Automatic one-loop calculations with OpenLoops

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In Collaboration with
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4 Irreducible background to $H \rightarrow WW^* + 0,1$ jet
The NLO Frontier: Automation

The list of completed $\geq 6$ particle processes keeps growing . . .

$pp \rightarrow WWb\bar{b}$
[Denner, Dittmaier, Kallweit, Pozzorini ‘11]
[Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek ‘11]

$pp \rightarrow t\bar{t}b\bar{b}$
[Bredenstein, Denner, Dittmaier, Pozzorini ‘08, ‘09, ‘10]
[Bevilacqua, Czakon, Papadopoulos, Pittau, Worek ‘09]

$pp \rightarrow t\bar{t}jj$
[Bevilacqua, Czakon, Papadopoulos, Pittau, Worek ‘10]

$pp \rightarrow t\bar{t}t\bar{t}$
[Bevilacqua, Worek ‘12]

$pp \rightarrow WW + 2j$
[Melia, Melnikov, Rontsch, Zanderighi ‘10]
[Greiner, Heinrich, Mastrolia, Ossola, Reiter, Tramontano ‘12]

$pp \rightarrow W + 3j$
[Ellis, Melnikov, Zanderighi ‘09]

$pp \rightarrow \gamma^*/Z/W + 3j$
[Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maître ‘09, ‘10]

$pp \rightarrow Z/W + 4j$
[Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maître ‘10, ‘11]

$pp \rightarrow W^\pm + 5j$
[Bern, Dixon, Febres Cordero, Höche, Ita, Kosower, Maître, Ozeren ‘13]

$pp \rightarrow 4j$
[Bern, Diana, Dixon, Febres Cordero, Höche, Ita, Kosower, Maître, Ozeren ‘11]

$pp \rightarrow b\bar{b}b\bar{b}$
[Greiner, Guffanti, Reiter, Reuter ‘11]

$pp \rightarrow W\gamma\gamma j$
[Campanario, Englert, Rauch, Zeppenfeld ‘11]

$pp \rightarrow WZjj$
[Campanario, Kerner, Ninh, Zeppenfeld ‘13]

$e^+e^- \rightarrow 7j$
[Becker, Goetz, Reuschle, Schwan, Weinzierl ‘11]

. . . but NLO automation is still a challenge.

- Focus on speed and usability.
- Arbitrary processes, decays, electroweak corrections, . . .
- Progress in Monte Carlo generators: beyond parton level NLO.
From Loop Amplitudes to Scalar Integrals

\[ \int d^d q \frac{\mathcal{N}(q)}{D_0 D_1 \ldots D_{N-1}}, \quad D_i = (q + \sum_{\ell=0}^i p_\ell)^2 - m_i^2 \]

Tensor integral reduction to a linear combination of scalar basis integrals

\[ \int d^d q \left[ \sum_{i_1} \frac{a_{i_1}}{D_{i_1}} + \sum_{i_1, i_2} \frac{b_{i_1 i_2}}{D_{i_1} D_{i_2}} + \sum_{i_1, i_2, i_3} \frac{c_{i_1 i_2 i_3}}{D_{i_1} D_{i_2} D_{i_3}} + \sum_{i_1, i_2, i_3, i_4} \frac{d_{i_1 i_2 i_3 i_4}}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} \right] \]

Tensor integral reduction combined with off-shell current recursion can compete with on-shell methods in gluon scattering with up to 10 gluons. [van Hameren ‘09]
Colour and Tensor Reduction

For each Feynman diagram separate **colour factors** and **tensor coefficients** from **tensor integrals**.

\[ A = C \cdot \sum_{r=0}^{R} N_{r}^{\mu 1 \ldots \mu r} \cdot \int d^{d}q \frac{q_{\mu 1} \ldots q_{\mu r}}{D_{0} \cdot D_{1} \ldots D_{N-1}} \]

- Algebraic colour reduction and summation only once per process.
- Reduce tensor integrals to scalar basis integrals [Melrose; Passarino, Veltman; Denner, Dittmaier; Binoth et al.; Fleischer, Riemann; & many others].
  
  We use **Collier** [Denner, Dittmaier, Hofer]: cures numerical instabilities, e.g. by applying expansions in small Gram determinants.
- Alternatively use OPP reduction [Ossola, Papadopoulos, Pittau]: requires multiple evaluations of \( N_{r}^{\mu 1 \ldots \mu r} q_{\mu 1} \ldots q_{\mu r} \) for complex \( q \).

