

粲介子半轻衰变的格点QCD研究

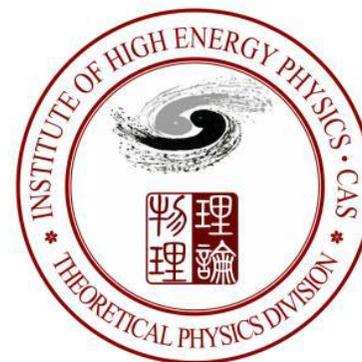
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BESIII实验物理研讨会

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内容提纲

- 格点QCD
- 粲介子半轻衰变
 - $D \rightarrow \pi/Kl\nu$
 - $D_s \rightarrow \phi l\nu$

T. Shen, ..., ZL et al., arXiv:2503.01219, PoS LAT2024 (2025) 294

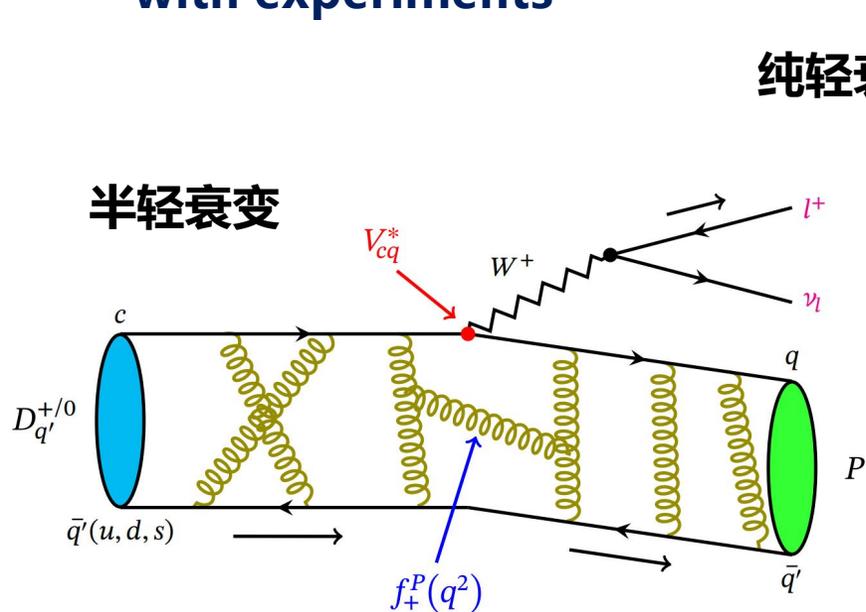
G. Fan, Y. Meng, C. Liu, ZL et al., arXiv:2510.14478, submitted to JHEP
FLAG review 2024, arXiv:2411.04268

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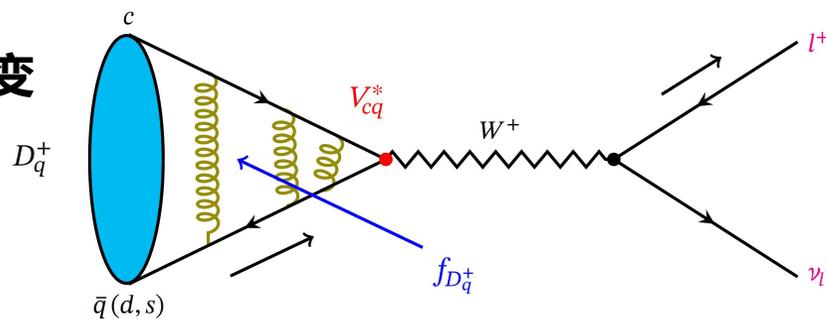
粲介子含轻衰变与LQCD

- LQCD can calculate form factors and meson decay constants appearing in weak decays of hadrons
- Combined with experiments, they can give us CKM matrix elements
- Test the SM (is the CKM matrix unitary?)
- Or use V_{ab} from elsewhere to compare QCD/SM results with experiments

V_{ud}	V_{us}	V_{ub}
$\pi \rightarrow \ell \nu$	$K \rightarrow \ell \nu$	$B \rightarrow \pi \ell \nu$
	$K \rightarrow \pi \ell \nu$	
V_{cd}	V_{cs}	V_{cb}
$D \rightarrow \ell \nu$	$D_s \rightarrow \ell \nu$	$B \rightarrow D \ell \nu$
$D \rightarrow \pi \ell \nu$	$D \rightarrow K \ell \nu$	$B \rightarrow D^* \ell \nu$
V_{td}	V_{ts}	V_{tb}
$B_d \leftrightarrow \bar{B}_d$	$B_s \leftrightarrow \bar{B}_s$	



纯轻衰变



CKM矩阵元

LQCD计算

例如:

$$\Gamma(P \rightarrow \ell \nu) = \frac{G_F^2 |V_{q_1 q_2}|^2 f_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2}{8\pi}$$

实验测量

格点QCD (1973, Wilson; 1979, Creutz) 用数值模拟研究QCD的非微扰性质

• 4维闵氏时空 \rightarrow 4维欧氏空间 ($\tau = it$)

$$\langle O \rangle = \frac{\int DA_\mu D\bar{\psi} D\psi O[A, \bar{\psi}, \psi] e^{-\int \mathcal{L}_{QCD} d^4x}}{\int DA_\mu D\bar{\psi} D\psi e^{-\int \mathcal{L}_{QCD} d^4x}}, \quad \mathcal{L}_{QCD} = \bar{\psi} M[A] \psi + \mathcal{L}_G$$

$$M = \gamma \cdot D + m_q$$

$$\langle O \rangle = \frac{\int DU_\mu O[U, M^{-1}[U]] \text{Det}[M[U]] e^{-S_G}}{\int DU_\mu \text{Det}[M[U]] e^{-S_G}} \sim \frac{\int dx f(x) \rho(x)}{\int dx \rho(x)} \quad \rightarrow \quad \sim \frac{1}{N} \sum_{n=1}^N f(x_n)$$

离散的 x_n 按 $\rho(x_n)$ 分布

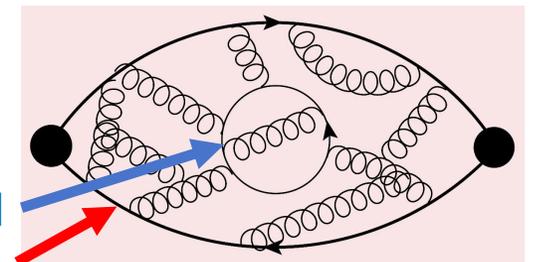
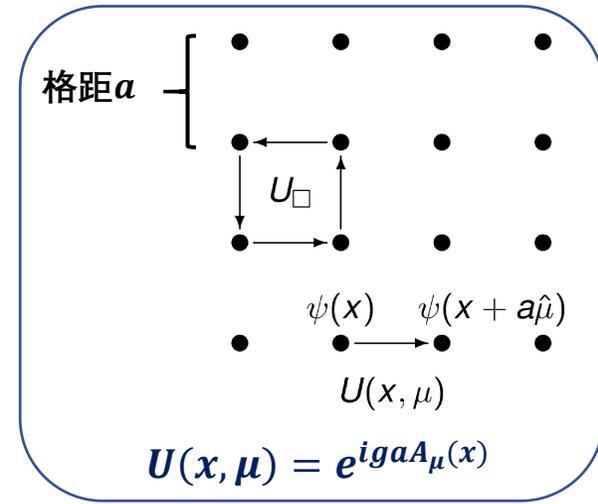
上式为带权重 $\text{Det}[M[U]] e^{-S_G}$ 的平均，类似Boltzmann系综平均

- 在有限体积4维超立方格子上，自由度个数可数，路径积分具有良好定义
- 巨大高维积分，无法直接计算；用重点抽样按权重分布产生 U (组态)

• 路径积分变为对组态的统计平均： $\langle O \rangle = \frac{1}{N} \sum_{i=1}^N O_i$

N 有限，统计误差 $\sim 1/\sqrt{N}$

输入若干实验测量值，确定自由参数：
格距 a 和夸克质量 m_q 。计算其他结果



介子两点关联函数

强子矩阵元的格点计算

$$C(t) = \sum_{\vec{x}} \langle \Omega | O(\vec{x}, t) O^\dagger(0) | \Omega \rangle \xrightarrow{t \rightarrow \infty} |\langle \Omega | O | P \rangle|^2 e^{-m_P t} \equiv A e^{-m_P t}$$

- 强子衰变常数从两点函数抽取, 例如 $O = \bar{q} \gamma_0 \gamma_5 c$

$$\langle 0 | \bar{q}(0) \gamma_\mu \gamma_5 c(0) | P(p) \rangle = i f_P p_\mu \quad q = d, s$$

- 结合两点和三点函数可抽取辐射/半轻过程强子矩阵元 (形状因子) :

$$C_3(\vec{p}, \vec{p}', T, t) = \sum_{\vec{z}} \sum_{\vec{y}} \langle 0 | O_P(\vec{z}, T) J(\vec{y}, t) O_D^\dagger(\vec{x}, 0) | 0 \rangle e^{-i\vec{p} \cdot \vec{z}} e^{i\vec{q} \cdot \vec{y}}$$

$$\xrightarrow[T \rightarrow \infty]{t \rightarrow \infty, (T-t) \rightarrow \infty} \langle 0 | O_P | P \rangle \langle P | J | D \rangle \langle D | O_D^\dagger | 0 \rangle e^{-m_D t} e^{-m_P (T-t)}$$

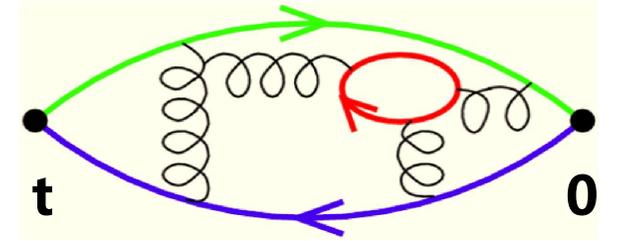
- 复合算符重正化常数

- 由于离散效应, 格子上的局域(轴)矢量流 $\bar{q} \gamma_\mu c$ ($\bar{q} \gamma_\mu \gamma_5 c$) 需要归一化常数 $Z_{V,A}$
- 标量及张量流算符随能标跑动, $Z_{S,T}$

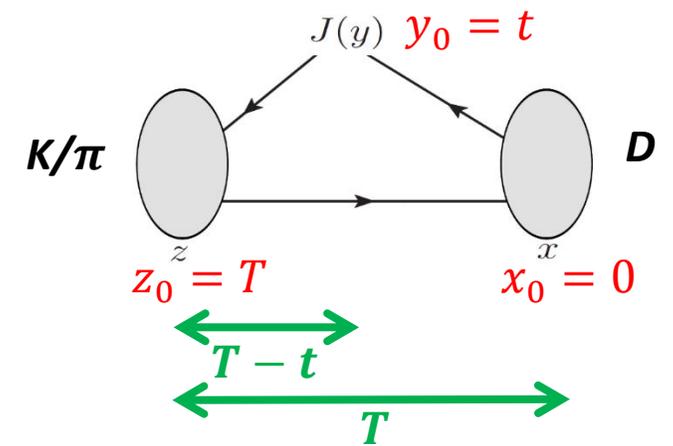
- 使用手征格点费米子有时可避免重正化常数的计算 ($Z_S Z_m = 1$)

- PCAC: $(m_q + m_c) \langle 0 | \bar{q}(0) \gamma_5 c(0) | P(p) \rangle = f_P m_{PS}^2$

- PCVC: $\langle K | S | D \rangle = f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{m_c - m_s}$



