Three-loop beta-functions and anomalous dimensions in the SM

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ArXiv: 1212.6829
ArXiv: 1303.4364

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Some of our motivations

- The discovery of the Higgs boson "finalizes" the SM

[ATLAS, CMS '11-12]

- No clear experimental hints of New Physics

More precise studies of the SM is required

- We have all necessary tools

Why not to try? :)

NB: The three-loop Renormalization Group Equations (RGE) in the MSSM are know:

[Jack, Jones,Kord,'05]

What to calculate?

$D = 4 - 2\epsilon$

- Renormalization constants in $\overline{MS}$ scheme Z_Γ of certain dimensionally regularized 2-,3-, and 4-point Green functions Γ at 1, 2, and 3 loops
- From Z_Γ extract Z_{a_i} - ren. const. for the SM parameters a_i
- Find <u>Beta-functions</u> from Z_{a_i} and <u>anomalous dimensions</u> from Z_Γ

SM (running $\overline{\text{MS}}$) parameters

$$a_i = \left(\frac{5}{3}\frac{g_1^2}{16\pi^2}, \frac{g_2^2}{16\pi^2}, \frac{g_s^2}{16\pi^2}, \frac{y_t^2}{16\pi^2}, \frac{y_b^2}{16\pi^2}, \frac{y_\tau^2}{16\pi^2}, \frac{\lambda}{16\pi^2}\right)$$

$\frac{\lambda}{16\pi^2} \equiv \hat{\lambda}$

U(1) SU(2) SU(3)

Relation to bare parameters in $\overline{\text{MS}}$ scheme:

$$a_{k,\text{Bare}}\mu^{-2\rho_k\epsilon} = Z_{a_k} a_k(\mu) = a_k + \sum_{n=1}^{\infty} c_k^{(n)} \frac{1}{\epsilon^n},$$

$\rho_k = 1/2$ For gauge and Yukawa

$\rho_k = 1$ For Higgs self-coupling

$$\beta_i(a_k) = \left.\frac{da_i(\mu,\epsilon)}{d\ln\mu^2}\right|_{\epsilon=0}, \qquad \beta_i = \beta_i^{(1)} + \beta_i^{(2)} + \beta_i^{(3)} + \ldots$$

Known results

(courtesy to M.Steinhauser)

- Gauge couplings:
  - 1 loop:

    [Gross,Wilczeck'73; Politzer'73]

  - 2loop:

    [Jones'74,Caswell'74; Tarasov,Vladimirov'77;Egorian,Tarasov'79; Jones'81;Fischler,Hill'82;Machacek,Vaughn'83;Jack,Osborn'84]

  - 3 loop:

    [Curtright'80; Jones'80; Steinhauser'98; Pickering,Gracey,Jones'01]

Mihaila,Salomon,Steinhauser'12

(full result for the first time)

Known results

(courtesy to M.Steinhauser)

- Yukawa couplings:
  - 2 loop:

[Fischler,Oliensis'82; Machacek,Vaughn'83; Jack,Osborn'84]

  - 3 loop:

Chetyrkin, Zoller'12

(no electroweak couplings, only top Yukawa)

Known results

(courtesy to M.Steinhauser)

- Higgs sector:
  - 2 loop:

    [Machacek,Vaughn'84; Jack,Osborn'84;

    Ford, JackJones'92; Luo,Xiao'02]]
  - 3 loop:

Chetyrkin, Zoller'12

(no electroweak couplings, only top Yukawa)

Chetyrkin, Zoller'13

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Problems ...

- A lot of Feynman diagrams $\mathcal{O}\left(10^4 - 10^6\right)$

  calculation by hand – impossible...

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  calculation by hand – impossible...

- γ_5 treatment in dimensional regularization

  Antocommute or not anticommute?

$$\{\gamma_\mu, \gamma_5\} \stackrel{?}{=} 0 \quad \text{vs} \quad \mathrm{tr}\left(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5\right) \stackrel{?}{=} -4i\epsilon^{\mu\nu\rho\sigma}$$

Our solutions (I)

$\overline{MS}$ scheme!

- InfraRed Rearrangement (IRR)
  - We are interesting in UV divergencies only, so it is possible to change IR structure of the diagrams
  - This should be done without introduction of spurious IR divergencies.

[Vladimirov'80]

NB: R*-operation for dealing with both IR and UV

[Chetyrkin, Tkachov,Smirnov'82-'84]

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- InfraRed Rearrangement (IRR)
- Calculation in the unbroken phase of the SM (massless fields)
- Background field-gauge fixing in order to extract gauge beta-functions solely from self-energies

QED-like Ward identities!

see e.g. [Abbot '82], [Denner, Weiglein, Dittmaier,'95]

Our solutions (I)

- InfraRed Rearrangement (IRR) $\overline{\text{MS}}$ scheme!
- Calculation in the unbroken phase of the SM (massless fields) SU(2) unbroken!
- Background field-gauge fixing in order to extract gauge beta-functions solely from self-energies
- High level of automatization

IRR trick

- Variant 1: "MINCER"
  - Set all masses to zero
  - Set n external momenta in all relevant diagrams with 2+n legs to zero

<u>single-scale propagator-type integrals</u>

- Pro: Multiplicative renormalizability of Green functions can be used
- Con: naïve application can introduce spurios IR (infrared R* is needed)

IRR trick

- Variant II: "Bubbles" (BAMBA/MATAD)
  - Expand in external momenta around zero to a sufficient order
  - Introduce an auxilarily mass in each propagator [Misiak,Munz'94] [Chetyrkin,Misiak,Munz'97]

Single-scale vacuum integrals

- Pro: No spurious IR divergencies
- Con: Requires explicit introduction of mass counter terms of gauge and scalar fields

IRR tricks. Which one?

- Variant I:
  - Gauge and Yukawa coupling Beta-functions
  - Field anomalous dimensions
- Variant II:
  - Beta-function for Higgs self-coupling
  - Beta-function for Higgs mass parameter

The SM in BFG

$$\mathcal{L} = \mathcal{L}_{\rm G} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm F} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}.$$

$$\begin{aligned}
\mathcal{L}_{\rm G} &= -\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^i_{\mu\nu} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu},\\
G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu,\\
W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_2 \epsilon^{ijk} W^j_\mu W^k_\nu,\\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu,
\end{aligned}$$

Quantum field

Background field

$$G^a_\mu = \tilde{G}^a_\mu + \hat{G}^a_\mu, \qquad a = 1, \ldots, 8$$

$$W^i_\mu = \tilde{W}^i_\mu + \hat{W}^i_\mu, \qquad i = 1, \ldots, 3$$

$$D_\mu = \partial_\mu - i g_s T^a G^a_\mu - i g_2 \tau^i W^i_\mu + i g_1 \frac{Y_W}{2} B_\mu.$$

The SM in BFG

$$\mathcal{L} = \mathcal{L}_{\rm G} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm F} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}.$$

$$\mathcal{L}_{\rm F} \quad = \quad -y_t(\bar{Q}\Phi^c)t_R - y_b(\bar{Q}\Phi)b_R - y_\tau(\bar{L}\Phi)\tau_R + \text{h.c.}$$

SU(2) doublets

SU(2) singlets

$$Q = \begin{pmatrix} t \\ b \end{pmatrix}_L \qquad L = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$\Phi = \left(\begin{array}{c} \phi^+(x) \\ \frac{1}{\sqrt{2}}(h + i\chi) \end{array} \right), \quad \Phi^c = i\sigma^2\Phi^\dagger = \left(\begin{array}{c} \frac{1}{\sqrt{2}}(h - i\chi) \\ -\phi^- \end{array} \right)$$

The SM in BFG

$$\mathcal{L} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{FP}}.$$

$$\mathcal{L}_{\mathrm{H}} \quad = \quad (D_\mu\Phi)^\dagger (D_\mu\Phi) + m^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2$$

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$$\mathcal{L} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{FP}}.$$

$$\mathcal{L}_{\mathrm{H}} = (D_\mu \Phi)^\dagger (D_\mu \Phi) + m^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2$$

The renormalization constant and corresponding beta-function can be extracted from the renormalization of the composite operator

$$\Phi^\dagger \Phi = \left(\frac{h^2 + \chi^2}{2} + \phi^+ \phi^- \right)$$

The SM in BFG

$$\mathcal{L} = \mathcal{L}_{\rm G} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm F} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}.$$

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi_G} G_G^a G_G^a - \frac{1}{2\xi_W} G_W^i G_W^i - \frac{1}{2\xi_B} G_B^2,$$

Gauge-fixing parameters (parameter Beta-functions should NOT depend on them)

$$G_G^a = \partial_\mu \tilde{G}_\mu^a + g_s f^{abc} \hat{G}_\mu^b \tilde{G}_\mu^c$$

$$G_W^i = \partial_\mu \tilde{W}_\mu^i + g_2 \epsilon^{ijk} \hat{W}_\mu^j \tilde{W}_\mu^k$$

$$G_B = \partial_\mu \tilde{B}_\mu$$

For non-abelian quantum fields ordinary derivative is substituted by the covariant one involving corresponding background fields

The SM in BFG

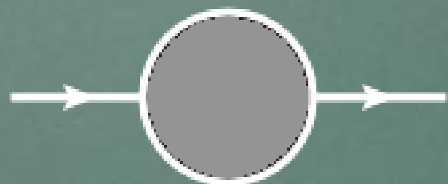
$$\mathcal{L} = \mathcal{L}_{\rm G} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm F} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}.$$

$$\mathcal{L}_{\rm FP} = -\bar{c}_\alpha \frac{\delta G_\alpha}{\delta \theta^\beta} c_\beta \qquad \alpha, \beta = (G, W, B)$$

$$\begin{aligned}
\delta \tilde{G}^a_\mu &= (D_\mu \theta_G)^a = \partial_\mu \theta^a_G + g_s f^{abc} G^b_\mu \theta^c_G\,, \\
\delta \tilde{W}^i_\mu &= (D_\mu \theta_W)^i = \partial_\mu \theta^i_W + g_2 \epsilon^{ijk} W^j_\mu \theta^k_W\,, \\
\delta \tilde{B}_\mu &= \partial_\mu \theta_B\,.
\end{aligned}$$

Infinitesimal quantum gauge transformation

What we calculate?



$f = t,\ b,\ \tau$

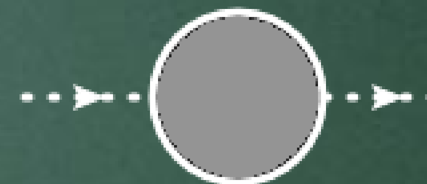
$\swarrow \quad \searrow$

$Z_{f_L} \qquad Z_{f_R}$

$V = G^a, W^i, B$

$\swarrow \quad \searrow$

$Z_{\hat{V}} \qquad Z_{\tilde{V}}$



$h,\ \phi^{\pm},\ \chi$

$\downarrow$

$Z_h = Z_{\phi^{\pm}} = Z_{\chi}$

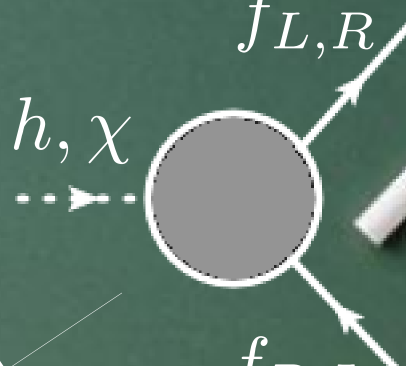
$$Z_{g_1} = Z_{\hat{B}}^{-1/2},\ Z_{g_2} = Z_{\hat{W}}^{-1/2},\ Z_{g_s} = Z_{\hat{G}}^{-1/2}$$

BFG

$$Z_{y_f} = \frac{Z_{ff\phi}}{\sqrt{Z_{f_L} Z_{f_R} Z_{\phi}}}$$

MINCER

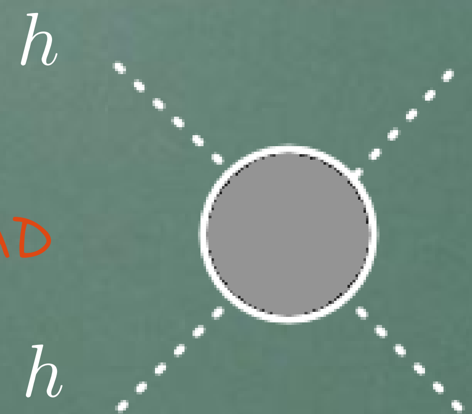
$\phi = h, \chi$

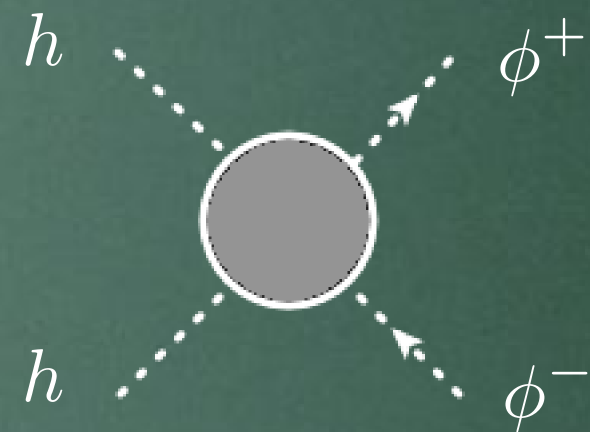


$\swarrow$ $Z_{ff\phi}$

What we calculate?

