Strong Backreaction of Gauge Quanta Produced During Inflation

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Outline

Model Setup

- Lagrangian:

$$\mathcal{L} = \frac{M_{\mathrm{pl}}^2}{2} R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{2}(\partial\chi)^2 - U(\chi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\chi}{4f} \tilde{F}^{\mu\nu} F_{\mu\nu}.$$

- Fields:
  - Inflaton ϕ drives expansion.
  - Spectator axion χ with potential $U(\chi) = c\chi$.
  - $U(1)$ gauge field A_μ coupled through Chern–Simons term.
- Hierarchy: $H \ll f \ll M_{\mathrm{pl}}$.
- Axion is a spectator: $U(\chi) \ll V(\phi)$.

Background Dynamics

- FLRW metric:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2 = a^2(\tau)(-\mathrm{d}\tau^2 + \mathrm{d}\mathbf{x}^2)$$

- Friedmann equations:

$$H^2 = \frac{\rho}{3M_{\rm pl}^2}, \qquad \frac{\ddot{a}}{a} + \frac{1}{2}H^2 = -\frac{P}{2M_{\rm pl}^2}.$$

- Inflaton EoM:

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0.$$

- Axion EoM with backreaction:

$$\ddot{\chi} + 3H\dot{\chi} + U_{,\chi} = \frac{1}{f}\langle \mathbf{E}\cdot\mathbf{B}\rangle.$$

- Total energy density ρ in and total pressure P:

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + V(\phi) + U(\chi) + \frac{1}{2}\langle \mathbf{E}^2 + \mathbf{B}^2\rangle,$$
$$P = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 - V(\phi) - U(\chi) + \frac{1}{6}\langle \mathbf{E}^2 + \mathbf{B}^2\rangle.$$

Gauge Field Excitation

- Mode expansion:

$$\hat{A}_i(t,\mathbf{x}) = \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} \sum_{\lambda=\pm} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}} \epsilon_i^\lambda(\hat{\mathbf{k}}) A^\lambda(t,\mathbf{k}) \hat{a}^\lambda(\mathbf{k}) + \text{h.c.}$$

- Mode equation in conformal time:

$$A''_\pm + \left(k^2 \pm 2\xi\, aH\, k\right) A_\pm = 0,$$

$$\xi \equiv -\frac{\dot{\chi}}{2fH}, \quad \text{with} \quad \dot{\chi} < 0.$$

where the polarization vector basis $\epsilon_i^\lambda(\hat{k})$ satisfying

$$k_i \epsilon_i^\pm(\hat{\mathbf{k}}) = 0, \quad \varepsilon_{ijk} k_j \epsilon_k^\pm(\hat{\mathbf{k}}) = \mp i k \epsilon_i^\pm(\hat{\mathbf{k}}), \quad \epsilon_i^\pm(\hat{\mathbf{k}}) \epsilon_i^\pm(\hat{\mathbf{k}}) = 0, \quad \epsilon_i^\pm(\hat{\mathbf{k}}) \epsilon_i^\mp(\hat{\mathbf{k}}) = 1.$$

- Tachyonic instability for negative-helicity modes:

$$k/(aH) < 2\xi \quad \Rightarrow \quad A_- \text{ grows exponentially.}$$

- Initial condition: Bunch–Davies vacuum:

$$A_\pm(k) = \frac{e^{-ik\tau}}{\sqrt{2k}}, \quad k \gg aH.$$

Expectation Values

$$\langle \mathbf{E} \cdot \mathbf{B} \rangle = -\frac{1}{4\pi^2 a^4} \sum_{\lambda=\pm} \lambda \int_0^\infty \mathrm{d}k k^3 \frac{\mathrm{d}}{\mathrm{d}\tau} |A^\lambda(k)|^2,$$

$$\langle E^2 \rangle = \frac{1}{2\pi^2 a^4} \sum_{\lambda=\pm} \int_0^\infty \mathrm{d}k k^2 \left| \frac{\mathrm{d}A^\lambda(k)}{\mathrm{d}\tau} \right|^2,$$

$$\langle B^2 \rangle = \frac{1}{2\pi^2 a^4} \sum_{\lambda=\pm} \int_0^\infty \mathrm{d}k k^4 |A^\lambda(k)|^2.$$

