Automatic calculation in Quarkonium Physics

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Outline

1. FDC project and its progress
   - Brief introduction
   - One-loop calculation
2. Work completed in quarkonium physics with FDC
   - Quarkonium production at the B factories
   - Quarkonium production at hadron colliders
3. Summary

Brief introduction

- FDC = Feynman Diagram Calculation
- Purpose: automatic calculation of physical processes
- First developed by Prof. J.X. Wang since 1993
- First version of FDC has been presented at AIHENP93.
- Written in REDUCE and RLISP to generate Fortran Code
- Including some additional parts for certain physical research e.g. FDC-PWA: Partial Wave Analysis application for Experiment

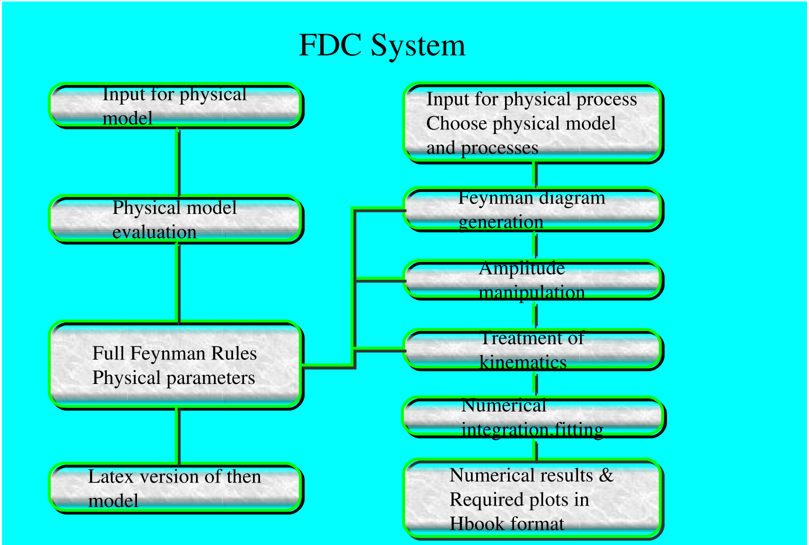


FIG.1: FDC system flow chart

To prepare phenomenological model

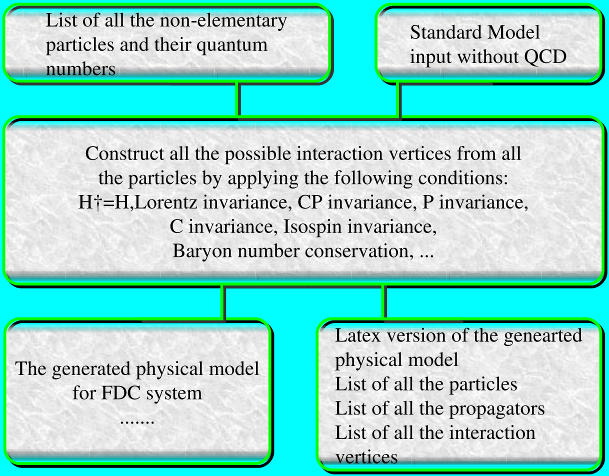


FIG.3: System flow chart for physical process

Special treatment for NRQCD

- NRQCD is now the most common framework to study quarkonium productions and decays.
- Following projection operators

$$\Pi_0(P,p) = \frac{1}{2\sqrt{2}E(E+m)}\left(\frac{1}{2}\not{P}+m+\not{p}\right)\frac{\not{P}+2E}{4E}\gamma_5\left(\frac{1}{2}\not{P}-m-\not{p}\right)$$
$$\Pi_1(P,p,\epsilon) = \frac{-1}{2\sqrt{2}E(E+m)}\left(\frac{1}{2}\not{P}+m+\not{p}\right)\frac{\not{P}+2E}{4E}\not{\epsilon}\left(\frac{1}{2}\not{P}-m-\not{p}\right)$$

have been implemented in FDC to deal with the quarkonium.