MATAD





BAMBA

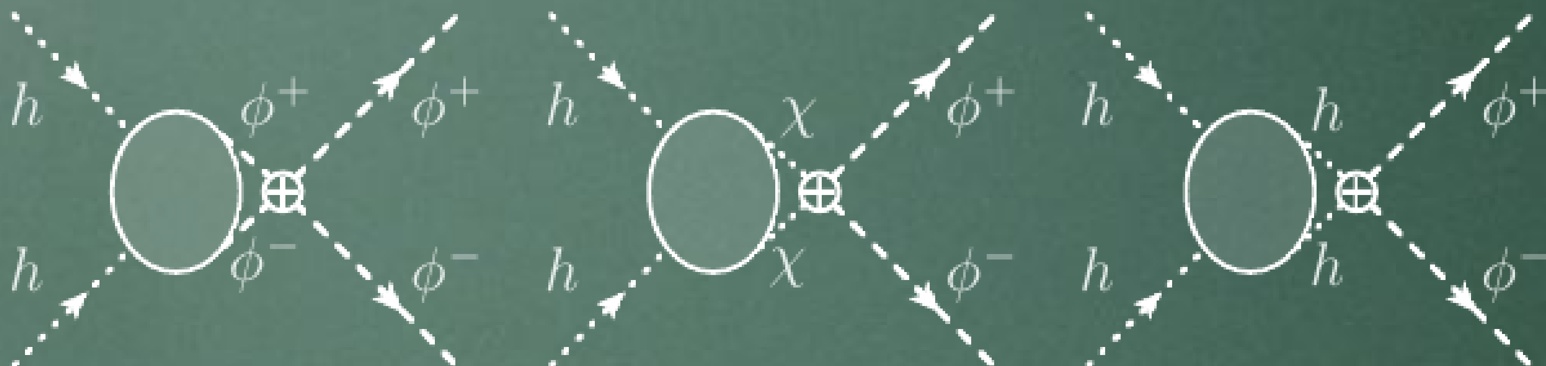
$$Z_{hhhh} = Z_{hh\phi^+\phi^-}$$

SU(2)

$$Z_\lambda = \frac{Z_{hhhh}}{Z_h^2} = \frac{Z_{hh\phi^+\phi^-}}{Z_h Z_\phi}$$

A comment on mass anomalous dimension

Mass parameter m^2 anomalous dimension can be found by considering the following diagrams:



i.e., by selecting the diagrams which contribute to $hh\phi^+\phi^-$ Green function and have ϕ^+, ϕ^- external particles connected to a four-vertex

$$Z_{m^2} = \frac{Z_{hh\,[\phi^+\phi^-]}}{Z_h}$$

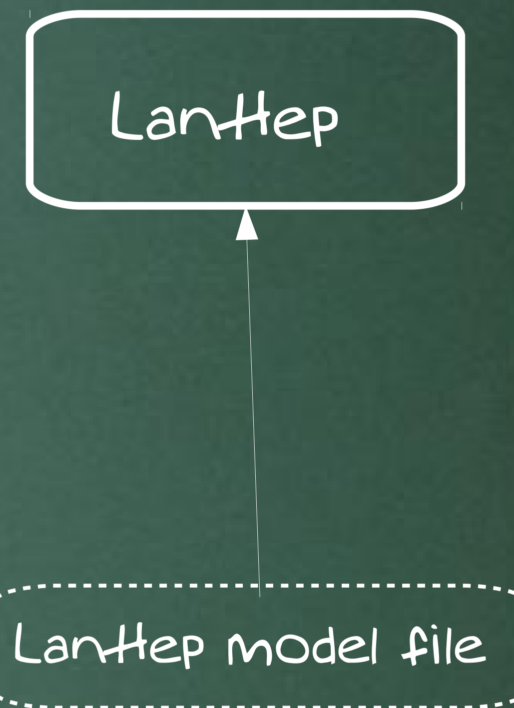
By V.N.Velizhanin

ACAT 2013, Beijing, 16 May 2013

Our setup

- Public and private computer codes:
  - LanHEP [A.Semenov]
  - FeynArts [Kublbeck,Eck,Mertig,Hahn]
  - Diana (QGRAF)[Fleischer,Tentyuokov]([Nogueira])
  - MINCER [Gorishii, Larin,Surguladze,Tkachov,Vermaseren]
  - COLOR [van Ritbergen,Schellekens,Vermaseren]
  - BAMBA [Velizhanin]
  - MATAD [Steinhauser]
  - Some awk/python/bash magic

Automatization...



FeynRules was used by [Mihaila et al,'12]

Variant I (MINCER)

Automatization...

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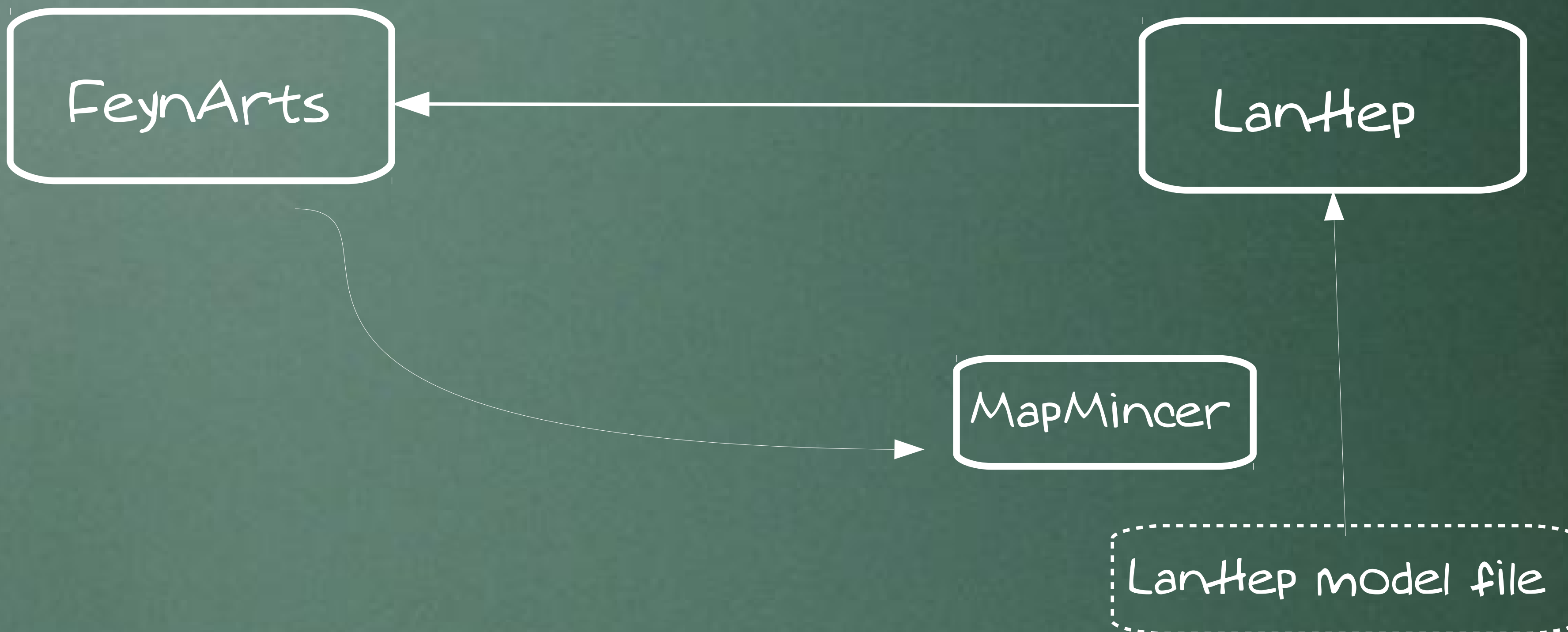

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Variant 1 (MINCER)

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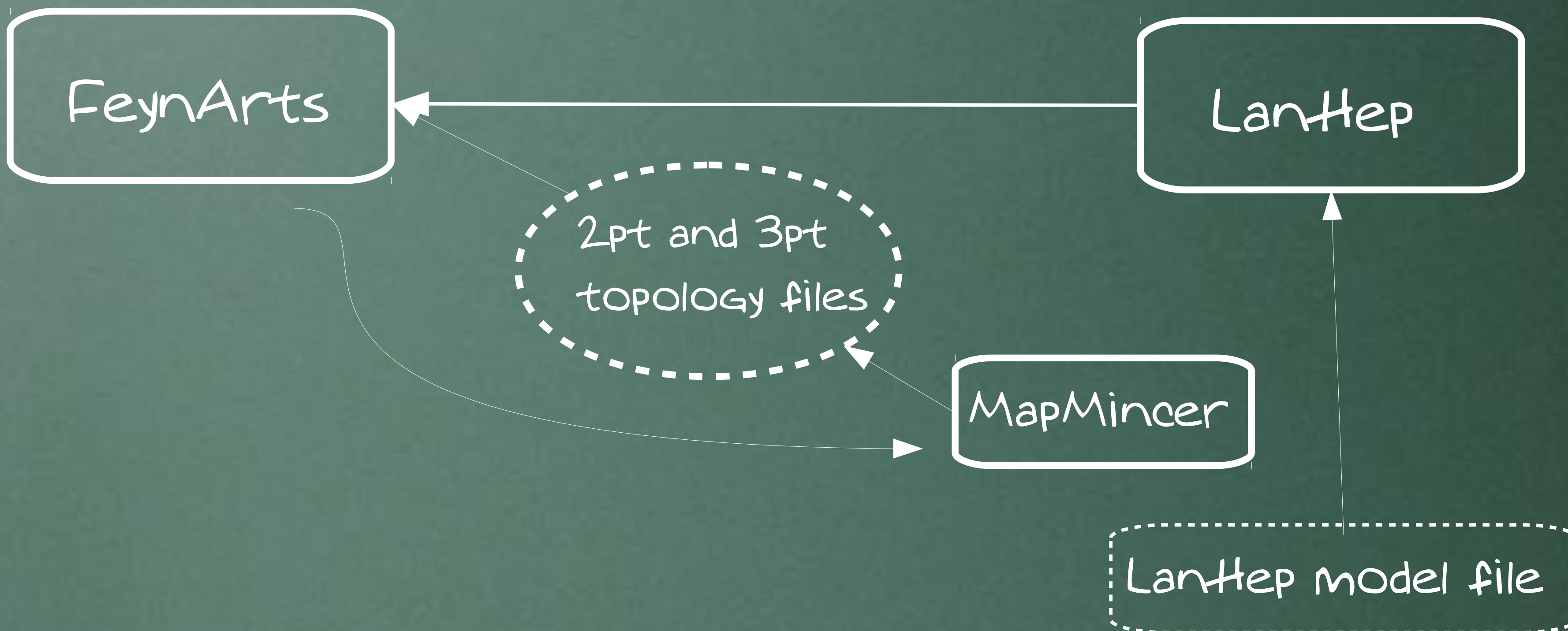

