What do experiments tell us about supersymmetry?

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An 11-year research program addressing the naturalness problem of low-energy supersymmetry (a mechanism to protect scalar mass from large radiative corections) in the post-LHC era.

The work focuses on extensions of the MSSM, with the goal of constructing a theory that maintains fine-tuning at the percent level even in the face of prospective experimental constraints.

1. Naturalness
2. Neutron Electric Dipole Moment
3. LHC searches for SUSY
4. Dark Matter Direct Detection Experiments
5. B-L NMSSM - Kill three birds with one stone!
6. Conclusions

Conclusion: MSSM

MSSM is disfavored by naturalness argument!

- Owing to intrinsic structure in its theoretical framework, **the fine-tuning of MSSM** is at 0.01% level, i.e., $\Delta_{FT}^{\mathrm{BG}} \sim 10000$;
  - Barbieri-Giudice Measurement:

    $$\Delta_{FT}^{\mathrm{BG}} \equiv max\left(\frac{\partial \ln O}{\partial \ln p_i}\right)$$

    O: physics observable, such as m_Z, Ωh^2, etc.;
    p_i: ith input of the theory.
  - Expected $\Delta_{FT}^{\mathrm{BG}}$ during LEP era: $\Delta_{FT}^{\mathrm{BG}} \lesssim 10$;
  - Acceptable $\Delta_{FT}^{\mathrm{BG}}$ in post-LHC era: $\Delta_{FT}^{\mathrm{BG}} \lesssim 300$.
- Constraints from **only** Ωh^2 measurement and LZ experiment **have excluded** Z- and h-resonant annihilation mechanisms;
- **Furthermore**, LHC searches for compressed spectrum have set a lower bound on DM mass, $m_{\tilde{\chi}_1^0} \gtrsim 200$ GeV.

Conclusion: Beyond MSSM

Unexpected theoretical advantages in gauge extensions;
B-L extended NMSSM: the most economic extension to significantly alleviate FT problem of MSSM.

1. **Economic extension of MSSM** - No dimensional parameters in its superpotential;
2. **Elegant solution to μ-problem** - **Free from domain-wall problem with minimal framework**;
3. **Explaining the origin of R-symmetry;**
4. **Generate dynamically neutrino mass and mixing;**
5. **Singlino-modulated and sequestered DM** - **Naturally suppressed DM-nucleon scattering**;
6. **Well-motivated DM annihilation** - **BLino - Higgsino coannihilation**;
7. **Simple phenomenology** - **Similar to that of MSSM with Higgsino acting as DM candidate.** $\cdots$

Conclusion: WIMP

Recent advancements in WIMP DM theory:

- **Keep the idea of freeze-out and thermal relic;**
- **More complex structure to break crossing symmetry between DM annihilation and DM-nucleon scattering;**
- **Phenomenology significantly differs from traditional MSSM prediction due to gauge singlet nature of DM, which results in lengthened decay chain of sparticles.**
  - No hints of DM and sparticles in experiments are consistent!

Examples

1. **Secluded DM Sector**
   Singlino-dominated DM in GNMSSM:
   $$pp \to \tilde{\chi}_i^0 \tilde{\chi}_j^\pm \to \mathbf{4\{W/Z/h\}} E_T^{\rm Miss}$$
   τ-flavored righthanded sneutrino DM in seesaw extension:
   $$pp \to \tilde{\chi}_i^0 \tilde{\chi}_j^\pm \to \mathbf{2\tau 2\{W/Z/h\}} E_T^{\rm Miss}$$

Conclusion: WIMP

Example

2. **Co-annihilation**
   - **Most natural co-annihilation: Same gauge multiplet;**
     - ✓ Higgsino $SU(2)_L$ doublet, $m_{\tilde{\chi}_1^0} \simeq 1.1$ TeV for measured Ωh^2;
     - ✓ Wino $SU(2)_L$ triplet, $m_{\tilde{\chi}_1^0} \simeq 3.0$ TeV for measured Ωh^2, **excluded by DM indirect search experiments**.
   - **Another natural co-annihilation: Higgsino-partnered. BLino-Higgsino co-annihilation in B-L NMSSM, well-motivated, any mass value for measured Ωh^2.**

Phenomenology:

Similar to that of MSSM with **Higgsino** acting **as DM candidate** except for the subtlety about Higgsino signal:

(1) Missing momentum; **(2) Displaced vertex;**

(3) Missing track; **(4) “Orphan” signal,**

which depends on Higgsino mass spectrum and lifetime, and also on LHC search strategies.

Section I

Naturalness

Naturalness

My opinion about naturalness in particle physics:
unifies **'t Hooft's technical naturalness** with **Hierarchy Problem.**

Based on current knowledge of the relevant physical degrees of freedom, evaluate the principal contributions to a given observable. A result is natural if and only if:

- **the principal contributions and the measured value have comparable magnitudes;**
  Comparable: to what extent? Currently: around 10^2.
- **the measured value does not depend on accidentally precise cancellations between independent contributions.**
  Cancellation: how precisely? Currently: around 1%.

