BESIII实验物理研讨会
2026.2.5-8，华北电力大学，北京

Tau半轻衰变相关的物理

Zhi-Hui Guo (郭志辉)

Hebei Normal University (河北师范大学)

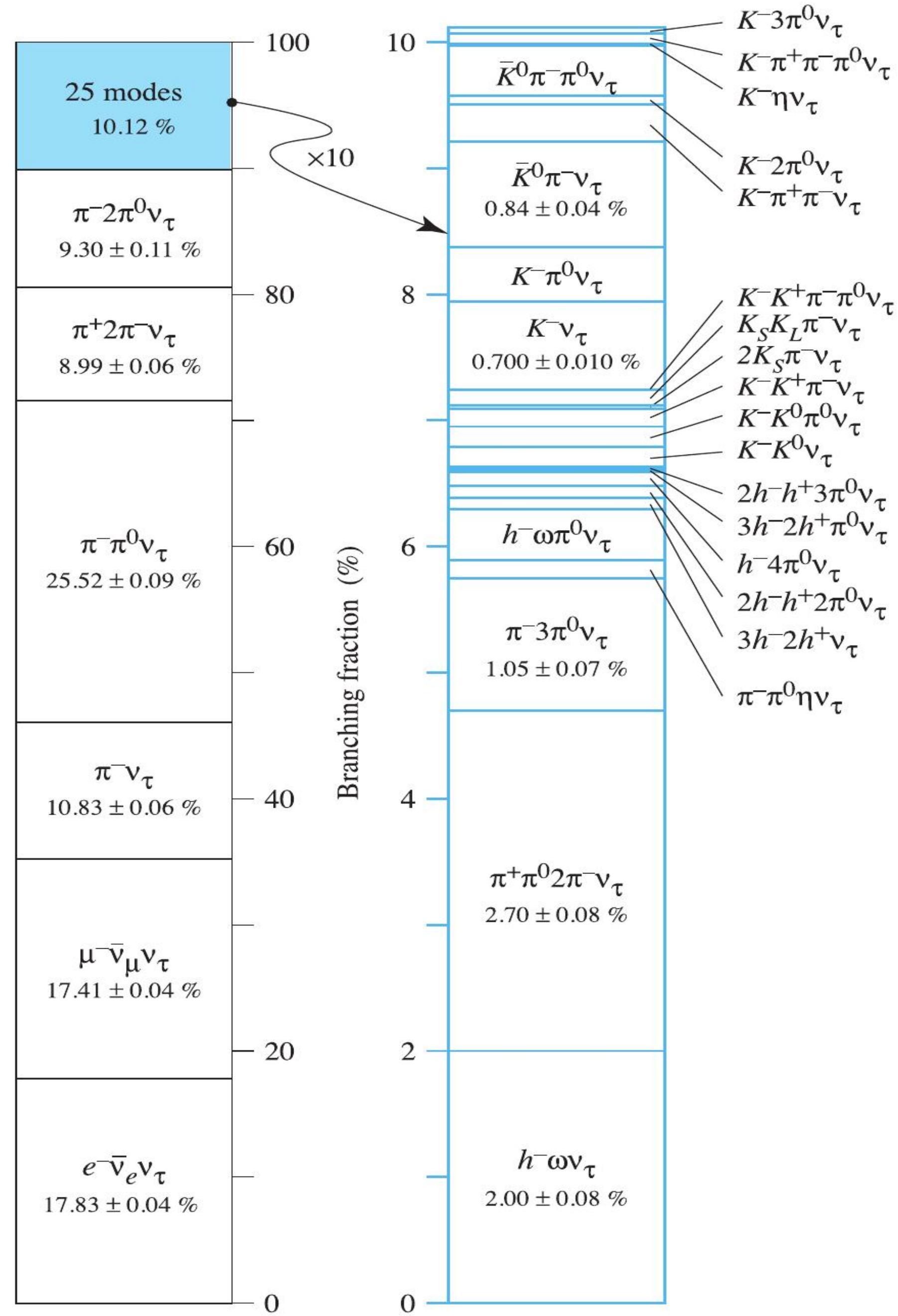
Tau衰变分支比概览

- ➢ $Br(\tau \to e v_\tau \overline{v_e})$: 17.8%

  $Br(\tau \to \mu v_\tau \overline{v_\mu})$: 17.4%

- ➢ $Br(\tau \to v+$Cabbibo allowed hadrons$)$ ~ 62%
- ➢ $Br(\tau \to v+$Cabbibo suppressed hadrons$)$: ~3%

- □ $Br(\tau \to v\pi\pi)$ ~ 25%, 单举衰变中分支比最大
- □ tau的衰变末态只有轻味强子，不涉及重味粒子 ($m_\tau < m_D$)
- □ 在重子数守恒的假设下，tau不能衰变至含有重子的末态($m_\tau < 2m_N$)

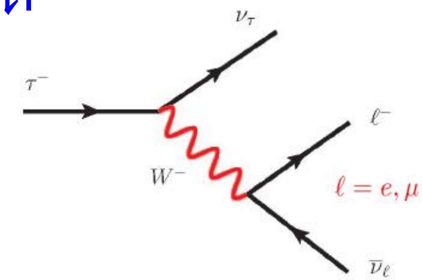


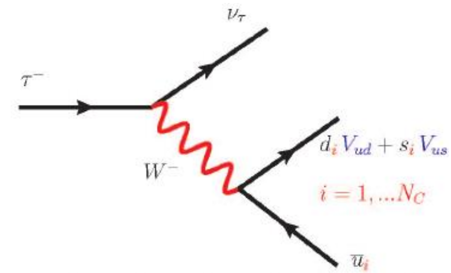
名词澄清：

- 单举(exclusive): 只包含某一个具体物理过程
- 遍举(inclusive): 包含所有可能的单举过程或者包含某一类单举过程

例如， Cabbibo允许的inclusive过程是指末态不含奇数个K介子的所有exclusive过程

基于SM的粗略分支比估计





universal coupling *g*
of W and fermions

V_{ud}=0.974， V_{us}=0.224

	粗略理论估计	实验值
➢ Br($\tau \to e \nu_\tau \overline{\nu_e}$) Br($\tau \to \mu \nu_\tau \overline{\nu_\mu}$)	$\frac{1}{2 + N_C\left(\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2\right)} \simeq 20\,\%$	17.8% 17.4%
➢ Br($\tau \to \nu$+Cabbibo allowed channels)	$\frac{N_C \lvert V_{ud}\rvert^2}{2 + N_C\left(\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2\right)} \simeq 57\%$	(62±4)%
➢ Br($\tau \to \nu$+Cabbibo suppressed channels)	$\frac{N_C \lvert V_{us}\rvert^2}{2 + N_C\left(\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2\right)} \simeq 3\%$	（2.6±0.7）%

❖ 简单计算成功的原因：tau的强衰变属于非常典型的inclusive过程，夸克-强子对偶很大程度上可以适用。

❖ 对于exclusive过程，因为QCD非微扰效应，其理论预言要复杂的多！

强衰变过程

tau的强衰变可以给我们提供什么信息？

- **inclusive衰变：（某类）所有的强子末态**

$$\tau^- \to \nu_\tau (\bar{u}d, \bar{u}s)$$

可以用来研究标准模型的基本参数：α_S, V_{us}, ...

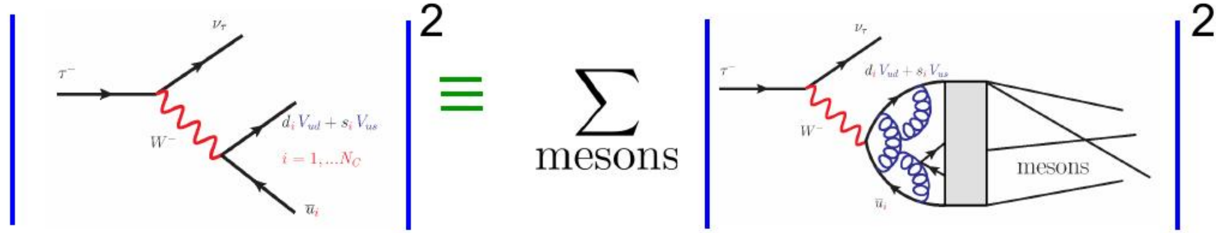
- **exclusive衰变：衰变至特定的强子末态**

$$\tau^- \to \nu_\tau (P,\ PP',\ P_1P_2P_3,\ \ldots)$$

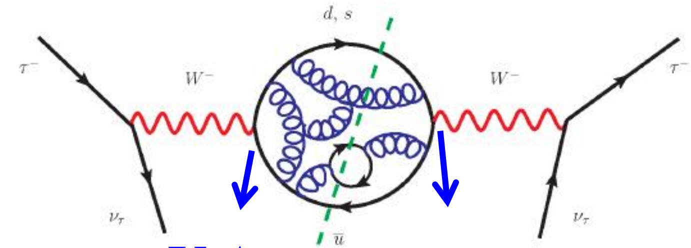
- 强作用形状因子，强子共振态，手征对称性，...
- CPV、轻子味道破坏、... ...

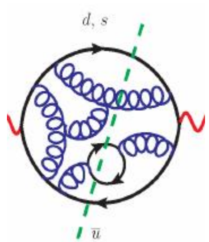
tau inclusive 强衰变

tau的
inclusive衰变

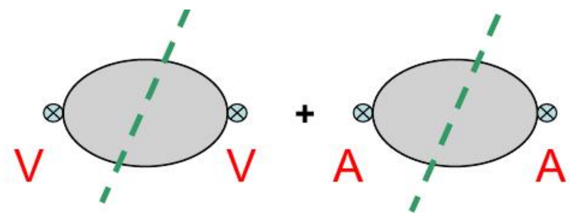


$$\Gamma\left(\tau \longrightarrow \nu_\tau \,\text{mesons}\right) \quad \propto$$



V  **A =0** **（强作用下宇称守恒）**

$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau \,\text{mesons}\right)}{\Gamma\left(\tau^- \to e^- \overline{\nu}_e \nu_\tau\right)} \quad \propto$$



S=0 S=1

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S} \simeq \frac{N_C}{2}|V_{ud}|^2 + \frac{N_C}{2}|V_{ud}|^2 + N_C|V_{us}|^2$$

Cabbibo allowed **Cabbibo suppressed**

$$R_\tau \simeq N_C$$

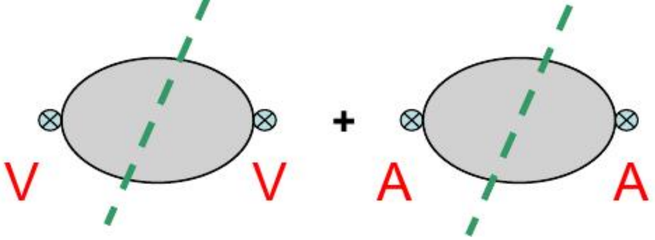
$$R_\tau^{\rm exp} = \frac{\sum_i \Gamma(\tau \to \nu_\tau h_i)}{\Gamma(\tau^- \to \nu_\tau e^- \overline{\nu}_e)} = 3.6355 \pm 0.0081$$

$$R_\tau^{\rm exp} = \frac{1 - \mathrm{B}_e - \mathrm{B}_\mu}{\mathrm{B}_e} = 3.6370 \pm 0.0075$$

QCD corrections amount to 20%!

Sensitive to α_s !