**“Traditional” approach**: construct \( N_{r}^{\mu 1 \ldots \mu r} \) analytically in \( d = 4 - 2\epsilon \). Huge expressions & expensive algebraic simplifications limit applicability.

**OpenLoops**: Recursive numerical construction of \( N_{r}^{\mu 1 \ldots \mu r} \) in \( d = 4 \).
Wave functions $w^\alpha$ of “sub-trees” are 4-tuples (for the spinor/Lorentz index) which are built by recursively connecting lower sub-trees with vertices $X_{\gamma\delta}^\beta$ and propagators, starting from external legs.

$$w^\beta(i) = \frac{X_{\gamma\delta}^\beta}{p_i^2 - m_i^2} w^\gamma(j) w^\delta(k)$$

A one-loop diagram is an ordered set of sub-trees $I_n = \{i_1, \ldots, i_n\}$

Connect sub-trees along the loop to build the numerator $N = N^\alpha_\alpha$:

$$N^\beta_\alpha (I_n; q) = X_{\gamma\delta}^\beta N^\gamma_\alpha (I_{n-1}; q) w^\delta(i_n)$$
Open Loops Recursion

Separation of the loop momentum $q$

$$\mathcal{N}_{\alpha}^{\beta}(I_n; q) = \sum_{r=0}^{n} \mathcal{N}_{\mu_1...\mu_r;\alpha}(I_n) q^{\mu_1} \cdots q^{\mu_r}, \quad X_{\gamma\delta}^{\beta} = Y_{\gamma\delta}^{\beta} + q^{\nu} Z_{\nu;\gamma\delta}^{\beta}$$

leads to the recursion formula for “Open loops” polynomials $\mathcal{N}_{\mu_1...\mu_r;\alpha}^{\beta}$:

$$\mathcal{N}_{\mu_1...\mu_r;\alpha}(I_n) = \left[ Y_{\gamma\delta}^{\beta} \mathcal{N}_{\mu_1...\mu_r;\alpha}(I_{n-1}) + Z_{\mu_1;\gamma\delta}^{\beta} \mathcal{N}_{\mu_2...\mu_r;\alpha}(I_{n-1}) \right] w^{\delta}(i_n)$$

- Retains functional dependence on the loop momentum.
- $\mathcal{N}_{\mu_1...\mu_r;\alpha}^{\alpha}$ are the coefficients of the tensor integrals.
- Also, once the polynomials are known, multiple evaluations of

$$\mathcal{N}(q) = \sum_{r=0}^{n} \mathcal{N}_{\mu_1...\mu_r;\alpha}^{\alpha} q^{\mu_1} \cdots q^{\mu_r}$$

are very fast. $\Rightarrow$ boosts OPP

Open loops can be interfaced with both tensor integrals and OPP in a straightforward way.
Recycling and Helicity Summation

Open loops recycling
Lower-point open-loops can be shared between diagrams if the cut it put appropriately.

Helicity summation
Perform interference with the Born amplitude $M$, colour and helicity sums and the sum over the set of diagrams $\Delta$ with identical denominator structure on the level of open-loop coefficients.

$$
\delta W^\Delta = \sum_{\text{hel, col}} 2 \text{Re} \left[ M^* \left( \sum_{d' \in \Delta} \delta M^{(d')} \right) \right]
$$

$$
\delta W^\Delta_{\mu_1...\mu_R} = \sum_{\text{hel, col}} 2 \times \left[ M^* \left( \sum_{d' \in \Delta} C^{(d')} N^{(d')}_{\mu_1...\mu_R} \right) \right]
$$

Helicity sums with OPP as efficient as with tensor integrals
Implementation

User input: process definition file

- FeynArts [Hahn] generates Feynman diagrams.
- Mathematica organises recursion and recycling, reduces colour factors and generates Fortran 90 code.
- Numerical routines for QCD corrections to Standard Model processes implemented in Fortran 90.
- Symmetrising $N_{\mu_1...\mu_r;\alpha}^\beta$ keeps the number of components manageable.
- Rational terms $R_2$ are calculated using the tree generator. 
[Draggiotis, Garzelli, Malamos, Papadopoulos, Pittau ‘09, ‘10; Shao, Zhang, Chao ‘11]
- No user interaction required: process definition $\rightarrow$ compiled library.

