Numerical multi-loop calculations with SecDec

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in collaboration with

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MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
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Motivation

- A lot of progress has been achieved towards the goal of describing hadron collider processes consistently at NLO
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  1. extraction of IR and UV singularities
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- SecDec 2.0 offers a solution to the second problem

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- SecDec 1.0 offers a solution to the first problem
- SecDec 2.0 offers a solution to the second problem
- SecDec 2.1 improves the third problem (+new features)

The program SecDec

http://secdec.hepforge.org

SecDec is hosted by Hepforge, IPPP D

- Home
- Subversion
- Tracker
- Wiki

SecDec

Sophia Borowka, Jonathon Carter, Gudrun Heinrich

A program to evaluate dimensionally regulated parameter integrals numerically

Download Program FAQ ChangeLog

NEW: Version 2.1 of the program can be downloaded as SecDec-2.1.tar.gz.

Version 2.0 of the program can be downloaded as SecDec-2.0.tar.gz.

To install the program:

- tar xzvf SecDec-2.1.tar.gz
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- ./install

Prerequisites: Mathematica (version 6 or higher), Perl, Fortran/C++ compiler

Sector Decomposition

- allows to extract UV and IR singularities from (dimensionally regulated) parameter integrals in an automated way
- produces a Laurent series in ϵ
- coefficients are finite parameter integrals
  $\Rightarrow$ integrate numerically
- can be applied in various contexts
  (e.g. multi-loop integrals, NNLO phase space integrals)

Sector Decomposition

history:

- originally devised by K. Hepp 1966

  (proof of Bogolyubov-Parasiuk theorem on renormalization)

  also used by Denner, Roth 1996
- construction of a general algorithm to isolate infared divergences from multi-loop integrals: Binoth, GH 2000
- meanwhile applied succesfully in various contexts, in particular NNLO real radiation

  [Anastasiou et al, Binoth et al, Boughezal, Czakon, Denner/Pozzorini et al, Kunszt et al, Passarino et al, Melnikov, Petriello, Smirnov et al, Somogyi/Trocsanyi et al, Weinzierl et al, . . .]

Sector Decomposition

public programs:

- sector_decomposition (uses Ginac) Bogner, Weinzierl '07

  supplemented with CSectors Gluza, Kajda, Riemann, Yundin '10

  for construction of integrand in terms of Feynman parameters
- FIESTA (uses Mathematica, C) A. Smirnov, V. Smirnov, M. Tentyukov '08, '09
- SecDec (uses Mathematica, Fortran/C++)

  J. Carter, GH '10; S. Borowka, J. Carter, GH '12; S. Borowka, GH '13

  http://secdec.hepforge.org

Parametric integrals

parameter integrals are ubiquitous when calculating higher order corrections

- (multi-) loop Feynman integrals
- subtraction terms for IR singular real radiation at NNLO
- Wilson loop polygons
- ...

Multi-loop integrals

general form of scalar L-loop integral with N propagators (to powers ν_j) after Feynman parametrisation:

$$G = \frac{(-1)^N}{\prod_{j=1}^{N}\Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} dx_j \; x_j^{\nu_j-1}\, \delta(1-\sum_{l=1}^{N} x_l) \; \frac{\mathcal{U}(x)^{N-(L+1)D/2}}{\mathcal{F}(x)^{N-LD/2}}$$

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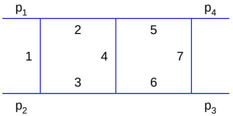
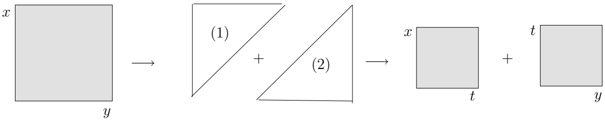
example planar double box with $p_1^2 = p_2^2 = p_3^2 = 0,\, p_4^2 \neq 0$: $N = 7,\, L = 2,\, D = 4 - 2\epsilon$

$$\begin{aligned}
\mathcal{F} &= -s\,(x_2 x_3 x_{4567} + x_5 x_6 x_{1234} + x_2 x_4 x_6 + x_3 x_4 x_5) \\
&\quad -t\, x_1 x_4 x_7 - p_4^2\, x_7 (x_2 x_4 + x_5 x_{1234}) + \mathcal{U} \sum_i x_i\, m_i^2 - i\,\delta \\
\mathcal{U} &= x_{123} x_{567} + x_4 x_{123567} \\
x_{ijk\ldots} &= x_i + x_j + x_k + \ldots
\end{aligned}$$



