Flavour Physics Lecture 味物理讲座

北京航空航天大學 BEIHANG UNIVERSITY

Toward a Precise Tomography of Heavy Mesons from Lattice QCD

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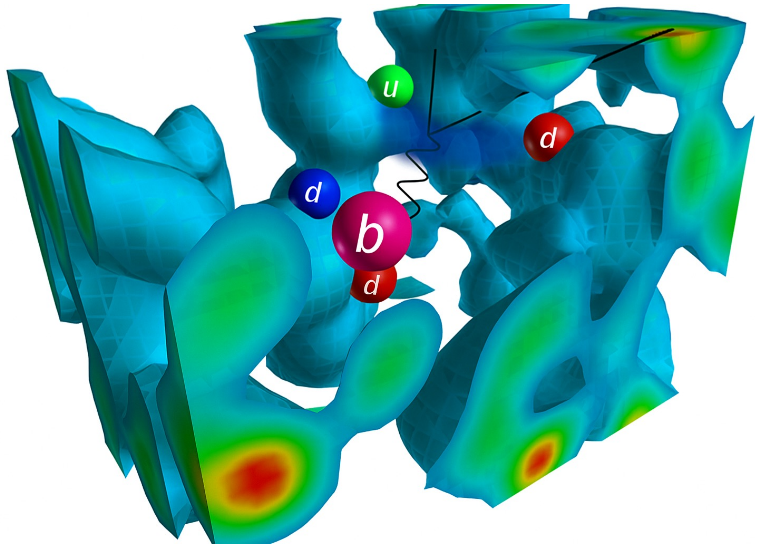
LPC Lattice Parton Collaboration



Based on PRD111, L111503 (2025); PRD111, 034503 (2025); PRD112, 054044 (2025); 2411.07101 [hep-ph]

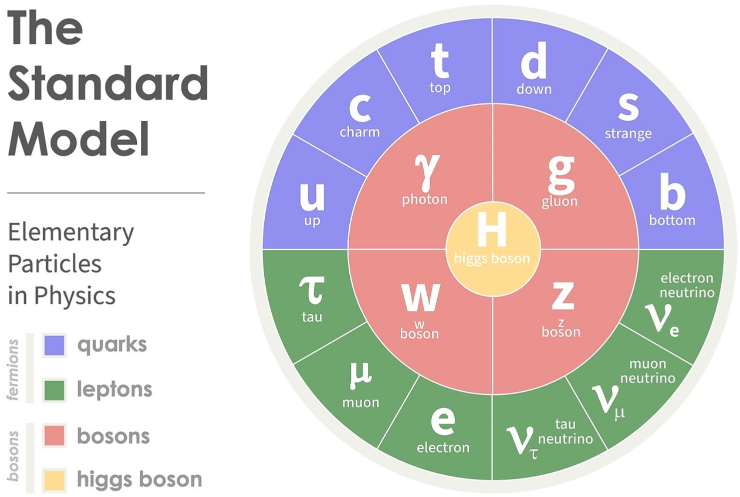
Dec.18, 2025

Outline

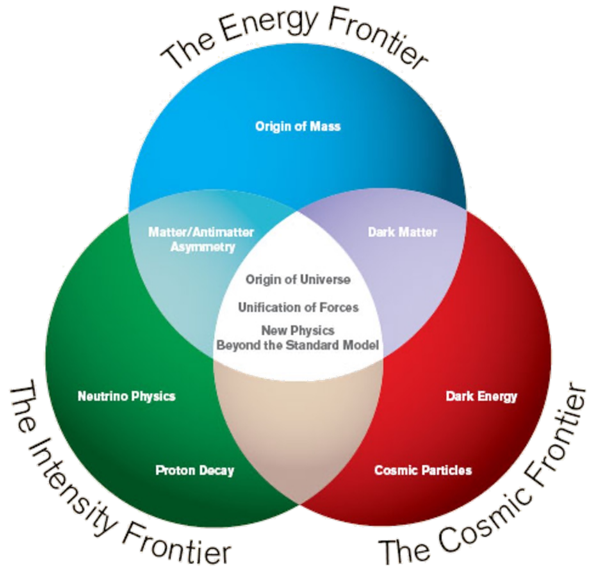
- Motivation
- Recipes for Profiling the Heavy Mesons
- Lattice QCD Implementation of Heavy Meson LCDAs
  - First-principle Calculations of QCD LCDAs: LaMET vs OPE?
  - Model-Independent Prediction for HQET LCDAs
- Theoretical Frameworks for Heavy Meson Shape Functions
- Summary and Outlook

Heavy Quark Physics

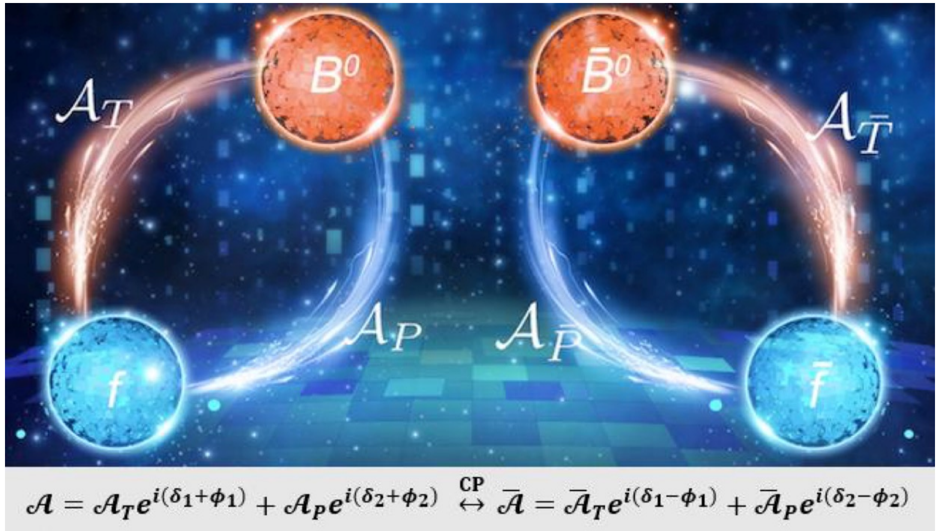
- **Precisely testing standard model**



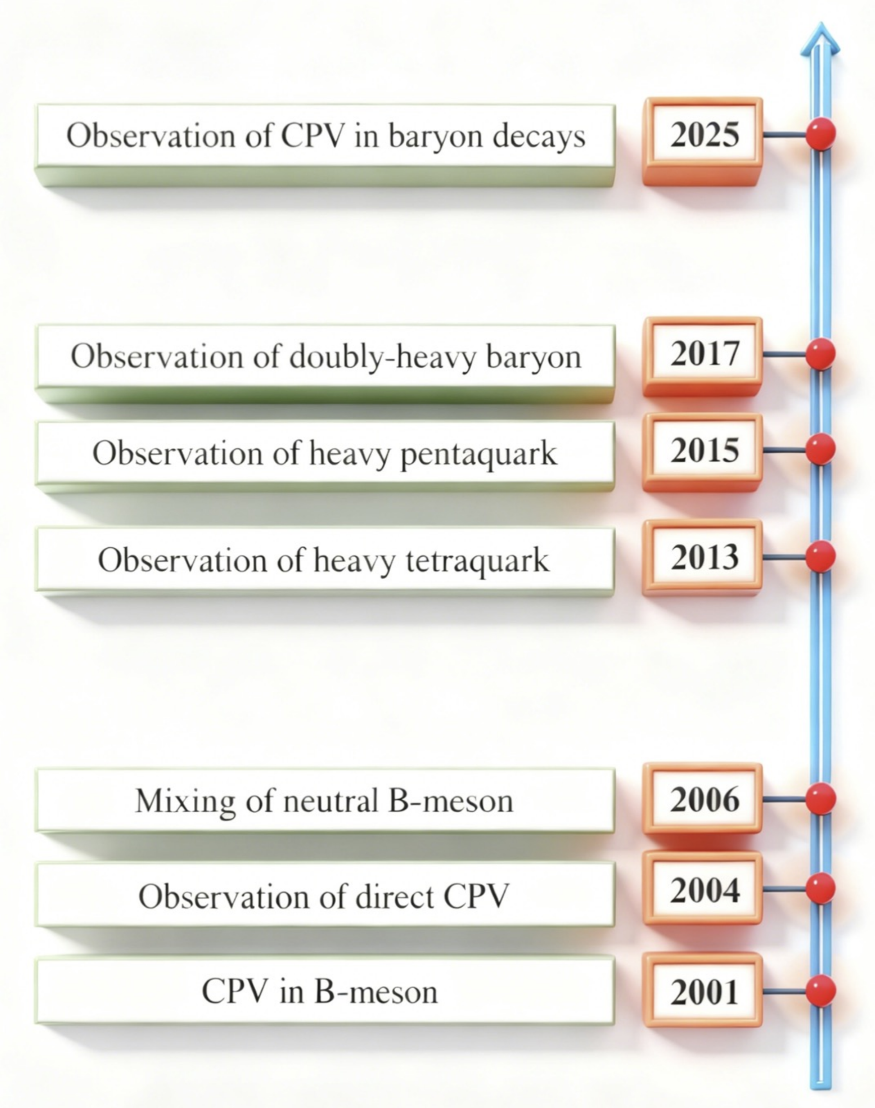
- **Indirect search for new physics**

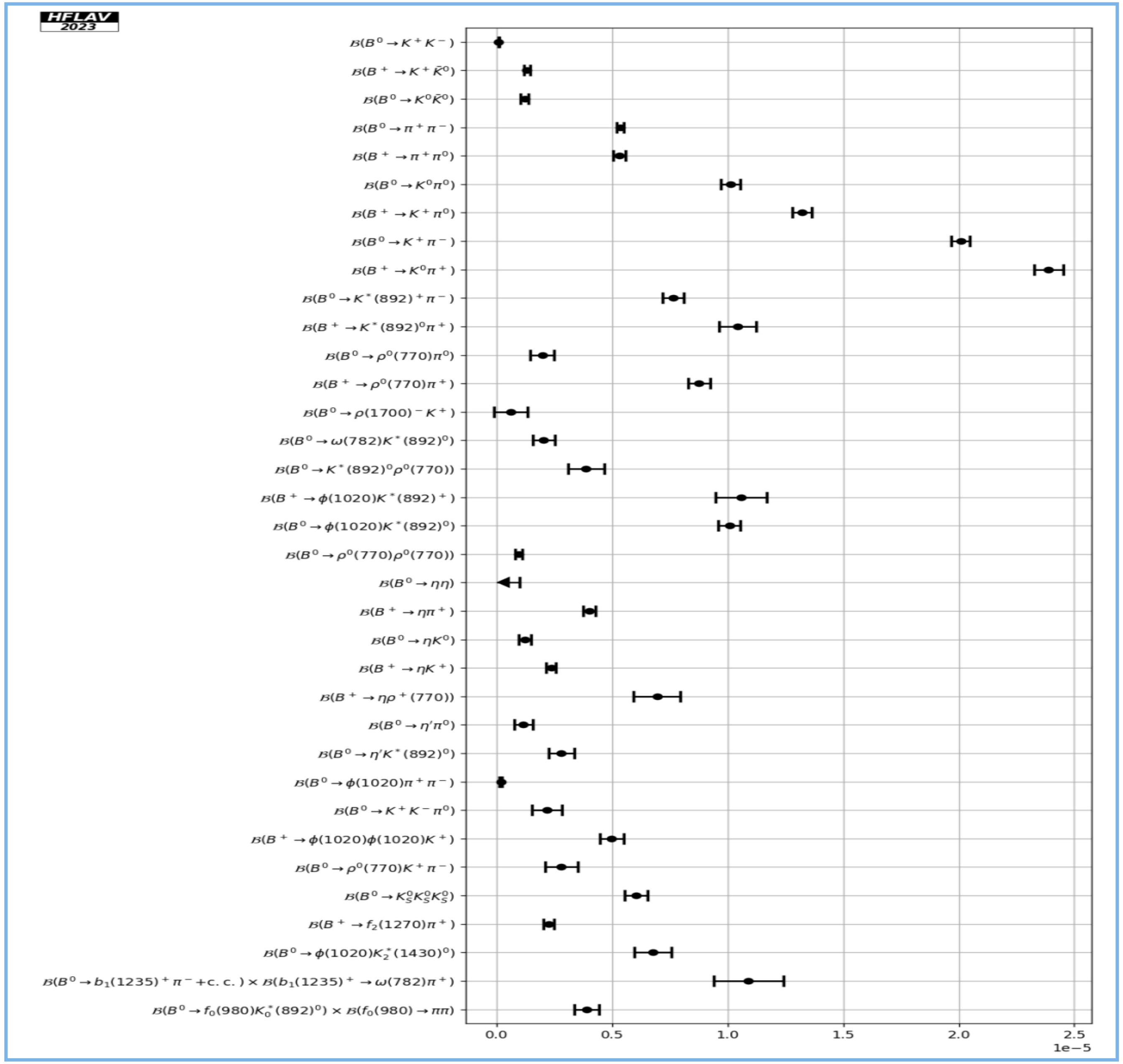


- **Study on CP violation**



Heavy Quark Physics

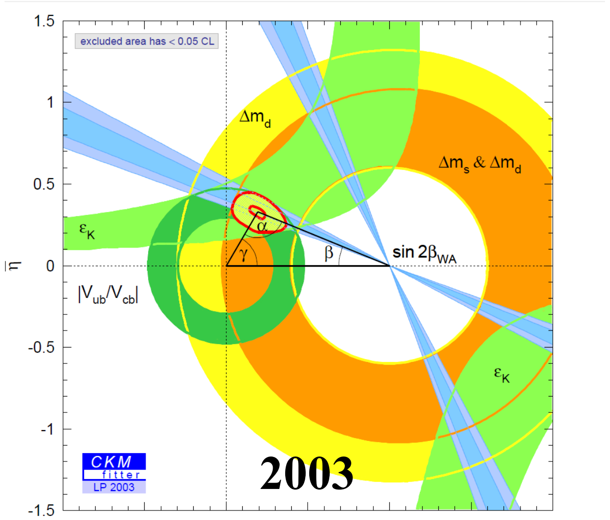


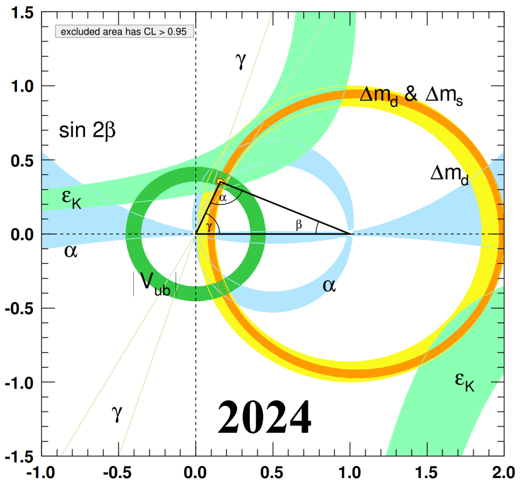


Decades of progress and many important achievements…

- **Test on Unitarity Triangle**

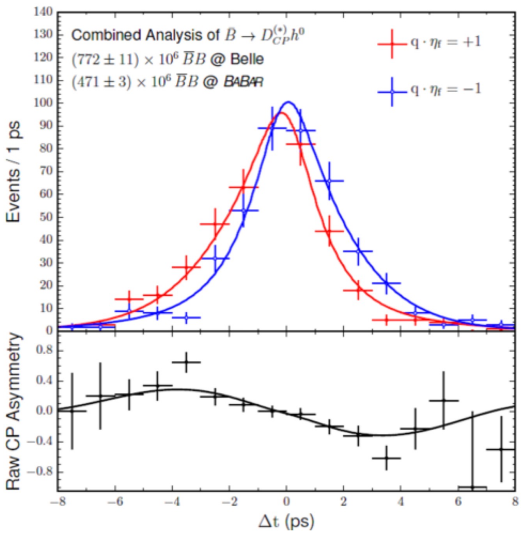
(2003 → 2024)

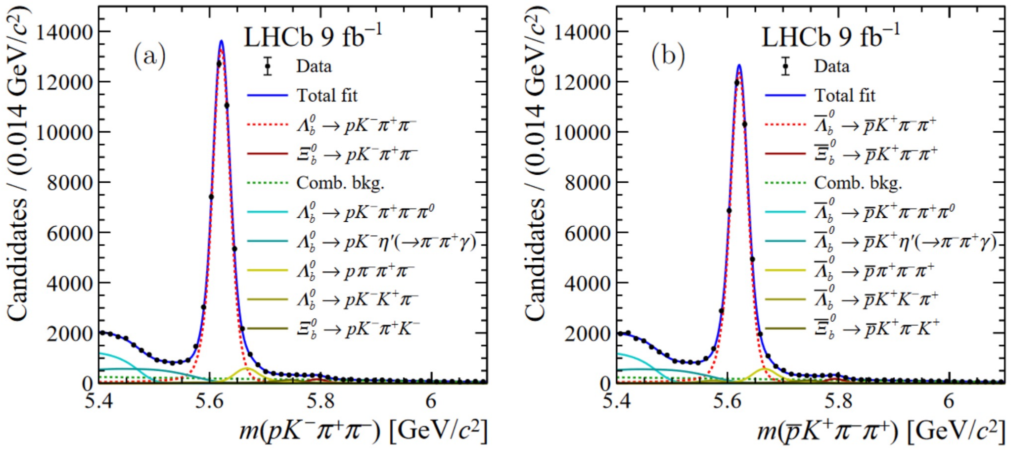




- **CP violation**

(meson, 2001 → baryon, 2025)





Heavy Quark Physics

… but fundamental puzzles still persist.