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau \,\mathrm{mesons})}{\Gamma(\tau^- \to e^- \overline{\nu}_e \nu_\tau)} \propto$$



两点关联函数 $V_{ij}^\mu = \bar{\psi}_j \gamma^\mu \psi_i \qquad A_{ij}^\mu = \bar{\psi}_j \gamma^\mu \gamma_5 \psi_i$

$$\Pi_{ij,J}^{\mu\nu}(q) \equiv i \int d^4x\; e^{iqx} \langle 0 | T[J_{ij}^\mu(x) J_{ij}^\nu(0)^\dagger] | 0 \rangle$$

$$= \left(-g^{\mu\nu} q^2 + q^\mu q^\nu\right) \Pi_{ij,J}^{(1)}(q^2) + q^\mu q^\nu\; \Pi_{ij,J}^{(0)}(q^2)$$

于是有：

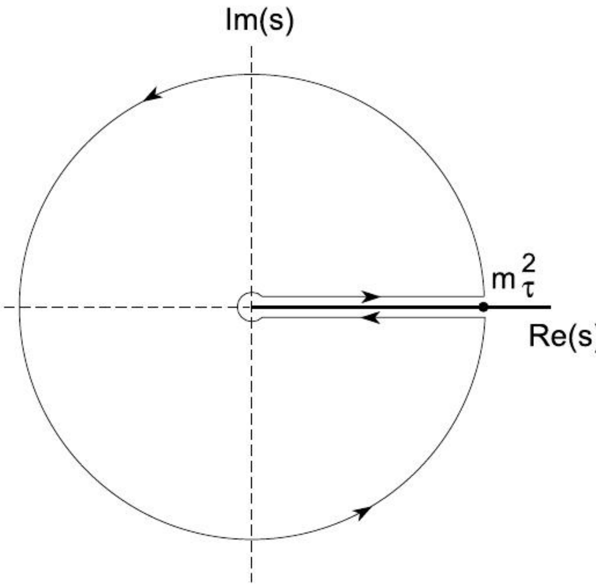
$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{M_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

谱函数 （两点关联函数的虚部）

说明：

- 谱函数ImΠ(s)实验可测：tau的inclusive衰变过程
- 谱函数ImΠ(s) 在 s~(0,m_τ^2) 区间内的理论计算完全涉及非微扰QCD，很难有可靠的计算
- 理论出路？

利用函数Π(s)的解析性质



- 柯西定理
- Π(s)在除去正实轴以外的其他地方解析
- *f*(s)为任一解析函数

$$\frac{1}{\pi}\int_0^{s_0} ds\, f(s)\,\mathrm{Im}\Pi(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, f(s)\,\Pi(s)$$

$$R_\tau = 12\pi\int_0^{M_\tau^2}\frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2\left[\left(1+2\frac{s}{M_\tau^2}\right)\mathrm{Im}\Pi^{(1)}(s)+\mathrm{Im}\Pi^{(0)}(s)\right]$$

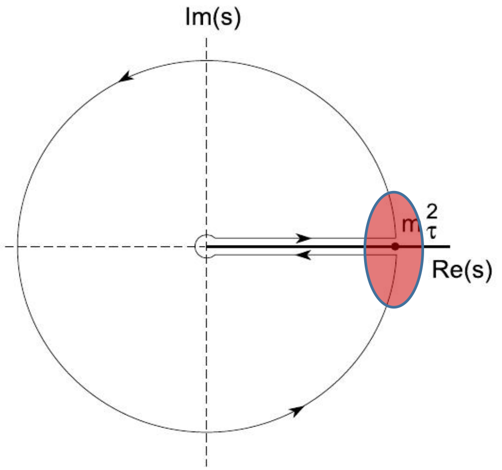
$$= 6\pi i\oint_{|s|=M_\tau^2}\frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2\left[\left(1+2\frac{s}{M_\tau^2}\right)\Pi^{(0+1)}(s)-\frac{2s}{M_\tau^2}\Pi^{(0)}(s)\right]$$

✓ 在|s|=m_τ^2的圆周(在能量复平面上)，利用算符乘积展开(operator product expansion, OPE)，可对Π(s)进行可靠的理论计算。

算符乘积展开(OPE)

$$\Pi^{(J)}(s) = \sum_{D=0,2,4,\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim \mathcal{O}=D} C_D^{(J)}(s,\mu) \langle \mathcal{O}_D(\mu) \rangle$$

- **D=0，QCD微扰部分（以α_s为参数进行展开）**
- **D>0, QCD非微扰部分（以各种凝聚量为展开）**
- **可能的Quark-hadron duality violation (DV) 效应**



$$R_\tau = N_C S_{EW} \left(1 + \delta_P + \delta_{NP}\right)$$

$S_{EW} = 1.0201\ (3)$; $\delta_{NP} = -0.0064 \pm 0.0013$

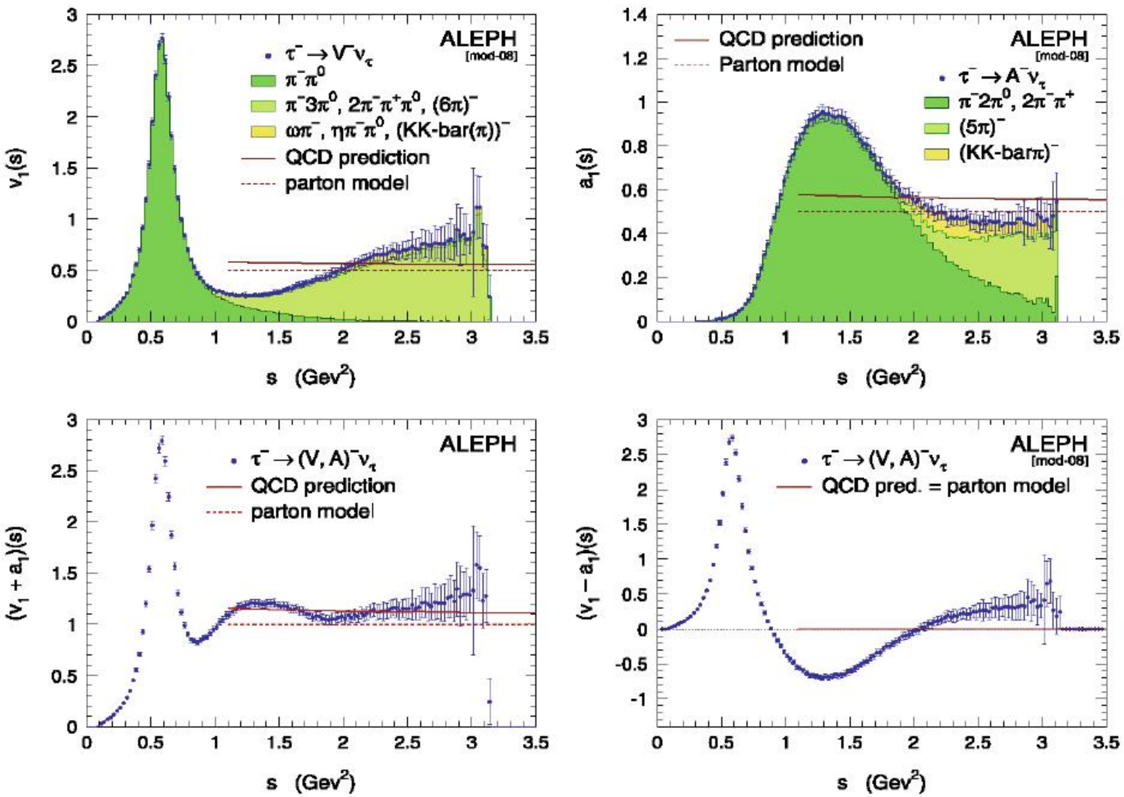
Marciano-Sirlin, Braaten-Li, Erler **Fitted from data (Davier et al)**

$\delta_P = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \ldots \approx 20\%$; $a_\tau \equiv \alpha_s(m_\tau)/\pi$

Baikov-Chetyrkin-Kühn

- **tau inclusive衰变的微扰修正非常重要，其对α_s依赖敏感，因此可以有效地确定α_s数值。**

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2 \left[\left(1+2\frac{s}{M_\tau^2}\right)\mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right] = N_C\, S_{EW}\,(1+\delta_P+\delta_{NP})$$



- **矢量谱函数(V)：**

$I^G J^P = 1^+ 1^-$

末态为偶数个π

- **轴矢谱函数(A)：**

$I^G J^P = 1^- 1^+$

末态为奇数个π

- **末态含K介子时：**

需要通过理论模型分别抽取V、A谱函数

$$(V-A)\big|_\chi \quad \propto \quad \text{non-perturbative} \qquad (m_u = m_d = m_s = 0)$$

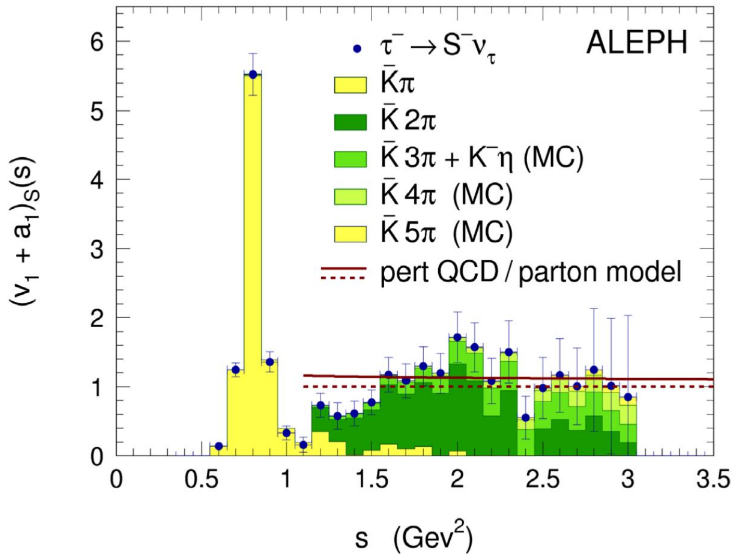
$$(V+A) \qquad \propto \quad \left(\text{perturbative} + \frac{1}{M_\tau^6}\,\text{non-perturbative}\right)$$

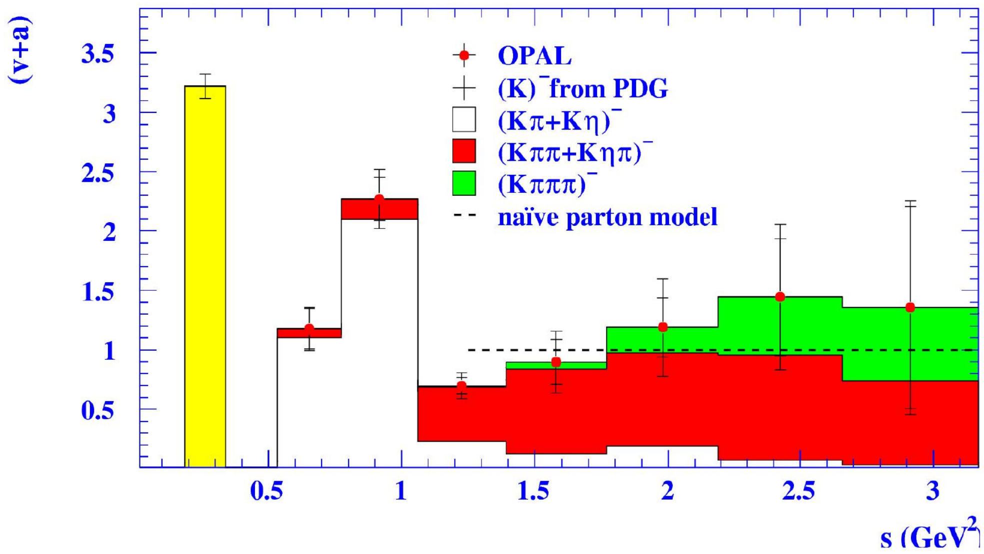
$\alpha_S(M_\tau)$

(微扰贡献大致20%，非微扰贡献大致在 0.5%)