Problem 1: Factorisation of endpoint singularities



UV and IR singularities will show up as endpoint singularities of type

$$I = \int_0^1 dx \int_0^1 dy\, x^{-1-\epsilon} (a_1 x + a_2 y)^{-1} [\underbrace{\Theta(x-y)}_{(1)} + \underbrace{\Theta(y-x)}_{(2)}]$$

subst. (1) $y = x\,z$ (2) $x = y\,z$ to remap to unit cube

$$I = \int_0^1 dx\, x^{-1-\epsilon} \int_0^1 dz\, (a_1 + a_2 z)^{-1} + \int_0^1 dy\, y^{-1-\epsilon} \int_0^1 dz\, z^{-1-\epsilon} (a_1 z + a_2)^{-1}$$

singularities are factorized, number of integrals doubled

Problem 2: Dealing with integrable singularities

now consider an integral of type

$$I = \int_0^1 dx \int_0^1 dy \, x^{-1-\epsilon} \, (a_1 \, x - a_2 \, y)^{-1}$$

(e.g. loop integral containing Lorentz invariants with different sign)

- limitation of SecDec 1.0:
  integrand should not change sign
  $\Rightarrow$ multi-scale integrals limited to Euclidean region where integrand is positive definite
- NEW (version $\geq$ 2.0):

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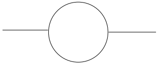
- NEW (version $\geq$ 2.0):

extension of SecDec to general kinematics

method: deformation of integration contour into complex plane
Soper '99, Nagy, Binoth; Kurihara/Kaneko et al, Anastasiou et al, Weinzierl et al.

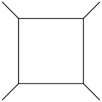
Integrable singularities: examples

$\mathcal{F}_{\rm bubble} = -p^2\, x\,(1-x) + m^2\; - i\,\delta$



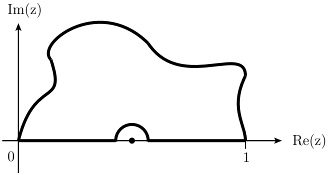
$\mathcal{F}_{\rm bubble}$ vanishing at $x = 1/2, p^2 = 4\, m^2 \Rightarrow$ threshold

$\mathcal{F}_{\rm box} = -s_{12}\, x_1 x_3 - s_{23} x_2 x_4 - i\,\delta$



$\mathcal{F}_{\rm box}$ can vanish inside integration region if s_{12} and s_{23} have different sign
("Euclidean region" if both s_{12} and s_{23} are negative)

Contour deformation



Cauchy: integral over closed contour is zero if no poles are enclosed

$$\int_0^1 \prod_{j=1}^{N} x_j \, \mathcal{I}(\vec{x}) = \int_0^1 \prod_{j=1}^{N} x_j \left| \frac{\partial z_k(\vec{x})}{\partial x_l} \right| \mathcal{I}(\vec{z}(\vec{x}))$$

$i\,\delta$ prescription for Feynman propagators $\Rightarrow Im(\mathcal{F})$ should be < 0

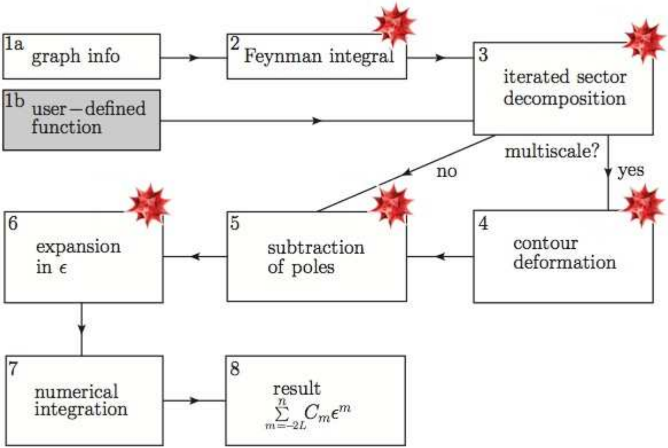
complexify:

$$\vec{z}(\vec{x}) = \vec{x} - i\,\vec{\tau}(\vec{x})\ ,\ \tau_k = \lambda\, x_k(1 - x_k)\, \frac{\partial \mathcal{F}(\vec{x})}{\partial x_k}$$