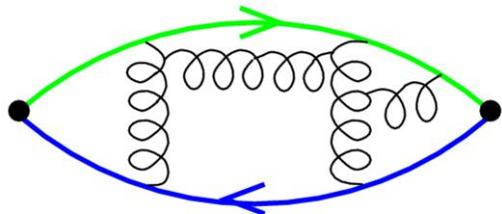
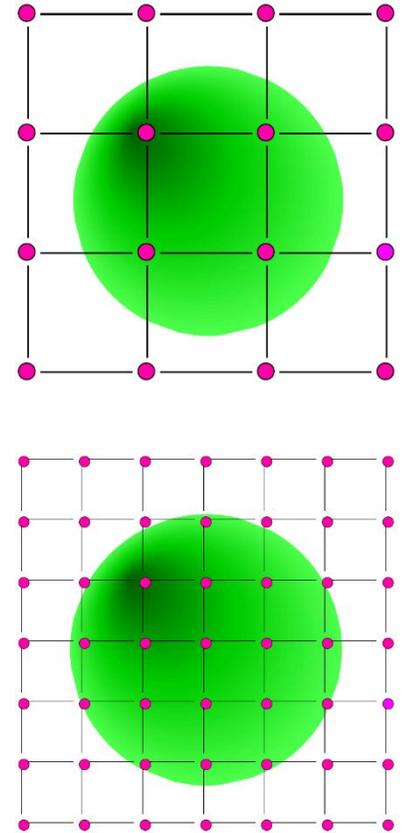
两点关联函数



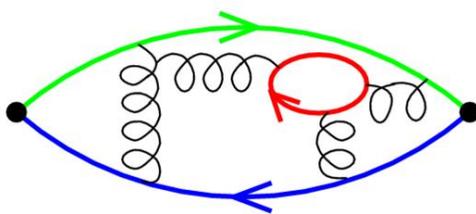
三点关联函数

格点QCD

- 系统误差来源
 - 非零格距 a 引入离散误差, 改进作用量, 取连续极限 $a \rightarrow 0$
 - 多数情况下 $m_\pi > 135$ MeV, 手征外推到物理质量点
 - 有限体积效应 $\propto \exp(-m_\pi L)$ (pion是质量最轻的色单态), 格子边长 L 需远大于 π 介子康普顿波长 $1/m_\pi$ ($m_\pi L > 5$ 时, $\exp(-m_\pi L) < 1\%$)
- 多个不同的 a 、 L 、 m_π 下作计算, 量化和控制系统误差
- 其他误差来源: 重正化常数的误差, 激发态污染,
- 已有少量 $m_\pi^{(\text{sea})} = 135$ MeV 组态样本
- 目前 $a \gtrsim 0.04$ fm (a 越小, 同样物理体积需要格子数越多, 计算量越大)
- 海夸克味道数 $N_f = 2 + 1 + 1$ (l, s, c), $2 + 1$ (l, s), 2 (简并的 u 和 d), 0 (淬火近似)



(A) Quenched QCD: quark loops neglected



(B) Full QCD

粲介子半轻衰变

- $D \rightarrow \pi l \nu$, $D \rightarrow K l \nu$ 可用于确定 $|V_{cd}|$ 和 $|V_{cs}|$

$$\frac{d\Gamma(D \rightarrow K l \nu)}{dq^2} = (\text{known}) |\mathbf{p}_K|^3 |V_{cs}|^2 |f_+^{D \rightarrow K}(q^2)|^2$$

- 非微扰输入量: 形状因子 $f_{+/0}(q^2)$

$$\langle K | V^\mu | D \rangle = f_+(q^2) \left(p_D^\mu + p_K^\mu - \frac{m_D^2 - m_K^2}{q^2} q^\mu \right) + f_0(q^2) \frac{m_D^2 - m_K^2}{q^2} q^\mu$$

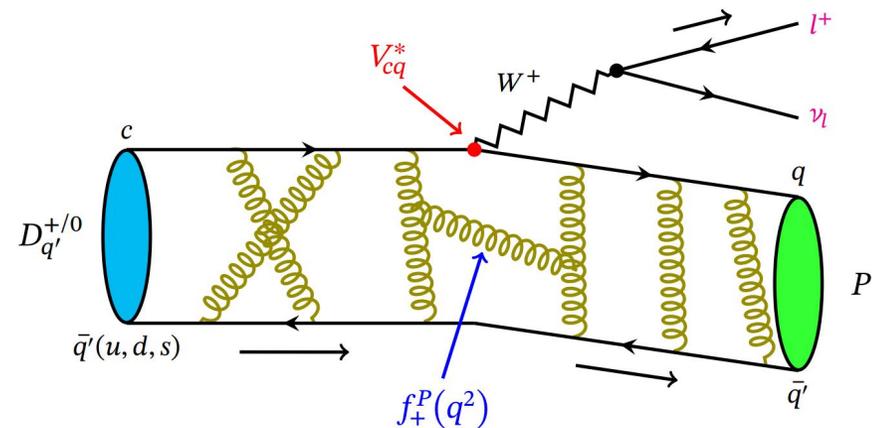
$f_+(0) = f_0(0)$

对于 $l = e, \mu$, 形状因子 f_0 对衰变宽度的贡献较小 (正比于 m_l^2)

- 标量流形状因子 $\langle K | S | D \rangle = f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{m_c - m_s}$

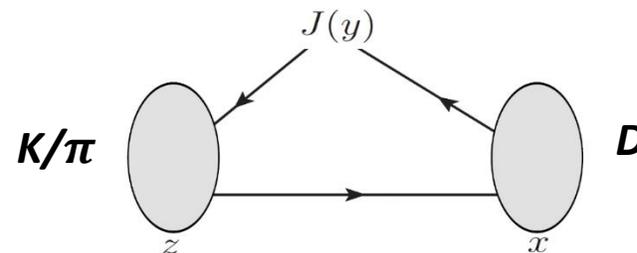
- 初末态强子四动量: p, p'

$q^2 = (p - p')^2$, 格点计算中3-动量取分立值



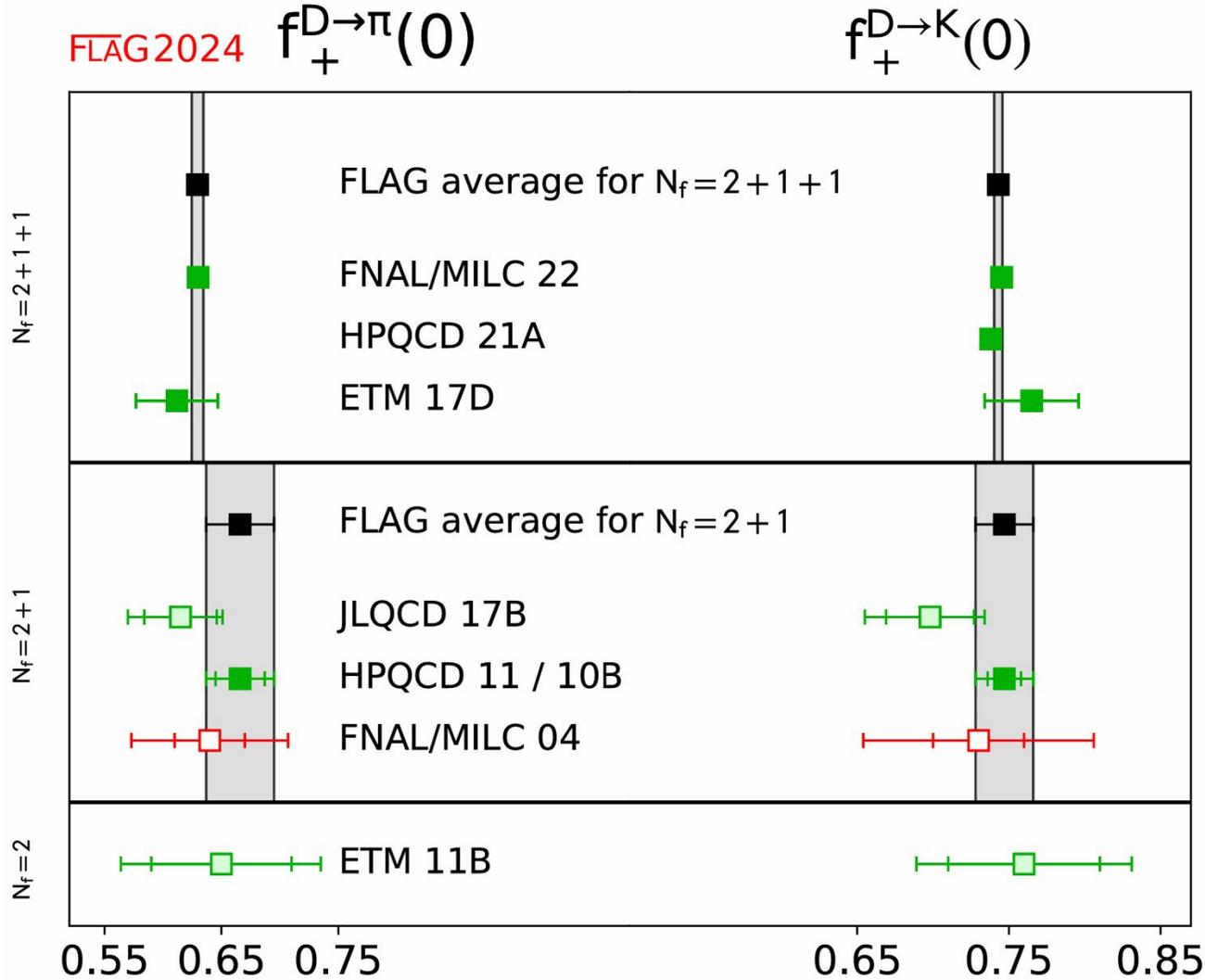
courtesy arXiv:2103.00908

(手征格点费米子有 $Z_S Z_m = 1$, 无需计算重整化常数)



三点关联函数

$f_+(q^2 = 0)$ for $D \rightarrow \pi/K$



FLAG Review 2024, arXiv:2411.04268

Results up to 2024.4.30

2+1+1味

- $f_+^{D\pi}(0) = 0.6296(50)$ [[ETM 17D: PRD96 \(2017\) 054514, 1706.03017](#)];
- [FNAL/MILC 22: 2212.12648, PRD107.094516](#)
- $f_+^{DK}(0) = 0.7430(27)$ [[ETM 17D: PRD96 \(2017\) 054514, 1706.03017](#). [HPQCD 21A: PRD104 \(2021\) 034505, 2104.09883](#). [FNAL/MILC 22: 2212.12648, PRD107.094516](#)]

2+1味

- JLQCD 17: [LAT2017 \[1711.11235\]](#)
- FNAL/MILC/HPQCD 04: 一个格距, m_π 大于 500 MeV
- $f_+^{D\pi}(0) = 0.666(29)$ [[HPQCD 11, PRD84,114505, 1109.1501](#)]
- $f_+^{DK}(0) = 0.747(19)$ [[HPQCD 10B, PRD82,114506, 1008.4562](#)]

