Analitical calculation of heavy quarkonia production processes in computer

V.V. Braguta

Institute for High Energy Physics, Protvino, Russia

ACAT 2013, 16-21 May 2013

Collaborators:

A.V. Berezhnoy
A.K. Likhoded
A.V. Luchinsky
A.A. Novoselov
S.V. Poslavsky

Outline:

- Introduction
  - Nonrelativistic QCD
  - Light cone expansion formalism
- Analitical calculation of heavy quarkonia production
  - Kinematics
  - Feynman diagrams
  - Projection operators
  - Results
- Application
  - Exclusive production
  - Inclusive production
- Conclusion

Heavy quarkonia production processes

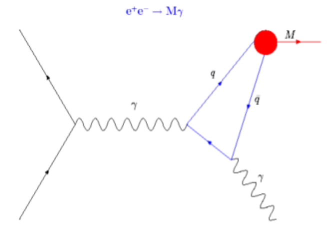
- Decays: $(\bar{b}b) \to (\bar{c}c) + X$
- e^+e^- annihilation: $e^+e^- \to (\bar{b}b) + X, (\bar{c}c) + X$
- pp-collision: $pp \to (\bar{b}b) + X, (\bar{b}c) + X, (\bar{c}c) + X, ...$

General property

- Small relative velocity $v \ll 1$ ($v^2 \sim 0.3$ for $(\bar{c}c)$, $v^2 \sim 0.1$ for $(\bar{b}b)$)
- Scales: $M_Q \gg M_Q v \gg M_Q v^2$
- Amplitudes can be expanded in v

Nonrelativistic QCD (NRQCD)

Factorization



Factorization formula :

$$T = \sum_n \overbrace{C_n}^{Short\ Dist.} \times \underbrace{<M|O_n(0)|0>}_{Large\ Dist.}$$

Nonperturbative effects: $\langle M|O_n|0\rangle$

- Infinite number of operators: $\hat{O} \sim \chi^+\psi, \chi^+\vec{D}\vec{\sigma}\psi, \chi^+\mathbf{D}^2\psi, \chi^+\vec{H}\vec{\sigma}\psi, \ldots$
- Velocity scaling rules: $\chi^+\psi \sim v^3$, $\chi^+\mathbf{D}^2\psi \sim v^5$,...
- At given accuracy finite number of operators contribute
- At leading order only $\langle\eta_c|\chi^+\psi|0\rangle \sim \Psi(0)$ contributes

Factorization

$$T(e^+e^- \to \eta_c\gamma) = \langle \eta_c|\chi^+\psi|0\rangle \left(C_0 + C_2\langle q^2\rangle + C_4\langle q^4\rangle + \dots \right)$$

- Relativistic corrections: $\langle q^n\rangle = m_c^n\langle v^n\rangle = \frac{\langle \eta_c|\chi^+(-\frac{i}{2}\mathrm{D})^n\psi|0\rangle}{\langle \eta_c|\chi^+\psi|0\rangle}$
- Radiative corrections: $C_n = c_n^{(0)} + c_n^{(1)}\alpha_s + c_n^{(2)}\alpha_s^2 + \dots$

Process independent matrix elements: $\langle v^n\rangle$ (QCD sum rules)

- 1S-states $(\eta_c, J/\psi)$ $\langle v^2\rangle = 0.21 \pm 0.04$, $\langle v^4\rangle = 0.06 \pm 0.02$, $\langle v^6\rangle = 0.022 \pm 0.08$
(V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys.Lett. B646, 80; Phys.Rev. D75, 094016)
- 2S-states (η_c', ψ') $\langle v^2\rangle = 0.54 \pm 0.35$
(V.V. Braguta, Phys.Rev. D77, 034026)
- 1P-states $(\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c)$ $\langle v^2\rangle = 0.30 \pm 0.10$, $\langle v^4\rangle = 0.12 \pm 0.04$, $\langle v^6\rangle = 0.051 \pm 0.018$
(V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys.Rev. D79, 074004)

Aim: CALCULATION OF THE c_n^0 FOR ANY PROCESS

Hard exclusive processes

- Decays: $\Upsilon \to \rho\pi, \eta_b \to J/\psi J/\psi, \chi_{b0} \to J/\psi\psi' \ldots$
- Annihilations: $e^+e^- \to J/\psi\eta_c, J/\psi J/\psi, \chi_{c0}\gamma, \ldots$
- Different formfactors: $F(Q^2)$

- General property: $E_h \gg \Lambda_{QCD}, M$
- Expansion parameter $\sim \frac{M^2}{E_h^2} \sim \frac{1}{10}$
- $\sigma = \frac{a_n(E_h{=}\infty)}{E_h^n} + \frac{a_{n+1}(E_h{=}\infty)}{E_h^{n+1}} + \ldots$

Light cone expansion formalism (LCF)

Factorization

$$T = \sum_n C_n \langle M | O_n | 0 \rangle$$

Operators that contribute to pseudoscalar meson production

- $\bar{Q}\gamma_\mu\gamma_5 Q,\ \bar{Q}\gamma_\mu\gamma_5 D_{\mu_1} Q,\ \bar{Q}\gamma_\mu\gamma_5 D_{\mu_1} D_{\mu_2} Q,\ \ldots$
- $\bar{Q}\sigma_{\alpha\beta}\gamma_5 Q,\ \bar{Q}\sigma_{\alpha\beta}\gamma_5 D_{\mu_1} Q,\ \bar{Q}\sigma_{\alpha\beta}\gamma_5 D_{\mu_1} D_{\mu_2} Q,\ \ldots$
- $\bar{Q}\gamma_\mu\gamma_5 G_{\alpha\beta} Q,\ \bar{Q}\gamma_\mu\gamma_5 G_{\alpha\beta} D_{\mu_1} Q,\ \bar{Q}\gamma_\mu\gamma_5 G_{\alpha\beta} D_{\mu_1} D_{\mu_2} Q,\ \ldots$

$$\ldots$$

$$\sigma = \frac{\sigma_0}{s^n} + \frac{\sigma_1}{s^{n+1}} + \frac{\sigma_1}{s^{n+1}} + \ldots$$

