Automated computation of scattering amplitudes

Giovanni Ossola

New York City College of Technology
City University of New York (CUNY)

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Introduction

Scattering Amplitudes turn our theories into predictions:

- The Evaluation of **Scattering Amplitudes** is necessary to test our theoretical models and **compare their prediction with the experiments**
- There is a **long tradition** and **extensive literature**
- The **understanding of the structure** of Scattering Amplitudes goes in parallel with the **development of tools** for computations

Introduction

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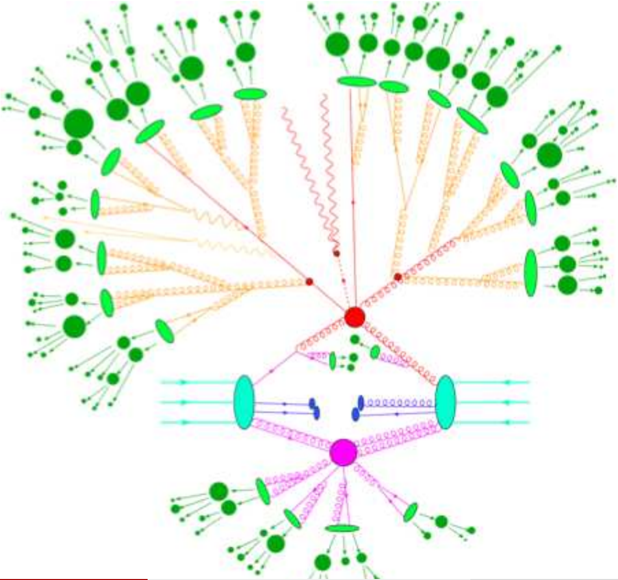
Outline of this talk:

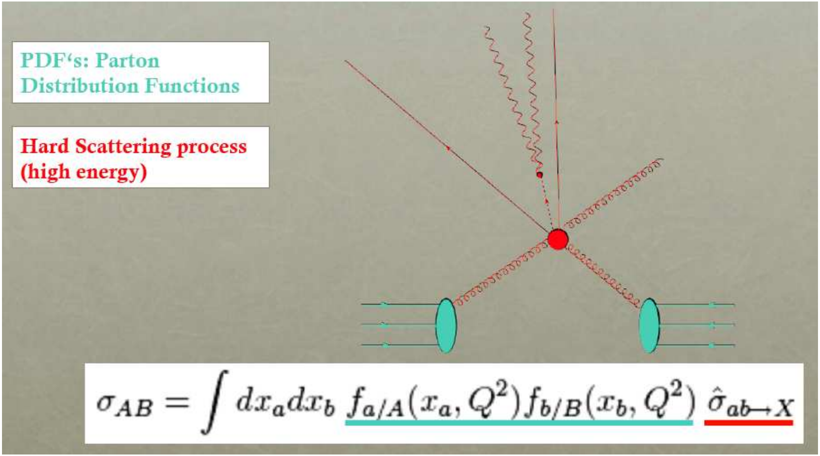
- Motivation for higher order calculations
- An overview of techniques employed for virtual one-loop amplitudes
- Can we extend what we learned at one-loop to study a more general problem?

For more details $\rightarrow$ several talks in **Track 3**

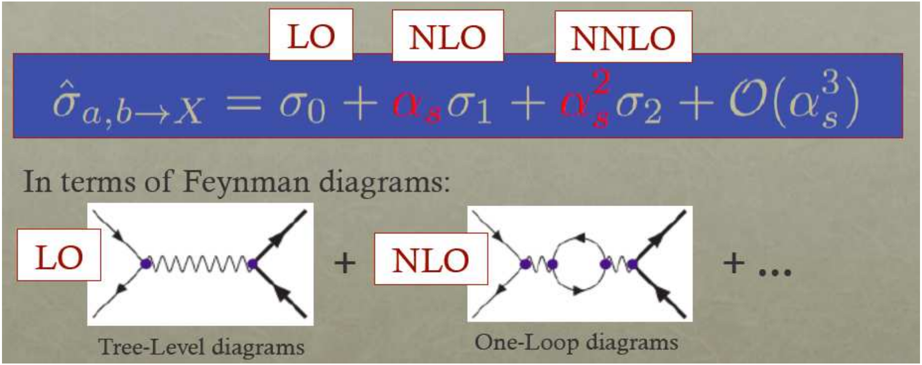
THE "BIG PICTURE"