- Here m is the mass of heavy quark, P is the momentum of quarkonium, $p = (p_Q - p_{\bar{Q}})/2$ is the "relative momentum" between heavy quark pair, and $E \equiv \sqrt{m^2 - p^2}$ can be regarded as half of the "mass" of quarkonium.

One-loop calculation

- With the development of experiment in high energy physics, accuracy of the measurement has been gradually improved
- Make it more and more important to include higher order corrections in theoretical predictions.
- The development of one-loop calculation part in FDC started from 2002, and finished in 2007.
- The results are obtained analytically.
  - At the level of amplitude square, before the integration of phase space.
  - Usually they are still in numerical form (Fortran Code), as in most cases, they are too complicated to read.

Real Corrections

A two-cutoff phase space slicing method [Harris and Owens (2002)] is realized in FDC to deal with IR divergence.

- Two cutoffs, δ_s and δ_c are introduced to separate the phase space into three parts: soft, hard-collinear, hard-noncollinear
- The hard-noncollinear part is finite, and can be calculated numerically with traditional Monte-Carlo method.
- Both the soft and hard-collinear part are factorized in soft/collinear limit, and added to corresponding virtual correction processes.
- All the divergence (including those in virtual corrections) are separated analytically, and then sum up to check if they are really cancelled with others.

Virtual Corrections

- Counter term diagrams are generated automatically (after the input of renormalization constant)
- FDC have two ways to generate square of amplitude:
  - square the amplitude analytically
  - generate numerical result (Fortran Code) of amplitude, then square it.
  - these two ways will lead to different tensor reduction and then are a cross-check inside FDC
- All the divergence (both UV and IR) are separated during the calculation of amplitude analytically
- The cutoff (δ_s and δ_c) independence has to be checked after summing up both real and virtual corrections.

Reduction and calculation of loop integrals

- Feynman parametrization is used directly to derive the analytical results for one and two point integrals. (without tensor reduction)
- Passarino-Veltman reduction method is used for tensors of $N(N \geq 3)$ point
- Some other relations are used in the reduction of $N(N \geq 5)$ point integrals
- Loop integrals are calculated analytically under dimensional regularization.

N-point scalar integral in D-dimension can be defined as

$$T_0^{(N)}(p_1, \ldots, p_{N-1}, m_0, m_1, \ldots, m_{N-1}) = \mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{N_0 \ldots N_{N-1}},$$

where $N_n = (q + p_n)^2 - m_n^2 + i\epsilon, \quad n = 0, \ldots, N-1,$
According to [Dittmaier (2003)], the IR singularities part can be expressed as sum of a few 3-point with IR singularities

$$T_0|_{\text{sing}}^{(N)} = \sum_{n=0}^{N-1} \sum_{\substack{k=0 \\ k\neq n, n+1}}^{N-1} A_{nk}\, C_0(p_0, \ldots, p_k, m_n, m_{n+1}, m_k).$$

We can evaluate the scalar integral by

$$T_0^D = T_0^\epsilon - T_0|_{\text{sing}}^\epsilon + T_0|_{\text{sing}}^D.$$

Where T_0^ϵ, $T_0|_{\text{sing}}^\epsilon$ means to use $i\epsilon$ to regularize the singularities

$i\epsilon$-regularization

Here is the procedure to deal with the integral under $i\epsilon$ regularization:

- keep $i\epsilon$ in the propagators to make the scalar integrals well defined.
- let the dimension back to 4
- do the integration following the standard way described in ['t Hooft and Veltman (1979)]
- expand the results in $i\epsilon$
- This way is suitable to program and realized in FDC package.