At a given accuracy some operators can be omitted

The leading twist distribution amplitudes

Operators that contribute at the leading twist approximation:

$$\langle M(p)|\bar{Q}\gamma_+\gamma_5(D_+)^n Q|0\rangle_\mu = p_+^{(n+1)}\int d\xi \xi^n \phi(\xi,\mu)$$

$v_+ = v_0 + v_z,\ \xi = x_1 - x_2,$

The distribution amplitude (DA) $\phi(\xi)$ can be considered as a meson's wave function

Exclusive processes at the leading twist

$$T = \int d\xi H(\xi,\mu) \times \phi(\xi,\mu), \quad \mu \sim E_h$$

- Resume infinite series of operators
- No double logarithmic corrections ($\sim \alpha_s(E_h) \cdot \log^2 E_h$)
- Resume leading logarithmic corrections ($\sim \alpha_s(E_h) \cdot \log E_h$) in all loops

A.V. Efremov, A.V. Radyushkin, Phys.Lett. B94 (1980) 245,
G.P. Lepage, S. J. Brodsky, Phys.Rev. D22 (1980) 2157

Factorization

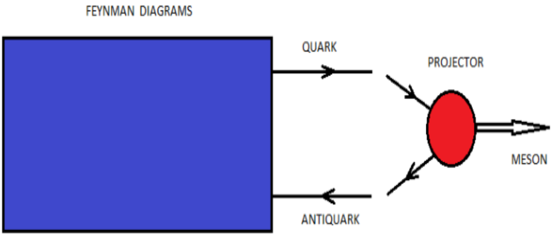
$$T = \int d\xi H(\xi, \mu) \times \phi(\xi, \mu)$$

Process independent distribution amplitudes: $\phi(\xi, \mu)$

Models for charmonia distribution amplitudes are proposed in papers

- V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys.Lett. B646 (2007) 80 (η_c meson)
- V.V. Braguta, Phys.Rev. D75 (2007) 094016 (J/ψ meson)
- V.V. Braguta, Phys.Rev. D77 (2008) 034026 (η_c', ψ' mesons)
- V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys.Rev. D79 (2009) 074004 (χ_{c0}, χ_{c1}, χ_{c2}, h_c mesons)

Aim: CALCULATION OF THE $H(\xi)$ FOR ANY PROCESS



"Factorization"of analitical calculation

Amplitude of meson production:

- Diagrams with $Q\bar{Q}$ production
- Projection of $Q\bar{Q}$ to state with definite quantum numbers

FeynCalc

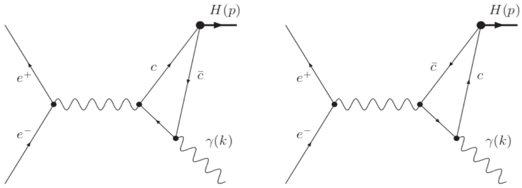
Tools and Tables for Quantum Field Theory Calculations

Introduction

FeynCalc is a Mathematica package for algebraic calculations in elementary particle physics.

Some of the features of FeynCalc are:

- Passarino-Veltman reduction of one-loop amplitudes to standard scalar integrals
- Tools for frequently occuring tasks like Lorentz index contraction, color factor calculation, Dirac matrix manipulation and traces, etc.
- Tensor and Dirac algebra manipulations (including traces) in 4 or D dimensions
- Generation of Feynman rules from a lagrangian
- Tools for non-commutative algebra
- SU(N) algebra
- Tables of integrals, convolutions and Feynman rules
- Special convolution, Mellin transform and other integral tables
- Tools for calculating 2-loop propagator-type diagrams
- FORM and FORTRAN code generation



Amplitude can be expressed through the $\langle H(p)\gamma(k)|J^{\mu}_{em}|0\rangle$

NRQCD kinematics

- p-momentum of meson, q-relative momentum of $Q\bar{Q}$ pair ($pq = 0$)
- $p_Q = \frac{1}{2}p + q$ quark momentum, $\bar{p}_Q = \frac{1}{2}p - q$ antiquark momentum
- meson mass $M = 2E(q) = \sqrt{m_c^2 - q^2}$

LCF kinematics ($s \to \infty$)

- No transverse motion ($p^{\mu} = \frac{\sqrt{s}}{2}(1, 0, 0, 1)$)
- Quark momentum $p_Q = x_1 p$, antiquark momentum $p_Q = x_2 p$ ($x_1 + x_2 = 1$)
- Relative momentum $q = \xi p$, $\xi = x_1 - x_2$

Kinematics (NRQCD)

```
p[a_] = FourVector[p, a]; (*  H -- momentum     *)
k[a_] = FourVector[k, a];(*  photon momentum  *)
q[a_] = FourVector[q, a]; (* relative momentum of quark-antiquark pair  *)
A[a_] = FourVector[A, a]; (* photon polarization *)

M = Sqrt[4 * (mc ^ 2 - ScalarProduct[q, q])];
ScalarProduct[p, q] = 0;
ScalarProduct[k, k] = 0;
ScalarProduct[p, p] = M ^ 2;
ScalarProduct[p, k] = s / 2 - M ^ 2 / 2;

hp = GS[p];
hk = GS[k];
hq = GS[q];
hA = GS[A];
```

