Hadronic Decays of Charm Mesons at BESIII

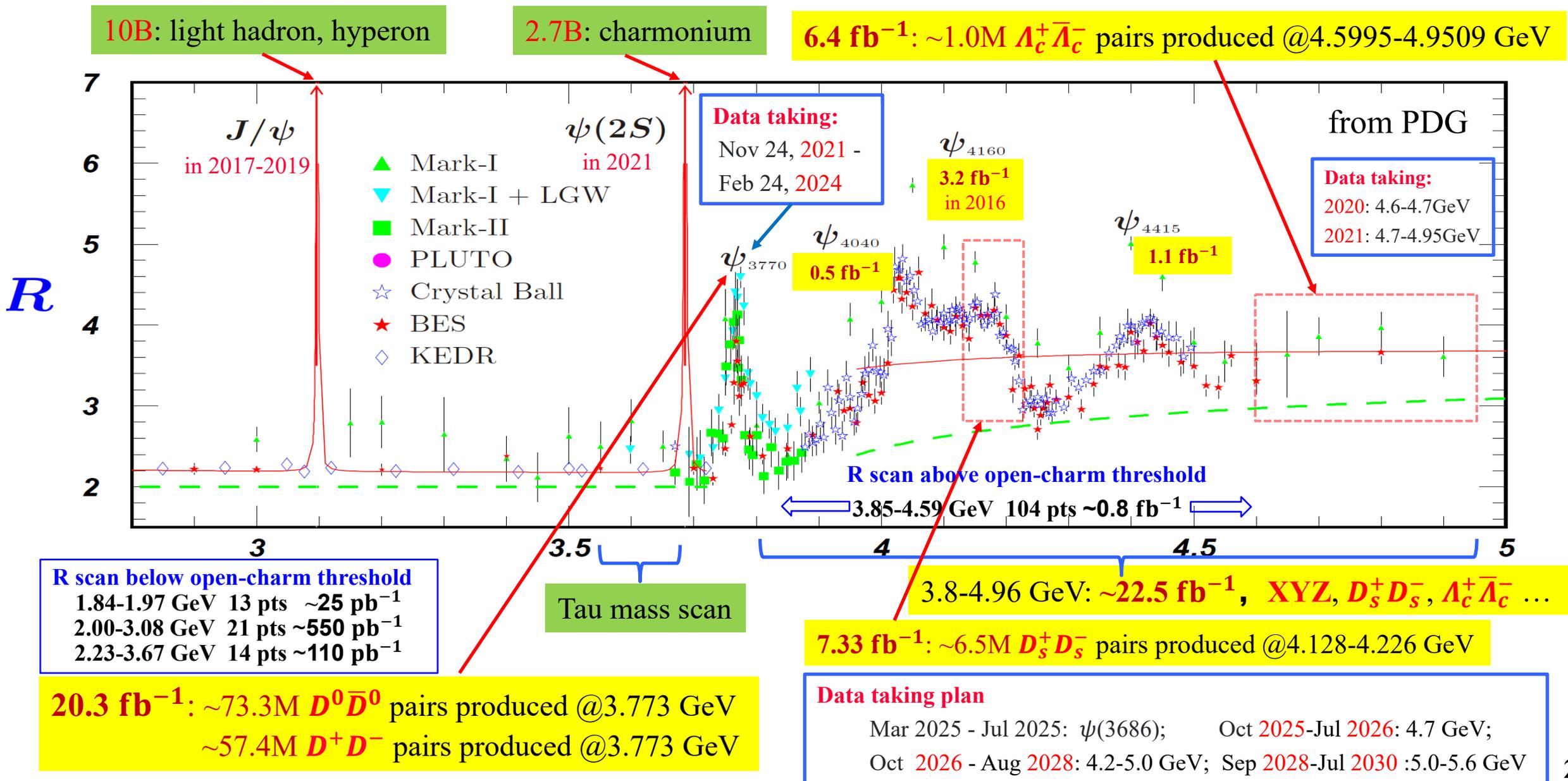## 董燎原（代表BESIII合作组）

中国科学院高能物理研究所

BESIII实验物理研讨会

北京昌平 2026年2月6日

Datasets (totally ~50 fb^{-1} from 1.84 − 4.95 GeV)

10B: light hadron, hyperon

2.7B: charmonium

6.4 fb^{-1}: ~1.0M $\Lambda_c^+\bar{\Lambda}_c^-$ pairs produced @4.5995-4.9509 GeV

J/ψ in 2017-2019

$\psi(2S)$ in 2021

from PDG

- ▲ Mark-I
- ▼ Mark-I + LGW
- ■ Mark-II
- ● PLUTO
- ☆ Crystal Ball
- ★ BES
- ◇ KEDR

Data taking: Nov 24, 2021 - Feb 24, 2024

ψ_{4160} **3.2 fb^{-1}** in 2016

Data taking: 2020: 4.6-4.7GeV 2021: 4.7-4.95GeV

ψ_{4415} **1.1 fb^{-1}**

ψ_{4040} **0.5 fb^{-1}**

ψ_{3770}

R

R scan above open-charm threshold
3.85-4.59 GeV 104 pts ~0.8 fb^{-1}

R scan below open-charm threshold
1.84-1.97 GeV 13 pts ~25 pb^{-1}
2.00-3.08 GeV 21 pts ~550 pb^{-1}
2.23-3.67 GeV 14 pts ~110 pb^{-1}

Tau mass scan

3.8-4.96 GeV: ~22.5 fb^{-1}, XYZ, $D_s^+D_s^-$, $\Lambda_c^+\bar{\Lambda}_c^-$ ...

7.33 fb^{-1}: ~6.5M $D_s^+D_s^-$ pairs produced @4.128-4.226 GeV

20.3 fb^{-1}: ~73.3M $D^0\bar{D}^0$ pairs produced @3.773 GeV
~57.4M D^+D^- pairs produced @3.773 GeV

Data taking plan
Mar 2025 - Jul 2025: $\psi(3686)$;   Oct 2025-Jul 2026: 4.7 GeV;
Oct 2026 - Aug 2028: 4.2-5.0 GeV; Sep 2028-Jul 2030 :5.0-5.6 GeV

# Charmed mesons hadronic decays

✿ Measurements of the BFs

→ Are important component of heavy-flavor physics program.

→ Probe and calibrate non-perturbative QCD

→ Understand SU(3) flavor symmetry and its breaking effect

→ Test theoretical calculations of BFs and improve theoretical predictions of *CP* violation

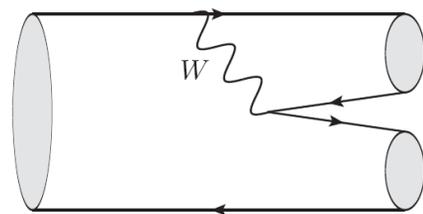## ✿ Amplitude analysis of multi-body decays

→ Information of $D \rightarrow VP, PP, SP, SV, VV, AP, AV, TP$ ...

→ Light hadron spectroscopy

P: pseudo-scalar
V: vector
S: scalar
A: axial-vector
T: tensor

Topological Diagrammatic for D/D_s^+ decays:

H. -Y. Cheng, *et al*. PRD 85, 034036
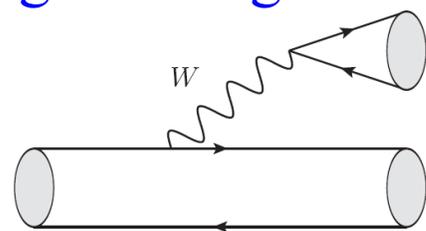
Color-allowed tree T Color-suppressed tree C W-exchange E W-annihilation A
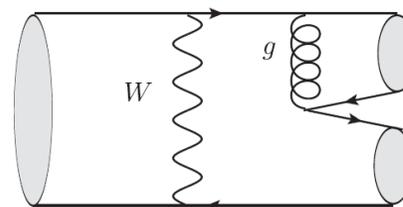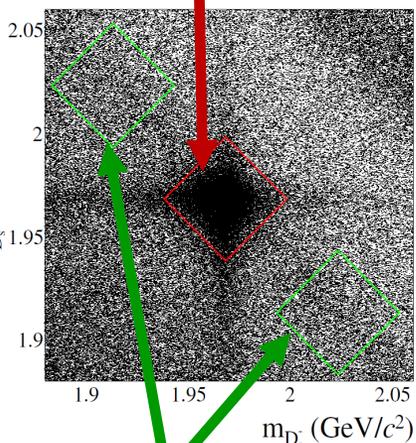
Calculation is not reliable, need exp. input

BF of D_s^+ hadronic decays

BES III

🌀 BF Measurement of D_s^+ hadronic decays

17 Amplitude analyses published

42965 events in signal region

14728 events in combined sideband

Mode	$\mathcal{B}$ (%)	$\mathcal{A}_{CP}$ (%)
$D_s^+ \to K_S^0 K^+$	$1.502 \pm 0.012 \pm 0.009$	$0.29 \pm 0.50 \pm 0.21$
$D_s^+ \to K^+ K^- \pi^+$	$5.49 \pm 0.04 \pm 0.07$	$0.48 \pm 0.26 \pm 0.24$
$D_s^+ \to K_S^0 K^+ \pi^0$	$1.47 \pm 0.02 \pm 0.02$	$-0.85 \pm 1.97 \pm 0.46$
$D_s^+ \to K_S^0 K_S^0 \pi^+$	$0.73 \pm 0.01 \pm 0.01$	$1.14 \pm 1.58 \pm 0.44$
$D_s^+ \to K^+ K^- \pi^+ \pi^0$	$5.50 \pm 0.05 \pm 0.11$	$-0.66 \pm 0.91 \pm 0.33$
$D_s^+ \to K_S^0 K^+ \pi^+ \pi^-$	$0.93 \pm 0.02 \pm 0.01$	$2.00 \pm 2.37 \pm 0.70$
$D_s^+ \to K_S^0 K^- \pi^+ \pi^+$	$1.56 \pm 0.02 \pm 0.02$	$-0.24 \pm 1.05 \pm 1.07$
$D_s^+ \to \pi^+ \pi^+ \pi^-$	$1.09 \pm 0.01 \pm 0.01$	$-0.88 \pm 1.17 \pm 0.38$
$D_s^+ \to \pi^+ \eta$	$1.69 \pm 0.02 \pm 0.02$	$-0.44 \pm 0.89 \pm 0.19$
$D_s^+ \to \pi^+ \pi^0 \eta$	$9.10 \pm 0.09 \pm 0.15$	$1.05 \pm 1.45 \pm 0.62$
$D_s^+ \to \pi^+ \pi^+ \pi^- \eta$	$3.08 \pm 0.06 \pm 0.05$	$2.42 \pm 2.85 \pm 0.78$
$D_s^+ \to \pi^+ \eta'$	$3.95 \pm 0.04 \pm 0.07$	$-0.59 \pm 0.76 \pm 0.20$
$D_s^+ \to \pi^+ \pi^0 \eta'$	$6.17 \pm 0.12 \pm 0.14$	$-1.60 \pm 2.57 \pm 0.64$
$D_s^+ \to K_S^0 \pi^+ \pi^0$	$0.51 \pm 0.02 \pm 0.01$	$-2.17 \pm 4.65 \pm 1.10$
$D_s^+ \to K^+ \pi^+ \pi^-$	$0.620 \pm 0.009 \pm 0.006$	$1.81 \pm 2.01 \pm 0.45$

Agree with PDG with much improved precision

🌀 Observation of $D_s^+ \to \omega \pi^+ \eta$ ➡️

$\mathcal{B}(D_s^+ \to \omega \pi^+ \eta) = (0.54 \pm 0.12 \pm 0.04)\%$

7.6σ
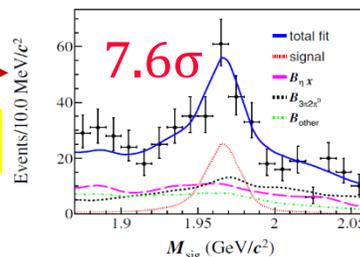
Mode	Reference
$D_s^+ \to K^+ K^- \pi^+$	Phys. Rev. D **104** (2021) 012016
$D_s^+ \to K_S^0 K^+ \pi^0$	Phys. Rev. Lett. **129** (2022) 182001
$D_s^+ \to \pi^+ \pi^+ \pi^-$	Phys. Rev. D **106** (2022) 112006
$D_s^+ \to \pi^+ \pi^+ \pi^- \eta$	Phys. Rev. D **104** (2021) L071101
$D_s^+ \to \pi^+ \pi^0 \eta'$	**JHEP 04** (2022) 058
$D_s^+ \to \pi^+ \pi^0 \pi^0$	**JHEP 01** (2022) 052
$D_s^+ \to K^+ \pi^+ \pi^-$	**JHEP 08** (2022) 196
$D_s^+ \to K^+ \pi^+ \pi^- \pi^0$	**JHEP 09** (2022) 242
$D_s^+ \to K_S^0 K_S^0 \pi^+$	Phys. Rev. D **105** (2022) L051103
$D_s^+ \to K_S^0 K^- \pi^+ \pi^+$	Phys. Rev. D **103** (2021) 092006
$D_s^+ \to K^+ K^- \pi^+ \pi^0$	Phys. Rev. D **104** (2021) 032011
$D_s^+ \to K^+ K^- \pi^+ \pi^+ \pi^-$	**JHEP 07** (2022) 051
$D_s^+ \to K_S^0 \pi^+ \pi^0$	**JHEP 06** (2021) 181
$D_s^+ \to \pi^+ \pi^0 \eta$	Phys. Rev. Lett. **123** (2019) 112001
$D_s^+ \to \pi^+ \pi^+ \pi^- \pi^0$	Phys. Rev. Lett. **134** (2025) 011904
$D_s^+ \to \pi^+ \pi^+ \pi^- \pi^0 \pi^0$	Phys. Rev. Lett. **134** (2025) 201902
$D_s^+ \to K_S^0 K_L^0 \pi^+$	Phys. Rev. Lett. **135** (2025) 161902
$D_s^+ \to \pi^+ \pi^0 \pi^0 \eta$	**to be submitted soon**

分支比测量和振幅分析为理解D_s^+介子强子衰变机制提供了完整的实验信息

7.33fb^{-1} @ 4.178 $-$ 4.226GeV , PRL 134, 011904 (2025)

1552 candidates with >75% purity

🌀 Observation of $D_s^+ \to f_0(980)\rho(770)^+$

（Mainly involves W-external-emission diagram）

$$\mathcal{B}(D_s^+ \to \pi^+\pi^+\pi^-\pi^0|_{\text{non}-\eta}) = (2.04 \pm 0.08_{\text{stat.}} \pm 0.05_{\text{syst.}})\%$$