$D \rightarrow \pi/K, D_s \rightarrow K$ 形状因子

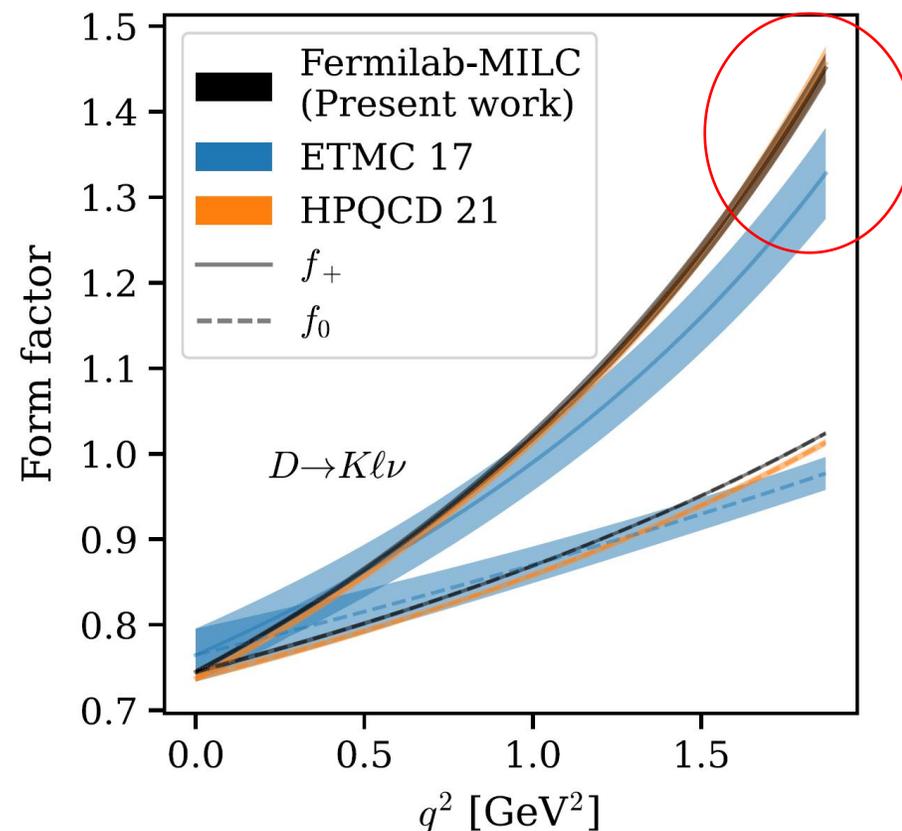
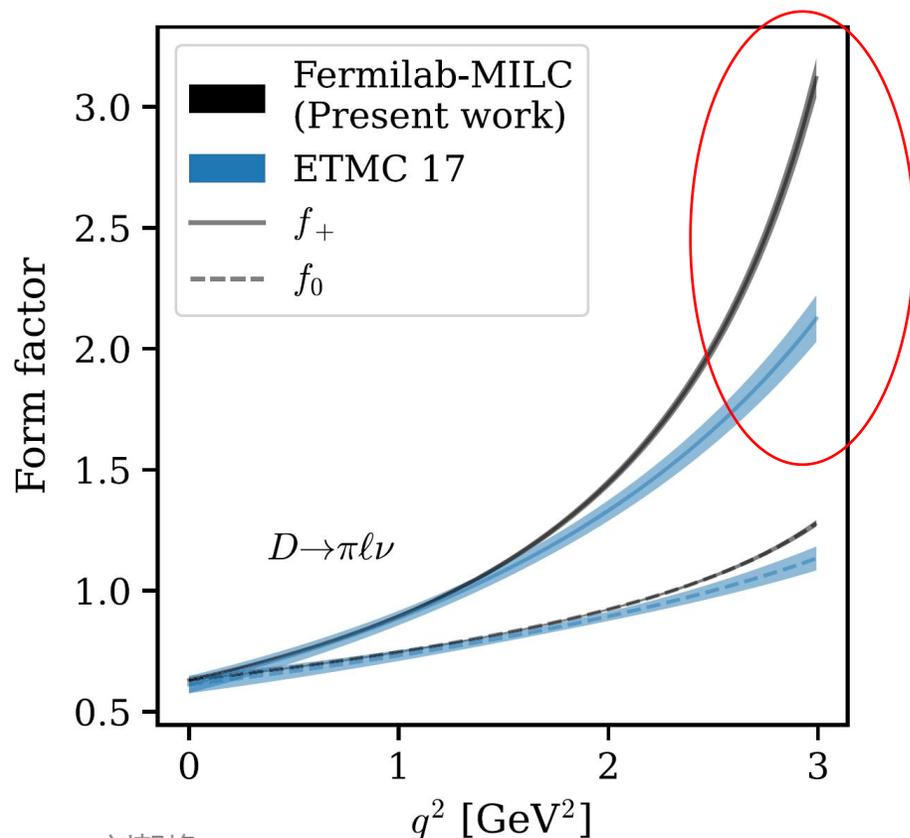
Fermilab/MILC, 2+1+1 味 HISQ

A. Bazavov et al., 2212.12648, PRD107.094516

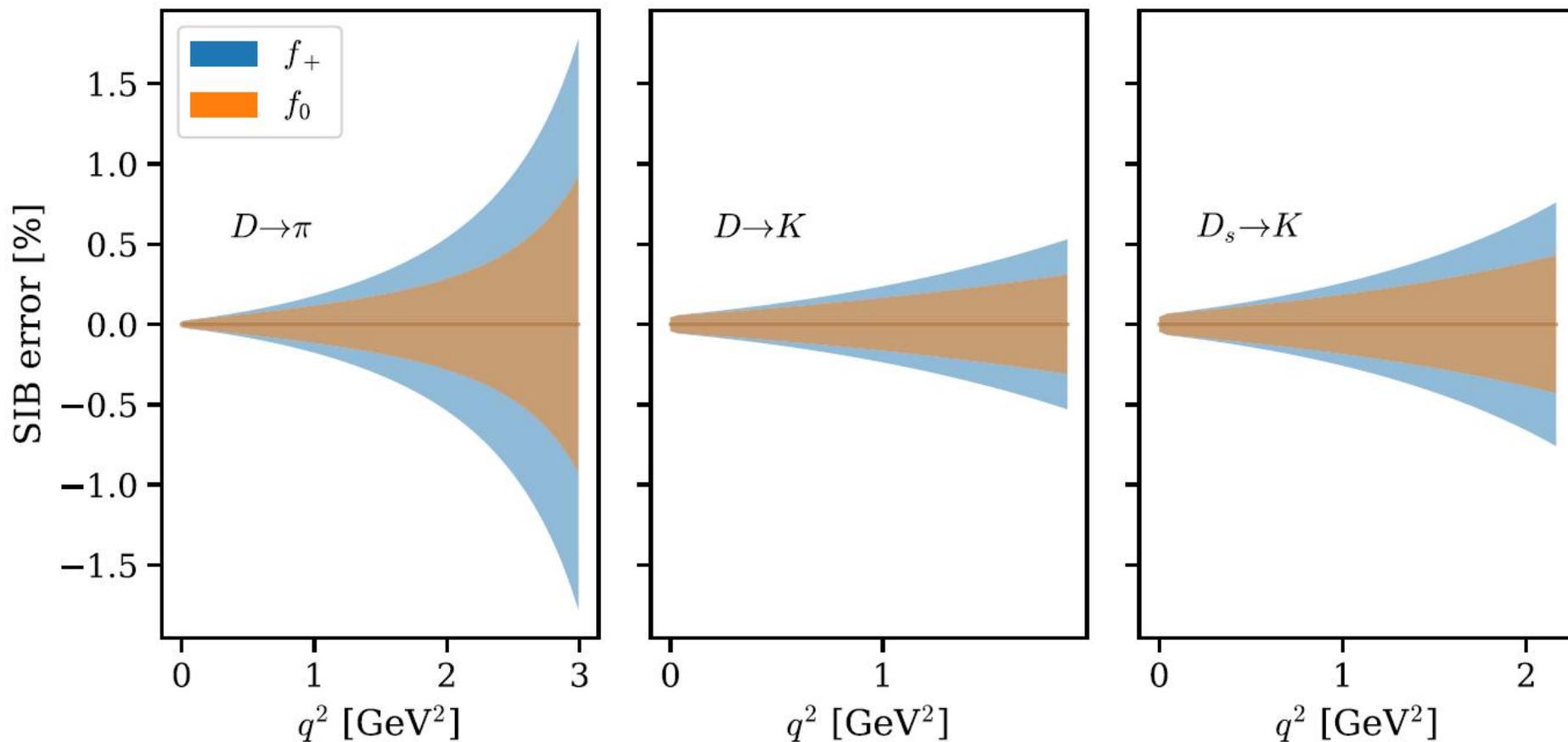
- 四个格距: ~ 0.12 fm - 0.04 fm
- $\mathcal{O}(1000)$ 组态数, 多次测量/组态
- 两或三个体积@两个格距
- 物理轻夸克质量@三个格距

$$\begin{aligned} f_+^{D\pi}(0) &= 0.6300(51) \\ f_+^{DK}(0) &= 0.7452(31) \\ f_+^{D_s K}(0) &= 0.6307(20) \end{aligned}$$

Not included: systematic uncertainties associated with QED, isospin, and electroweak corrections

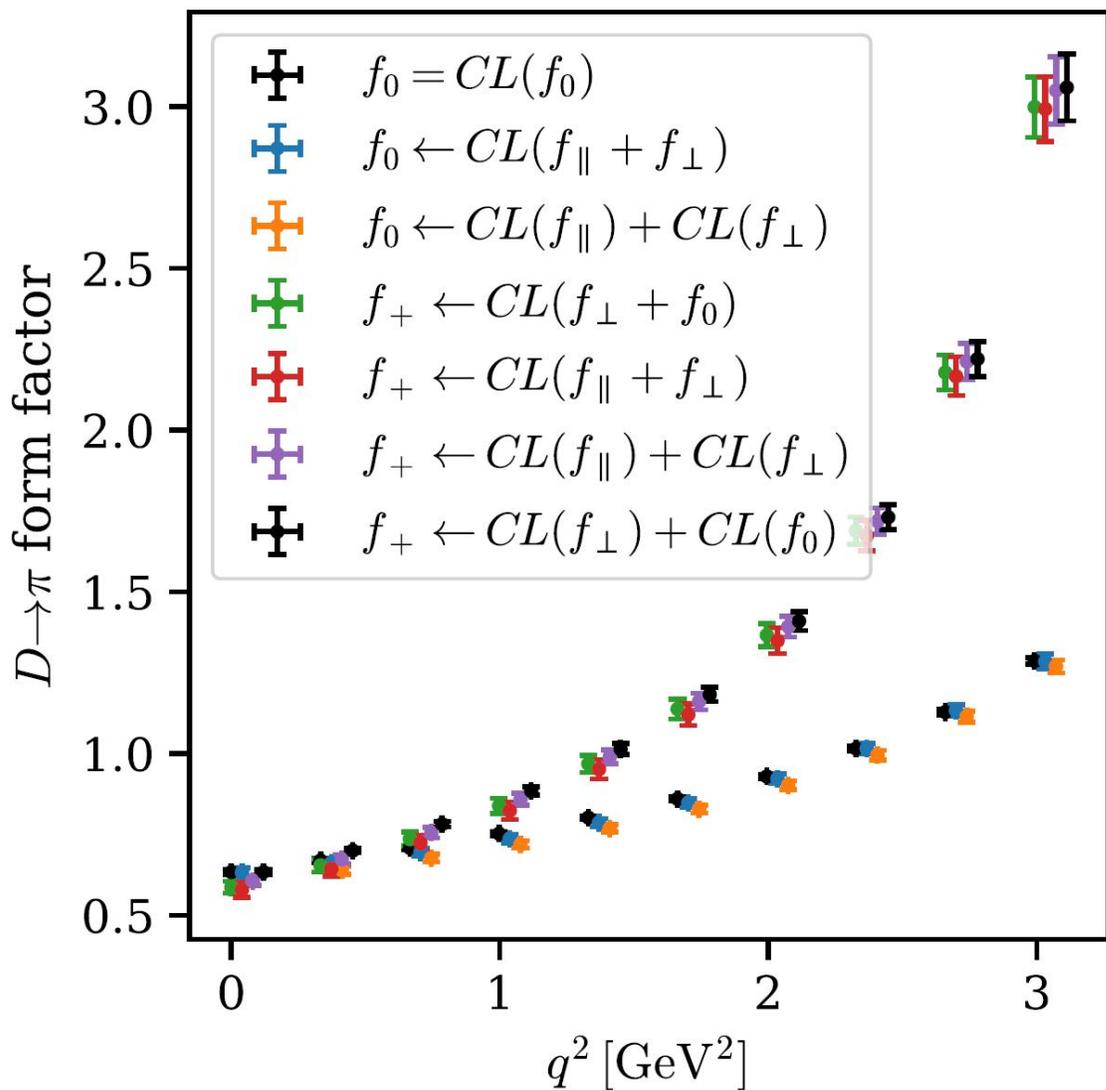


- 格点QCD模拟：上、下夸克简并 $m_l = (m_u + m_d)/2$ ，不带电荷
- 现实世界： $m_u \neq m_d$ ，带电
- 误差估计：使用中性介子质量 vs 带电介子质量，结果差异 $(1 - f_{+,0}^{\text{neutral}}/f_{+,0}^{\text{charged}})$