Kinematics (LCF)

```
In[52]:= p[a_] = FourVector[p, a];   (*  H -- momentum    *)
         p1[a_] = x1 * FourVector[p, a];  (*  p1=x1*p  quark momentum    *)
         p2[a_] = x2 * FourVector[p, a];  (*  p2=x2*p  antiquark momentum  *)
         k[a_] = FourVector[k, a];   (*  photon momentum  *)
         A[a_] = FourVector[A, a];  (* photon polarization *)

         ScalarProduct[k, k] = 0;
         ScalarProduct[p, p] = 0;
         ScalarProduct[p1, p1] = 0;
         ScalarProduct[p2, p2] = 0;
         ScalarProduct[p, k] = s / 2;
         ScalarProduct[p1, k] = x1 * s / 2;
         ScalarProduct[p2, k] = x2 * s / 2;
         ScalarProduct[A, k] = 0;

         (* Dirac matrixes *)
         hp = GS[p];
         hp1 = x1 * GS[p];
         hp2 = x2 * GS[p];
         hk = GS[k];
         hA = GS[A];
```

Diagrams (NRQCD)

```
In[36]:= T =
  qc ^ 2 * e ^ 2 *
   ExpandScalarProduct[SpinorUBar[1 / 2 p + q, mc].GA[mu].(hq - hk - 1 / 2 * hp + mc).hA.SpinorV[1 / 2 p - q, mc] /
      (ScalarProduct[q - k - p / 2, q - k - p / 2] - mc ^ 2) +
     SpinorUBar[1 / 2 p + q, mc].hA.(1 / 2 * hp + hk + hq + mc).GA[mu].SpinorV[1 / 2 p - q, mc] /
      (ScalarProduct[q + k + p / 2, q + k + p / 2] - mc ^ 2)] *
   (SUNTrace[1] / Sqrt[SUNTrace[1]]) /. {SUNN → 3}
```

Out[36]= $\sqrt{3}\; e^2\; \text{qc}^2 \left(\frac{\varphi\left(\frac{p}{2}+q,\ \text{mc}\right).\gamma^{\text{mu}}.\left(-\gamma\cdot k+\text{mc}-\frac{\gamma\cdot p}{2}+\gamma\cdot q\right).(\gamma\cdot A).\varphi\left(q-\frac{p}{2},\ \text{mc}\right)}{-2\; k\cdot q-2\left(\text{mc}^2-q^2\right)+\frac{s}{2}} + \frac{\varphi\left(\frac{p}{2}+q,\ \text{mc}\right).(\gamma\cdot A).\left(\gamma\cdot k+\text{mc}+\frac{\gamma\cdot p}{2}+\gamma\cdot q\right).\gamma^{\text{mu}}.\varphi\left(q-\frac{p}{2},\ \text{mc}\right)}{2\; k\cdot q-2\left(\text{mc}^2-q^2\right)+\frac{s}{2}} \right)$

Diagrams (LCF)

```
In[52]:= T =
   qc ^ 2 * e ^ 2 * ExpandScalarProduct[SpinorUBar[p1].GA[mu].(-hp2 - hk).hA.SpinorV[p2] / (ScalarProduct[p2 + k, p2 + k]) +
      SpinorUBar[p1].hA.(hp1 + hk).GA[mu].SpinorV[p2] / (ScalarProduct[p1 + k, p1 + k])] *
    (SUNTrace[1] / SUNTrace[1]) /. {SUNN → 3}
```

Out[52]= $e^2\; \text{qc}^2 \left(\frac{\varphi(\text{p1}).(\gamma\cdot A).(\gamma\cdot k+\text{x1}\;\gamma\cdot p).\gamma^{\text{mu}}.\varphi(\text{p2})}{s\;\text{x1}} + \frac{\varphi(\text{p1}).\gamma^{\text{mu}}.(-(\gamma\cdot k)-\text{x2}\;\gamma\cdot p).(\gamma\cdot A).\varphi(\text{p2})}{s\;\text{x2}} \right)$

In some calculation FeynArts is used

Projection operators (LCF)

$$J = \bar{\Psi} T \Psi = Tr\left[TP\right]$$

- the η_c meson: $P_{\beta\alpha} = (\hat{p}\gamma_5)_{\beta\alpha}\frac{f_P}{4}$
- the χ_{c0} meson: $P_{\beta\alpha} = (\hat{p})_{\beta\alpha}\frac{f_{\chi 0}}{4}$
- the χ_{c1} meson: $P_{\beta\alpha} = (\hat{p}\gamma_5)_{\beta\alpha}\frac{f_{\chi 1}}{4}$
- the χ_{c2} meson: $P_{\beta\alpha} = (\hat{p})_{\beta\alpha}\frac{f_{\chi 2}}{4}$

V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys.Rev. D80 (2009) 094008

```
In[21]:= Project = {fP / 4 * hp.GA[5], fc0 / 4 * hp, fc1 / 4 * hp.GA[5], fc2 / 4 * hp}
```

Out[21]= $\left\{\frac{1}{4}\ \text{fP}\ (\gamma\cdot p).\gamma^5,\ \frac{\text{fc0}\ \gamma\cdot p}{4},\ \frac{1}{4}\ \text{fc1}\ (\gamma\cdot p).\gamma^5,\ \frac{\text{fc2}\ \gamma\cdot p}{4}\right\}$

