Radiative (QCD and Electroweak) Corrections to Drell–Yan Process for Experiments at the Large Hadron Collider

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Introduction

Despite the fact that the Standard Model (SM) keeps for oneself the status of consistent and experimentally confirmed theory, the search of New Physics (NP) manifestations is continued.

The possible traces of NP can be production of high-mass dilepton resonances, extra spatial dimensions and so on.

One of powerful tool in the modern experiments at LHC from this point of view is the experimental investigation of Drell–Yan lepton-pair production, i.e. data on the cross section and the forward-backward asymmetry of the reaction

$$pp \to \gamma, Z \to l^+ l^- X \tag{1}$$

at large invariant mass of lepton-pair ($M \geq 1$ TeV).

The studies of NP effects is impossible without the exact knowledge of ElectroWeak Corrections (EWKC) and QCD corrections (QCDC):

♢ Pure QED corrections with covariant approach: V. Mosolov, N. Shumeiko, Nucl.Phys. B **186**, 394 (1981), A. Soroko, N. Shumeiko, Yad. Fiz **52**, 514 (1990).
♢♢ EWKC of ZGRAD2: U. Baur et al., Phys. Rev. D **65**: 033007, (2002),
♢♢♢ EWKC of SANC group: A. Andonov et al., Comput. Phys. Commun. **174**, 481 (2006); Eur. Phys. J. C. **54**, 451 (2008)
♢♢♢♢ EWKC at extra large M: V. Zykunov, Phys. Atom Nucl. **69**, 1557 (2006); Phys. Rev. D **75**, 073019 (2007); Phys. Atom Nucl. **71**, 757 (2008)

♢ QCD NNLO corrections: R. Hambert, W. L. van Neerven, T. Matsuura, Nucl. Phys. B **359**, 343 (1991).
♢♢ other QCD corr.: H. Baer, *et al.*, Phys. Rev. D 40, 2844 (1989); Phys. Rev. D **42**, 61 (1990); W. Giele, E. Glover, Phys. Rev. D **46**, 1980 (1992); C. Anastasiou *et al.*, Phys. Rev. D **69**, 094008 (2004); A. Andonov *et al.*, Phys. Atom Nucl. **73**, 1761 (2010).
♢♢♢ QCD corr. to fully diff.: K. Melnikov and F. Petriello, Phys. Rev. D **74**, 114017 (2006). S. Catani *et al.*, Phys. Rev. Lett. **103**, 082001 (2009). V. Zykunov, Phys. Atom Nucl. **73**, 1269 (2010); **74**, 72 (2011).

Some codes for NLO and NNLO NC DY at hadronic colliders

- DYNNLO
- FEWZ
- HORACE
- MC@NLO
- RADY
- READY
- SANC
- ZGRAD / ZGRAD2

Current experimental situation at CMS LHC

Now we have measurement of the

- differential $\frac{d\sigma}{dM}$ (M is dilepton invariant mass)
- and double-differential $\frac{d^2\sigma}{dM\,dy}$ (y is dilepton rapidity)

Drell–Yan cross sections at $\sqrt{S}$ = 7 TeV, $M \leq 1.5$ TeV and integrated luminosity 4.5 fb^{-1} (CMS PAS EWK-11-007).

Uncertainty due to the QCD and EWK corrections was calculated using FEWZ and POWHEG. Measurements are in agreement with the SM predictions:

- all of them with NNLO of FEWZ (MSTW2008 parton density functions),
- and double-differential with NLO of POWHEG (CT10 parton density functions).

Both accurate and fast! Mathematical content

At the edges of kinematical region (extra large $\sqrt{S}$, M) the important task is make the correction procedure of background both accurate and fast. For the latter it is desirable to obtaint the set of as much compact as possible formulas for the EWK and QCD corrections.

To get leading effect of Weak corrections in the region of large invariant dilepton mass we actively used the so-called Sudakov logarithms (SL) (V. Sudakov, Sov. Phys. JETP **3**, 65 (1956)),

$$l_{i,x} = \log \frac{m_i^2}{|x|} \quad (i = Z, W; \;\; x = s, t, u). \tag{2}$$

Collinear logarithms play leading role in QED- and QCD-corrections

$$\log \frac{m_f^2}{|x|} \quad (f = e, \mu, q; \;\; x = s, t, u). \tag{3}$$

Notations and Born amplitude

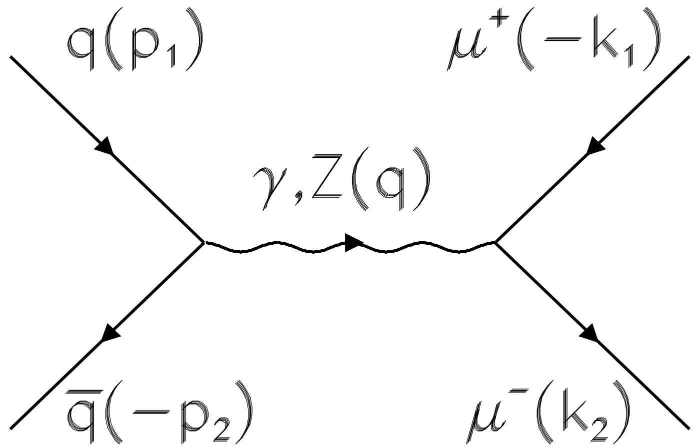

Figure 1: **The lowest order graph giving contribution to the DY scattering at parton level**

The standard set of Mandelstam invariants for the partonic elastic scattering:

$$s = (p_1 + p_2)^2, \; t = (p_1 - k_1)^2, \; u = (k_1 - p_2)^2. \quad (4)$$

Common convolution formula for Born and V-contribution

$$\sigma_V^H = \frac{1}{3}\int\limits_0^1 dx_1 \int\limits_0^1 dx_2 \int\limits_{-S}^0 dt \sum_{q=u,d,s,c,b} [f_q^A(x_1,Q^2) f_{\bar{q}}^B(x_2,Q^2)\sigma_V^{q\bar{q}}(t) +$$

$$+ f_{\bar{q}}^A(x_1,Q^2) f_q^B(x_2,Q^2)\sigma_V^{\bar{q}q}(t)]\theta(s+t)\theta_M\theta_D,$$

where $V = \{0, \text{BSE}, \text{LV}, \text{HV}, b, \text{fin}\}$, $b = \{\gamma\gamma, \gamma Z, ZZ, WW\}$,
θ_M, θ_D are kinematical factors,
hadronic invariant $S = (P_A + P_B)^2$,
invariant dilepton mass $M = \sqrt{(k_1 + k_2)^2}$.