大 q^2 区域受相空间压低，同位旋破缺效应对唯象结果的影响较小

形状因子不同抽取方式的差别 (离散效应)



$$\langle K|V^\mu|D\rangle = f_+(q^2) \left(p_D^\mu + p_K^\mu - \frac{m_D^2 - m_K^2}{q^2} q^\mu \right) + f_0(q^2) \frac{m_D^2 - m_K^2}{q^2} q^\mu$$

$$\langle L|\mathcal{V}^\mu|H\rangle \equiv \sqrt{2M_H} [v^\mu f_{||}(q^2) + p_\perp^\mu f_\perp(q^2)]$$

k^μ 为初态介子四动量, $v^\mu = k^\mu/M_H$

p^μ 为末态介子四动量, $p_\perp^\mu = p^\mu - (p \cdot v)v^\mu$

$$f_+^{\text{alt}}(q^2) = \frac{1}{\sqrt{2M_H}} [f_{||}(q^2) + (M_H - E_L)f_\perp(q^2)]$$

$$f_0^{\text{alt}}(q^2) = \frac{\sqrt{2M_H}}{M_H^2 - M_L^2} \times [(M_H - E_L)f_{||}(q^2) + (E_L^2 - M_L^2)f_\perp(q^2)]$$

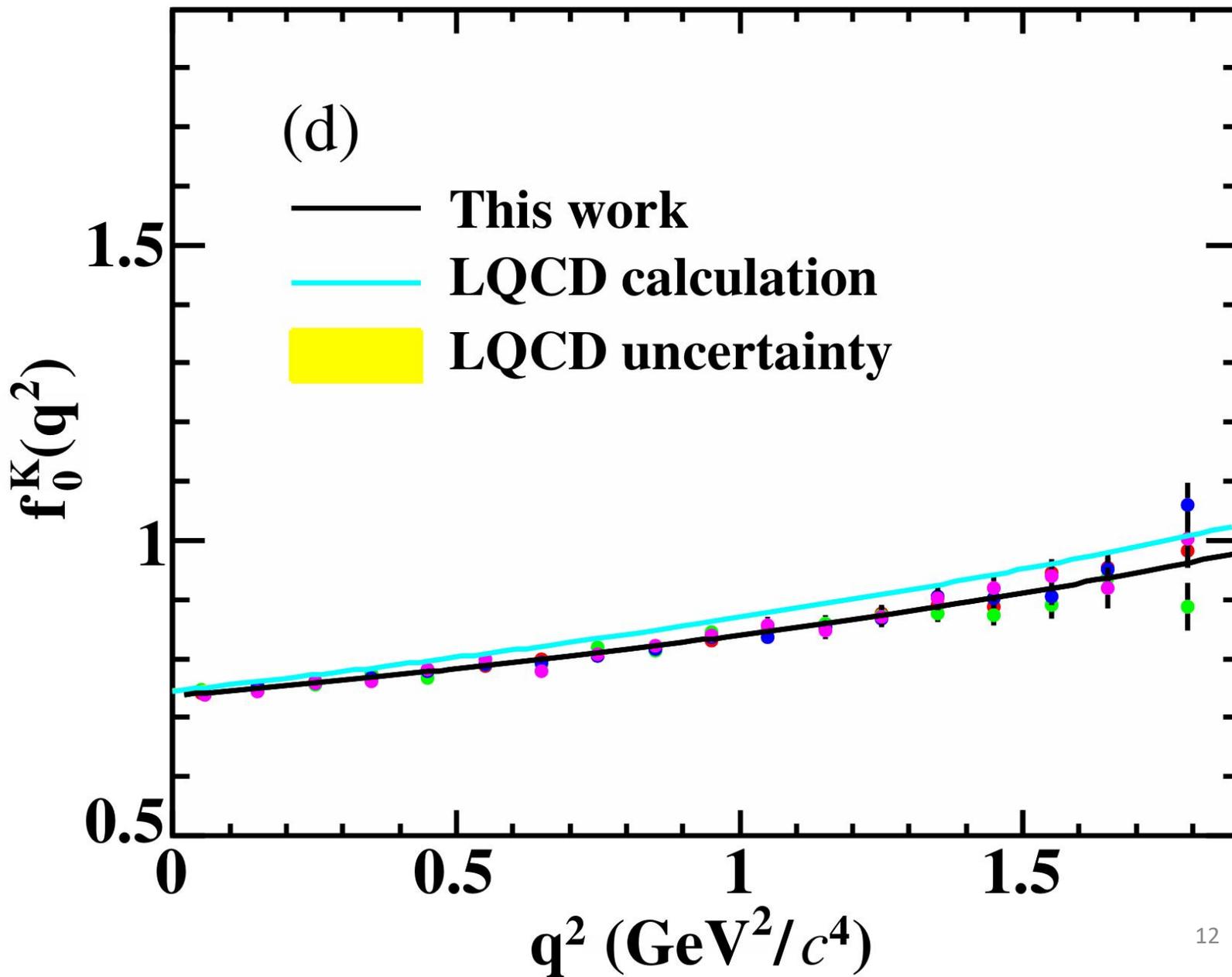
A. Bazavov et al., 2212.12648, PRD107.094516

$D \rightarrow Kl\nu$ 形状因子 $f_0(q^2)$

BESIII, PRD110.112006 (2024)

BESIII VS LQCD

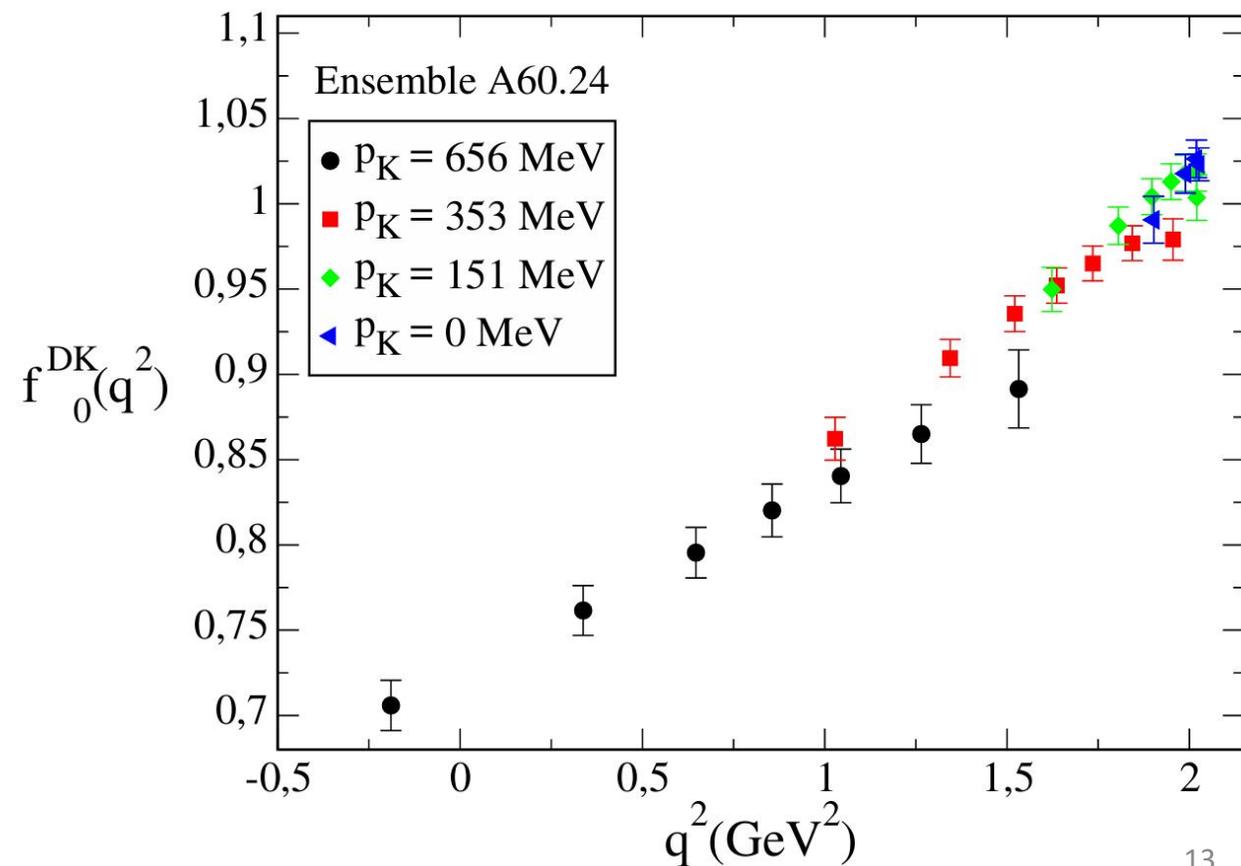
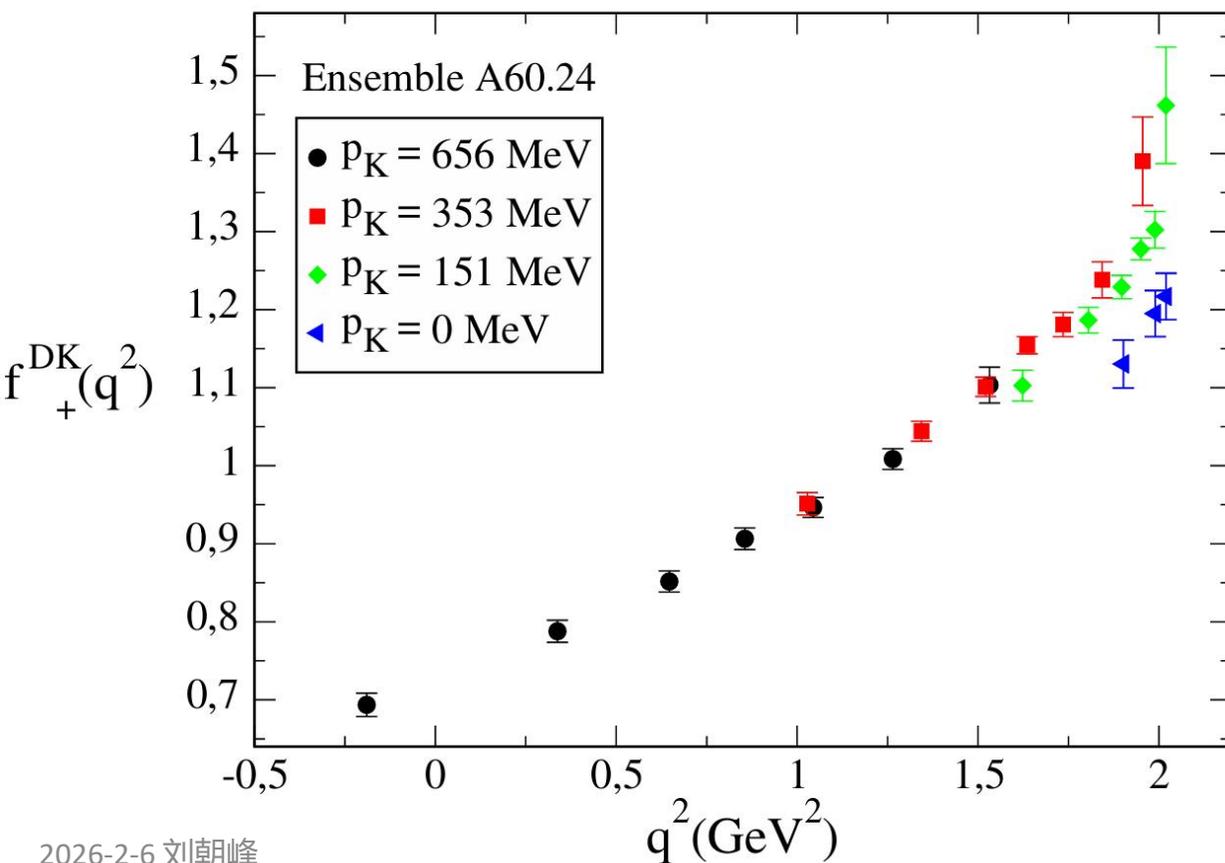
A. Bazavov et al., 2212.12648,
PRD107.094516