A theory satisfying these conditions is free from fine-tuning problems.

Naturalness Crises ⟶ Sig. Advancements in Physics

Solved:

- **Lamb Shift (1947)** ⟶ Renormalization in Filed Theory;
- **Electron Anomalous Magnetic Moment (1948)** ⟶ HPC;
- **Energy Spectrum in β-decay (1930)** ⟶ Neutrino;
- **Fermion Constant (1934)** ⟶ Unified EW Thoery;
- **Suppressed FCNC in K Mesons (1960's)** ⟶ Charm Quark;
- **Enhanced $B^0 - \bar{B}^0$ Mixing (1987)** ⟶ Top Quark;
- **Uranus' Orbit (1841)** ⟶ Neptune;
- **Mercury's Perihelion Precession (1910's)** ⟶ GR;

Unsolved:

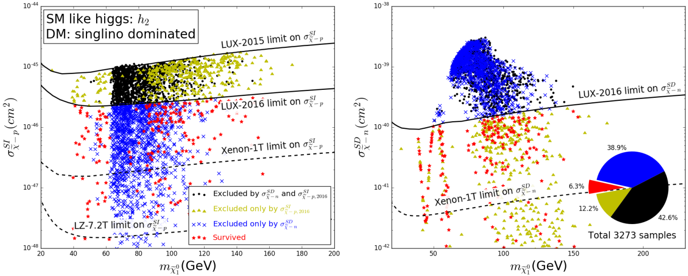
- **Cosmological Constant** ⟶ Landscape + Anthropic Principle?
- **Hierarchy Problem** ⟶ SUSY? Composite Theory? $\cdots$
- **Strong CP Problem** ⟶ Axion? Nelson-Barr Model? LR? $\cdots$

Naturalness and DM DD experiments（2016)

- Conclusion: about 95% parameter points excluded, 1% FT!

J. Cao, et. al. JHEP 1610 (2016) 136, arXiv: 1609.00204;

applying LUX data before Sept. 2016 to constrain natural NMSSM.



The 7th Corss-Strait Conference,
Shandong University, April in 2017.

- Solutions:
  - Blind Spots: another type of FT;
  - Multi-component DM: More complex theory!
  - New theory, new DM candidate? SUSY facing great challenge!

Section II

Neutron Electric Dipole Moment

Neutron EDM

Experimental Result: $|d_n| < 1.8 \times 10^{-26}$ e $\cdot$ cm.

SM Prediction:

- $d_n^{\rm CKM} \sim 10^{-31}$ e·cm; $d_n^{\theta} \approx 2.4 \times 10^{-14} \times \theta$ e·cm $\Rightarrow |\theta| < \mathbf{10^{-10}}$.

Solutions: **Axion**, **Nelson-Barr model**, **Left-Right model**, $\cdots$

Axion: seemingly most **natural**, **simplest**, and **testable** solution.

- Goldstone Theorem & Dynamically drive θ to zero;
- DM candidate, misalignment mechanism (**Robust**) for relic density (**Conditional**):

$$\Omega h^2 \approx 0.18 \times \left(\frac{f_a}{10^{12}\ \mathrm{GeV}}\right)^{1.19} \times \left(\frac{3\theta_i^2}{\pi^2}\right);$$

- Rich applications in cosmology and astronomy;
- **Difficult to detect in the past, now technically feasible**;
- **Quality problem (QP), domain wall, $\cdots$**.

Neutron EDM

QP: Fine tuning (FT) induced by quantum gravity.

Assuming: $\phi = \frac{\rho + f_a}{\sqrt{2}} e^{i\frac{a}{f_a}}$;

Broken PQ symmetry: $V_0(\phi) = \lambda \left(|\phi|^2 - \frac{f_a^2}{2} \right)^2$;

Instanton effects: $V_{\rm QCD}(a) = \left(0.4 \frac{f_\pi m_\pi}{f_a}\right)^2 f_a^2 \left[1 - \cos\left(\frac{a}{f_a} + \theta\right)\right]$;

PQ mechanism: $\mathcal{L} = \frac{g_s^2}{32\pi^2} \left(\theta + \frac{a}{f_a}\right) G^{\mu\nu,a} \tilde{G}^a_{\mu\nu}$;

Gravity effects: $V_g(\phi) = \frac{|g| e^{i\delta}}{M_{Pl}^{2m+n-4}} |\phi|^{2m} \phi^n + h.c.$;

$$V_g(a) = \left(|g| M_{Pl}^2 \left(\frac{f_a}{\sqrt{2} M_{Pl}} \right)^{2m+n-2} \right)^2 f_a^2 \left[1 - \cos\left(\frac{na}{f_a} + \delta\right)\right];$$

Axion potential: $V(a) = V_{\rm QCD}(a) + V_g(a)$;

FT problem: $|g| \lesssim 10^{-55}$,

for dimension-5 symmetry breaking operator.

