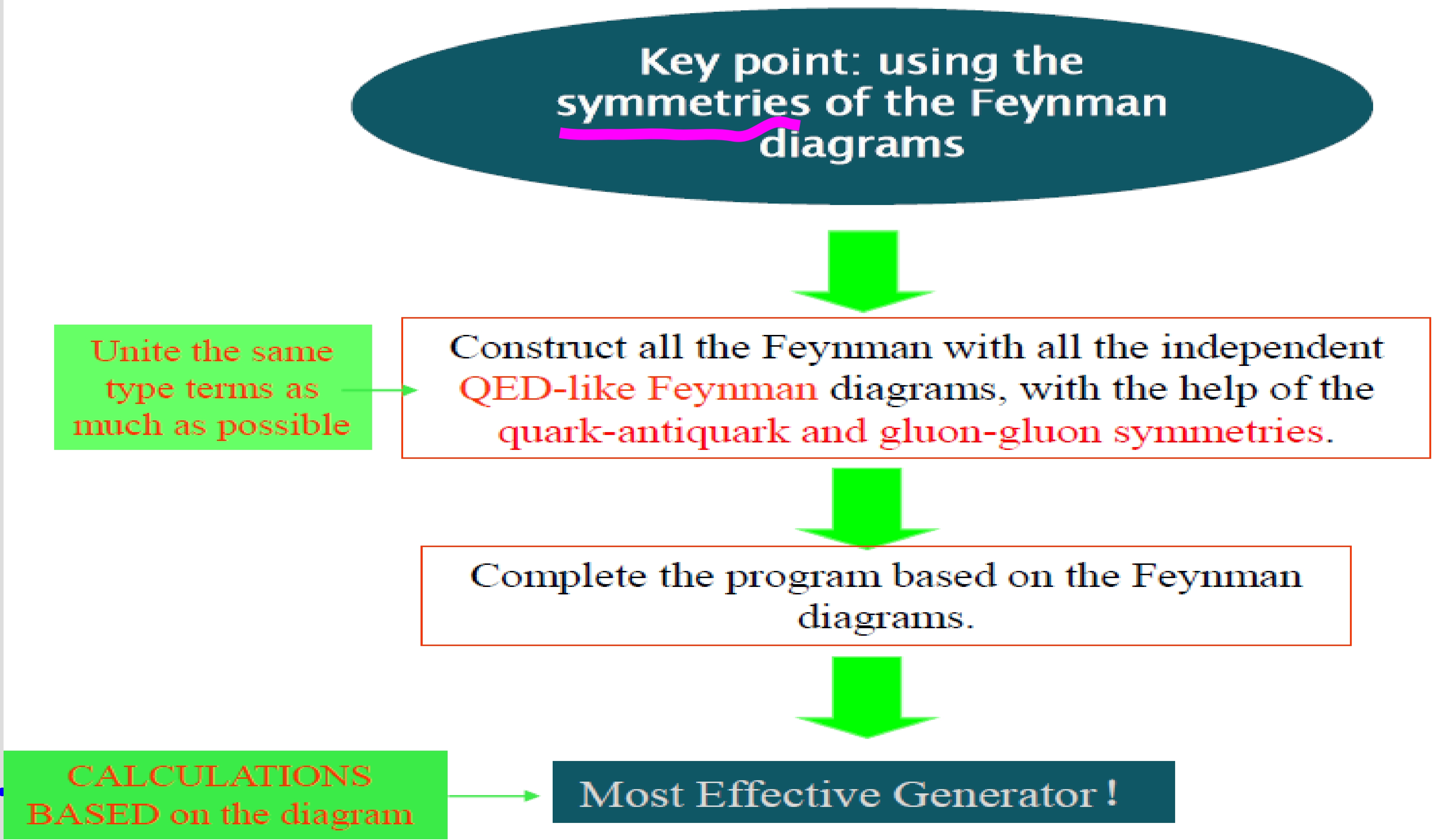
Unite the same type terms as much as possible → Construct all the Feynman with all the independent QED-like Feynman diagrams, with the help of the quark-antiquark and gluon-gluon symmetries.

Complete the program based on the Feynman diagrams.

CALCULATIONS BASED on the diagram → Most Effective Generator !