$D \rightarrow \pi/K$ 形状因子

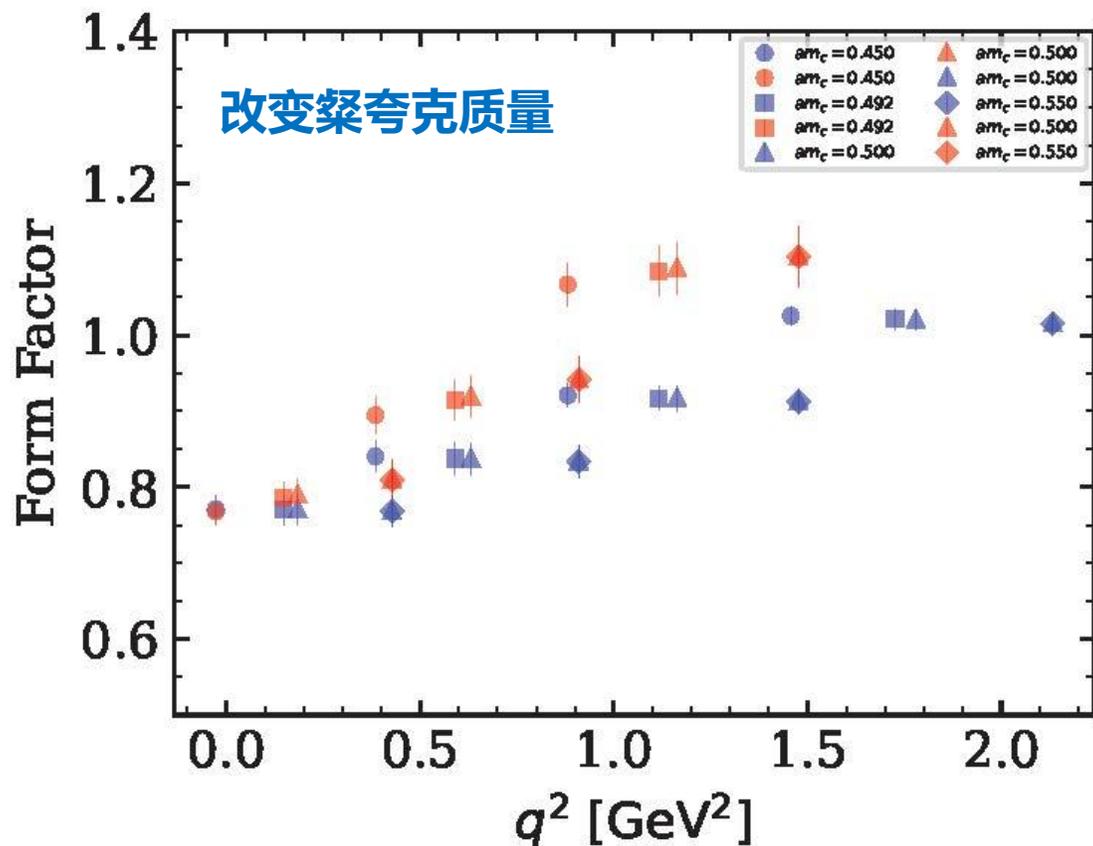
- 三个格距: $\sim 0.062, 0.082, 0.089$ fm
- 多套组态
- 多个体积@每个格距
- $m_\pi^{\text{sea}} \in [210, 450]$ MeV
- Hypercubic effects analyzed

$$f_+^{D\pi}(0) = 0.612(35)$$
$$f_+^{DK}(0) = 0.765(31)$$



$D \rightarrow Kl\nu$ 形状因子 $f_+(q^2)$ 和 $f_0(q^2)$

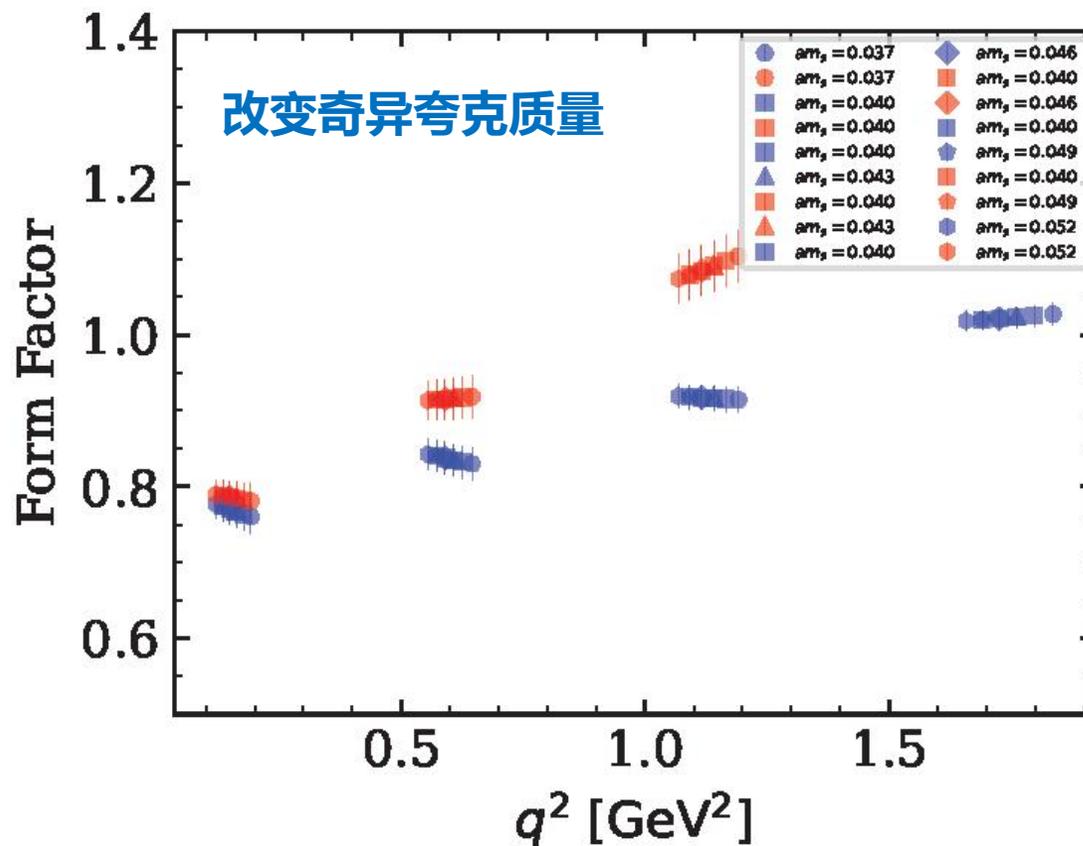
- Overlap价夸克质量点分别有 4(u/d)、6(s)、4(c)个
- 海夸克质量点3个 ($m_\pi^{\text{sea}} \approx 302, 360, 412$ MeV)
 - f004和f008组态上的分析正在进行
- 内插/外推到物理点



$1/a$ (GeV)	Label	am_l/am_s	Volume	$N_{\text{conf}} \times N_{\text{src}}$
2.383(9)	f004	0.004/0.03	$32^3 \times 64$	628×1
	f006	0.006/0.03	$32^3 \times 64$	42×16
	f008	0.008/0.03	$32^3 \times 64$	49×16

($m_\pi^{\text{sea}} \approx 360$ MeV)

RBC-UKQCD, 2+1味domain-wall fermion组态



$f_+(q^2)$ 和 $f_0(q^2)$ 的参数化及夸克质量依赖

定义:
$$z(q^2; t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- 取 $t_0 = 0$, 此时 $q^2 = 0$ 与 $z = 0$ 对应

$$f_+(q^2; m_c, m_s, m_l) = \frac{1}{1 - q^2/m_{D_s^*}^2} \sum_{i=0}^n a_i D_i z^i$$

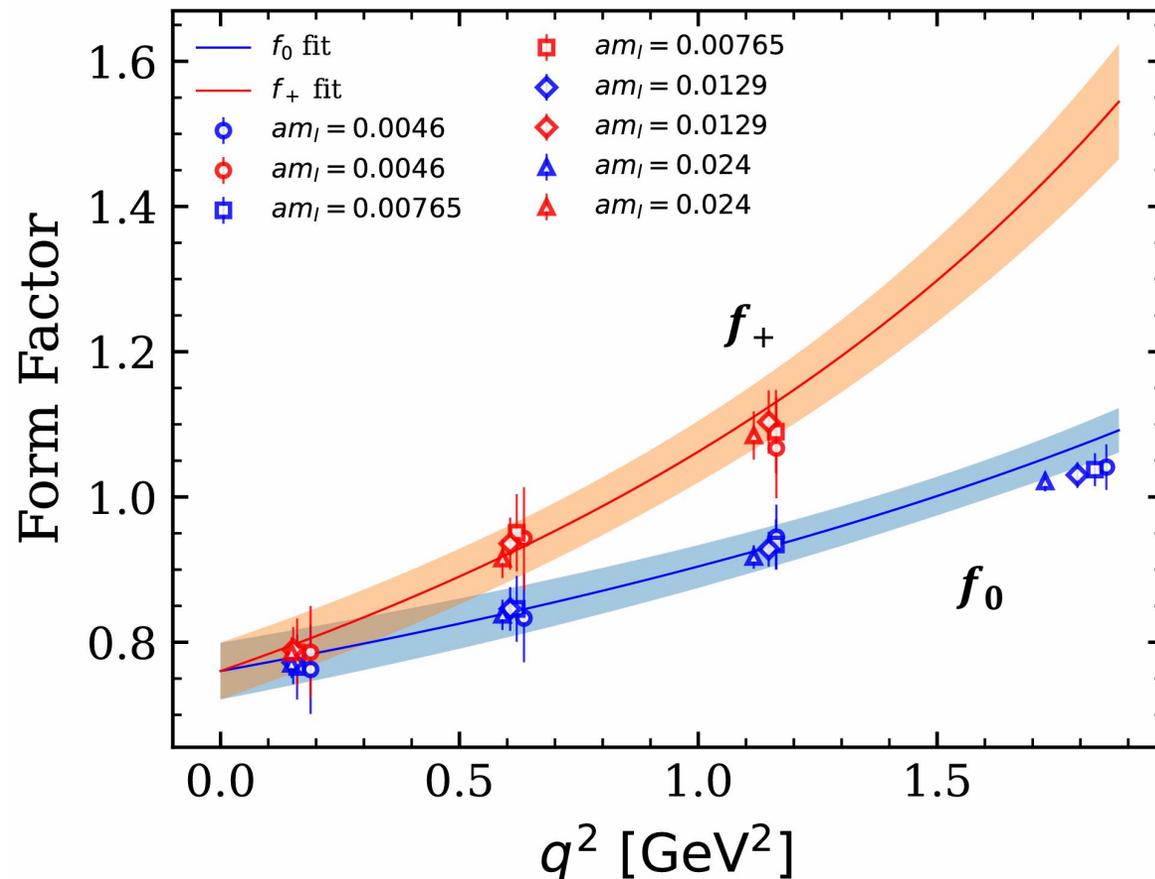
$$f_0(q^2; m_c, m_s, m_l) = \frac{1}{1 - q^2/m_{D_{s0}^*}^2} \sum_{i=0}^n b_i D_i z^i$$

$$D_i = 1 + c_{i1} (m_\pi^2 - (m_\pi^{\text{phys}})^2) + c_{i2} (m_{\eta_s}^2 - (m_{\eta_s}^{\text{phys}})^2) + c_{i3} (m_{J/\psi} - m_{J/\psi}^{\text{phys}})$$