- **Cabbibo压低（含奇数个K介子）的谱函数**

统计量低，实验误差大





➢ 这些谱函数可以用来研究： m_s和V_{us}

举例： V+A 型的R_τ

$$R_{\tau,V+A} = N_C\,|V_{ud}|^2\,S_{\rm EW}\,\{1+\delta_P+\delta_{NP}\}$$

$$= 6\pi i \oint_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2\left[\left(1+2\frac{s}{M_\tau^2}\right)\Pi^{(0+1)}(s)-\frac{2s}{M_\tau^2}\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) = \sum_{D=0,2,4,\dots}\frac{1}{(-s)^{D/2}}\sum_{\dim\mathcal{O}=D} C_D^{(J)}(s,\mu)\,\langle\mathcal{O}_D(\mu)\rangle$$

Perturbative contribution

$m_q = 0$

$$\delta_P = \sum_{n=1} K_n\,A^{(n)}(\alpha_S)$$

$$A^{(n)}(\alpha_S) \equiv \frac{1}{2\pi i}\oint_{|s|=M_\tau^2}\frac{ds}{s}\left(\frac{\alpha_S(-s)}{\pi}\right)^n\left(1-2\frac{s}{M_\tau^2}+2\frac{s^3}{M_\tau^6}-\frac{s^4}{M_\tau^8}\right)$$

K_n known up to $\mathcal{O}\left({\alpha_S}^4\right) \longrightarrow N_F = 3$

$$K_0 = K_1 = 1$$
$$K_2 = 1.63982$$
$$K_3^{\overline{\rm MS}} = 6.37101$$
$$K_4^{\overline{\rm MS}} = 49.07570$$

$$a_\tau \equiv \alpha_S(M_\tau)/\pi$$

Contour-improved perturbation theory (CIPT)
Using the exact solution for $\alpha_S(s)$ given by the RG β-function equation

$$\alpha_s(m_\tau^2)^{\rm CIPT} = 0.339\,^{+0.019}_{-0.017}$$

Fixed-order perturbation theory (FOPT)
Expansion of $A^{(n)}(\alpha_S)$ in powers of $\alpha_S(M_\tau^2)$

$$\alpha_s(m_\tau^2)^{\rm FOPT} = 0.319\,^{+0.017}_{-0.015}$$

$$\longrightarrow \alpha_s(m_\tau^2) = 0.329\,^{+0.020}_{-0.018}$$

[Pich, et al., PRD'16]

Non-perturbative contributions

$$\delta_{NP} = \frac{-1}{2\pi i}\oint_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+2\frac{s}{M_\tau^2}\right)\sum_{D\geq 2}\frac{1}{(-s)^{D/2}}C_D(s,\mu)\,\langle\mathcal{O}_D(\mu)\rangle$$

$$\delta_{NP}\Big|_{C_D=\text{constant}} \simeq \frac{1}{M_\tau^2}C_2\langle\mathcal{O}_2\rangle - \frac{3}{M_\tau^6}C_6\,\langle\mathcal{O}_6\rangle - \frac{2}{M_\tau^8}C_8\,\langle\mathcal{O}_8\rangle + \ldots$$

$$C_2\,\langle\mathcal{O}_2\rangle \propto \left[1+\frac{16}{3}\frac{\alpha_S(M_\tau)}{\pi}\right]\left(m_u^2(M_\tau)+m_d^2(M_\tau)\right)$$

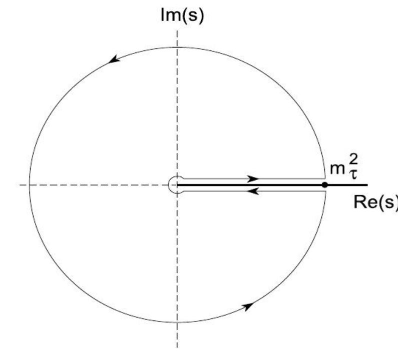
$$C_4\,\langle\mathcal{O}_4\rangle \propto \left(\frac{\alpha_S(M_\tau)}{\pi}\right)^2 \langle(\alpha_S/\pi)G_{\mu\nu}G^{\mu\nu}\rangle\,, \quad \langle m_u\overline{\psi}_u\psi_u + m_d\overline{\psi}_d\psi_d\rangle\,, \ldots.$$

$$C_6\,\langle\mathcal{O}_6\rangle \propto \frac{\alpha_S(M_\tau^2)}{\pi}\,\langle\overline{\psi}_u\Gamma\psi_d\overline{\psi}_d\Gamma\psi_u\rangle\,, \ldots.$$

$$C_8\,\langle\mathcal{O}_8\rangle \propto \langle(\alpha_S/\pi)G_{\mu\nu}G^{\mu\nu}\rangle^2\,, \ldots.$$

- **非微扰贡献大致在 0.5%， 微扰贡献大致20%**

另一个例子：V-A 型的R_τ



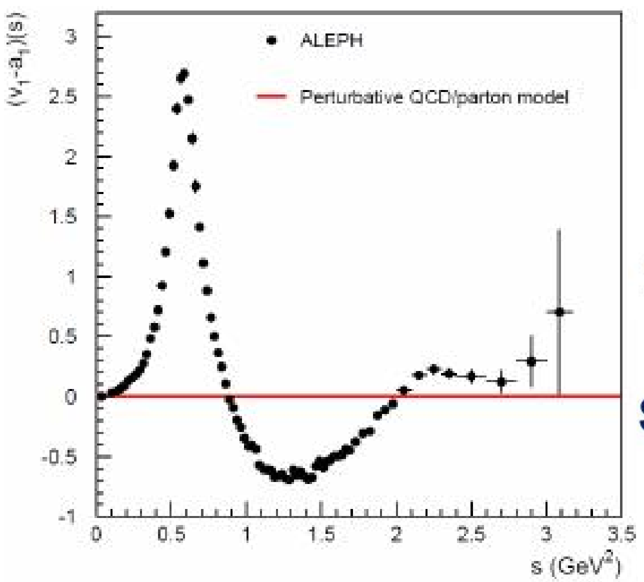
$$\Pi_{LR}(q) = \Pi_{VV-AA}(q) \equiv$$

$$i\int d^4x\, e^{-iqx}\langle 0|J_V(x)J_V^\dagger(0) - J_A(x)J_A^\dagger(0)|0\rangle\,,\ J_{V(A)} = \bar{d}\gamma_\mu(\gamma_5)u$$

- 在手征极限下，QCD的微扰贡献恒为零，只有非微扰部分才有贡献！

$$\int_{s_{\rm th}}^{s_0} ds\,\omega(s)\,\frac{1}{\pi}\,{\rm Im}\,\Pi(s) + \frac{1}{2\pi i}\oint_{|s|=s_0} ds\,\omega(s)\,\Pi(s) \;=\; 2\,f_\pi^2\,\omega(m_\pi^2) + {\rm Res}[\omega(s)\Pi(s), s=0]$$

$$\omega(s) = \frac{1}{s}\,,\quad \frac{1}{s^2}$$



$$\lim_{s\to\infty} s^2\,\Pi(s) = 0 \quad \rightarrow \quad \Pi^{\rm OPE}(s) = -\frac{O_6}{s^3} + \frac{O_8}{s^4} - \cdots$$

χPT (s→0): $\Pi(s) = \dfrac{2F^2}{s} - 8L_{10}^r(\mu^2) + \dfrac{1}{16\pi^2}\left(\dfrac{5}{3} - \ln\dfrac{-s}{\mu^2}\right) + 16C_{87}^r(\mu^2)\,\dfrac{s}{F^2} + \cdots$

Statistical analysis:

González-Pich-Rodríguez, 1602.06112

$$C_{87}^{\rm eff} = (8.40 \pm 0.18)\cdot 10^{-3}\,{\rm GeV}^{-2}$$
$$L_{10}^{\rm eff} = (-6.48 \pm 0.05)\cdot 10^{-3}\,.$$

$$\mathcal{O}_6 = (-3.6 \pm 0.7)\cdot 10^{-3}\,{\rm GeV}^6$$
$$\mathcal{O}_8 = (-1.0 \pm 0.4)\cdot 10^{-2}\,{\rm GeV}^8$$

再来一个例子：利用Cabbibo压低的谱函数抽取V_{us}

$$R_\tau \equiv \frac{\Gamma(\tau\to\nu_\tau \text{hadrons})}{\Gamma(\tau\to\nu_\tau e^-\bar{\nu}_e)} \qquad R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$$\mathrm{Im}\,\Pi_V^{(1)}(s) = \frac{N_c|V_{ud}|^2}{12\pi}, \ldots \to \frac{R_{\tau,V}}{|V_{ud}|^2} = \frac{R_{\tau,A}}{|V_{ud}|^2} = \frac{R_{\tau,S}}{2|V_{us}|^2} = \frac{N_c}{2}$$

$$\frac{R^{exp}_{\tau,V}}{|V_{ud}|^2} = 1.877 \pm 0.009 \qquad \frac{R^{exp}_{\tau,A}}{|V_{ud}|^2} = 1.784 \pm 0.011 \qquad \frac{R^{exp}_{\tau,S}}{2|V_{us}|^2} = 1.614 \pm 0.028$$

ALEPH, HFLAV

In the $SU(3)_V$ limit $\frac{R_{\tau,V+A}}{|V_{ud}|^2} - \frac{R_{\tau,s}}{|V_{us}|^2} = 0$

$SU(3)_V$ breaking by $m_s \neq m_d$

$$\Rightarrow \delta R_\tau \equiv \frac{R_{\tau,V+A}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2} \propto \frac{m_s^2 - m_d^2}{m_\tau^2}$$

（理论上可算的物理量）

$$|V_{us}| = \left(\frac{R_{\tau,S}}{\frac{R_{\tau,V+A}}{|V_{ud}|^2} - \delta R_{\tau,\text{th}}} \right)^{1/2}$$

$$\mathrm{Br}\left(\tau^- \to \nu_\tau \bar{u} s\right) = (2.908 \pm 0.048)\%$$

$$\mathrm{Br}\left(\tau^- \to \nu_\tau \bar{u} d\right) = (61.83 \pm 0.10)\%$$

$$\mathrm{Br}\left(\tau^- \to \nu_\tau \bar{\nu}_e e^-\right)_{\text{univ}} = (17.812 \pm 0.022)\%$$