Axion solution worsens FT of strong CP problem: $10^{10} \to 10^{55}$**!**

Neutron EDM

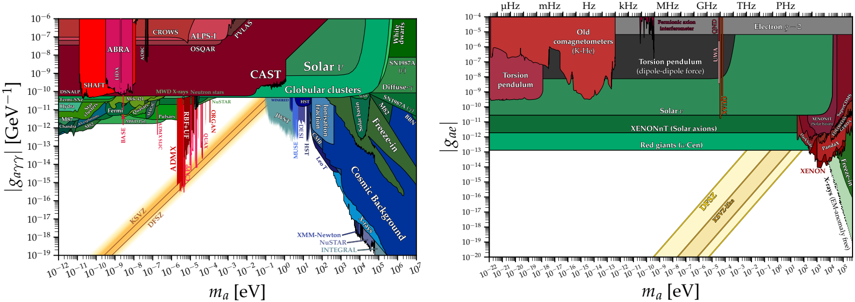


Figure: Current Experimental Constraints on Axion Properties.

If Axion is discovered in near future:

1. Property measurement by various experiments/observations;
2. QP should be explained: **quite complicated!**
   - Forbidden low-dimensional operators by symmetry (Z_N or String);
   - Suppressing the gravity effect by clockwork/alignment mechanisms.
3. A process similar to that after the Higgs discovery, but **more difficult** since ultra-high energy scale physics is involved!

If not, does not matter, just examine other mechanisms!

Roadmap of New physics

Remarks on Axion theory:

- **Not the sole solution** to the strong CP problem;
- **Not simple and natural** in explaining the problem;
- **No hints** in experiments;
- **Unlike WIMP**, **some assumptions** in explaining measured DM density;
- **Almost decoupled from EWSB/collider phy., affect model-building?**

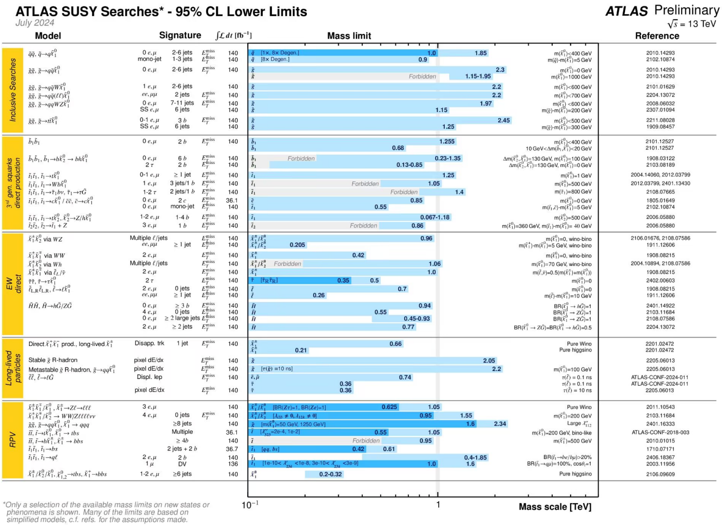
Personal insights about NP based on urgency of solutions:

1. **Top problems before Axion is discovered**
    - **The nature of (non Axion-like) DM**;
    - Neutrino mass and mixing;
    - Matter-antimatter asymmetry, and related issues such as EWPT and Vacuum Stability;
    - **Stabilization of the EW scale**;
    - **GUT is favored; Naturalness is preferred!**
2. **Minor problems: too complex to be solved in near future**
    - Origin of CP violation:
        - **Strong CP problem**;
        - CP violations in hadrons and leptons;
    - Quark-lepton flavor structure.