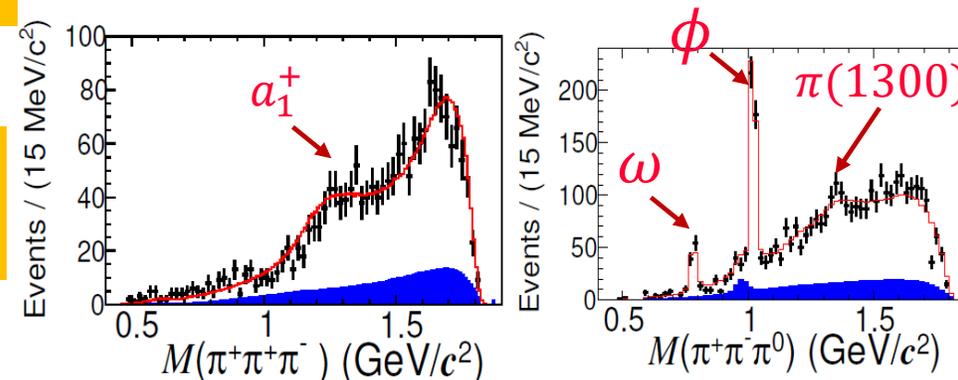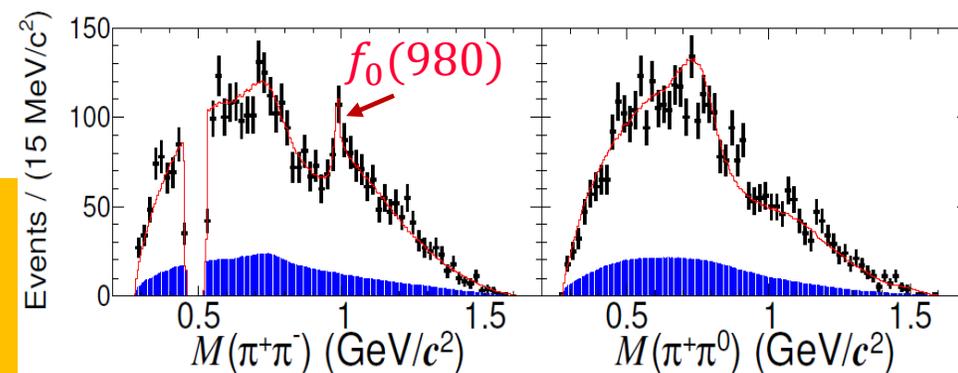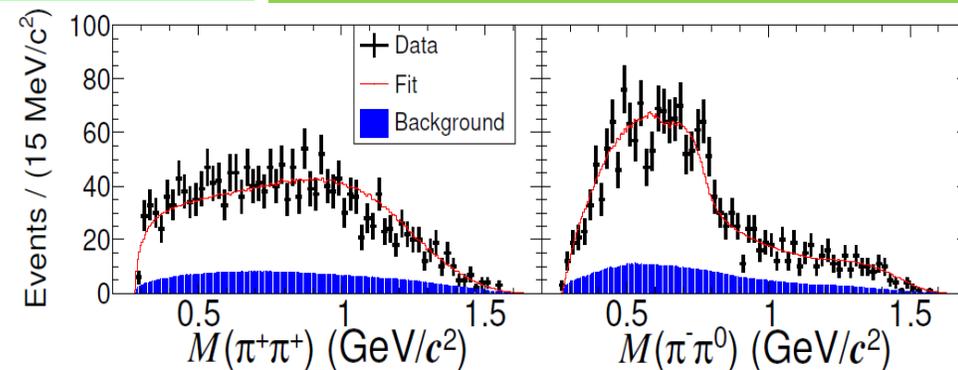$$\mathcal{B}(D_s^+ \to \eta\pi^+) = (1.56 \pm 0.09_{\text{stat.}} \pm 0.04_{\text{syst.}})\%$$

$$\frac{\mathcal{B}(\phi(1020) \to \pi^+\pi^-\pi^0)}{\mathcal{B}(\phi(1020) \to K^+K^-)} = 0.230 \pm 0.014_{\text{stat.}} \pm 0.010_{\text{syst.}}$$

Taking from $D_s^+ \to K^+K^-\pi^+$
BESIII, PRD 104, 012016 (2021)

Deviates from PDG value (0.313±0.010) by >4σ

| Component	Phase (rad)	BF (10^{-3})
$f_0(1370)\rho^+$	0.0(fixed)	$5.08 \pm 0.80 \pm 0.43$
$f_0(980)\rho^+$	$3.99 \pm 0.13 \pm 0.07$	$2.57 \pm 0.44 \pm 0.20$
$f_2(1270)\rho^+$	$1.11 \pm 0.10 \pm 0.10$	$1.94 \pm 0.36 \pm 0.12$
$(\rho^+\rho^0)_S$	$1.10 \pm 0.18 \pm 0.10$	$0.71 \pm 0.25 \pm 0.12$
$(\rho(1450)^+\rho^0)_S$	$0.43 \pm 0.18 \pm 0.17$	$0.94 \pm 0.27 \pm 0.16$
$(\rho^+\rho(1450)^0)_P$	$4.58 \pm 0.16 \pm 0.09$	$1.75 \pm 0.27 \pm 0.08$
$\phi((\rho\pi) \to \pi^+\pi^-\pi^0)\pi^+$	$2.90 \pm 0.15 \pm 0.18$	$5.08 \pm 0.32 \pm 0.10$
$\omega((\rho\pi) \to \pi^+\pi^-\pi^0)\pi^+$	$3.22 \pm 0.21 \pm 0.09$	$1.41 \pm 0.17 \pm 0.06$
$a_1^+(\rho^0\pi^+)_S\pi^0$	$3.78 \pm 0.16 \pm 0.12$	$2.55 \pm 0.34 \pm 0.20$
$a_1^0((\rho\pi)_S \to \pi^+\pi^-\pi^0)\pi^+$	$4.82 \pm 0.15 \pm 0.12$	$1.29 \pm 0.39 \pm 0.24$
$\pi(1300)^0((\rho\pi)_P \to \pi^+\pi^-\pi^0)\pi^+$	$2.22 \pm 0.14 \pm 0.08$	$2.39 \pm 0.48 \pm 0.45$

W-annihilation decay
$BF = (1.92 \pm 0.30) \times 10^{-3}$
(PDG)

Amplitude analysis of $D_s^+ \to \pi^+\pi^+\pi^-\pi^0\pi^0$
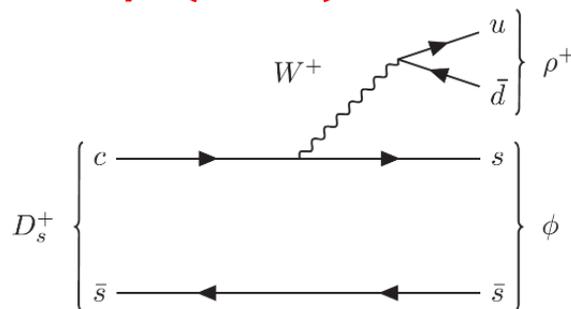
7.33fb^{-1} @ 4.178 − 4.226GeV , PRL 134, 201902 (2025) | 1888 candidates with >79% purity

✿ Observation of $D_s^+ \to \omega\rho(770)^+$
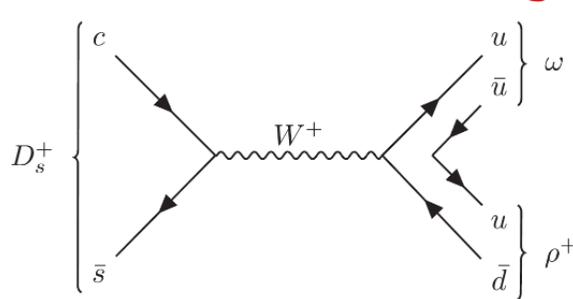
Pure *W*-annihilation   Pure *W*-external emission

✿ Polarization puzzle

Amplitude	BF (%)
$D_s^+ \to \rho(1450)^+\pi^0$, $\rho(1450)^+ \to \omega\pi^+$	$0.39 \pm 0.04 \, ^{+0.03}_{-0.03}$
$D_s^+[S] \to a_1(1260)^0\rho^+$, $a_1(1260)^0[S] \to \rho^+\pi^-$	$0.23 \pm 0.02 \, ^{+0.01}_{-0.01}$
$D_s^+[P] \to a_1(1260)^0\rho^+$, $a_1(1260)^0[S] \to \rho^+\pi^-$	$0.50 \pm 0.04 \, ^{+0.02}_{-0.02}$
$D_s^+ \to a_1(1260)^0\rho^+$, $a_1(1260)^0 \to \rho^+\pi^-$	$0.50 \pm 0.04 \, ^{+0.02}_{-0.02}$
$D_s^+[S] \to a_1(1260)^0\rho^+$, $a_1(1260)^0[S] \to \rho^-\pi^+$	$0.16 \pm 0.02 \, ^{+0.01}_{-0.01}$
$D_s^+[P] \to a_1(1260)^0\rho^+$, $a_1(1260)^0[S] \to \rho^-\pi^+$	$0.17 \pm 0.01 \, ^{+0.01}_{-0.01}$
$D_s^+ \to a_1(1260)^0\rho^+$, $a_1(1260)^0 \to \rho^-\pi^+$	$0.33 \pm 0.02 \, ^{+0.02}_{-0.02}$
$D_s^+[S] \to a_1(1260)^+\rho^0$, $a_1(1260)^+[S] \to \rho^+\pi^0$	$0.41 \pm 0.05 \, ^{+0.05}_{-0.05}$
$D_s^+[P] \to a_1(1260)^+\rho^0$, $a_1(1260)^+[S] \to \rho^+\pi^0$	$0.31 \pm 0.02 \, ^{+0.02}_{-0.02}$
$D_s^+ \to a_1(1260)^+\rho^0$, $a_1(1260)^+ \to \rho^+\pi^0$	$0.73 \pm 0.07 \, ^{+0.07}_{-0.07}$
$D_s^+ \to b_1(1235)^+\pi^0$, $b_1(1235)^+[S] \to \omega\pi^+$	$0.53 \pm 0.05 \, ^{+0.03}_{-0.03}$
$D_s^+ \to b_1(1235)^0\pi^+$, $b_1(1235)^0[S] \to \omega\pi^0$	$0.72 \pm 0.06 \, ^{+0.05}_{-0.05}$

Amplitude	BF (%)	
$D_s^+[S] \to \omega\rho^+$	$0.30 \pm 0.07 \, ^{+0.02}_{-0.03}$	
$D_s^+[P] \to \omega\rho^+$	$0.25 \pm 0.04 \, ^{+0.04}_{-0.04}$	*D>S>P*
$D_s^+[D] \to \omega\rho^+$	$0.52 \pm 0.07 \, ^{+0.04}_{-0.07}$	
$D_s^+ \to \omega\rho^+$	$0.99 \pm 0.08 \, ^{+0.05}_{-0.07}$	
$D_s^+[S] \to \phi\rho^+$	$3.32 \pm 0.29 \, ^{+0.19}_{-0.17}$	*S>P>D*
$D_s^+[P] \to \phi\rho^+$	$0.63 \pm 0.12 \, ^{+0.05}_{-0.06}$	
$D_s^+ \to \phi\rho^+$	$3.98 \pm 0.33 \, ^{+0.21}_{-0.19}$	

Naive prediction: PRL 128,011803

transverse dominates than longitudinal in charm decays
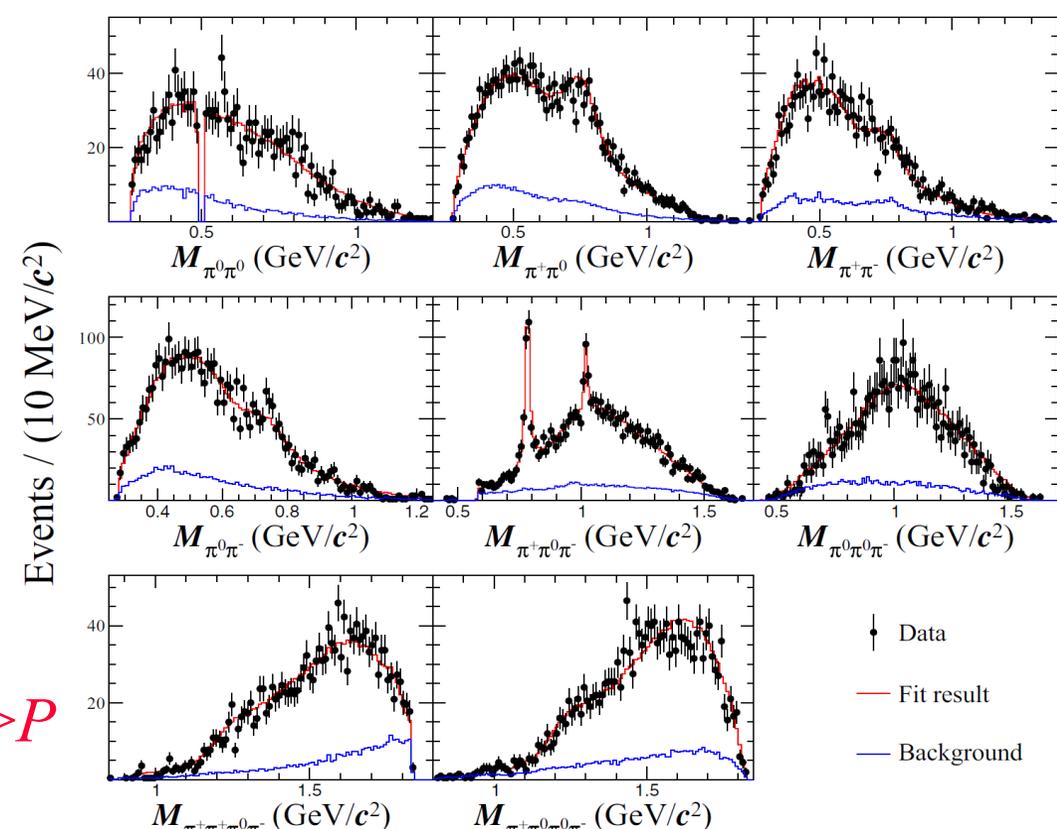
$$\frac{\mathcal{B}(\phi(1020) \to \pi^+\pi^-\pi^0)}{\mathcal{B}(\phi(1020) \to K^+K^-)} = 0.222 \pm 0.019_{\text{stat}} \pm 0.016_{\text{syst}}$$

Taking from $D_s^+ \to K^+K^-\pi^+\pi^0$
BESIII, PRD 104, 032011 (2021)

Deviates from PDG value (0.313 ± 0.010) by >3σ

6

Amplitude analysis of $D_s^+ \to \eta\pi^+\pi^+\pi^-$

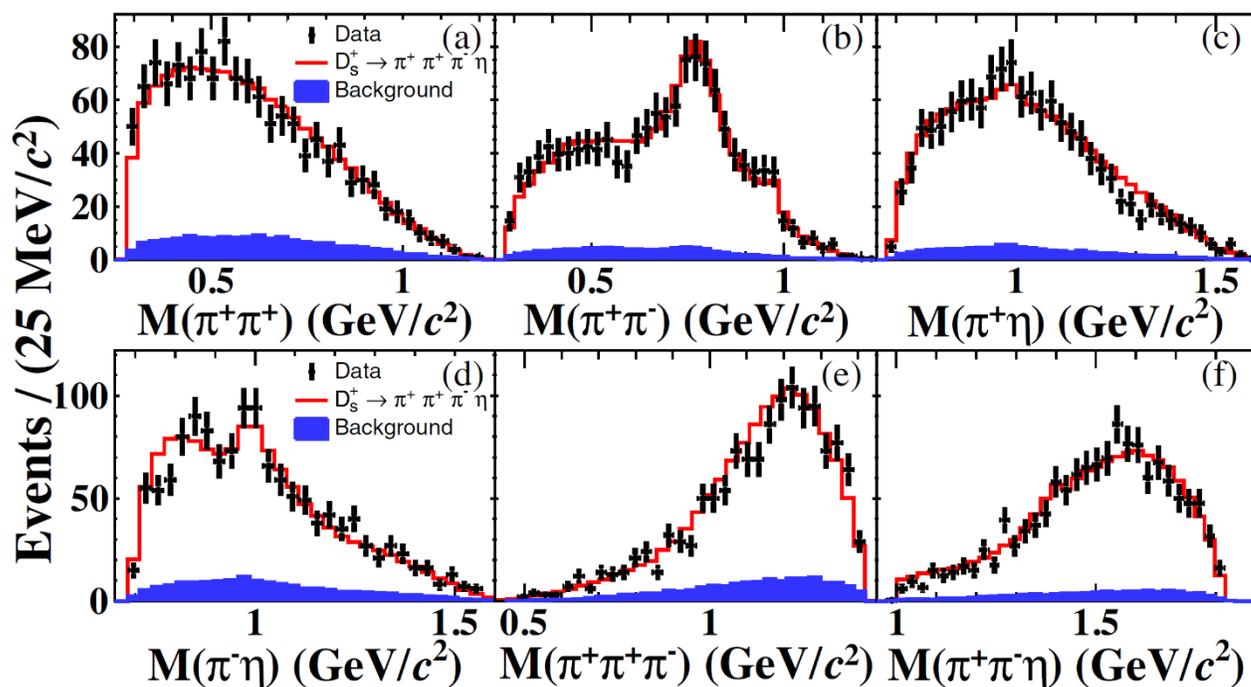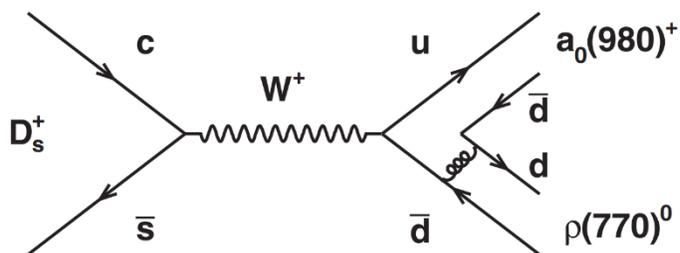⟳ $D_s^+ \to \eta\pi^+\pi^+\pi^-$