Partons and Protons



Perturbative expansion

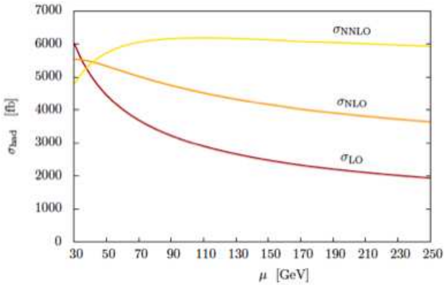


Theoretical predictions should match the needs of the experiments

For many analyses, Leading-Order (LO) predictions are not sufficient $\rightarrow$ we need higher orders in the perturbative expansion

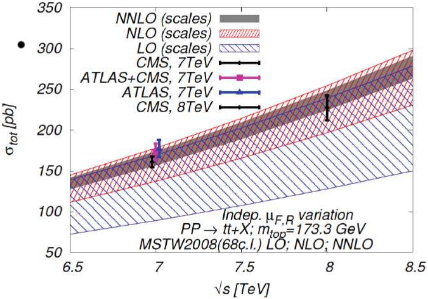
Why Higher Orders?

- Reduce the **Theoretical Error**
  - Parametric Error (input parameters: PDF, masses, couplings...)
  - Truncation Error (missing Higher Orders)
  - Estimated by looking at the Scale-Dependence



Higgs + jet at NNLO

Boughezal at al. (2013)



Top-pair production at NNLO

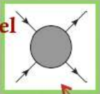
Czakon, Fiedler, Mitov (2013)

Why Higher Orders?

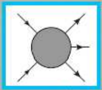
- Reduce the **Theoretical Error**
  - Parametric Error (input parameters: PDF, masses, couplings...)
  - Truncation Error (missing Higher Orders)
  - Estimated by looking at the Scale-Dependence
- Effects of Higher Order Corrections (NLO, NNLO, etc)
  - Large effects in the cross-section (in particular in QCD)
  - Changes in the shape of distributions
  - Loop-induced effects
- At the LHC:
  - Distinguish the Signal from Backgrounds
  - Provide the "theory answers" to anomalies/new physics signals

From LO to NLO

Tree-Level (Born)



Virtual Part (+1 loop)



Real Emission (Born +1 leg)

$$\sigma_{NLO} = \int_n \left(d\sigma^B + d\sigma^V + \int_1 d\sigma^A\right) + \int_{n+1} \left(d\sigma^R - d\sigma^A\right)$$

Virtual Part and Real Emission contributions are IR divergent

Subtraction terms

Needed to cancel infrared singularities numerically.
Idea: Add zero in suitable way to cancel infrared singularities from real and virtual parts.

Several **Automated** Implementations for **Tree-Level** and **Subtraction Terms**

Automated tools for Real & Subtraction

- ❑ Born & Real radiation: tree level matrix element

✓ Many tools available on the market, too many to mention all of them:

GRACE	SHERPA
HELAS/MADGRAPH/MADEVENT	HELAC/PHEGAS
WHIZARD	CompHep

....

- ❑ Subtractions: automated in several programs:

✓ MadDipole [Frederix, Greiner, Gehrmann]

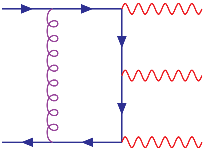
✓ AutoDipole (SuperAutoDipole) [Hasegawa, Moch, Uwer]

✓ Dipoles in SHERPA [Gleisberg, Krauss]

✓ HELAC-DIPOLES [Czakon, Papadopoulos, Worek]

In the following: we focus on the computation of the Virtual Part of NLO calculations