Section III

LHC searches for SUSY

LHC searches for SUSY



ATLAS SUSY Searches* - 95% CL Lower Limits
July 2024

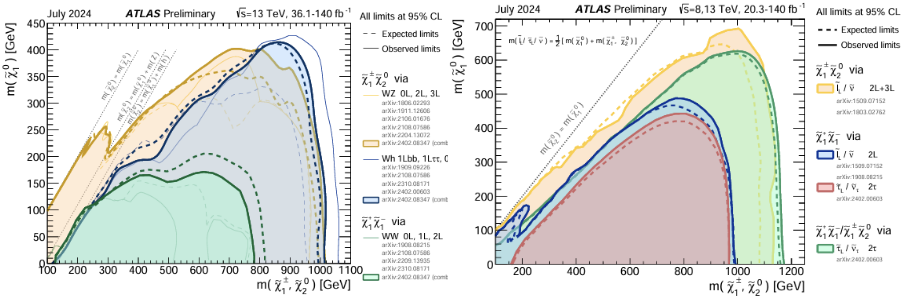
ATLAS Preliminary
$\sqrt{s}$ = 13 TeV

	Model	Signature			$\int\mathcal{L}\,dt$ [fb^{-1}]		Mass limit [TeV]		Reference
Inclusive Searches	$\tilde q\tilde q$, $\tilde q\to q\tilde\chi^0_1$	0 e,μ	2-6 jets	$E_T^{\rm miss}$	140	$\tilde q$ [1×, 8× Degen.]	1.0 1.85	m($\tilde\chi^0_1$)<400 GeV	2010.14293
		mono-jet	1-3 jets	$E_T^{\rm miss}$	140	$\tilde q$ [8× Degen.]	0.9	m($\tilde q$)-m($\tilde\chi^0_1$)=5 GeV	2102.10874
	$\tilde g\tilde g$, $\tilde g\to q\bar q\tilde\chi^0_1$	0 e,μ	2-6 jets	$E_T^{\rm miss}$	140	$\tilde g$	2.3	m($\tilde\chi^0_1$)=0 GeV	2010.14293
						$\tilde g$	Forbidden 1.15-1.95	m($\tilde\chi^0_1$)=1000 GeV	2010.14293
	$\tilde g\tilde g$, $\tilde g\to q\bar q W\tilde\chi^0_1$	1 e,μ	2-6 jets		140	$\tilde g$	2.2	m($\tilde\chi^0_1$)<600 GeV	2101.01629
	$\tilde g\tilde g$, $\tilde g\to q\bar q(\ell\ell)\tilde\chi^0_1$	$ee,\mu\mu$	2 jets	$E_T^{\rm miss}$	140	$\tilde g$	2.2	m($\tilde\chi^0_1$)<700 GeV	2204.13072
	$\tilde g\tilde g$, $\tilde g\to qqWZ\tilde\chi^0_1$	0 e,μ	7-11 jets	$E_T^{\rm miss}$	140	$\tilde g$	1.97	m($\tilde\chi^0_1$) <600 GeV	2008.06032
		SS e,μ	6 jets		140	$\tilde g$	1.15	m($\tilde g$)-m($\tilde\chi^0_1$)=200 GeV	2307.01094
	$\tilde g\tilde g$, $\tilde g\to t\bar t\tilde\chi^0_1$	0-1 e,μ	3 b	$E_T^{\rm miss}$	140	$\tilde g$	2.45	m($\tilde\chi^0_1$)<500 GeV	2211.08028
		SS e,μ	6 jets		140	$\tilde g$	1.25	m($\tilde g$)-m($\tilde\chi^0_1$)=300 GeV	1909.08457
3rd gen. squarks direct production	$\tilde b_1\tilde b_1$	0 e,μ	2 b	$E_T^{\rm miss}$	140	$\tilde b_1$	1.255	m($\tilde\chi^0_1$)<400 GeV	2101.12527
						$\tilde b_1$	0.68	10 GeV<Δm($\tilde b_1$,$\tilde\chi^0_1$)<20 GeV	2101.12527
	$\tilde b_1\tilde b_1$, $\tilde b_1\to b\tilde\chi^0_2\to bh\tilde\chi^0_1$	0 e,μ	6 b	$E_T^{\rm miss}$	140	$\tilde b_1$	Forbidden 0.23-1.35	Δm($\tilde\chi^0_2$,$\tilde\chi^0_1$)=130 GeV, m($\tilde\chi^0_1$)=100 GeV	1908.03122
		2 τ	2 b	$E_T^{\rm miss}$	140	$\tilde b_1$	0.13-0.85	Δm($\tilde\chi^0_2$,$\tilde\chi^0_1$)=130 GeV, m($\tilde\chi^0_1$)=0 GeV	2103.08189
	$\tilde t_1\tilde t_1$, $\tilde t_1\to t\tilde\chi^0_1$	0-1 e,μ	≥ 1 jet	$E_T^{\rm miss}$	140	$\tilde t_1$	1.25	m($\tilde\chi^0_1$)=1 GeV	2004.14060, 2012.03799
	$\tilde t_1\tilde t_1$, $\tilde t_1\to Wb\tilde\chi^0_1$	1 e,μ	3 jets/1 b	$E_T^{\rm miss}$	140	$\tilde t_1$	Forbidden 1.05	m($\tilde\chi^0_1$)=500 GeV	2012.03799, 2401.13430