$6.32\text{fb}^{-1}@E_{cm} = 4.178 - 4.226\text{GeV}$

1306 candidates with >85% purity

Observe W-annihilation decay $D_s^+ \to a_0(980)^+\rho(770)^0$

- First measurement:

$$\mathcal{B}_{D_s^+\to\eta\pi^+\pi^+\pi^-} = (3.12 \pm 0.13 \pm 0.09)\%$$

- Dominant process:

$$\mathcal{B}_{D_s^+\to a_1(1260)^+\eta, a_1(1260)^+\to\rho^0\pi^+}$$
$$= (1.73 \pm 0.14 \pm 0.08)\%$$

- W-annihilation contribution:

$$\mathcal{B}_{D_s^+\to a_0(980)^+\rho^0, a_0(980)^+\to\eta\pi^+}$$
$$= (0.21 \pm 0.08 \pm 0.05)\%$$

Significantly larger than the BFs of most other measured pure W-annihilation decays

Amplitude analysis of $D_s^+ \to K_S^0 K_L^0 \pi^+$

7.33fb^{-1} @ $4.178 - 4.226\text{GeV}$
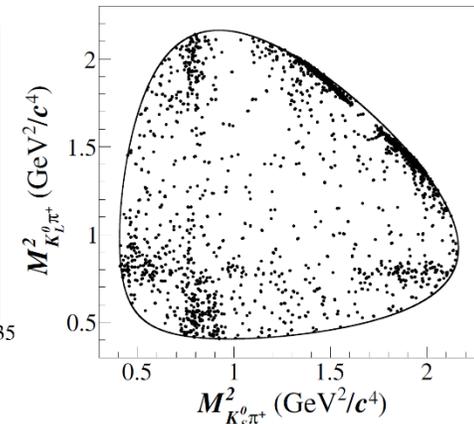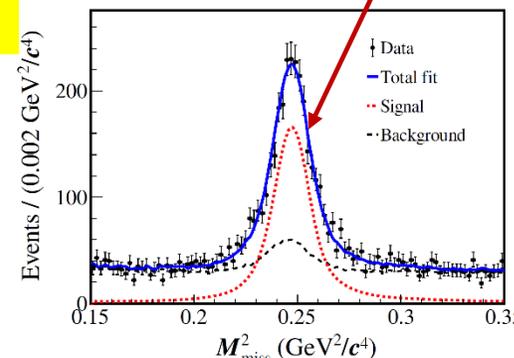
2310 candidates with >78% purity

Measurement of $\phi \to K_S^0 K_L^0$ PRL 135, 161902 (2025)

$$\mathcal{B}_{D_s^+ \to K_S^0 K_L^0 \pi^+} = (1.86 \pm 0.06_{\text{stat}} \pm 0.003_{\text{syst}})\%$$

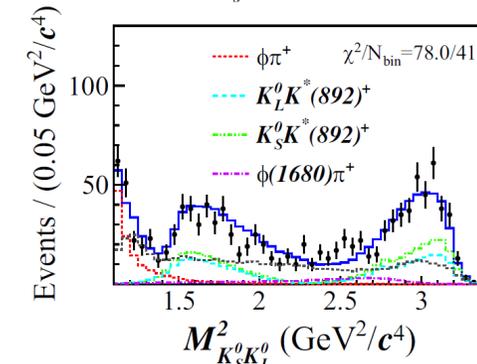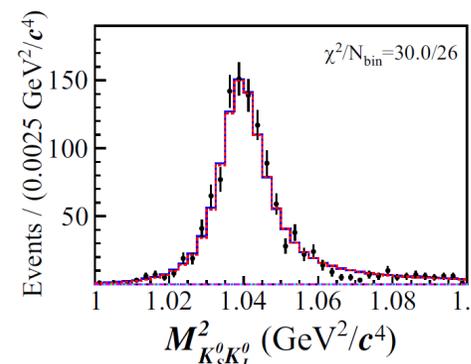| Amplitude	Phase (rad)	BF (%)
$D_s^+ \to \phi\pi^+$	0.0 (fixed)	$1.31 \pm 0.05 \pm 0.03$
$D_s^+ \to K_L^0 K^*(892)^+$	$1.10 \pm 0.20 \pm 0.20$	$0.36 \pm 0.03 \pm 0.02$
$D_s^+ \to K_S^0 K^*(892)^+$	$-1.93 \pm 0.20 \pm 0.25$	$0.27 \pm 0.02 \pm 0.02$

$$\frac{\mathcal{B}(\phi(1020) \to K_S^0 K_L^0)}{\mathcal{B}(\phi(1020) \to K^+K^-)} = (0.593 \pm 0.023_{\text{stat}} \pm 0.014_{\text{syst}} \pm 0.016_{\phi\pi})$$

Taking from PDG

$\mathcal{B}(D_s^+ \to \phi\pi^+, \phi \to K^+K^-)$

Deviates from PDG value (0.740±0.031) by >3σ
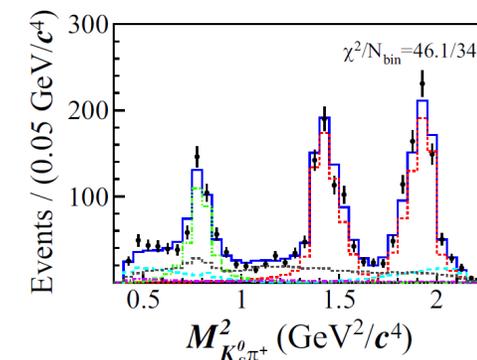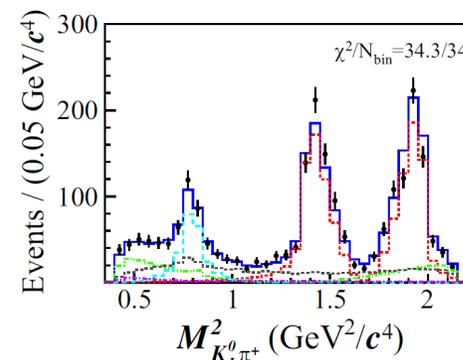
First observation of $K_S^0 - K_L^0$ asymmetry

$$\frac{\mathcal{B}(D_s^+ \to K_S^0 K^*(892)^+) - \mathcal{B}(D_s^+ \to K_L^0 K^*(892)^+)}{\mathcal{B}(D_s^+ \to K_S^0 K^*(892)^+) + \mathcal{B}(D_s^+ \to K_L^0 K^*(892)^+)} = (-14.5 \pm 5.1_{\text{stat}} \pm 1.8_{\text{syst}})\%$$

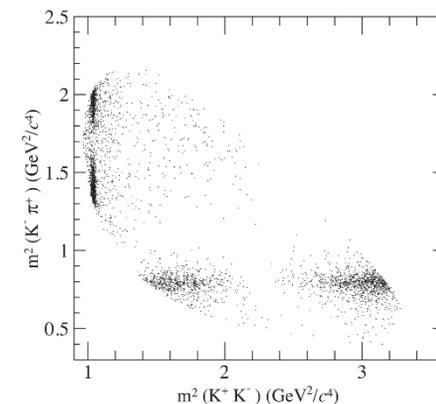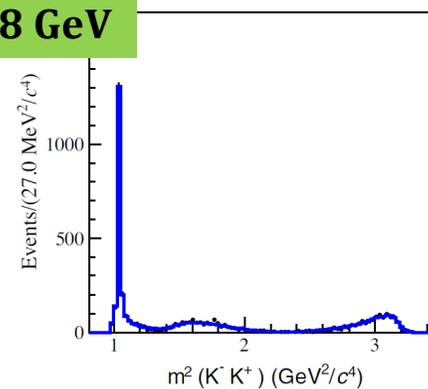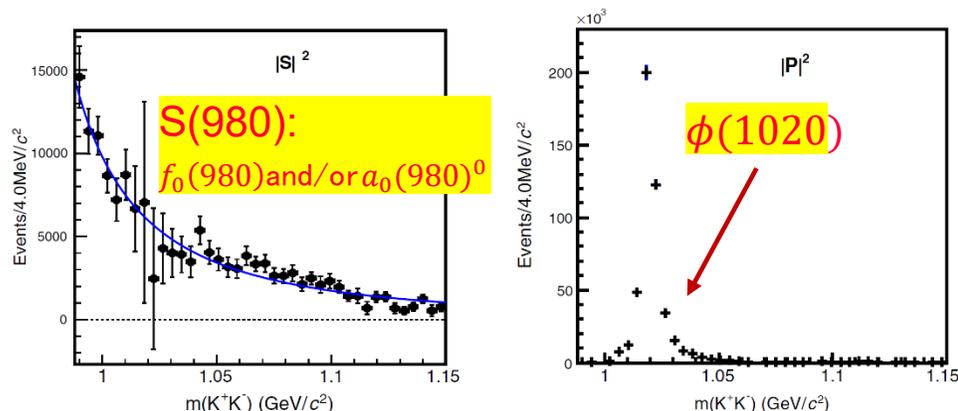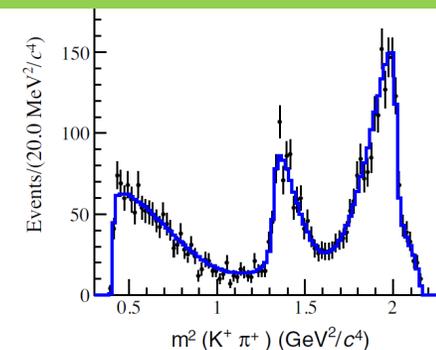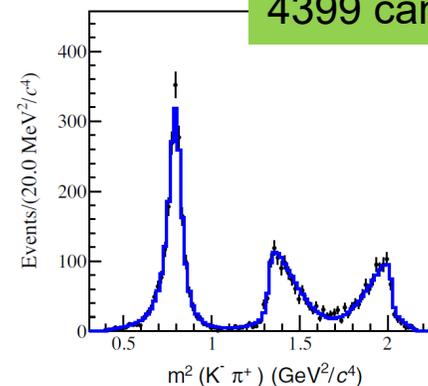| Model	DAT(F4)	DAT(F1')
$D_s^+ \to \bar{K}^0 K^{*+}$	-0.164 ± 0.032	-0.159 ± 0.028

Predictions by H.-Y. Cheng et al., PRD109, 073008（2024）

Amplitude analysis of $D_s^+ \to K^+ K^- \pi^+$

🌀 $D_s^+ \to K^+ K^- \pi^+$ PRD 104, 012016 (2021) $3.19\text{fb}^{-1}@E_{\text{cm}} = 4.178$ GeV

Gold normalization channel, 4399 candidates with 99.6% purity.

Model-independent PWA to determine the KK S-wave line shape:

S(980):
$f_0(980)$ and/or $a_0(980)^0$

$\phi(1020)$

4399 candidates with 99.6% purity

Measured BFs and quoted from PDG:

| Process	BF (%)	PDG 2024
$D_s^+ \to \bar{K}^*(892)^0 K^+,\ \bar{K}^*(892)^0 \to K^- \pi^+$	$2.64 \pm 0.06_{\text{stat}} \pm 0.07_{\text{sys}}$	2.58 ± 0.06
$D_s^+ \to \phi(1020)\pi^+,\ \phi(1020) \to K^+ K^-$	$2.21 \pm 0.05_{\text{stat}} \pm 0.07_{\text{sys}}$	2.21 ± 0.06
$D_s^+ \to S(980)\pi^+,\ S(980) \to K^+ K^-$	$1.05 \pm 0.04_{\text{stat}} \pm 0.06_{\text{sys}}$	1.11 ± 0.19
$D_s^+ \to \bar{K}_0^*(1430)^0 K^+,\ \bar{K}_0^*(1430)^0 \to K^- \pi^+$	$0.16 \pm 0.03_{\text{stat}} \pm 0.03_{\text{sys}}$	0.18 ± 0.03
$D_s^+ \to f_0(1710)\pi^+,\ f_0(1710) \to K^+ K^-$	$0.10 \pm 0.02_{\text{stat}} \pm 0.03_{\text{sys}}$	0.07 ± 0.03
$D_s^+ \to f_0(1370)\pi^+,\ f_0(1370) \to K^+ K^-$	$0.07 \pm 0.02_{\text{stat}} \pm 0.01_{\text{sys}}$	0.07 ± 0.03
$D_s^+ \to K^+ K^- \pi^+$ total BF	$5.47 \pm 0.08_{\text{stat}} \pm 0.13_{\text{sys}}$	5.37 ± 0.10

BFs of Intermediate processes:

$$\mathcal{B}(D_s^+ \to \bar{K}^*(892)^0 K^+) = (3.94 \pm 0.12)\%$$

$$\mathcal{B}(D_s^+ \to \phi(1020)\pi^+) = (4.60 \pm 0.17)\%$$

Consistent with latest calculations:

Hai-Yang Cheng *et al.*, Phys. Rev. D **109**, 114027 (2024)

Amplitude analysis of $D_s^+ \to K_S^0 K^+ \pi^0, K_S^0 K_S^0 \pi^+$
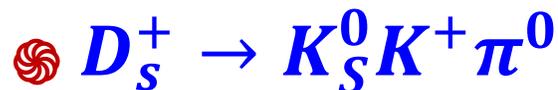
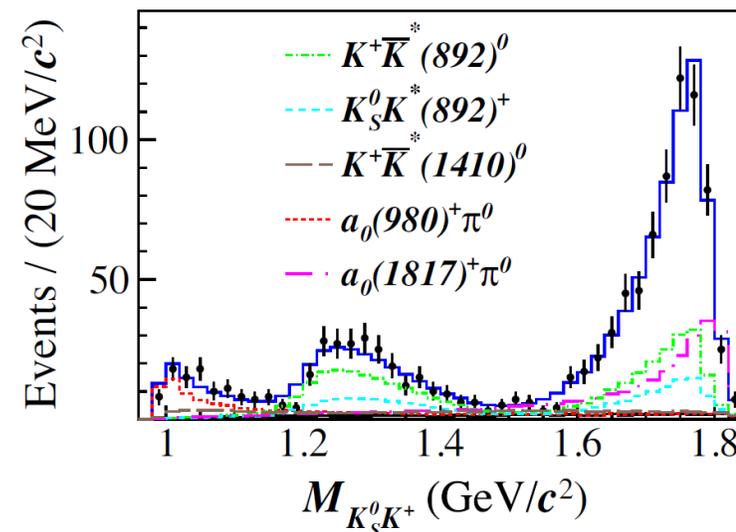❂ $D_s^+ \to K_S^0 K^+ \pi^0$ PRL 129, 182001 (2022)