- **$|V_{ub}|$ and $|V_{cb}|$ Puzzle**

  discrepancy between inclusive and exclusive processes extracxtions

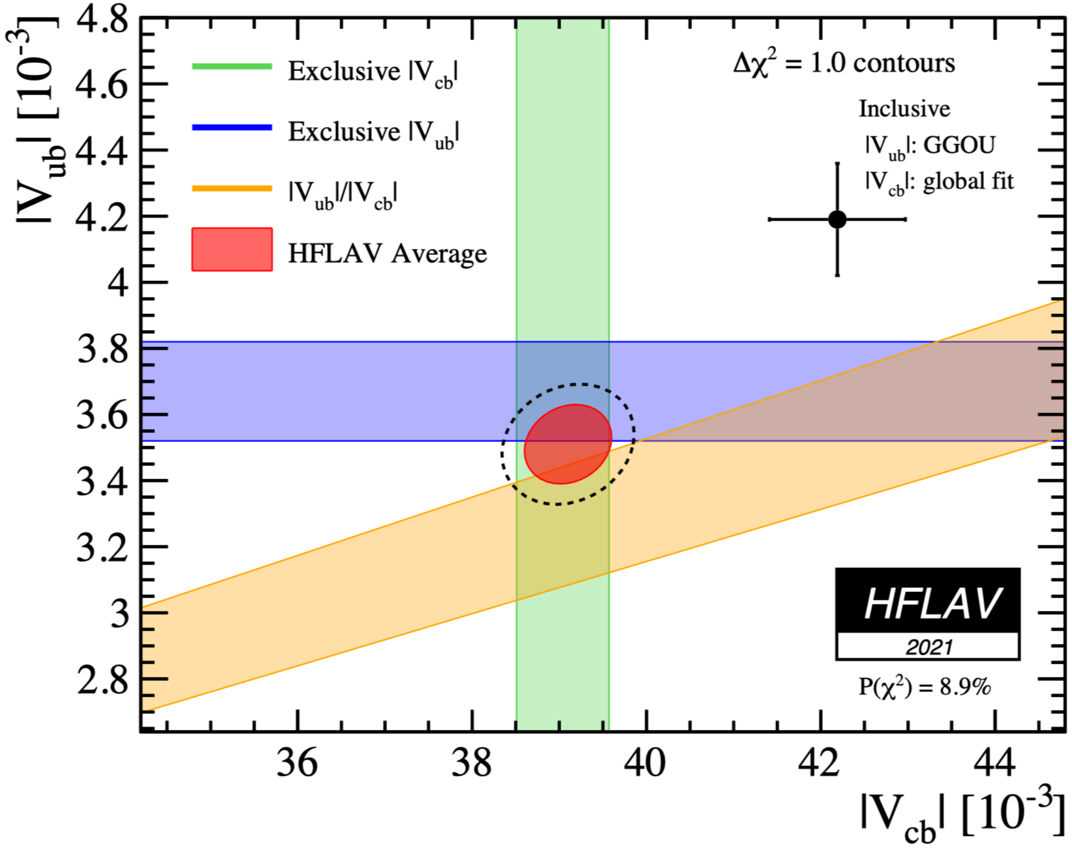
  - From inclusive *B* decays

$$|V_{ub}^{\rm incl.}| = (4.19 \pm 0.17) \times 10^{-3}$$
$$|V_{cb}^{\rm incl.}| = (42.19 \pm 0.78) \times 10^{-3}$$

  - From exclusive *B*, B_s and Λ_b decays

$$|V_{ub}^{\rm excl.}| = (3.51 \pm 0.12) \times 10^{-3}$$
$$|V_{cb}^{\rm excl.}| = (39.10 \pm 0.50) \times 10^{-3}$$

HFLAV, PRD 107, 052008 (2023)

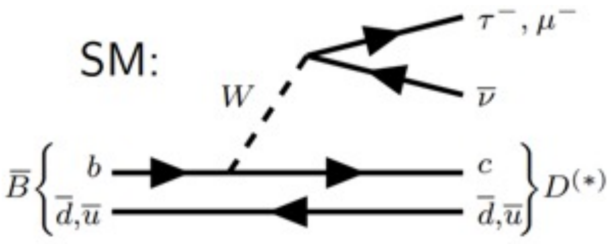


3.3σ discrepancy!

… but fundamental puzzles still persist.

- $R(D)$ and $R(D^*)$

$$\mathcal{R}(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)}$$



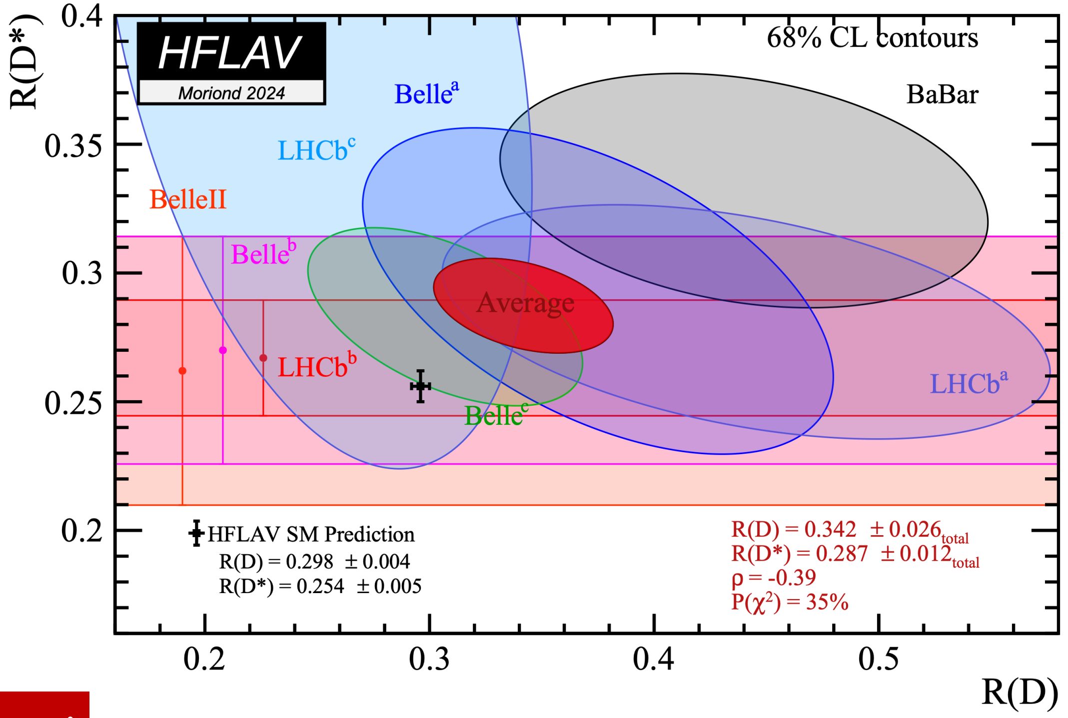
Precisely determined both experimentally and theoretically.

$$\mathcal{R}(D)_{\text{exp}} = 0.342 \pm 0.026$$
$$\mathcal{R}(D^*)_{\text{exp}} = 0.287 \pm 0.012$$

vs

$$\mathcal{R}(D)_{\text{SM}} = 0.298 \pm 0.004$$
$$\mathcal{R}(D^*)_{\text{SM}} = 0.254 \pm 0.005$$

HFLAV, preliminary average for Moriond 2024

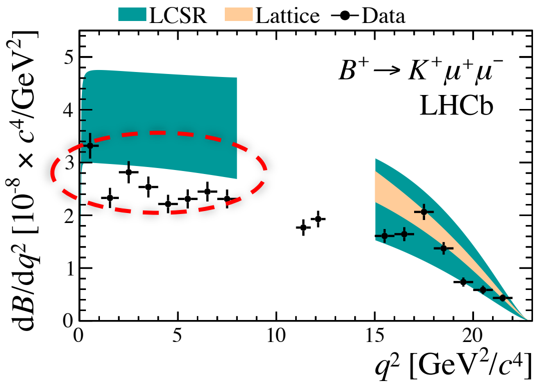


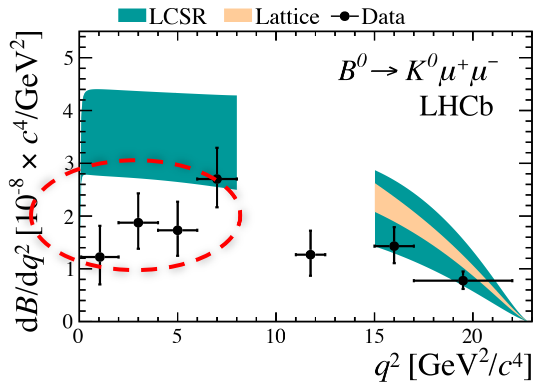
3.31σ discrepancy!

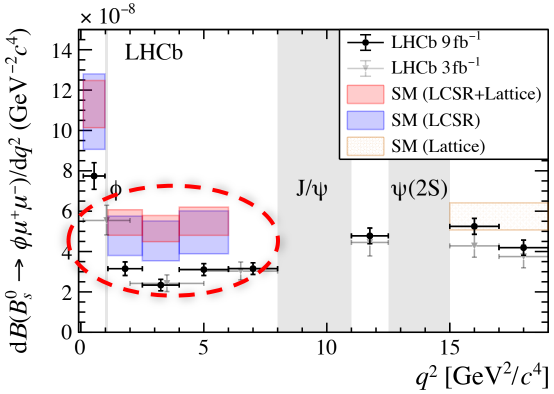
… but fundamental puzzles still persist.

- $b \to s\ell\ell$ **Processes**

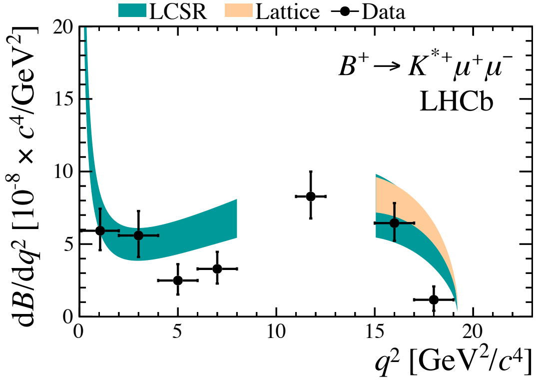
FCNC processes: Small SM amplitude $\Rightarrow$ Excellent place to search for NP