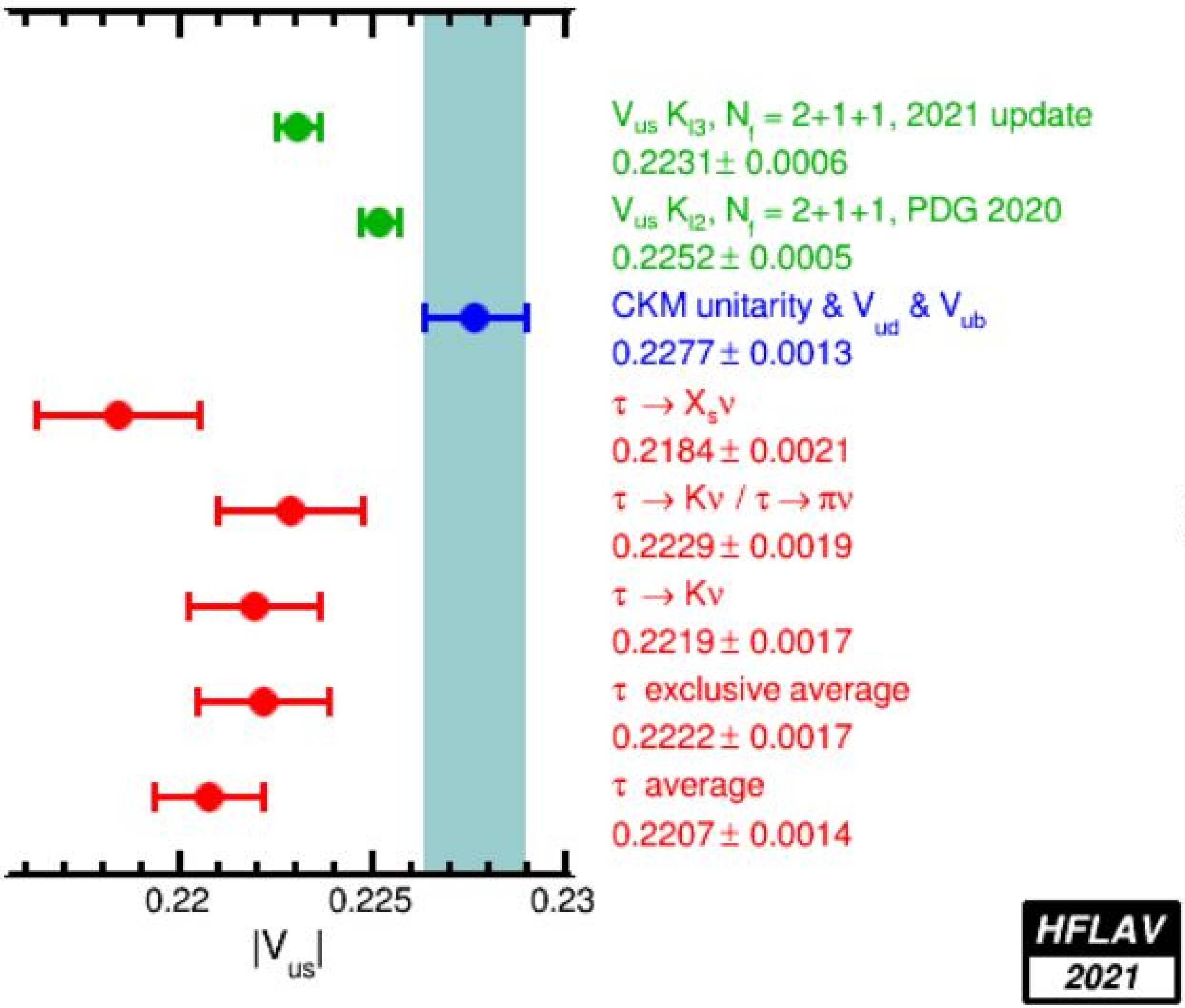
$$V_{ud} = 0.97373 \pm 0.00031$$

$$\Rightarrow |V_{us}| = 0.2184 \pm 0.0018_{\text{exp}} \pm 0.0011_{\text{th}}$$

$R_{\tau,S}$贡献大部分实验误差

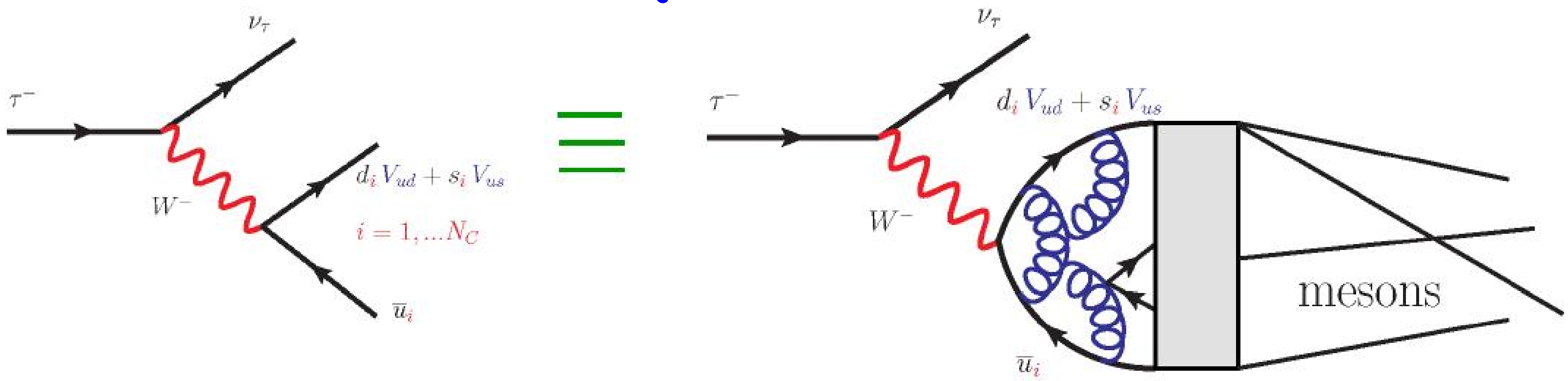
Cabbibo anomaly: CKM矩阵第一行的幺正性问题

$$\Delta_{CKM}=|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2-1=-1.48(53)\times10^{-3}$$



tau exclusive 强衰变

Tau exclusive decays



$$\mathcal{M}\left(\tau \to \nu_\tau \mathrm{H}\right) = \frac{G_F}{\sqrt{2}} V_{\mathrm{CKM}}\, \overline{u}_{\nu_\tau} \gamma^\mu \left(1 - \gamma_5\right) u_\tau \,\langle H | \left(V_\mu - A_\mu\right) e^{i\,\mathrm{L_{QCD}}} |\Omega_{\mathrm{H}}\rangle$$

Hadronic V-A currents

Chiral EFT is the low energy realization of QCD:

$$e^{i\mathcal{Z}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}q \mathcal{D}\bar{q} \mathcal{D}G_\mu \; e^{i \int d^4x \mathcal{L}_{QCD}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}u \; e^{i \int d^4x \mathcal{L}_{EFT}(v_\mu, a_\mu, s, p)}$$

$$\mathcal{L}^{QCD} = \mathcal{L}_0^{QCD} + \bar{q}\gamma^\mu (v_\mu + a_\mu \gamma_5) q - \bar{q}(s - i\gamma_5 p) q$$

v_μ, a_μ, s, p are the external source fields .

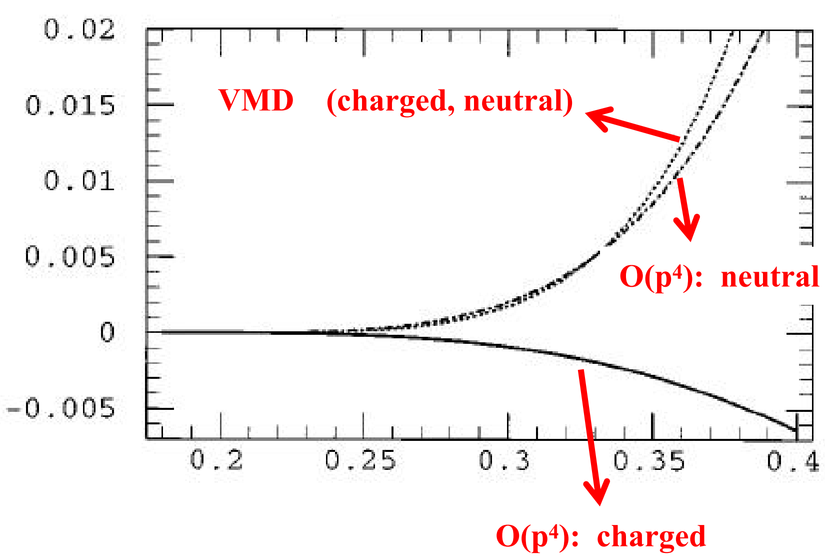
Chiral symmetry is RELEVANT to tau decays

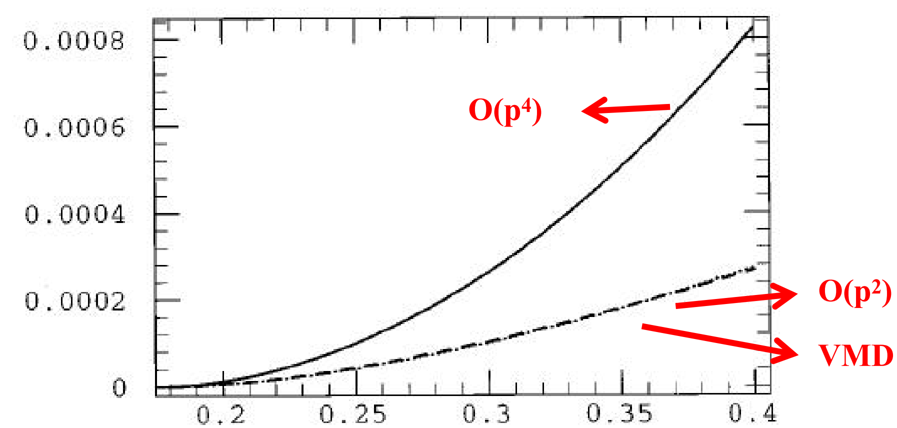
Example: $\tau\rightarrow \nu_\tau \pi\pi\pi$ transition amplitudes in the low energy region
VMD models do not automatically respect chiral symmetry.

$$J_\alpha = -i\frac{2\sqrt{2}}{3f_\pi}\mathrm{BW}_a(Q^2)(B_\rho(s_2)V_{1\alpha}+B_\rho(s_1)V_{2\alpha})$$

[Kuhn,Santamaria, ZPC'90]

W_D structure function | **W_{SA} structure function (neutral channel)**





[Colangelo, et al., PRD'96]

- **Resonance chiral theory** **implements the constraint of chiral symmetry from the very beginning in the construction of the Lagrangians.**

Resonance chiral theory (RχT)

Chiral group: $G = SU(3)_L \times SU(3)_R,\ H = SU(3)_V,\ u(\phi) = G/H$

Resonances : $R \stackrel{G}{\Longrightarrow} h\, R\, h^\dagger, \quad h \in H$

pNGB and external sources : $X = u_\mu,\ \chi_\pm,\ f_\pm^{\mu\nu},\ h_{\mu\nu}$

Operators	P	C	h.c.	chiral order
u	$u^\dagger$	u^T	$u^\dagger$	1
Γ_μ	Γ^μ	$-{\Gamma_\mu}^T$	$-\Gamma_\mu$	p
u_μ	$-u^\mu$	u_μ^T	u_μ	p
$\chi_\pm$	$\pm\chi_\pm$	$\chi_\pm^T$	$\pm\chi_\pm$	p^2
$f_{\mu\nu\,\pm}$	$\pm f_\pm^{\mu\nu}$	$\mp f_{\mu\nu\,\pm}^T$	$f_{\mu\nu\,\pm}$	p^2
$h_{\mu\nu}$	$-h^{\mu\nu}$	$h_{\mu\nu}^T$	$h_{\mu\nu}$	p^2

Operators	P	C	h.c.
$V_{\mu\nu}$	$V^{\mu\nu}$	$-V_{\mu\nu}^T$	$V_{\mu\nu}$
$A_{\mu\nu}$	$-A^{\mu\nu}$	$A_{\mu\nu}^T$	$A_{\mu\nu}$
S	S	S^T	S
P	$-P$	P^T	P

Minimal RχT Lagrangian [Ecker, *et al.*, '89]

$$\mathcal{L}_{2V} = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$

$$\mathcal{L}_{2A} = \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle ,$$

$$\mathcal{L}_{2S} = c_d\langle S u_\mu u^\mu \rangle + c_m\langle S\chi_+ \rangle ,$$

$$\mathcal{L}_{2P} = i d_m\langle P\chi_- \rangle .$$

Operators beyond minimal
[Cirigliano, *et al.*, '04]:

$$\mathcal{L}_{VAP} = \lambda_1^{VA}\langle [V^{\mu\nu}, A_{\mu\nu}]\chi_-\rangle + \dots ,$$

[Ruiz-Femenia, Pich and Portolés, '03]

$$\mathcal{L}_{VVP} = d_1 \varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \dots ,$$

$$\mathcal{L}_{VJP} = \frac{c_1}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \dots ,$$

$$d_4 \varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, V^{\rho\alpha}\} u_\alpha\rangle$$

QCD dynamics in RχT

- Low energy QCD: implemented from the construction of RχT
- Intermediate energy: explicit resonance states
- High energy information: to match the same physical objects in RχT and QCD, $\langle J(x_n)\cdot\cdot J(0)\rangle^{\mathrm{R}\chi\mathrm{T}} = \langle J(x_n)\cdot\cdot J(0)\rangle^{\mathrm{QCD}}$.