Observation of $a_0(1817)^+$

$$m(a_0) = (1.817 \pm 0.008 \pm 0.020) \text{ GeV}/c^2$$
$$\Gamma(a_0) = (0.097 \pm 0.022 \pm 0.015) \text{ GeV}/c^2$$
$$\mathcal{B}(D_s^+ \to a_0(1817)^+ \pi^0, a_0(1817)^+ \to K_S^0 K^+) =$$
$$(3.44 \pm 0.52 \pm 0.32) \times 10^{-4}$$

→ 结合$D_s^+ \to K_S^0 K_S^0 \pi^+$的分析，
　　支持存在新的a_0同位旋三重态

❂ $D_s^+ \to K_S^0 K_S^0 \pi^+$ PRD 105, L051103 (2022)

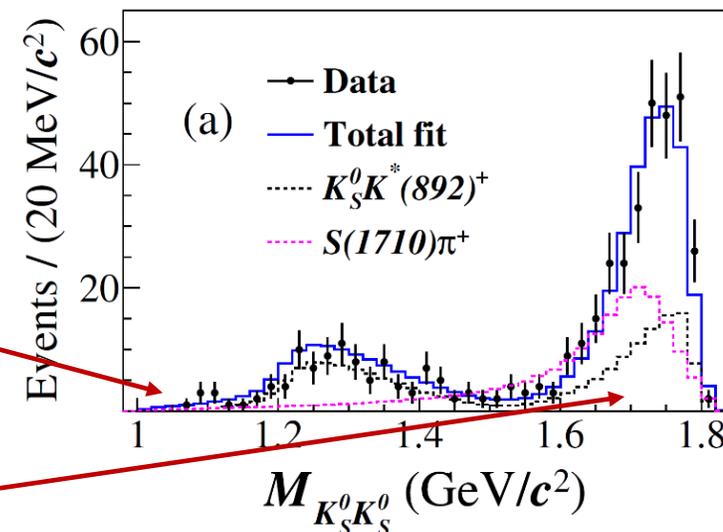$$\mathcal{B}(D_s^+ \to S(980)\pi^+) < 1.8 \times 10^{-4}$$
$$@ 90\% \text{ CL}$$

→ $f_0(980)$和$a_0(980)$
相消干涉

$$\mathcal{B}(D_s^+ \to S(1710)\pi^+) = (0.31 \pm 0.03 \pm 0.01)\%$$

比预期值大1个量级 →

$f_0(1710)$和$a_0(1710)$
相加干涉

$D_s^+ \to K^+ K^- \pi^+$, PRD 104, 012016 (2021)
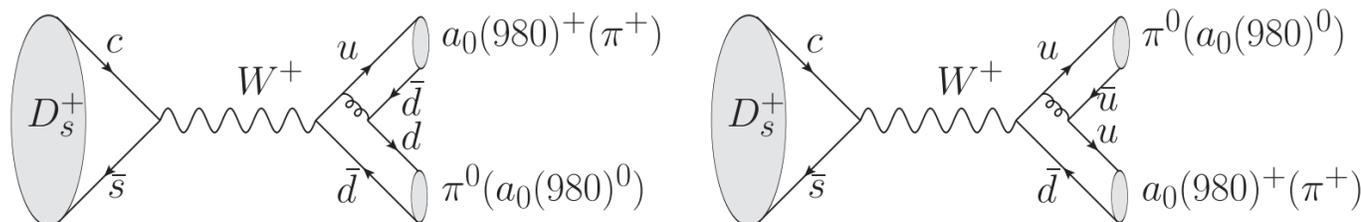
Amplitude analysis of $D_s^+ \to \pi^+ \pi^0 \eta$

⚙ $D_s^+ \to \pi^+ \pi^0 \eta$ PRL 123, 112001 (2019)

3.19fb^{-1}@$E_{cm} = 4.178$ GeV

1239 candidates with 97.7% purity

Observe W-annihilation decay:

$\mathcal{B}(D_s^+ \to a_0(980)^{+(0)} \pi^{0(+)}, a_0(980)^{+(0)} \to \pi^{+(0)} \eta)$
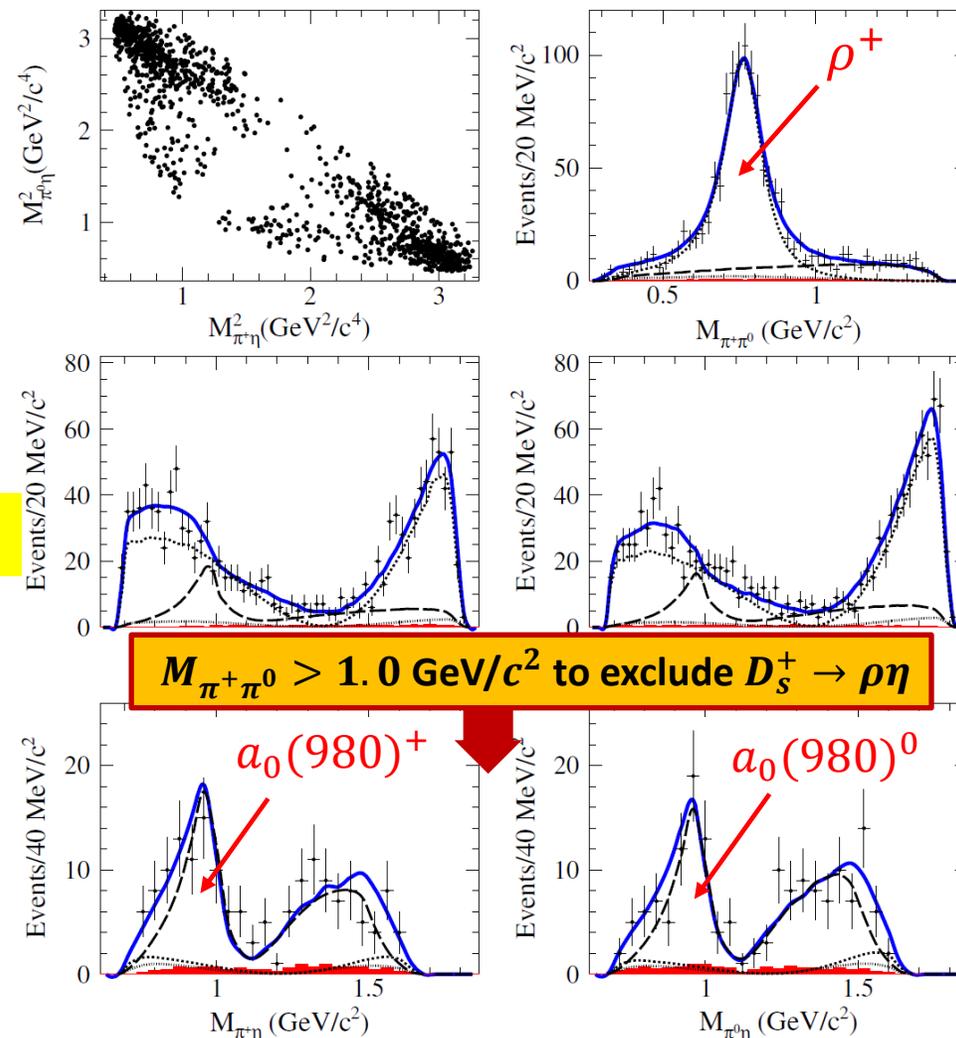$= (1.46 \pm 0.15_{stat} \pm 0.23_{sys})\%$

Significantly larger than other W-annihilation decays by one order

⚙ BF measurement:

B$(D_s^+ \to \pi^+ \pi^0 \eta) = (9.50 \pm 0.28 \pm 0.41)\%$

⚙ Dominant process:

B$(D_s^+ \to \rho^+ \eta) = (7.44 \pm 0.48 \pm 0.44)\%$

$M_{\pi^+\pi^0} > 1.0$ GeV/c^2 to exclude $D_s^+ \to \rho\eta$

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Amplitude analysis of $D^0 \to \pi^+\pi^-\eta$, $D^+ \to \pi^+\pi^0\eta$

BESⅢ

🌀 Observation of $D \to a_0(980)\pi$

7.9 fb^{-1}@$E_{cm} = 3.773$GeV, PRD 110, L111102 (2024)

$\mathcal{B}(D^0 \to \pi^+\pi^-\eta) = (1.24 \pm 0.04 \pm 0.03)\%$

$\mathcal{B}(D^+ \to \pi^+\pi^0\eta) = (2.18 \pm 0.12 \pm 0.03)\%$

Amplitude	Phase (in unit rad)	BF ($\times 10^{-3}$)
$D^0 \to \rho^0\eta$	0 (fixed)	$0.19 \pm 0.02 \pm 0.01$
$D^0 \to a_0(980)^-\pi^+$	$0.06 \pm 0.16 \pm 0.12$	$0.07 \pm 0.02 \pm 0.01$
$D^0 \to a_0(980)^+\pi^-$	$-1.06 \pm 0.12 \pm 0.10$	$0.55 \pm 0.05 \pm 0.07$
$D^0 \to a_2(1320)^+\pi^-$	$-1.16 \pm 0.25 \pm 0.23$	$0.03 \pm 0.01 \pm 0.01$
$D^0 \to a_2(1700)^+\pi^-$	$0.08 \pm 0.17 \pm 0.23$	$0.07 \pm 0.02 \pm 0.03$
$D^0 \to (\pi^+\pi^-)_{S-\text{wave}}\eta$	$-0.92 \pm 0.29 \pm 0.14$	$0.05 \pm 0.02 \pm 0.03$
$D^+ \to \rho^+\eta$	$-4.03 \pm 0.19 \pm 0.13$	$0.20 \pm 0.07 \pm 0.05$
$D^+ \to (\pi^+\pi^0)_V\eta$	$-0.64 \pm 0.22 \pm 0.19$	$0.34 \pm 0.11 \pm 0.11$
$D^+ \to a_0(980)^+\pi^0$	0 (fixed)	$0.95 \pm 0.12 \pm 0.05$
$D^+ \to a_0(980)^0\pi^+$	$2.44 \pm 0.20 \pm 0.10$	$0.37 \pm 0.10 \pm 0.04$
$D^+ \to a_2(1700)^+\pi^0$	$0.92 \pm 0.20 \pm 0.14$	$0.09 \pm 0.05 \pm 0.02$
$D^+ \to a_0(1450)^+\pi^0$	$0.63 \pm 0.41 \pm 0.30$	$0.15 \pm 0.06 \pm 0.02$

$\mathcal{B}(D^0 \to a_0(980)^+\pi^-)/\mathcal{B}(D^0 \to a_0(980)^-\pi^+) = 7.5^{+2.5}_{-0.8\,\text{stat.}} \pm 1.7_{\text{syst.}}$

$\mathcal{B}(D^+ \to a_0(980)^+\pi^0)/\mathcal{B}(D^+ \to a_0(980)^0\pi^+) = 2.6 \pm 0.6_{\text{stat.}} \pm 0.3_{\text{syst.}}$

→ **Disagrees with theoretical predictions by orders of magnitude**

$D^0 \to \pi^+\pi^-\eta$

1678 candidates with 74% purity

$D^+ \to \pi^+\pi^0\eta$
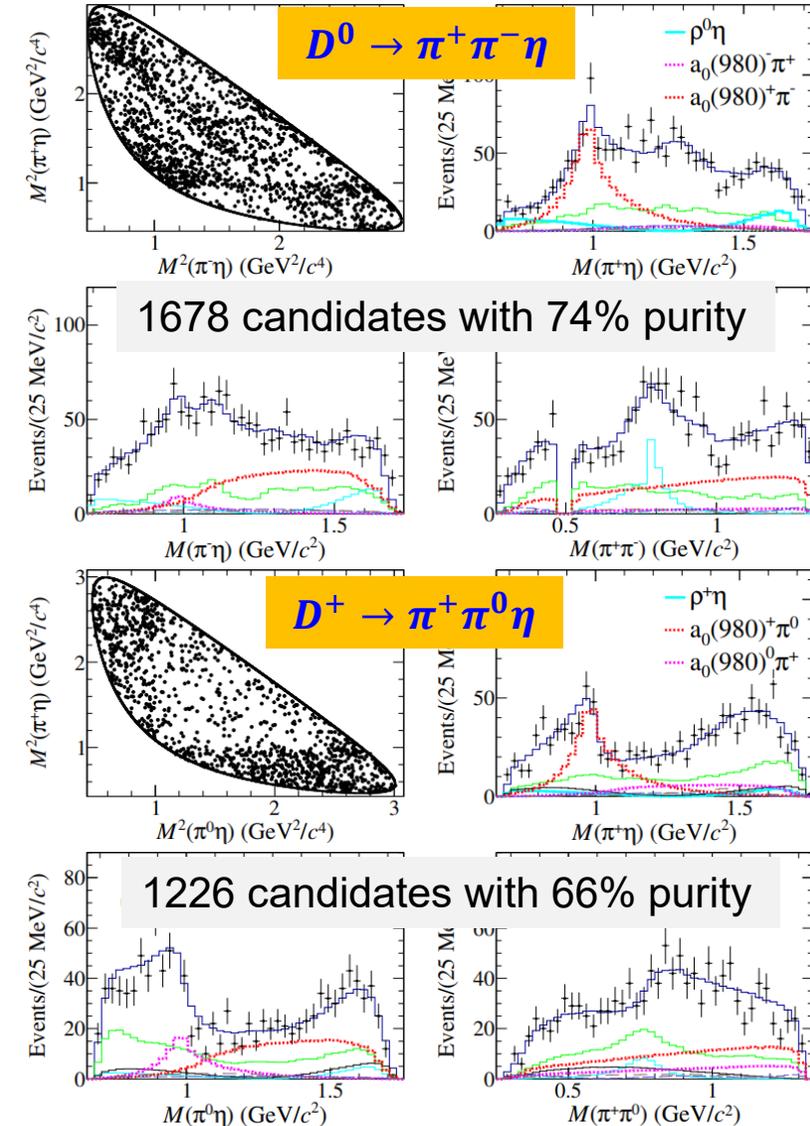
1226 candidates with 66% purity

Amplitude analysis of $D^+ \to \pi^+ \eta \eta$

BESⅢ

1624 candidates with 85% purity

🌀 First observation of an

   altered $a_0(980)$ line-shape

   due to triangle loop rescattering
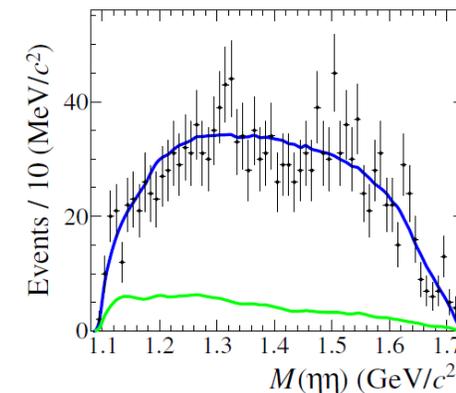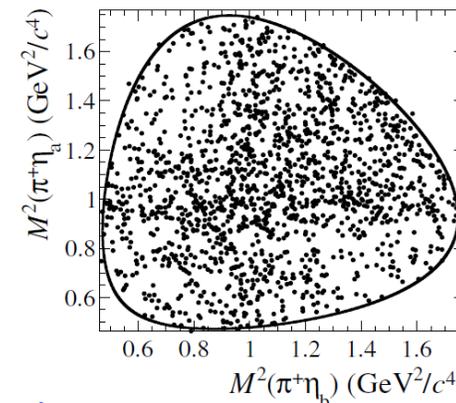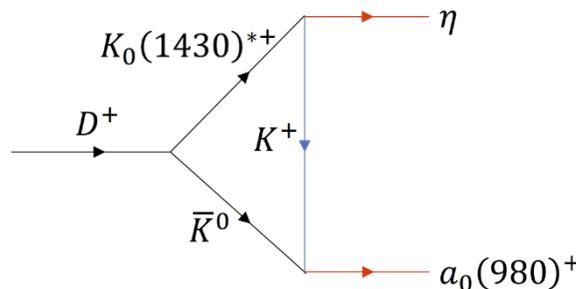
Fit1: $P_{a_0(980)}$ three-channel coupled Flatte formulae,

   the fitted pole position is inconsistent with previous measurement.