For small λ correct sign of Im part is guaranteed:

$$\mathcal{F}(\vec{z}(\vec{x})) = \mathcal{F}(\vec{x}) - i\,\lambda \sum_j x_j(1 - x_j) \left(\frac{\partial \mathcal{F}}{\partial x_j} \right)^2 + \mathcal{O}(\lambda^2)$$

The program SecDec



numerical integration: Cuba [T. Hahn et al], Bases [S. Kawabata]

Installation and Usage

- installation:
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  - prerequisites:
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- user input: two files:
  - parameter.input: parameters for the integrand specification and numerical integration (text file)
  - graph.m: definition of the integrand (Mathematica syntax)

Usage: parameter.input

Bubble2Lrank3.input - /Users/gudrun/SecDec-2.1/loop/demos/

File Edit Search Preferences Shell Macro Windows Help

```
################ input parameters for sector decomposition ################
#
# ############### all lines beginning with # are comments ##################
#------------------------------------------------------------------------------------
# subdirectory for the mathematica output files (will be created if non-existent):
subdir=2loop
#------------------------------------------------------------------------------------
# graphname (can contain underscores, numbers, but should not contain commas)
graph=Bubble2Lrank3
#------------------------------------------------------------------------------------
# number of propagators:
propagators=5
#------------------------------------------------------------------------------------
# number of external legs:
legs=2
#------------------------------------------------------------------------------------
# number of loops:
loops=2
#------------------------------------------------------------------------------------
# construct integrand (F and U) via topological cuts (only for scalar integrals)
# default is 0 (no cut construction used)
cutconstruct=0
#########################################################
# parameters for subtractions and epsilon expansion
#########################################################
# epsord: level up to which expansion in eps is desired
# (default is epsord=0: Laurent series is cut after finite part eps^0)
# series will be calculated from eps^(-maxpole) to eps^epsord
# note that epsord is negative if only some pole coeffs are required
epsord=0
#----------------------------------------------------------------------------------
#  flag for prefactor:
```

Usage: graph.m

Bubble2Lrank3.m - /Users/gudrun/SecDec-2.1/loop/demos/

File Edit Search Preferences Shell Macro Windows Help

```
(* USER INPUT: *)

(* give -list of loop momenta (momlist)
          -list of propagators (proplist):
          -numerator: list of scalar products of loop momenta contracted with
           external vectors or loop momenta;
          -list of propagator powers (powerlist), default is 1:
           powers of propagators as listed in proplist         *)

(* example is 2-loop 2-point integral with k1.k2 k1.p1 in the numerator *)

momlist={k1,k2};

proplist={k1^2-ms[1],(k1+p1)^2-ms[1],(k1-k2)^2,(k2+p1)^2-ms[2],k2^2-ms[2]};

numerator={k1*k2,k1*p1};

(* optional: give propagator powers if different from one *)

powerlist=Table[1,{i,Length[proplist]}];

(* optional: give on-shell conditions *)
(* note that in constructing F, (pi+pj)^2 will automatically be called sp[i,j];
    pi^2 will be called ssp[i];
    masses m_i^2 must be called ms[i];
    for the numerator, only the replacements given explicitly in onshell will be made  *)

onshell={};

(* Dim can be changed, but symbol for epsilon must be the same *)
Dim=4-2*eps;
```

Usage

to launch one run:

```
./launch -p parameter.input -t graph.m
```

to scan over a set of parameter values:

- do decomposition once (exeflag=1 in parameter.input)
- define parameter values in multiparam.input

```
perl multinumerics.pl -p multiparam.input
```