Consistency checks

- UV/IR cancellations and Ward identities
- Tensor integrals / OPP reduction with different libraries
- “pseudo-tree”: fix loop momentum and compare to tree generator
Speed and Flexibility

Time to generate code: seconds to minutes
Compiled library size: 100 kB to a few MB
Runtime per phase space point: < 1 s for a $2 \rightarrow 4$ process (i7-750 single core, ifort 10.1)

Fractions of the runtime for scalar integrals, tensor reduction, coefficients

Full helicity sums cost only a factor $\sim 2$ for a $2 \rightarrow 4$ process.
Numerical Stability

The numerical precision can be estimated by a scaling test:

\[ m_i \rightarrow \xi m_i, \quad p_i^\mu \rightarrow \xi p_i^\mu \]

leads to

\[ \delta \mathcal{W} \rightarrow \delta \mathcal{W}' = \xi^K \delta \mathcal{W} \]

\[ \Rightarrow \text{precision } \Delta = \left| \frac{\xi^{-K} \delta \mathcal{W}'}{\delta \mathcal{W}} - 1 \right|, \quad \text{rsp. } d = -\log_{10} \Delta \text{ decimal digits.} \]

12 processes, \( 10^6 \) phase space points each;

\[ \sqrt{s} = 1 \text{ TeV}, \quad p_T > 50 \text{ GeV, } \Delta R_{ij} > 0.5; \]

using tensor integrals, in double precision;

11-15 digits on average;

1 permille with <5 digits in the worst 2 \( \rightarrow \) 4 case.
Automated one-loop calculations with OpenLoops

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Automation of NLO Calculations

Combine OpenLoops with multi-purpose Monte Carlo programs

- aMC@NLO, POWHEG, Sherpa:
  IR subtraction, real emission, phase space integration,
  NLO matching with shower, jet merging, hadronisation.
- OpenLoops provides an easy to use API to directly access
  initialisation and matrix element routines.
- Seamless integration of tools desired.

Done: Sherpa+OpenLoops interface

- Use OpenLoops to generate and compile process libraries.
- Steered by standard Sherpa run cards.
- No hard-wiring or interface code generation required.
- Perform on-the-fly consistency and stability checks.
Process libraries for ATLAS and CMS

Libraries for a wide range of processes are available to ATLAS and CMS.

<table>
<thead>
<tr>
<th>W/Z</th>
<th>jets</th>
<th>HQ pairs</th>
<th>single-top</th>
<th>Higgs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V + 3j$</td>
<td>$\gamma + 3j$</td>
<td>$tt + 1j$</td>
<td>$tb + 1j$</td>
<td>$(H + 2j)$</td>
</tr>
<tr>
<td>$VV + 2j$</td>
<td>$\gamma + 1(2)j$</td>
<td>$t\bar{t}V + 0(1)j$</td>
<td>$t + 1(2)j$</td>
<td>$VH + 1j$</td>
</tr>
<tr>
<td>$gg \rightarrow VV + 1j$</td>
<td>$V\gamma + 2j$</td>
<td>$b\bar{b}V + 0(1)j$</td>
<td>$tW + 0(1)j$</td>
<td>$t\bar{t}H$</td>
</tr>
<tr>
<td>$VVV + 0(1)j$</td>
<td></td>
<td></td>
<td></td>
<td>$qq \rightarrow Hqq + 0(1)j$</td>
</tr>
</tbody>
</table>

(including lower jet multiplicities)

- Validated process-by-process.
- All contributing 1-loop diagrams, full colour.
- Off-shell leptonic W/Z decays (complex masses).
- First step towards a public OpenLoops release.
Signal: two opposite sign leptons $+ E_T^{\text{miss}}$, binned in jet multiplicities. Data driven analysis: normalise background (from MC simulation) to data in control region (right) and extrapolate to signal region (left).
**H → WW* → e⁻νₑμ⁺νμ** in exclusive 0-/1-jet bins