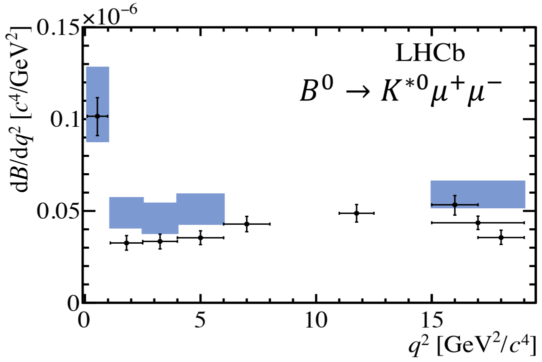


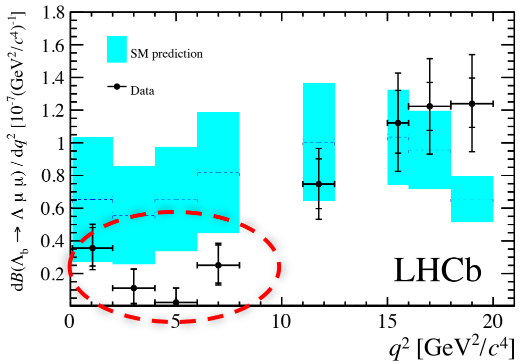


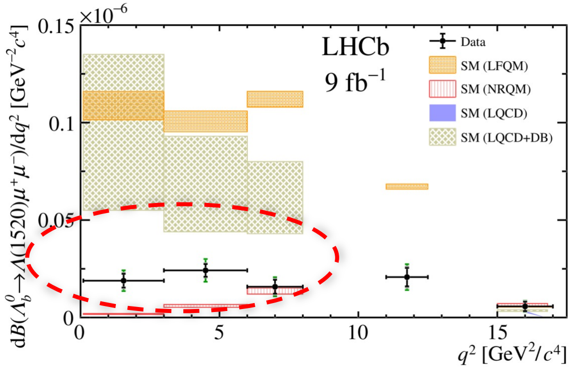


significant deviations

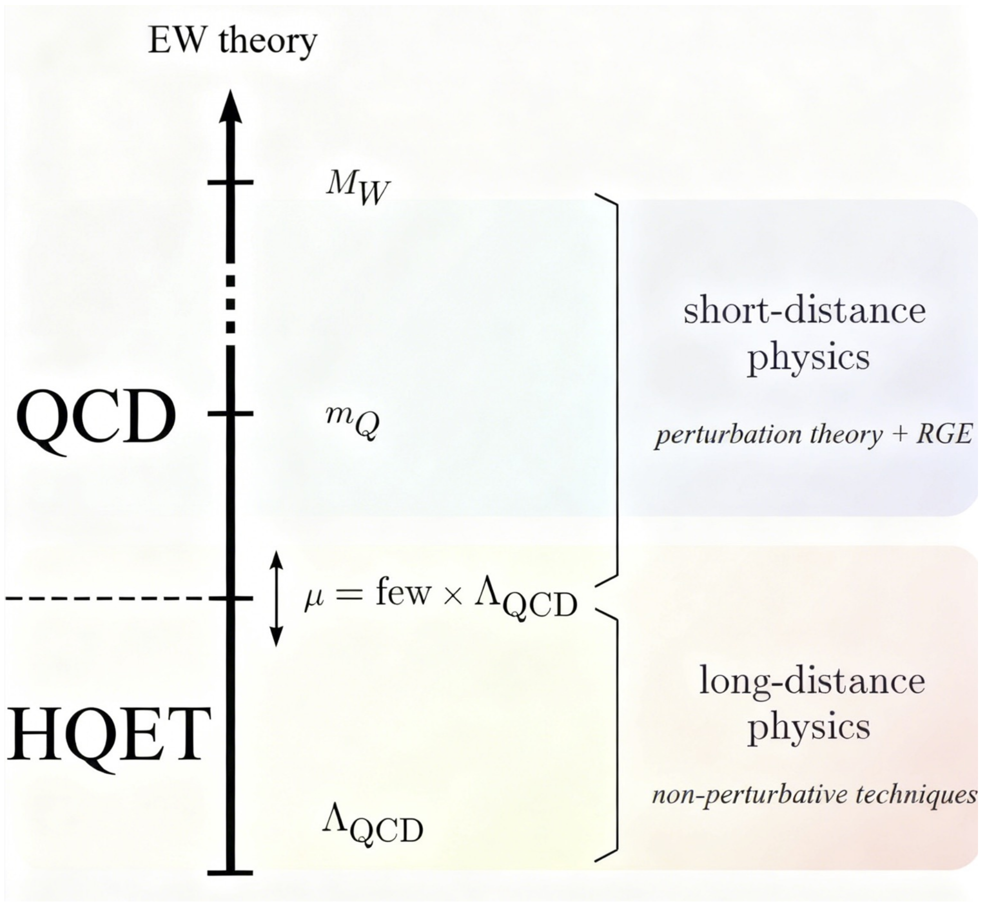


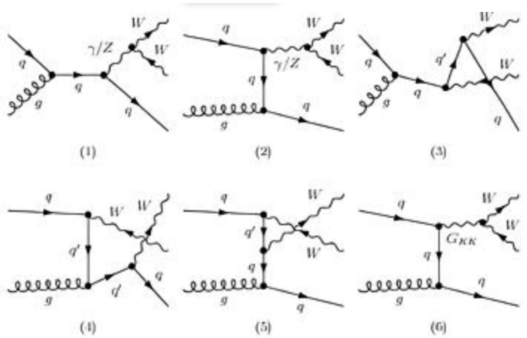






Factorization Methods





Perturbative

<< 0.1 fm

High energy
Small distance

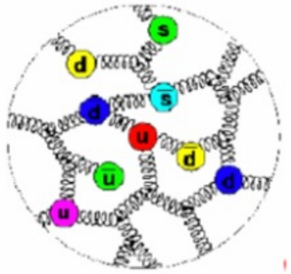
Asymptotic freedom

Nonperturbative

> 1 fm

Low energy
Large distance

Confinement

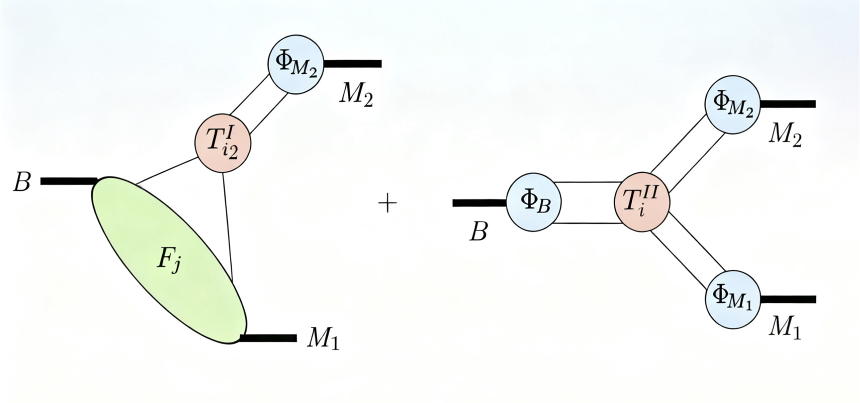


Factorization Methods

- **Naïve factorization and Generalized factorization**

[BSW (1985)] [Cheng etc., Ali, Kramer, Lü (1998)]

$$\langle M_1 M_2|O_i|\bar{B}\rangle = \mathrm{C}_1 \mathrm{F}^{\mathrm{B}\to M_1} f_{M_2} + C_2 \mathrm{F}^{\mathrm{B}\to M_2} f_{M_1}$$



- **QCD factorization and SCET**

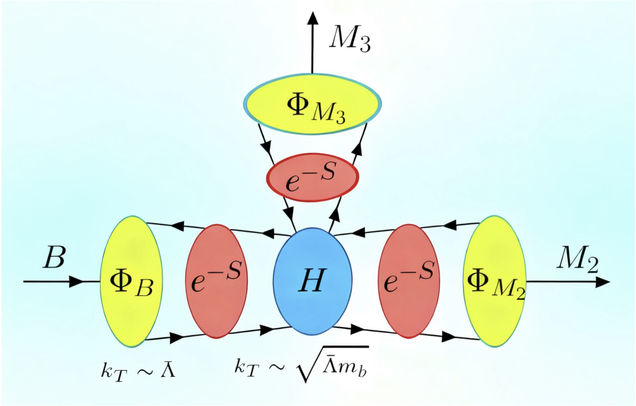
[Bauer et.al, (2001)] [Beneke, Buchalla, Neubert, Sachrajda (1999)]

$$\langle M_1 M_2|O_i|\bar{B}\rangle = \mathrm{F}^{\mathrm{B}\to M_1} \mathrm{T}_\mathrm{i}^\mathrm{I} \otimes \phi_{M_2} + \mathrm{T}_\mathrm{i}^\mathrm{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

- **Perturbative QCD approach (PQCD)**

[Li, Lü, Xiao etc.]

$$\langle M_1 M_2|O_i|\bar{B}\rangle = H \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2} \otimes \mathrm{e}^{-\sum S_i}$$



Factorization of Heavy Meson Decays

- Nonleptonic decays, such as $B^0/\bar{B}^0 \to \pi^+\pi^-$, are important to explore the CP violation

$$\langle \pi^+\pi^- | Q_i | \bar{B}^0 \rangle = f^0_{B\to\pi}(m_\pi^2) \int_0^1 dx\, T_i^I(x,\mu)\phi_\pi(x,\mu) + \int d\omega\, dx\, dy\, T_i^{II}(\xi,x,y,\mu)\varphi_+^B(\omega,\mu)\phi_\pi(x,\mu)\phi_\pi(y,\mu)$$

Light-cone distribution amplitudes

- FCNC processes, such as $B^0 \to K^*\ell\ell$, are sensitive to new physics:

$$\langle K_a^*\ell^+\ell^- | \mathcal{H}_{\rm eff} | B \rangle = T_a^I(q^2)\xi_a(q^2) + \sum_\pm \int_0^\infty \frac{d\omega}{\omega}\varphi_\pm^B(\omega,\mu) \int_0^1 du\, \phi_{K^*}(u,\mu)\, T^{II}_{a,\pm}(\omega,u,q^2)$$

- Semileptonic $B \to M\ell\bar{\nu}$ decays, $M = \pi, D, \ldots$, contribute to the determination of $|V_{ub}^{\rm excl}|$, $\mathcal{R}(D^{(*)})$:

$$F_i^{B\to M}(q^2) = C_i^{(A0)}(q^2)\xi_a(q^2) + \int_0^\infty \frac{d\omega}{\omega}\int_0^1 dv\, T_i(\ln\omega, v, q^2)\, \varphi_\pm^B(\omega,\mu)\phi_M(v,\mu)$$

- Inclusive decays, such as $\bar{B} \to X_s\gamma$ or $\bar{B} \to X_u\ell\bar{\nu}$, will contribute to $|V_{ub}^{\rm incl}|$:

Shape function

$$\frac{d\Gamma^{\bar{B}\to X_s\gamma}}{dx} = \Gamma(b\to s\gamma)\frac{M}{v^+}\int d\ell_+\, S(\ell_+)\, J(P_+ - \ell_+)\, H(P_-), \qquad P_+ = \frac{M}{\sqrt{2}}(1-x)$$

Nonperturbative Structures of Mesons

- Light-cone distribution amplitudes (LCDAs) of light meson, contribute to the exclusive processes and describe the partonic structures of final-state light meson:

$$\phi(x,\mu) = \frac{1}{i f_H}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \langle H(P_H)\,|\,\bar{q}(\xi^-)\not{n}_+\gamma_5\, W_c(\xi^-,0) q(0)\,|\,0\rangle$$

- LCDAs of heavy meson with QCD fields (QCD LCDAs), contribute to the heavy meson final state of exclusive processes, such as $e^+e^- \to B\bar{B}$, $B \to D\ell\nu$:

$$\phi(x,\mu) = \frac{1}{i f_{H_Q}}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \langle H_Q(P_{H_Q})\,|\,\bar{q}(\xi^-)\not{n}_+\gamma_5\, W_c(\xi^-,0) Q(0)\,|\,0\rangle$$

- LCDAs of heavy meson with HQET fields (HQET LCDAs), contribute to the heavy meson initial state of exclusive processes, such as $B \to D\ell\nu$, $B \to K^*\ell\ell$:

$$\varphi^+(\omega,\mu) = \frac{1}{i\,\tilde{f}_{H_Q} m_{H_Q} n_+\cdot v}\int \frac{dt}{2\pi} e^{-i\omega t n_+\cdot v} \langle 0\,|\,\bar{q}(tn_+)\not{n}_+\gamma_5\, W_c(tn_+,0) h_v(0)\,|\,H_Q(v)\rangle$$

- Shape functions (SFs) of heavy meson with HQET fields, contribute to the heavy meson initial state of inclusive processes, such as $\bar{B} \to X_s\gamma$, $\bar{B} \to X_u\ell\bar{\nu}$:

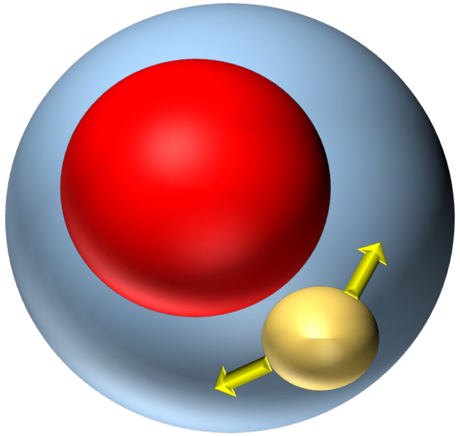
$$S(\omega,\mu) = \frac{1}{2m_{H_Q}}\int \frac{dt}{2\pi} e^{-i\omega t n_+\cdot v} \langle H_Q(v)|\,\bar{h}_v(tn_+,0) W_c(tn_+,0) h_v(0)\,|\,H_Q(v)\rangle$$

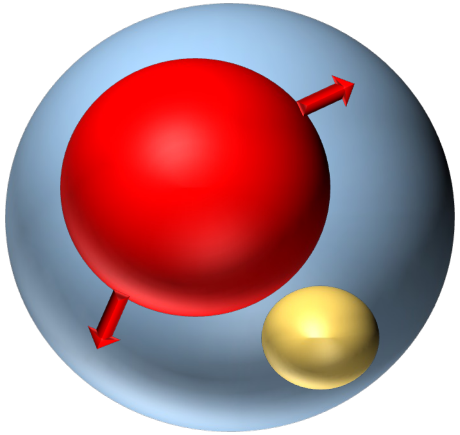
Tomography of Partonic Structures of Heavy Meson

A heavy flavor meson consists of a pair of heavy and light quarks.

Light-cone distribution Amplitudes (LCDAs)

Shape function





LCDAs describe the momentum distribution amplitude of the light quark.

Shape function characterizes the momentum distribution function of the heavy quark.

- **They provide the most essential information about the profile of heavy mesons.**

LCDAs of Light Mesons Have Been Extensively Studied

- Asymptotic form

  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979; ……

- QCD Sum rules

  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989; ……

- Dyson-Schwinger Equation

  Chang, Cloet, et.al, 2013; Gao, Chang, et.al, 2014; ……

- Global Fits

  Cheng, et.al, 2020; Hua, Li, Lu, Wang, Xing, 2021; ……

- Models

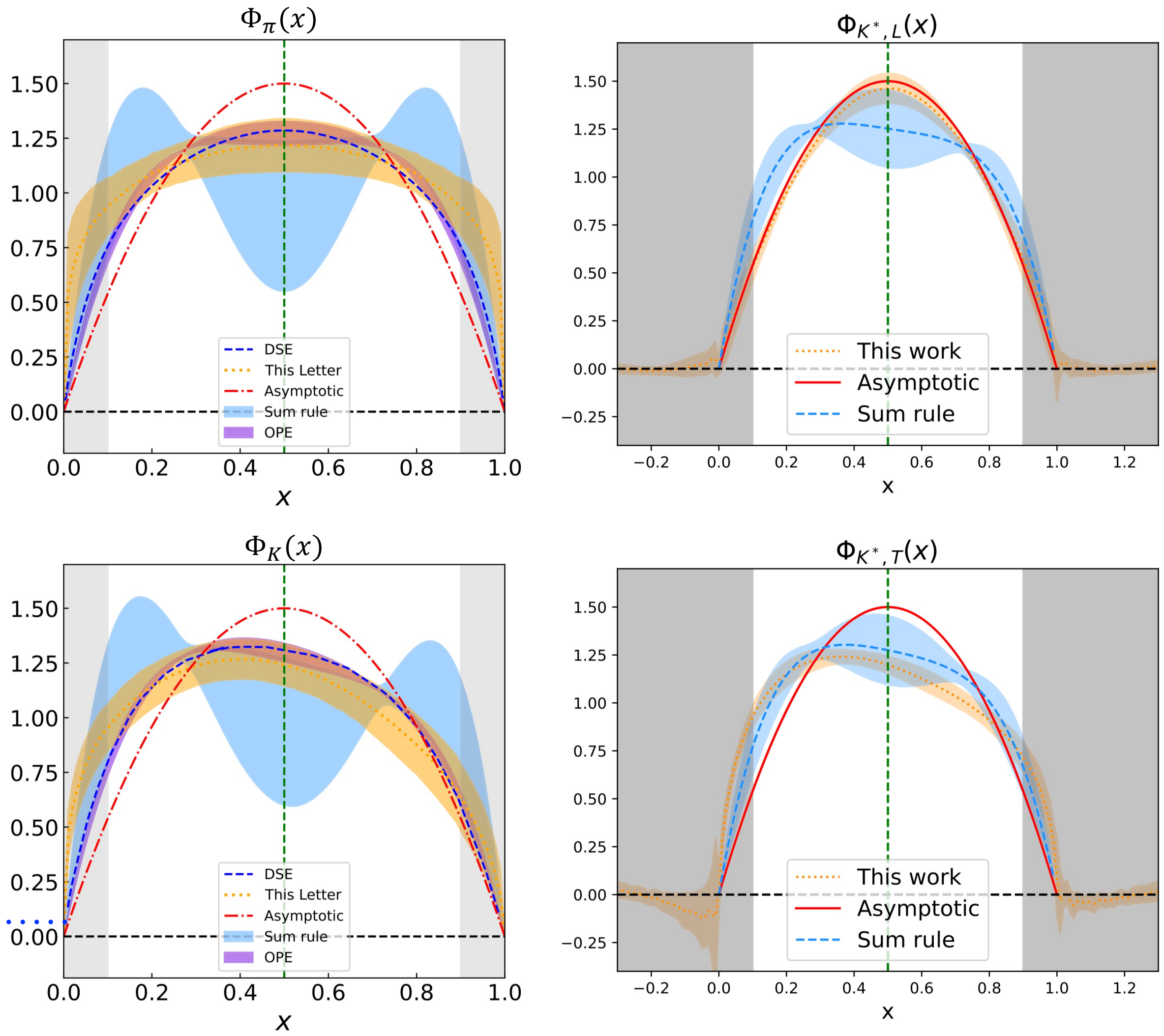
  Arriola, Broniowski, 2002; Zhong, Zhu, et.al, 2021; ……

- Lattice QCD with OPE

  Braun, Bruns, et al., 2016; RQCD collaboration, 2019, 2020; ……

- Lattice QCD with LaMET

  LP3, 2019; LPC, 2021, 2022; ……



PRL127, 062002 (2021); PRL129, 132001 (2022)

When Extending to Heavy Mesons……

- **Heavy Meson LCDAs**

2001 — **Equation of motion** [Kawamura, Kodaira, Qiao, Tanaka, PLB 523 (2001)]

2003 — **Evolution equations** [Lange, Neubert, PRL 91 (2003)]

2005 — **Perturbative constraint for large ω** [Lee, Neubert, PRD 72 (2005)]

2008 — **Subleading Evolution equations** [Bell, Feldmann, JHEP 04 (2008)]

2013 — **Solution of evolution equations** [Bell, Feldmann, Wang and Yip, JHEP 11 (2013)]

2019 — **Two loop RG equations** [Braun, Ji, Manashov, PRD 100 (2019)]