  For example: $\pi\pi$ vector form factor

$$\left[\mathcal{F}^{v}_{\pi\pi}(q^2)\right]^{\mathrm{R}\chi\mathrm{T}} = 1 + \frac{F_V G_V}{F^2}\frac{q^2}{M_V^2 - q^2}\,,$$
$$\left[\mathcal{F}^{v}_{\pi\pi}(q^2)\right]^{\mathrm{QCD}} \to 0, \qquad \text{for } q^2 \to \infty$$

  This leads to

$$[\mathcal{F}^{v}_{\pi\pi}(q^2)]^{\mathrm{R}\chi\mathrm{T}} = [\mathcal{F}^{v}_{\pi\pi}(q^2)]^{\mathrm{QCD}} \Longrightarrow F_V G_V = F^2$$

例1： $\tau \to P + \nu_\tau$

辐射修正项

$$\Gamma(\tau \to H\nu_\tau) = \frac{m_\tau^3 f_H^2 G_F^2 |V_{uD}|^2}{16\pi}\left(1 - \frac{m_H^2}{m_\tau^2}\right)^2 (1 + \delta_{RC}^{(H)})$$

$\delta_{\tau\pi} = (-0.24 \pm 0.56)\%$
$\delta_{\tau K} = (-0.15 \pm 0.57)\%$

轻子普适性检验：

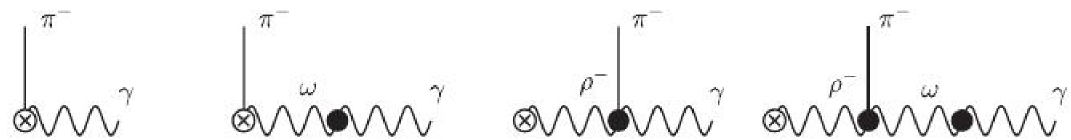
[Arroyo-Urena, et al., PRD'21]

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1 - m_P^2/M_\tau^2)^2}{(1 - m_\mu^2/m_P^2)^2}(1 + \delta R_{\tau/P})$$

$\delta R_{\tau/\pi} = (0.18 \pm 0.57)\%$
$\delta R_{\tau/K} = (0.97 \pm 0.58)\%$

Predictions to $\tau \to \pi/K\, \gamma\nu_\tau$ [ZHG,Roig,PRD'10]

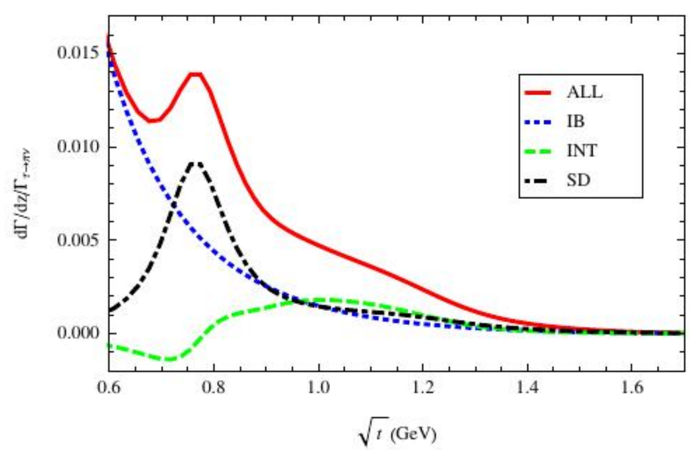
vector currents:



axial-vector currents:

	E_γ=50 MeV	E_γ=400MeV
IB	13.09×10^{-3}	1.48×10^{-3}
$IB-V$	0.02×10^{-3}	0.04×10^{-3}
$IB-A$	0.34×10^{-3}	0.29×10^{-3}
VV	0.99×10^{-3}	0.73×10^{-3}
VA	~ 0	0.02×10^{-3}
AA	0.15×10^{-3}	0.14×10^{-3}
ALL	14.59×10^{-3}	2.70×10^{-3}

in units of $\mathrm{Br}(\Gamma_{\tau\to\pi\nu_\tau}) \sim 11\%$



例2： $\tau \to PP' + \nu_\tau$

$$\langle P_1P_2|\bar{D}\gamma^\mu u|0\rangle = \left[(p_2-p_1)^\mu - \frac{\Delta_{P_2P_1}}{s}q^\mu\right] F_+^{P_1P_2}(s) + \frac{\Delta_{Du}}{s} q^\mu \widehat{F}_0^{P_1P_2}(s)$$

(vector FF) (scalar FF)

$$\Delta_{P_2P_1} = m_{P_2}^2 - m_{P_1}^2\,, \qquad \Delta_{Du} = B_0(m_D - m_u)\,, \qquad q_\mu = (p_1+p_2)_\mu, \qquad s = q^2.$$

- **Invariant-mass distribution of P_1P_2**

$$\frac{d\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{48\pi^3 s} S_{\rm EW}\,|V_{uD}|^2 \left(1-\frac{s}{M_\tau^2}\right)\left\{\left(1+\frac{2s}{M_\tau^2}\right) q_{P_1P_2}^3(s)\left|F_+^{P_1P_2}(s)\right|^2 + \frac{3\Delta_{Du}^2}{4s} q_{P_1P_2}(s)\left|\widehat{F}_0^{P_1P_2}(s)\right|^2\right\}$$

- **Forward-Backward (FB) asymmetry distribution**

$$A_{FB}(s) = \frac{\int_0^1 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha} - \int_{-1}^0 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha}}{\int_0^1 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha} + \int_{-1}^0 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha}} = \frac{\Delta_{Du}\, q_{P_1P_2}(s)\, \Re\left[F_+^{P_1P_2}(s)\, \widehat{F}_0^{P_1P_2*}(s)\right]}{\frac{2\sqrt{s}}{3}\left(1+\frac{2s}{M_\tau^2}\right) q_{P_1P_2}^2(s)\left|F_+^{P_1P_2}(s)\right|^2 + \frac{\Delta_{Du}^2}{2\sqrt{s}}\left|\widehat{F}_0^{P_1P_2}(s)\right|^2}$$

α：angle between the momenta of P_1 and τ in the P_1P_2 rest frame

$\tau \to \pi^-\pi^0\nu_\tau$：$\Delta_{PP'}\to 0$ (同位旋破坏项)，标量F_0项可忽略，矢量F_+绝对主导!

$\tau \to K\pi\nu_\tau$：$\Delta_{PP'}\neq 0$，标量形状因子F_0项以及矢量形状因子F_+项都有贡献!

Calculation of P_1P_2 form factors

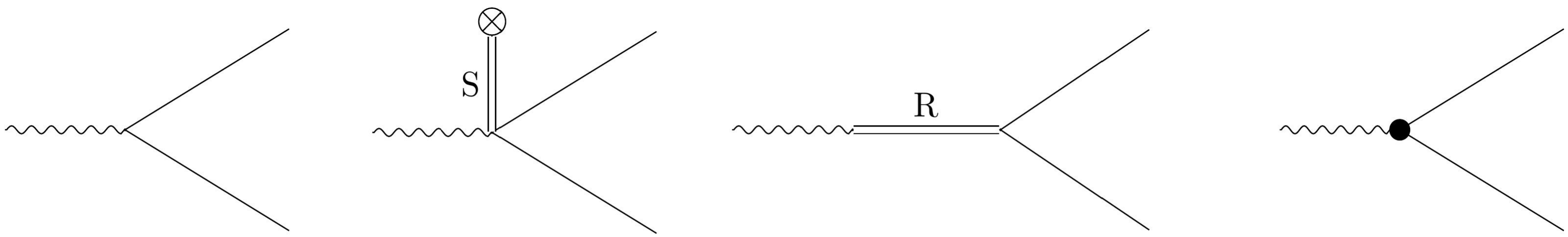
[Hao, Duan, ZHG, FOP'26]

$$\mathcal{L}^{\rm LO} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{12}M_0^2 X^2$$

$$\mathcal{L}_\Lambda^{\rm NLO,U(3)} = -\frac{F^2\Lambda_1}{12}D^\mu X D_\mu X - \frac{F^2\Lambda_2}{12}X\langle\chi_-\rangle$$

$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu}u^\mu u^\nu\rangle\,,$$

$$\mathcal{L}_S = c_d\langle S u_\mu u^\mu\rangle + c_m\langle S\chi_+\rangle\,,$$



(a) (b) (c) (d)

(a) (b)

Mixing relations of π^0-η-η'-a (axion)

[Gao,ZHG,Oller,Zhou, JHEP'23]
[Gao,Hao,ZHG,Oller,Zhou, EPJC'25]

$$\begin{pmatrix}\hat\pi^0\\ \hat\eta\\ \hat\eta'\\ \hat a\end{pmatrix} = \begin{pmatrix} 1+z_{11} & c_\theta(-v_{12}+z_{12})+s_\theta(-v_{13}+z_{13}) & -s_\theta(-v_{12}+z_{12})+c_\theta(-v_{13}+z_{13}) & -v_{14}+z_{14}\\ v_{12}+z_{21} & c_\theta(1+z_{22})+s_\theta(z_{23}-v_{23}) & -s_\theta(1+z_{22})+c_\theta(z_{23}-v_{23}) & -v_{24}+z_{24}\\ v_{13}+z_{31} & c_\theta(z_{32}+v_{23})+s_\theta(1+z_{33}) & -s_\theta(z_{32}+v_{23})+c_\theta(1+z_{33}) & -v_{34}+z_{34}\\ v_{41}+z_{41} & c_\theta(v_{42}+z_{42})+s_\theta(v_{43}+z_{43}) & -s_\theta(v_{42}+z_{42})+c_\theta(v_{43}+z_{43}) & 1+v_{44}+z_{44}\end{pmatrix}\begin{pmatrix}\pi^0\\ \eta_8\\ \eta_0\\ a\end{pmatrix}$$

v_{ij}: LO terms

z_{ij}: NLO terms (L_5, L_8, Λ_1, Λ_2) or [(L_5, L_8)~ (c_dc_m, c_mc_m)/M^2_S , Λ_1, Λ_2]

Some explicit expressions for Form Factors

- **VFF-ππ**

$$F_+^{\pi^-\pi^0}(s) = -\frac{\sqrt{2}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s)$$

$$G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) = \frac{G_V F_V s + F^2(M_\rho^2 - s)}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} - \frac{G_V' F_V' s}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} - \frac{G_V'' F_V'' s}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)},$$

$$\Gamma_\rho(s) = \frac{M_\rho s}{96\pi F_\pi^2}\left[\sigma_{\pi\pi}^3(s) + \frac{1}{2}\sigma_{KK}^3(s)\right], \quad \Gamma_{\rho',\rho''}(s) = \Gamma_{\rho',\rho''}\frac{s}{M_{\rho',\rho''}^2}\frac{\sigma_{\pi\pi}^3(s)}{\sigma_{\pi\pi}^3(M_{\rho',\rho''}^2)}, \quad \sigma_{P_1P_2}(s) = \frac{2q_{P_1P_2}(s)}{\sqrt{s}}\theta[s - (m_{P_1} + m_{P_2})^2]$$

- **VFF-πη/πη’/πa**

$$F_+^{\pi^-\eta}(s) = -\frac{\sqrt{2}v_{12}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) - \sqrt{2}\left(y_{12} - v_{13}y_{23}^{(0)}\right),$$

$$F_+^{\pi^-\eta'}(s) = -\frac{\sqrt{2}v_{13}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) - \sqrt{2}\left(y_{13} + v_{12}y_{23}^{(0)}\right),$$

$$F_+^{\pi^- a}(s) = -\frac{\sqrt{2}v_{41}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) - \sqrt{2}\left(y_{14} + v_{12}\,y_{24}^{(0)} + v_{13}\,y_{34}^{(0)}\right)$$