Previously available predictions for \( pp \rightarrow e⁻νₑμ⁺νμ + 0/1\)jets

<table>
<thead>
<tr>
<th></th>
<th>NLO</th>
<th>gg induced</th>
<th>NLO+PS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 jets</td>
<td>[Campbell, Ellis, Williams '11]</td>
<td>[Binoth et al. '05]</td>
<td>[Melia et al. '11]</td>
</tr>
<tr>
<td></td>
<td>[Campbell, Ellis, Williams '11]</td>
<td></td>
<td>[Frederix et al. '11]</td>
</tr>
<tr>
<td>1 jet</td>
<td>[Dittmaier, Kallweit, Uwer '07]</td>
<td>[Melia et al. '12]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[Campbell, Ellis, Zanderighi '07]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[Agrawal, Shivaji '12]</td>
<td></td>
</tr>
</tbody>
</table>

**Sherpa+OpenLoops (preliminary)**

- \( ℓℓνν + 0/1\)jets MEPS@NLO [Höche, Krauss, Schönherr, Siegert '12]: parton shower and jet merging, NLO+LL accuracy in 0- and 1-jet bins.
- More realistic error estimates, including \( p_T^{veto} \) logs.
- Compare to NLO (no resummation), and MC@NLO (LO in 1-jet bin).
- Include all spin correlation, off-shell, and interference effects.
- Studies for ATLAS and CMS experimental analysis.
- Gluon induced channels in progress.
Transverse $WW$ mass distributions ($CMS @ 8$ TeV)

- 20% level agreement between NLO/MC@NLO/MEPS@NLO in 0-jet bin.
- 20% discrepancies between MC@NLO and MEPS@NLO in 1-jet bin.
- Shape distortions are small.
Cross sections in 0-jet and 1-jet bins (CMS @ 8 TeV)

... in the signal and control regions for NLO/MC@NLO/MEPS@NLO.

<table>
<thead>
<tr>
<th>0-jets bin</th>
<th>NLO $\pm \Delta_{QCD}$</th>
<th>MC@NLO</th>
<th>MEPS@NLO $\pm \Delta_{QCD}$ $\pm \Delta_{res}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_s$ [fb]</td>
<td>159.34(18) $^{+1.8%}_{-1.7%}$</td>
<td>150.6(2)</td>
<td>160.3(3) $^{+2.6%}<em>{-3.8%}$ $^{+1.4%}</em>{-0.5%}$</td>
</tr>
<tr>
<td>$\sigma_C$ [fb]</td>
<td>60.08(15) $^{+1.5%}_{-1.4%}$</td>
<td>56.60(11)</td>
<td>60.31(22) $^{+3.6%}<em>{-3.5%}$ $^{+0.7%}</em>{-0.2%}$</td>
</tr>
<tr>
<td>$\sigma_s/\sigma_C$</td>
<td>2.65</td>
<td>2.66</td>
<td>2.66</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1-jet bin</th>
<th>NLO $\pm \Delta_{QCD}$</th>
<th>MC@NLO</th>
<th>MEPS@NLO $\pm \Delta_{QCD}$ $\pm \Delta_{res}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_s$ [fb]</td>
<td>45.01(7) $^{+1.3%}_{-2.6%}$</td>
<td>34.75(9)</td>
<td>44.88(23) $^{+3.0%}<em>{-2.7%}$ $^{+0.1%}</em>{-0.3%}$</td>
</tr>
<tr>
<td>$\sigma_C$ [fb]</td>
<td>22.09(5) $^{+1.2%}_{-3.2%}$</td>
<td>17.41(7)</td>
<td>22.30(18) $^{+3.0%}<em>{-2.7%}$ $^{+0.5%}</em>{-0.4%}$</td>
</tr>
<tr>
<td>$\sigma_s/\sigma_C$</td>
<td>2.04</td>
<td>2.00</td>
<td>2.01</td>
</tr>
</tbody>
</table>

- Error estimation from QCD scales and resummation scale.
- Good agreement between NLO and MEPS@NLO, small scale uncertainties $\rightarrow$ Sudakuv logarithms turn out to be small.
- MC@NLO $\sim$ 20% smaller in 1-jet bin (only LO accuracy).
Summary

OpenLoops

- Diagrammatic, tree-like recursion for loop momentum polynomials to calculate one-loop amplitudes.
- Automatic, fast code generation, compact libraries.
- Fast and numerically stable evaluation of matrix elements.

Sherpa+OpenLoops

- Fully automated interface, NLO matching with parton shower and jet merging.
- Process libraries available to ATLAS and CMS.

MEPS@NLO predictions for $H \rightarrow WW^*$ background in 0/1-jet bins

- NLO accuracy and LL Sudakov resummation in individual jet bins.
- Small and more reliably estimated theoretical uncertainties.