To consider the rescattering process $D^+ \to \bar{K}_0^*(1430)^0 K^+ \to a_0(980)^+ \eta$

we perform Fit2 and Fit3

Fit2: $(1 + |C|e^{i\phi_C} A_{\text{loop}}) P_{a_0(980)}$

Fit3: $(1 + |C| A_{\text{loop}}) P_{a_0(980)}$ with ϕ_C fixed to zero.

$$|C| = 0.113 \pm 0.015_{\text{stat.}} \pm 0.048_{\text{syst.}}$$

Fit2 and Fit3 give good descriptions of the altered $a_0(980)$ line-shape.

🌀 **BF measurement**

$$\left. \frac{\mathcal{B}(D^+ \to \pi^+ \eta \eta)}{\mathcal{B}(D^+ \to a_0(980)^+ \eta) \; \mathcal{B}(a_0(980)^+ \to \pi^+ \eta)} \right\} = (3.67 \pm 0.12_{\text{stat.}} \pm 0.06_{\text{syst.}}) \times 10^{-3}$$

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Amplitude analysis of $D^+ \to K_S^0 \pi^+ \eta$

BES III

2.93fb^{-1} @3.773GeV
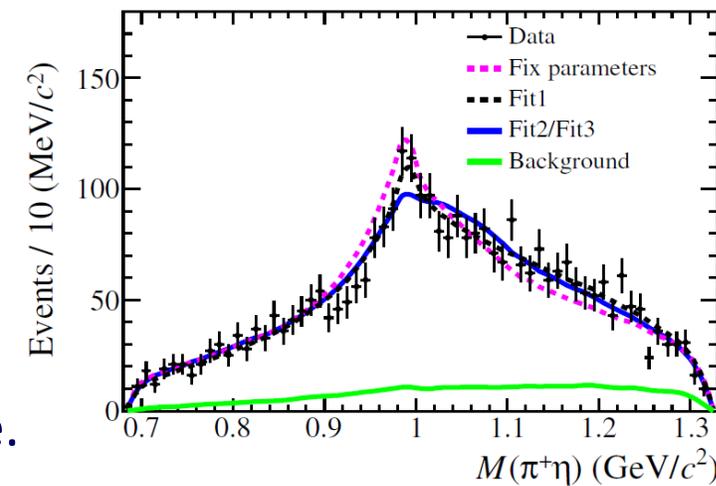
☸ Observation of W-annihilation-free decay $D^+ \to K_S^0 a_0(980)^+$

1113 candidates with 98% purity
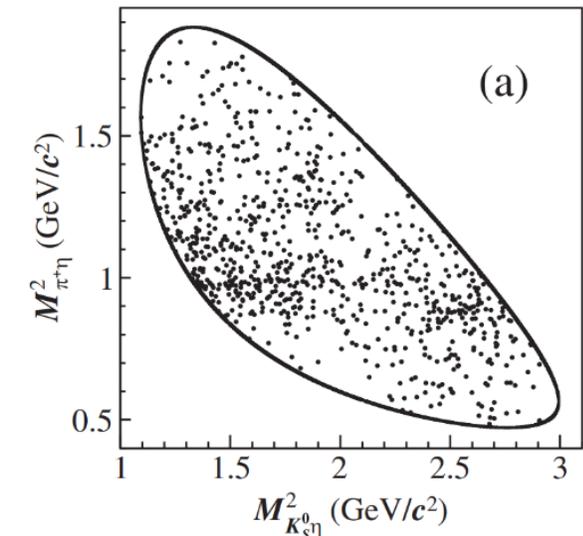
$$\mathcal{B}(D^+ \to K_S^0 a_0(980)^+, a_0(980)^+ \to \pi^+\eta) = (1.33 \pm 0.05_{\text{stat}} \pm 0.04_{\text{syst}})\%$$
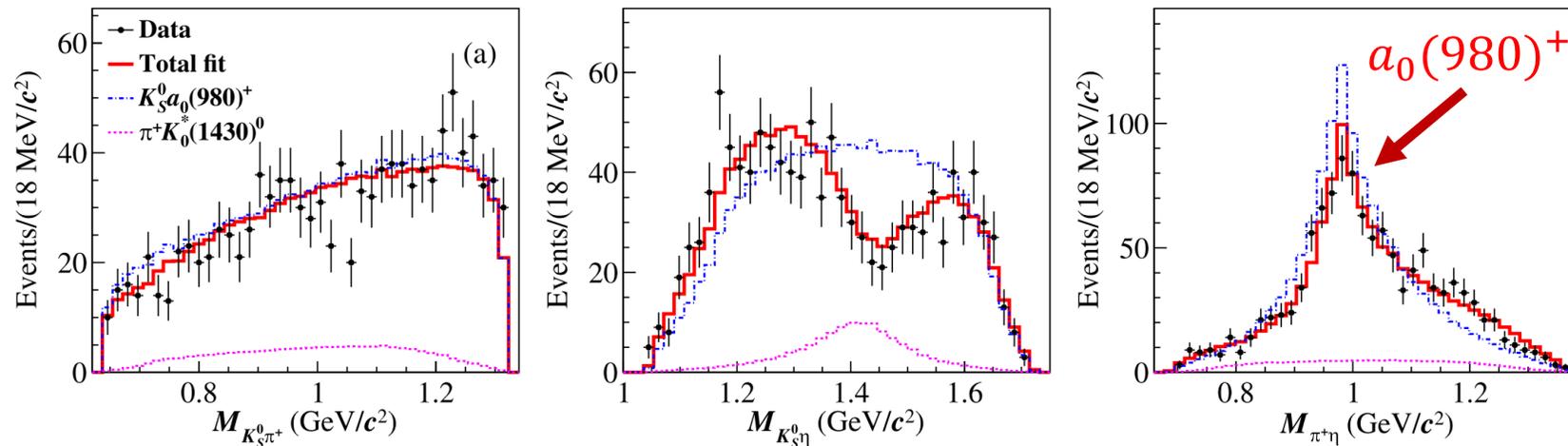
$$\mathcal{B}(D^+ \to \bar{K}_0^*(1430)^0 \pi^+, \bar{K}_0^*(1430)^0 \to K_S^0\eta) = (0.14 \pm 0.02_{\text{stat}} \pm 0.02_{\text{syst}})\%$$

$$\mathcal{B}(D^+ \to K_S^0 \pi^+ \eta) = (1.27 \pm 0.04_{\text{stat}} \pm 0.03_{\text{syst}})\%$$

Provide sensitive constraints in the extraction of contributions from internal W-emission diagrams →
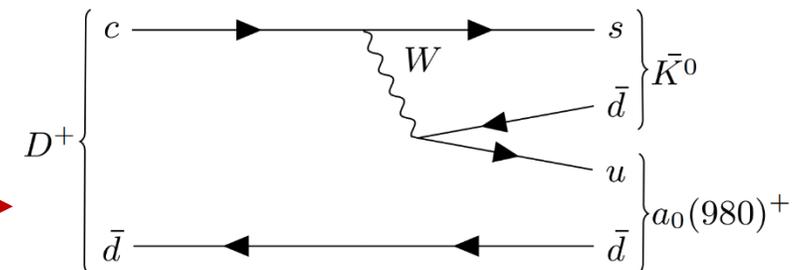
Amplitude analysis of $D^+ \rightarrow K^- \pi^+ \pi^- \pi^0$

🌀 $D^+ \rightarrow K^- \pi^+ \pi^- \pi^0$ **Golden tag mode**

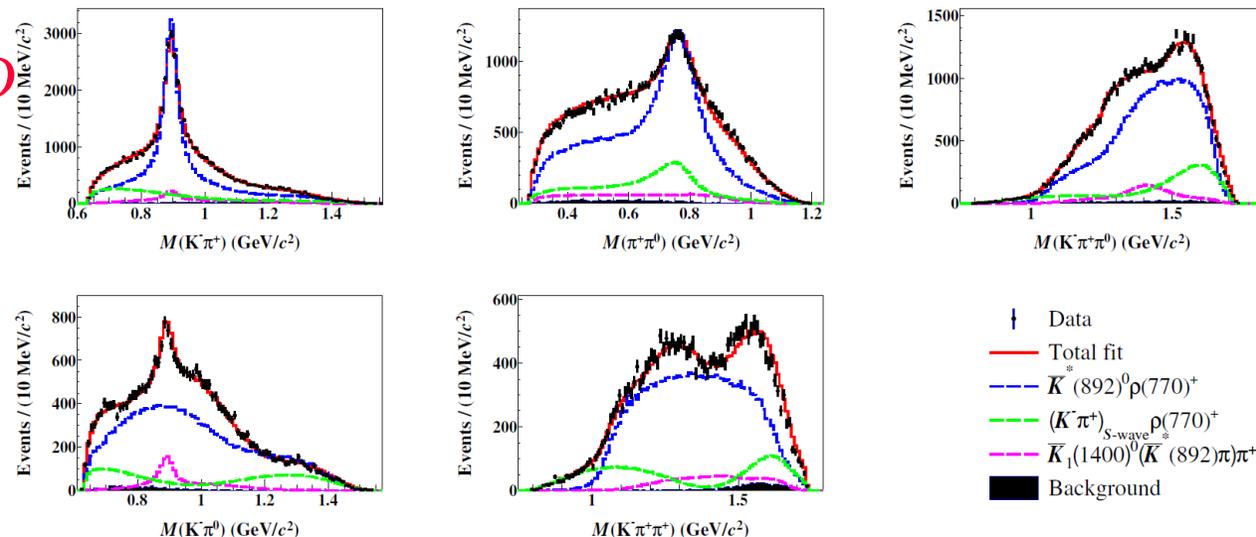7.9fb^{-1} @3.773GeV，JHEP05(2025)195

Amplitude	Phase (rad)	FFs (%)
$D^+[S] \rightarrow \bar{K}^*(892)^0 \rho(770)^+$	0.0(fixed)	$66.5 \pm 1.1 \pm 3.0$
$D^+[P] \rightarrow \bar{K}^*(892)^0 \rho(770)^+$	$1.45 \pm 0.04 \pm 0.08$	$1.9 \pm 0.2 \pm 0.2$
$D^+ \rightarrow \bar{K}^*(892)^0 \rho(770)^+$	—	$68.4 \pm 1.1 \pm 2.6$
$D^+ \rightarrow \bar{K}_1(1270)^0[S]\pi^+$, $\bar{K}_1(1270)^0 \rightarrow K^- \rho(770)^+$	$-0.09 \pm 0.03 \pm 0.03$	$3.8 \pm 0.3 \pm 0.3$
$D^+ \rightarrow \bar{K}_1(1400)^0[S]\pi^+$	$0.40 \pm 0.02 \pm 0.04$	$7.5 \pm 0.2 \pm 0.3$
$D^+ \rightarrow \bar{K}_1(1400)^0[D]\pi^+$	$-2.42 \pm 0.04 \pm 0.04$	$0.5 \pm 0.1 \pm 0.1$
$D^+ \rightarrow \bar{K}_1(1400)^0 \pi^+$, $\bar{K}_1(1400)^0 \rightarrow \bar{K}^*(892)\pi$	—	$7.3 \pm 0.2 \pm 0.3$
$D^+ \rightarrow \bar{K}(1460)^0 \pi^+$, $\bar{K}(1460)^0 \rightarrow \bar{K}^*(892)\pi$	$0.41 \pm 0.04 \pm 0.07$	$5.1 \pm 0.2 \pm 0.3$
$D^+ \rightarrow \bar{K}^*(1680)^0 \pi^+$, $\bar{K}^*(1680)^0 \rightarrow \bar{K}^*(892)\pi$	$1.13 \pm 0.04 \pm 0.09$	$3.8 \pm 0.4 \pm 0.8$
$D^+ \rightarrow (K^- \pi^+)_{S-wave} \rho(770)^+$	$2.90 \pm 0.02 \pm 0.04$	$18.3 \pm 0.7 \pm 0.7$
$D^+ \rightarrow \bar{K}(1460)^0 \pi^+$, $\bar{K}(1460)^0 \rightarrow K^- [\pi^+ \pi^0]^{L=1}\pi$	$-1.29 \pm 0.08 \pm 0.06$	$8.6 \pm 0.8 \pm 0.5$
$D^+ \rightarrow \bar{K}(1460)^0 \pi^+$, $\bar{K}(1460)^0 \rightarrow [K^- \pi^-]^{L=1}\pi$	$-2.31 \pm 0.07 \pm 0.06$	$3.4 \pm 0.5 \pm 0.3$

$S>P>D$

$$\mathcal{B}(D^+ \rightarrow K^- \pi^+ \pi^+ \pi^0) = (6.35 \pm 0.04_{stat.} \pm 0.07_{syst.})\%$$

Decay channel and Collaboration	$\mathcal{B}(D^+ \rightarrow \bar{K}^*(892)^0 \rho(770)^+)$ ($\times 10^{-2}$)
$D^+ \rightarrow K^- \pi^+ \pi^+ \pi^0$, current analysis	$6.52 \pm 0.11 \pm 0.26$
$D^+ \rightarrow K^- \pi^+ \pi^+ \pi^0$, MARK-III [4]	$7.2 \pm 1.8 \pm 2.1$
$D^+ \rightarrow K^0_S \pi^+ \pi^0 \pi^0$, BESIII [21]	$5.82 \pm 0.49 \pm 0.29$

Data
Total fit
$\bar{K}^*(892)^0 \rho(770)^+$
$(K^- \pi^+)_{S-wave} \rho(770)^+$
$\bar{K}_1(1400)^0 (\bar{K}^*(892)\pi)^+ \pi^+$
Background

$D \rightarrow K3\pi$ 7个四体衰变振幅分析研究现状

$D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$	**BESIII** PRD95, 072010 (2017); LHCb EPJC 78, 443 (2018)
$D^0 \rightarrow K^0_S \pi^+ \pi^- \pi^0$	在研：memo审核阶段
$D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$	**BESIII** PRD99, 092008 (2019)
$D^0 \rightarrow K^0_S \pi^0 \pi^0 \pi^0$	在研：memo审核阶段
$D^+ \rightarrow K^0_S \pi^+ \pi^+ \pi^-$	**BESIII** PRD100, 072008(2019)
$D^+ \rightarrow K^- \pi^+ \pi^+ \pi^0$	**BESIII** **JHEP05 (2025) 195**
$D^+ \rightarrow K^0_S \pi^+ \pi^0 \pi^0$	**BESIII** JHEP 09 (2023) 077