Projection operators to to definite S (NRQCD)

$$J = \bar{\Psi} T \Psi = Tr\Big[TP\Big]$$

- Spin-triplet of $Q\bar{Q}$ pair (spin polarization ϵ)

$$\hat{P} = \frac{1}{4\sqrt{2}E(q)(E(q)+m_Q)}(\hat{p}_{\bar{Q}} - m_Q)\hat{\epsilon}^*(\hat{p} + 2E(q))(\hat{p}_Q + m_c)$$

- Spin-singlet of $Q\bar{Q}$ pair (spin polarization ϵ)

$$\hat{P} = \frac{1}{4\sqrt{2}E(q)(E(q)+m_Q)}(\hat{p}_{\bar{Q}} - m_Q)\gamma_5(\hat{p} + 2E(q))(\hat{p}_Q + m_c)$$

G.T.Bodwin, A.Petrelli, Phys.Rev.D 66,094011

Projection operators to definite J (NRQCD)

- $T(S=0) = A + B_\sigma q^\sigma + \ldots, \quad T(S=1) = (C_\rho + D_{\sigma\rho} q^\sigma + \ldots)\epsilon^\rho$
- the η_c meson: $T(\eta_c) = \sqrt{\frac{\langle O_1 \rangle_{\eta_c}}{2m_c}} A$
- the χ_{c0} meson: $T(\chi_{c0}) = \sqrt{\frac{\langle O_1 \rangle_{\chi_{c0}}}{2m_c}} D_{\rho\sigma} \frac{1}{\sqrt{3}} \left(-g^{\rho\sigma} + \frac{p^\rho p^\sigma}{4m_c^2} \right)$
- the χ_{c1} meson: ...
- the χ_{c2} meson: ...

E.Braaten, J.Lee, Phys.Rev.D 67,054007

Relativistic corrections

$\overline{T} = \int \frac{do}{4\pi} T$

$T(\eta_c) = \sqrt{\frac{4E(q)}{2Nc}} \langle \eta_c | \chi^+ \psi | 0 \rangle \sum_n \frac{\langle v^{2n} \rangle}{n!} \left(\frac{\partial}{\partial q^2} \right)^n \left[\frac{\overline{T}}{4E(q)} \right]$

G. T. Bodwin, J. L. Lee, C. Yu, Phys.Rev. D77 (2008) 094018

Results for J^{μ} (LCF)

In[19]:=

```
Factor[Tr[T.Project[[1]]]] /. {x1 + x2 → 1}
Factor[Tr[T.Project[[2]]]] /. {x1 + x2 → 1}
Factor[Tr[T.Project[[3]]]] /. {x1 + x2 → 1}
Factor[Tr[T.Project[[4]]]] /. {x1 + x2 → 1}
```

Out[19]= $-\frac{i\, e^2\, \mathrm{fP}\, \mathrm{qc}^2\, \epsilon^{\mathrm{mu}\ A\ k\ p}}{s\ \mathrm{x1}\ \mathrm{x2}}$

Out[20]= $\frac{e^2\, \mathrm{fc0}\, \mathrm{qc}^2\, (\mathrm{x1}-\mathrm{x2})\,(s\, A^{\mathrm{mu}} - 2\, k^{\mathrm{mu}}\, A\cdot p)}{2\ s\ \mathrm{x1}\ \mathrm{x2}}$

Out[21]= $-\frac{i\, e^2\, \mathrm{fc1}\, \mathrm{qc}^2\, \epsilon^{\mathrm{mu}\ A\ k\ p}}{s\ \mathrm{x1}\ \mathrm{x2}}$

Out[22]= $\frac{e^2\, \mathrm{fc2}\, \mathrm{qc}^2\, (\mathrm{x1}-\mathrm{x2})\,(s\, A^{\mathrm{mu}} - 2\, k^{\mathrm{mu}}\, A\cdot p)}{2\ s\ \mathrm{x1}\ \mathrm{x2}}$

Results for J^{μ} (NRQCD)

```
Sqrt[O1 / 2 / 3 / mc] * (Factor[Tr[Dot[T, Project[[1]]]]] /. {ScalarProduct[q, q] → 0,
    ScalarProduct[k, q] → 0, Eq → mc})
```

$$\frac{4\, i\, e^2\, \mathrm{qc}^2 \sqrt{\frac{\mathrm{O1}}{\mathrm{mc}}}\, \epsilon^{\mathrm{mu}\, A\, k\, p}}{4\, \mathrm{mc}^2 - s}$$

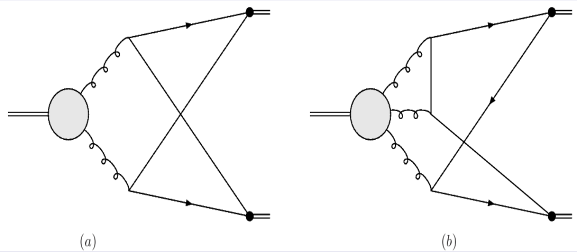
```
R =
  D[((Tr[Dot[T, Project[[2]]]] /. {ScalarProduct[q, q] → 0, Eq → mc, ScalarProduct[A, k] → 0}) /.
      {Pair[a_, Momentum[q]] → t * Pair[a, LorentzIndex[si]]}), t] /. {t → 0};
Factor[Sqrt[O1 / 2 / 3 / mc] * Contract[(-MetricTensor[si, ro] + p[si] * p[ro] / 4 / mc ^ 2) * R]] /.
 {ScalarProduct[q, q] -> 0, ScalarProduct[A, k] → 0}
```