One-loop – Notations



Any m-point one-loop amplitude can be written, before integration, as

$$A(\bar{q}) = \frac{N(\bar{q})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

where

$$\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2 \quad , \quad \bar{q}^2 = q^2 - \mu^2 \quad , \quad \bar{D}_i = D_i - \mu^2$$

Our task is to calculate, for each phase space point:

$$\mathcal{M} = \int d^n\bar{q}\ A(\bar{q}) = \int d^n\bar{q} \frac{N(\bar{q})}{\bar{D}_0 \bar{D}_1 \ldots \bar{D}_{m-1}}$$

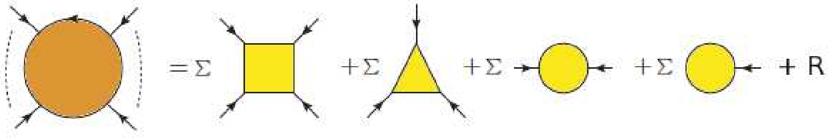
The traditional one-loop "master" formula

Passarino, Veltman (1979)

$$
\begin{aligned}
\int d^n\bar{q}\, \frac{N(\bar{q})}{\bar{D}_{i_0}\bar{D}_{i_1}\ldots\bar{D}_{m-1}} &= \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) \int d^n\bar{q}\, \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}\bar{D}_{i_3}} \\
&+ \sum_{i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) \int d^n\bar{q}\, \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}} \\
&+ \sum_{i_0<i_1}^{m-1} b(i_0 i_1) \int d^n\bar{q}\, \frac{1}{\bar{D}_{i_0}\bar{D}_{i_1}} \\
&+ \sum_{i_0}^{m-1} a(i_0) \int d^n\bar{q}\, \frac{1}{\bar{D}_{i_0}} \\
&+ \text{rational terms}
\end{aligned}
$$

$$\int d^n\bar{q}\,\frac{N(\bar{q})}{\bar{D}_{i_0}\bar{D}_{i_1}\ldots\bar{D}_{m-1}} = \sum_i d_i\,\mathrm{Box}_i + \sum_i c_i\,\mathrm{Triangle}_i + \sum_i b_i\,\mathrm{Bubble}_i + \sum_i a_i\,\mathrm{Tadpole}_i + \mathrm{R}\,,$$

1) **Generation**: Compute the unintegrated amplitudes for all diagrams
2) **Reduction**: Extract all coefficients and rational terms
3) **Master Integrals**: Calculate the Master Integrals (scalar integrals) and combine with the coefficients



One-Loop as a 3 step process

$$
\begin{aligned}
\int d^n\bar{q}\,\frac{N(\bar{q})}{\bar{D}_{i_0}\bar{D}_{i_1}\ldots\bar{D}_{m-1}} &= \sum_i d_i\ \mathrm{Box}_i + \sum_i c_i\ \mathrm{Triangle}_i \\
&+ \sum_i b_i\ \mathrm{Bubble}_i + \sum_i a_i\ \mathrm{Tadpole}_i + \mathrm{R}\,,
\end{aligned}
$$

1) **Generation**: Compute the unintegrated amplitudes for all diagrams
2) **Reduction**: Extract all coefficients and rational terms
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There are several techniques available for Generation+Reduction and available codes to compute the one-loop Scalar Integrals

One-Loop Master Integrals: **Ellis, Zanderighi; van Oldenborgh; van Hameren; Binoth et al.; Hahn et al.**

Tensorial Reduction

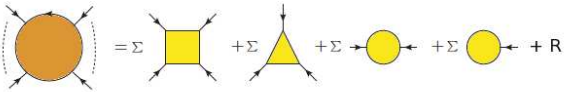
- The numerator function of any amplitude is a **polynomial** in the integration momentum q