The propagator for j-boson has the form

$$D^{js} = \frac{1}{s - m_j^2 + im_j\Gamma_j}. \qquad (5)$$

BORN and coupling constants

Born cross section looks like

$$\sigma_0^{q\bar{q}}(t) = \frac{2\pi\alpha^2}{s^2} \sum_{i,j=\gamma,Z} D^i D^{j*} (b_+^{i,j} t^2 + b_-^{i,j} u^2), \tag{6}$$

where

$$b_\pm^{n,k} = \lambda_{q+}^{n,k} \lambda_{l+}^{n,k} \pm \lambda_{q-}^{n,k} \lambda_{l-}^{n,k}, \tag{7}$$

$$\lambda_{f+}^{i,j} = v_f^i v_f^j + a_f^i a_f^j,\ \lambda_{f-}^{i,j} = v_f^i a_f^j + a_f^i v_f^j, \tag{8}$$

$$v_f^\gamma = -Q_f,\ a_f^\gamma = 0,\ v_f^Z = \frac{I_f^3 - 2s_W^2 Q_f}{2s_W c_W},\ a_f^Z = \frac{I_f^3}{2s_W c_W}. \tag{9}$$

The Feynman rules from papers
M. Böhm, H. Spiesberger, W. Hollik, Forschr. Phys. – 1986. – V. 34. – P. 687
M. Böhm, H. Spiesberger, Nucl. Phys. B. – 1987. – V. 294. – P. 1081
were used.

Main features of our calculation for QCD and EWK corrections

- the t'Hooft-Feynman gauge,
- on-mass renormalization scheme ($\alpha, \alpha_s, m_W, m_Z, m_H$ and the fermion masses are independent parameters),
- ultrarelativistic limit.

QCD result can be obtained from QED case by substitution:

$$Q_q^2 \alpha \to \sum_{a=1}^{N^2-1} t^a t^a \alpha_s = \frac{N^2-1}{2N} I \alpha_s \to \frac{4}{3}\alpha_s, \qquad (10)$$

here $2t^a$– Gell-Man matrices, $N = 3$.

Boson Self Energies (EWK)

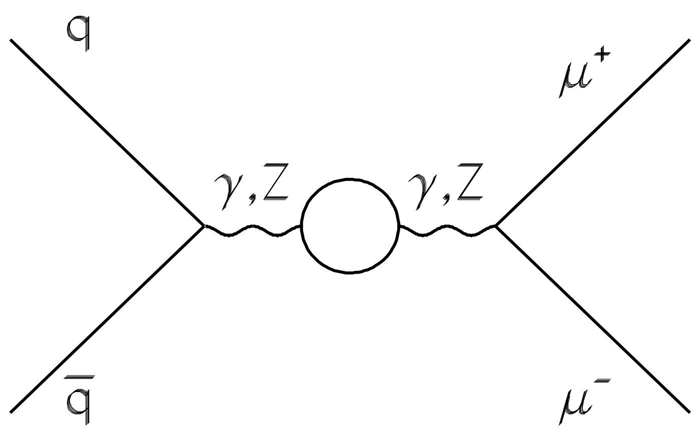

Figure 2: $\gamma\gamma$ **– ,** γ **Z – and ZZ – Self Energy diagrams**

$$\sigma^{q\bar{q}}_{\rm BSE}(t) \quad = \quad -\frac{4\alpha^2\pi}{s^2}\Big[\sum_{i,j=\gamma,Z} \Pi^i_S D^i D^{j*} \sum_{\chi=+,-} \lambda^{i,j}_{q_\chi}\lambda^{i,j}_{l_\chi}(t^2+\chi u^2) + $$

$$+\Pi^{\gamma Z}_S D^Z \sum_{i=\gamma,Z} D^{j*} \sum_{\chi=+,-} (\lambda^{\gamma,j}_{q_\chi}\lambda^{Z,j}_{l_\chi} + \lambda^{Z,j}_{q_\chi}\lambda^{\gamma,j}_{l_\chi})(t^2+\chi u^2)\Big] \quad (11)$$

$\Pi^{\gamma,Z,\gamma Z}$ are connected with the renormalized γ–, Z– and γZ–self energies as

$$\Pi^\gamma = \frac{\hat{\Sigma}^\gamma}{s}, \; \Pi^Z = \frac{\hat{\Sigma}^Z}{s-m_Z^2}, \; \Pi^{\gamma Z} = \frac{\hat{\Sigma}^{\gamma Z}}{s}.$$

Light and Heavy Vertices (QCD and EWK)

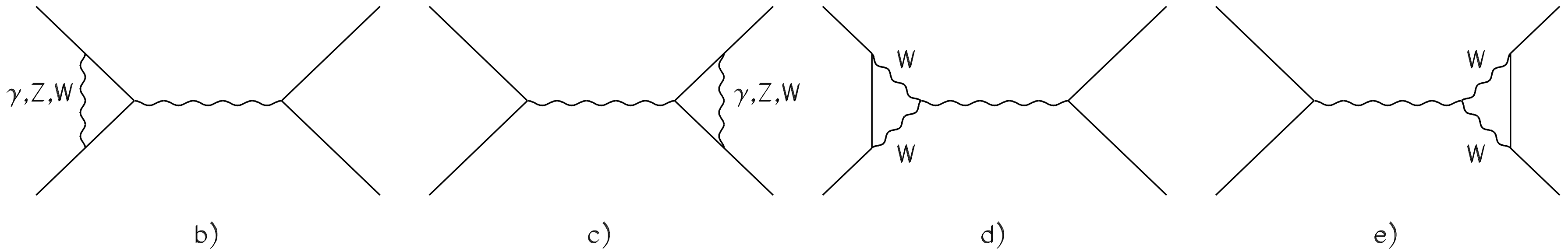

Figure 3: **Feynman graphs for Vertices diagrams. Unsigned helix lines mean γ or Z.**

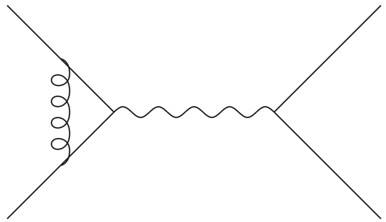

Figure 4: **Feynman graph for Vertex diagram with virtual gluon.**

The results are presented as the form factor set to the Born vertices (as, for example, in (M. Böhm *et al.*, Fortschr. Phys. **34**, 687 (1986)), so we can easily use they to construct the cross section: all that we need is to replace the coupling constants in Born vertex to the corresponding form factors:

$$v_f^j \to \delta F_V^{jf}, a_f^j \to \delta F_A^{jf}. \tag{12}$$

Electroweak form factors $\delta F_{V,A}^{if}$ in ultrarelativistic limit depend on the Sudakov logarithms by means of functions $\Lambda_{2,3}(m_i)$ as:

$$\Lambda_2(m_i) = \frac{\pi^2}{3} - \frac{7}{2} - 3l_{i,s} - l_{i,s}^2, \quad \Lambda_3(m_i) = \frac{5}{6} - \frac{1}{3} l_{i,s}. \tag{13}$$

Then HV (LV) contribution to cross section looks like

$$\sigma_{\mathrm{HV(LV)}}^{q\bar{q}}(t) = \frac{4\pi\alpha^2}{s^2}\mathrm{Re} \sum_{i,j=\gamma,Z} D^i D^{j*} \sum_{\chi=+,-} (\lambda_{q\,\chi}^{\mathrm{F}\,i,j} \lambda_{l\,\chi}^{i,j} + \lambda_{q\,\chi}^{i,j} \lambda_{l\,\chi}^{\mathrm{F}\,i,j})(t^2 + \chi u^2). \tag{14}$$

Light and Heavy Boxes (EWK)