	$\tilde t_1\tilde t_1$, $\tilde t_1\to\tilde\tau_1 b\nu$, $\tilde\tau_1\to\tau\tilde G$	1-2 τ	2 jets/1 b	$E_T^{\rm miss}$	140	$\tilde t_1$	Forbidden 1.4	m($\tilde\tau_1$)=800 GeV	2108.07665
	$\tilde t_1\tilde t_1$, $\tilde t_1\to c\tilde\chi^0_1$ / $\tilde c\tilde c$, $\tilde c\to c\tilde\chi^0_1$	0 e,μ	2 c	$E_T^{\rm miss}$	36.1	$\tilde c$	0.85	m($\tilde\chi^0_1$)=0 GeV	1805.01649
		0 e,μ	mono-jet	$E_T^{\rm miss}$	140	$\tilde t_1$	0.55	m($\tilde t_1$,$\tilde c$)-m($\tilde\chi^0_1$)=5 GeV	2102.10874
	$\tilde t_1\tilde t_1$, $\tilde t_1\to t\tilde\chi^0_2$, $\tilde\chi^0_2\to Z/h\tilde\chi^0_1$	1-2 e,μ	1-4 b	$E_T^{\rm miss}$	140	$\tilde t_1$	0.067-1.18	m($\tilde\chi^0_2$)=500 GeV	2006.05880
	$\tilde t_2\tilde t_2$, $\tilde t_2\to\tilde t_1+Z$	3 e,μ	1 b	$E_T^{\rm miss}$	140	$\tilde t_2$	Forbidden 0.86	m($\tilde\chi^0_1$)=360 GeV, m($\tilde t_1$)-m($\tilde\chi^0_1$)= 40 GeV	2006.05880
EW direct	$\tilde\chi^\pm_1\tilde\chi^0_2$ via WZ	Multiple ℓ/jets		$E_T^{\rm miss}$	140	$\tilde\chi^\pm_1/\tilde\chi^0_2$	0.96	m($\tilde\chi^0_1$)=0, wino-bino	2106.01676, 2108.07586
		$ee,\mu\mu$	≥ 1 jet	$E_T^{\rm miss}$	140	$\tilde\chi^\pm_1/\tilde\chi^0_2$	0.205	m($\tilde\chi^\pm_1$)-m($\tilde\chi^0_1$)=5 GeV, wino-bino	1911.12606
	$\tilde\chi^\pm_1\tilde\chi^\mp_1$ via WW	2 e,μ		$E_T^{\rm miss}$	140	$\tilde\chi^\pm_1$	0.42	m($\tilde\chi^0_1$)=0, wino-bino	1908.08215
	$\tilde\chi^\pm_1\tilde\chi^0_2$ via Wh	Multiple ℓ/jets		$E_T^{\rm miss}$	140	$\tilde\chi^\pm_1/\tilde\chi^0_2$	Forbidden 1.06	m($\tilde\chi^0_1$)=70 GeV, wino-bino	2004.10894, 2108.07586
	$\tilde\chi^\pm_1\tilde\chi^\mp_1$ via $\tilde\ell_L/\tilde\nu$	2 e,μ		$E_T^{\rm miss}$	140	$\tilde\chi^\pm_1$	1.0	m($\tilde\ell$,$\tilde\nu$)=0.5(m($\tilde\chi^\pm_1$)+m($\tilde\chi^0_1$))	1908.08215
	$\tilde\tau\tilde\tau$, $\tilde\tau\to\tau\tilde\chi^0_1$	2 τ		$E_T^{\rm miss}$	140	$\tilde\tau$ [$\tilde\tau_R\tilde\tau_R$]	0.35 0.5	m($\tilde\chi^0_1$)=0	2402.00603
	$\tilde\ell_{L,R}\tilde\ell_{L,R}$, $\tilde\ell\to\ell\tilde\chi^0_1$	2 e,μ	0 jets	$E_T^{\rm miss}$	140	$\tilde\ell$	0.7	m($\tilde\chi^0_1$)=0	1908.08215
		$ee,\mu\mu$	≥ 1 jet	$E_T^{\rm miss}$	140	$\tilde\ell$	0.26	m($\tilde\ell$)-m($\tilde\chi^0_1$)=10 GeV	1911.12606
	$\tilde H\tilde H$, $\tilde H\to h\tilde G/Z\tilde G$	0 e,μ	≥ 3 b	$E_T^{\rm miss}$	140	$\tilde H$	0.94	BR($\tilde\chi^0_1\to h\tilde G$)=1	2401.14922
		4 e,μ	0 jets	$E_T^{\rm miss}$	140	$\tilde H$	0.55	BR($\tilde\chi^0_1\to Z\tilde G$)=1	2103.11684
		0 e,μ	≥ 2 large jets	$E_T^{\rm miss}$	140	$\tilde H$	0.45-0.93	BR($\tilde\chi^0_1\to Z\tilde G$)=1	2108.07586