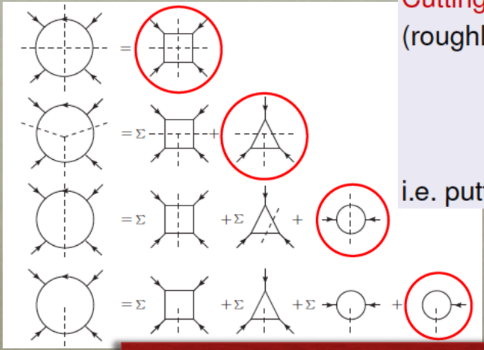
$$\mathcal{N}(\bar{q}) = \sum_{r=0}^{R} C_{\mu_1 \ldots \mu_r} \bar{q}^{\mu_1} \ldots \bar{q}^{\mu_r}$$

- Each amplitude can be decomposed in a linear combination of **kinematic factors**, which are **q-independent**, multiplied by **tensors of various ranks in q** sitting on sets of Denominators
- Tensor integrals can be written in terms of scalar integrals (or not!)
- PROS: **control over spurious singularities** (Gram); fully **algebraic**.
- Needs **efficient computer algebra** and smart book-keeping/caching

Denner, Dittmaier; Binoth et al.;
Hahn et al.; Fleisher, Riemann, Yundin.

On-Shell Methods / Unitarity Cuts





Cutting a loop propagator (roughly) means

$$\frac{1}{D_i} \rightarrow \delta(D_i)$$

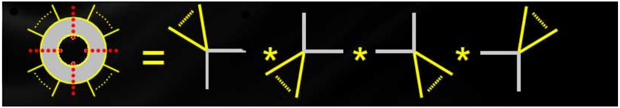
i.e. putting it on-shell

In the Loop-Integral = Generalized Unitarity Methods

In the Integrand = OPP integrand-level reduction

Generalized Unitarity

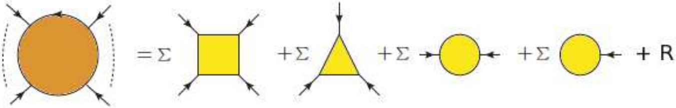
- Generate **loop momenta configurations that satisfy the cut conditions** (complex momenta)
- For each configuration, **compute and multiply the trees** at the corner of the cut diagram
- Combine the results appropriately to **get all the coefficients** of the scalar integrals



- More subtleties for the triple cut (leakage from higher point functions)
- Effectively **reduces a loop computation to tree computation** ("fusing tree amplitudes into loop amplitudes")

Bern, Dixon, Dunbar, Kosower;
Britto, Cachazo, Feng; Ellis, Giele, Kunszt, Melnikov

Rational Term **R**



Cut-Constructible part vs **Rational Term**

R is "automatic" in the tensorial reduction (algebra in dimension d)

On-shell methods offer different options for calculation of **R**

- Higher integer dimensions Giele, Kunszt, Melnikov
- Loop-level on shell recursions Berger, Bern, Dixon, Forde and Kosower
- Mass continuation method Badger
- Tree-level like Feynman Rules G.O., Papadopoulos, Pittau; Draggiotis, Garzelli, Malamos, Pittau; Shao, Zhang, Chao

Integral-level vs Integrand-level

Description in terms of Master Integrals I_i $\rightarrow$ Integral Level

$$\int d^n\bar{q}\ A(\bar{q}) = \int d^n\bar{q} \frac{N(\bar{q})}{\bar{D}_0\bar{D}_1\ldots\bar{D}_{m-1}} = c_0 I_0 + c_1 I_1 + \ldots + C_n I_n$$

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Integrand Level $\rightarrow$ The $N = N$ identity

$$N(\bar{q}) = ???$$

Challenge: write a complete expression at the l.h.s. for $N(\bar{q})$

- powers of q and μ^2
- scalar products
- reconstructed denominators $\bar{D}_i$

Integrand-Level Approach

Integrand level decomposition:

$$
\begin{aligned}
N(\bar{q}) &= \sum_{i<<m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h\neq i,j,k,\ell,m}^{n-1} \bar{D}_h + \sum_{i<<\ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h\neq i,j,k,\ell}^{n-1} \bar{D}_h + \\
&+ \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k}^{n-1} \bar{D}_h + \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j}^{n-1} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i}^{n-1} \bar{D}_h
\end{aligned}
$$

Integrand-Level Approach

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&+ \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k}^{n-1} \bar{D}_h + \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j}^{n-1} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i}^{n-1} \bar{D}_h
\end{aligned}
$$