$$-\frac{2\, e^2\, \mathrm{qc}^2 \sqrt{\frac{\mathrm{O1}}{\mathrm{mc}}} \left(24\, \mathrm{mc}^4\, k^{\mathrm{mu}}\, A\cdot p - 2\, \mathrm{mc}^2\, s\, k^{\mathrm{mu}}\, A\cdot p + 48\, \mathrm{mc}^6\, A^{\mathrm{mu}} - 16\, \mathrm{mc}^4\, s\, A^{\mathrm{mu}} + \mathrm{mc}^2\, s^2\, A^{\mathrm{mu}}\right)}{\mathrm{mc}^3 \left(4\, \mathrm{mc}^2 - s\right)^2}$$

Exclusive processes

- Heavy quarkonia production at B-factories
  - Single production: $e^+e^- \to \eta_c\gamma, \chi_{c0}\gamma, ...$
  - Double production: $e^+e^- \to J/\psi\eta_c, J/\Psi\chi_{c0}, J/\psi J/\psi, ...$
  - Production in decays: $\chi_{b0} \to J/\Psi J/\Psi, \Upsilon \to J/\psi\eta_c, ...$
- Heavy quarkonia production at pp-collision
  - Production in decays:
    $\chi_{b0} \to J/\Psi J/\Psi, \Upsilon \to J/\psi\eta_c, B_c \to J/\Psi e\nu ...$

Bottomonia decays to double charmonia



All leading twist C-even bottomonia decays were considered in paper
V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys.Rev. D80 (2009) 094008,
Erratum-ibid. D85 (2012) 119901

Approximately 30 processes

Leading twist decays of the η_b meson

reaction	$\Gamma_{\rm NRQCD}$, eV	$\Gamma_{\rm LC}$, eV	$\mathrm{Br}_{\rm LC}, 10^{-5}$
$\eta_b \to h_c \psi$	$16.^{+2.3}_{-1.5} \pm 8.4 \pm 8.1$	$32. \pm 2.6 \pm 6.1 \pm 8.2$	0.33
$\eta_b \to h_c \psi(2S)$	$7.8^{+1.1}_{-0.72} \pm 6.5 \pm 3.9$	$16. \pm 1.4 \pm 3.1 \pm 4.2$	0.17
$\eta_b \to \eta_c \chi_{c0}$	$13.^{+3.5}_{-2.7} \pm 6.8 \pm 6.5$	$9.1 \pm 0.73 \pm 4.6 \pm 2.3$	0.092
$\eta_b \to \eta_c(2S) \chi_{c0}$	$6.3^{+1.7}_{-1.3} \pm 5.2 \pm 3.1$	$4.3 \pm 0.36 \pm 3. \pm 1.1$	0.043
$\eta_b \to \eta_c \chi_{c2}$	$3.6^{+1.1}_{-1.1} \pm 8.4 \pm 1.8$	$18. \pm 1.4 \pm 8.7 \pm 4.5$	0.18
$\eta_b \to \eta_c(2S) \chi_{c2}$	$1.7^{+0.54}_{-0.53} \pm 4.1 \pm 0.86$	$8.2 \pm 0.7 \pm 5.6 \pm 2.1$	0.083
$\eta_b \to \chi_{c0} \chi_{c1}$	$2.3^{+0.21}_{-0.29} \pm 2.2 \pm 1.2$	$4.4 \pm 0.38 \pm 2.3 \pm 1.1$	0.045
$\eta_b \to \chi_{c1} \chi_{c2}$	$0.93^{+0.22}_{-0.21} \pm 2.9 \pm 0.46$	$8.6 \pm 0.73 \pm 4.3 \pm 2.2$	0.087

Leading twist decays of the χ_{b0} meson

$\chi_{b0} \to \eta_c \chi_{c1}$	$1.9^{+0.23}_{-0.27} \pm 1.9 \pm 0.93$	$9.8 \pm 0.25 \pm 4.8 \pm 2.5$	1.2
$\chi_{b0} \to \eta_c(2S) \chi_{c1}$	$0.9^{+0.11}_{-0.13} \pm 1.1 \pm 0.45$	$5.9 \pm 1. \pm 4. \pm 1.5$	0.73
$\chi_{b0} \to \chi_{c0} \chi_{c2}$	$0.00015^{+0.0007}_{-0.00014} \pm 0.038 \pm 7.6 \times 10^{-5}$	$0.14 \pm 0.034 \pm 0.07 \pm 0.034$	0.017
$\chi_{b0} \to \eta_c \eta_c$	$7.9^{+0.69}_{-0.57} \pm 5.6 \pm 4.$	$10. \pm 0.45 \pm 4.9 \pm 2.5$	1.3
$\chi_{b0} \to \eta_c \eta_c(2S)$	$7.8^{+0.68}_{-0.56} \pm 7.5 \pm 3.9$	$12. \pm 2.1 \pm 8.3 \pm 3.$	1.5
$\chi_{b0} \to \eta_c(2S) \eta_c(2S)$	$1.9^{+0.16}_{-0.14} \pm 2.8 \pm 0.94$	$3.6 \pm 1.4 \pm 3. \pm 0.91$	0.45
$\chi_{b0} \to \psi \psi$	$4.3^{+0.28}_{-0.25} \pm 5.7 \pm 2.2$	$15. \pm 0.68 \pm 0.51 \pm 3.8$	1.9
$\chi_{b0} \to \psi \psi(2S)$	$4.3^{+0.28}_{-0.25} \pm 6.3 \pm 2.1$	$20. \pm 3.5 \pm 0.62 \pm 5.$	2.5
$\chi_{b0} \to \psi(2S) \psi(2S)$	$1.^{+0.068}_{-0.06} \pm 1.9 \pm 0.52$	$6.5 \pm 2.5 \pm 0.18 \pm 1.6$	0.81
$\chi_{b0} \to h_c h_c$	$0.014^{+0.0025}_{-0.0035} \pm 0.021 \pm 0.0071$	$0.3 \pm 0.074 \pm 0.079 \pm 0.075$	0.037
$\chi_{b0} \to \chi_{c0} \chi_{c0}$	$0.006^{+0.0076}_{-0.0041} \pm 0.022 \pm 0.003$	$0.035 \pm 0.0087 \pm 0.018 \pm 0.0088$	0.0044
$\chi_{b0} \to \chi_{c1} \chi_{c1}$	$0.087^{+0.037}_{-0.025} \pm 0.63 \pm 0.043$	$2.4 \pm 0.12 \pm 1.2 \pm 0.6$	0.3
$\chi_{b0} \to \chi_{c2} \chi_{c2}$	$0.0032^{+0.0038}_{-0.0012} \pm 0.035 \pm 0.0016$	$0.13 \pm 0.033 \pm 0.066 \pm 0.033$	0.017