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BESIII

$D^0 \to \pi^+\pi^-\pi^+\pi^-, \pi^+\pi^-\pi^0\pi^0$

CPC 48, 083001 (2024)   2.93 fb^{-1} @3.773GeV

Component	Branching fraction (%)	
	$\pi^+\pi^-\pi^+\pi^-$	$\pi^+\pi^-\pi^0\pi^0$
$D^0 \to a_1(1260)^+\pi^-$	$0.566 \pm 0.024 \pm 0.008 \pm 0.110$	$0.546 \pm 0.027 \pm 0.011 \pm 0.069$
$D^0 \to a_1(1260)^-\pi^+$	$0.071 \pm 0.010 \pm 0.001 \pm 0.017$	$0.068 \pm 0.011 \pm 0.001 \pm 0.021$
$D^0 \to a_1(1260)^0\pi^0$	-	$0.313 \pm 0.031 \pm 0.007 \pm 0.082$
$D^0 \to a_1(1640)^+\pi^-$	$0.012 \pm 0.003 \pm 0.000 \pm 0.006$	$0.010 \pm 0.003 \pm 0.000 \pm 0.007$
$D^0 \to h_1(1170)^0\pi^0$	-	$0.012 \pm 0.006 \pm 0.000 \pm 0.010$
$D^0 \to \pi(1300)^+\pi^-$	$0.222 \pm 0.018 \pm 0.003 \pm 0.031$	$0.148 \pm 0.014 \pm 0.003 \pm 0.025$
$D^0 \to \pi(1300)^-\pi^+$	$0.162 \pm 0.016 \pm 0.002 \pm 0.028$	$0.108 \pm 0.011 \pm 0.002 \pm 0.021$
$D^0 \to \pi(1300)^0\pi^0$	-	$0.221 \pm 0.027 \pm 0.005 \pm 0.033$
$D^0 \to \pi_2(1670)^0\pi^0$	-	$0.010 \pm 0.002 \pm 0.000 \pm 0.004$
$D^0 \to \rho(770)^0\rho(770)^0$	$0.193 \pm 0.013 \pm 0.003 \pm 0.022$	-
$D^0 \to \rho(770)^0\rho(770)^0[S]$	$0.012 \pm 0.004 \pm 0.000 \pm 0.003$	-
$D^0 \to \rho(770)^0\rho(770)^0[P]$	$0.067 \pm 0.007 \pm 0.001 \pm 0.006$	-
$D^0 \to \rho(770)^0\rho(770)^0[D]$	$0.159 \pm 0.015 \pm 0.002 \pm 0.017$	-
$D^0 \to \rho(770)^0\rho(1450)^0$	$0.017 \pm 0.006 \pm 0.000 \pm 0.008$	-
$D^0 \to \rho(770)^0\rho(1450)^0[P]$	$0.007 \pm 0.003 \pm 0.000 \pm 0.003$	-
$D^0 \to \rho(770)^0\rho(1450)^0[D]$	$0.010 \pm 0.006 \pm 0.000 \pm 0.008$	-
$D^0 \to \rho(770)^+\rho(770)^-$	-	$0.864 \pm 0.040 \pm 0.018 \pm 0.075$
$D^0 \to \rho(770)^+\rho(770)^-[S]$	-	$0.124 \pm 0.019 \pm 0.003 \pm 0.033$
$D^0 \to \rho(770)^+\rho(770)^-[P]$	-	$0.186 \pm 0.013 \pm 0.004 \pm 0.019$
$D^0 \to \rho(770)^+\rho(770)^-[D]$	-	$0.342 \pm 0.029 \pm 0.007 \pm 0.024$
$D^0 \to \rho(770)^+\rho(1450)^-[D]$	-	$0.016 \pm 0.008 \pm 0.000 \pm 0.016$

D>P>S

$D^0 \to \pi^+\pi^-\pi^+\pi^-$

+ data
— Fit
$a_1(1260)\pi$
$\pi(1300)\pi$
$(\pi^+\pi^-)_S(\pi^+\pi^-)_S$
$\rho(770)^0\rho(770)^0$
others
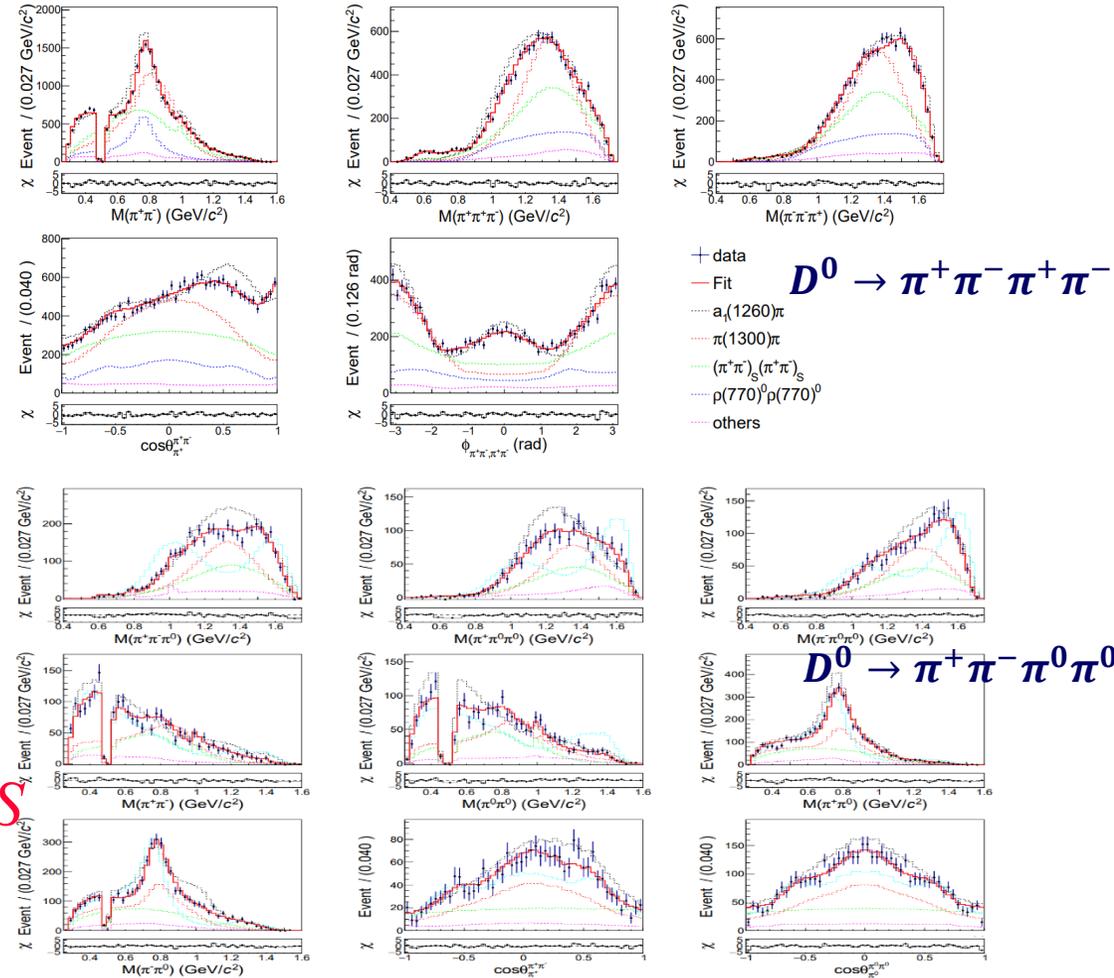
$D^0 \to \pi^+\pi^-\pi^0\pi^0$

	Branching fractions	PDG
$D^0 \to \pi^+\pi^-\pi^+\pi^-$	$(0.688 \pm 0.010 \pm 0.010)\%$	$(0.756 \pm 0.020)\%$
$D^0 \to \pi^+\pi^-\pi^0\pi^0$ (non-η)	$(0.951 \pm 0.025 \pm 0.021)\%$	$(1.005 \pm 0.090)\%$

CP-even fractions

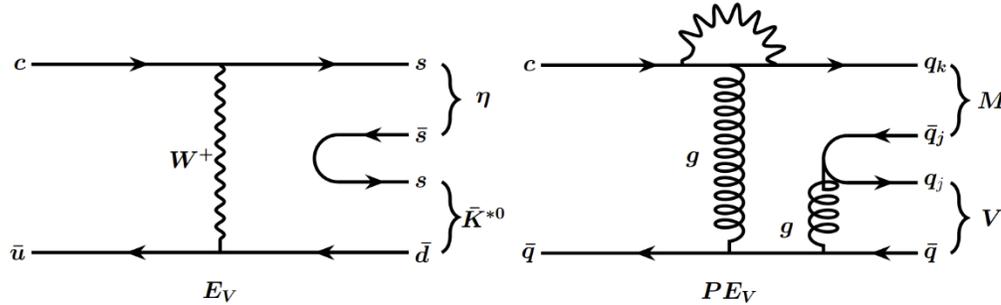| $F_+^{\pi^+\pi^-\pi^+\pi^-}$	$F_+^{\pi^+\pi^-\pi^0\pi^0}$ (non-η)
$(75.2 \pm 1.1_{\text{stat.}} \pm 1.5_{\text{syst.}})\%$	$(68.9 \pm 1.5_{\text{stat.}} \pm 2.4_{\text{syst.}})\%$

Amplitude analysis of $D^0 \to K_S^0 \pi^0 \eta$

$D^0 \to \eta \bar{K}^*(892)^0$

20.3fb^{-1} @ 3.773GeV, arXiv:2512.23389

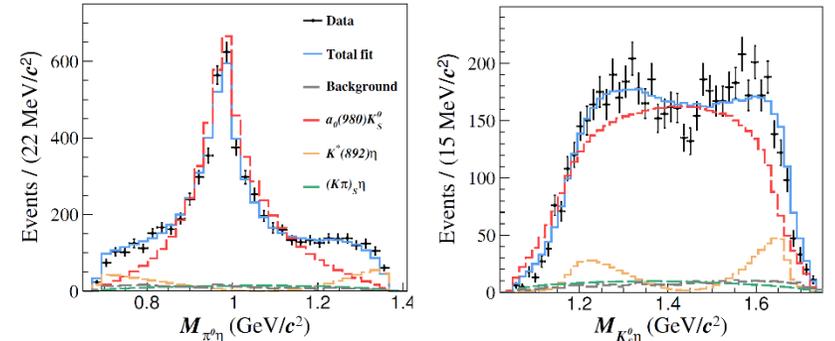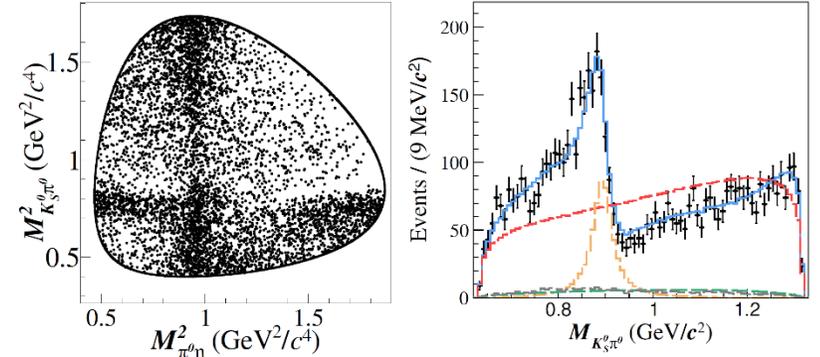6070 candidates with 94.6$\pm$ 0.5% purity

Impacts predictions from the topological diagram approach,

affecting CP violation and DCS branching fraction estimates.

The BF determines the magnitude of E_V

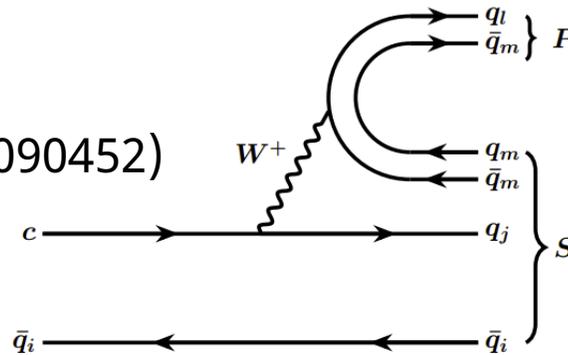E_V determines the magnitude of the penguin diagram PE_V

PE_V affects predictions for $\overline{DCS}$ decays and CP violation calculations.

$D^0 \to K_S^0 a_0(980)^0$

→ **Consistent with prediction** (PRD 110, 090452) assuming $a_0(980)^0$ **tetraquark state.**

→ **Helps determine the T-like diagram contribution under the tetraquark scenario.**

$$\mathcal{B}(D^0 \to K_S^0 a_0(980)^0, a_0(980)^0 \to \pi^0 \eta) = (9.88 \pm 0.37_{\text{stat.}} \pm 0.42_{\text{syst.}}) \times 10^{-3}$$
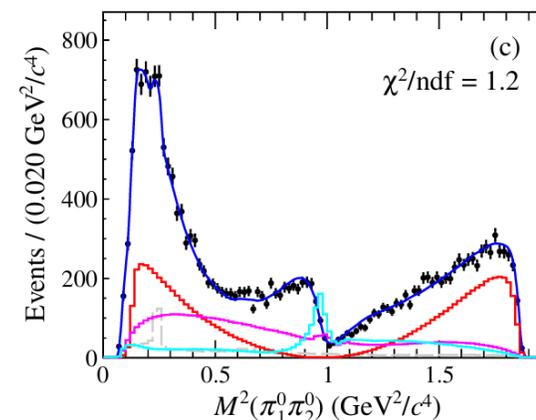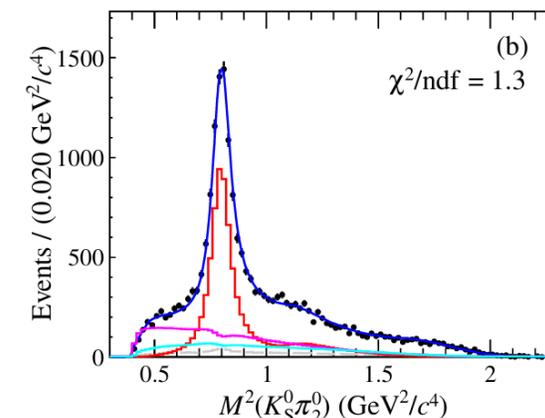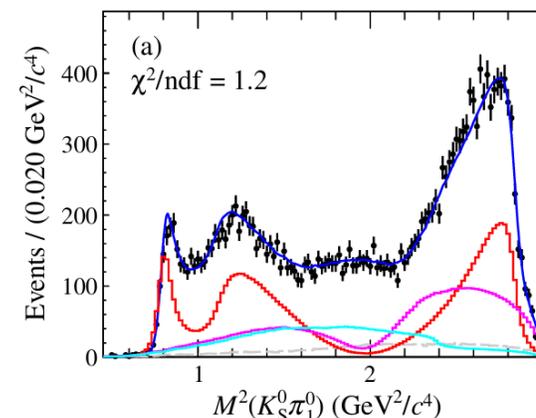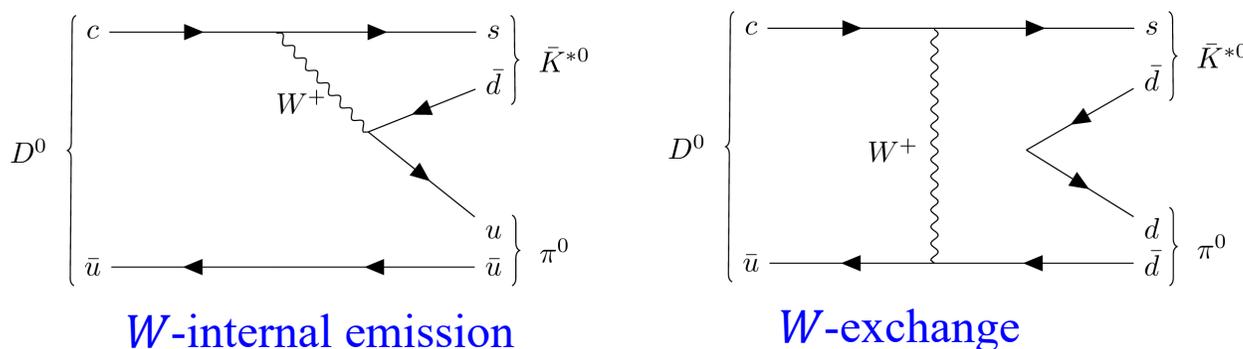
Amplitude	BF (10^{-3})
$D^0 \to K_S^0 a_0(980)^0, a_0(980)^0 \to \pi^0 \eta$	$9.88 \pm 0.37 \pm 0.42$
$D^0 \to K_S^0 a_2(1320)^0, a_2(1320)^0 \to \pi^0 \eta$	$0.08 \pm 0.11 \pm 0.11$
$D^0 \to \bar{K}^*(892)^0 \eta, \bar{K}^*(892)^0 \to K_S^0 \pi^0$	$1.22 \pm 0.09 \pm 0.11$
$D^0 \to \bar{K}^*(1410)^0 \eta, \bar{K}^*(1410)^0 \to K_S^0 \pi^0$	$0.56 \pm 0.14 \pm 0.16$
$D^0 \to (K_S^0 \pi^0)_{S-\text{wave}} \eta$	$0.64 \pm 0.10 \pm 0.10$
$D^0 \to \bar{K}^*(1680)^0 \pi^0, \bar{K}^*(1680)^0 \to K_S^0 \eta$	$0.19 \pm 0.07 \pm 0.05$
$D^0 \to \bar{K}_2^*(1980)^0 \pi^0, \bar{K}_2^*(1980)^0 \to K_S^0 \eta$	$0.45 \pm 0.15 \pm 0.16$