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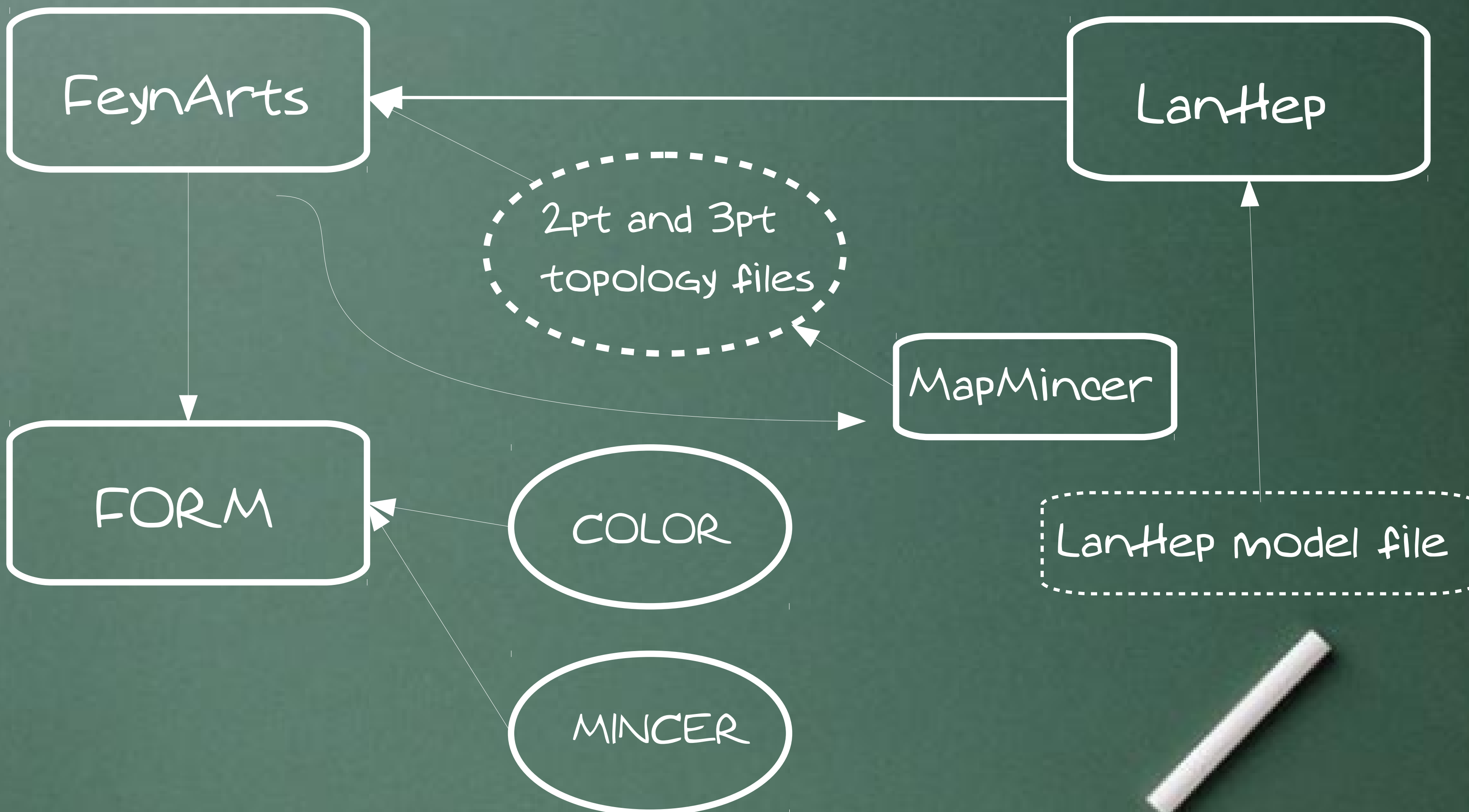

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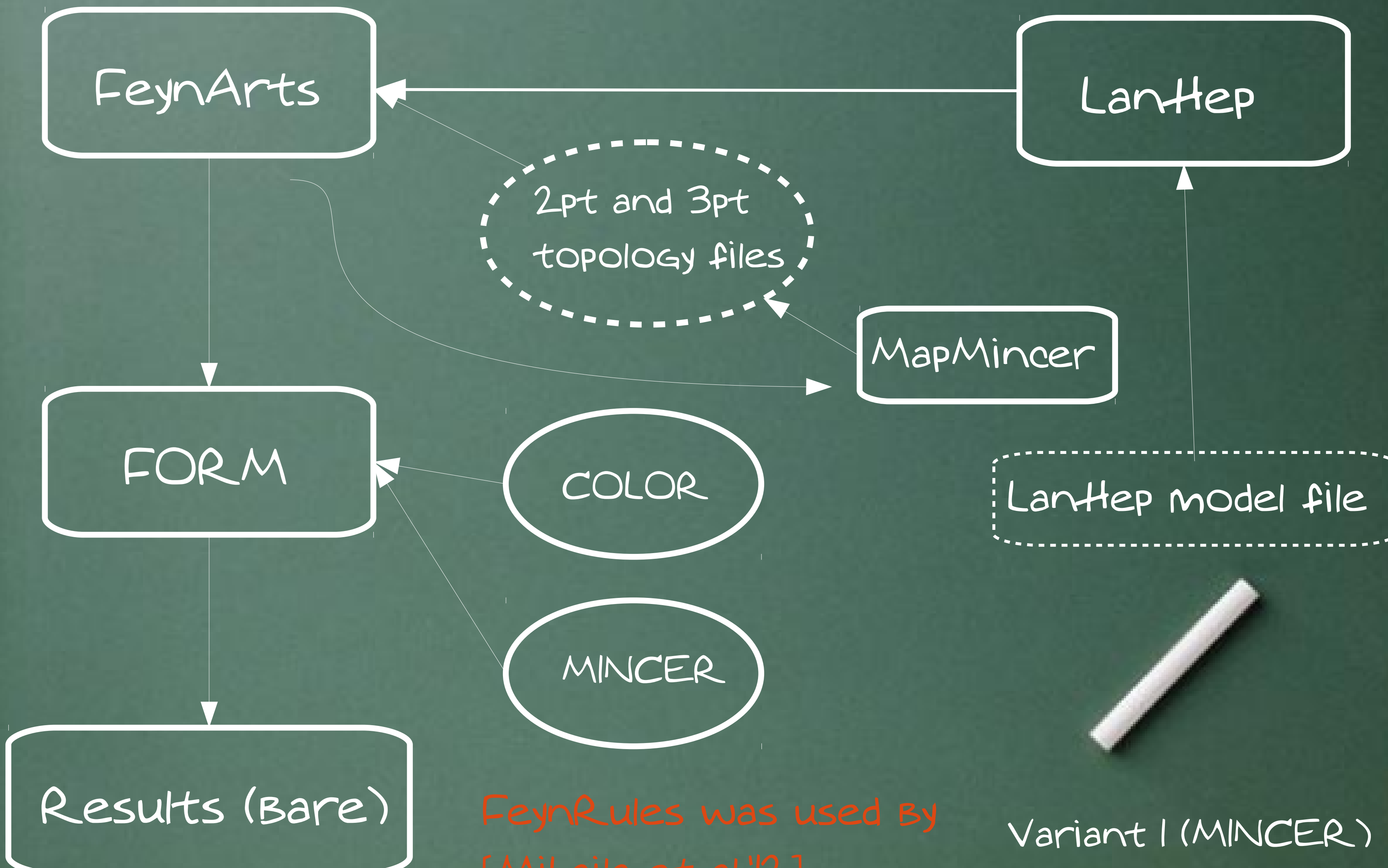

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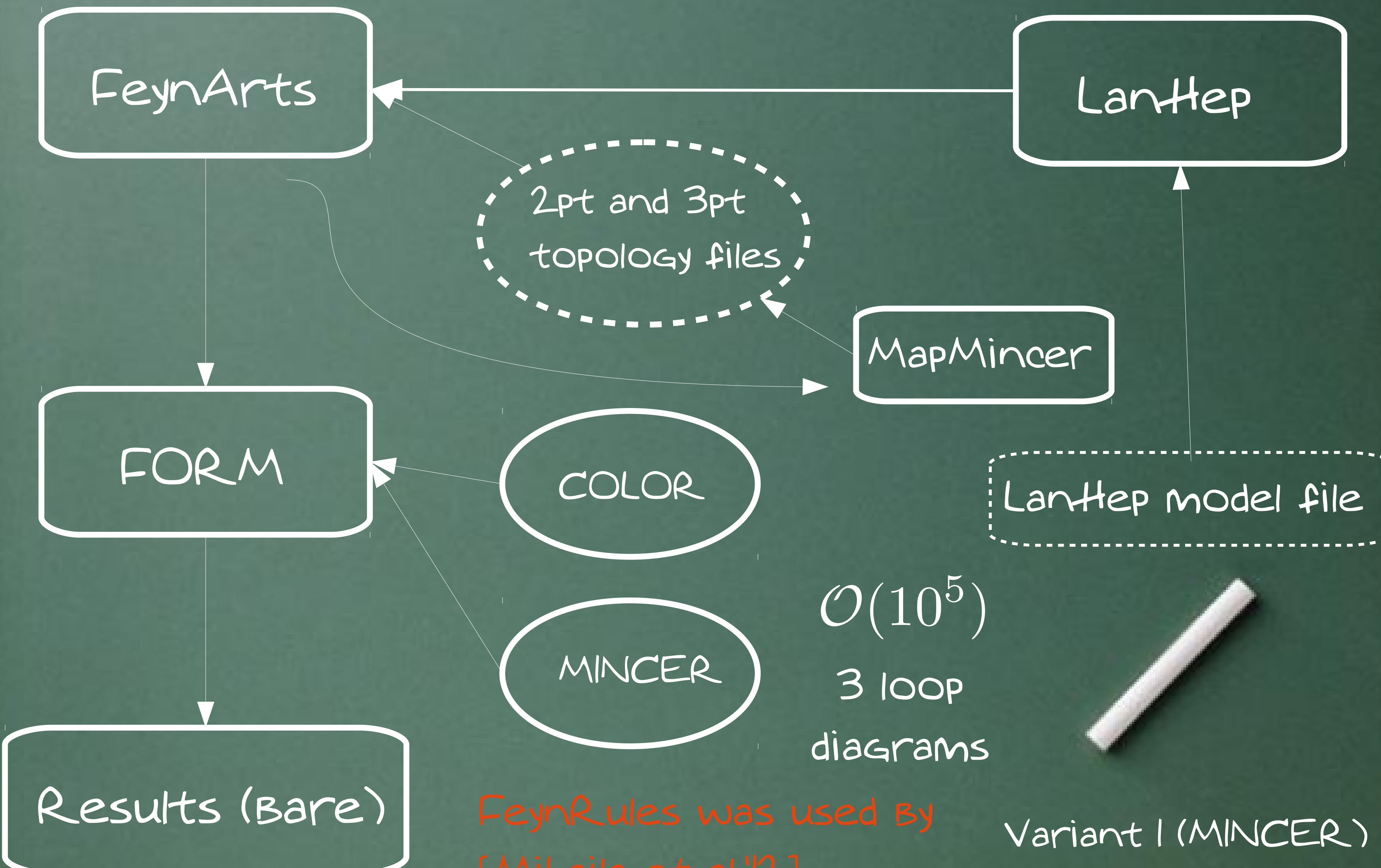

$\mathcal{O}(10^5)$

3 loop diagrams

FeynRules was used by [Mihaila et al,'12]

Variant 1 (MINCER)

Automatization...

Diana
(QGRAF)

Variant II (BUBBLES)

Automatization...

Variant II (bubbles)

Automatization...