- **SFF-πη**

$$F_0^{\pi^-\eta}(s) = \sqrt{\frac{2}{3}}(c_\theta - \sqrt{2}s_\theta) + \frac{1}{\sqrt{3}}(\Lambda_1 - 2\Lambda_2)s_\theta - \frac{1}{\sqrt{3}}y_{23}(2c_\theta + \sqrt{2}s_\theta) + 4\sqrt{\frac{2}{3}}\frac{c_\theta - \sqrt{2}s_\theta}{F^2}\Bigg\{$$
$$\left[\frac{c_m(c_m - c_d)2m_\pi^2 + c_m c_d(s + m_\pi^2 - m_\eta^2)}{M_{a_0}^2 - s - iM_{a_0}\Gamma_{a_0}(s)} - \frac{2c_m(c_m - c_d)(2m_K^2 - m_\pi^2)}{M_S^2}\right]$$
$$+ \left[c_{m,d}, M_{a_0}, \Gamma_{a_0}, M_S \to c'_{m,d}, M_{a_0'}, \Gamma_{a_0'}, M_{S'}\right]\Bigg\},$$

Fits to experimental spectra and BRs

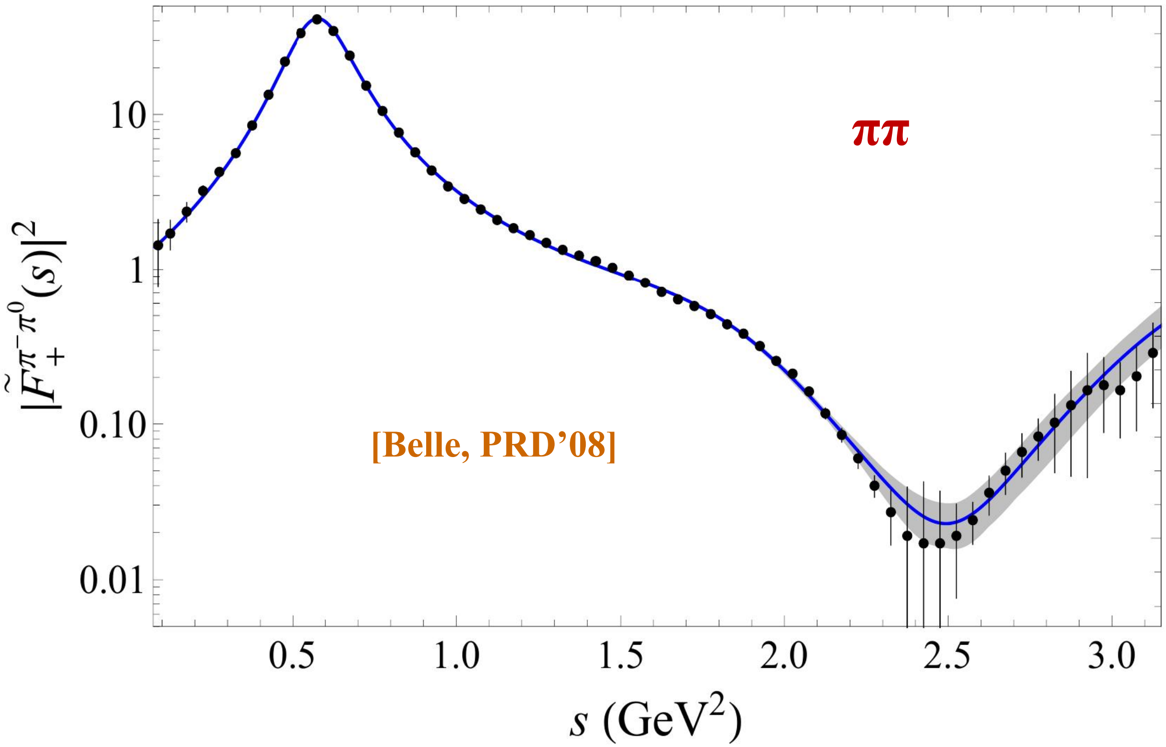
$G_V\, F_V(\mathrm{GeV}^2)\times 10^3$	$10.26^{+0.01}_{-0.01}$	$G'_V\, F'_V(\mathrm{GeV}^2)\times 10^3$	$0.64^{+0.03}_{-0.02}$
$G''_V\, F''_V(\mathrm{GeV}^2)\times 10^3$	$-0.94^{+0.05}_{-0.05}$	$M_\rho(\mathrm{GeV})$	$0.7738^{+0.0003}_{-0.0003}$
$M_{\rho'}(\mathrm{GeV})$	$1.409^{+0.004}_{-0.004}$	$\Gamma_{\rho'}(\mathrm{GeV})$	$0.338^{+0.012}_{-0.010}$
$M_{\rho''}(\mathrm{GeV})$	$1.842^{+0.012}_{-0.013}$	$\Gamma_{\rho''}(\mathrm{GeV})$	$0.268^{+0.025}_{-0.026}$
$c'_m(\mathrm{GeV})$	$0.053^{+0.007}_{-0.009}$	$M_{K^*}(\mathrm{GeV})$	$0.8956^{+0.0002}_{-0.0002}$
$\Gamma_{K^*}(\mathrm{GeV})$	$0.0477^{+0.0005}_{-0.0005}$	$M_{K^{*\prime}}(\mathrm{GeV})$	$1.339^{+0.009}_{-0.009}$
$\overline{B}_{K_S\pi^-}\times 10^3$	$3.98^{+0.04}_{-0.04}$	$\overline{B}_{K^-\eta}\times 10^4$	$1.34^{+0.04}_{-0.04}$
$\chi^2/\mathrm{d.o.f}$	$271.5/(182-14)=1.61$		

$$c_m c_d + c'_m c'_d = \frac{F^2}{4}$$

c_m= 27 MeV, c_d=15 MeV

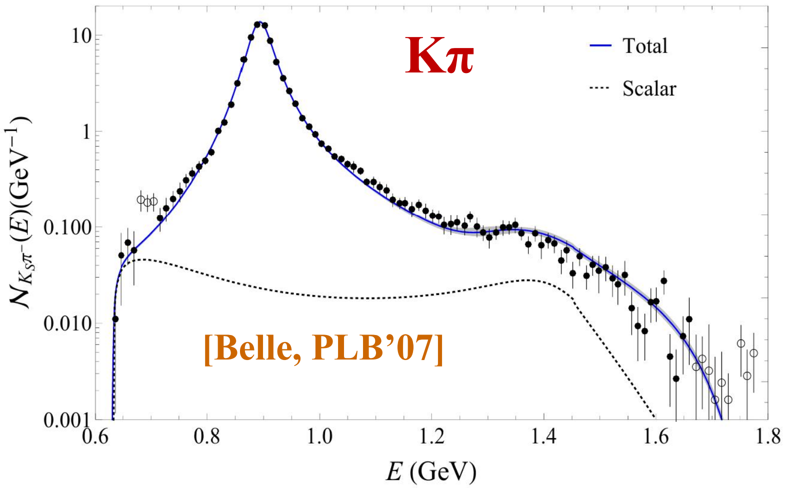
[ZHG,Oller, PRD'11]

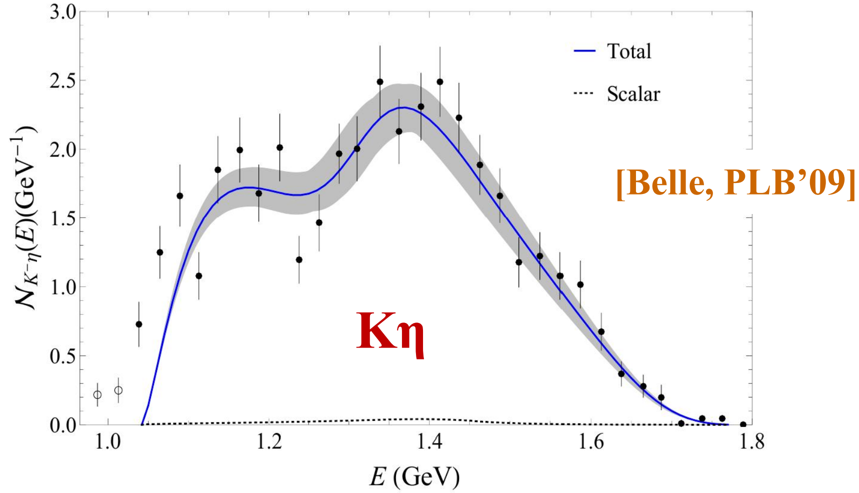
Parameters for a_0(980)/a_0(1450)/K_0*(700)//K_0*(1430) are fixed to their pole positions .



- **Crucial inputs to address muon g-2**
- **Most precise spectra is from Belle;**

  but most precise BR is from ALEPH: 25.47(13)%
- **Coherent precise measurements of both spectra and BR from one Exp would be invaluable!**

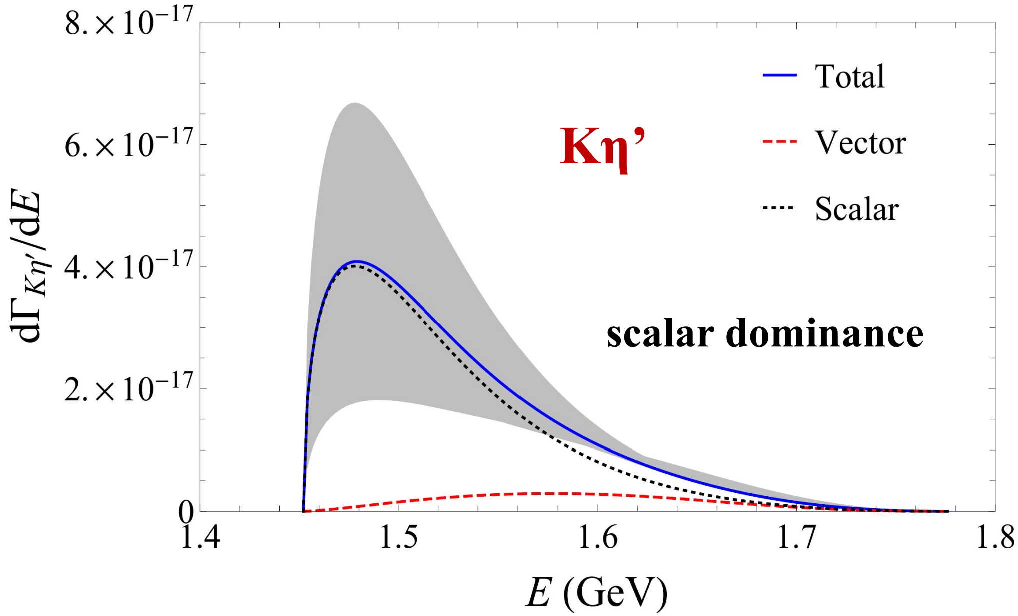




F_+^{KP}: K*, K*(1410) , K*(1680)

F_0^{KP}: κ, $K^*_0(1430)$

prediction to Kη'