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Amplitude analysis of $D^0 \to K_S^0 \pi^0 \pi^0$

20.3fb^{-1} @ 3.773GeV, arxiv:2510.25111 | 19117 candidates with >93% purity

❀ Dominant contribution $D^0 \to \bar{K}^*(892)^0 \pi^0$

W-internal emission   *W*-exchange

Amplitude	$\mathcal{B}(D^0 \to \bar{K}^{*0}\pi^0)$ (%)
TDA	3.61±0.18
FAT [mix]	3.25
pole	2.9±1.0
CLEO [from $D^0 \to K^-\pi^+\pi^0$]	2.74±0.23(stat.)±0.41(syst.)
CLEO [from $D^0 \to K_S^0\pi^0\pi^0$]	4.16±0.37(stat.)±0.33(syst.)
This work	2.46 ± 0.06(stat.) ± 0.04(syst.)

❀ Precise BF measurement of $D^0 \to K_S^0 \pi^0 \pi^0$

$$\mathcal{B}(D^0 \to K_S^0\pi^0\pi^0) = \left(1.026 \pm 0.008_{\text{stat.}} \pm 0.009_{\text{syst.}}\right)\%$$

consistent with PDG value (0.91 ± 0.11)% with precision improved by a factor of 10
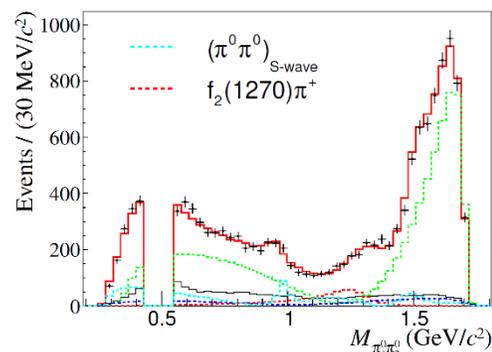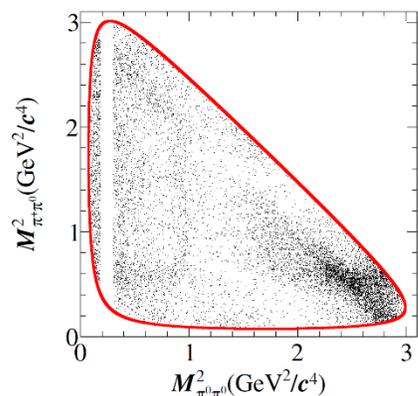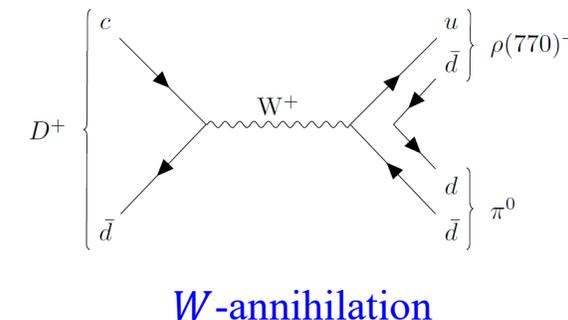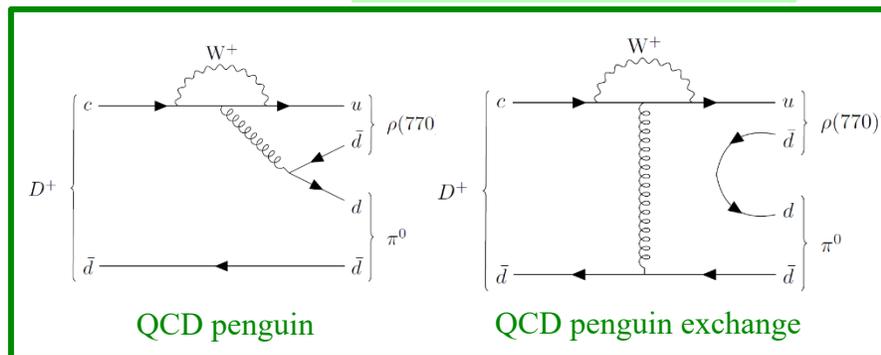
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Amplitude analysis of $D^+ \to \pi^+\pi^0\pi^0$

First observation of $D^+ \to \rho(770)^+\pi^0$

20.3fb^{-1} @ 3.773GeV, arxiv:2512.12397

13498 candidates with >87% purity

W-external emission   *W*-internal emission

QCD penguin   QCD penguin exchange

W-annihilation
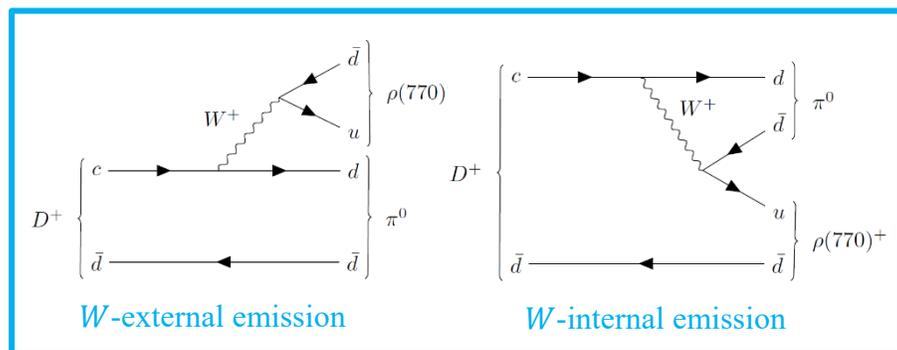
$$\mathcal{B}(D^+ \to \rho(770)^+\pi^0) = (3.08 \pm 0.10_{\text{stat.}} \pm 0.05_{\text{syst.}})\%$$

CP asymmetry

Decay mode	$D^+(\%)$	$D^-(\%)$	$\mathcal{A}_{\text{FF}}(\%)$
$D^{+(-)} \to \rho(770)^{+(-)}\pi^0$	$64.3 \pm 2.3 \pm 0.9$	$63.2 \pm 2.1 \pm 0.6$	$0.9 \pm 1.8 \pm 0.6$
$D^{+(-)} \to \rho(1450)^{+(-)}\pi^0$	$4.9 \pm 0.8 \pm 0.8$	$5.6 \pm 0.8 \pm 0.8$	$-7.2 \pm 6.8 \pm 6.8$
$D^{+(-)} \to f_2(1270)\pi^{+(-)}$	$4.3 \pm 0.4 \pm 0.2$	$4.7 \pm 0.4 \pm 0.2$	$-5.0 \pm 5.7 \pm 2.7$
$D^{+(-)} \to (\pi^0\pi^0)_{S-\text{wave}}\pi^{+(-)}$	$11.6 \pm 0.9 \pm 0.5$	$11.4 \pm 0.9 \pm 0.5$	$0.8 \pm 1.8 \pm 1.1$

$$A_{\text{CP}} = (-1.4 \pm 1.0_{\text{stat.}} \pm 0.6_{\text{syst.}})\%$$

No evidence for CP violation is observed!

Doubly Cabibbo-suppressed Decays

$$R = \mathcal{B}(DCS)/\mathcal{B}(CF) \sim (0.5 - 2.0)\tan^4\theta_C$$

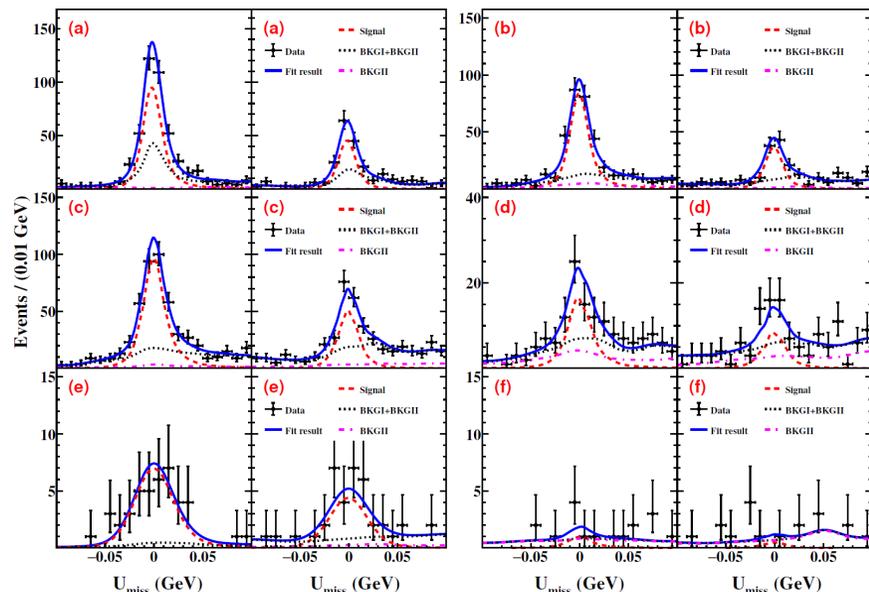| Signal decay	$\mathcal{B}_{\mathrm{DCS}}^{\mathrm{This\ work}}$ ($\times 10^{-4}$)	$\mathcal{B}_{\mathrm{DCS}}^{\mathrm{PDG}}$ ($\times 10^{-4}$)	$\mathcal{B}_{\mathrm{DCS}}^{\mathrm{This\ work}}/\mathcal{B}_{\mathrm{CF}}$ (%)	$\times \tan^4\theta_C$
$D^0 \to K^+\pi^-$	$1.30 \pm 0.09 \pm 0.04$	1.50 ± 0.07	0.328 ± 0.027	1.14 ± 0.09
$D^0 \to K^+\pi^-\pi^-\pi^+$	$2.38 \pm 0.19 \pm 0.12$	2.65 ± 0.06	0.289 ± 0.028	1.00 ± 0.10
$D^0 \to K^+\pi^-\pi^0$	$3.06 \pm 0.21 \pm 0.10$	3.06 ± 0.16	0.212 ± 0.021	0.74 ± 0.07
$D^0 \to K^+\pi^-\pi^0\pi^0$	$1.40 \pm 0.27 \pm 0.09$	< 3.6	0.158 ± 0.036	0.55 ± 0.12
$D^0 \to K^+\pi^-\eta$	$1.04 \pm 0.16 \pm 0.08$	$-$	0.555 ± 0.092	1.93 ± 0.32
$D^0 \to K^+\pi^-\pi^0\eta$	< 0.7	$-$	< 1.78	< 6.19
$D^+ \to K^+\pi^+\pi^-$	$4.50 \pm 0.12 \pm 0.35$	4.91 ± 0.09	0.480 ± 0.019	1.67 ± 0.07
$D^+ \to K^+\pi^+\pi^-\eta$	$1.56 \pm 0.22 \pm 0.04$	$-$	$-$	$-$
$D^+ \to K^+(\pi^+\pi^-\eta)_{\mathrm{non}-\eta'}$	$0.67 \pm 0.18 \pm 0.02$	$-$	5.0 ± 1.4	17.3 ± 4.8
$D^+ \to K^+K^+K^-$	$0.51 \pm 0.05 \pm 0.01$	0.614 ± 0.011	$-$	$-$
$D^+ \to K^+\eta\eta$	$0.59 \pm 0.23 \pm 0.02$	$-$	$-$	$-$

$$\mathcal{B}_{D_s^+ \to K^+K^+\pi^-} = (1.24^{+0.28}_{-0.26}(\mathrm{stat}) \pm 0.06(\mathrm{syst})) \times 10^{-4}$$

$$\mathcal{B}_{D_s^+ \to K^+K^+\pi^-\pi^0} < 1.7 \times 10^{-4}$$
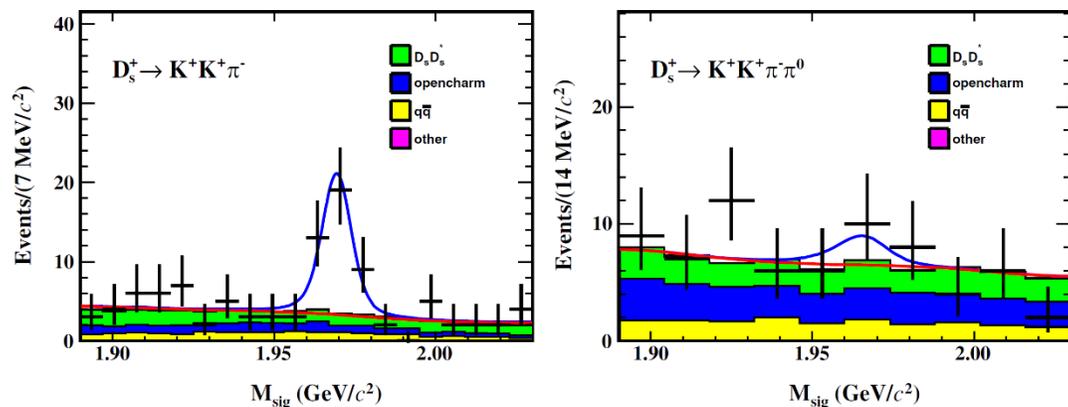
DCS decay	$\mathcal{B}_{\mathrm{DCS}}^{\mathrm{this\ work}}/\mathcal{B}_{\mathrm{CF}}^{\mathrm{PDG}}$ ($\times 10^{-3}$)	$\times \tan^4\theta_C$
$D_s^+ \to K^+K^+\pi^-$	$2.31^{+0.52}_{-0.48}$	$0.80^{+0.18}_{-0.16}$
$D_s^+ \to K^+K^+\pi^-\pi^0$	< 3.09	< 1.07