Recent improvement

In 2011, new reduction method for loop integrals realized in FDC [Duplancic and Nizic (2004)].
It has many advantages:

- Can reduce integrals with abnormal dimension and denominators (e.g. P-wave states)
- Can reduce tensors with high (4^+) rank and more (5^+) external momenta at same time (where P-V reduction fails)
- Can further reduce some integrals (e.g. scalar integrals containing Coulomb singularities)
- Easy to obtain certain Lorentz term

Based on above progress in FDC, many important work are completed. Mainly,

- Quarkonium production at the B factories.
- Quarkonium production and polarization puzzle at hadron colliders.

$e^+e^- \to J/\psi + \eta_c$

Experimantal Data

BELLE: $\sigma[J/\psi + \eta_c] \times B^{\eta_c}[\geq 2] = (25.6 \pm 2.8 \pm 3.4)$ fb

BARAR: $\sigma[J/\psi + \eta_c] \times B^{\eta_c}[\geq 2] = (17.6 \pm 2.8^{+1.5}_{-2.1})$ fb

Abe et al. (2002); Pakhlov (2004); Aubert et al. (2005)

LO NRQCD Predictions

$2.3 \sim 5.5$ fb

Braaten and Lee (2003); Liu et al. (2003); Hagiwara et al. (2003)

NLO QCD corrections

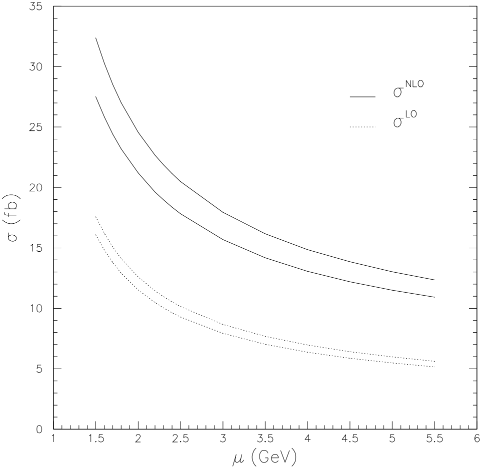
$K \equiv \sigma^{NLO}/\sigma^{LO} \sim 2$

First given in PRL96 (2006) Y. J. Zhang, Y. J. Gao and K. T. Chao

Confirmed analytically in PRD77 (2008), BG and J.X. Wang

m_c(GeV)	μ	$\alpha_s(\mu)$	σ_{LO}(fb)	σ_{NLO}(fb)	σ_{NLO}/σ_{LO}
1.5	m_c	0.369	16.09	27.51	1.710
1.5	$2m_c$	0.259	7.94	15.68	1.975
1.5	$\sqrt{s}/2$	0.211	5.27	11.14	2.114
1.4	m_c	0.386	19.28	34.92	1.811
1.4	$2m_c$	0.267	9.19	18.84	2.050
1.4	$\sqrt{s}/2$	0.211	5.76	12.61	2.189

Cross sections with different charm quark mass m_c and renormalization scale μ. $\sqrt{s} = 10.6\ \mathrm{GeV}$ is the c.m. energy.



Cross sections as function of the renormalization scale μ with $|R_s(0)|^2 = 0.978\ \mathrm{GeV}^3$, $\Lambda = 0.338\ \mathrm{GeV}$ and c.m. energy $10.6\ \mathrm{GeV}$. The charm quark mass is chosen as 1.4 GeV (upper curves) and 1.5 GeV (lower curves).

$e^+e^- \to J/\psi + J/\psi$

Problem

LO NRQCD prediction indicates that the cross section of this process is large than that of $J/\psi + \eta_c$ production by a factor of 1.8, but no evidence for this process was found at the B factories.
Bodwin et al. (2003); Abe et al. (2004)

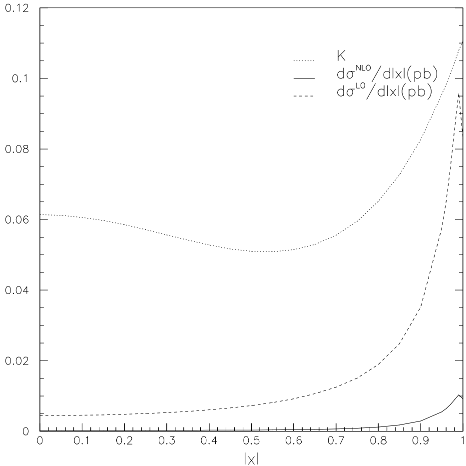
NLO QCD corrections

- Greatly decreased, with a K factor ranging from $-0.31 \sim 0.25$ depending on the renormalization scale.
- Might explain the situation.