2020 — **LCDA in QCD and HQET** [Ishaq, Jia, Xiong, Yang, PRL 125 (2020); Beneke, Finauri, Vos, Wei, JHEP 09 (2023)]

2004 — **Evolution equations** [Bauer, Manohar, PRD 70 (2004)]

2005 — **Perturbative constraint for large ω** [Bosch, Lange, Neubert, Paz, NPB 699 (2005)]

2008 — **Relation between LCDA and SF** [Yaouanc, Oliver, Raynal, PRD 77 (2008)]

2009 — **Two loop in SF region** [Beneke, Huber, Li, NPB 811 (2009)]

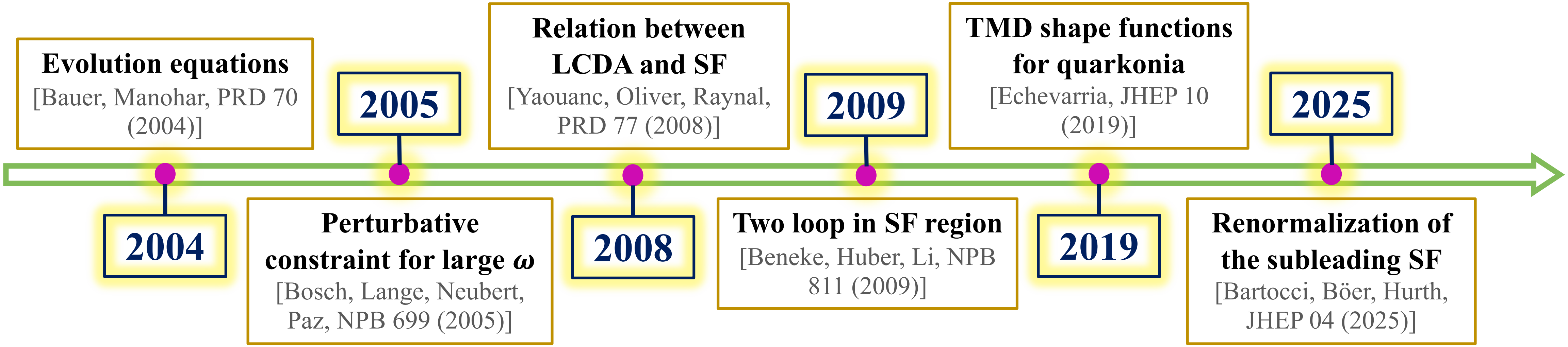
2019 — **TMD shape functions for quarkonia** [Echevarria, JHEP 10 (2019)]

2025 — **Renormalization of the subleading SF** [Bartocci, Böer, Hurth, JHEP 04 (2025)]

Adapted from Ji Xu

When Extending to Heavy Mesons……

- **Heavy Meson Shape Functions**



Adapted from Ji Xu

When Extending to Heavy Mesons……

- Limited understanding of the **nonperturbative** heavy meson LCDAs and SFs:

  - **Only models** for **heavy meson LCDAs**:

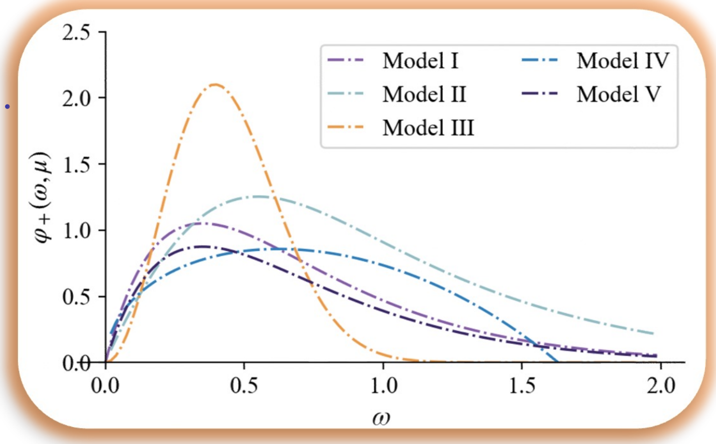
    Grozin, Neubert, 1997; Braun, Ivanov, Korchemsky, 2004; Beneke, Braun, Ji, Wei, 2018; ….

  - **Only models** for heavy meson **Shape Functions**:

    Korchemsky, Sterman, 1994; Bauer, Luke, Mannel, 2001; Neubert, 2005; Lee, Ligeti, Stewart, Tackmann, 2006; ……

  - Relations between LCDAs and SFs:

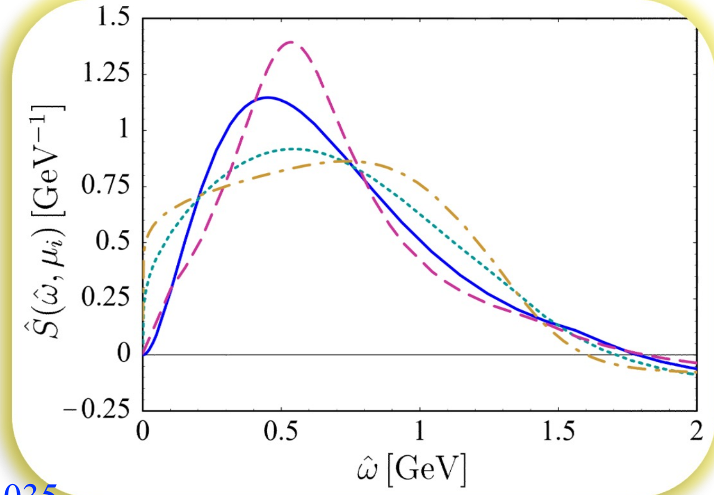
    Yaouanc, Oliver, Raynal, 2008



- Uncertainties from heavy meson LCDAs **dominate** the errors in theoretical calculation.

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

$$f^+_{B\to\pi}(0) = 0.122\times\Big[1\pm 0.07\Big|_{S_0^\pi}\pm 0.11\Big|_{\Lambda_q}\pm 0.02\Big|_{\lambda_E^2/\lambda_H^2}{}^{+0.05}_{-0.06}\Big|_{M^2}\pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2}$$
$$\qquad {}^{+0.06}_{-0.10}\Big|_{\mu_h}\pm 0.04\Big|_{\mu}{}^{+1.36}_{-0.56}\Big|_{\lambda_B}{}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\Big].$$



Gao, et al., PRD 101 (2020) 074035

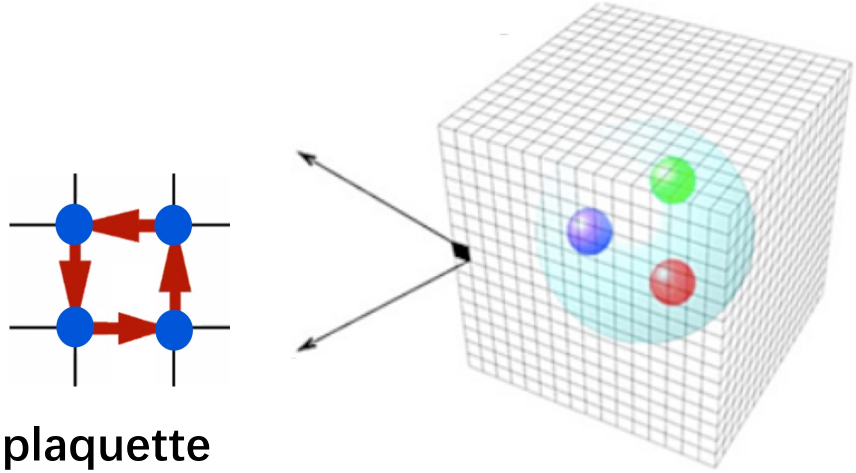
Cui, et al., JHEP 03 (2023) 140

Recipes for Heavy Meson Distributions from Lattice QCD

Lattice QCD

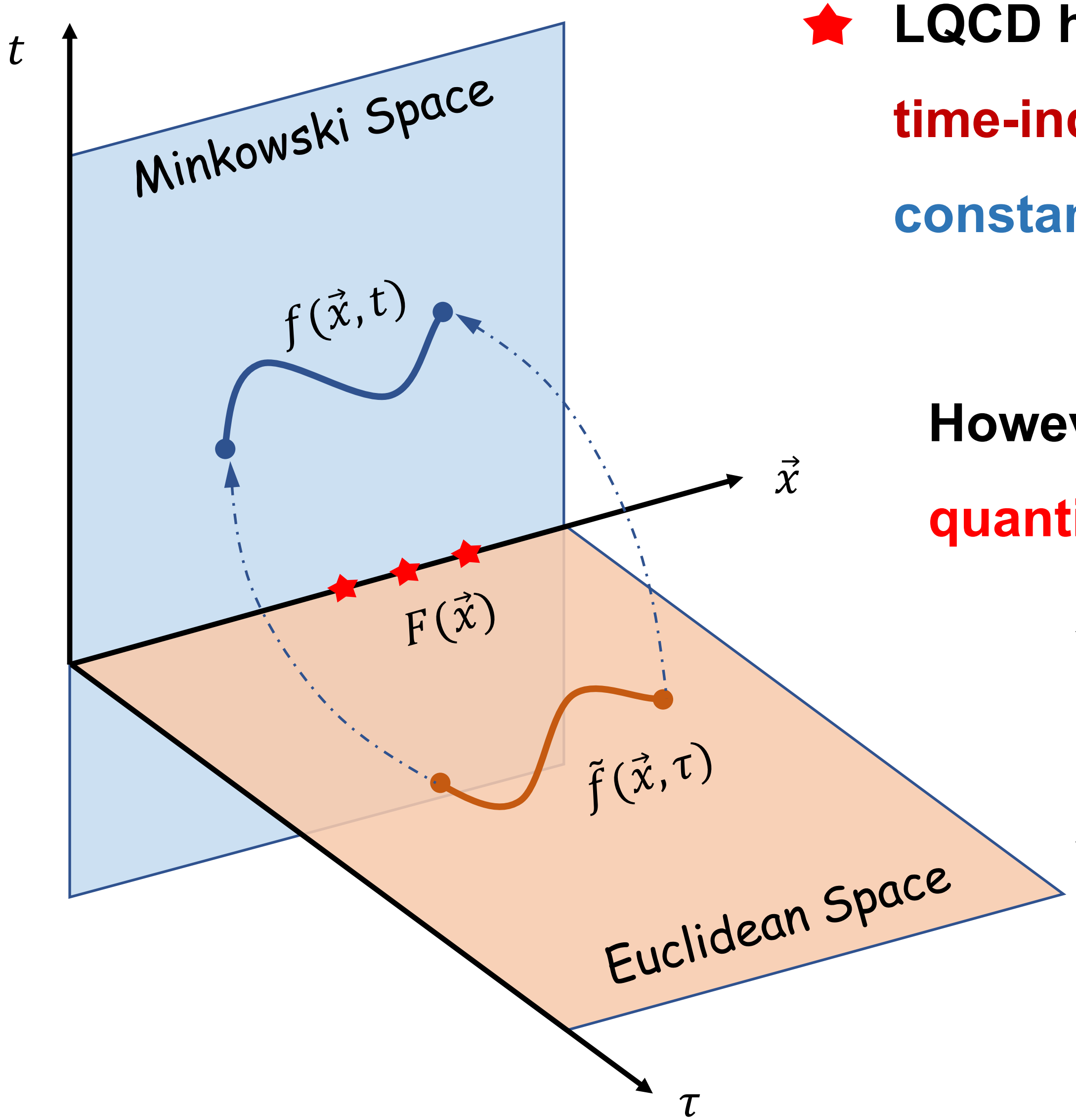
- **Discretizing the 4D spacetime;**
- **Wick rotation to Euclidean space to avoid sign problem:** $e^{iS} \rightarrow e^{-S_E}$
- **Quark fields on the lattice sites;**
- **Gauge links connecting lattice sites;**
- **Quark masses are benchmarked by the ground-state meson masses, e.g.** $m_\pi, m_K, m_D, \ldots\ldots$
- **Able to calculate the nonperturbative observables:**

$$\langle O \rangle = \frac{1}{Z} \int D\psi \, D\bar{\psi} \, DA \, e^{-S_E} \, O$$



plaquette

Challenges for Heavy Meson Distributions from Lattice QCD



★ LQCD have achieved great successes in calculating **time-independent** quantities: **hadron masses, decay constants, α_s, form factors** and so on.

However, LQCD cannot deal with **real-time dependent quantities** directly!

- ✓ Trans to time-independent variables:
  Lattice OPE
- ✓ Connecting the $f(\vec{x}, t)$ and $\tilde{f}(\vec{x}, \tau)$:
  Large-Momentum Effective Theory (LaMET)

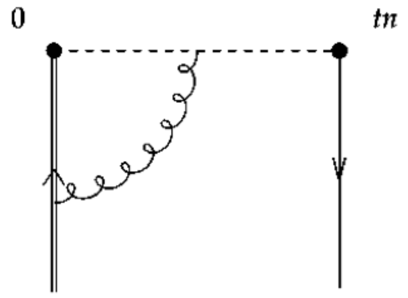
Challenges for Heavy Meson Distributions from Lattice QCD

- **Scheme I: Operator Product Expansion (OPE) for QCD quark bilinears:**

$$\bar{q}(z_2 n)\not{n}\gamma_5 W_c(z_2 n, z_1 n)\, q'(z_1 n) \;=\; \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k!l!} n^\rho n^{\mu_1} \ldots n^{k+l} \mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} + \cdots,$$

with $\mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} \;=\; \bar{q}(0)\overleftarrow{D}_{(\mu_1}\ldots\overleftarrow{D}_{\mu_k}\overrightarrow{D}_{\mu_{k+1}}\ldots\overrightarrow{D}_{\mu_{k+l}}\gamma_{\rho)}\gamma_5 q'(0).$

While for HQET fields, the OPE break down……



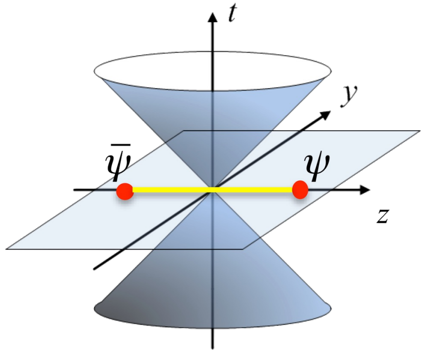
$$O_v^{\text{ren}}(t,\mu) \;=\; \frac{4}{\hat{\epsilon}} \ln(it\mu)\, O_v^{\text{bare}}(t) + \ldots \;\;\rightarrow\;\; \log 0! \;\;\Rightarrow$$ No Local Limit

Cusp Divergence!