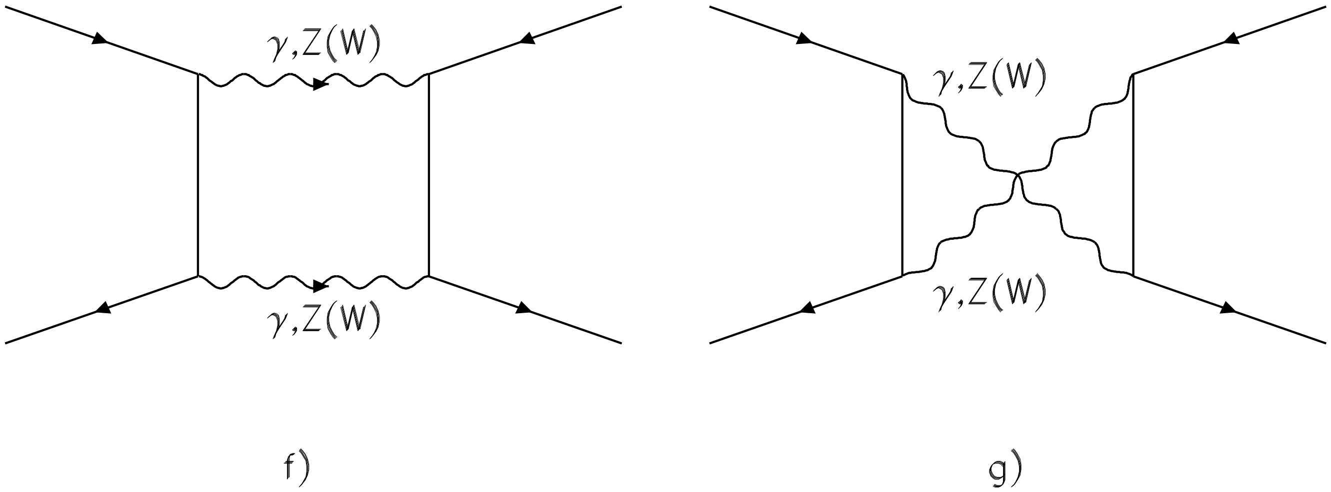

Figure 5: **Feynman graphs for Boxes.**

The calculation of two heavy boson contribution is more complicate procedure since it demands the integration of 4-point functions with complex masses in unlimited from above kinematical region of invariants (see pioneer paper: G.'t Hooft and M. Veltman, Nucl. Phys. B **153**, 365 (1979)). Fortunately there is a way to avoid many of troubles with the integration all of terms in box contribution - Asymptotic Approach (AAp).

First of all we construct the box cross section for $q\bar{q} \to l^+l^-$ using the standard Feynman rules:

$$d\sigma_{ZZ} = -\frac{4\alpha^3}{\pi s}\delta(q-p_1-p_2)\frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_2^0}\mathrm{Re}\frac{i}{(2\pi)^2}\int d^4k \sum_{k=\gamma,Z} D^{ks^*}(D^{ZZ}+C^{ZZ}), \tag{15}$$

here $D^{ZZ}(C^{ZZ})$ is contribution of direct (crossed) diagram.

Neglecting of fermion masses and polarization of particles we present the direct contribution in the form:

$$D^{ZZ} = \frac{\mathrm{Tr}[\gamma^\alpha \hat{p}_2\gamma_\mu(\hat{p}_1-\hat{k})\gamma_\nu \rho_q^{ZZ,k}(p_1)]\mathrm{Tr}[\gamma_\alpha \hat{k}_2\gamma^\mu(\hat{k}-\hat{k}_1)\gamma^\nu \rho_l^{ZZ,k}(k_1)]}{((q-k)^2-m_Z^2)(k^2-m_Z^2)(k^2-2k_1k)(k^2-2p_1k)}, \tag{16}$$

Combinations of the density matrices $\rho(p)$ and the coupling constants can be reduced to production of λ-factors (as in the "Born" formulas). For the crossed part:

$$C^{ZZ} = -D^{ZZ}\Big|_{t\leftrightarrow u}^{b_+^{ZZ,k}\leftrightarrow b_-^{ZZ,k}}, \; b_\pm^{n,k} = \lambda_{q_+}^{n,k}\lambda_{l_+}^{n,k} \pm \lambda_{q_-}^{n,k}\lambda_{l_-}^{n,k}. \tag{17}$$

To extract the part of cross section which predominates in region $s,\ |t|,\ |u| \gg m_Z^2$ we should make equivalent transformation based on the close connection of infrared divergency and SL terms:

$$D^{ZZ} = (D^{ZZ}_{k\to 0} + D^{ZZ}_{k\to q}) + (D^{ZZ} - D^{ZZ}_{k\to 0} - D^{ZZ}_{k\to q}) = D^{ZZ}_1 + D^{ZZ}_2. \qquad (18)$$

Integrating over k and retaining the terms which are proportional to the second ($\sim\ l^2_{i,x}$), first ($\sim\ l^1_{i,x}$) and zero ($\sim\ l^0_{i,x}$) power of Sudakov logarithms we get the asymptotic expressions

$$\frac{i}{(2\pi)^2}\int d^4k D^{ZZ}_1 \approx -\frac{2}{s}(b^{ZZ,k}_+ t^2 + b^{ZZ,k}_- u^2)(\frac{\pi^2}{3} + \frac{1}{2} l^2_{Z,t}), \text{t'Hooft and Veltman, 79} \qquad (19)$$

$$\frac{i}{(2\pi)^2}\int d^4k D^{ZZ}_2 \approx b^{ZZ,k}_- u \log\frac{s}{|t|} + (b^{ZZ,k}_- \frac{t^2+u^2}{2s} + b^{ZZ,k}_+ \frac{t^2}{s}) \log^2\frac{s}{|t|}, \text{Kahane, 64} \qquad (20)$$

Masses of fermions and UV parameter L are cancelled out.

Retaining only leading $\sim\ l^2_{i,x}$ term how it had been done in P. Ciafaloni and D. Comelli, Phys. Lett. B **446**, 278 (1999) we get coincidence with the results of this paper.

To obtain the WW-box contribution to Drell–Yan cross section one should:
1) to do the trivial substitution in all of indices of coupling constants and boson masses $Z \to W$,
2) to take into consideration that some parton diagrams are forbidden by the charge conservation law (direct WW-box: $d\bar{d} \to l^+l^-$ and $\bar{u}u \to l^+l^-$; crossed WW-box: $u\bar{u} \to l^+l^-$ and $\bar{d}d \to l^+l^-$). This second feature of WW-boxes explains the fact of domination of WW-contribution to Drell–Yan cross section in comparison with ZZ (or γZ) -contribution (see below in numerical analysis). Point is that the leading term of ZZ-contribution is proportional to difference

$$\delta^{ZZ,k}(t,u,b_+,b_-) - \delta^{ZZ,k}(u,t,b_-,b_+) \sim l^2_{Z,t} - l^2_{Z,u} = \log\frac{u}{t}(l^1_{Z,t} + l^1_{Z,u}), \quad (21)$$

i.e. leading terms of ZZ-box contribution $\sim l^1_{Z,x}$, whereas the leading parts of WW–cross section do not contain the difference (21) and are proportional to $l^2_{W,x}$. Let us remark here that the factorization property (21) is absent in heavy vertex part and takes place for infrared finite part of γZ–box contribution.