Recombining with the denominators:

$$
\begin{aligned}
A(\bar{q}) &= \sum_{i<<m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} + \sum_{i<<\ell}^{n-1} \frac{\Delta_{ijk\ell}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell} + \sum_{i<<k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k} + \\
&+ \sum_{i<j}^{n-1} \frac{\Delta_{ij}(\bar{q})}{\bar{D}_i \bar{D}_j} + \sum_{i}^{n-1} \frac{\Delta_{i}(\bar{q})}{\bar{D}_i} \; ,
\end{aligned}
$$

the decomposition exposes **the multi-pole nature of the integrand**

Integrand-Level Approach

Integrand level decomposition:

$$\begin{aligned} N(\bar{q}) &= \sum_{i<<m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h\neq i,j,k,\ell,m}^{n-1} \bar{D}_h + \sum_{i<<\ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h\neq i,j,k,\ell}^{n-1} \bar{D}_h + \\ &+ \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k}^{n-1} \bar{D}_h + \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j}^{n-1} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i}^{n-1} \bar{D}_h \end{aligned}$$

- the functional form is **process-independent**, the process-dependent coefficients are contained in the Δ's
- the **Rational Term is automatically included**
- **polynomial fitting replaces the integration** (both sides are polynomial)
- we can **extract the coefficients going on-shell**: we only need the **Numerator Function evaluated on the cuts**

GO, Papadopoulos, Pittau;
Mastrolia, GO, Reiter, Tramontano

Example: 4-particles process (in 4-dim)

$$
\begin{aligned}
N(q) &= d + \tilde{d}(q) + \sum_{i=0}^{3} \left[c(i) + \tilde{c}(q;i) \right] D_i + \sum_{i_0 < i_1}^{3} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] D_{i_0} D_{i_1} \\
&+ \sum_{i_0=0}^{3} \left[a(i_0) + \tilde{a}(q; i_0) \right] D_{i \neq i_0} D_{j \neq i_0} D_{k \neq i_0}
\end{aligned}
$$

We look for a q such that

$$D_0 = D_1 = D_2 = D_3 = 0$$

$\rightarrow$ there are two solutions $q_0^{\pm}$

Example: 4-particles process (in 4-dim)

$$N(q) \;=\; d + \tilde{d}(q)$$

Our "master formula" for $q = q_0^{\pm}$ is:

$$N(q_0^{\pm}) = [d + \tilde{d}\,(w \cdot q_0^{\pm})]$$

$\rightarrow$ solve to extract the coefficients d and $\tilde{d}$

Example: 4-particles process (in 4-dim)

$$N(q) - d - \tilde{d}(q) = \sum_{i=0}^{3} \left[c(i) + \tilde{c}(q;i) \right] D_i + \sum_{i_0<i_1}^{3} \left[b(i_0 i_1) + \tilde{b}(q;i_0 i_1) \right] D_{i_0} D_{i_1}$$
$$+ \sum_{i_0=0}^{3} \left[a(i_0) + \tilde{a}(q;i_0) \right] D_{i\neq i_0} D_{j\neq i_0} D_{k\neq i_0}$$

Then we can move to the extraction of c coefficients using

$$N'(q) = N(q) - d - \tilde{d}(w \cdot q)$$

and setting to zero three denominators (ex: $D_1 = 0$, $D_2 = 0$, $D_3 = 0$)

Example: 4-particles process (in 4-dim)

$$N(q) - d - \tilde{d}(q) \quad = \qquad [c(0) + \tilde{c}(q;0)]\, D_0$$

We have infinite values of q for which

$$D_1 = D_2 = D_3 = 0 \quad \text{and} \quad D_0 \neq 0$$

$\rightarrow$ Here we need 7 of them to determine $c(0)$ and $\tilde{c}(q;0)$

Integrand-Reduction via Laurent Expansion

- The coefficients of the integrand can be extracted by performing a Laurent expansion with respect to one of the free parameters which appear in the solutions of the cut
- corrections at the coefficient level replace the subtractions at the integrand level