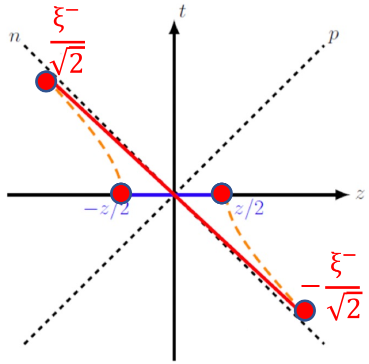
Braun, Ivanov, Korchemsky, PRD69, 034014 (2004)

Challenges for Heavy Meson Distributions from Lattice QCD

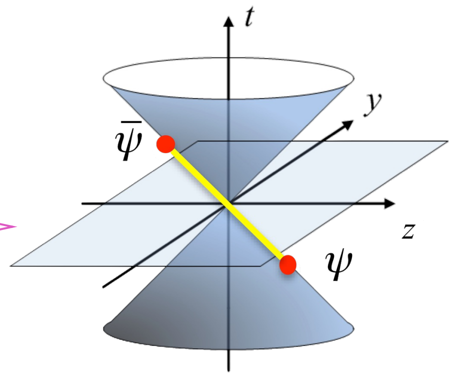
- **Scheme II: Large-momentum Effective Theory (LaMET): connecting the Euclidean and Minkowski correlators in the large momentum limit:**



Boosting P^z



EFT



Quasi distributions:
Directly calculable on the lattice

Ji,PRL110 (2013), 262002
Ji, et al., Rev.Mod.Phys. 93 (2021) 3, 035005

Light-cone distributions

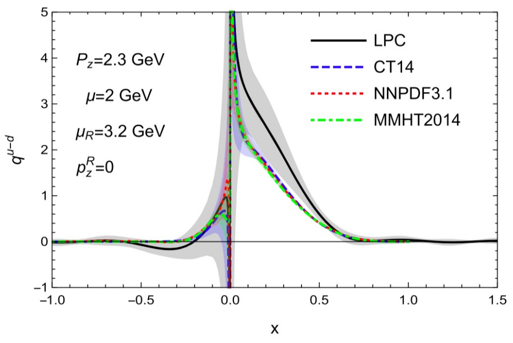
Effective field theory: one can perform an expansion for large but finite P^z:

$$\tilde{f}(y, P^z, \tilde{\mu}) = \int_{-1}^{1} \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}, \frac{\tilde{\mu}}{\mu}\right) f(x, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(yP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-y)P^z)^2}\right)$$

Challenges for Heavy Meson Distributions from Lattice QCD

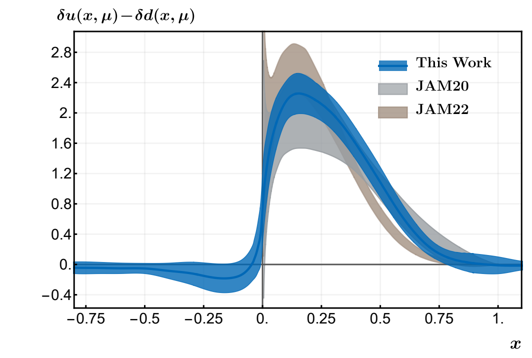
Many Progress has been made on PDFs, LCDAs, TMDs, GPDs, *et al.*:

Nucleon unpolarized PDF



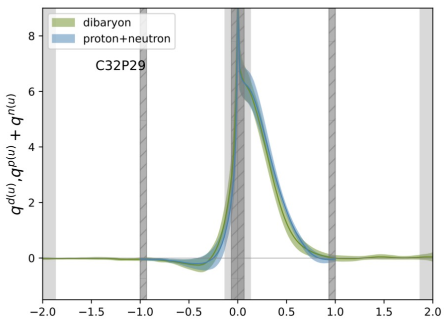
LPC, 2020

Nucleon transversity PDF



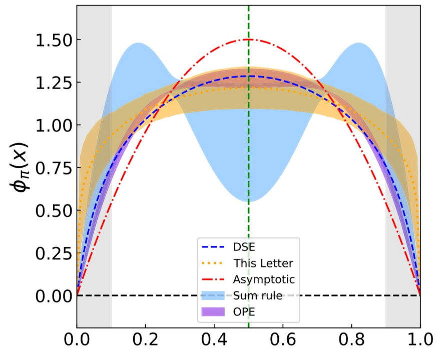
LPC, 2024

Deuteron PDF



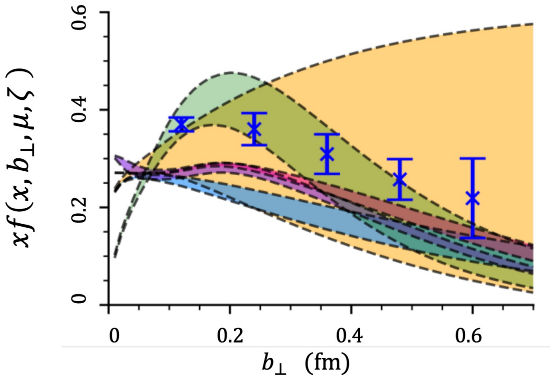
LPC, 2025

Light meson LCDA



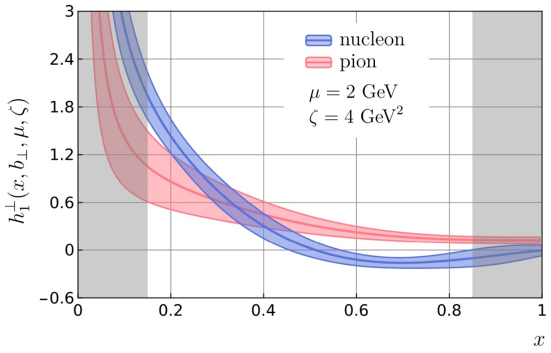
LPC, 2022

Nucleon unpolarized TMDPDF



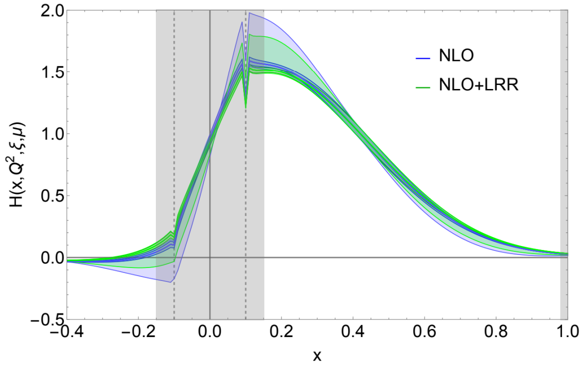
LPC, 2024

Nucleon&pion Boer-Mulders function



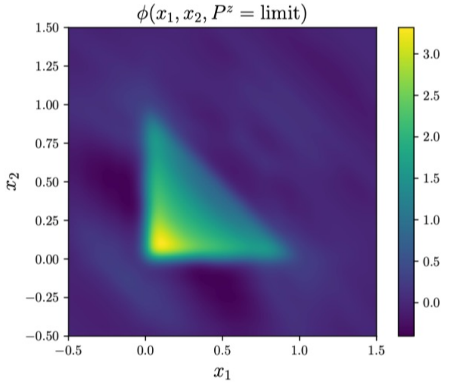
LPC, 2025

Nucleon GPD



Holligan, Lin, 2024

Light baryon LCDA



LPC, 2025

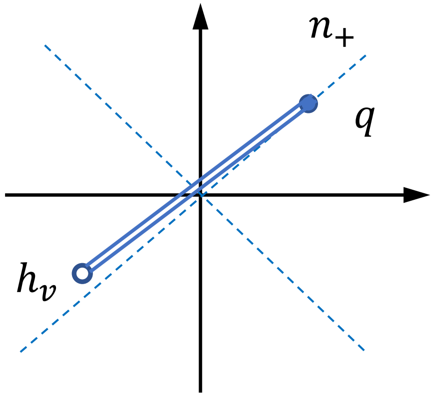
Challenges for Heavy Meson Distributions from Lattice QCD

Turns to correlators with HQET fields……

- An intuitive approach: adopt **off light-cone Wilson line** to avoid cusp divergence

$$\langle H_Q(P_{H_Q})\,|\,\bar{q}(z)\not{n}_z\gamma_5\,W_c(z,0)h_v(0)\,|\,0\rangle \qquad \langle H_Q(v)\,|\,\bar{h}_v(z,0)W_c(z,0)h_v(0)\,|\,H_Q(v)\rangle$$

Wang, Wang, Xu, Zhao, PRD 102, 011502 (2020);
Xu, Zhang, PRD 106, 114019 (2022);
Hu, Xu, Zhao, EPJC 84, 502 (2024); ……



Need to realize the **boosted HQET fields on lattice**.

Challenges for Heavy Meson Distributions from Lattice QCD

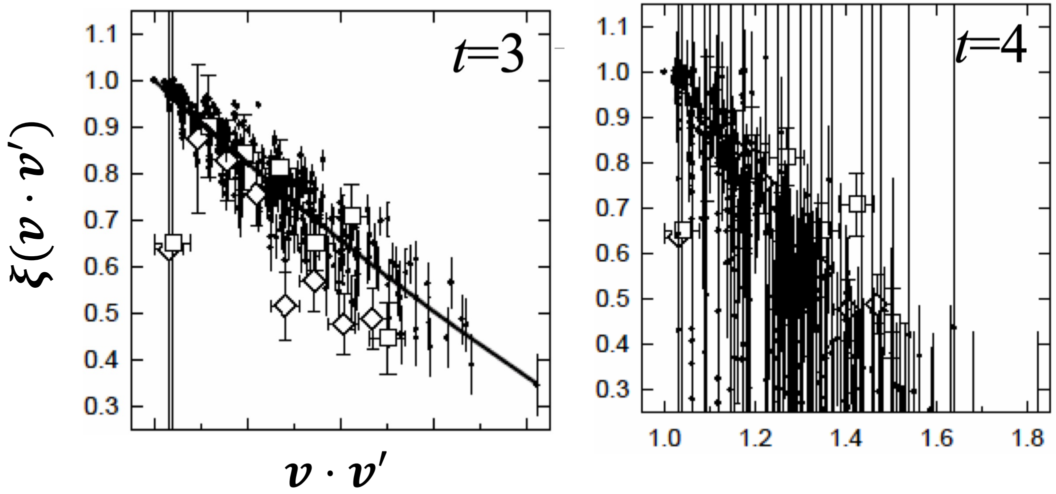
- **Boosted HQET on Lattice**
  - HQET: $\mathcal{L}_{\mathrm{HQET}} = \bar{h}_v(x)\, i v\cdot D h_v(x)$ $\xrightarrow{\text{Discretization}}$ $\bar{h}_v(x)\,(-v_0 D_0 + i\vec{v}\cdot\vec{D})\, h_v(x)$
  - The QCD field is recovered by: $\psi(x) = e^{-imv\cdot x}\dfrac{1+\not{v}}{2} h_v(x)$
  - Evolution equation of HQET propagator:

$$G(x+\hat{t}) = U_4^\dagger(x)\,(1 + i\vec{v}\cdot\vec{D})G(x)$$

Significant signal-to-noise (StN) problem

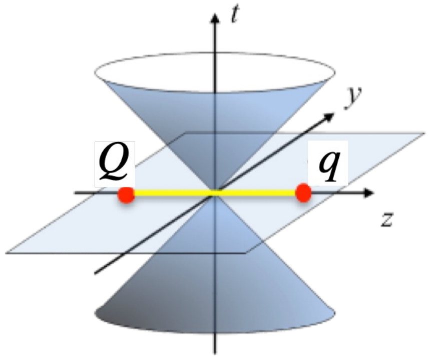
e.g. Isgur-Wise function:

Mandula, Ogilvie, PRD 45, 2183-2187 (1992); NPB 34, 480-482 (1994)



Recipes for Heavy Meson Distributions from LQCD

Heavy meson LCDA as an example:

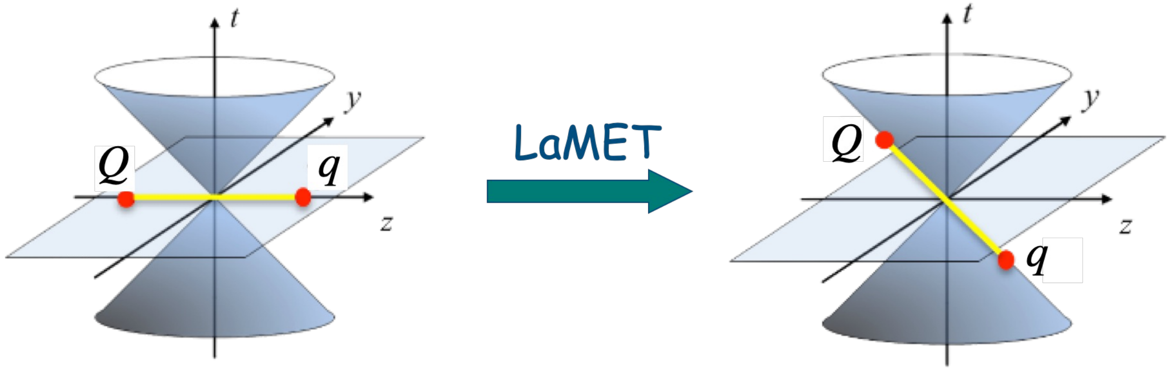


Equal-time correlators

- **Start from the equal-time correlators with QCD fields:**
  - Equal-time correlations ensure that it is directly calculable from lattice QCD;
  - Without h_v, the issues of **cusp divergence** and **StN problem** are both resolved.

Recipes for Heavy Meson Distributions from LQCD

Heavy meson LCDA as an example:



Equal-time correlators

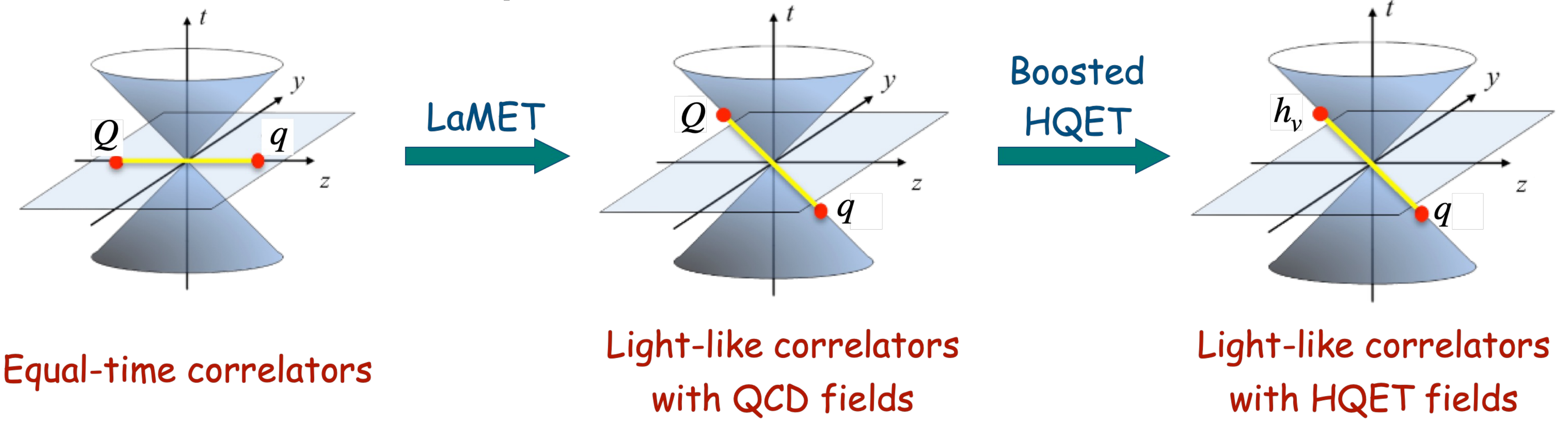
Light-like correlators with QCD fields

➢ **Large-momentum effective theory (LaMET) provides a connection between equal-time correlators and light-cone observables.**

- Integrating out large-momentum scale, one can obtain the heavy meson LCDA in QCD.

Recipes for Heavy Meson Distributions from LQCD

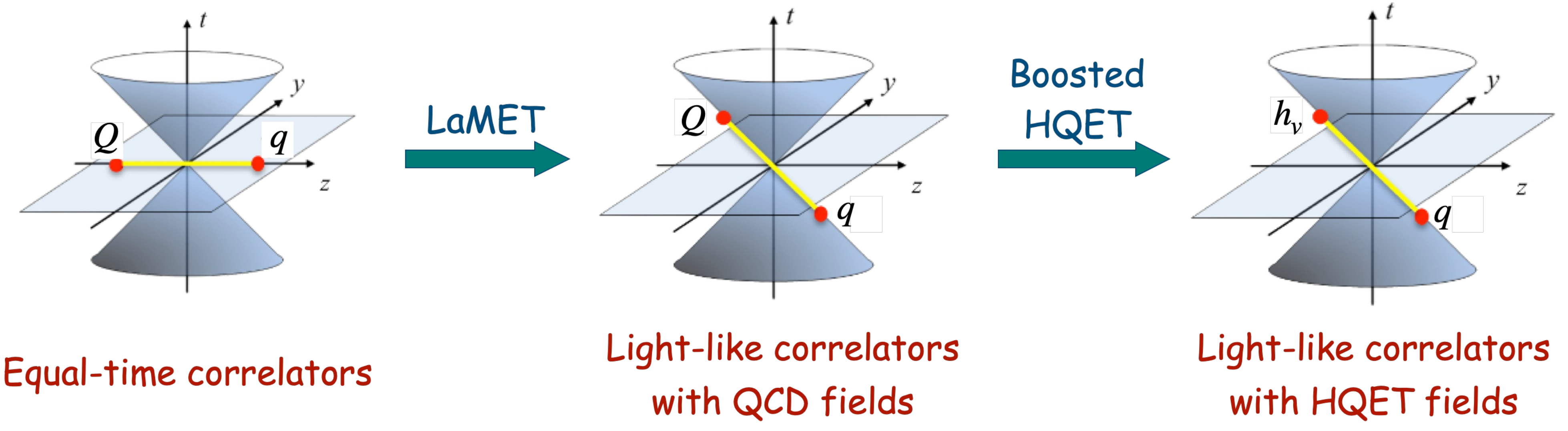
Heavy meson LCDA as an example:



- **In the Isgur-Wise limit $(m_Q \to \infty)$, QCD LCDAs are related to the HQET LCDAs:**
  - The boosted HQET provides the matching between them.