New features of SecDec 2

- **loop** integrals:
  - no restriction on the kinematics!
  - tensors of (in principle) arbitrary rank
  - several options for the user to tune the numerical integration
  - can be parallelized easily (also thanks to `CUBA-3.x` [T. Hahn])
  - extension to non-standard loop integrals
    (useful e.g. if some parameter(s) have been integrated out analytically already)

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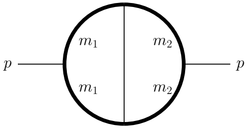
- **general** parameter integrals: (extraction of endpoint singularities from general parametric functions)
  - option to define implicit functions
- **both** parts:
  - loops over ranges of numerical values automated

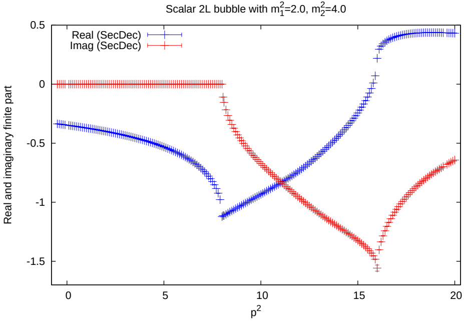
Results: tensor integrals

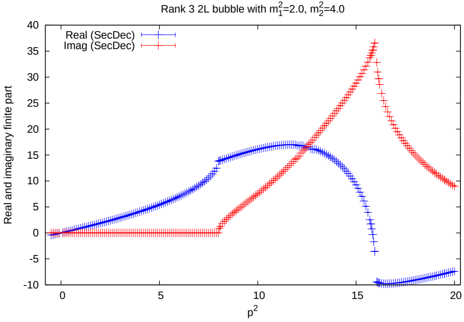


a two-mass two-loop bubble

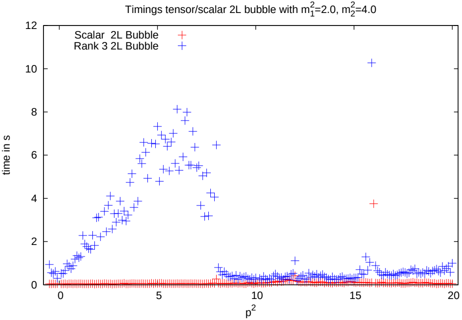
(analytical result:1-dim integral representation)

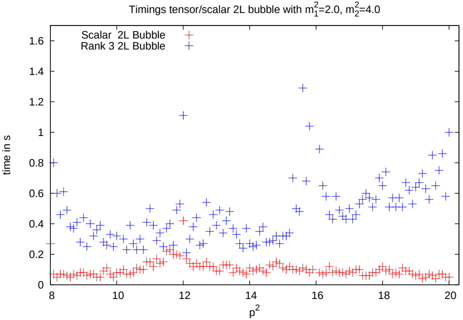
[Bauberger, Böhm '95]





Timings for tensor integrals





relative accuracy 0.1%

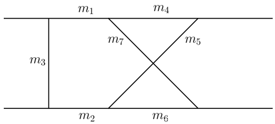
Intel core i7 processor

below 1st threshold at $p^2 = 8$: Imaginary part zero

$\Rightarrow$ artificially increases integration time

good timings for tensor integrals $\Rightarrow$ no need for reduction

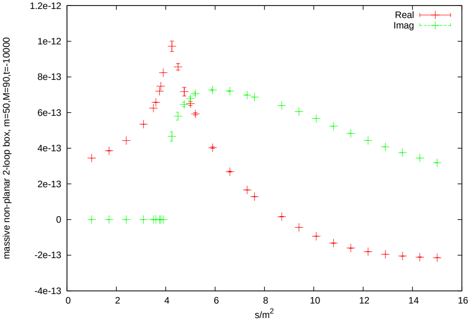
Non-planar four-point functions



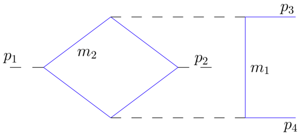
compared with numerical result from Yuasa et al, CPC 183 (2012)

$m_1 = m_2 = m_5 = m_6 = m = 50,\ m_3 = m_4 = m_7 = M = 90,\ p_1^2 = p_2^2 = p_3^2 = p_4^2 = m^2,\ s_{23} = -10^4$