PRL100 (2008) BG and J.X. Wang

m_c(GeV)	μ	$\alpha_s(\mu)$	σ_{LO}(fb)	σ_{NLO}(fb)	σ_{NLO}/σ_{LO}
1.5	m_c	0.369	7.409	-2.327	-0.314
1.5	$2m_c$	0.259	7.409	0.570	0.077
1.5	$\sqrt{s}/2$	0.211	7.409	1.836	0.248
1.4	m_c	0.386	9.137	-3.350	-0.367
1.4	$2m_c$	0.267	9.137	0.517	0.057
1.4	$\sqrt{s}/2$	0.211	9.137	2.312	0.253

Cross sections with different charm quark mass m_c and renormalization scale μ, and $\sqrt{s} = 10.6\ \mathrm{GeV}$.



Differential cross section as function of $|x|$ where $x = \cos(\theta)$.

θ is the angle between the J/ψ and the beam, and $K = \frac{\mathrm{d}\sigma^{\mathrm{NLO}}}{\mathrm{d}|x|} / \frac{\mathrm{d}\sigma^{\mathrm{LO}}}{\mathrm{d}|x|}$ is the ratio of differential cross section of NLO to LO. m_c is set as 1.5 GeV and $\mu = \sqrt{s}$ is taken.

$e^+e^- \to J/\psi + gg$

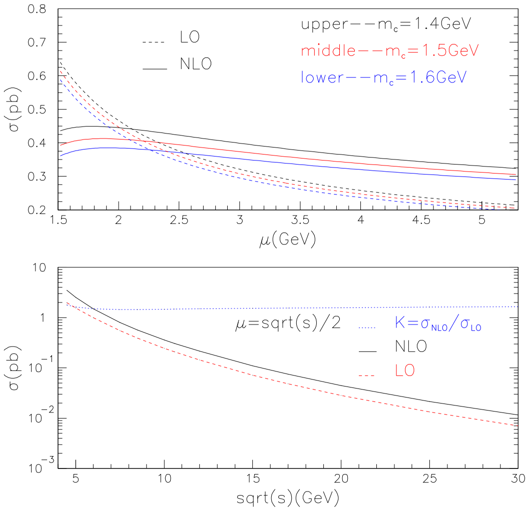
Cross section at NLO

$$\sigma^{(1)} = \sigma^{(0)} \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[a(\hat{s}) + \beta_0 \ln \left(\frac{\mu}{2m_c} \right) \right] \right\}$$

m_c(GeV)	$\alpha_s(\mu)$	$\sigma^{(0)}$(pb)	$a(\hat{s})$	$\sigma^{(1)}$(pb)	$\sigma^{(1)}/\sigma^{(0)}$
1.4	0.267	0.341	2.35	0.409	1.20
1.5	0.259	0.308	2.57	0.373	1.21
1.6	0.252	0.279	2.89	0.344	1.23

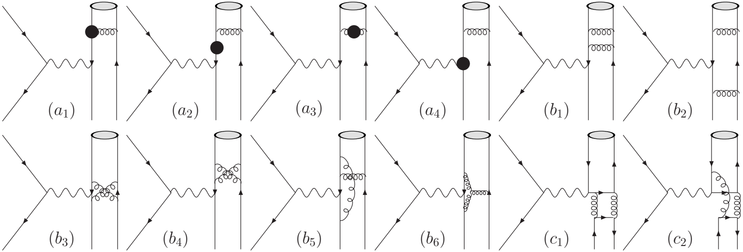
Cross sections with different charm quark mass m_c where the renormalization scale $\mu = 2m_c$ and $\sqrt{s} = 10.6$ GeV.