Infrared fifnite part (EWK and QCD RC)

Fin-part of cross section – sum of Virtual part and Soft photon part (G. 't Hooft, M. Veltman, Nucl. Phys. B. – 1979. – V. 153. – P. 365):

$$\sigma^{q\bar{q}}_{\rm fin,EWK}(t) = \frac{\alpha}{\pi}\,\delta_{\rm fin,EWK}\,\sigma^{q\bar{q}}_0(t),\quad \sigma^{q\bar{q}}_{\rm fin,QCD}(t) = \frac{4}{3}\frac{\alpha_s}{\pi}\delta_{\rm fin,QCD}\,\sigma^{q\bar{q}}_0(t), \qquad (22)$$

where corrections $\delta_{\rm fin}$ look like

$$\delta_{\rm fin,EWK} = 2\log\frac{2\omega}{\sqrt{s}}\Big(Q_q^2(\log\frac{s}{m_q^2} - 1) - 2Q_qQ_l\log\frac{t}{u} + Q_l^2(\log\frac{s}{m^2} - 1)\Big) +$$

$$+Q_l^2(\frac{3}{2}\log\frac{s}{m^2} - 2 + \frac{\pi^2}{3}) + Q_q^2(\frac{3}{2}\log\frac{s}{m_q^2} - 2 + \frac{\pi^2}{3})$$

$$-Q_qQ_l(\log\frac{s^2}{tu}\log\frac{t}{u} + \frac{\pi^2}{3} + \log^2\frac{t}{u} + 4{\rm Li}_2\frac{-t}{u}).$$

$$\delta_{\rm fin,QCD} = 2\log\frac{2\omega}{\sqrt{s}}(\log\frac{s}{m_q^2} - 1) + \frac{3}{2}\log\frac{s}{m_q^2} - 2 + \frac{\pi^2}{3} \qquad (23)$$

Comparison with existing results: BSE

$\sqrt{s}$, TeV	BSE, SANC, %	BSE, our, %
0.2	11.2259	12.2144
0.5	11.1526	11.9455
1.0	12.2096	12.9793
2.0	13.1993	13.9634
3.0	13.7682	14.5314
5.0	14.4811	15.2437
10.0	15.4456	16.2080

Table 1. The relative corrections (in per cents) from the BSE contribution to cross section at the parton level for $u\bar{u} \to \mu^+\mu^-$ as a functions of $\sqrt{s}$, calculated by different groups: SANC and our calculation.

Comparison with existing results: HV

$\sqrt{s}$, TeV	HV, SANC, %	HV, our , %
0.2	-1.6883	-1.6688
0.5	4.2958	4.2978
1.0	6.2447	6.2451
2.0	8.5534	8.5535
3.0	10.1709	10.1710
5.0	12.4894	12.4895
10.0	16.1160	16.1160

Table 2. The relative corrections (in per cents) from the HV– cross section at the parton level for $u\bar{u} \to \mu^+\mu^-$ as a functions of $\sqrt{s}$, calculated by SANC and our results.

Comparison with existing results: ZZ- and WW-boxes

$\sqrt{s}$, TeV	ZZ, SANC	ZZ, ZGRAD	ZZ, AAp	WW, SANC	WW, AAp
0.2	-0.0908	-0.0907	-0.007	-3.107	-4.874
0.5	-0.2144	-0.2145	-0.190	-10.777	-10.109
1.0	-0.3346	-0.3346	-0.325	-16.998	-16.572
2.0	-0.4638	-0.465	-0.461	-25.442	-25.250
3.0	-0.5423	-0.543	-0.541	-31.468	-31.355
5.0	-0.6432	-0.643	-0.642	-40.185	-40.131
10.0	-0.7787	-0.779	-0.778	-53.989	-53.970

Table 3. The relative corrections (in per cents) from the ZZ- and WW-box cross section at the parton level for $u\bar{u} \to \mu^+\mu^-$ as a functions of $\sqrt{s}$, calculated by different groups: SANC, program ZGRAD (U. Baur et al. `http://ubhex.physics.buffalo.edu/~baur/zgrad2.tar.gz`) and using the asymptotic formulas (AAp).

Photon bremsstrahlung

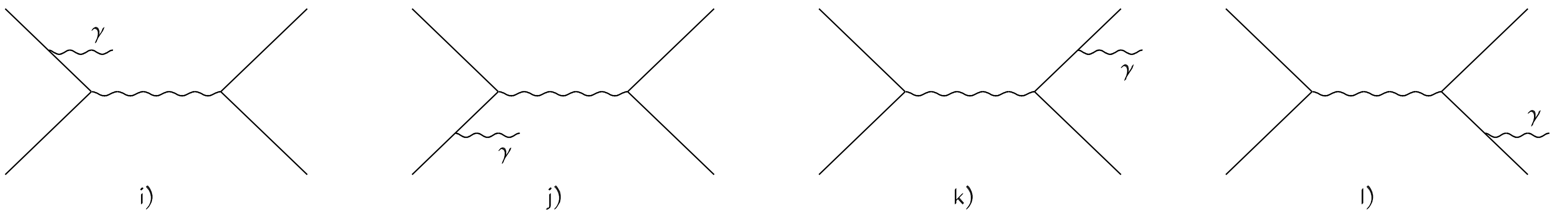

Figure 6: **Photon bremsstrahlung diagrams. Unsigned helix lines – γ or Z.**

$$I_\Omega^6[A] = \int_0^1 dx_1 \int_0^1 dx_2 \iiiint_\Omega dt dv dz du_1 \frac{1}{\pi\sqrt{R_{u_1}}} \theta(R_{u_1})\theta_M^R \theta_D^R \; A, \qquad (24)$$

with radiative invariants $z = 2k_1 p,\ v = 2k_2 p,\ z_1 = 2p_1 p,\ u_1 = 2p_2 p$, and p – 4-momenta of real photon.

For numerical integration we used Monte Carlo routine based on the VEGAS algorithm: G. Peter Lepage, J. Comput.Phys. **27**, 192 (1978)