Advantages:

- a "lighter" reduction algorithm where fewer coefficients
- 4-cut decoupled from triple-, double-, and single-cut.
- No more "sampling on the cuts"

This method has been implemented in the C++ library **Ninja** and interfaced with GoSam: Preliminary tests show an improvement in the computational performance

Mastrolia, Mirabella, Peraro

see also **Forde; Badger**

Different Multi-Purpose Codes/Tools

Several Strategies = Many Computational Tools:

- ★ BlackHat [Bern, Dixon, Febres Cordero, Hoeche, Ita, **Kosower**, Maitre, Ozeren]
- ★ Feynarts/Formcalc/LoopTools [**Hahn** et al.]
- ★ GoSam [Cullen, Greiner, **Heinrich**, Mastrolia, GO, Reiter, Tramontano, Luisoni]
- ★ Helac-NLO [Bevilacqua, Czakon, van Hameren, Papadopoulos, Pittau, Worek]
- ★ MadLoop [Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau]
- ★ NJet [Badger, Biedermann, Uwer, **Yundin**]
- ★ Openloops [Cascioli, **Maierhöfer**, Pozzorini]
- ★ Recola [Actis, Denner, Hofer, Scharf, Uccirati]
- ★ Rocket [Ellis, Giele, Kunszt, Melnikov, Zanderighi]

The "NLO industrial revolution" = NLO Automation

Different Multi-Purpose Codes/Tools

Several Strategies = Many Computational Tools:

- ★ BlackHat → Generalized Unitarity
- ★ Feynarts/Formcalc/LoopTools → Feynman Diagrams + Tens.Red./Integrand-Level
- ★ GoSam → Feynman Diagrams + Tens.Red./Integrand-Level
- ★ Helac-NLO → Tree-level recursion + Integrand-Level
- ★ MadLoop → Tree-level recursion + Integrand-Level
- ★ NJet → Generalized Unitarity
- ★ Openloops → Recursive Tensorial Reconstruction + Tens.Red./Integrand-Level
- ★ Recola → Recursive Tensorial Reconstruction + Tens.Red./Integrand-Level
- ★ Rocket → Generalized Unitarity

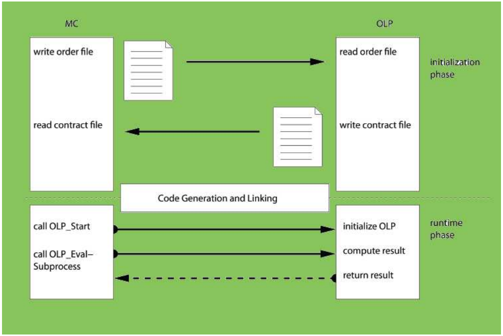
The "NLO industrial revolution" = NLO Automation

Modularity, Automation, BLHA

Automation is crucial for multi-lop NLO calculations:

- Optimization/Self-organization
- Avoid human mistakes
- Process-independent techniques

Different Levels of Automation: MC controls the OLP via BLHA



WHAT ABOUT HIGHER LOOPS?

Problem: at two loops (and higher), we do not have a
Standard Complete Basis of Master Integrals

recent work of Gluza, Kosower, Kajda; Schabinger

The most common (and successful) approach relies on:

- Amplitude generation via **Feynman diagrams**
- Reduction to a minimal set of MIs using **IBPs** (Laporta algorithm)
- Direct Computation of the MIs (analytically or numerically)

WHAT ABOUT HIGHER LOOPS?

Problem: at two loops (and higher), we do not have a **Standard Complete Basis of Master Integrals**

Can we extend what we learned at one-loop about to develop alternative approaches for multi-loop reduction?