When Does Backreaction Become Strong?

$$\ddot{\chi} + 3H\dot{\chi} + U_{,\chi} = \frac{1}{f}\langle \mathbf{E}\cdot\mathbf{B}\rangle\,,$$

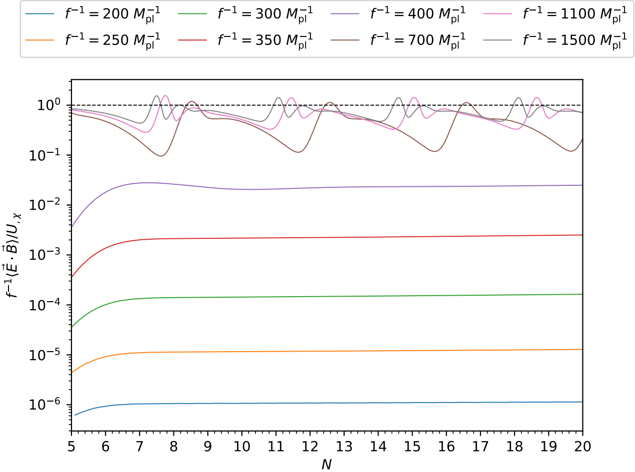
$$A_-'' + \left(k^2 - 2\xi\, aH\, k\right) A_- = 0\,, \quad \text{with} \;\; \xi \equiv -\frac{\dot{\chi}}{2fH}\,.$$

- Condition for strong backreaction:

$$\frac{1}{f}\langle E\cdot B\rangle \sim U_{,\chi} = c.$$

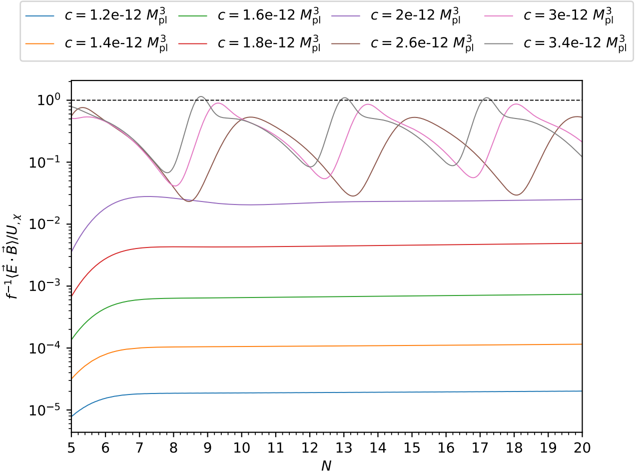
- In the strong backreaction regime,
  - Gauge-field production slows the axion.
  - Reduced $\dot{\chi}$ decreases ξ.
  - Production shuts off and the axion accelerate once more (feedback loop).
- As shown in Fig., when the backreaction term $f^{-1}\langle \mathbf{E}\cdot\mathbf{B}\rangle$ is much smaller than the slope U_{χ}, its influence on the dynamics of the system is negligible.
- However, when the backreaction term and the slope are of comparable magnitude, the backreaction induces strong oscillations in particle production.

Evolution of the ratio $f^{-1}\langle \mathbf{E}\cdot\mathbf{B}\rangle/U_{,\chi}$



Evolution of the ratio between the backreaction term and the axion slope $f^{-1}\langle \mathbf{E}\cdot\mathbf{B}\rangle/U_{,\chi}$ for different coupling constants with $c = 2\times 10^{-12}\ M_{\rm pl}^3$.

Evolution of the ratio $f^{-1}\langle \mathbf{E}\cdot\mathbf{B}\rangle/U_{,\chi}$



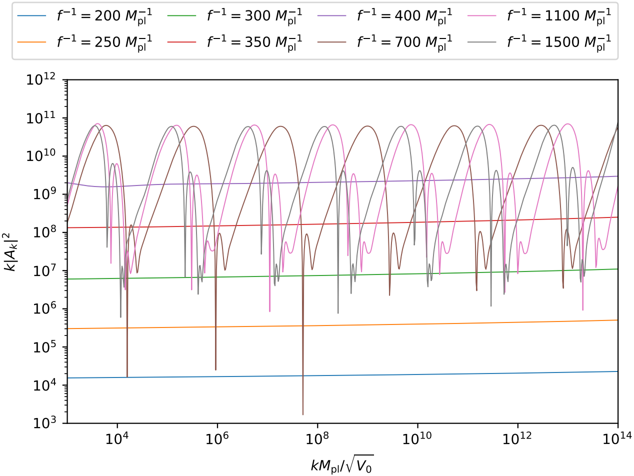
Evolution of the ratio between the backreaction term and the axion slope $f^{-1}\langle \mathbf{E}\cdot\mathbf{B}\rangle/U_{,\chi}$ for different slopes with $f^{-1} = 400\ M_{\rm pl}$.