Gluon and inverse gluon bremsstrahlung

Figure 7: **Gluon bremsstrahlung diagrams.**

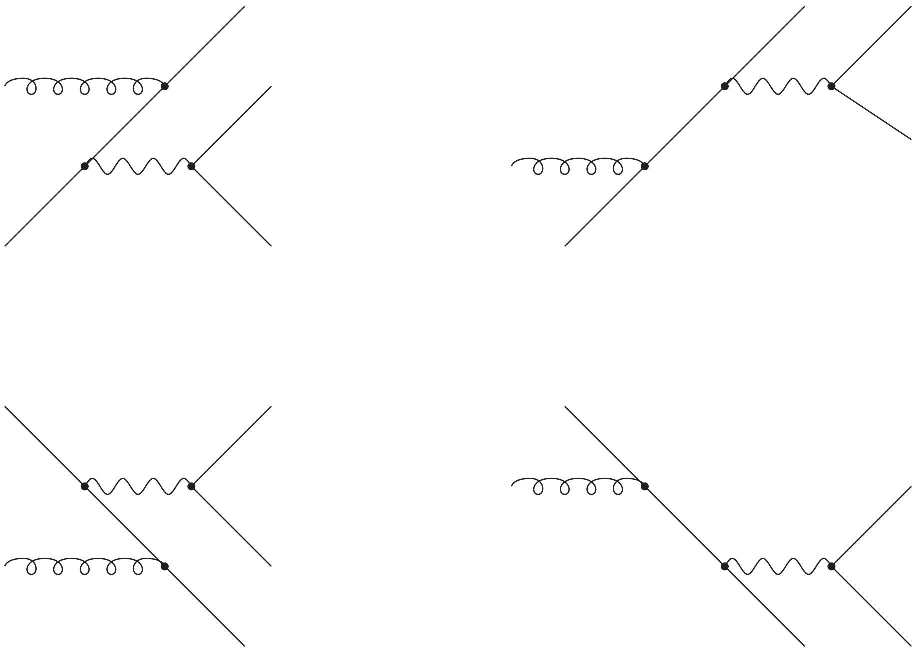

Figure 8: **Inverse gluon bremsstrahlung diagrams.**

Rebuilding to fully differential cross section

Here we rebuild all of the cross sections to completely differential form

$$\sigma_C \to \sigma_C^{(3)} \equiv \frac{d^3\sigma_C}{dMdyd\psi}, \tag{25}$$

where
$y \equiv |y(l^-l^+)|$ – dilepton rapidity,
ψ – cosine of angle between $\vec{P}_A$ and $\vec{k}_1$.

For non-radiative part the translation to differential form simply to do using the Jackobian J_N:

$$J_N = \frac{D(x_1, x_2, t)}{D(M, y, \psi)} = \frac{4M^3 e^{2y}}{S[1+\psi+(1-\psi)e^{2y}]^2}. \tag{26}$$

The radiative Jackobian can introduce in the following way

$$J_R^{(3)} = \frac{D(x_1, x_2, t)}{D(M, y, \psi)} = \frac{4Me^{2y}}{S} \frac{(v+M^2)(z_1+M^2)(u_1+M^2)}{[(1+\psi)(z_1+M^2)+(1-\psi)e^{2y}(u_1+M^2)]^2}. \tag{27}$$

Leading log part of EWK Hard bremsstrahlung

- u_1-peak of ISR: $p = (1-\eta)p_2$ i.e. photon is collinear to p_2

$$\sigma_R^{(3),\mathrm{u_1-peak}} = \frac{1}{3} J_N \frac{\alpha^3}{s_B^2} \sum_{\chi=+,-} \sum_{i,j=\gamma,Z} \sum_{q=u,d,s,c,b} Q_q^2 \lambda_{q_\chi}^{i,j} \lambda_{l_\chi}^{i,j} \Pi^i \Pi^{j^*} (t_B^2 + \chi u_B^2) \times$$

$$\times \int_{x_2^B}^{1-2\omega/M} \frac{d\eta}{1-\eta} [f_q^A(x_1^B) f_{\bar{q}}^B(\frac{x_2^B}{\eta}) + \chi f_{\bar{q}}^A(x_1^B) f_q^B(\frac{x_2^B}{\eta})](-2 + \frac{1+\eta^2}{\eta} \log \frac{s}{m_q^2}).$$

- z_1-peak of ISR: i.e. photon is collinear to p_1
- z-peak of FSR: $p = \frac{1-\eta}{\eta} k_1$ i.e. photon is collinear to k_1
- v-peak of FSR: i.e. photon is collinear to k_2
- and all interference terms.

Leading log part of QCD Hard gluon bremsstrahlung

- u_1-peak of ISR: i.e. gluon is collinear to p_2

$$\sigma_R^{(3),\mathrm{u_1-peak}} = \frac{1}{3} J_N \frac{4}{3} \frac{\alpha^2 \alpha_s}{s_B^2} \sum_{\chi=+,-} \sum_{i,j=\gamma,Z} \sum_{q=u,d,s,c,b} \lambda_{q_\chi}^{i,j} \lambda_{l_\chi}^{i,j} \Pi^i \Pi^{j^*} (t_B^2 + \chi u_B^2) \times$$

$$\times \int_{x_2^B}^{1-2\omega/M} \frac{d\eta}{1-\eta} [f_q^A(x_1^B) f_{\bar{q}}^B(\frac{x_2^B}{\eta}) + \chi f_{\bar{q}}^A(x_1^B) f_q^B(\frac{x_2^B}{\eta})](-2 + \frac{1+\eta^2}{\eta} \log \frac{s}{m_q^2}).$$

- z_1-peak of ISR: i.e. gluon is collinear to p_1

- and interference term.

Leading log part of Inverse Gluon Emission

- z_1-peak of gq-type

$$\sigma^{(3)}_{\rm IGE} = \frac{1}{3}J_N\frac{4}{3}\frac{\alpha^2\alpha_s}{s_B^2}\sum_{\chi=+,-}\sum_{i,j=\gamma,Z}\sum_{q=u,d,s,c,b}\Pi^i\Pi^{j^*}\chi\lambda_{q_\chi}^{i,j}\lambda_{l_\chi}^{i,j}(t_B^2+\chi u_B^2)\times$$

$$\times\int_{x_1^B}^{1}d\eta f_g^A(\frac{x_1^B}{\eta},Q^2)f_q^B(x_2^B,Q^2)S_\chi,\ \text{where} \quad (28)$$

$$S_+ = S_- - \frac{1-\eta(1-\eta)}{\eta^2}\frac{2t_Bu_B^2}{s_B(t_B^2+u_B^2)},$$

$$S_- = \frac{(1-\eta)^2+\eta^2}{\eta}\log\frac{(\eta-1)u_B}{\eta m_q^2}+2(1-\eta)-\frac{1-\eta}{\eta}\frac{u_B}{s_B}.$$

- $g\bar{q}$-type

- qg and $\bar{q}g$-types.

Discussion of numerical results. Code READY.

In the following the scale of radiative corrections and their effect on the observables of Drell–Yan processes will be discussed using FORTRAN program READY: (Radiative corrEctions to lArge invariant mass Drell-Yan process).

We used the following set of prescriptions:

- investigated reaction is (1) with the energy of LHC $\sqrt{S} = 7,\ 10,\ 14$ TeV,
- the set of SM input electroweak parameters: $\alpha = 1/137.03599911$, $m_Z = 91.1876$ GeV, $m_W = 80.37399$ GeV, $\Gamma_Z = 2.4924$ GeV, $\Gamma_W = 2.0836$ GeV, $m_H = 125$ GeV,
- muon mass $m_\mu = 0.105658369$ GeV, masses of fermions for loop contributions to the BSE: $m_e = 0.51099892$ keV, $m_\tau = 1.77699$ GeV, $m_u = 0.06983$ GeV, $m_c = 1.2$ GeV, $m_t = 174$ GeV, $m_d = 0.06984$ GeV, $m_s = 0.15$ GeV, $m_b = 4.6$ GeV (the light quark "effective" masses provide $\Delta\alpha^{(5)}_{had}(m_Z^2)$=0.0276),

- 5 active flavors of quarks in proton, their masses as regulators of the collinear singularity,

- "soft"-"hard" photon separator $\omega \leq 0.1$ GeV,

- CTEQ and MRST (2004QED) sets of parton distribution functions (with the choice $Q = M_{sc} = M$),

- we impose the experimental restriction conditions on the detected lepton angle $-\zeta^* \leq \zeta \leq \zeta^*$ and on the rapidity $|y(l)| \leq y(l)^*$; for CMS detector the cut values of ζ^* and $y(l)^*$ are determined as

$$y(l)^* = -\log\ \tan\frac{\theta^*}{2} = 2.5,\ \zeta^* = \cos\theta^* \approx 0.986614, \tag{29}$$

  also we used the second standard CMS restriction $p_T(l) \geq 20$ GeV,

- here we used so-called "bare" setup for muons identification requirements (no smearing, no recombination of muon and photon).