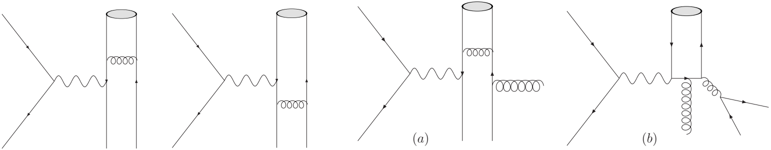
PRL102 (2009) BG and J.X. Wang



Cross sections as function of the renormalization scale μ and the center-of-mass energy of e^+e^- $\sqrt{s}$.

$e^+e^- \to J/\psi + c\bar{c}$





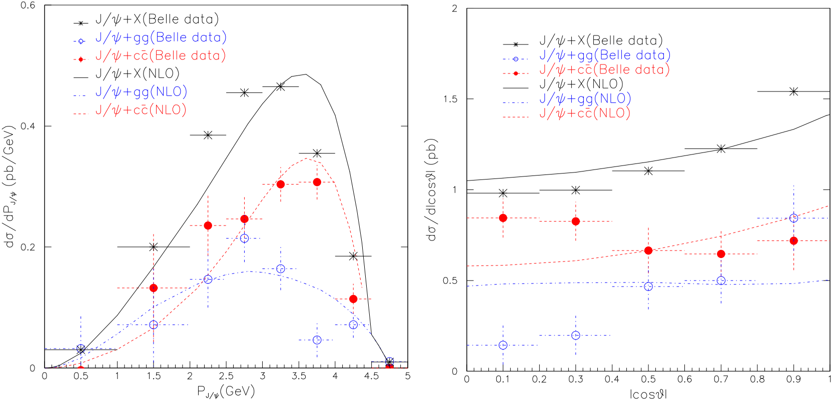
Cross section at NLO

$$\sigma^{(1)} = \sigma^{(0)} \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[a(\hat{s}) + \beta_0 \ln \left(\frac{\mu}{2m_c} \right) \right] \right\}$$

m_c(GeV)	$\alpha_s(\mu)$	$\sigma^{(0)}$(pb)	$a(\hat{s})$	$\sigma^{(1)}$(pb)	$\sigma^{(1)}/\sigma^{(0)}$
1.4	0.267	0.224	8.19	0.380	1.70
1.5	0.259	0.171	8.94	0.298	1.74
1.6	0.252	0.129	9.74	0.230	1.78

Cross sections with different charm quark mass m_c with the renormalization scale $\mu = 2m_c$ and $\sqrt{s} = 10.6$ GeV.

PRD80 (2009) BG and J.X. Wang



Momentum distribution of inclusive J/ψ production with $\mu = \mu^*$ and $m_c = 1.4$ GeV is taken for the $J/\psi cc$ channel. The contribution from the feed-down of ψ' has been added to all curves by multiplying a factor of 1.29.

Quarkonium production at hadron colliders

- The discrepancy between LO theoretical prediction and CDF measurement for J/ψ polarization has been a challenging topic for a long time.
- Higher order corrections are highly expected to further clarify the situation.
- Involving lots of loop diagrams
- Most important in my past work.
- Same problem with bottomonium

According to the NRQCD factorization formalism, the cross section for hadroproduction of quarkonium H can be expressed as

$$d\sigma[pp \to H + X] = \sum_{i,j,n} \int dx_1 dx_2 G_p^i G_p^j \hat{\sigma}[ij \to (Q\bar{Q})_n X] \langle \mathcal{O}_n^H \rangle$$

- p is either a proton or anti-proton, the indices i, j run over all the partonic species and n denotes the color, spin and angular momentum states of the intermediate $Q\bar{Q}$ pair.
- For J/ψ, up to LO in v^2, n can be ${}^3S_1^{[1]}, {}^3S_1^{[8]}, {}^1S_0^{[8]}$ and ${}^3P_J^{[8]}$.
- The short-distance contribution $\hat{\sigma}$ can be calculated perturbatively, while the long-distance matrix elements (LDMEs) $\langle \mathcal{O}_n^H \rangle$ are fully governed by non-perturbative QCD effects.