Influence of the coupling constant

Figure shows the normalized gauge field spectrum $k|A_k|^2$ at the end of inflation for various coupling constants.

- One can observe that when backreaction is weak, increasing the coefficient f^{-1} leads to an exponential increase in the gauge quanta production.
- However, once the system enters the strong backreaction regime, the spectrum begins to oscillate and further increases f^{-1} no longer exponentially increasing the maximum particle production. Instead, it will slightly suppress the particle production.

Influence of the coupling constant



Gauge field spectrum at the end of inflation for different coupling constants with $c = 2 \times 10^{-12}\ M_{\rm pl}^3$.

Influence of the coupling constant

Weak backreaction:

- When the coefficient f^{-1} is small, produced particles do not significantly affect the evolution of the axion field. Consequently, the velocity of the axion remains nearly unchanged, producing a fairly flat gauge field spectrum.
- **Increasing the coefficient f^{-1} leads to an exponential increase in the gauge quanta production.**

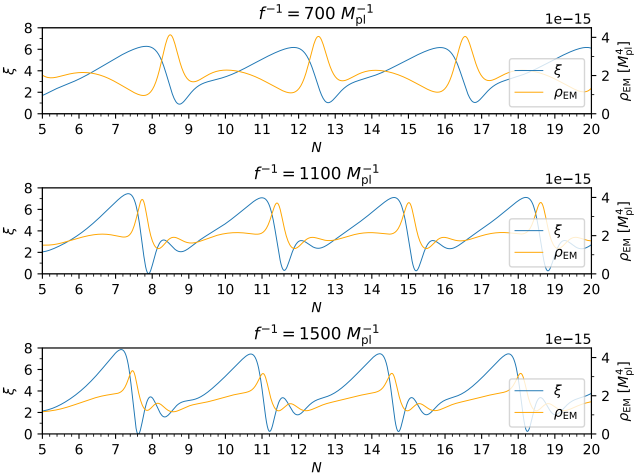
$$|A_k^-|^2 \propto \exp\left(c_1 \frac{|\dot{\chi}|}{fH}\right) \sim \exp(c_2/f)$$

Strong backreaction:

- Once the backreaction term $f^{-1}\langle \mathbf{E} \cdot \mathbf{B} \rangle$ becomes comparable to the potential slope $U_{,\chi}$, the axion decelerates, causing backreaction to weaken. As the backreaction subsides, the axion can accelerate again.
- The repetition of this cycle leads to the oscillatory behavior observed in ξ and in the energy density $\rho_{\rm EM}$.
- **Increasing f^{-1} will linearly decrease the energy density.**

$$\rho_{\rm EM,peak} \propto (cf)^2 \Rightarrow \frac{\partial \ln \rho_{\rm EM,peak}}{\partial(1/f)} < 0$$

Influence of the coupling constant



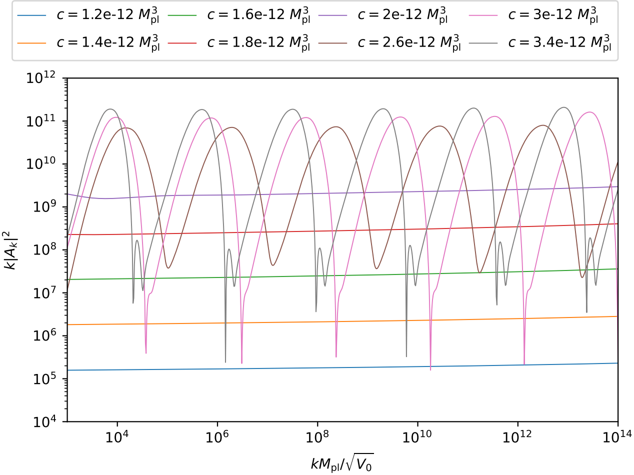
Evolution of ξ and $\rho_{\rm EM}$ for different coupling constants in the strong backreaction regime with $c = 2 \times 10^{-12}\ M_{\rm pl}^3$.