		2 e,μ	≥ 2 jets	$E_T^{\rm miss}$	140	$\tilde H$	0.77	BR($\tilde\chi^0_1\to Z\tilde G$)=BR($\tilde\chi^0_1\to h\tilde G$)=0.5	2204.13072
Long-lived particles	Direct $\tilde\chi^+_1\tilde\chi^-_1$ prod., long-lived $\tilde\chi^\pm_1$	Disapp. trk	1 jet	$E_T^{\rm miss}$	140	$\tilde\chi^\pm_1$	0.66	Pure Wino	2201.02472
						$\tilde\chi^\pm_1$	0.21	Pure higgsino	2201.02472
	Stable $\tilde g$ R-hadron	pixel dE/dx		$E_T^{\rm miss}$	140	$\tilde g$	2.05		2205.06013
	Metastable $\tilde g$ R-hadron, $\tilde g\to qq\tilde\chi^0_1$	pixel dE/dx		$E_T^{\rm miss}$	140	$\tilde g$ [$\tau(\tilde g)$ =10 ns]	2.2	m($\tilde\chi^0_1$)=100 GeV	2205.06013
	$\tilde\ell\tilde\ell$, $\tilde\ell\to\ell\tilde G$	Displ. lep		$E_T^{\rm miss}$	140	$\tilde e,\tilde\mu$	0.74	$\tau(\tilde\ell)$ = 0.1 ns	ATLAS-CONF-2024-011
						$\tilde\tau$	0.36	$\tau(\tilde\ell)$ = 0.1 ns	ATLAS-CONF-2024-011
		pixel dE/dx		$E_T^{\rm miss}$	140	$\tilde\tau$	0.36	$\tau(\tilde\ell)$ = 10 ns	2205.06013
RPV	$\tilde\chi^\pm_1\tilde\chi^\mp_1/\tilde\chi^0_1$, $\tilde\chi^\pm_1\to Z\ell\to\ell\ell\ell$	3 e,μ			140	$\tilde\chi^\mp_1/\tilde\chi^0_1$ [BR($Z\tau$)=1, BR(Ze)=1]	0.625 1.05	Pure Wino	2011.10543
	$\tilde\chi^\pm_1\tilde\chi^\mp_1/\tilde\chi^0_2\to WW/Z\ell\ell\ell\ell\nu\nu$	4 e,μ	0 jets	$E_T^{\rm miss}$	140	$\tilde\chi^\pm_1/\tilde\chi^0_2$ [$\lambda_{i33}\neq 0,\lambda_{12k}\neq 0$]	0.95 1.55	m($\tilde\chi^0_1$)=200 GeV	2103.11684
	$\tilde g\tilde g$, $\tilde g\to qq\tilde\chi^0_1$, $\tilde\chi^0_1\to qqq$		≥8 jets		140	$\tilde g$ [m($\tilde\chi^0_1$)=50 GeV, 1250 GeV]	1.6 2.34	Large λ''_{112}	2401.16333
	$\tilde t\tilde t$, $\tilde t\to t\tilde\chi^0_1$, $\tilde\chi^0_1\to tbs$		Multiple		36.1	$\tilde t$ [λ''_{323}=2e-4, 1e-2]	0.55 1.05	m($\tilde\chi^0_1$)=200 GeV, bino-like	ATLAS-CONF-2018-003
	$\tilde t\tilde t$, $\tilde t\to b\tilde\chi^\pm_1$, $\tilde\chi^\pm_1\to bbs$		≥ 4b		140	$\tilde t$	Forbidden 0.95	m($\tilde\chi^\pm_1$)=500 GeV	2010.01015
	$\tilde t_1\tilde t_1$, $\tilde t_1\to bs$		2 jets + 2 b		36.7	$\tilde t_1$ [qq, bs]	0.42 0.61		1710.07171
	$\tilde t_1\tilde t_1$, $\tilde t_1\to q\ell$	2 e,μ	2 b		140	$\tilde t_1$	0.4-1.85	BR($\tilde t_1\to be/b\mu$)>20%	2406.18367
		1 μ	DV		136	$\tilde t_1$ [1e-10< λ'_{23k} <1e-8, 3e-10< λ'_{23k} <3e-9]	1.0 1.6	BR($\tilde t_1\to q\mu$)=100%, cosθ_t=1	2003.11956
	$\tilde\chi^\pm_1/\tilde\chi^0_2/\tilde\chi^0_1$, $\tilde\chi^0_{1,2}\to tbs$, $\tilde\chi^+_1\to bbs$	1-2 e,μ	≥6 jets		140	$\tilde\chi^0_1$	0.2-0.32	Pure higgsino	2106.09609