Involving subprocesses and number of corresponding Feynman diagrams

	$^3S_1^{[1]}$	$^3S_1^{[8]}$	$^1S_0^{[8]}$	$^3P_J^{[8]}$
$gg \to J/\psi + g$	6/129	16/413	12/267	12/267
$gq \to J/\psi + q$	-	5/111	2/49	2/49
$q\bar{q} \to J/\psi + g$	-	5/111	2/49	2/49
$gg \to J/\psi + gg$	60	123	98	98
$gg \to J/\psi + q\bar{q}$	6	36	20	20
$gq \to J/\psi + gq$	6	36	20	20
$q\bar{q} \to J/\psi + gg$	6*	36	20	20
$q\bar{q} \to J/\psi + q\bar{q}$	-	14	4	4
$q\bar{q} \to J/\psi + q'\bar{q}'$	-	7	2	2
$qq \to J/\psi + qq$	-	14	4	4
$qq' \to J/\psi + qq'$	-	7	2	2

Summary of recent work

- J. Campbell, F. Maltoni, and F. Tramontano, PRL98, 252002 (2007)
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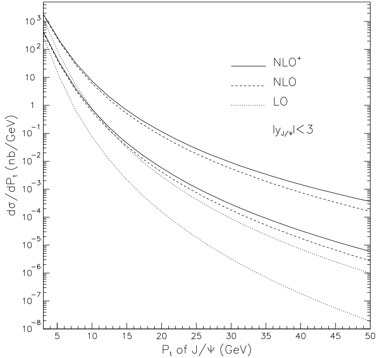
- Y.Q. Ma, K. Wang, and K.T. Chao, PRL106, 042002 (2011); PRD84, 114001 (2011)
- M. Butenschoen and B. A. Kniehl, PRL108, 172002 (2012)
- K.T. Chao, Y.Q. Ma, H.S. Shao, K. Wang, and Y.J. Zhang, PRL108, 242004 (2012)
- BG, L.P. Wan, J.X. Wang, H.F. Zhang, PRL110, 042002 (2013)

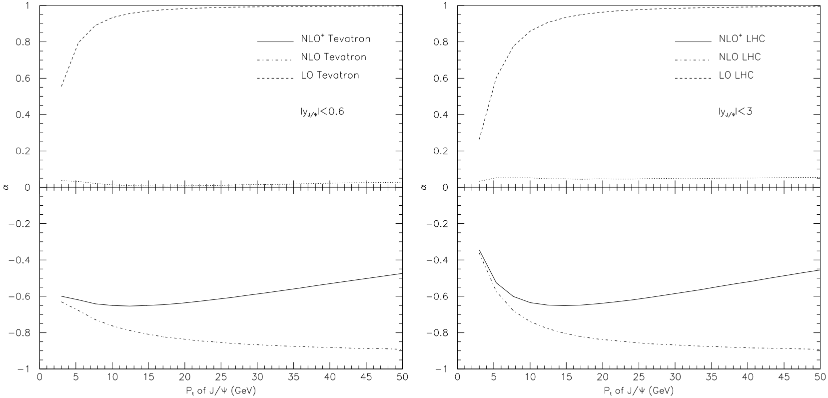
${}^3S_1^{[1]}$ channel



Transverse momentum distribution of differential cross section at the LHC (upper curves) and Tevatron (lower curves).