- 拟合参数: $a_i, b_i, c_{i1}, c_{i2}, c_{i3}$
- $f_+(q^2 = 0) = f_0(0)$
- $m_{D_s^*}, m_{D_{s0}^*}$ 取对应的格点计算结果或实验值
- $m_{D_{s0}^*} = 2.3178 \text{ GeV}$ (PDG)
- 截断到 z 的一次、二次或三次项

2026-2-6 刘朝峰

对于给定的夸克质量, $t_+ \equiv (m_D + m_K)^2$ 用对应的强子质量计算



- At the physical point (valence quark)
 $f_+(0) = f_0(0) = 0.760(39) (n = 1)$

Preliminary results

$D \rightarrow K$ 张量流形状因子 $f_T(q^2)$

• 张量流形状因子 $\langle P | T_{\mu\nu} | D \rangle = \frac{2}{m_D + m_P} [p_{P\mu} p_{D\nu} - p_{P\nu} p_{D\mu}] f_T(q^2)$

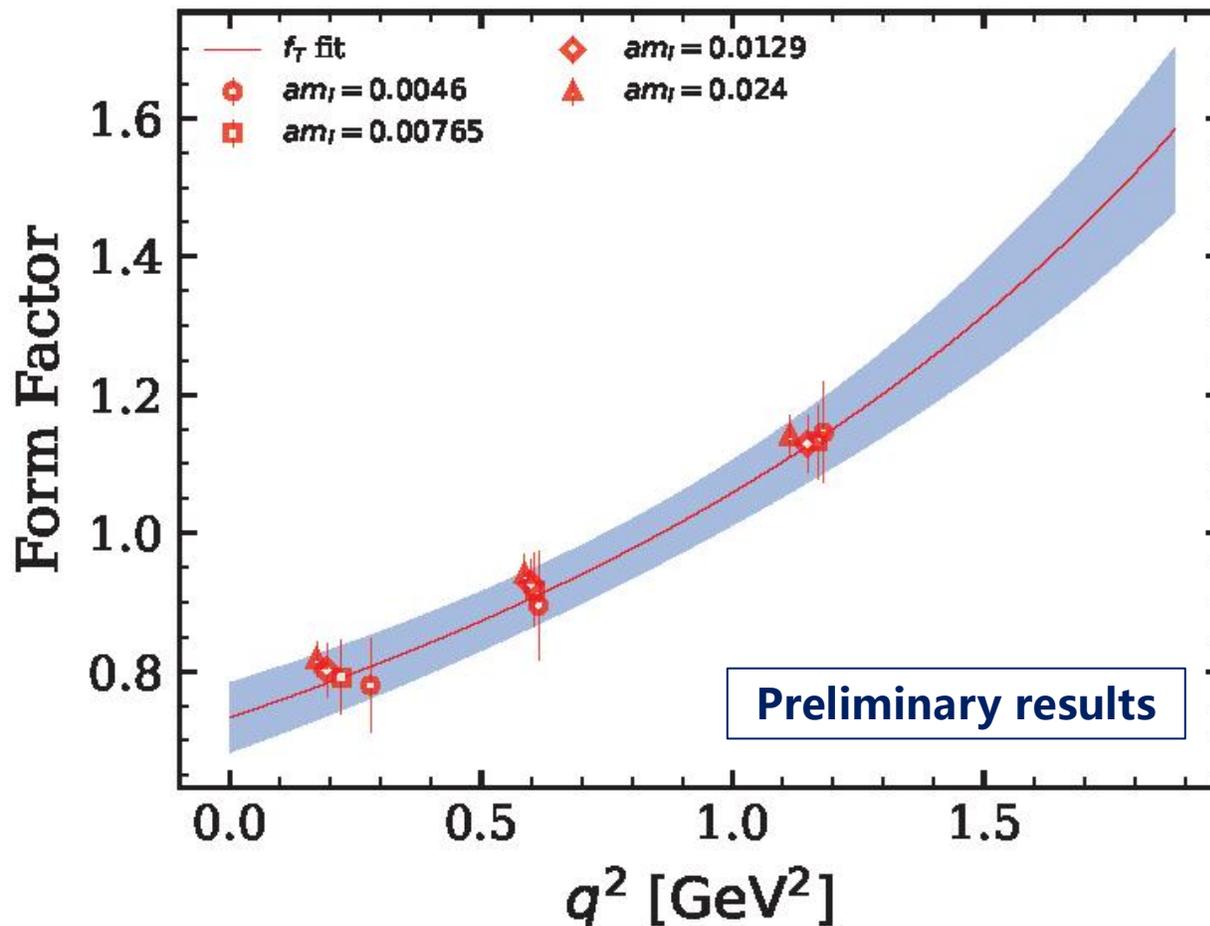
助力新物理寻找

- 以新物理方式影响 $D \rightarrow K/\pi lv$
- 影响 FCNC 稀有过程 $D \rightarrow K/\pi ll$

- At the physical point (valence quark)
 $f_T(0) = 0.733(50)$ ($n = 1$)

重整化常数: $Z_T^{\overline{MS}}(2 \text{ GeV}) = 1.157(11)$

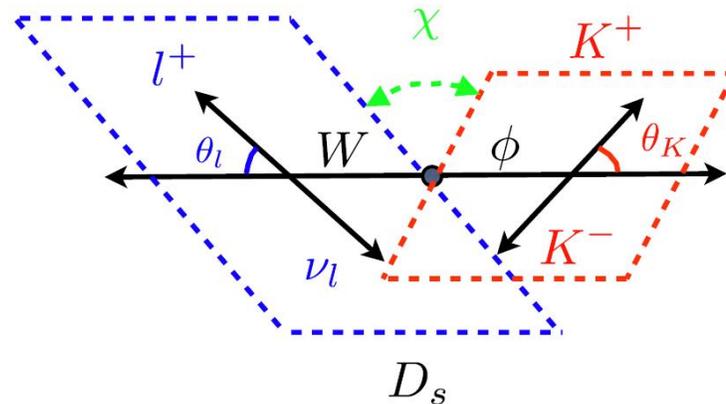
Y. Bi, ..., ZL et al., PRD108.054506 (2023)



$D_s \rightarrow \phi l \nu$

- 截面除了 q^2 依赖，还有三个角度依赖， $\phi \rightarrow K\bar{K}$
- HPQCD之前的计算 [PRD90, arXiv:1311.6669]
 - 忽略非连通图贡献 (OZI压低)
 - 模拟中 ϕ 不衰变； ϕ 宽度较窄，预期阈效应小
 - 形状因子 V, A_2, A_1, A_0 ；z-expansion
 - 2+1味；两个格距；HISQ价夸克作用量
 - 和BABAR ($D_s \rightarrow \phi e^+ \nu_e$)，CLEO ($D \rightarrow K^*$) 的结果做比较
(假设spectator quark的影响很小)

$$\frac{d\Gamma(D_s^+ \rightarrow \phi l^+ \nu_l)}{dq^2 d\chi d\cos\theta_\ell d\cos\theta_K} = \frac{G_F^2 |V_{cs}|^2 |\vec{p}| q^2}{12(2\pi)^4 M^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 W(\theta_K, \theta_\ell, \chi)$$



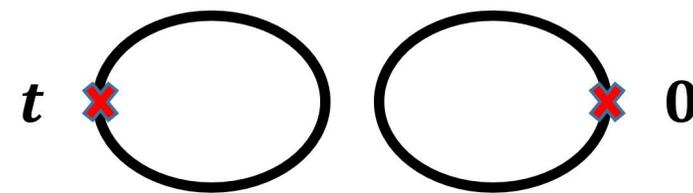
$$\langle \phi(p', \varepsilon) | V^\mu - A^\mu | D_s(p) \rangle$$

$$= \frac{2i\varepsilon^{\mu\nu\alpha\beta}}{M_{D_s} + M_\phi} \varepsilon_\nu^* p'_\alpha p_\beta V(q^2) - (M_{D_s} + M_\phi) \varepsilon^{*\mu} A_1(q^2)$$

$$+ \frac{\varepsilon^* \cdot q}{M_{D_s} + M_\phi} (p + p')^\mu A_2(q^2) + 2M_\phi \frac{\varepsilon^* \cdot q}{q^2} q^\mu A_3(q^2)$$

$$- 2M_\phi \frac{\varepsilon^* \cdot q}{q^2} q^\mu A_0(q^2).$$

$$A_3(q^2) = \frac{M_{D_s} + M_\phi}{2M_\phi} A_1(q^2) - \frac{M_{D_s} - M_\phi}{2M_\phi} A_2(q^2)$$



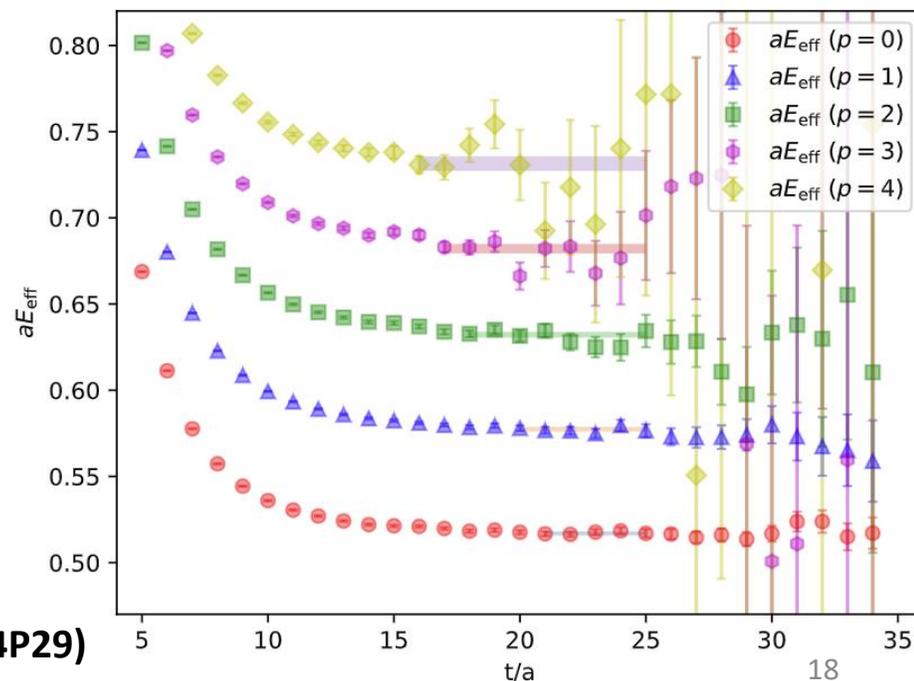
两点函数中的 (夸克线) 非连通图

$$A_0(0) = A_3(0)$$

	C24P29	C32P23	C32P29	F32P30	F48P21	G36P29	H48P32
a (fm)		0.10524(05)(62)		0.07753(03)(45)		0.06887(12)(41)	0.05199(08)(31)
am_l	-0.2770	-0.2790	-0.2770	-0.2295	-0.2320	-0.2150	-0.1850
am_s	-0.2400	-0.2400	-0.2400	-0.2050	-0.2050	-0.1926	-0.1700
am_s^V	-0.2356(1)	-0.2337(1)	-0.2358(1)	-0.2038(1)	-0.2025(1)	-0.1928(1)	-0.1701(1)
am_c^V	0.4159(07)	0.4190(07)	0.4150(06)	0.1974(05)	0.1997(04)	0.1433(12)	0.0551(07)
L (fm)	2.53	3.37	3.37	2.48	3.72	2.48	2.50
$L^3 \times T$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 64$	$32^3 \times 96$	$48^3 \times 96$	$36^3 \times 108$	$48^3 \times 144$
N_{mea}	$450 \times 72 \times 2$	$333 \times 64 \times 3$	$397 \times 64 \times 2$	$360 \times 96 \times 2$	$241 \times 48 \times 4$	$300 \times 54 \times 2$	$300 \times 72 \times 2$
m_π (MeV)	292.3(1.0)	227.9(1.2)	293.1(0.8)	300.4(1.2)	207.5(1.1)	297.2(0.9)	316.6(1.0)