- **Integrand-level Techniques**
  Mastrolia, GO (2011); Badger, Frellersvig, Zhang; Kleiss, Malamos, Papadopoulos, Verheyen; Zhang; Mastrolia, GO, Mirabella, Peraro; Feng, Huang; Huang, Zhang
- **Maximal Unitarity**
  Kosower, Larsen (2011); Johansson, Kosower, Larsen; Larsen, Caron-Huot

Integrand-Level Strategy

Let's consider a two-loop integral with n denominators:

$$\int dq\ dk\ \frac{N(q,k)}{\bar{D}_1 \bar{D}_2 \dots \bar{D}_n}$$

As in the one-loop case, we want to construct an identity for the integrands:

$$N(q,k) = \sum_{i_1 << i_8}^{n} \Delta_{i_1,\dots,i_8}(q,k) \prod_{h \neq i_1,\dots,i_8}^{n} \bar{D}_h + \dots + \sum_{i_1 << i_2}^{n} \Delta_{i_1,i_2}(q,k) \prod_{h \neq i_1,i_2}^{n} \bar{D}_h$$

$$A(q,k) = \sum_{i_1 << i_8}^{n} \frac{\Delta_{i_1,\dots,i_8}(q,k)}{\bar{D}_{i_1} \bar{D}_{i_2} \dots \bar{D}_{i_8}} + \sum_{i_1 << i_7}^{n} \frac{\Delta_{i_1,\dots,i_7}(q,k)}{\bar{D}_{i_1} \bar{D}_{i_2} \dots \bar{D}_{i_7}} + \dots + \sum_{i_1 << i_2}^{n} \frac{\Delta_{i_1,i_2}(q,k)}{\bar{D}_{i_1} \bar{D}_{i_2}}$$

- Which terms appear in the above expressions?
- What is the general form of the residues $\Delta_{i_1,\dots,i_m}$?

Collaboration with **P.Mastrolia, E.Mirabella, T.Peraro, U.Schubert**

Integrand-Level Strategy

Mastrolia, GO, Mirabella, Peraro

Let's look at the on-shell conditions, and impose

$$D_1 = D_2 = \ldots = D_n = 0$$

1) There are no solutions $\rightarrow$ **reducible**

$\rightarrow$ The n-denominator integrand can be written in terms of integrands with $(n-1)$ denominators
$\rightarrow$ it is **fully reducible in terms of lower point functions**
i.e. a six-point function at one-loop

Integrand-Level Strategy

Mastrolia, GO, Mirabella, Peraro

Let's look at the on-shell conditions, and impose

$$D_1 = D_2 = \ldots = D_n = 0$$

1) There are no solutions $\rightarrow$ **reducible**

2) The cut has solutions $\rightarrow$ there is a **residue** Δ

We divide the numerator modulo the Gröbner basis of the n-ple cut (a set of polynomials vanishing on the same on-shell cuts of the denominators).
$\rightarrow$ The *remainder* of the division is the *residue* of the n-ple cut.
$\rightarrow$ The *quotients* generate integrands with $(n-1)$ denominators.

Mastrolia, GO, Mirabella, Peraro

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1) There are no solutions $\rightarrow$ **reducible**
2) The cut has solutions $\rightarrow$ there is a **residue** Δ
3) Finite number of solutions n_s $\rightarrow$ **Maximum Cut**

$\rightarrow$ "Maximum Cut" i.e. a four-point function at one-loop (in 4-dim)
$\rightarrow$ its residue is a univariate polynomial parametrized by n_s coefficients
$\rightarrow$ the corresponding residue can always be reconstructed at the cut
$\rightarrow$ the residue is determined as in the previous case

"On-shell" in Algebraic Geometry Language

$$\mathcal{I}_{i_1\cdots i_n} = \frac{\mathcal{N}_{i_1\cdots i_n}(z)}{D_{i_1}(z)\cdots D_{i_n}(z)}$$

where z = components of the loop momenta

- Ideal: $\mathcal{J}_{i_1\cdots i_n} = \langle D_{i_1}, \cdots, D_{i_n}\rangle$
- Gröbner basis $\mathcal{G}_{i_1\cdots i_n}$: same zero as the denominators
- Multivariate division of $\mathcal{N}_{i_1\cdots i_n}$ modulo $\mathcal{G}_{i_1\cdots i_n}$

$$\mathcal{N}_{i_1\cdots i_n}(z) = \Gamma_{i_1\cdots i_n} + \Delta_{i_1\cdots i_n}(z)$$