*Only a selection of the available mass limits on new states or phenomena is shown. Many of the limits are based on simplified models, c.f. refs. for the assumptions made.

Mass reach of the ATLAS searches for Supersymmetry

LHC searches for SUSY



Latest LHC searches for Electroweakinos.

1. Simplified model for a specified process;
2. Generally, invalid for a specific theory: complex decay chain, multiple production processes, and various signals to be analyzed; Elaborated Monte Carlo simulations are necessary;
3. **Valid for many cases of the MSSM!**

Example: FT of MSSM

Superpotential of MSSM

$$W_{\rm MSSM} = y_u \hat{Q}\cdot\hat{H}_u\hat{U} + y_d\hat{H}_d\cdot\hat{Q}\hat{D} + \mu\hat{H}_u\cdot\hat{H}_d + \cdots$$

μ parameter: Natural values are $\mu = 0$ or $\mu = \Lambda_{\rm GUT}$.

Z-boson mass: $\mu \lesssim 1$ TeV, LHC: $\mu \gtrsim 180$ GeV.

Giudice-Maserio Mechanism:

Generate SUSY-conserving term by gravity-mediated SUSY-breaking.

SM: $m_h^2 = m_h^2|_{tree} + \delta m_h^2 = m_h^2|_{tree} - \frac{3y_t^2}{8\pi^2}\Lambda^2;$

BG Measurement:

$$\Delta_{\rm FT} = max\left(\frac{\partial \ln m_h^2}{\partial \ln p_i}\right)$$

MSSM:

$$\begin{aligned} m_h^2 &= m_h^2|_{tree} + \delta m_h^2 \\ &\simeq m_h^2|_{tree} - \frac{3y_t^2}{16\pi^2}(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)\log\frac{\Lambda^2}{m_{\tilde{t}_1}m_{\tilde{t}_2}} \\ &\simeq m_h^2|_{tree} - \frac{3y_t^2}{16\pi^2}\frac{2g_s^2}{3\pi^2}m_{\tilde{g}}^2\left(\log\frac{\Lambda^2}{m_{\tilde{t}_1}m_{\tilde{t}_2}}\right)^2; \end{aligned}$$

since $\delta m_{\tilde{t}_i}^2 \simeq \frac{g_s^2}{3\pi^2}m_{\tilde{g}}^2\log\frac{\Lambda^2}{m_{\tilde{t}_1}m_{\tilde{t}_2}}$. $M_3|_{\rm Weak\ scale} = 2.91 M_3|_{\rm GUT}$.

$m_h^2 \simeq m_h^2|_{tree} - 6.8\times m_{\tilde{g}}^2$, **assuming** $\Lambda = \Lambda_{\rm GUT}$ **and** $m_{\tilde{t}} = 1$ TeV.

LHC search for SUSY set a FT of 0.04%!

Example: FT of MSSM

Considering SUSY breaking at GUT scale and RGE running effects,

$$\begin{aligned} m_Z^2 \equiv & \left\{2(m_{H_d}^2 - m_{H_u}^2 \tan^2\beta)/(\tan^2\beta - 1) - 2\mu^2 \ \right\}\Big|_{\textbf{Weak scale}} \\ = & \left\{\left(3.84 M_3^2 + 0.32 M_3 M_2 + 0.047 M_1 M_3 - 0.42 M_2^2 + 0.01 M_2 M_1 \right.\right. \\ & -0.012 M_1^2 - 0.65 M_3 A_t - 0.15 M_2 A_t - 0.025 M_1 A_t + 0.22 A_t^2 \\ & +0.004 M_3 A_b - 1.27 m_{H_u}^2 - 0.053 m_{H_d}^2 + 0.73 m_{Q_3}^2 + 0.57 m_{U_3}^2 \\ & +0.049 m_{D_3}^2 - 0.052 m_{L_3}^2 + 0.053 m_{E_3}^2 + 0.051 m_{Q_2}^2 - 0.110 m_{U_2}^2 \\ & +0.051 m_{D_2}^2 - 0.052 m_{L_2}^2 + 0.053 m_{E_2}^2 + 0.051 m_{Q_1}^2 - 0.110 m_{U_1}^2 \\ & \left.\left. +0.051 m_{D_2}^2 - 0.052 m_{L_1}^2 + 0.053 m_{E_1}^2 \right) - 2.18\mu^2 \ \right\} \Big|_{\textbf{GUT}}, \\ & \text{for } \tan\beta = 10. \end{aligned}$$

$$m_Z^2 = \underline{\left(0.45 m_{\tilde g}^2 + 0.82 m_{\tilde t_L}^2 + 0.74 m_{\tilde t_R}^2 - 1.27 m_{H_u}^2|_{\mathrm{GUT}} + \cdots\right)} \boldsymbol{-2m_{\tilde H}^2}.$$

The first term is very large, implying severe FT.

Reason: MSSM relies on UV-scale mechan. to generate μ_{EW}.

Solutions:

- **Generate μ-parameter at the TeV scale**;
- **Change RGE and/or the initial conditions of RGE**.

Solutions: NMSSM and Dirac Gaugino models

Z_3-NMSSM: Simplest TeV-scale solution.

- Extend the MSSM by one or more gauge singlet fields;
- Dynamically generate μ_{eff} within low-energy theory;
- Z_3-NMSSM acts as a self-contained low-energy EFT of SUSY!
  Its UV-completion is **unimportant** for phenomenological study.

MRSSM: Soft-breaking terms → Supersoft operators;
$\underline{\text{Set a definite UV scale \& Change the RGE!}}$

$$\begin{aligned} m_h^2 &= m_h^2|_{tree} + \delta m_h^2 \\ &\simeq m_h^2|_{tree} - \frac{3y_t^2}{16\pi^2}(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)\log\frac{M_3^2}{m_{\tilde{t}_1} m_{\tilde{t}_2}}; \end{aligned}$$

M_3: $N=2$ **SUSY recovery scale!**

$$\delta m_{\tilde{t}_i}^2 = \frac{g_s^2}{3\pi^2} m_{\tilde{g}}^2 \log\frac{\tilde{m}^2}{m_{\tilde{g}}^2},$$

$\tilde{m}$: **Sgluon mass**.

$$m_Z^2 = a_1 M_3^2 + a_2 m_{Q_3}^2 + a_3 m_{U_3}^2 - 2\mu^2 + \cdots$$

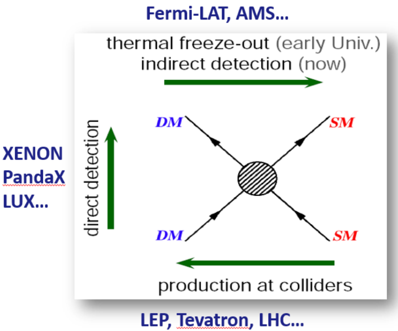
a_i: one-loop or two-loop suppressed, no log enhancement.