- 七套2+1味组态, Wilson-clover fermions [CLQCD]
- 四个格距, 多个pion 质量
- D_s 静止, 改变 ϕ 的三动量 $\vec{p} = \frac{2\pi}{La} (k_x, k_y, k_z)$
- $(k_x, k_y, k_z) = (100), (110), (111), (200), (210), (211)$
- (轴) 矢量流重正化常数 [CLQCD, arXiv:2408.03548]
- 组态 C32P29 用于检查有限体积效应

ϕ 两点函数能量平台



(组态 C24P29)

物理质量点及连续极限下的形状因子 $(D_s \rightarrow \phi lv)$

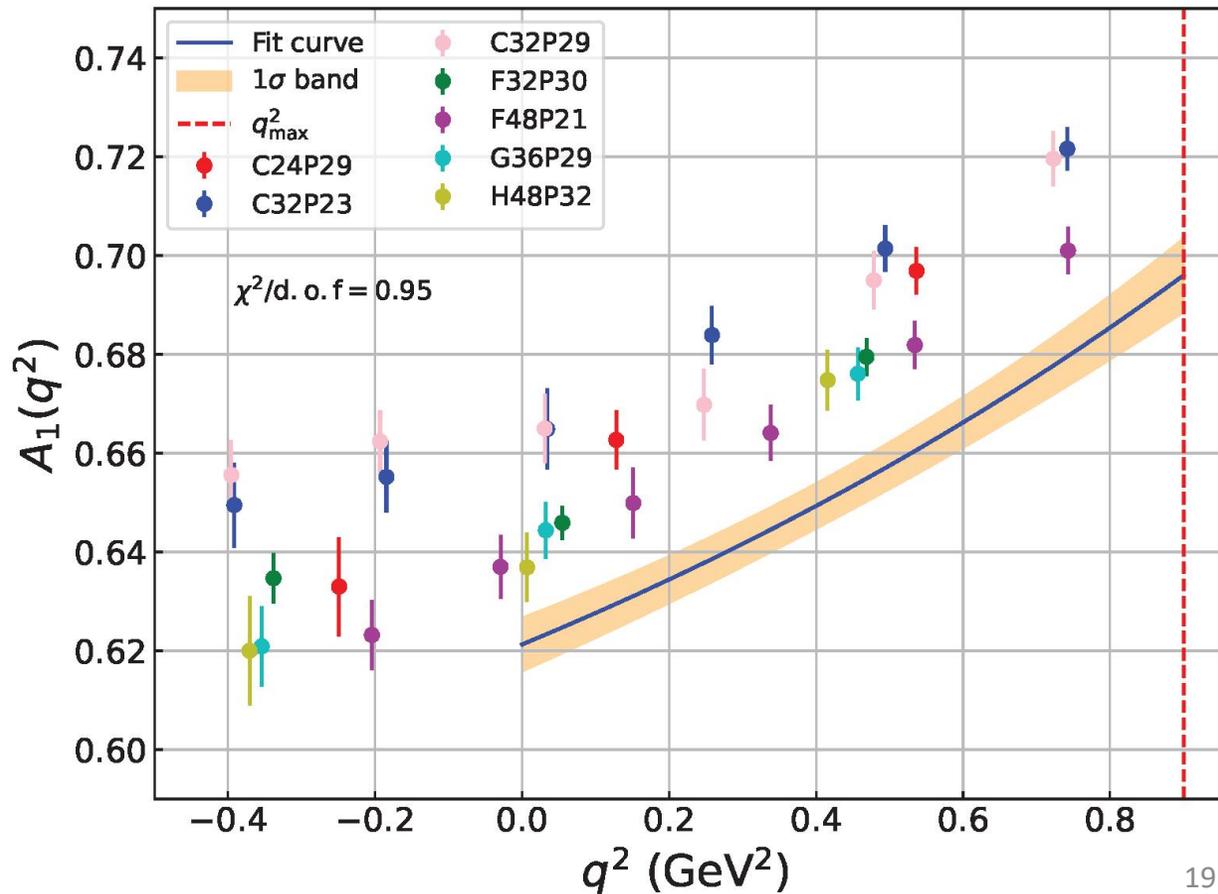
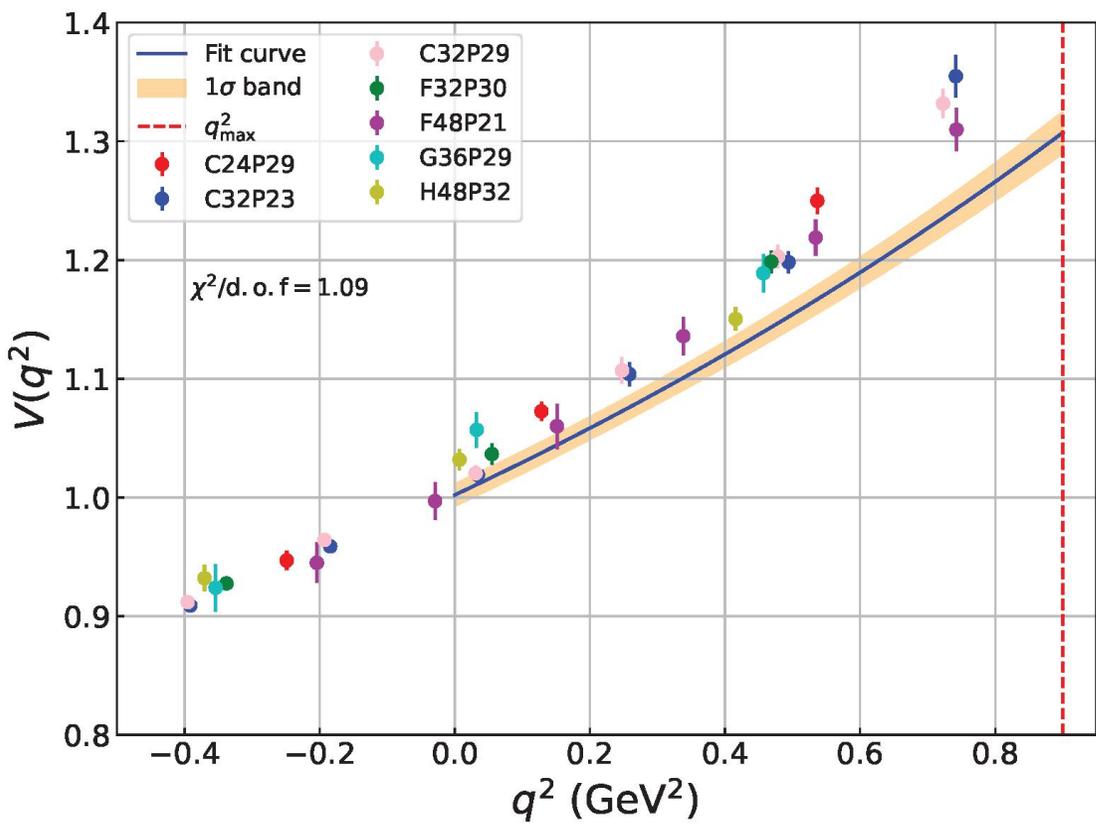
其它参数化形式给出误差内一致结果:

- Single pole form
- Modified pole form
- Phase moment form [Yao et al., PRD 101.034014 (2020)]

$$V(q^2, a, m_\pi) = \frac{1}{1 - q^2/m_{D_s^*}^2} \sum_{i=0}^2 (c_i + d_i a^2) \left[1 + f_i (m_\pi^2 - m_{\pi, \text{phys}}^2) + g_i (m_\pi^2 - m_{\pi, \text{phys}}^2)^2 \right] z^i$$

$$A_{0,1,2}(q^2, a, m_\pi) = \frac{1}{1 - q^2/m_{D_{s1}}^2} \sum_{i=0}^2 (c_i + d_i a^2) \left[1 + f_i (m_\pi^2 - m_{\pi, \text{phys}}^2) + g_i (m_\pi^2 - m_{\pi, \text{phys}}^2)^2 \right] z^i$$

$$m_{\pi, \text{phys}}^2 = 135.0 \text{ MeV}/c^2, m_{D_s^*}^2 = 2112.2 \text{ MeV}/c^2, m_{D_{s1}}^2 = 2459.5 \text{ MeV}/c^2$$

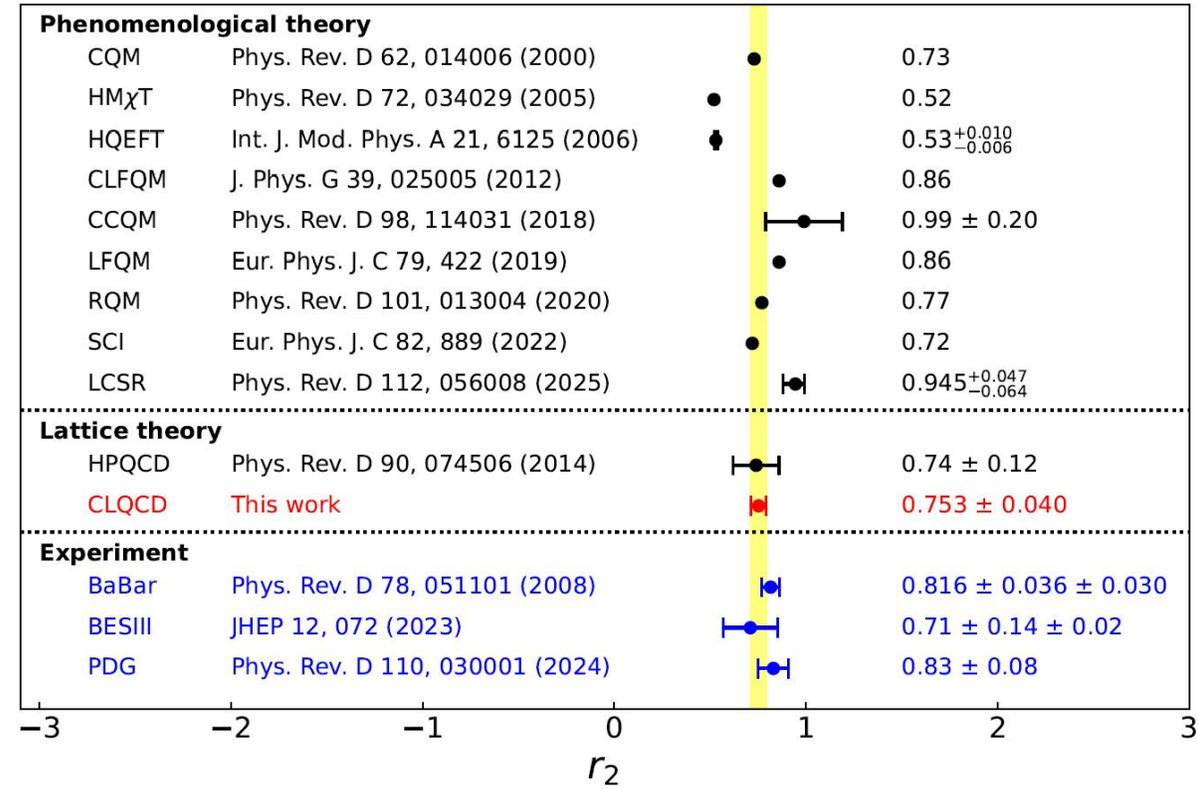
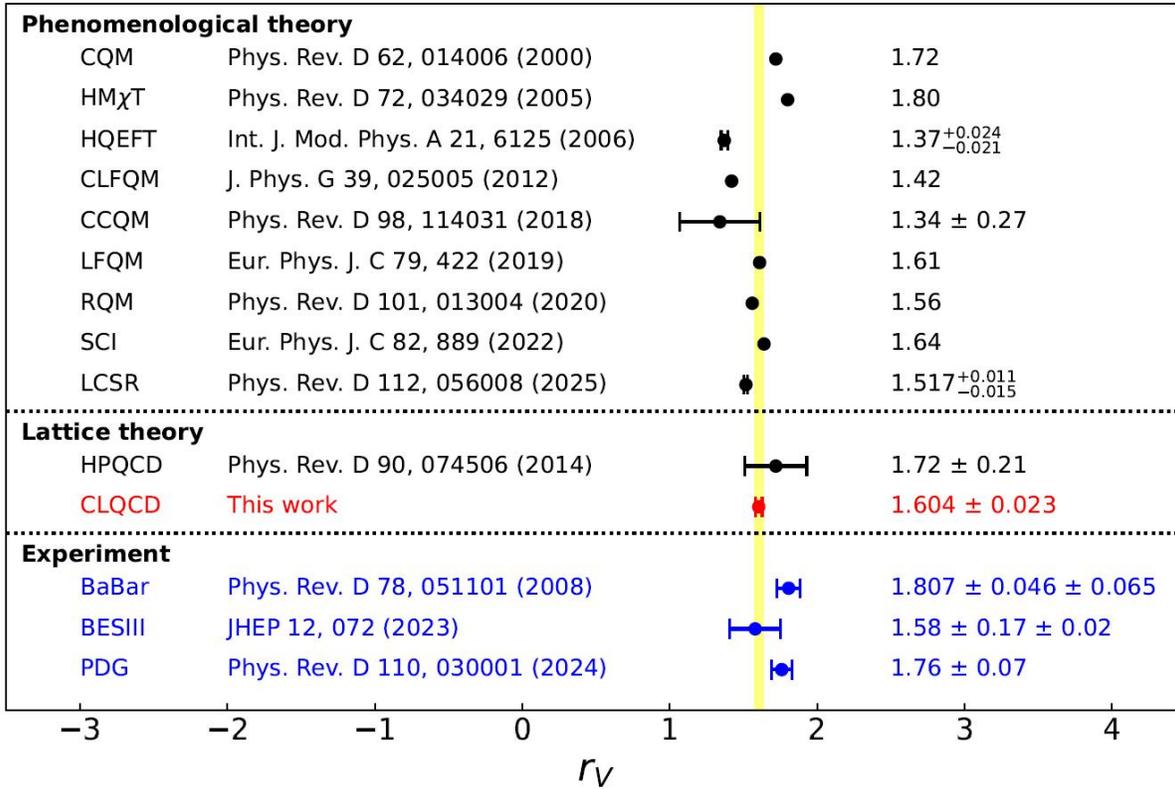