Leading twist decays of the χ_{b1} meson

$\chi_{b1} \to h_c \psi$	$0.18^{+0.0016}_{-0.0077} \pm 0.13 \pm 0.091$	$0.88 \pm 0.078 \pm 0.17 \pm 0.22$	0.68
$\chi_{b1} \to h_c \psi(2S)$	$0.089^{+0.00076}_{-0.0037} \pm 0.086 \pm 0.045$	$0.67 \pm 0.18 \pm 0.13 \pm 0.17$	0.52
$\chi_{b1} \to \eta_c \chi_{c0}$	$0.038^{+0.0048}_{-0.0055} \pm 0.038 \pm 0.019$	$0.25 \pm 0.022 \pm 0.12 \pm 0.061$	0.19
$\chi_{b1} \to \eta_c(2S) \chi_{c0}$	$0.019^{+0.0023}_{-0.0027} \pm 0.022 \pm 0.0093$	$0.17 \pm 0.046 \pm 0.12 \pm 0.043$	0.13
$\chi_{b1} \to \eta_c \chi_{c2}$	$0.11^{+0.0031}_{-0.0066} \pm 0.075 \pm 0.055$	$0.48 \pm 0.042 \pm 0.24 \pm 0.12$	0.37
$\chi_{b1} \to \eta_c(2S) \chi_{c2}$	$0.054^{+0.0015}_{-0.0032} \pm 0.051 \pm 0.027$	$0.33 \pm 0.089 \pm 0.23 \pm 0.083$	0.26
$\chi_{b1} \to \chi_{c0} \chi_{c1}$	$0.08^{+0.022}_{-0.018} \pm 0.061 \pm 0.04$	$0.12 \pm 0.015 \pm 0.06 \pm 0.03$	0.091
$\chi_{b1} \to \chi_{c1} \chi_{c2}$	$0.018^{+0.0015}_{-0.00087} \pm 0.028 \pm 0.0091$	$0.23 \pm 0.03 \pm 0.11 \pm 0.057$	0.18

Leading twist decays of the χ_{b2} meson

$\chi_{b2} \to \eta_c \chi_{c1}$	$0.26^{+0.0073}_{-0.015} \pm 0.18 \pm 0.13$	$0.63 \pm 0.011 \pm 0.31 \pm 0.16$	0.31
$\chi_{b2} \to \eta_c(2S) \chi_{c1}$	$0.13^{+0.0036}_{-0.0075} \pm 0.12 \pm 0.064$	$0.35 \pm 0.044 \pm 0.24 \pm 0.086$	0.17
$\chi_{b2} \to \chi_{c0} \chi_{c2}$	$0.076^{+0.02}_{-0.017} \pm 0.058 \pm 0.038$	$0.049 \pm 0.0075 \pm 0.025 \pm 0.012$	0.025
$\chi_{b2} \to \eta_c \eta_c$	$0.26^{+0.069}_{-0.069} \pm 0.69 \pm 0.13$	$0.64 \pm 0.02 \pm 0.31 \pm 0.16$	0.32
$\chi_{b2} \to \eta_c(2S) \eta_c$	$0.26^{+0.068}_{-0.068} \pm 0.7 \pm 0.13$	$0.71 \pm 0.092 \pm 0.48 \pm 0.18$	0.36
$\chi_{b2} \to \eta_c(2S) \eta_c(2S)$	$0.062^{+0.016}_{-0.017} \pm 0.18 \pm 0.031$	$0.2 \pm 0.068 \pm 0.17 \pm 0.051$	0.1
$\chi_{b2} \to \psi \psi$	$9.7^{+0.87}_{-0.73} \pm 6.9 \pm 4.9$	$9.6 \pm 0.42 \pm 0.33 \pm 2.4$	4.8
$\chi_{b2} \to \psi(2S) \psi$	$9.6^{+0.86}_{-0.72} \pm 9.3 \pm 4.8$	$11. \pm 1.9 \pm 0.35 \pm 2.8$	5.7
$\chi_{b2} \to \psi(2S) \psi(2S)$	$2.3^{+0.21}_{-0.17} \pm 3.5 \pm 1.2$	$3.4 \pm 1.4 \pm 0.094 \pm 0.84$	1.7
$\chi_{b2} \to h_c h_c$	$0.061^{+0.012}_{-0.012} \pm 0.17 \pm 0.031$	$0.48 \pm 0.034 \pm 0.13 \pm 0.12$	0.24
$\chi_{b2} \to \chi_{c0} \chi_{c0}$	$0.0021^{+0.00037}_{-0.00044} \pm 0.0037 \pm 0.0011$	$0.013 \pm 0.0019 \pm 0.0065 \pm 0.0032$	0.0063
$\chi_{b2} \to \chi_{c1} \chi_{c1}$	$0.026^{+0.0069}_{-0.0074} \pm 0.063 \pm 0.013$	$0.28 \pm 0.03 \pm 0.14 \pm 0.069$	0.14
$\chi_{b2} \to \chi_{c2} \chi_{c2}$	$0.028^{+0.0038}_{-0.0052} \pm 0.042 \pm 0.014$	$0.54 \pm 0.11 \pm 0.27 \pm 0.13$	0.27
$\chi_{b2} \to h_c \psi$	$1.1^{+0.12}_{-0.14} \pm 1. \pm 0.57$	$3.6 \pm 0.09 \pm 0.68 \pm 0.9$	1.8
$\chi_{b2} \to h_c \psi(2S)$	$0.56^{+0.057}_{-0.069} \pm 0.62 \pm 0.28$	$2.1 \pm 0.36 \pm 0.39 \pm 0.52$	1.
$\chi_{b2} \to \chi_{c1} \chi_{c2}$	$0.044^{+0.0008}_{-0.0015} \pm 0.036 \pm 0.022$	$0.49 \pm 0.1 \pm 0.24 \pm 0.12$	0.25