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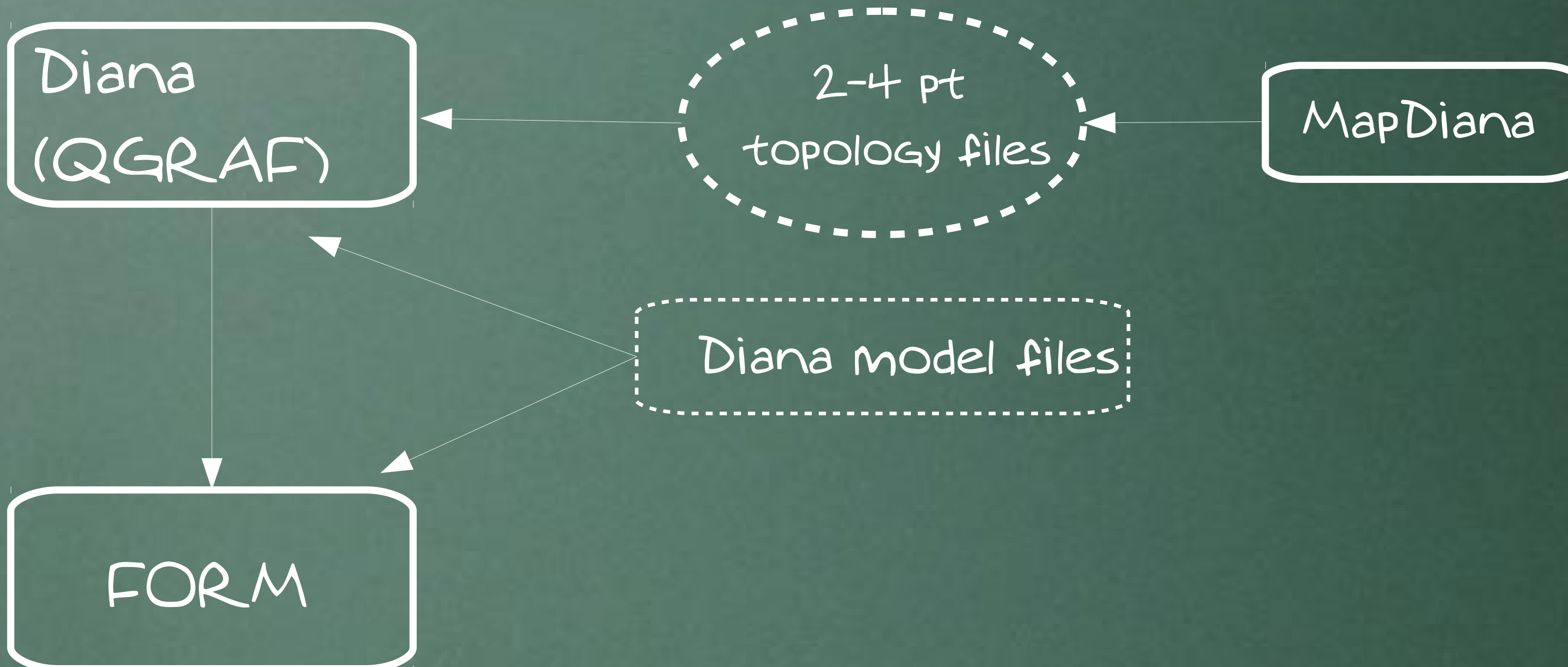

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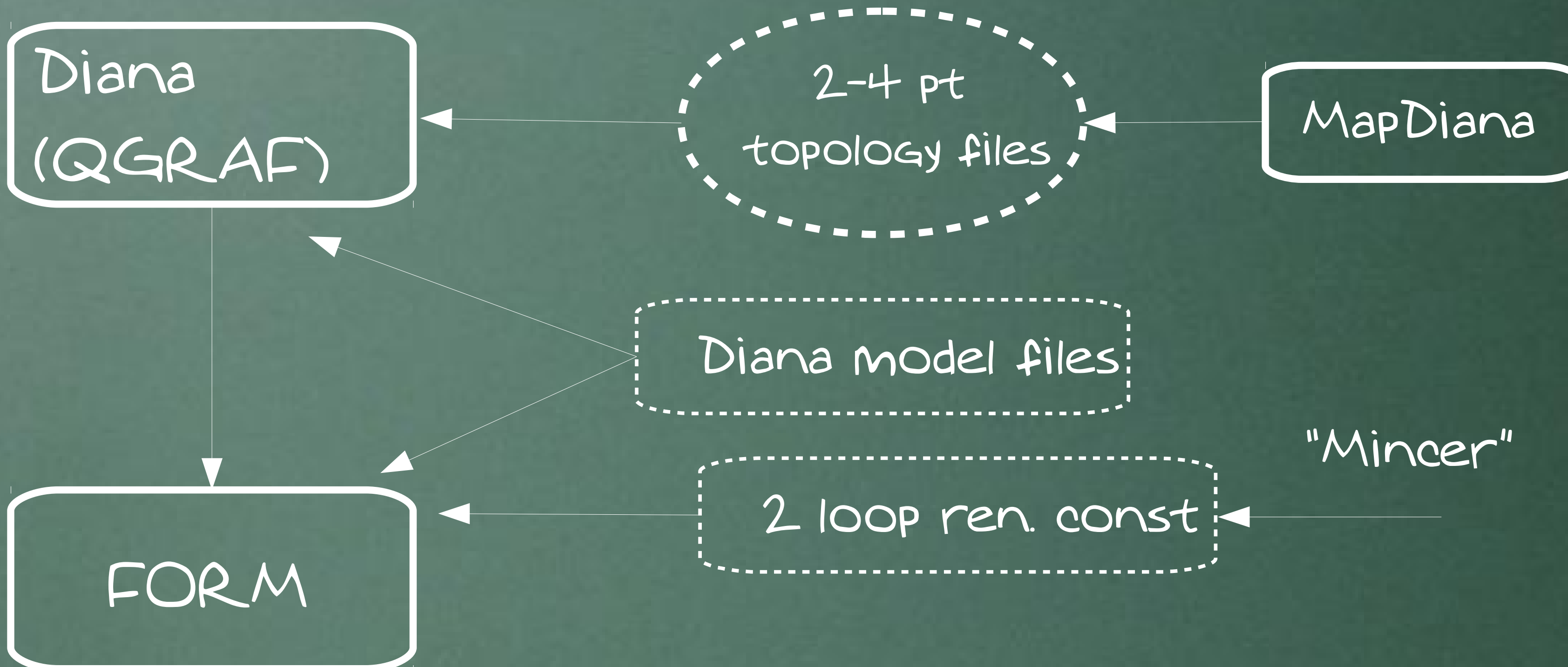
Automatization...



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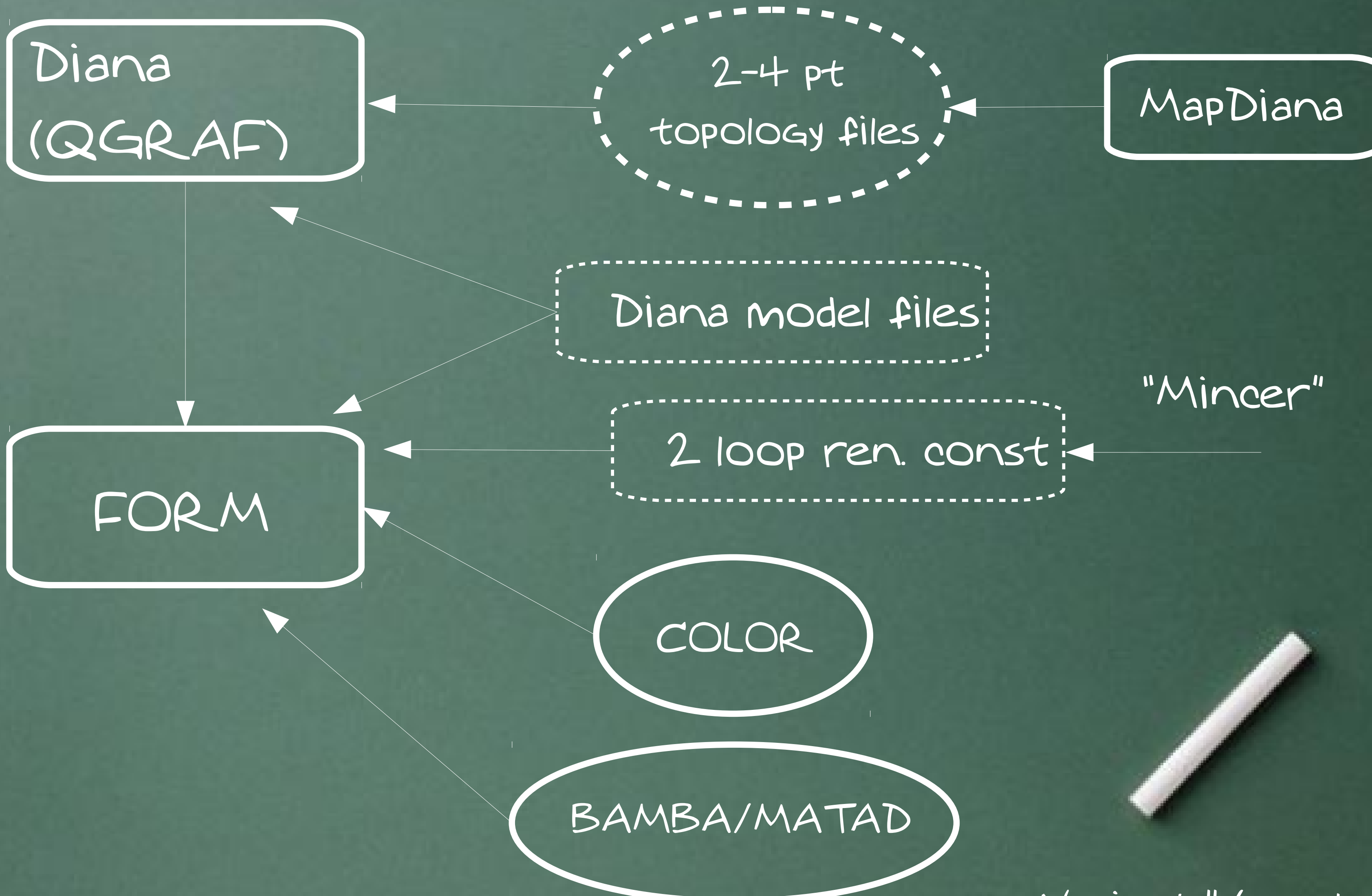

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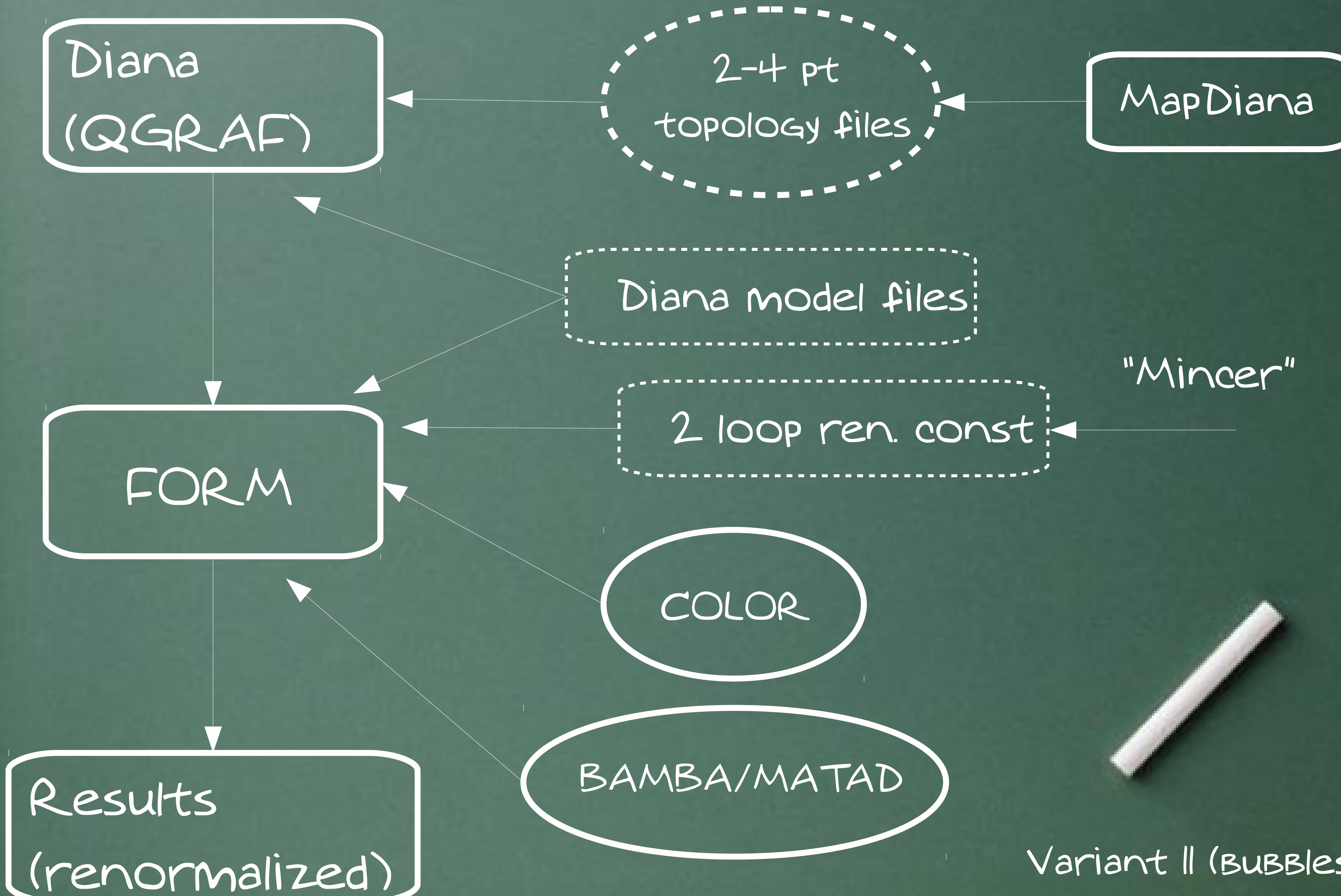