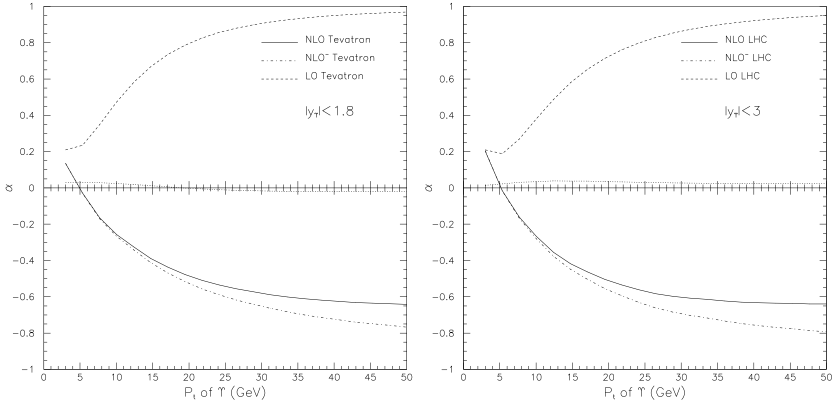
- NLO QCD corrections are found to be large in this channel.
- The polarization changed drastically at NLO.
- Shed a light on the puzzle.
- Unfortunately, contribution of this channel is too small to affect the total polarization.

PRL100 (2008) BG and J.X. Wang



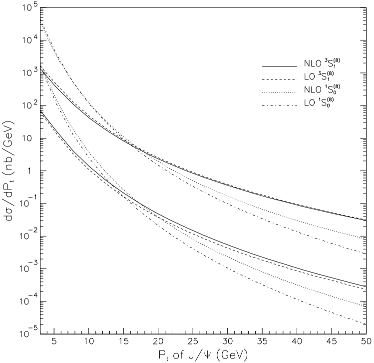
Transverse momentum distribution of polarization α at the Tevatron (left) and LHC (right).

Υ case



Transverse momentum distribution of polarization α at the Tevatron (left) and LHC (right).

${}^3S_1^{[8]}$ and ${}^1S_0^{[8]}$ channels



Transverse momentum distribution of differential cross section at the LHC (upper curves) and Tevatron (lower curves).

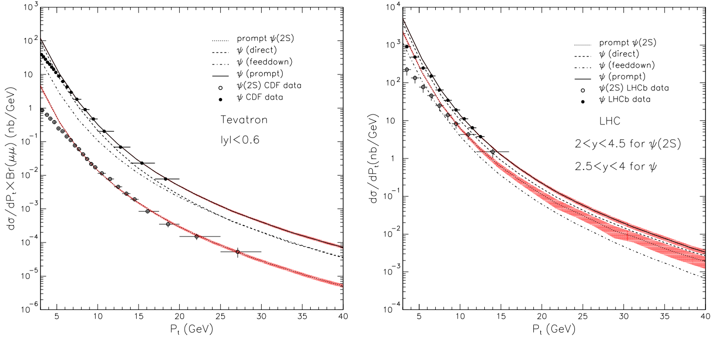
NLO QCD corrections are small in these two channels.

PLB673 (2009) BG, X.Q. Li and J.X. Wang

Complete NLO results

- All direct channels (including ${}^3P_J^{[8]}$) are included.
- Feeddown contributions from $\psi(2S)$ and χ_{cJ} are included
- This is first prediction for "prompt" J/ψ hadroproduction that can compare with experimental data.
- According to current results, it is hard to explain all the data (differential cross section and polarization) at same time in the frame work of NRQCD