Ishaq, Jia, Xiong, Yang, PRL125(2020)132001; Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

Recipes for Heavy Meson Distributions from LQCD



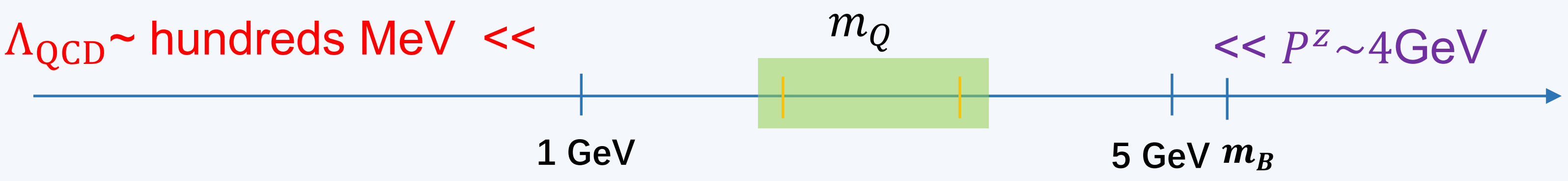
- **A sequential effective theory:**

  - LaMET: $\Lambda_{\rm QCD}, m_{\rm Q} \ll P^z$ and integrate out P^z
  - bHQET: $\Lambda_{\rm QCD} \ll m_{\rm Q}$ and integrate out $m_{\rm Q}$

$\Rightarrow$ Introduce a hierarchy $\boldsymbol{\Lambda_{\rm QCD} \ll m_Q \ll P^z}$

Recipes for Heavy Meson Distributions from LQCD

- **Lattice feasibility at this stage:**



Window for heavy meson mass

- ✓ Heavy quark flavor symmetry ensures that the HQET measurement is independent of heavy quark mass;
- ✓ m_Q (m_c or m_b) only contributes to the power corrections.

Power corrections: $\frac{\Lambda_{\rm QCD}^2}{(yP^z)^2}, \frac{m_Q^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}}{m_Q}$

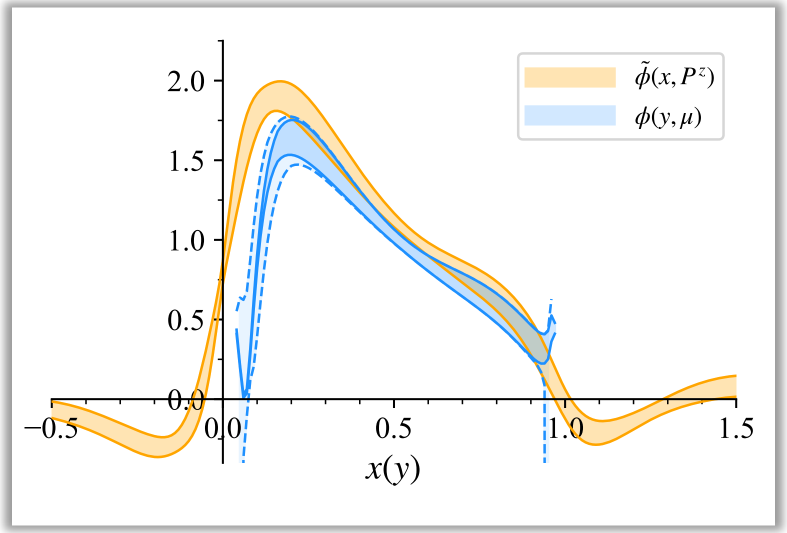
Heavy Meson LCDAs from Lattice QCD

The First LQCD Implementation of Heavy Meson LCDAs

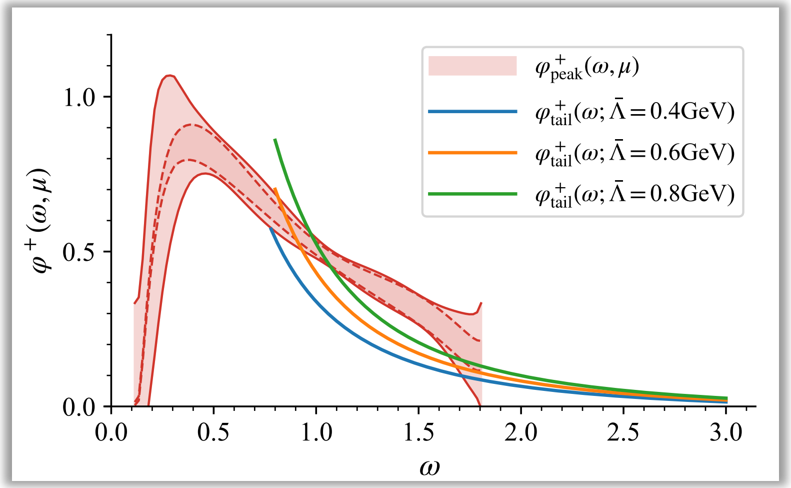
- ✓ A numerical simulation on the finest CLQCD ensemble ($a = 0.05187$ fm);
- ✓ Simulate the D meson quasi DA with $m_D \simeq 1.92\text{GeV}$, up to $P^z \simeq 3.98\text{GeV}$;
- ✓ Predictions for D meson QCD LCDA and heavy meson HQET LCDA at leading power accuracy.

LPC, Phys.Rev.D 111 (2025), L111503; PRD111 (2025), 034503

Quasi DA and QCD LCDA



HQET LCDA



Towards More Precise Predictions: What can be Improved?

- ✓ A numerical simulation on the finest CLQCD ensemble ($a = 0.05187$ fm); ➡ ☐ Multi lattice spacings and **continuum limit**
- ✓ Simulate the *D* meson quasi DA with $m_D \simeq 1.92\text{GeV}$, up to $P^z \simeq 3.98\text{GeV}$; ➡ ☐ **Physical mass** ☐ **Infinite momentum limit**
- ✓ Predictions for *D* meson QCD LCDA and heavy meson HQET LCDA at leading power accuracy. ➡ ☐ **Power corrections?**

$$\frac{\Lambda_{\text{QCD}}^2}{(yP^z)^2} \sim \frac{0.4^2}{(4y)^2} < 1\%$$

$$\frac{m_D^2}{(P^z)^2} \sim \frac{2^2}{4^2} \simeq 25\%$$

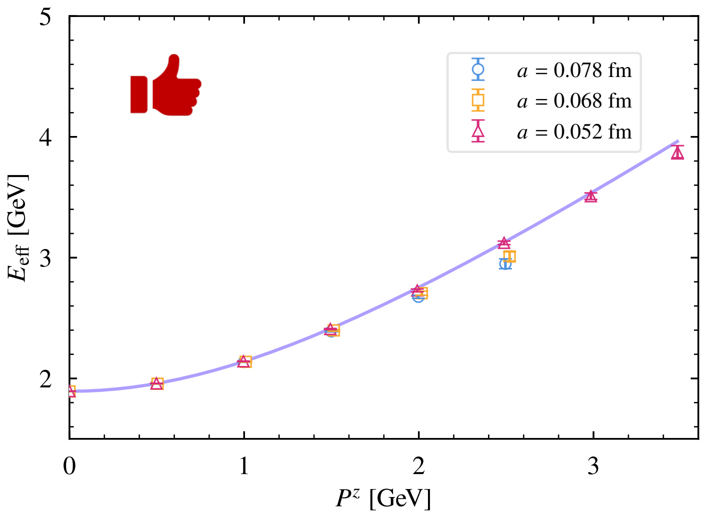
$$\frac{\Lambda_{\text{QCD}}}{m_D} \sim \frac{0.4}{2} \simeq 20\%$$

Towards More Precise Predictions

Large statistics calculations on CLQCD ensembles

Ensemble	$L^3 \times T$	a (fm)	m_π (GeV)	P^z (GeV)	N_{stats}
F32P30	$32^3 \times 96$	0.07750	303.2(1.3)	Up to 2.5	183600
F32P21	$32^3 \times 64$	0.07750	210.9(2.2)	Up to 2.5	29376
G36P29	$36^3 \times 108$	0.06826	295.1(1.2)	Up to 2.5	112832
H48P32	$48^3 \times 144$	0.05187	317.2(0.9)	Up to 3.5	108000

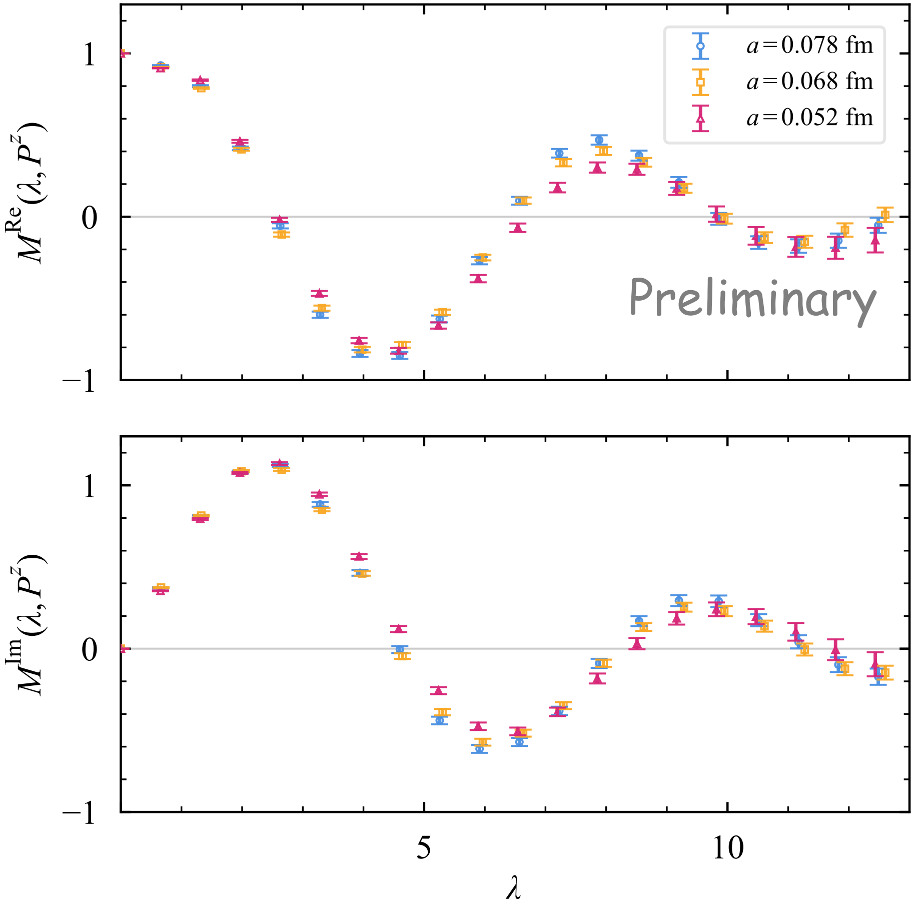
- ✓ Multi lattice spacings: Continuum extrapolation.
- ✓ Multi pion masses: Chiral extrapolation.
- ✓ Multi boost momenta: Infinite momentum extrapolation.
- ✓ Large statistic.
- ✓ Discretization effects well controlled.



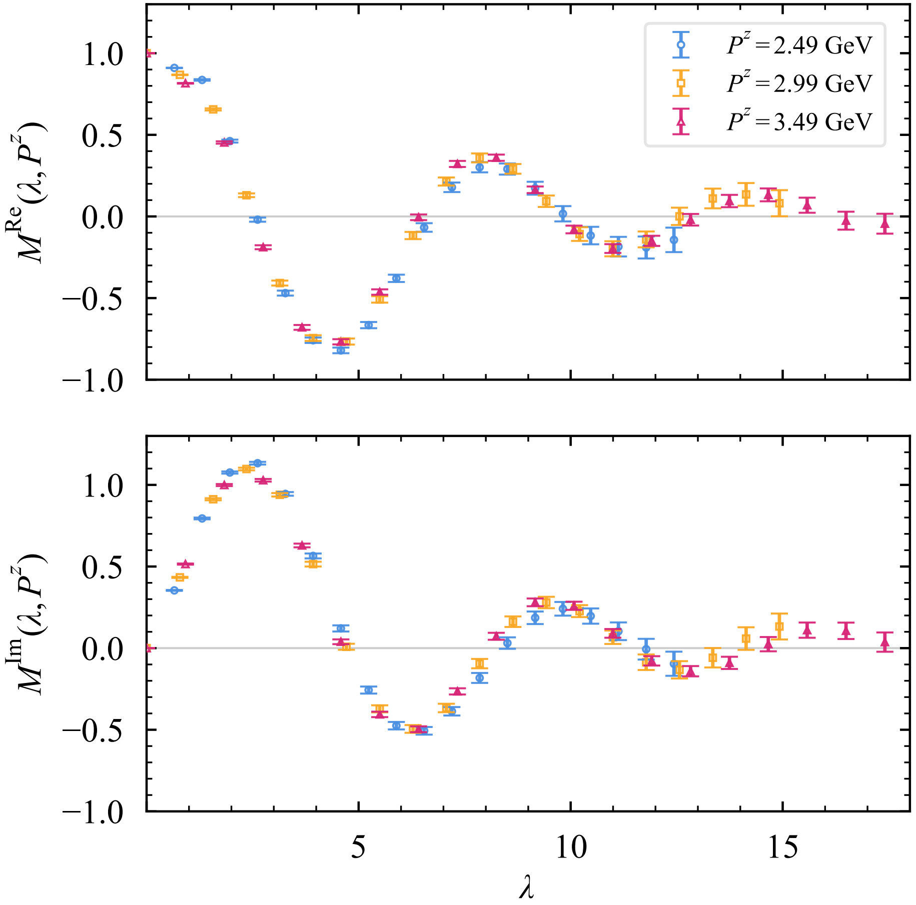
QCD LCDA of D Meson from LaMET

Quasi DAs:

- a-dependence, $P^z = 2.5\text{GeV}$



- P^z-dependence, $a = 0.052\text{fm}$



LPC, 2601.xxxxx

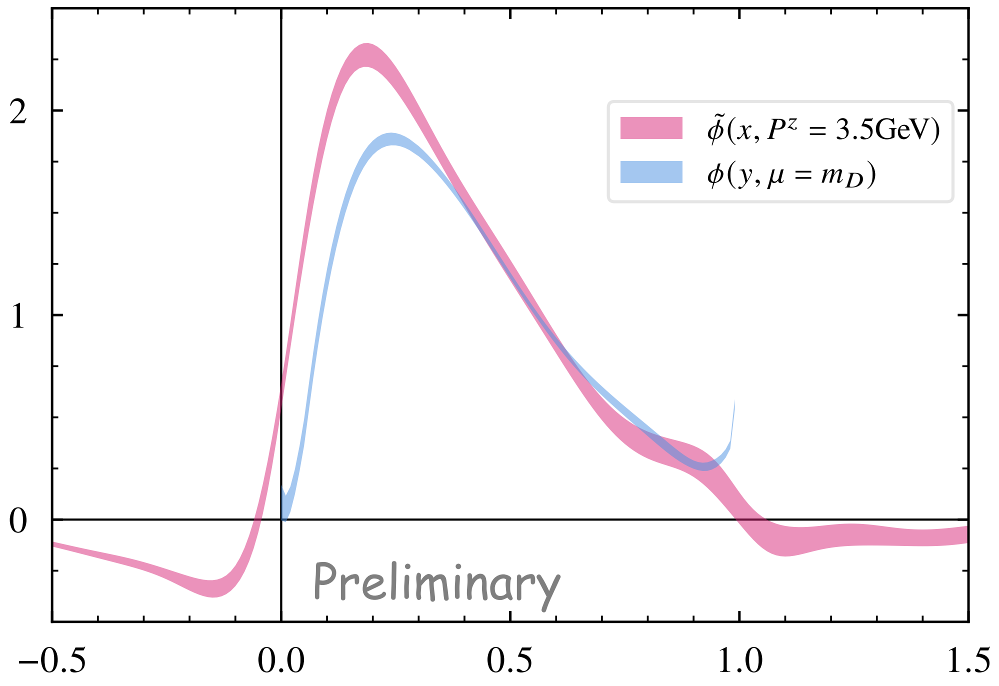
QCD LCDA of D Meson from LaMET

Matching from quasi-DA to QCD LCDA:

$$\phi(y,\mu) = \int_{-\infty}^{+\infty} \frac{dx}{|x|}\, \mathcal{C}\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \tilde{\phi}(x, P^z) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(yP^z, (1-y)P^z)^2}\right).$$