Comparison of different results

	$Br \cdot 10^{-5}$ NRQCD [1]	$Br \cdot 10^{-5}$ NRQCD [2]	$Br \cdot 10^{-5}$ LCF [3]	$Br \cdot 10^{-5}$ Exp. [4]
$\chi_{b0} \to 2J/\psi$	0.5	1.9	1.9 ± 0.5	< 7.1
$\chi_{b2} \to 2J/\psi$	3.4	17.5	4.8 ± 1.2	< 4.5
$\chi_{b0} \to J/\psi\ \psi(2S)$	—	—	2.5 ± 0.7	< 12
$\chi_{b2} \to J/\psi\ \psi(2S)$	—	—	5.7 ± 1.8	< 4.9
$\chi_{b0} \to 2\psi(2S)$	—	—	0.8 ± 0.4	< 3.1
$\chi_{b2} \to 2\psi(2S)$	—	—	1.7 ± 0.8	< 1.6

.

NRQCD calculation:
[1] Juan Zhang, Hairong Dong, Feng Feng, Phys.Rev. D84 (2011) 094031
[2] Wen-Long Sang, Reyima Rashidin, U-Rae Kim, Jungil Lee, Phys.Rev. D84 (2011) 074026

Light cone formalism calculation:
[3] V.V. Braguta, A.K. Likhoded, A.V. Luchinsky, Phys.Rev. D80 (2009) 094008
Erratum-ibid. D85 (2012) 119901

Belle experiment:
[4] Phys.Rev. D85 (2012) 071102

Inclusive processes

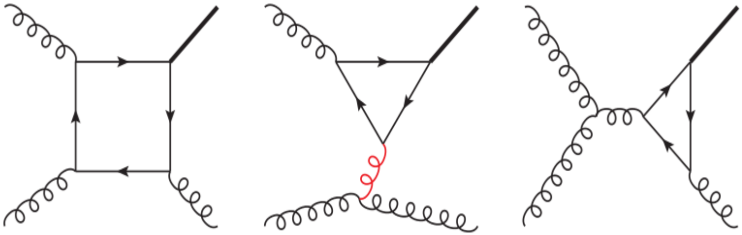
- Heavy quarkonia production at B-factories
  - Single production: $e^+e^- \to J/\Psi D\bar{D} + X, J/\Psi + X$
  - Production in decays: $\chi_{b0} \to J/\Psi D\bar{D} + X, \Upsilon \to J/\psi + X, ...$
- Heavy quarkonia production at pp-collision
  - $pp \to J/\Psi + X, B_c + X, \chi_{cP} + X, 2J/\Psi + X, J/\Psi D\bar{D} + X, ...$

$pp \to \chi_P + X$

$$\sigma(pp \to \chi_P + X) = \int dx_1 dx_2 f(x_1) f(x_2) \hat{\sigma}(gg \to \chi_P)$$

At the leading order approximation ($gg \to \chi_P$)

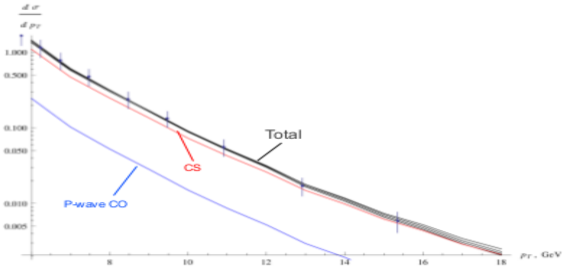
- χ_1 production is forbidden (Landau-Yang theorem)
- $\sigma(pp \to \chi_P + X)$ independent on p_T