PRL110 (2013) BG, L.P. Wan, J.X. Wang and H.F. Zhang

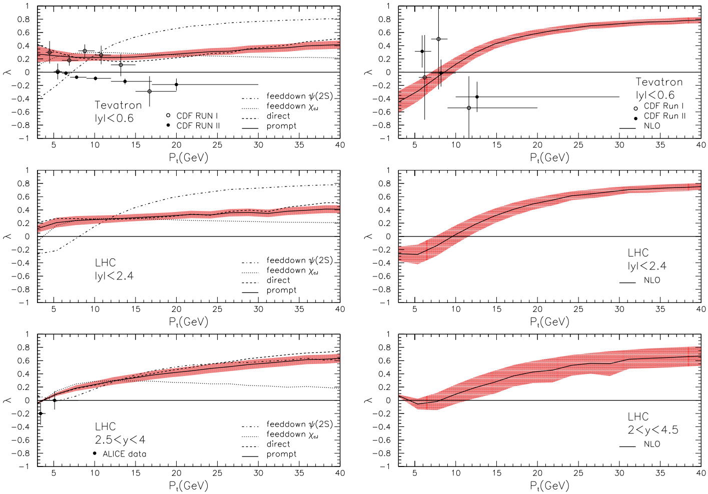


$$\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle = (9.7 \pm 0.9) \times 10^{-2}\ \mathrm{GeV}^3 \quad \langle \mathcal{O}^{\psi(2S)}({}^1S_0^{[8]})\rangle = (-0.01 \pm 0.87) \times 10^{-2}\ \mathrm{GeV}^3$$

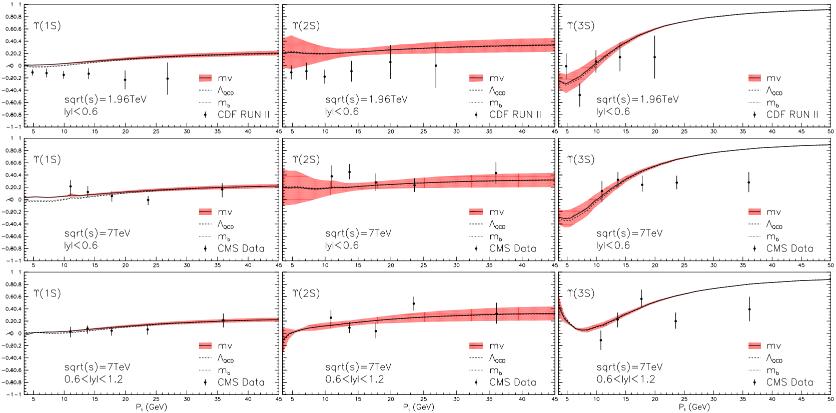
$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle = (-0.46 \pm 0.13) \times 10^{-2}\ \mathrm{GeV}^3 \quad \langle \mathcal{O}^{\psi(2S)}({}^3S_1^{[8]})\rangle = (0.34 \pm 0.12) \times 10^{-2}\ \mathrm{GeV}^3$$

$$\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]})\rangle/m_c^2 = (-0.95 \pm 0.25) \times 10^{-2}\ \mathrm{GeV}^3 \quad \langle \mathcal{O}^{\psi(2S)}({}^3P_0^{[8]})\rangle/m_c^2 = (0.42 \pm 0.24) \times 10^{-2}\ \mathrm{GeV}^3$$

$$\langle \mathcal{O}^{\chi_{c0}}({}^3S_1^{[8]})\rangle = (0.220 \pm 0.005) \times 10^{-2}\ \mathrm{GeV}^3$$



Polarizatoin of J/ψ (left) and $\psi(2S)$ (right) hadroproduction at the Tevatron and LHC



Polarization parameter λ for Υ hadroproduction at the Tevatron and LHC. From left to right: $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$. Rows from top to bottom are corresponding to different experimental conditions of CDF RUN II, CMS ($|y| < 0.6$) and CMS ($0.6 < |y| < 1.2$).

arXiv:1305.0748 BG, L.P. Wan, J.X. Wang and H.F. Zhang

Summary

- FDC can do one-loop calculation automatically now.
- Reduction method in FDC have been improved to deal with more cases.
- Many important work in quarkonium physics has been finished with FDC.

Thanks for your attention!

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