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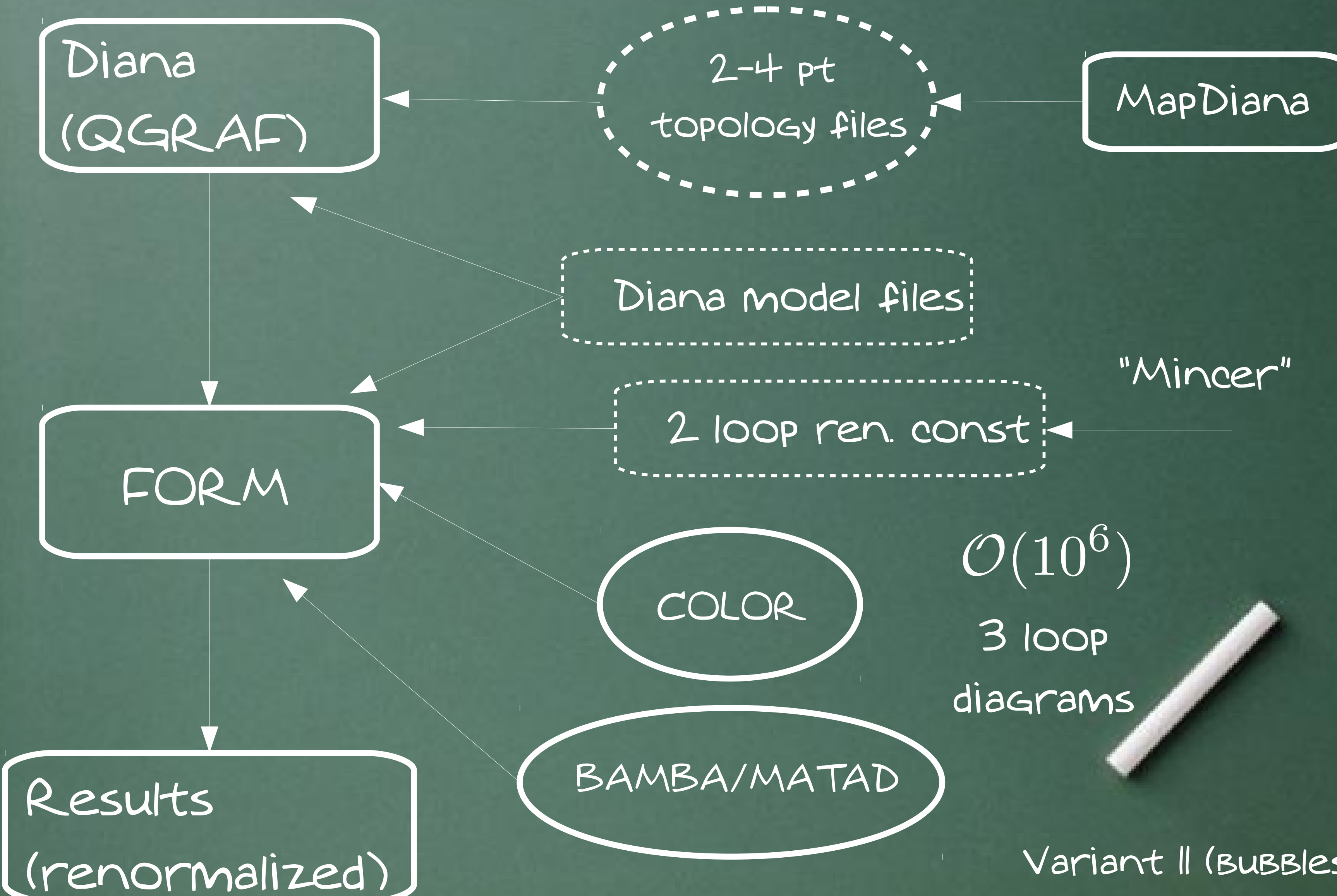


Variant II (bubbles)

...and (our) "solutions" (II)

$$\gamma_5 ?$$

In D dimensions

See review [Jegerlehner,'00]

$$\{\gamma_\mu, \gamma_5\} = 0 \Rightarrow \mathrm{tr}\,(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5) = 0$$

Naive DREG

...and (our) "solutions" (II)

$$\gamma_5 ?$$

In D dimensions

See review [Jegerlehner;'00]

$$\{\gamma_\mu, \gamma_5\} = 0 \quad \Rightarrow \quad \mathrm{tr}\,(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5) = 0$$

Naive DREG

How about?

$$\mathrm{tr}\,(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5) \stackrel{?}{=} -4i\epsilon^{\mu\nu\rho\sigma}$$

4D object!

Semi-naive Gamma5

- Semi-naive treatment of Gamma5:
  - Use $\{\gamma_5, \gamma^\mu\} = 0, \quad \gamma_5^2 = 1$

    to put all the gamma5's to the rightmost position in a fermion chain*
  - "Even" traces (no γ_5 left) pose no problem
  - In "Odd" traces (one γ_5 left) we use

$$\gamma_5 = -\frac{i}{4!}\epsilon_{\mu_1\mu_2\mu_3\mu_4}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}$$

$\mathcal{O}(\epsilon)$

Difference!

  - Contract Eps-tensors as in 4D!

Semi-naive Gamma5

Do we have a contibution ("EPS-contibution") from this?

$$\mathrm{tr}\left(\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}\gamma_5\right) = -4i\epsilon^{\mu\nu\rho\sigma}$$

Vector indices should be contracted either with external momenta or with each other

Two closed fermion loops are required

What kind of fermion loops appear in our calculations?

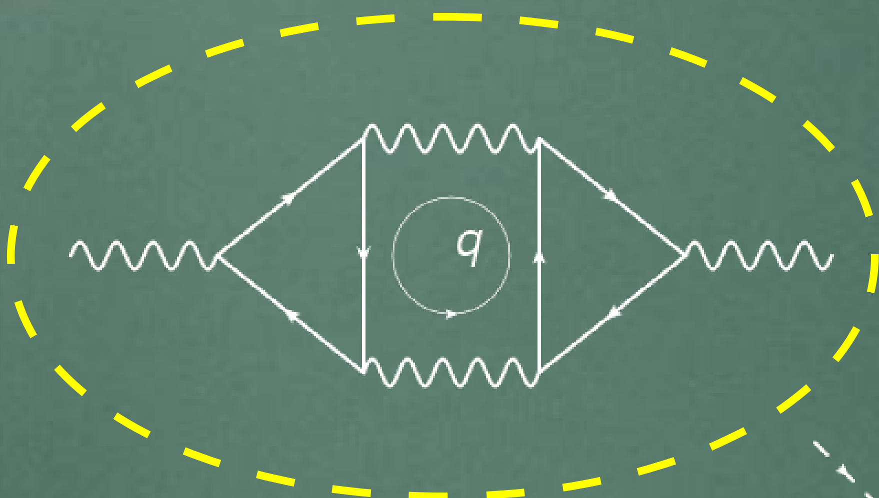
Semi-naive Gamma5

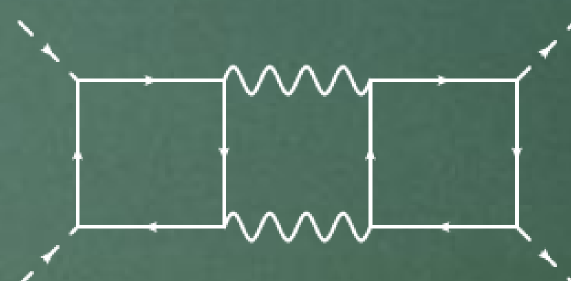
What kind of fermion loops appear in our calculations?

Two <u>internal</u> fermion loops

Only Bosonic external fields at three loops

Gauge-Anomaly cancelations: No EPS-contibution upon summation over the SM fermions





"Dangerous" diagrams

$$\frac{\mathcal{O}(\epsilon)}{\epsilon^2} \text{ error}$$

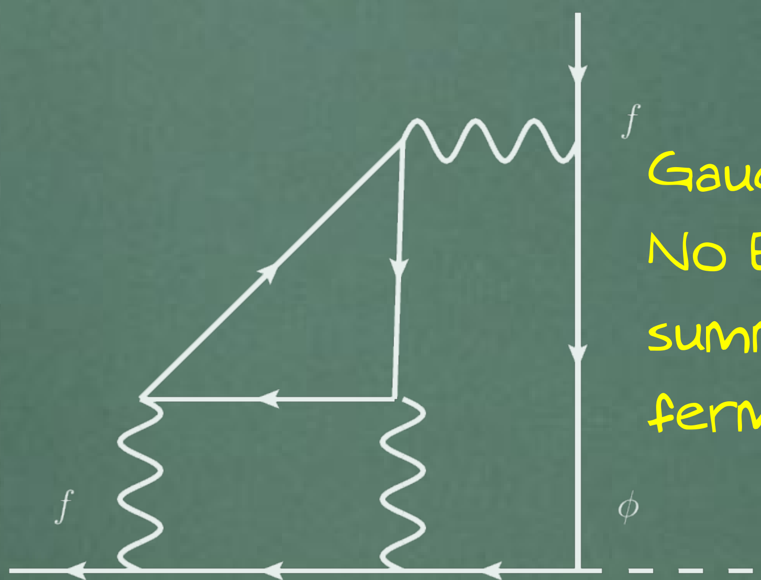
Semi-naive Gamma5

What kind of fermion loops appear in our calculations?

One internal loop and one external fermion chain

Relevant for our calculation of Yukawa coupling Z's

Become a loop in Dirac space upon contraction with a projector



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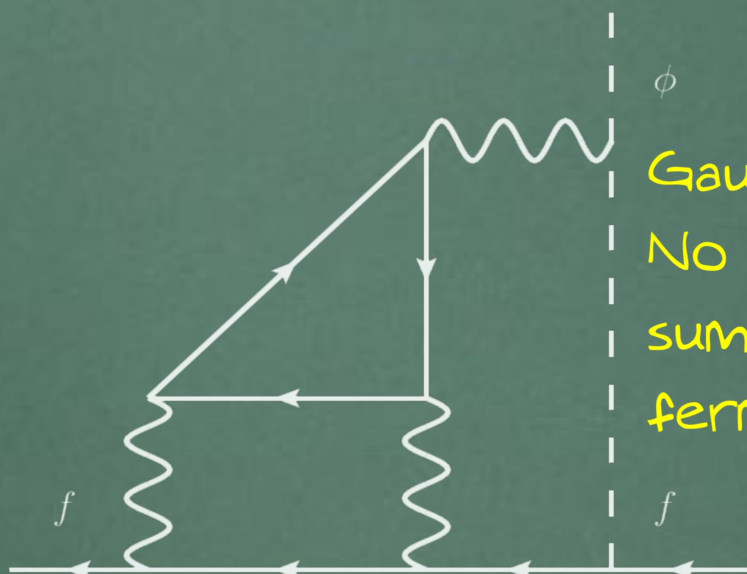

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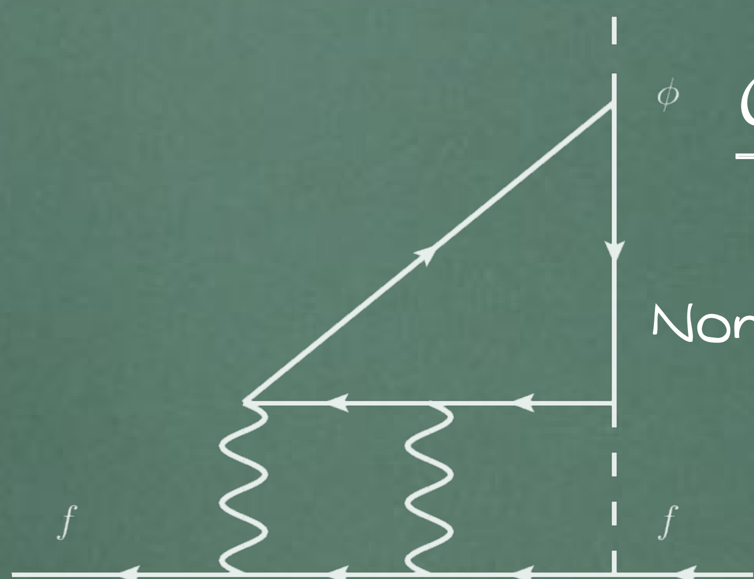


$\frac{\mathcal{O}(\epsilon)}{\epsilon}$ error → Not important

Non-trivial contribution!