$pp \to \chi_P + X$ at NLO

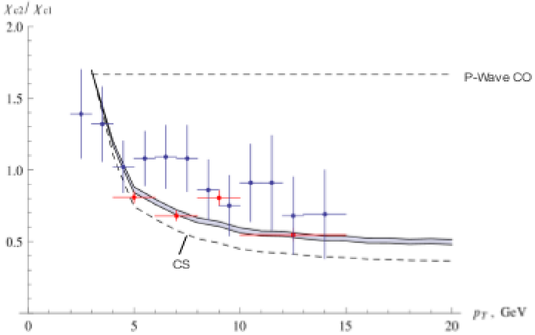
- χ_1 production is allowed
- $\sigma(pp \to \chi_P + X)$ dependence on p_T appears

CDF



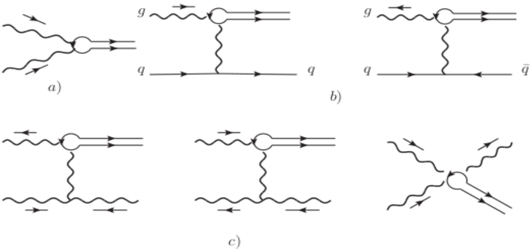
[CDF Collab., PRL 79 (1997) 578]

A.K. Likhoded, A.V. Luchinsky, S.V. Poslavsky, e-Print: arXiv:1305.2389

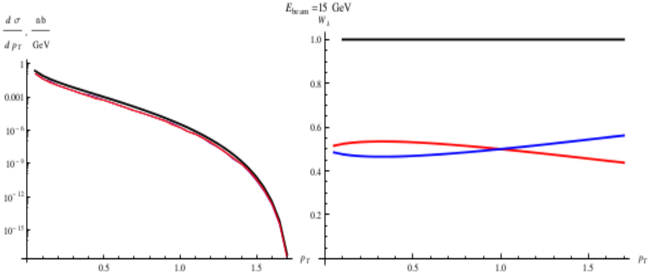


[CDF Collab., PRL 79 (1997) 578]
[LHCb Collab.. PL B 714 (2012) 215]

A.K. Likhoded, A.V. Luchinsky, S.V. Poslavsky, Phys.Rev. D86 (2012) 074027; e-Print:arXiv:1305.2389

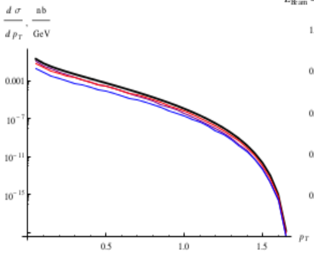


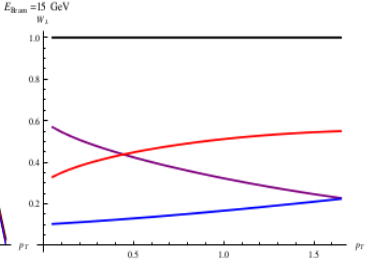
$p\bar{p} \to \chi_{cP} + X$ at FAIR



A.V. Luchinsky, S.V. Poslavsky, Phys.Rev. D85 (2012) 074016

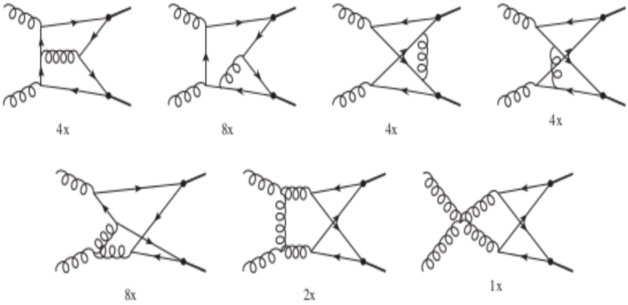
Cross sections are included in PandaRoot



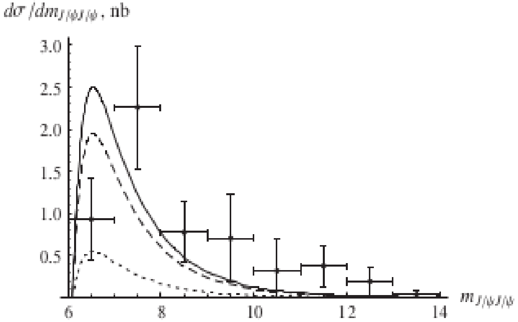


A.V. Luchinsky, S.V. Poslavsky, Phys.Rev. D85 (2012) 074016

Cross sections are included in PandaRoot

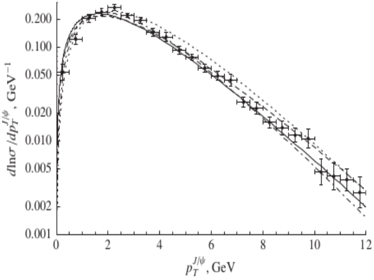


$$pp \to J/\psi J/\psi + X \text{ (30 diagrams)}$$



A.V. Berezhnoy, A.K. Likhoded, A.V. Luchinsky, A.A. Novoselov, Phys.Rev. D84 (2011) 094023

$$pp \to J/\psi D + X$$



A.V. Berezhnoy, A.K. Likhoded, A.V. Luchinsky, A.A. Novoselov, Phys.Rev. D86 (2011) 034017

Conclusion

- Algorithm for automatic calculation of heavy quarkonia production processes (NRQCD, LCF) is developed
- It can be used to calculate relativistic corrections (NRQCD) at any accuracy
- Algorithm is very simple (tools: Mathematica, FeynCalc, FeynArt)
- Successful applications for exclusive and inclusive production

THANK YOU