Quark Mass Singularity in QED- and QCD-corrections

In order to solve the problem of Quark mass Singularity (QS), we used the prescriptions of $\overline{\rm MS}$-scheme W. Bardeen *et al.*, Phys. Rev. D **18**, 3998 (1978), then $\log(s/m_q^2)$-terms are adsorpting into Parton Distribution Functions depending on M_{sc} – the factorization scale. The part that must be subtracted in order to avoid the QS-dependence:

$$\sigma_{\rm QS} = \frac{1}{3}\int\limits_0^1 dx_1 \int\limits_0^1 dx_2 \int\limits_{-S}^0 dt \int\limits_0^{1-2\omega/M} d\eta \sum_{q=u,d,s,c,b} \Big[\Big(f_q(x_1,Q^2)\Delta\bar{q}(x_2,\eta) + $$

$$+\Delta q(x_1,\eta) f_{\bar{q}}(x_2,Q^2)\Big)\sigma_0^{q\bar{q}}(t) + (q \leftrightarrow \bar{q}) \Big]\theta(s+t)\theta_M\theta_D,$$

$$\Delta q(x,\eta) = C_{\rm RC}\left[\frac{1}{\eta} f_q(\frac{x}{\eta}, M_{\rm sc}^2)\theta(\eta - x) - f_q(x, M_{\rm sc}^2)\right]\frac{1+\eta^2}{1-\eta}\left(\log\frac{M_{\rm sc}^2}{m_q^2(1-\eta)^2} - 1\right),$$

where $C_{\rm QED} = \frac{\alpha}{2\pi}Q_q^2$ and $C_{\rm QCD} = \frac{4}{3}\frac{\alpha_s}{2\pi}$.

For Inverse Gluon Emission (IGE) the result of QS-term substraction is trivial:

$$\sigma^{(3)}_{\rm IGE} - \sigma^{(3)}_{\rm IGE,QS} = \sigma^{(3)}_{\rm IGE}(m_q \to M_{\rm sc}).$$

Relative corrections

$$\delta_{\text{tot}} = \sigma^{(3)}_{\text{tot}} / \sigma^{(3)}_{0} \tag{30}$$

$$\sigma^{(3)}_{\text{tot}} = \sum_{V \neq 0} \sigma^{(3)}_{V} + \sigma^{(3),\ \text{hard}}_{R} - \sigma^{(3)}_{\text{QS}}, \tag{31}$$

$$\sigma^{(3),\ \text{hard}}_{R} = \sigma^{(3),\ \text{ISR}}_{R} + \sigma^{(3),\ \text{FSR}}_{R} + \sigma^{(3),\ \text{INT}}_{R}. \tag{32}$$

Independence of EWK corr. from ω (in GeV) and quark masses at $l = \mu$, $\sqrt{S} = 10$ TeV, $M = 2$ TeV, and $y = 0$, $\psi = 0$, MRST2004QED

ω	m_q/m_u	$\delta_{\rm fin}$	$\delta_{QS}^{\rm soft}$	$\delta^{\rm hard}$	$\delta_{QS}^{\rm hard}$	$\delta_{\rm fin}-\delta_{QS}^{\rm soft}$	$\delta^{\rm hard}-\delta_{QS}^{\rm hard}$	$\delta_{\rm tot}$
10	10.0	-0.4452	-0.1299	0.3142	0.1019	-0.3153	0.2123	-0.1030
	1.0	-0.4751	-0.1599	0.3367	0.1244	-0.3152	0.2123	-0.1029
	0.1	-0.5049	-0.1898	0.3592	0.1469	-0.3151	0.2123	-0.1028
1	10.0	-0.7030	-0.2319	0.5690	0.2033	-0.4711	0.3657	-0.1054
	1.0	-0.7507	-0.2796	0.6092	0.2435	-0.4711	0.3657	-0.1054
	0.1	-0.7984	-0.3274	0.6493	0.2837	-0.4711	0.3657	-0.1054
0.1	10.0	-0.9608	-0.3521	0.8265	0.3234	-0.6087	0.5031	-0.1056
	1.0	-1.0263	-0.4177	0.8845	0.3814	-0.6087	0.5031	-0.1056
	0.1	-1.0919	-0.4832	0.9425	0.4394	-0.6087	0.5031	-0.1056
0.01	10.0	-1.2186	-0.4902	1.0843	0.4615	-0.7284	0.6228	-0.1056
	1.0	-1.3020	-0.5735	1.1601	0.5373	-0.7284	0.6228	-0.1056
	0.1	-1.3854	-0.6569	1.2359	0.6131	-0.7284	0.6228	-0.1056
10^{-3}	10.0	-1.4764	-0.6461	1.3421	0.6174	-0.8303	0.7247	-0.1056
	1.0	-1.5776	-0.7473	1.4357	0.7110	-0.8303	0.7247	-0.1056
	0.1	-1.6788	-0.8485	1.5294	0.8047	-0.8303	0.7247	-0.1056

Independence of QCD RC from ω (in GeV) and quark masses at $l = \mu$, $\sqrt{S} = 10$ TeV, $M = 1$ TeV, and $y = 0$, $\psi = -0.4$, MRST2004QED

ω	$\frac{m_q}{m_u}$	$\delta_{\rm fin}$	$\delta^{\rm soft}_{QS}$	$\delta^{\rm hard}_{LL}$	$\delta^{\rm hard}_{QS}$	$\delta_{\rm fin}-\delta^{\rm soft}_{QS}$	$\delta^{\rm hard}_{LL}-\delta^{\rm hard}_{QS}$	$\delta_{\rm tot}$
10	10.0	-4.1325	-5.6712	3.4503	4.6402	1.5386	-1.1899	0.3487
	1.0	-5.5861	-7.1339	4.6085	5.7984	1.5478	-1.1899	0.3579
	0.1	-7.0397	-8.5966	5.7667	6.9566	1.5569	-1.1899	0.3670
1	10.0	-7.2432	-11.0686	6.5170	10.0013	3.8254	-3.4842	0.3412
	1.0	-9.7553	-13.5816	8.7182	12.2024	3.8263	-3.4842	0.3421
	0.1	-12.2674	-16.0946	10.9193	14.4035	3.8272	-3.4842	0.3430
0.1	10.0	-10.3539	-17.5605	9.6233	16.4887	7.2065	-6.8654	0.3411
	1.0	-13.9245	-21.1311	12.8814	19.7468	7.2066	-6.8654	0.3412
	0.1	-17.4951	-24.7018	16.1394	23.0048	7.2067	-6.8654	0.3413
0.01	10.0	-13.4647	-25.1152	12.7336	24.0429	11.6505	-11.3093	0.3412
	1.0	-18.0937	-29.7443	17.0500	28.3593	11.6506	-11.3093	0.3412
	0.1	-22.7228	-34.3733	21.3663	32.6757	11.6506	-11.3093	0.3412
10^{-3}	10.0	-16.5754	-33.7289	15.8442	32.6566	17.1535	-16.8123	0.3412
	1.0	-22.2629	-39.4164	21.2191	38.0314	17.1535	-16.8123	0.3412
	0.1	-27.9504	-45.1040	26.5940	43.4063	17.1535	-16.8123	0.3412