Influence of the potential slope

Figure shows the normalized gauge field spectrum $k|A_k|^2$ at the end of inflation for different slopes of the axion potential.

- A small slope leads to weak backreaction, which does not significantly affect the velocity of the axion field, resulting in a straight line in the logarithmic plot of the gauge field spectrum.
- By contrast, a larger slope causes faster rolling of the axion and strengthens backreaction. Increasing the slope further eventually makes the backreaction term comparable to the potential slope, causing both the axion velocity and the gauge field energy density to decrease. As the backreaction term then subsides, the axion field accelerates once more. Repetition of this cycle leads to oscillations in both ξ and the gauge field energy density.

Influence of the potential slope



Gauge field spectrum at the end of inflation with different slopes, where we fix the coupling constant as $f^{-1} = 400\ M_{\rm pl}$.

Influence of the potential slope

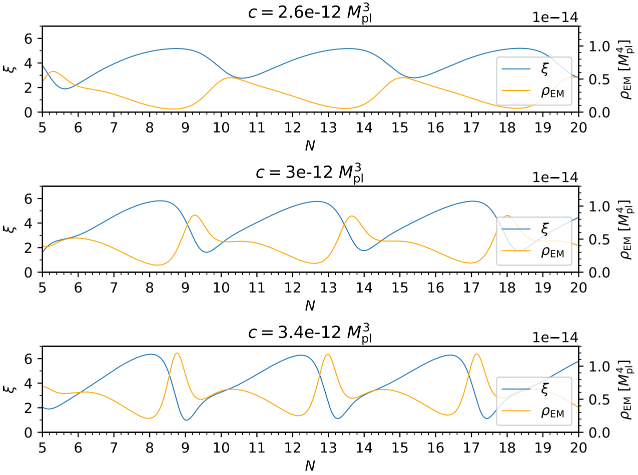
In the weak backreaction regime,

- the slope of the axion potential influences the gauge field spectrum in a manner similar to the coupling constant: increasing either the slope $U_{,\chi}$ or the coupling coefficient f^{-1} leads to an exponential increase in the gauge field spectrum.

In the strong backreaction regime,

- Increasing the slope can still enhance the peak value of the spectrum, although the efficiency is much lower compared to the weak backreaction regime.

Influence of the potential slope



Evolution of ξ and $\rho_{\rm EM}$ for different slopes in the strong backreaction regime, where we fix the coupling costant as $f^{-1} = 400\ M_{\rm pl}$.

Tensor Perturbations with Source

- Metric perturbation:

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right],$$

$$h_{ii} = 0, \quad \partial_j h_{ij} = 0.$$

- Sourced GW equation:

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - \nabla^2 h_{ij} = \frac{2}{M_{\rm pl}^2}\Pi_{ij}^{\ \ lm} T_{lm}^{(EM)}.$$

- GW power spectrum:

$$\mathcal{P}_h(k) \sim \frac{k^3|h(k)|^2}{2\pi^2 M_{\rm pl}^2}.$$

Tensor Perturbation & Source

Metric:

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right], \quad h_i{}^i = 0,\ \partial^j h_{ij} = 0$$

EoM (conformal time):

$$\boxed{h_{ij}'' + 2Hh_{ij}' - \nabla^2 h_{ij} = \frac{2}{M_{\rm pl}^2}\Pi_{ij}{}^{lm} T_{lm}^{\rm EM}}$$

Gauge field stress tensor (dominant part):

$$T_{ij}^{\rm EM} \approx -a^2\left(E_i E_j + B_i B_j\right)$$

Only the tachyonic enhanced minus mode $A^-(k)$ contributes significantly.