- The quotient $\Gamma_{i_1\cdots i_n}$ can be expressed in terms of denominators

$$\Gamma_{i_1\cdots i_n} = \sum_{\kappa=1}^{n} \mathcal{N}_{i_1\cdots i_{\kappa-1} i_{\kappa+1}\cdots i_n}(\zeta)\bar{D}_{i_\kappa}(\zeta)$$

- Which provides the Recursive Formula

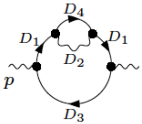
$$\mathcal{I}_{i_1\cdots i_n} = \sum_{\kappa=1}^{n} \mathcal{I}_{i_1\cdots i_{\kappa-1} i_{\kappa+1} i_n} + \frac{\Delta_{i_1\cdots i_n}}{\bar{D}_{i_1}\cdots\bar{D}_{i_n}}$$

Two-loop Photon Self-Energy in QED

Mastrolia, Peraro (2013)

⇝ higher powers of propagators are not problematic

- Example: QED photon self energy @ two loops



$$\mathcal{I} = \frac{\Delta_{11234}}{D_1^2 D_2 D_3 D_4} + \frac{\Delta_{1234}}{D_1 D_2 D_3 D_4} + \frac{\Delta_{1124}}{D_1^2 D_2 D_4} + \frac{\Delta_{234}}{D_2 D_3 D_4} + \frac{\Delta_{124}}{D_1 D_2 D_4}$$

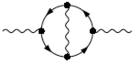
$$\mathcal{N} = 16[\mu_{11}^2 - \mathrm{k}_1^2\,(\mathrm{k}_1 \cdot p)] + \cdots$$

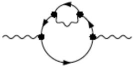
d dimensions: $\bar{\mathrm{k}}_i^\mu = \mathrm{k}_i^\mu + \vec{\mu}_i \qquad \mu_{ij} \equiv \vec{\mu}_i \cdot \vec{\mu}_j$

- Reduction completed after five steps ...

$$\mathcal{I} = \frac{8\mu_{11}(2\mu_{11} - p^2)}{D_1^2 D_2 D_3 D_4} - \frac{8(\mu_{11} + p^2)}{D_1 D_2 D_3 D_4} + \frac{8\mu_{11}}{D_1^2 D_2 D_4} - \frac{8}{D_2 D_3 D_4} + \frac{8}{D_1 D_2 D_4}$$

- ... performed for the full $\mathcal{N}$ ...
- ... and for the other diagrams





Conclusions/Outlook

- Mini-collection of "Conclusions" form the past 5 years:
  "Can we achieve at NLO the same degree of automation of the LO?"
  "This method is potentially a good candidate for NLO automation"
  "A generic NLO calculator seems feasible"

Conclusions/Outlook

- At present, there is a great variety of methods available for NLO scattering amplitudes: Old ideas merged with New Ideas

Tensorial Reduction

Generalized Unitarity

Integrand-level reduction

Techniques for the Rational Terms

Trees "recycled" into loops

Automation of Feynman Diagrams

On-shell tree-amplitudes

Off-shell currents

Recursion Relations

Tensorial Reconstruction

Conclusions/Outlook

- At present, there is a great variety of methods available for NLO scattering amplitudes: Old ideas merged with New Ideas

Tensorial Reduction	Automation of Feynman Diagrams
Generalized Unitarity	On-shell tree-amplitudes
Integrand-level reduction	Off-shell currents
Techniques for the Rational Terms	Recursion Relations
Trees "recycled" into loops	Tensorial Reconstruction

- OLP integrated inside the MC (or via BLHA) to generate theoretical prediction for a large variety of processes: high multiplicities, several scales, effective vertices . . .

Conclusions/Outlook

- At present, there is a great variety of methods available for NLO scattering amplitudes: Old ideas merged with New Ideas

Tensorial Reduction

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Integrand-level reduction

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Ideas + Automation = NLO Revolution

"Will we achieve at NNLO the same degree of automation of the NLO?"

(not a serious question, but...)