BESIII

✸ **CKM angle γ can be determined directly in $B^{\pm} \to DK^{\pm}$ (golden mode) decay**

$$DK^-$$
$$r_D e^{i(\delta_D)}$$
$$B^- \quad r_B e^{i(\delta_B - \gamma)} \quad (f)_D K^-$$
$$\overline{D}K^-$$
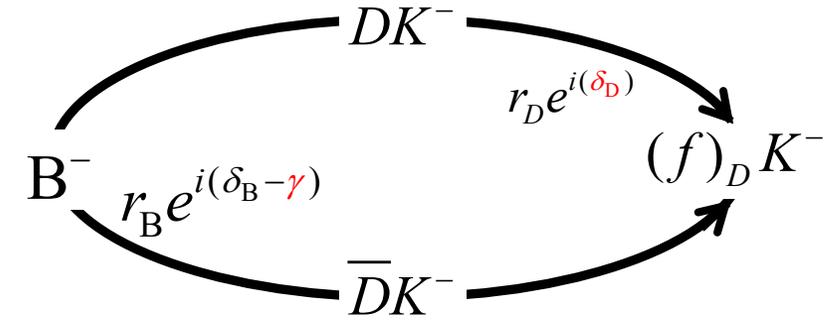
→ **Phase-space integrated decay rate:**

$$\Gamma(B^{\mp} \to DK^{\mp}) \propto (r_D^f)^2 + r_B^2 + 2 r_B r_D^f R_D^f \cos(\delta_B \mp \gamma - \delta_D^f)$$

Magnitude ratio between $D^0 \to f$ and $\overline{D}^0 \to f$ amplitudes

Coherence factor

Strong-phase difference between $D^0 \to f$ and $\overline{D}^0 \to f$ amplitudes

<mark>May limit precision to γ</mark>

✸ **Extract γ using different D decays:**

→ Self-conjugate decays (e.g. $K_S^0 h^+ h^-$): "GGSZ " → c_i, s_i (amplitude-weighted $\cos[\sin]\Delta\delta_D$ in phase-space i)
[PRD 68 (2003) 054018; PRD 67 (2003) 071301]

→ CF and DCS decays (e.g. $K\pi, K\pi\pi^0, K\pi\pi\pi$): "ADS" → δ_D^f, R_D^f
[PRL 78 (1997) 3257; PRD 63 (2001) 036005]

→ (Quasi-)CP eigenstates (e.g. $h^+ h^-$): "GLW" → CP fraction F_+; ...
[PLB 265 (1991) 172; PLB 253 (1991) 483]

✸ D strong-phase parameters provide the crucial inputs for γ measurement

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✸ Studies of quantum correlations in
$$e^+e^- \to \gamma^* \to XD\overline{D}$$

→ Quantum correlated C-odd $D^0\overline{D}^0$ pairs used at
$\psi(3770)$ for D-decay strong phase measurements

✸ First observation of quantum correlations in $e^+e^- \to XD\overline{D}$
and C-even constrained $D\overline{D}$ pairs

$$\hat{C}\,|D\overline{D}\rangle = \kappa C_{\exp}\,|D\overline{D}\rangle - (1-\kappa)C_{\exp}\,|D\overline{D}\rangle$$

Production mechanism	κ
$D\overline{D}$	1.015 ± 0.066
$D^*\overline{D} \to \gamma D\overline{D}$	1.044 ± 0.044
$D^*\overline{D} \to \pi^0 D\overline{D}$	1.028 ± 0.024
$D^*\overline{D}^* \to \gamma\pi^0 D\overline{D}$	1.027 ± 0.017
$D^*\overline{D}^* \to \gamma\gamma/\pi^0\pi^0 D\overline{D}$	0.963 ± 0.060

κ=1: coherent
κ=0.5: incoherent

Production mechanism	C
$e^+e^- \to D\overline{D}$	-1
$e^+e^- \to D^*\overline{D} \to D\overline{D}\,\gamma$	$+1$
$e^+e^- \to D^*\overline{D} \to D\overline{D}\,\pi^0$	-1
$e^+e^- \to D^*\overline{D}^* \to D\overline{D}\,\gamma\gamma$	-1
$e^+e^- \to D^*\overline{D}^* \to D\overline{D}\,\pi^0\gamma$	$+1$
$e^+e^- \to D^*\overline{D}^* \to D\overline{D}\,\pi^0\pi^0$	-1

$$C_\gamma = -1$$
$$C_{\pi^0} = +1$$

$D\overline{D} \to$

| CP eigenvalue | -1 | -1 | $+1$ | -1 | $+1$ | - |
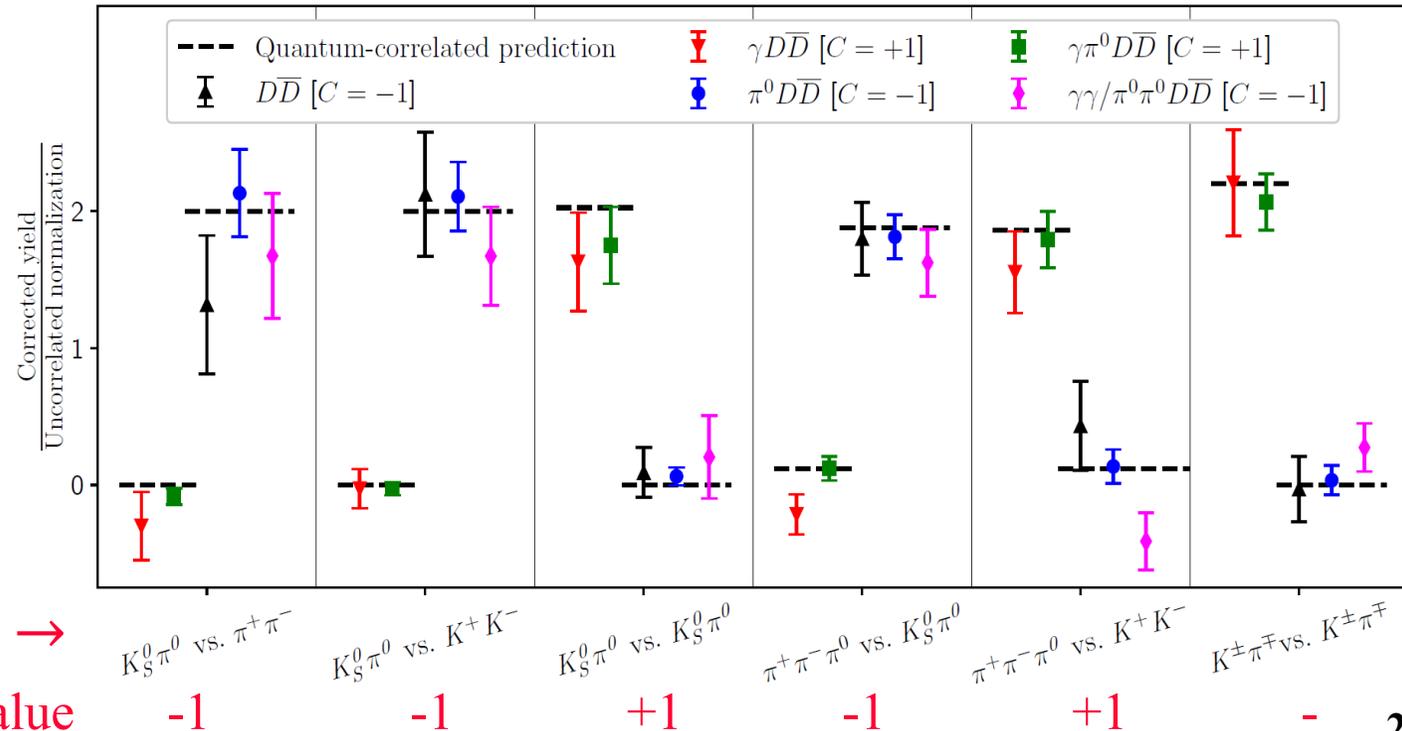
Measurement of $\delta_{K\pi}$

※ $\delta_{K\pi}$ is the strong-phase difference between $D^0 \to K^-\pi^+$ and $\overline{D}^0 \to K^-\pi^+$ decays

$$r_{K\pi}^D e^{-i\delta_{K\pi}^D} = \frac{A(\overline{D}^0 \to K^-\pi^+)}{A(D^0 \to K^-\pi^+)}, \quad r_{K\pi}^D \text{ is the magnitude of the ratio}$$

$\to$ is an key input for measurements of the charm mixing parameters and CKM angle γ

※ Use the quantum correlated pairs to measure $\delta_{K\pi}$

$\to$ CP tags ($K^+K^-, \pi^+\pi^-, \pi^+\pi^-\pi^0, K_S^0\pi^0$)

($r_{K\pi}\cos\delta_{K\pi}$)

$\to$ $K_S^0\pi^+\pi^-$ tags ($r_{K\pi}\cos\delta_{K\pi}$ and $r_{K\pi}\sin\delta_{K\pi}$)

Source	Observable	Value
$D \to CP$	$r_{K\pi}^D \cos\delta_{K\pi}^D$	$-0.070 \pm 0.008 \pm 0.0015$
$D \to K_S^0\pi^+\pi^-$	$r_{K\pi}^D \cos\delta_{K\pi}^D$	$-0.044 \pm 0.014 \pm 0.0018$
$D \to K_S^0\pi^+\pi^-$	$r_{K\pi}^D \sin\delta_{K\pi}^D$	$-0.022 \pm 0.017 \pm 0.0031$

※ Extract $\delta_{K\pi}$

$\delta_{K\pi}^D = \left(192.8^{+11.0+1.9}_{-12.4-2.4}\right)^\circ$ agree with global average $\delta_{K\pi}^D = (190.2 \pm 2.8)^\circ$

[LHCb, CERN-LHCb-CONF-2024-004]

※ BESIII combination (combined fit with BESIII measurement using 2.93 fb^{-1} ψ(3770) data)

$\delta_{K\pi}^D = \left(189.2^{+6.9+3.4}_{-7.4-3.8}\right)^\circ$ $\delta_{K\pi}^D = \left(187.6^{+8.9+5.4}_{-9.7-6.4}\right)^\circ$

[BESIII, EPJC 82,1009(2022)]

Strong-phase measurement of $D \to K^0_{S/L}\pi^+\pi^-$

✦ Measurement of $D \to K^0_{S/L}\pi^+\pi^-$ strong phase parameters:

$c_i[s_i] \equiv$ amplitude-weighted $\cos[\sin]\Delta\delta_D$ in phase-space i

→ is an key input for direct measurement of CKM angle γ

✦ **Impact** on the γ measurement

Model-predicted $c'_i(s'_i) - c_i(s_i)$ between $K^0_S\pi^+\pi^-$ and $K^0_L\pi^+\pi^-$

→ Uncertainty: **0.9°** with constraints
→ Uncertainty: **1.5°** without constraints
} with optimal binning

Statistical uncertainty of current γ measurement: **5.1°** [LHCb, JHEP02(2021)169]

✦ **Expected** γ precision

Belle II [PTEP 2019, 123C01 (2019)]:
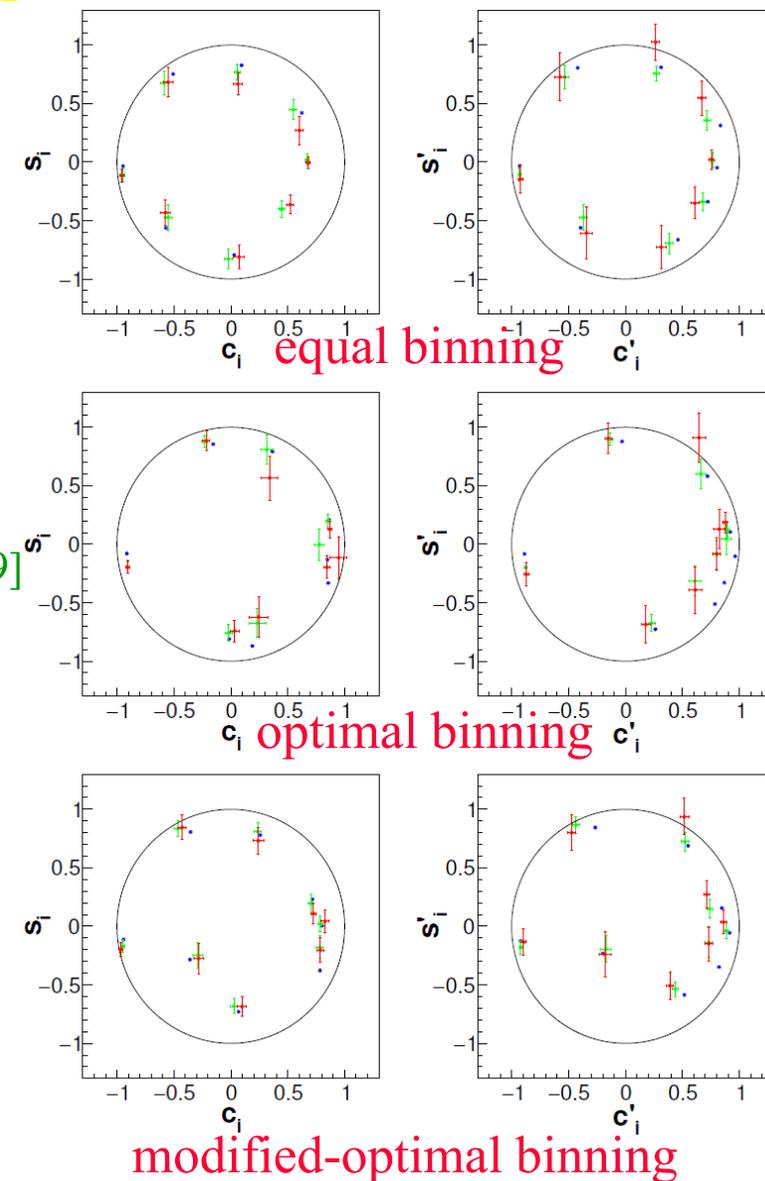 **1.5°** with 50 ab^{-1}
LHCb [LHCb-PUB-2016-025 (2016)]:
  < 1° with 50 fb^{-1}, phase-1 upgrade (2030),
  < 0.4° with 300 fb^{-1}, phase-2 upgrade (>2035)

Uncertainties due to strong-phases
in $D \to K^0_S h^+ h^-$ decays with 20fb^{-1}data → **0.5°**

BESIII white paper

7.93 fb^{-1} @3.773 GeV, JHEP 06(2025)086

equal binning

optimal binning

modified-optimal binning

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Summary and Prospects

🌀 BESIII made **significant contributions** to charmed flavor physics and hadron physics.

🌀 Charm hadronic decays are key labs to understand non-perturbative QCD;

  study the properties of light hadron ($a_0(980)$, $f_0(980)$, $f_0(500)$…);

  and provide crucial inputs to model-independent determination of γ and charm mixing/CPV

  Strong-phase measurement: 0.9° on γ ($<0.5^\circ$ with 20fb^{-1})

🌀 BESIII's Future prospects

→ More interesting results are coming using $20.3\ \text{fb}^{-1}\psi(3770)$ data.

→ BEPCII-U will extend the lifetime of BESIII (**will continue to run till ~2030**).
  3× luminosity above 4 GeV & max energy to 5.6 GeV

谢谢！