Comparison with existing results at hadronic level

Comparing our results for relative EWK corrections to $d\sigma/dM$ with the results of

- HORACE (C. M. Carloni Calame, G. Montagna, O. Nicrosini, A. Vicini // JHEP. 2007. Vol. 10. P. 109, arXiv:0710.1722)

- SANC (A. Andonov *et al.* Comput. Phys. Commun. 2006. Vol. 174. P. 481 (hep-ph/0411186))

- ZGRAD2 (U. Baur *et al.* Phys. Rev. D. 2002. Vol. 65, 033007, P. 1–19. (hep-ph/0108274))

published in paper C. Buttar *et al.*, Proc. of Les Houches 2007, Physics at TeV colliders, arXiv:0803.0678 (hep-ph) we have a good agreement (on level ~ 1 %) at $M \geq 1$ TeV.

Numbers for relative radiative corrections to fully-differential Drell–Yan cross sections

Main features of our numbers:

- Our motto is "ALL FOR USERS!"
- All Tables were developed as working tool for RDMS CMS (JINR, Dubna) NP group (A.Lanev, S.Shmatov et al.) and concentrates on extra large M (and $\sqrt{S}$) region;
- Tables with content depending on all (three) variables of Drell–Yan reaction;
- There is possibility to take any integrated distribution (2-differential or 1-differential). For this it is necessary to numerically integrate the cross section with relative correction as weight.

Relative corrections to fully differential cross sections: EWK RC, $l = \mu$ and $\sqrt{S} = 14$ TeV, MRST2004QED

M	y	ψ										
		-0.985	-0.800	-0.600	-0.400	-0.200	0.000	0.200	0.400	0.600	0.800	0.985
0.5	0.0	0.007	0.018	0.011	0.012	0.016	0.018	0.016	0.012	0.011	0.018	0.007
	0.6		0.038	0.034	0.026	0.021	0.019	0.020	0.021	0.019	0.009	0.028
	1.2						0.024	0.035	0.033	0.031	0.026	0.004
	1.8											-0.006
1.0	0.0	-0.027	-0.024	-0.033	-0.033	-0.029	-0.026	-0.029	-0.033	-0.033	-0.024	-0.027
	0.6		0.018	0.006	-0.005	-0.012	-0.015	-0.017	-0.019	-0.027	-0.042	-0.030
	1.2						0.010	0.018	0.014	0.007	-0.016	-0.068
	1.8											-0.083
2.0	0.0	-0.109	-0.104	-0.110	-0.107	-0.102	-0.099	-0.102	-0.107	-0.110	-0.104	-0.109
	0.5		-0.055	-0.071	-0.079	-0.082	-0.083	-0.085	-0.092	-0.108	-0.128	-0.126
	1.0				-0.046	-0.035	-0.038	-0.041	-0.049	-0.069	-0.112	-0.174
	1.5									-0.013	-0.069	-0.213
	1.9											-0.309
3.0	0.0	-0.196	-0.175	-0.173	-0.164	-0.155	-0.151	-0.155	-0.164	-0.173	-0.175	-0.196
	0.5		-0.132	-0.138	-0.139	-0.136	-0.134	-0.136	-0.148	-0.169	-0.196	-0.212
	1.0				-0.058	-0.055	-0.060	-0.068	-0.088	-0.127	-0.190	-0.271
	1.5									-0.135	-0.218	-0.408
4.0	0.0	-0.286	-0.241	-0.229	-0.213	-0.199	-0.193	-0.199	-0.213	-0.229	-0.241	-0.286
	0.4		-0.207	-0.204	-0.196	-0.188	-0.182	-0.186	-0.202	-0.227	-0.255	-0.280
	0.8			-0.093	-0.098	-0.103	-0.111	-0.129	-0.161	-0.209	-0.265	-0.333
	1.2						-0.092	-0.107	-0.133	-0.185	-0.286	-0.427
5.0	0.0	-0.370	-0.303	-0.280	-0.257	-0.238	-0.231	-0.238	-0.257	-0.280	-0.303	-0.370
	0.3		-0.271	-0.259	-0.244	-0.230	-0.224	-0.231	-0.251	-0.279	-0.311	-0.352
	0.6		-0.153	-0.158	-0.160	-0.163	-0.174	-0.197	-0.234	-0.277	-0.323	-0.386
	0.9			-0.124	-0.129	-0.149	-0.175	-0.213	-0.270	-0.344	-0.424	-0.480

Relative corrections to fully differential cross sections: EWK RC, $l = e$ and $\sqrt{S} = 14$ TeV, MRST2004QED

M	y	ψ										
		-0.985	-0.800	-0.600	-0.400	-0.200	0.000	0.200	0.400	0.600	0.800	0.985
0.5	0.0	-0.040	-0.016	-0.026	-0.024	-0.019	-0.016	-0.019	-0.024	-0.026	-0.016	-0.040
	0.6		0.003	-0.001	-0.011	-0.017	-0.019	-0.017	-0.014	-0.015	-0.027	0.002
	1.2						-0.027	-0.007	-0.008	-0.008	-0.009	-0.032
	1.8											-0.050
1.0	0.0	-0.079	-0.063	-0.075	-0.073	-0.068	-0.065	-0.068	-0.073	-0.075	-0.063	-0.079
	0.6		-0.021	-0.032	-0.046	-0.054	-0.058	-0.058	-0.059	-0.066	-0.084	-0.065
	1.2						-0.045	-0.029	-0.033	-0.040	-0.060	-0.114
	1.8											-0.145
2.0	0.0	-0.173	-0.152	-0.161	-0.157	-0.150	-0.147	-0.150	-0.157	-0.161	-0.152	-0.173
	0.5		-0.101	-0.120	-0.130	-0.134	-0.135	-0.136	-0.143	-0.159	-0.180	-0.175
	1.0				-0.111	-0.092	-0.097	-0.101	-0.109	-0.129	-0.171	-0.235
	1.5									-0.094	-0.153	-0.296
	1.9											-0.461
3.0	0.0	-0.270	-0.232	-0.232	-0.223	-0.213	-0.208	-0.213	-0.223	-0.232	-0.232	-0.270
	0.5		-0.188	-0.198	-0.201	-0.200	-0.197	-0.199	-0.209	-0.231	-0.259	-0.275
	1.0				-0.136	-0.130	-0.137	-0.147	-0.167	-0.208	-0.271	-0.352
	1.5									-0.292	-0.383	-0.562
4.0	0.0	-0.367	-0.308	-0.297	-0.280	-0.265	-0.259	-0.265	-0.280	-0.297	-0.308	-0.367
	0.4		-0.274	-0.274	-0.269	-0.261	-0.255	-0.258	-0.273	-0.298	-0.327	-0.352
	0.8			-0.173	-0.182	-0.188	-0.198	-0.217	-0.250	-0.299	-0.354	-0.422
	1.2						-0.215	-0.236	-0.266	-0.323	-0.429	-0.553
5.0	0.0	-0.459	-0.379	-0.357	-0.334	-0.314	-0.306	-0.314	-0.334	-0.357	-0.379	-0.459
	0.3		-0.349	-0.339	-0.325	-0.312	-0.305	-0.311	-0.330	-0.359	-0.391	-0.432
	0.6		-0.240	-0.248	-0.252	-0.256	-0.268	-0.292	-0.328	-0.371	-0.416	-0.479
	0.9			-0.267	-0.269	-0.295	-0.327	-0.370	-0.432	-0.510	-0.590	-0.622