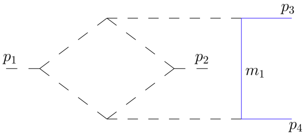
timings: (relative accuracy 10^{-3}): far from threshold: $\sim 20\,s$, close to threshold: $\sim 500\,s$



Non-planar four-point functions for $pp \to t\bar{t}$@NNLO



ggtt1: finite
analytic result unknown
(blue lines are massive)

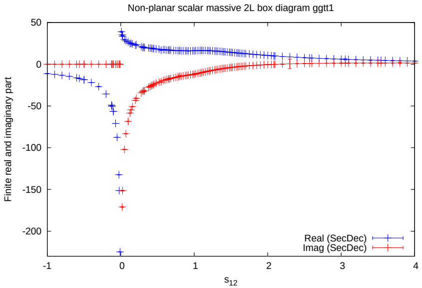


ggtt2: $1/\epsilon^4$ poles
analytical manipulations and new type of transformations to assist SecDec
$\rightarrow$ triggered development of code to allow non-standard input

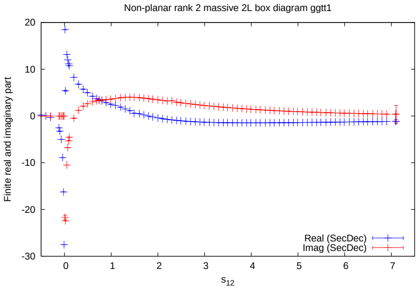
Non-planar four-point functions: ggtt1

$m_1 = m_2 = 1,\ p_1^2 = p_2^2 = 0,\ p_3^2 = p_4^2 = m_1^2,\ s_{23} = -1.25$

analytical result unknown



scalar integral



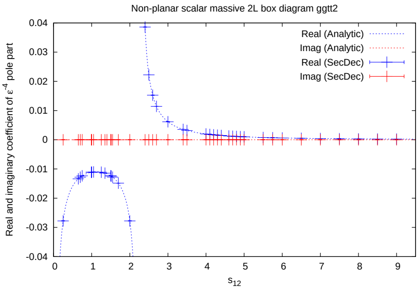
rank two tensor integral
$(\mathcal{N} = k_1 \cdot k_2)$

timings per phase space point: (relative accuracy 10^{-3})

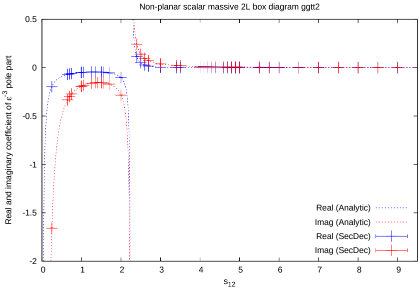
far from threshold: $\mathcal{O}(10\,s)$, very close to threshold: $\mathcal{O}(500\,s)$

Non-planar four-point functions: ggtt2

$m_1 = 1,\ p_1^2 = p_2^2 = 0,\ p_3^2 = p_4^2 = m_1^2,\ s_{23} = -1.25$



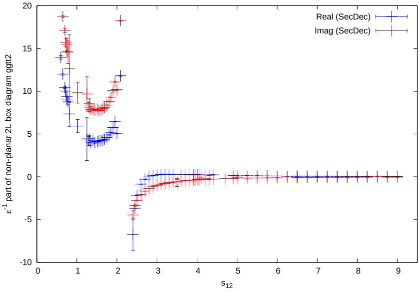
leading pole



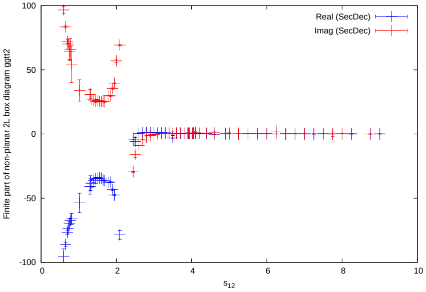
subleading pole

analytical result by Manteuffel, Studerus '12

Non-planar four-point functions: ggtt2



$1/\epsilon$ part



finite part

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- phenomenological application (NNLO)
- combination with unitarity-inspired reduction of two-loop amplitudes