II. Applications

12) Xing-Gang Wu: *Generators BCVEGPY and GENXICC for doubly heavy mesons and baryons*



II. Applications

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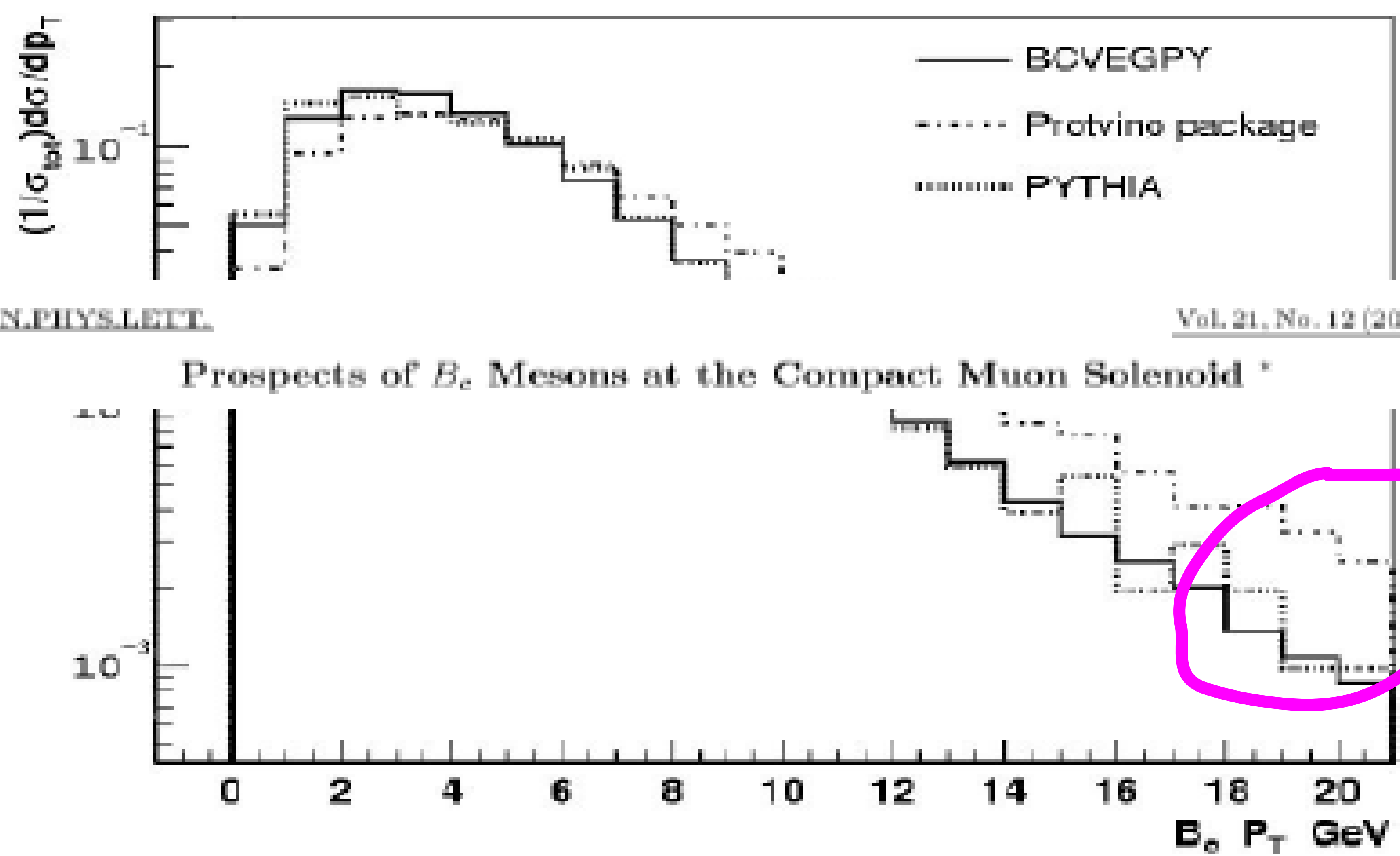

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Prospects of B_c Mesons at the Compact Muon Solenoid *

ACAT 2013, Track 3

Concluding remarks (Part II)

Computing issues:

Laptops – PC – Workstations – Clusters – Supercomputers

Parallelization (vectorization)

Data storge

GPU?

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Concluding remarks (Part II)

Codes:

Stand-alone – System of interfaced programs

Private – Public

(licence, attribution, citation questions)