Solution in Fourier Space

Helicity decomposition:

$$h^{\pm}(\tau, \mathbf{k}) = \sqrt{2}\, \Pi^{\pm}_{ij}(\mathbf{k})\, h_{ij}(\tau, \mathbf{k})$$

Green-function solution (de Sitter):

$$h^{\pm}(\tau, \mathbf{k}) = \frac{2H^2}{M_{\rm pl}^2} \int_{-\infty}^{\tau} d\tau' \; G_k(\tau, \tau')\, \tau'^2 \times$$
$$\Pi^{\pm}_{lm}(\mathbf{k}) \left[\tilde{A}'^{-}_{l}(\mathbf{q}, \tau') \tilde{A}'^{-}_{m}(\mathbf{k} - q, \tau') + (\text{BB term}) \right]$$

Green function (exact in de Sitter):

$$G_k(\tau, \tau') = \frac{1}{k^3 \tau'^2} \left[(1 + k^2 \tau \tau') \sin k(\tau - \tau') - k(\tau - \tau') \cos k(\tau - \tau') \right] \Theta(\tau - \tau')$$

Key paper approximation: Main contribution from $\mathbf{q} \simeq \mathbf{k}$ $\rightarrow$ super-horizon contribution exponentially suppressed $\rightarrow$ $\mathcal{P}_h(k) \propto |A^{-}(k)|^4$

Gravitational-Wave Energy Spectrum Today

Final tensor power spectrum at end of inflation:

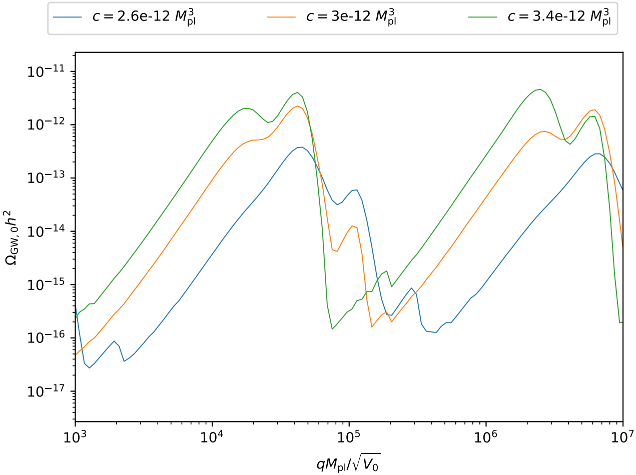
$$\mathcal{P}_h^\lambda = \frac{H^4 k^3}{2^4 M_{\rm pl}^4} \int_0^\infty q^2 \mathrm{d}q \int_{-1}^1 \mathrm{d}u \left[\left| \epsilon_i^\lambda(k) \epsilon_i^-(-q) \right|^2 \left| \epsilon_j^\lambda(k) \epsilon_j^-(q-k) \right|^2 \right.$$

$$\times \left| \int_{-\infty}^0 \mathrm{d}\tau' \tau'^2 G_k(\tau_{\rm end}, \tau') \Big(A'^-(\tau', q) A'^-(\tau', k-q) + q|k-q| A^-(\tau', q) A^-(\tau', k-$$

Present-day energy density:

$$\boxed{\Omega_{\rm GW}(f) h^2 = \frac{\Omega_{r,0} h^2}{24} (\mathcal{P}_h^+ + \mathcal{P}_h^\times) \approx \frac{\Omega_{r,0} h^2}{12} \mathcal{P}_h(k)} \qquad (\Omega_{r,0} h^2 \simeq 4.2 \times 10^{-5})$$

Energy spectrum of GWs



Energy spectrum of GWs for different slopes, where we fix the coupling constant as $f^{-1} = 400\ M_{\rm pl}$.

Summary

1. **Strong backreaction saturates particle production**
   Larger Chern-Simons coupling $1/f \Rightarrow$ **linearly suppresses** peak gauge field energy density and spectrum in the strong backreaction regime.
2. **Steeper linear axion slope enhances production**
   Larger slope $c \Rightarrow$ **linearly increases** peak ρ_{EM} and gauge spectrum, even under strong backreaction.
3. **Weak vs Strong backreaction: completely different scaling**

Regime	Gauge field production
Weak backreaction	Exponential in $1/f$ and c
Strong backreaction	$\propto (cf)^2$ (saturated)

4. **Gravitational waves are extremely sensitive** $\mathcal{P}_h(k) \propto |A^-(k)|^4 \longrightarrow$ **fourth-power dependence** on the gauge field enhancement.