Relative corrections to fully differential cross sections: QCD(qq) RC, $l = e, \mu$ and $\sqrt{S} = 14$ TeV, MRST2004QED

M	y	ψ										
		-0.985	-0.800	-0.600	-0.400	-0.200	0.000	0.200	0.400	0.600	0.800	0.985
0.5	-1.5	0.627	0.654	0.670								
	-1.0	0.510	0.515	0.531	0.541	0.542	0.546	0.547	0.546			
	-0.5	0.405	0.405	0.406	0.411	0.417	0.422	0.425	0.426	0.427	0.427	
	0.0	0.302	0.302	0.302	0.302	0.302	0.302	0.302	0.302	0.302	0.302	0.302
1.0	-1.5	0.648	0.665	0.673								
	-1.0	0.517	0.518	0.528	0.540	0.542	0.543	0.548	0.547			
	-0.5	0.411	0.406	0.414	0.416	0.419	0.424	0.426	0.428	0.430	0.428	
	0.0	0.321	0.321	0.321	0.321	0.321	0.321	0.321	0.321	0.321	0.321	0.321
2.0	-1.5	0.817	0.827	0.831								
	-1.0	0.589	0.593	0.597	0.599	0.602	0.605	0.606	0.608			
	-0.5	0.461	0.462	0.466	0.465	0.470	0.471	0.471	0.475	0.475	0.473	
	0.0	0.380	0.380	0.380	0.380	0.380	0.380	0.380	0.380	0.380	0.380	0.380
3.0	-1.2	0.839	0.838	0.842	0.846	0.850	0.851					
	-0.8	0.620	0.619	0.626	0.629	0.634	0.632	0.642	0.634	0.633		
	-0.4	0.507	0.509	0.510	0.508	0.508	0.506	0.509	0.508	0.510	0.510	
	0.0	0.446	0.446	0.446	0.446	0.446	0.446	0.446	0.446	0.446	0.446	0.446
4.0	-0.9	0.811	0.806	0.815	0.822	0.821	0.824	0.828	0.829	0.831		
	-0.6	0.652	0.644	0.646	0.654	0.657	0.662	0.666	0.666	0.671	0.670	
	-0.3	0.562	0.559	0.565	0.562	0.559	0.562	0.566	0.573	0.571	0.574	
	0.0	0.522	0.522	0.522	0.522	0.522	0.522	0.522	0.522	0.522	0.522	0.522
5.0	-0.4	0.670	0.670	0.673	0.678	0.694	0.708	0.708	0.715	0.718	0.718	
	-0.2	0.622	0.614	0.619	0.625	0.622	0.633	0.637	0.644	0.642	0.637	
	0.0	0.610	0.610	0.610	0.610	0.610	0.610	0.610	0.610	0.610	0.610	0.610

Relative corrections to fully differential cross sections: QCD (qg) RC, $l = e, \mu$ and $\sqrt{S} = 14$ TeV, CTEQ6M

M,	y	ψ										
		-0.985	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8	0.985
0.5	-1.5	-0.025	-0.121	-0.302								
	-1.0	-0.101	-0.026	-0.105	-0.198	-0.270	-0.327	-0.378	-0.429			
	-0.5	-0.210	-0.087	-0.053	-0.065	-0.105	-0.153	-0.202	-0.249	-0.301	-0.371	
	0.0	-0.371	-0.210	-0.150	-0.106	-0.077	-0.066	-0.077	-0.106	-0.150	-0.210	-0.371
1.0	-1.5	-0.031	-0.100	-0.236								
	-1.0	-0.065	-0.042	-0.093	-0.155	-0.206	-0.246	-0.281	-0.316			
	-0.5	-0.116	-0.069	-0.060	-0.070	-0.094	-0.122	-0.149	-0.176	-0.205	-0.245	
	0.0	-0.208	-0.134	-0.107	-0.089	-0.076	-0.072	-0.076	-0.089	-0.107	-0.134	-0.208
2.0	-1.5	-0.040	-0.089	-0.188								
	-1.0	-0.054	-0.049	-0.080	-0.122	-0.158	-0.187	-0.212	-0.237			
	-0.5	-0.074	-0.059	-0.059	-0.067	-0.081	-0.097	-0.113	-0.127	-0.143	-0.164	
	0.0	-0.123	-0.092	-0.082	-0.075	-0.071	-0.070	-0.071	-0.075	-0.082	-0.092	-0.123
3.0	-1.2	-0.050	-0.057	-0.103	-0.150	-0.187	-0.216					
	-0.8	-0.057	-0.053	-0.064	-0.086	-0.110	-0.132	-0.152	-0.171	-0.191		
	-0.4	-0.070	-0.062	-0.061	-0.065	-0.072	-0.081	-0.090	-0.099	-0.109	-0.122	
	0.0	-0.098	-0.080	-0.074	-0.071	-0.069	-0.068	-0.069	-0.071	-0.074	-0.080	-0.098
4.0	-0.9	-0.056	-0.055	-0.070	-0.095	-0.122	-0.146	-0.168	-0.189	-0.212		
	-0.6	-0.061	-0.058	-0.062	-0.072	-0.086	-0.102	-0.117	-0.130	-0.145	-0.165	
	-0.3	-0.069	-0.063	-0.062	-0.064	-0.068	-0.074	-0.079	-0.085	-0.091	-0.099	
	0.0	-0.085	-0.073	-0.070	-0.068	-0.067	-0.066	-0.067	-0.068	-0.070	-0.073	-0.085
5.0	-0.4	-0.063	-0.060	-0.061	-0.066	-0.073	-0.082	-0.092	-0.101	-0.110	-0.121	
	-0.2	-0.068	-0.063	-0.062	-0.063	-0.066	-0.070	-0.074	-0.077	-0.081	-0.087	
	0.0	-0.079	-0.070	-0.067	-0.066	-0.065	-0.065	-0.065	-0.066	-0.067	-0.070	-0.079

Conclusions

- The one-loop EWK and QCD RC to Drell–Yan process at extra large M in fully differential form have been studied.
- The results for 1) weak, 2) QED, 3) QCD parts are the compact expressions, they expand in Sudakov and collinear logarithms.
- At the parton/hadron level we compare the investigated radiative corrections with the existing results and obtain a rather good coincidence at $M > 1$ TeV.
- The numerical analysis is performed using FORTRAN code READY.
- It has been ascertained that the considered radiative corrections change the cross section very significantly: for example, in the central region $y = \psi = 0$ at $M = 2$ TeV,

$$\text{EWK} + \text{QCD(qq)} + \text{QCD(qg)} = -0.099 + 0.380 - 0.070$$

- The exact NNLO QCD and $\mathcal{O}(\alpha\alpha_s)$ would be desirable for a better control of theory vs. experiment

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