$A_0(0) - A_3(0) = -0.004(12)$
和零一致, 符合运动学限制

$D_s \rightarrow \phi l \nu$

$$r_V = V(0)/A_1(0)$$

$$r_2 = A_2(0)/A_1(0)$$

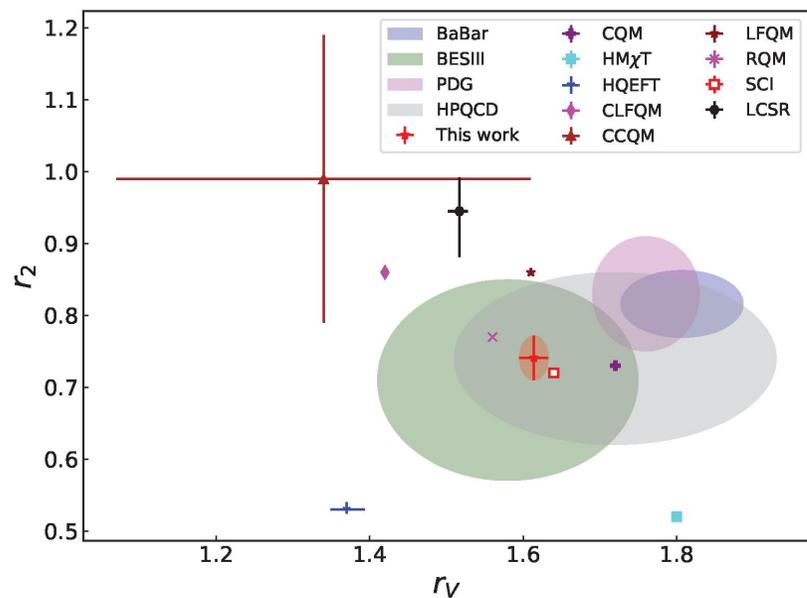


分支比及比值:
(精度比实验高)

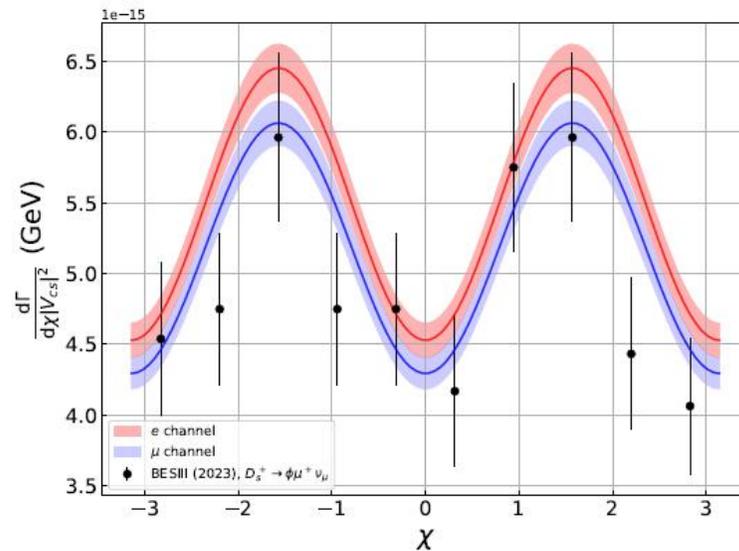
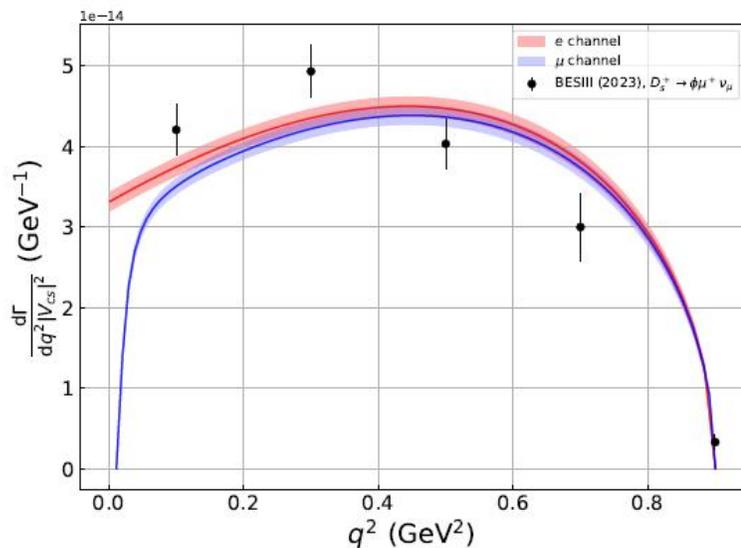
$\mathcal{B}(D_s \rightarrow \phi l \nu_e) \times 10^2$	e channel	μ channel	$\mathcal{R}_{\mu/e}$
This work (z -expansion)	2.476(80)	2.336(74)	0.9435(13)
BaBar [9]	2.61(17)	—	—
CLEO [10]	2.14(19)	—	—
BESIII (2018) [11]	2.26(46)	1.94(54)	0.86(29)
BESIII (2023) [12]	—	2.25(11)	—
PDG [41]	2.34(12)	2.24(11)	0.957(68)

BABAR, PRD78.051101(R) (2008)
CLEO, PRD92.012009 (2015)
BESIII, PRD97.012006 (2018)
BESIII, JHEP12 (2023) 072
PDG, PRD110.030001 (2024)

与实验及其它理论计算的比较

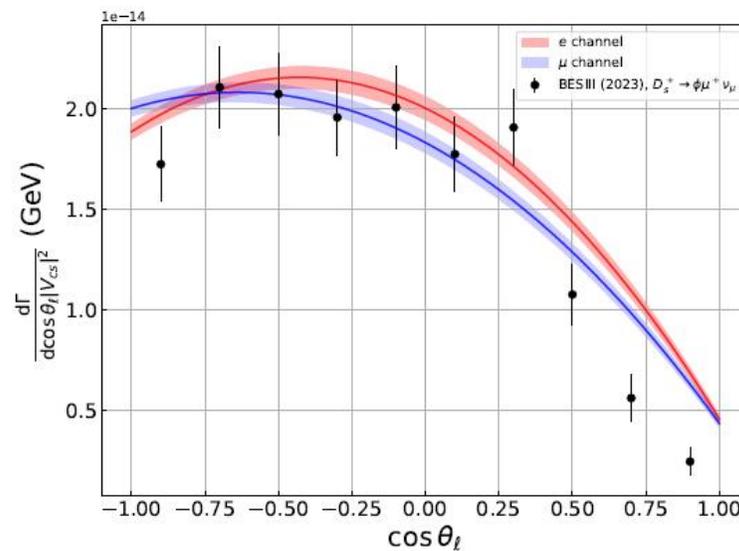
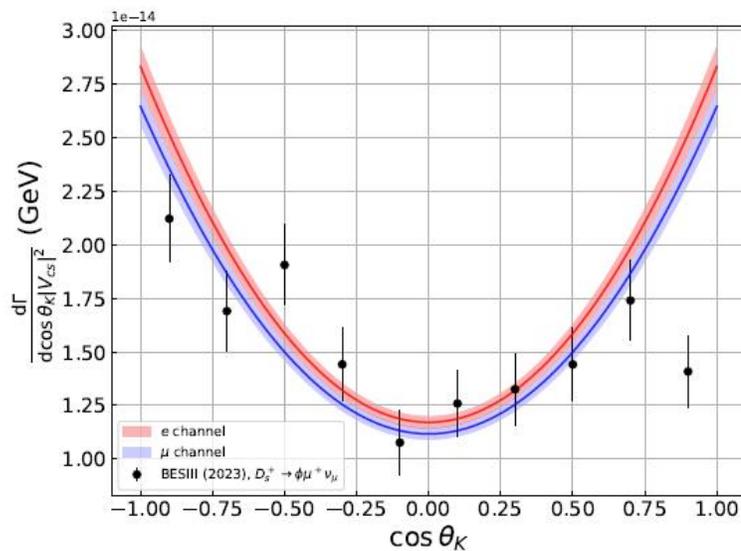


$$\frac{d\Gamma(D_s^+ \rightarrow \phi l^+ \nu_l)}{dq^2 d\chi d\cos\theta_\ell d\cos\theta_K} = \frac{G_F^2 |V_{cs}|^2 |\vec{p}| q^2}{12(2\pi)^4 M^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 W(\theta_K, \theta_\ell, \chi)$$



Unitarity	●	0.97349 ± 0.00016	Phys. Rev. D 110, 030001 (2024)
HFLAV	●	0.9701 ± 0.0081	Phys. Rev. D 113, 012008 (2026)
PDG	●	0.975 ± 0.006	Phys. Rev. D 110, 030001 (2024)
BESIII	●	0.9623 ± 0.0046	Phys. Rev. D 110, 112006 (2024)
HPQCD	●	1.017 ± 0.063	Phys. Rev. D 90, 074506 (2014)
CLQCD	◆	0.948 ± 0.028	This work, $D_s \rightarrow \phi e \nu_e$ channel
CLQCD	◆	0.955 ± 0.027	This work, $D_s \rightarrow \phi \mu \nu_\mu$ channel

Y-axis label: $|V_{cs}|$



总结

- 粲介子半轻衰变：精确检验标准模型、寻找新物理
- 格点QCD可提供强子矩阵元（形状因子）
 - $D \rightarrow \pi/Kl\nu$
 - $D_s \rightarrow \phi l\nu$
- 控制误差，提高精度
 - 统计误差
 - 激发态污染
 - 离散误差
 - 同位旋破缺效应
 - 重整化常数误差
 - 格距不确定性
 -

谢谢！