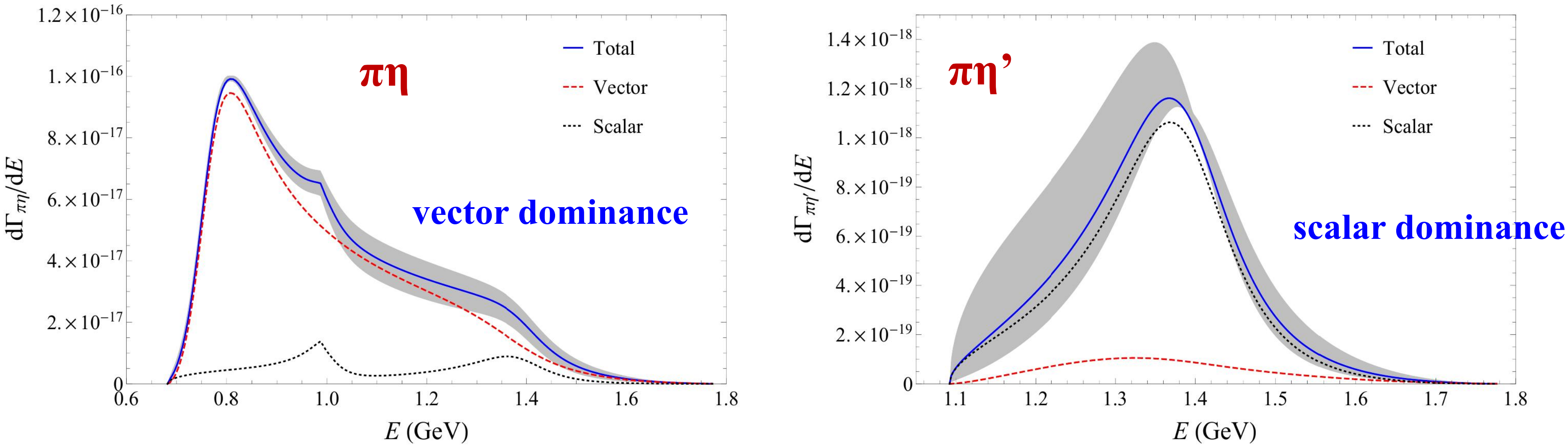
$BR(K^-\eta')^{Theo} = (2.0\pm1.0)\times10^{-6}$

$BR(K^-\eta')^{Exp,\ BaBar} < 2.4\times10^{-6}$

Predictions to $\tau \to \pi^- \eta^{(\prime)} \nu_\tau$ (Cabibbo allowed): second-class currents

Processes driven by second-class currents are suppressed by isospin breaking:

$$\langle \pi^- P | \bar{d}\gamma^\mu u | 0 \rangle = \left[(p_P - p_\pi)^\mu - \frac{\Delta_{P\pi}}{s} q^\mu \right] F_+^{\pi^- P}(s) + \frac{\Delta_{du}^{\text{Phy}}}{s} q^\mu F_0^{\pi^- P}(s)$$

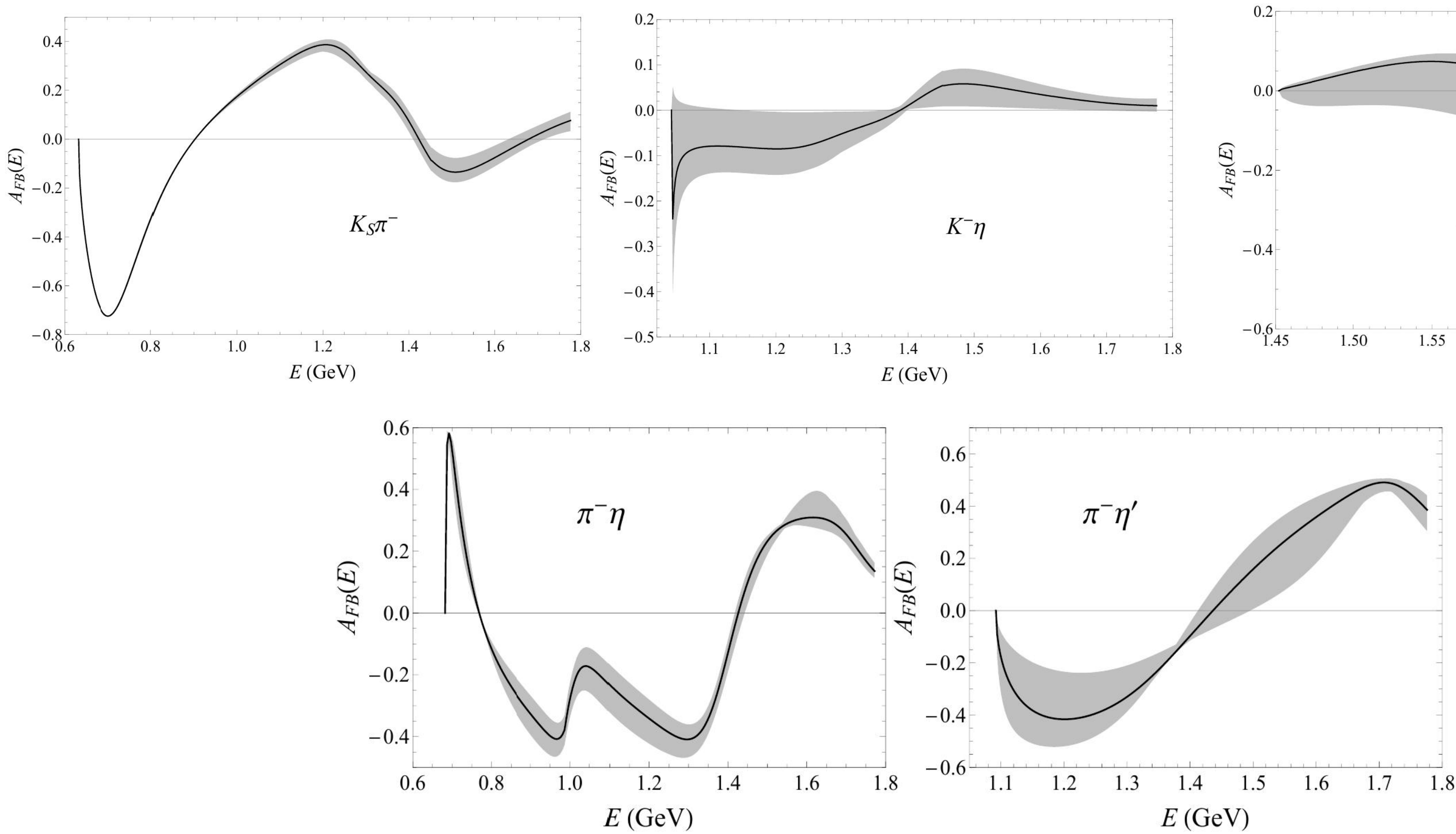


Channel	Total	Vector	Scalar	Exp Limits
$\tau^- \to \pi^- \eta \nu_\tau$ ($\times 10^5$)	$1.63^{+0.14}_{-0.14}$	$1.43^{+0.18}_{-0.21}$	$0.20^{+0.07}_{-0.04}$	< 9.9 (BaBar) [69] < 7.3 (Belle) [70]
$\tau^- \to \pi^- \eta' \nu_\tau$ ($\times 10^7$)	$1.17^{+0.36}_{-0.07}$	$0.14^{+0.09}_{-0.08}$	$1.03^{+0.44}_{-0.16}$	< 40 (BaBar) [71]

[Hao, Duan, ZHG, FOP'26]

Predictions to Forward-Backward asymmetries

$$\frac{\Delta_{Du}\, q_{P_1P_2}(s)\, \Re\left[F_+^{P_1P_2}(s)\, \widehat{F}_0^{P_1P_2*}(s)\right]}{\frac{2\sqrt{s}}{3}\left(1+\frac{2s}{M_\tau^2}\right) q_{P_1P_2}^2(s)\left|F_+^{P_1P_2}(s)\right|^2 + \frac{\Delta_{Du}^2}{2\sqrt{s}}\left|\widehat{F}_0^{P_1P_2}(s)\right|^2}$$

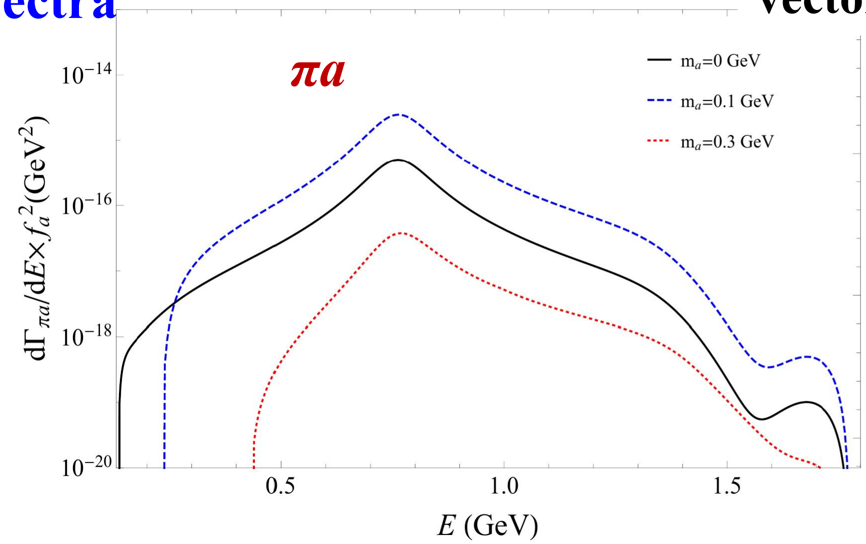


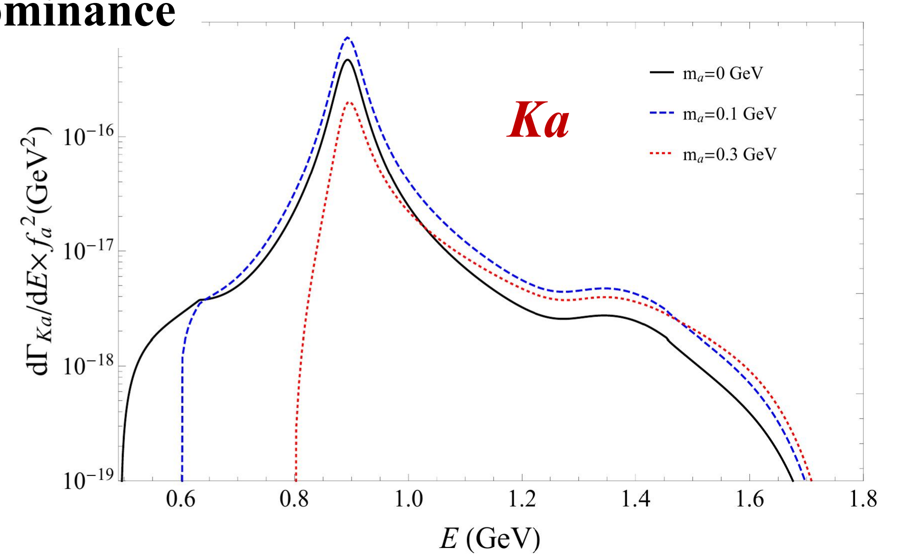
- **Measurement on A_{FB} can determine the curcial inputs for CPV**