Liu, Wang, Xu, **QAZ**, Zhao, PRD 99, 094036 (2019)

Han, **QAZ**, *et.al.*, PRD111, 034503, (2025)



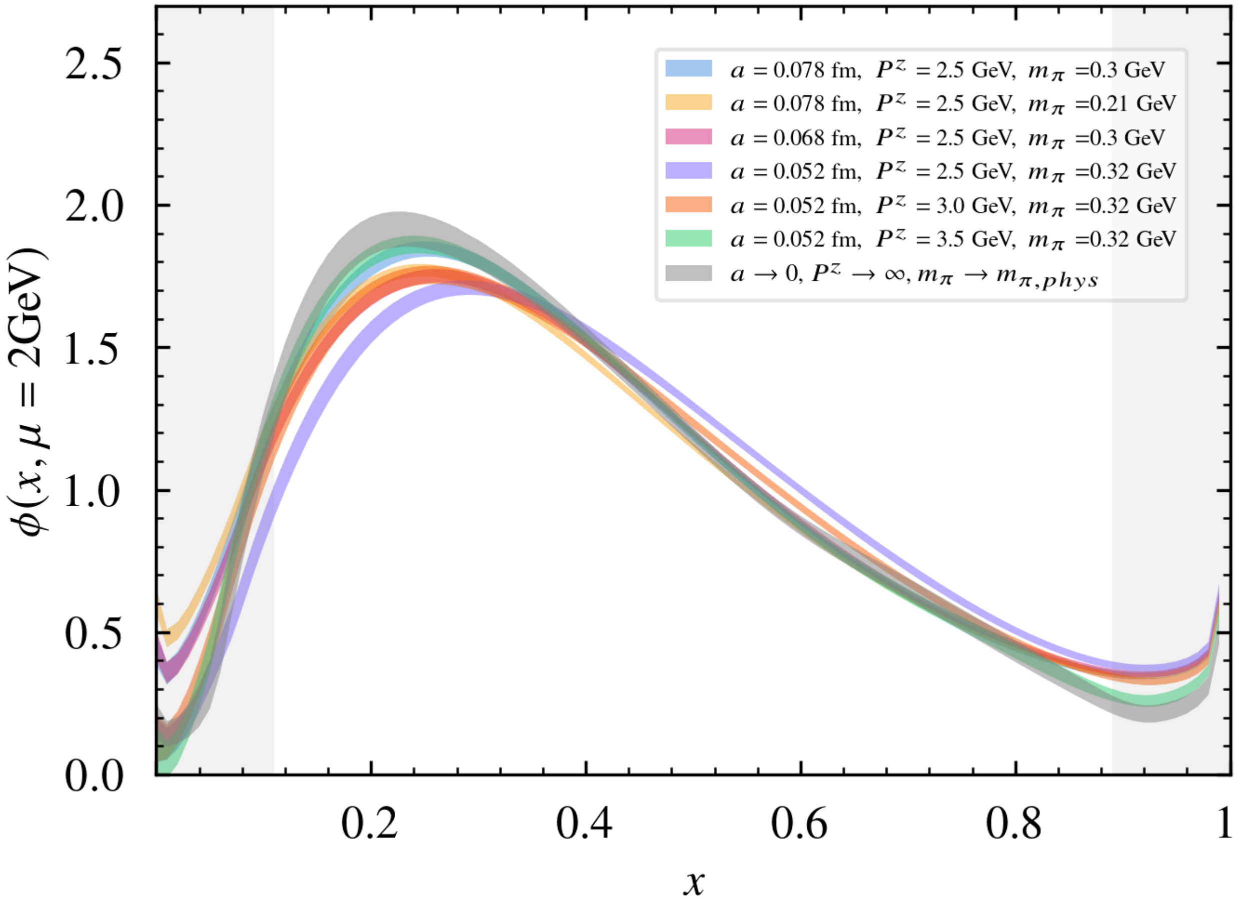
- Power corrections:

$\frac{\Lambda_{\rm QCD}^2}{(yP^z)^2} \sim \mathcal{O}(1\%)$: **Subordinate**, only contribute to end-point region.

$\frac{m_D^2}{(P^z)^2} \sim \mathcal{O}(10\%)$: **Dominant**, contribute to **all region**.

QCD LCDA of D Meson from LaMET

- Full distribution of D meson QCD LCDA from LaMET:



Power correction $\Lambda_{\text{QCD}}^2/(xP^z)^2$:

- Significant at end-point region ($x \to 0, 1$) of the QCD LCDA;
- Can be improved by considering the renormalon resummation.

Su, Holligan, Ji, Yao, Zhang, Zhang, NPB 991, 116201 (2023)

Recap: LaMET vs. Lattice OPE

- For QCD LCDA:

	LaMET	Lattice OPE
Pros	1) Full distributions 2) Directly comparison to phenomenology 3) ~~Detailed partonic structure~~	1) No need for large momentum 2) Low moments can be precise ~~→ good for benchmarks~~
Cons	1) Need large momentum 2) ~~Sizable power corrections~~	1) Only provides a finite number of moments

- The power correction $\frac{m_D^2}{(P^z)^2}$ in LaMET can be benchmarked by the **lowest order moments** calculated from lattice OPE

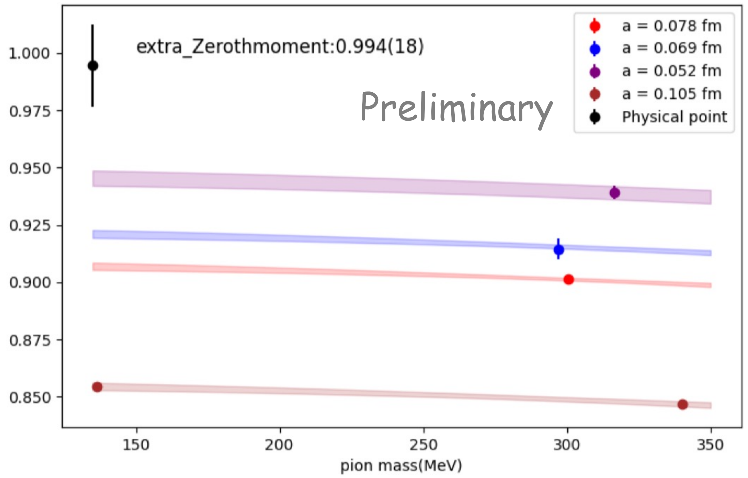
D Meson Moments from Lattice OPE

- Mellin moments from lattice QCD

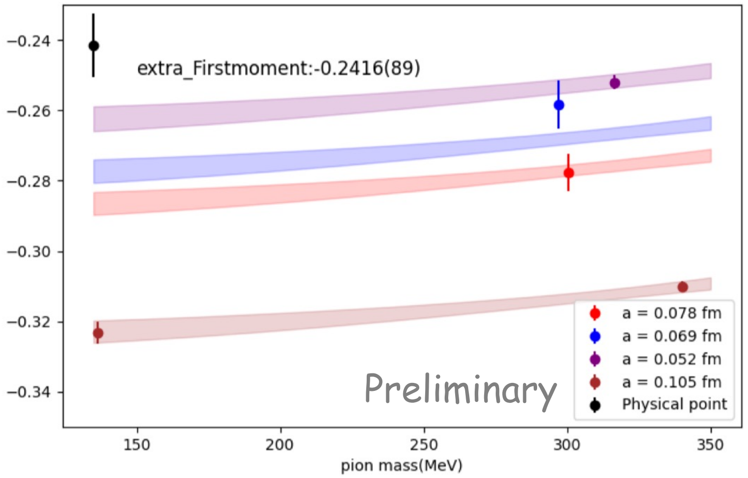
$$\begin{aligned}
\langle 0|O_\rho|D(P)\rangle &= i f_M P_\rho \langle 1\rangle, \\
\langle 0|O^-_{\rho\mu}|D(P)\rangle &= f_M P_\rho P_\mu \langle \xi\rangle, \\
\langle 0|O^+_{\rho\mu\nu}|D(P)\rangle &= -i f_M P_\rho P_\mu P_\nu \langle 1\rangle, \\
\langle 0|O^-_{\rho\mu\nu}|D(P)\rangle &= -i f_M P_\rho P_\mu P_\nu \langle \xi^2\rangle, \\
&\cdots
\end{aligned}$$

$$\begin{aligned}
O_\rho &= \bar{q}\gamma_\rho\gamma_5 c, \\
O^-_{\rho\mu} &= \bar{q}[\overleftarrow{D}_{(\mu} - \overrightarrow{D}_{(\mu}]\gamma_{\rho)}\gamma_5 c, \\
O^-_{\rho\mu\nu} &= \bar{q}[\overleftarrow{D}_{(\mu}\overleftarrow{D}_\nu - 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_\nu + \overrightarrow{D}_{(\mu}\overrightarrow{D}_\nu]\gamma_{\rho)}\gamma_5 c, \\
O^+_{\rho\mu\nu} &= \bar{q}[\overleftarrow{D}_{(\mu}\overleftarrow{D}_\nu + 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_\nu + \overrightarrow{D}_{(\mu}\overrightarrow{D}_\nu]\gamma_{\rho)}\gamma_5 c, \\
&\cdots
\end{aligned}$$

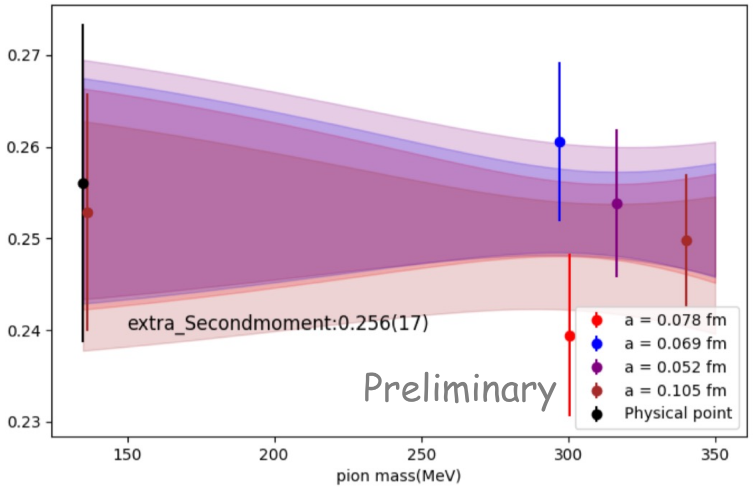
Zeroth moment



First moment



Second moment



D Meson Moments

- Gegenbauer moments from LaMET: vs.
- Moments from Lattice OPE

$$\phi(x,\mu^2) = 6x(1-x)\left[1+\sum_{n=1}^{\infty} a_n(\mu^2)\, C_n^{3/2}(2x-1)\right],$$

	n	a_1	a_2	a_3	a_4
LaMET (fit up to n^{th} order)	2	-0.393(19)	0.130(14)		
	4	-0.401(22)	0.129(22)	-0.012(16)	0.013(12)
	6	-0.390(25)	0.109(29)	0.013(28)	-0.032(23)
	8	-0.369(33)	0.091(52)	0.044(39)	-0.055(52)
Lattice OPE		-0.403(23)	0.167(61)		

Preliminary

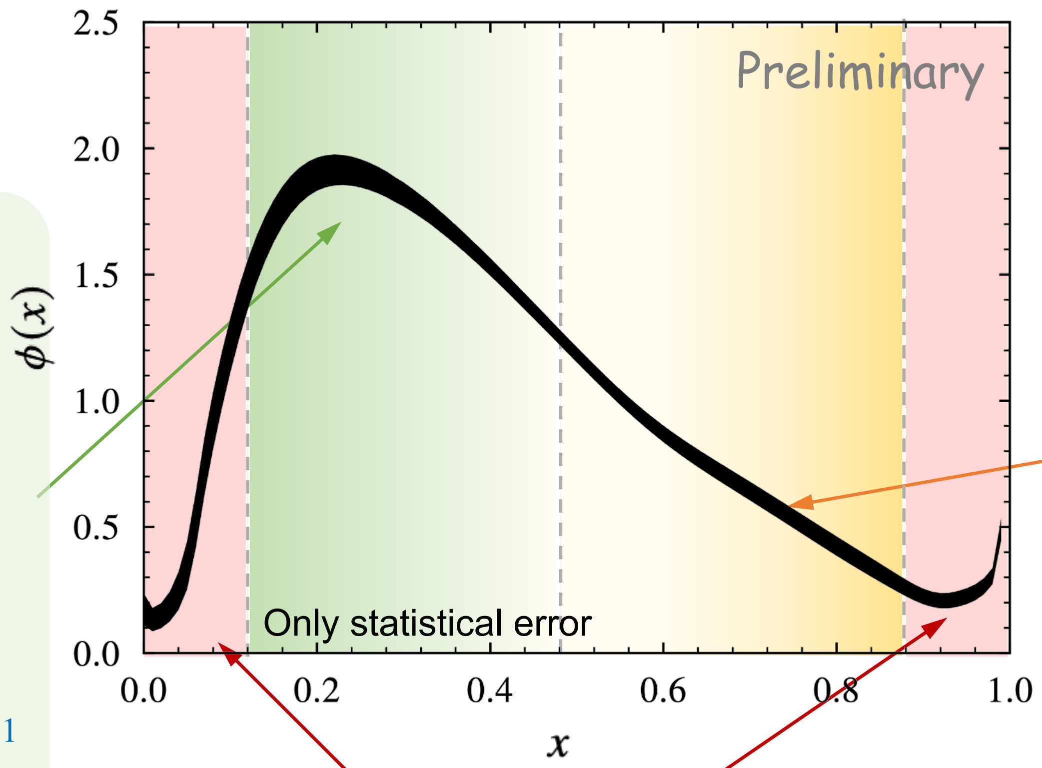
- The power correction $\frac{m_D^2}{(P^z)^2}$ can be benchmarked by a_1 and a_2 terms, and contribute to systematic errors.

Regions of QCD LCDA

Peak region: $y \sim \frac{\Lambda_{\text{QCD}}}{m_H}$

- Light quark carries small momentum fraction;
- Related to the HQET LCDA.

Ishaq, Jia, Xiong, Yang, PRL125(2020)132001
Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)



Tail region: $y \sim 1$

- Contain only hard-collinear physics;
- Suppressed in LCDA.