[Chetyrkin,Zoller'12]

Semi-naive Gamma5

A comment about non-cyclicity of Trace operation...

The price of simultaneous application of the above-mentioned rules is the fact that there is an ambiguity in positioning of gamma5 in a trace, e.g.

$$g^{\mu_1\mu_6}\left[\mathrm{Tr}\left(\gamma_5\gamma_{\mu_1}\ldots\gamma_{\mu_6}\right)-\mathrm{Tr}\left(\gamma_{\mu_6}\gamma_5\gamma_{\mu_1}\ldots\gamma_{\mu_5}\right)\right]\propto(D-4)\epsilon_{\mu_1\ldots\mu_5}$$

[Kreimer'94]

Can spoil gauge anomaly cancelations if treated non-consistently...

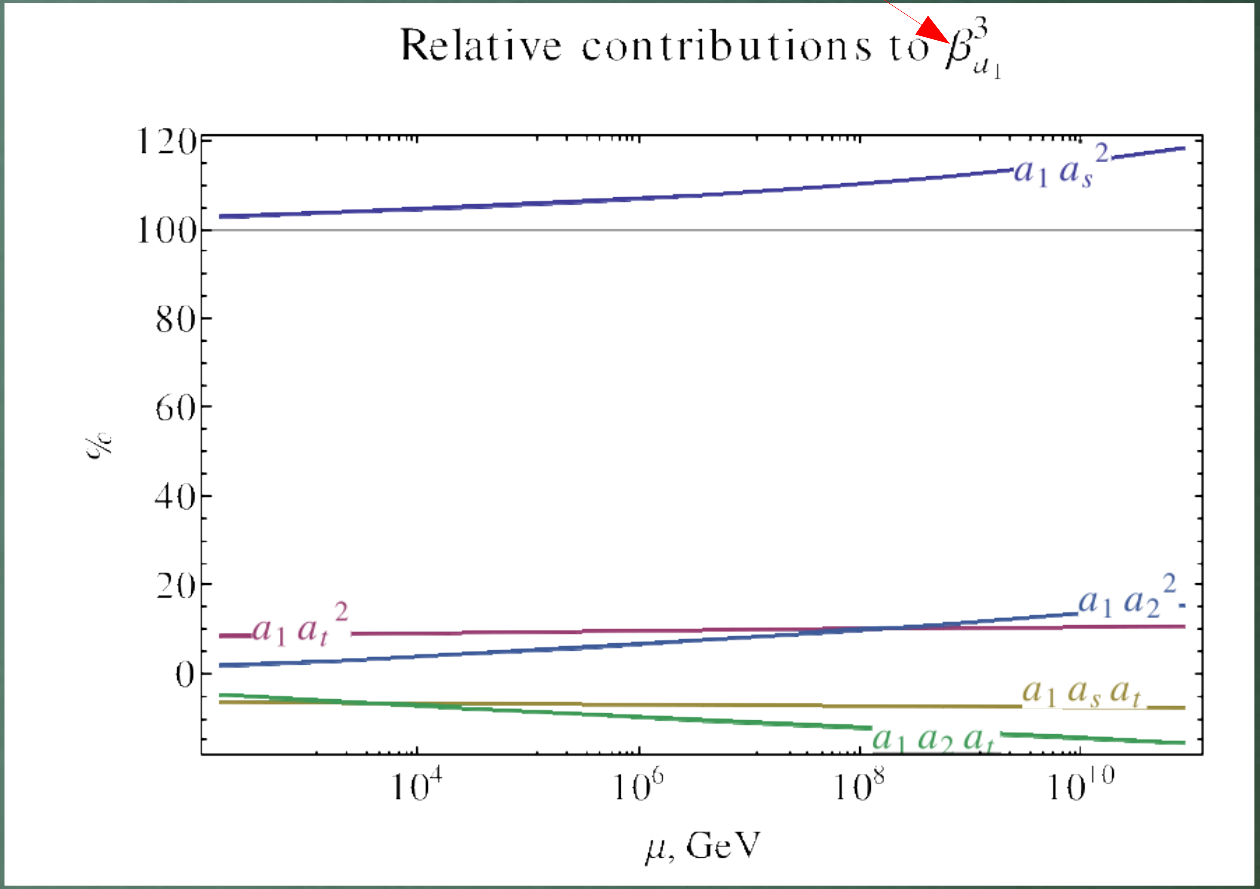
We are lucky! FeynArts and DIANA uniquely define "cut" points of closed fermion chains for all the diagrams with the same "Generic" propotype

Results. U(1) gauge coupling

Three-loop contribution to the three-loop beta-function:)

Initial values:

$$g_1 = 0.3576$$
$$g_2 = 0.6514$$
$$g_s = 1.2063$$
$$y_t = 0.9665$$
$$y_b = 0.016$$
$$y_\tau = 0.01$$
$$\lambda = 0.13$$
$$\mu = 100\ \text{GeV}$$



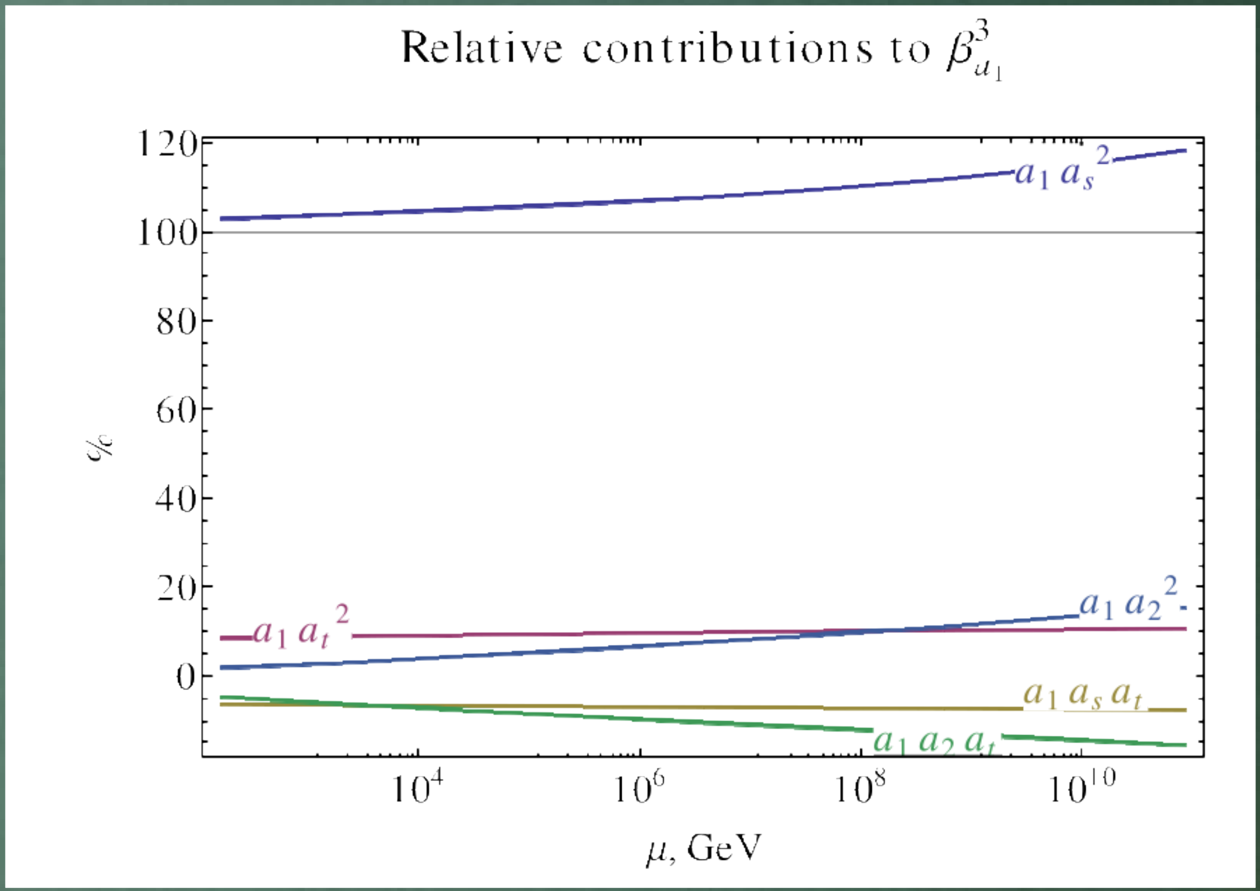
F.Bezrukov: http://www.inr.ac.ru/~fedor/SM

http://arxiv.org/src/1210.6873/anc

Results. U(1) gauge coupling

Initial values:

$$a_1 = 0.00134996$$
$$a_2 = 0.00268702$$
$$a_s = 0.009215$$
$$a_t = 0.00592$$
$$a_b = 1.62 \cdot 10^{-6}$$
$$a_\tau = 6.33 \cdot 10^{-7}$$
$$\hat{\lambda} = 0.00088$$

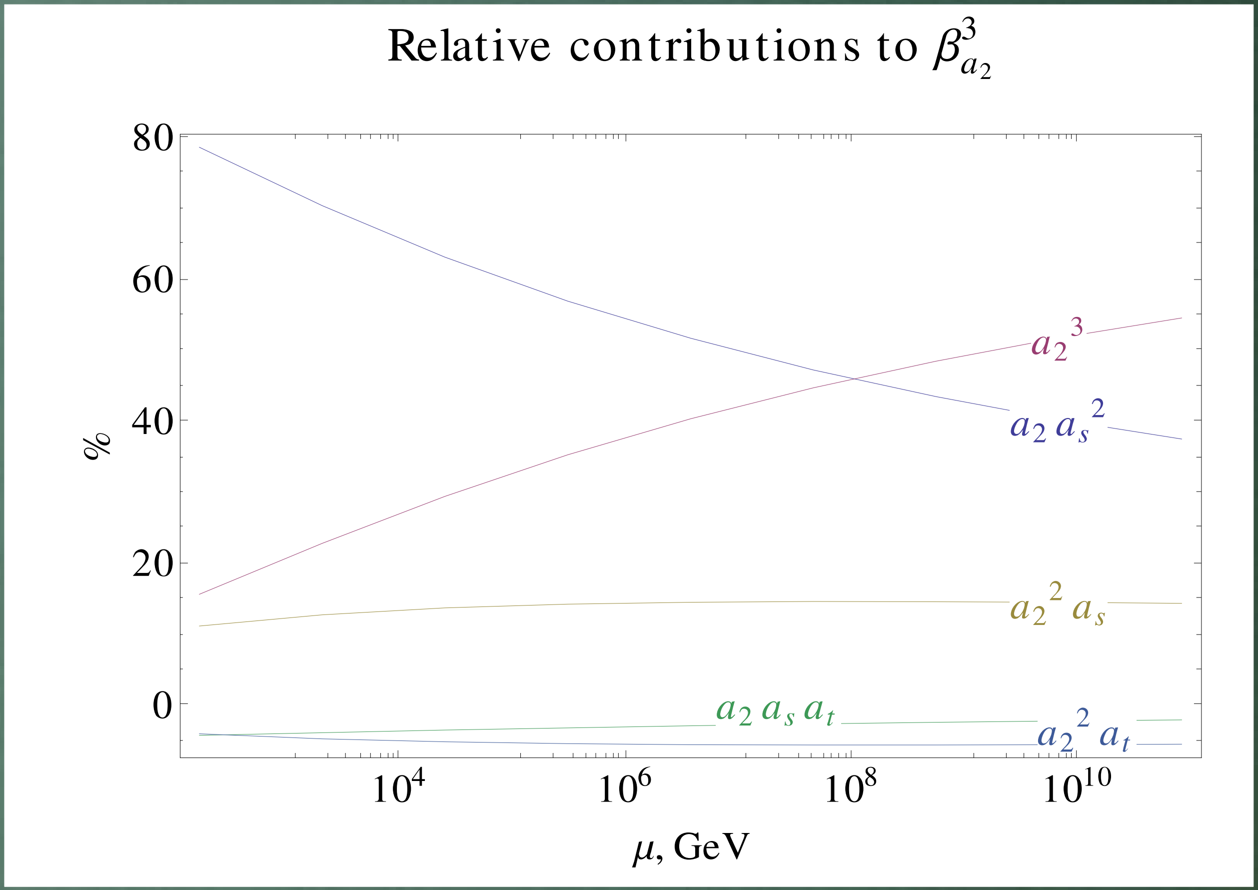


http://arxiv.org/src/1210.6873/anc

Results. SU(2) gauge coupling

Initial values:

$$a_1 = 0.00134996$$
$$a_2 = 0.00268702$$
$$a_s = 0.009215$$
$$a_t = 0.00592$$
$$a_b = 1.62 \cdot 10^{-6}$$
$$a_\tau = 6.33 \cdot 10^{-7}$$
$$\hat{\lambda} = 0.00088$$



http://arxiv.org/src/1210.6873/anc

Results. SU(3) gauge coupling

Initial values:

$$a_1 = 0.00134996$$

$$a_2 = 0.00268702$$

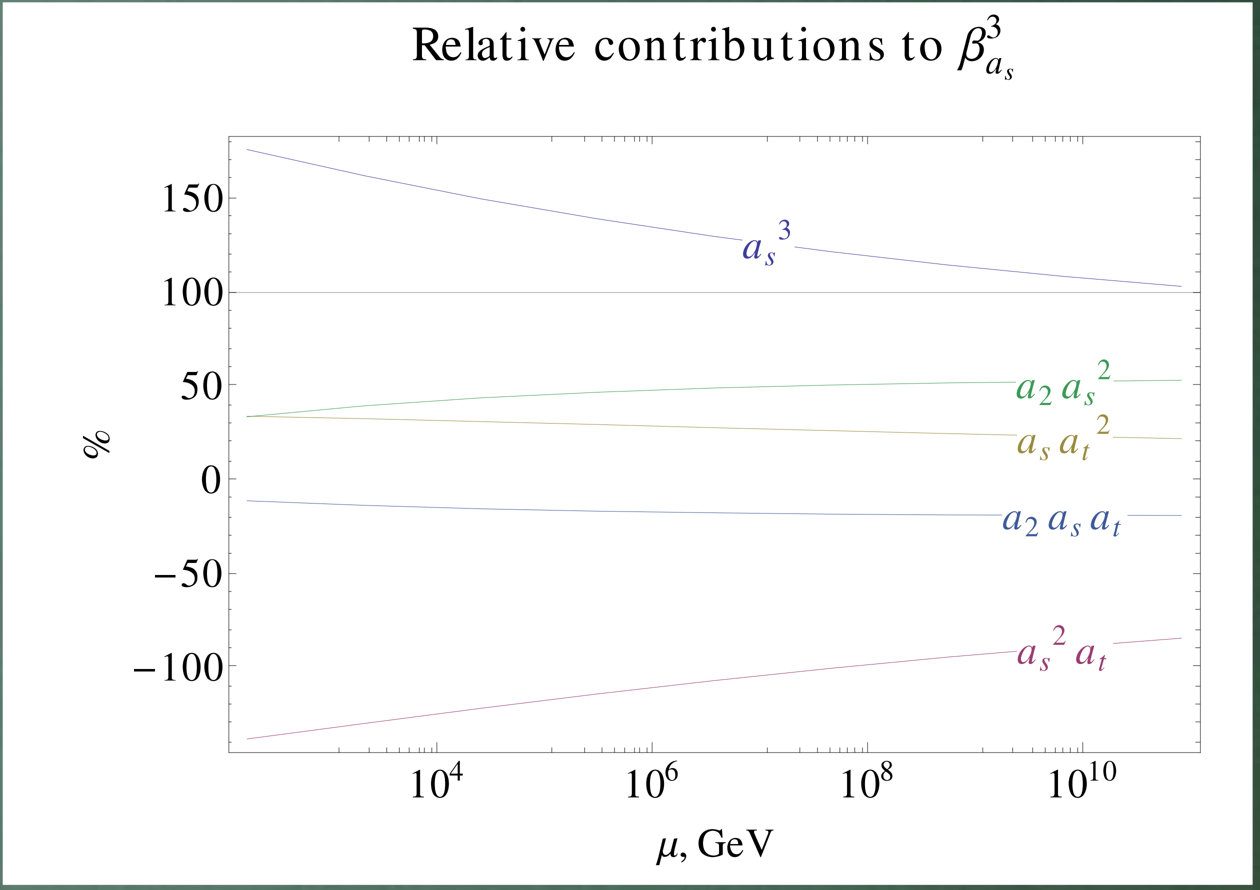
$$a_s = 0.009215$$

$$a_t = 0.00592$$

$$a_b = 1.62 \cdot 10^{-6}$$

$$a_\tau = 6.33 \cdot 10^{-7}$$

$$\hat{\lambda} = 0.00088$$



http://arxiv.org/src/1210.6873/anc

Results. Top Yukawa

Initial values:

$$a_1 = 0.00134996$$

$$a_2 = 0.00268702$$

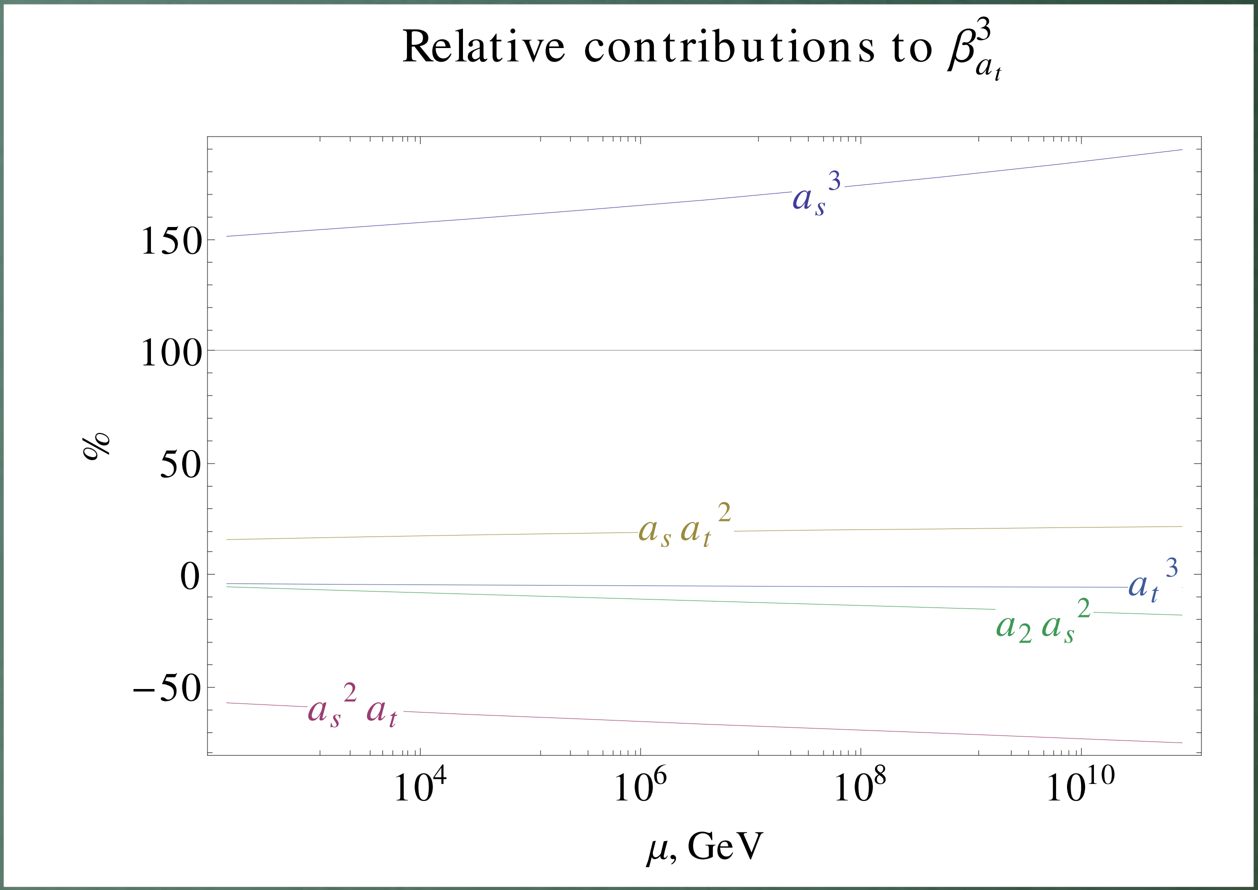
$$a_s = 0.009215$$

$$a_t = 0.00592$$

$$a_b = 1.62 \cdot 10^{-6}$$

$$a_\tau = 6.33 \cdot 10^{-7}$$

$$\hat{\lambda} = 0.00088$$



http://arxiv.org/src/1212.6829/anc

Results. Bottom Yukawa

Initial values:

$$a_1 = 0.00134996$$

$$a_2 = 0.00268702$$

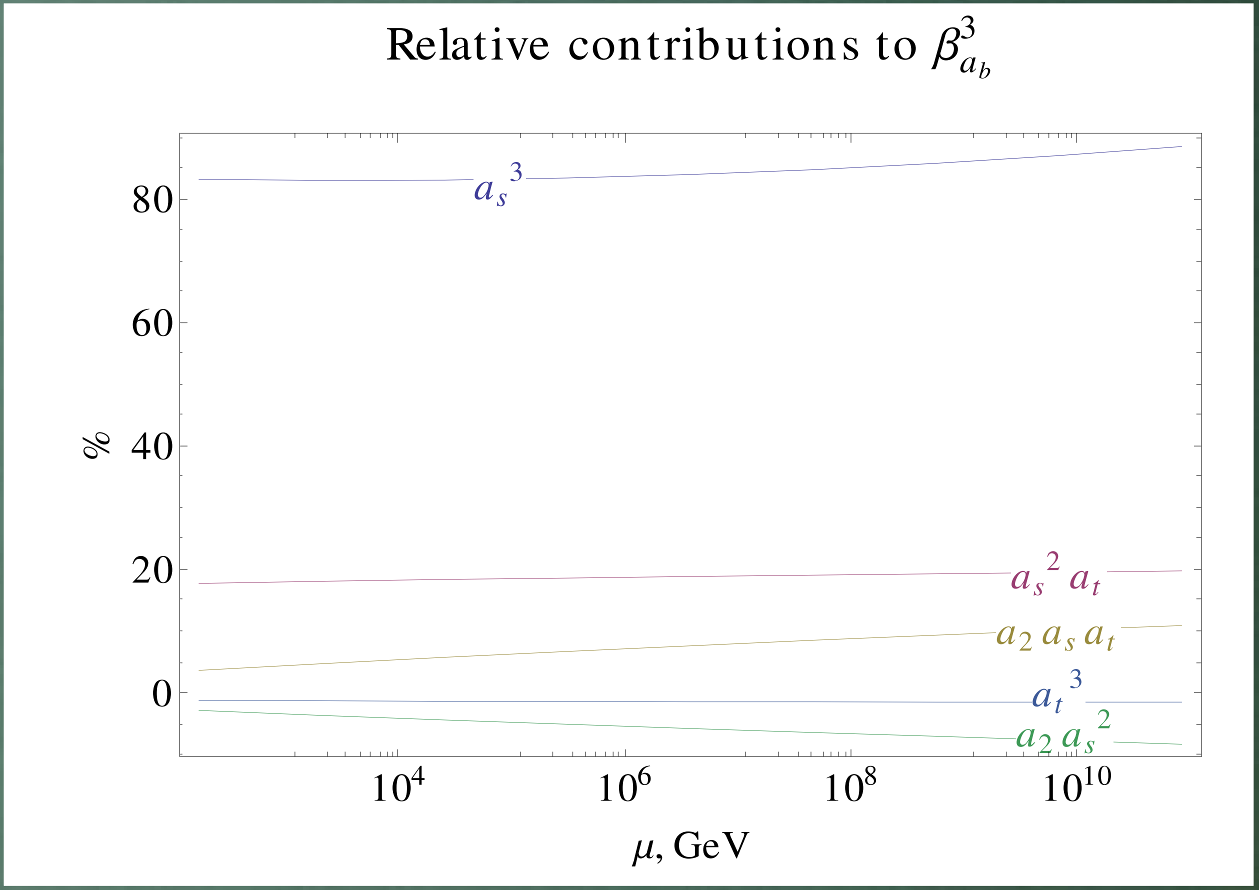
$$a_s = 0.009215$$

$$a_t = 0.00592$$

$$a_b = 1.62 \cdot 10^{-6}$$

$$a_\tau = 6.33 \cdot 10^{-7}$$

$$\hat{\lambda} = 0.00088$$

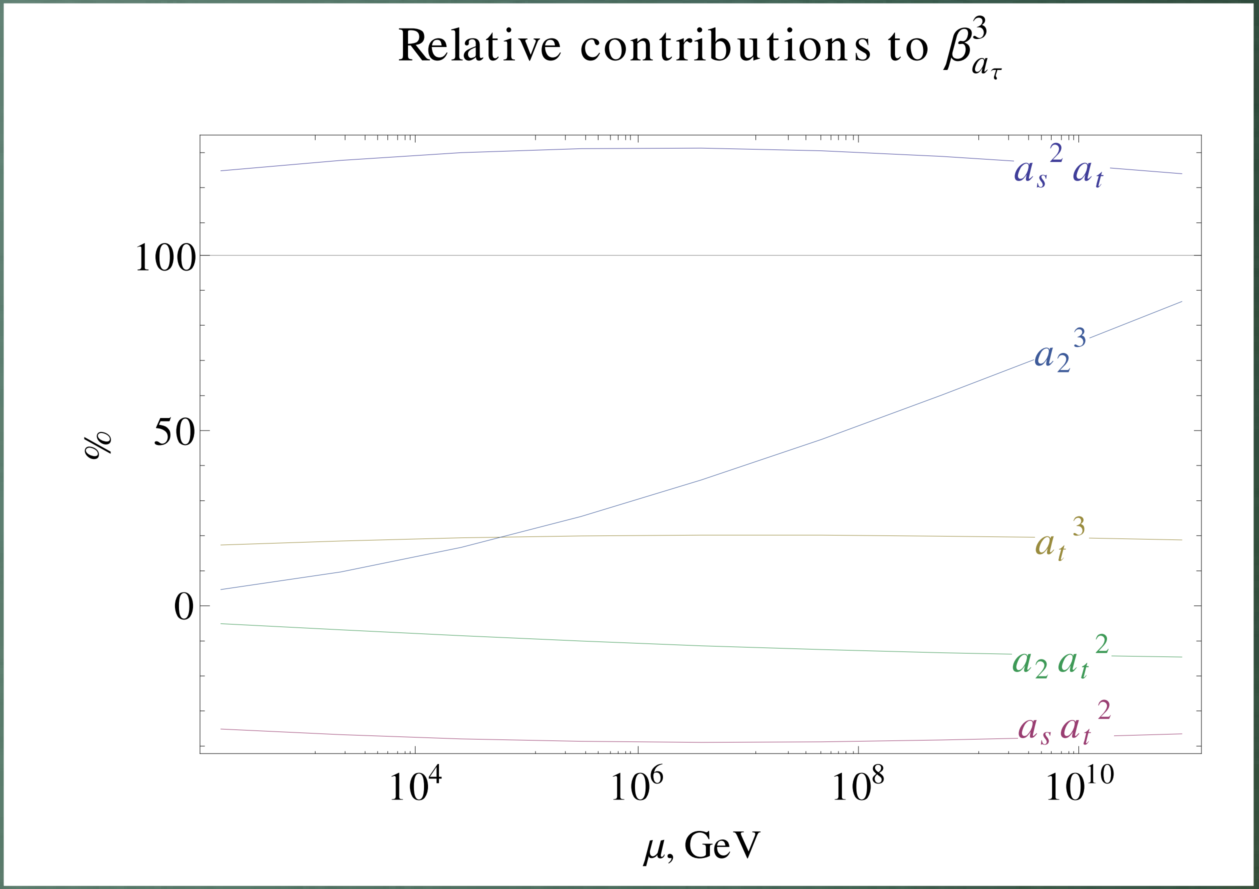


http://arxiv.org/src/1212.6829/anc

Results. Tau Yukawa

Initial values:

$$a_1 = 0.00134996$$
$$a_2 = 0.00268702$$
$$a_s = 0.009215$$
$$a_t = 0.00592$$
$$a_b = 1.62 \cdot 10^{-6}$$
$$a_\tau = 6.33 \cdot 10^{-7}$$
$$\hat{\lambda} = 0.00088$$

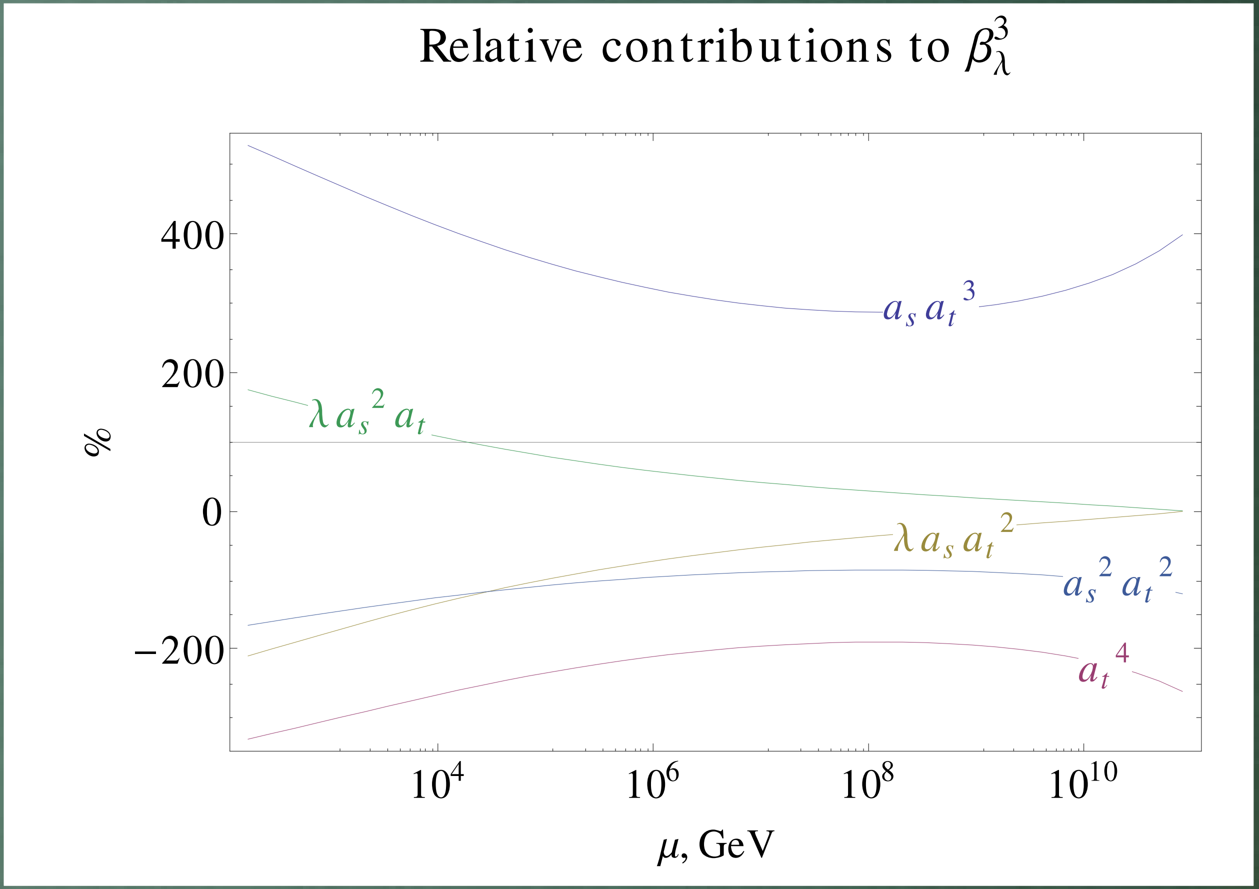


http://arxiv.org/src/1212.6829/anc

Results. Higgs self-coupling

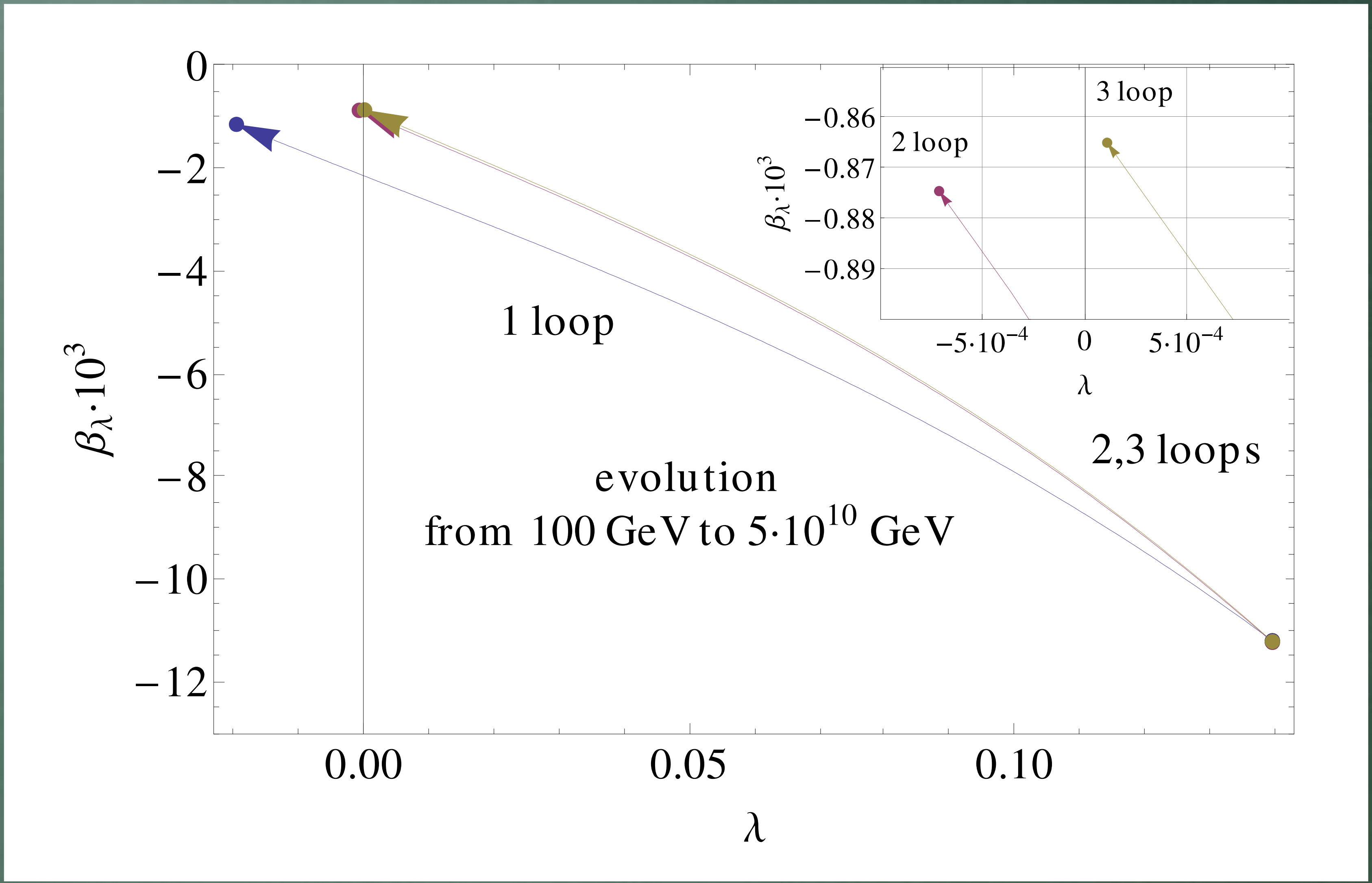
Initial values:

$$a_1 = 0.00134996$$
$$a_2 = 0.00268702$$
$$a_s = 0.009215$$
$$a_t = 0.00592$$
$$a_b = 1.62 \cdot 10^{-6}$$
$$a_\tau = 6.33 \cdot 10^{-7}$$
$$\hat{\lambda} = 0.00088$$



http://arxiv.org/src/1303.4364/anc

Results. Higgs self-coupling



http://arxiv.org/src/1303.4364/anc

Conclusions

- 3-loop Beta-functions for all the fundamental parameters of the SM are obtained and a full agreement is found with
  [Mihaila, Salomon, Steinhauser,'12]
  [Chetyrkin, Zoller,'12-'13]
  (3-loop Yukawa Beta-functions - new result :)
- A framework is established for calculation of three-loop RGEs within "arbitrary" QFT model
  (with the help of LanHEP/FeynRules)

Conclusions

- All the results can be found online as ancillary files of the arXiv versions of the corresponding papers

- But: do not forget about another big problem: two-loop "matching"...

See., e.g., [Bezrukov, Kalmykov, Kniehl, Shaposhnikov,'12]
and [Degrassi,Di Vita,Elias-Miro,Espinosa,Giudice,Isidori,Strumia,'12]

Thank you for your attention!