Predictions to ALP-meson production in tau decays

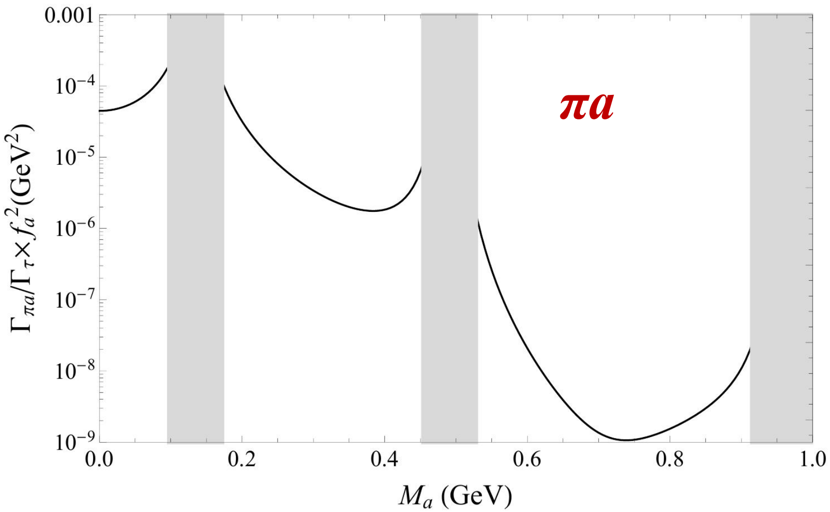
Spectra

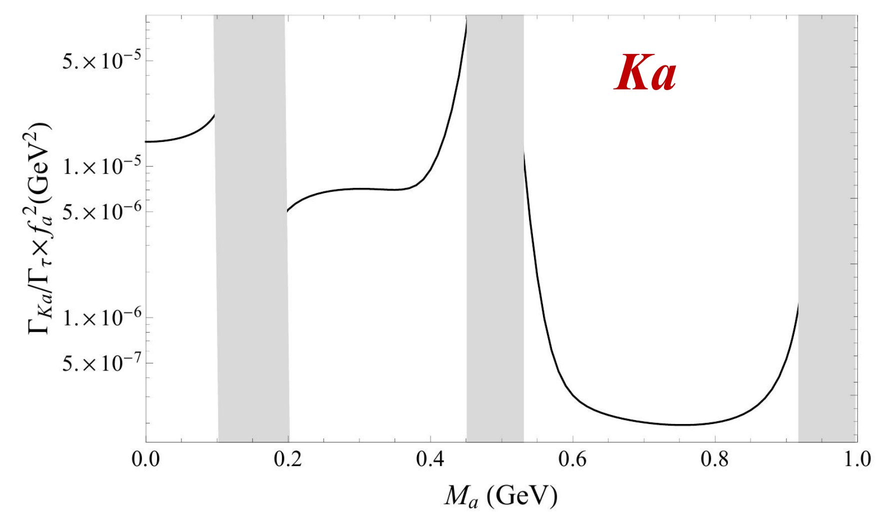
vector dominance





BRs





- **Hadronic resonances enhance the decay widths involving ALPs by around one order of magnitudes, with respect to the results from leading-order χPT.**

例3： $\tau \to P_1 P_2 P_3 + \nu_\tau$

$$\langle P_1^- P_2^- P_3^+ | (V_\mu - A_\mu)\, e^{i L_{\rm QCD}}) | \Omega_h \rangle =$$

$$Q = p_1 + p_2 + p_3, \quad s = (p_2 + p_3)^2, \quad t = (p_1 + p_3)^2$$

$$\left(g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} \right) \left[F_1(Q^2, s, t)\,(p_1 - p_3)^\nu + F_2(Q^2, s, t)\,(p_2 - p_3)^\nu \right]$$
$$+ F_3(Q^2, s, t)\, Q_\mu + i F_4(Q^2, s, t)\, \varepsilon_{\mu\alpha\beta\gamma}\, p_3^\alpha p_2^\beta p_1^\gamma$$

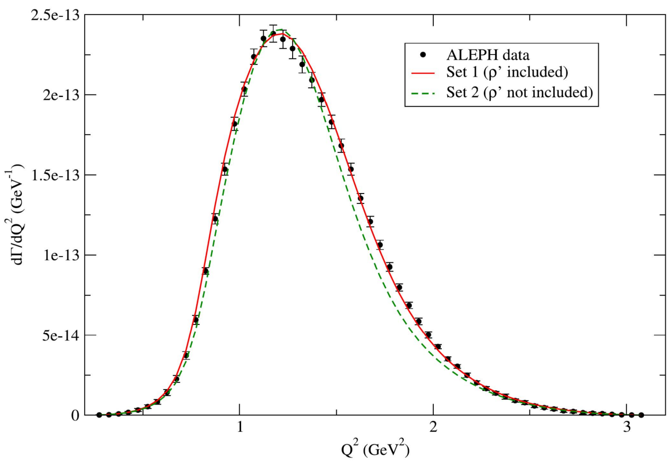
$\tau \to \pi\pi\pi\nu_\tau$
$F_2(Q^2, s, t) = F_1(Q^2, t, s)$
Bose symmetry, Axial-Vector only

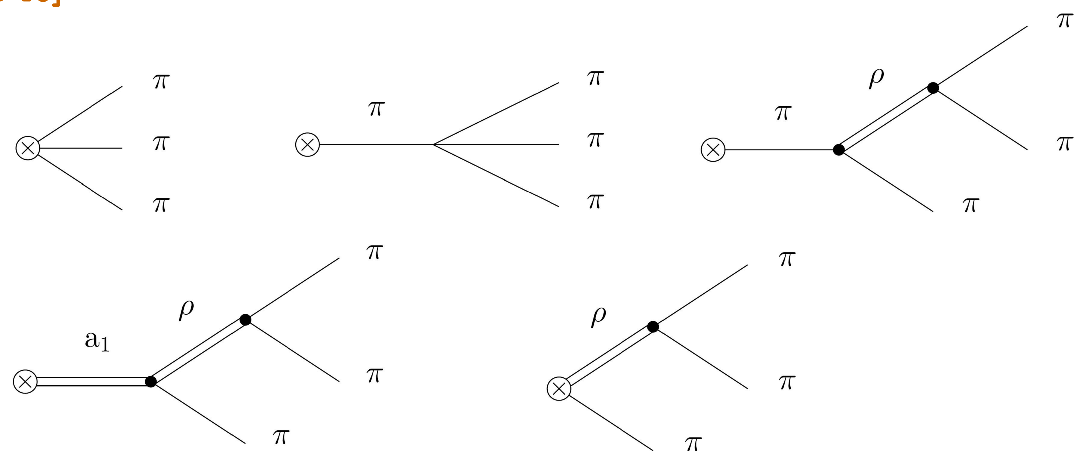
$\tau \to K K \pi \nu_\tau$
Vector and Axial-Vector

F_3 is suppressed by m_π/Q; F_4 is suppressed by IB.

$\tau^- \to \pi^- \pi^- \pi^+ \nu_\tau$

[Gomez Dumm, et al., PLB'10]

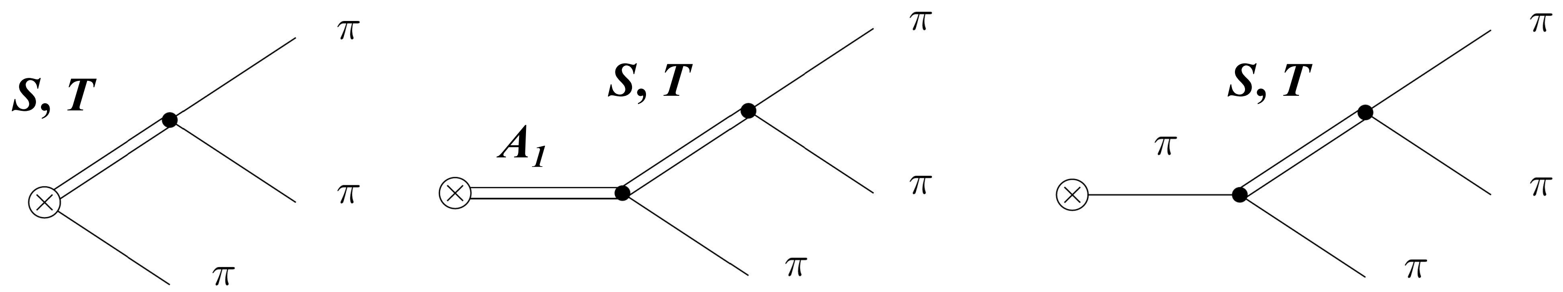




- **To describe the ALEPH data, it seems good enough to include vector and axial-vector resonances in chiral EFT.**

➤ Additional contributions from scalar and tensor resonances and extra axial-vector resonance ?

[CLEO, PRD'99] [Nugent (BaBar), NPBSP'13] [Nugent, et al., PRD'13] [Sanz-Cillero, et al., JHEP'17] [Rabusov, et al., 2405.19264] ......

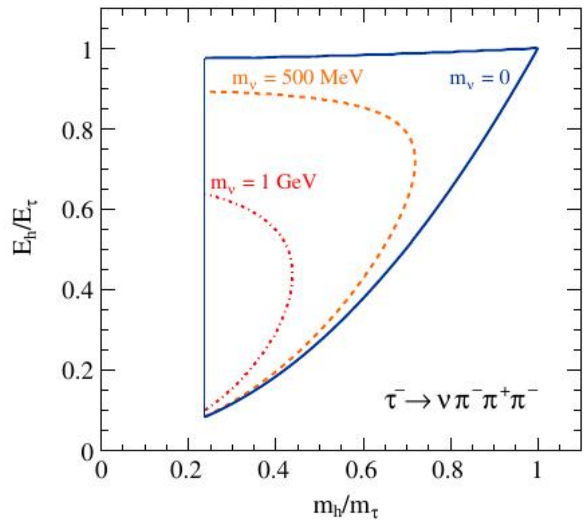


S: $f_0(500)$、 $f_0(980)$、 $f_0(1370)$; T: $f_2(1270)$; A_1: $a_1(1420)$

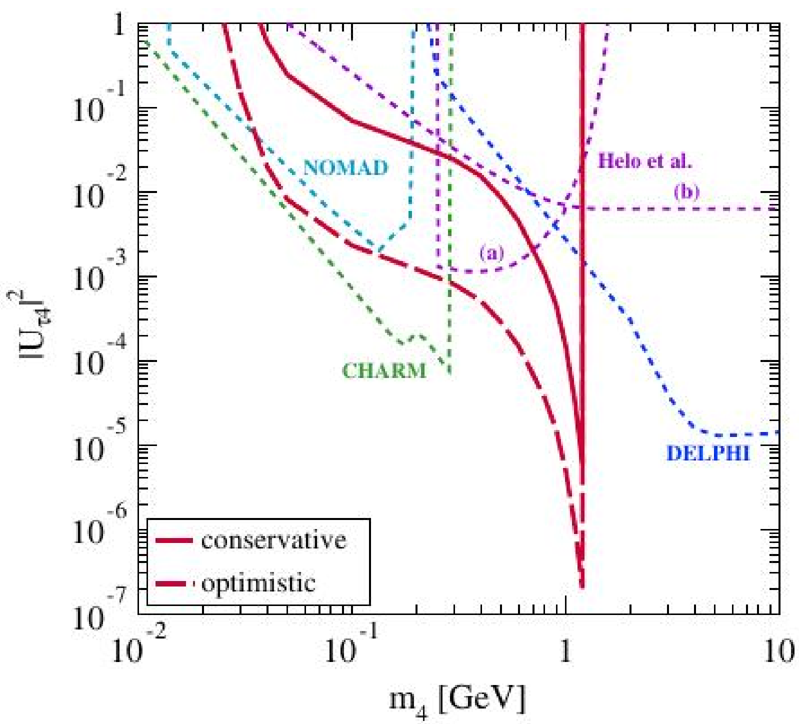
- It is still under debate about the effects of scalar and tensor resonances.
- Measurements of the $\pi\pi$ line shapes could be very helpful to address this issue.
- It can be crucial to do partial-wave analyses.

Proposal to search for massive neutrino in tau decay

$\tau \to \pi\pi\pi\nu_4$ [Kobach, Dobbs, PRD'15]



$$\frac{d\Gamma_{\rm tot}(\tau^- \to \nu h^-)}{dm_h dE_h} = (1 - |U_{\tau 4}|^2)\frac{d\Gamma(\tau^- \to \nu h^-)}{dm_h dE_h}\Big|_{m_\nu = 0} + |U_{\tau 4}|^2 \frac{d\Gamma(\tau^- \to \nu h^-)}{dm_h dE_h}\Big|_{m_\nu = m_4}$$



Strong interaction of the $\pi\pi\pi$ [dominated by $a_1(1260)$] system will greatly affect the final results!

总结

Tau衰变为粒子物理提供了一个非常广阔的平台:

- ✓ **电弱物理的精确检验:**

  V_{CKM}, 轻子普适性, $(g\text{-}2)_{\mu}$, ...

- ✓ **强相互作用相关的物理:**

  强作用基本耦合常数α_s, 强子共振态, 手征对称性, 形状因子, ...

- ✓ **可能的新物理现象:**

  超出标准模型的GeV以下轻粒子, 轻子味道破坏(LFV), CPV, ...

谢谢大家!