End-point region:

- LaMET matching kernel suffer large power corrections.
- Lattice QCD predictions fail

Prediction for HQET LCDA

- Matching to leading twist HQET LCDA:

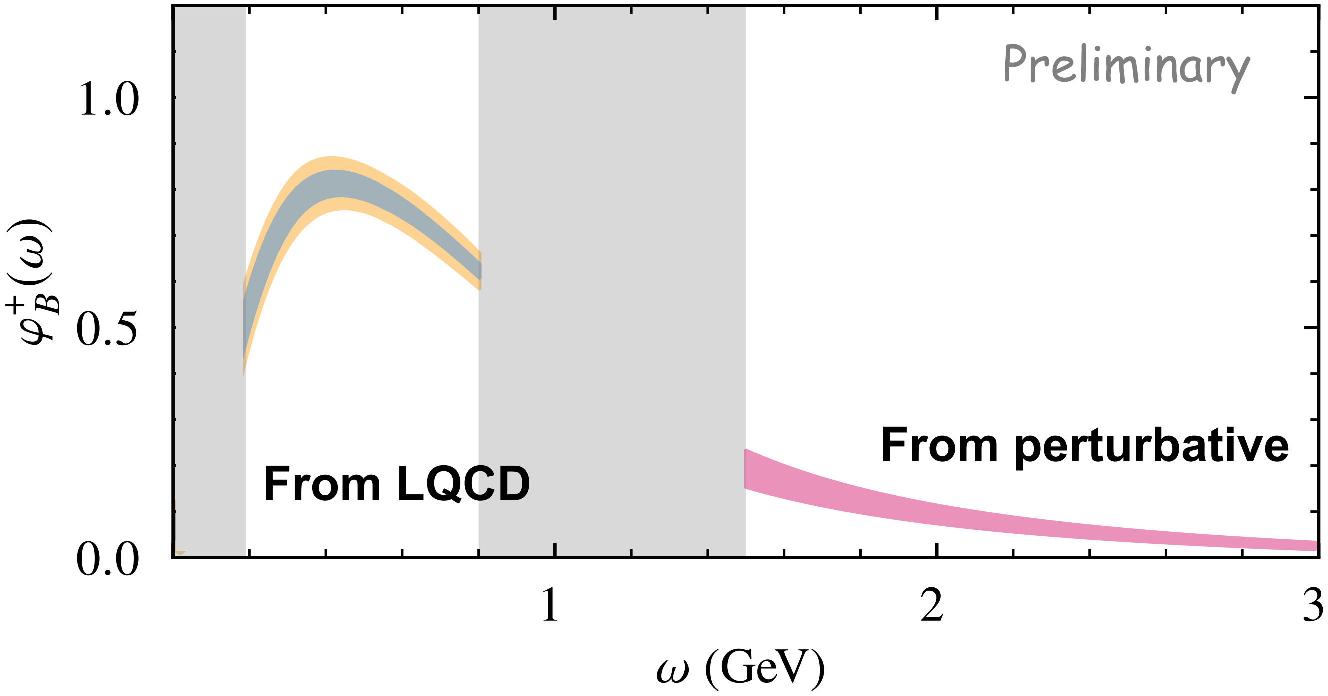
QCD LCDA, nonperturbative

$$\varphi^+(\omega,\mu) = \begin{cases} \frac{1}{m_H}\left[1 - \frac{\alpha_s C_F}{4\pi}\left(\frac{1}{2}\ln^2\frac{\mu^2}{m_H^2} + \frac{1}{2}\ln\frac{\mu^2}{m_H^2} + \frac{3}{2}\ln\frac{\mu^2}{m_Q^2} + \frac{\pi^2}{12} + 4\right) + \mathcal{O}\left(\alpha_s^2\right)\right]\phi(\frac{\omega}{m_H},\mu), & \omega \sim \Lambda_{\rm QCD} \\ \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right] + \mathcal{O}\left(\alpha_s^2\right), & \omega \sim m_H \end{cases}$$

Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

Lee, Neubert, PRD 72, 094028 (2005)

Prediction for HQET LCDA from first-principle:



Prediction for HQET LCDA

- Matching to leading twist HQET LCDA:

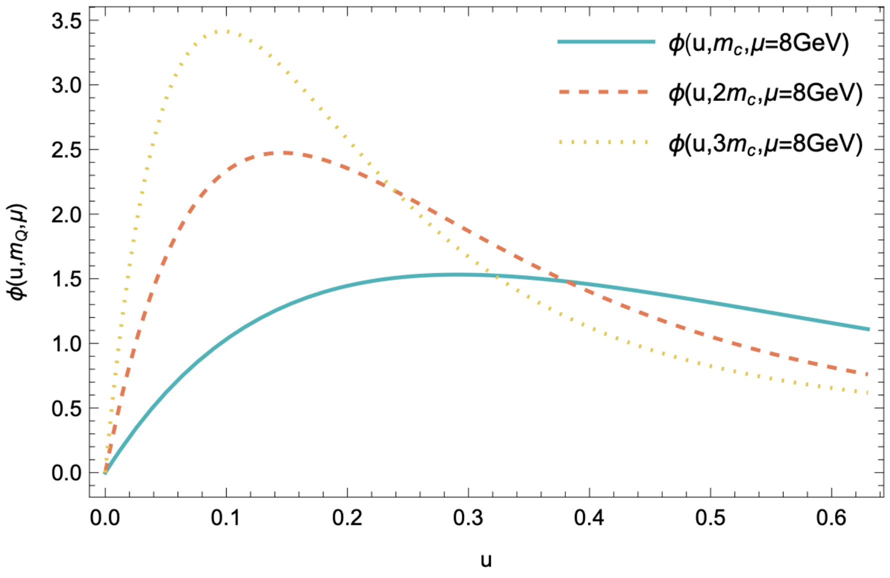
$$+\mathcal{O}\left(\frac{\Lambda_{QCD}}{m_Q}\right)$$

$$\varphi^+(\omega,\mu) = \begin{cases} \frac{1}{m_H}\left[1 - \frac{\alpha_s C_F}{4\pi}\left(\frac{1}{2}\ln^2\frac{\mu^2}{m_H^2} + \frac{1}{2}\ln\frac{\mu^2}{m_H^2} + \frac{3}{2}\ln\frac{\mu^2}{m_Q^2} + \frac{\pi^2}{12} + 4\right) + \mathcal{O}(\alpha_s^2)\right]\phi(\frac{\omega}{m_H},\mu), & \omega \sim \Lambda_{\rm QCD} \\ \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right] + \mathcal{O}(\alpha_s^2), & \omega \sim m_H \end{cases}$$

- m_Q is a **tunable scale** in the lattice simulation!

- The $\boldsymbol{m_Q}$**-RGE** of heavy meson LCDAs: Wang, Xu, **QAZ**, Zhao, arXiv:2411.07101 [hep-ph]

$$\left[m_Q\frac{\partial}{\partial m_Q} - u\frac{\partial}{\partial u} - 1 - \gamma(m_Q,\mu)\right]\phi(u,\mu;m_Q) = 0$$



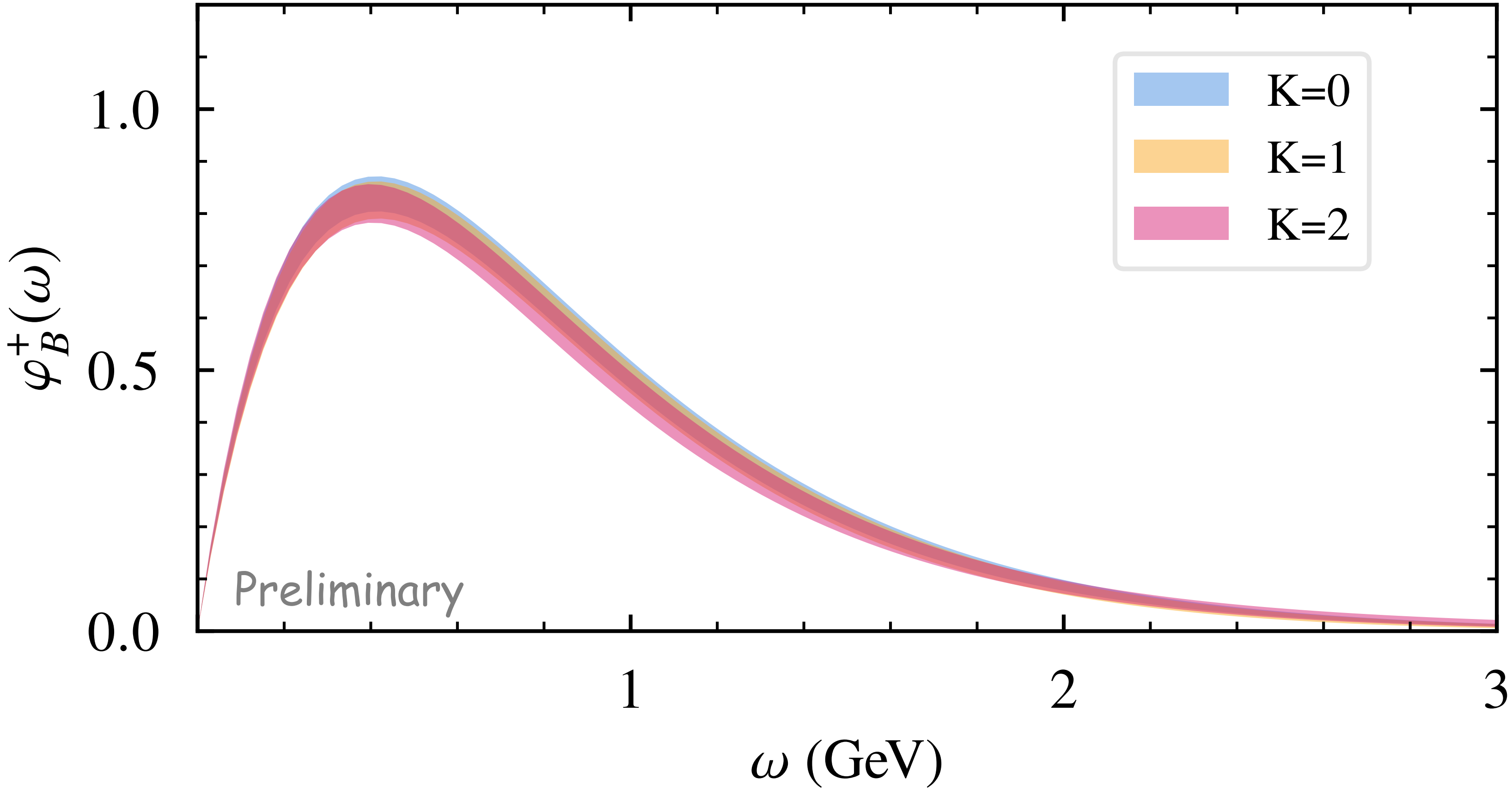
- The power correction $\propto \Lambda_{\rm QCD}/m_Q$ can controlled by computing at a range of m_Q and extrapolate to the $m_Q \to \infty$ limit.

Model-independent Parametrization of HQET LCDA

- Rebuild the full distribution of the HQET LCDA from a **model-independent parametrization**:

Feldmann, Lughausen, Dyk, JHEP10, 162 (2020)

$$\varphi^+(\omega,\mu) = \frac{\omega e^{-\omega/\omega_0}}{\omega_0^2} \sum_{k=0}^{K} \frac{a_k(\mu)}{1+k} L_k^{(1)}(2\omega/\omega_0),$$



Inverse and inverse-logarithmic moments

Predictions of inverse and inverse-logarithmic moments from the full distribution of HEQT LCDA:

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\varphi^+(\omega,\mu),$$

$$\sigma_B^{(n)}(\mu) = \lambda_B(\mu)\int_0^\infty \frac{d\omega}{\omega}\ln\left(\frac{\mu}{\omega}\right)^{(n)}\varphi^+(\omega,\mu).$$

	λ_B (GeV)	$\sigma_B^{(1)}$
Our work	0.414 (16)	1.451 (19)
PRD98 (2018)	>0.24	
JHEP (2020)	0.383 (153)	
PRD72 (2005)	0.48 (11)	1.6 (2)
PRD69 (2004)	0.46 (11)	1.4 (4)
PRD55 (1997)	0.35 (15)	
PRD101 (2020)	$0.343^{+0.064}_{-0.079}$	
PLB848 (2024)	0.338 (68)	

Preliminary, only statistic error

Heavy Meson Shape Functions from Lattice QCD

Heavy Meson Shape Functions

- Connections between SFs with HQET and QCD fields:

$$S^{\rm QCD}(x,\mu) = \begin{cases} Z_{\rm peak}(x,\omega,\mu) \otimes S^{\rm HQET}(\omega,\mu)\,, & x \sim 1 \\ Z_{\rm tail}(x,\mu)\,, & x \sim 0 \end{cases}$$

with NLO matching kernel in peak region:

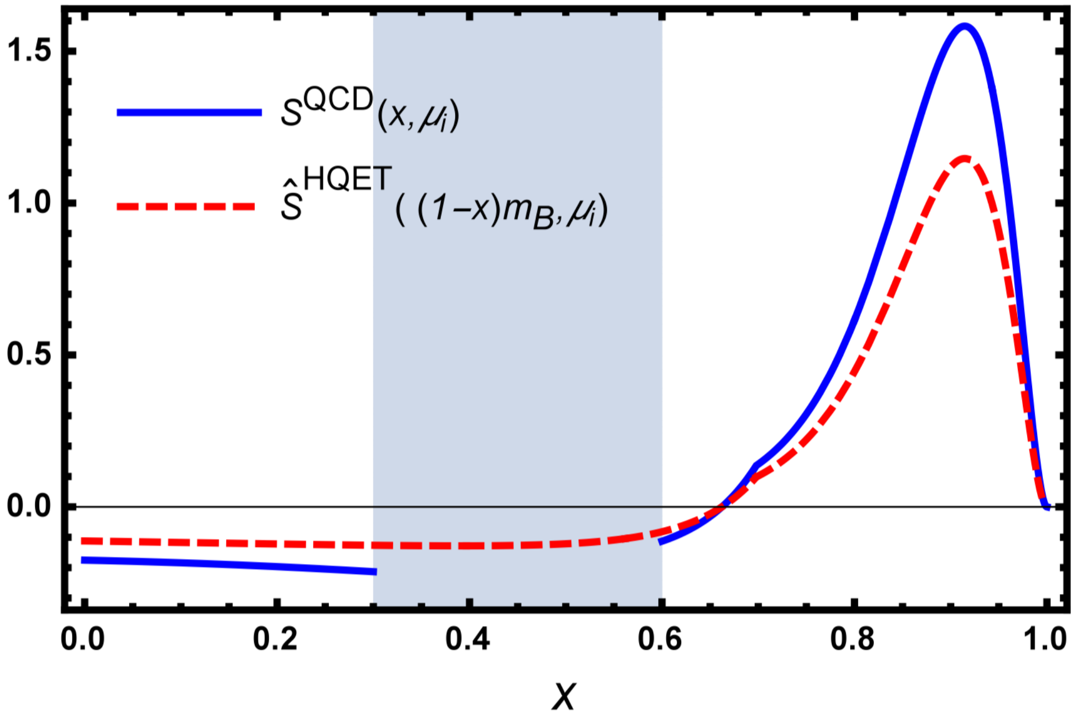
$$Z^{(1)}_{\rm peak}(x,\omega,\mu) = \left(\frac{1}{2}\ln^2\frac{\mu^2}{m_b^2} - \frac{3}{2}\ln\frac{\mu^2}{m_b^2} + \frac{\pi^2}{12} - 2\right) \times\delta(xm_Bv^+ - m_bv^+ - \omega v^+)\,.$$

and tail region

$$Z^{(1)}_{\rm tail}(x,\mu) = \frac{1}{m_bv^+}\,\frac{1+x^2}{1-x}\left[-1+\ln\frac{\mu^2}{(1-x)^2m_b^2}\right].$$

- Lattice QCD calculations are in progress.

Illustration of the matching based on modelized QCD SF



Wang, Xu, **QAZ**, Zhao, PRD112 (2025), 054044

Summary and Outlook

- ✓ We propose a practical first-principles strategy to predict heavy meson structure distributions (LCDAs and shape functions) from lattice QCD.

- **Heavy meson LCDAs:**
  - ✓ Factorization has been established;
  - ✓ Systematic lattice QCD calculation is now complete;
  - ✓ The next step is to properly control the systematic uncertainties, especially from power corrections.

- **Heavy meson shape function:**
  - ✓ Factorization has been proposed;
  - ☐ Expecting further validation in lattice QCD calculations.

Thanks for your attention

- ✓ A new era of lattice QCD calculations of heavy meson distributions, with the prospect of providing more precise nonperturbative inputs for both theory and experiment.