μ: May be light without conflicting with the LHC restrictions.

Section IV

Dark Matter Direct Detection Experiments

DM direct detection experiments



Fermi-LAT, AMS...

thermal freeze-out (early Univ.)
indirect detection (now)

XENON
PandaX
LUX...

direct detection

production at colliders

LEP, Tevatron, LHC...

No new physics signal !

arXiv:2410.17036

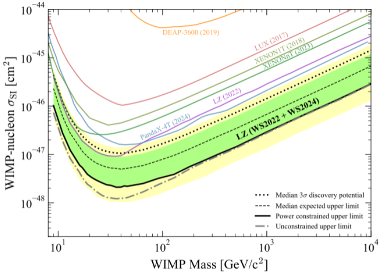


FIG. 5

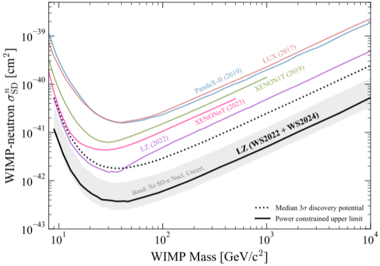


FIG. 6

Simple WIMP DM Theory:

DM mass: $m_{\rm DM} \sim 100$ GeV

Relic density: $\langle \sigma v \rangle \simeq 10^{-26} {\rm cm}^3 s^{-1}$

SI scattering: $\sigma^{\rm SI}_{\tilde{\chi}-N} \sim 10^{-45} {\rm cm}^2$,

SD scattering: $\sigma^{\rm SD}_{\tilde{\chi}-N} \sim 10^{-39} {\rm cm}^2$.

Solutions: Secluded DM sector

Table: **Implications of DD experiments in DM EFT framework, noting six mechanisms to suppress DM-nucleon scattering rates.**

DM EFTs	Examples	DM Abundance	$\tilde{\chi} - N$ Scattering	Remarks
SM+DM	SM+S_{real}	Weak/contact interactions	$\sigma_{\rm SI} \gtrsim 10^{-45}{\rm cm}^2$ and/or $\sigma_{\rm SD} \gtrsim 10^{-39}{\rm cm}^2$	Experimentally excluded.
			Suppressed by cancellation	Symmetry!
		Feeble interaction: h/Z funnels	Suppressed	Increasingly Fine-tuned: $\Delta > 150$.
SM+DM+X	MSSM with Light Gauginos	Coannihilation/Mediator	Suppressed	Fine-tuning: $\Delta > 30$; Tight LHC constraints.
SM+DM+XY	GNMSSM ISS-NMSSM	May form **secluded DM sector**	Suppressed	No tuning; **three portals to SM**.

Why is the DM still called WIMP?

Interactions of "Weak" magnitude in DM sector for Ωh^2, feeble connections between SM and DM sectors to suppress ...

At least two directions to build models:

- **Naturally solve μ-problem: MSSM → Z_3-NMSSM → General NMSSM.**
- **Generate neutrino mass: Type-I NMSSM → ISS-NMSSM → B-L NMSSM.**

Examples-I: no secluded DM sector

Scenario: MSSM, $\tilde{\chi}_1^0 \lesssim 100$ GeV. Ωh^2: **$\tilde{\chi}_1^0$ should be Bino-like.**

Effective Couplings of DM:

$$C_{\tilde{B}\tilde{B}H_{\rm SM}} = g_1^2 \frac{v}{\mu}\left(\sin 2\beta + \frac{m_{\tilde{\chi}_1^0}}{\mu}\right), \quad C_{\tilde{B}\tilde{B}H_{\rm NSM}} = g_1^2 \cos 2\beta \frac{v}{\mu},$$

$$-iC_{\tilde{B}\tilde{B}A_{\rm NSM}} = g_1^2 \frac{v}{\mu}\left(1 + \sin 2\beta \frac{m_{\tilde{\chi}_1^0}}{\mu}\right), \quad C_{\tilde{B}\tilde{B}Z} = \frac{g_1^3 \cos 2\beta}{2 \sin\theta_W} \frac{v^2}{\mu^2}.$$

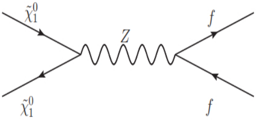
Formula for Direct Detection:

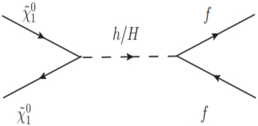
- Spin-independent scattering:

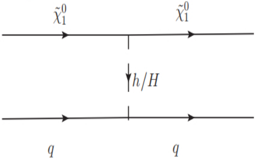
$$\sigma^{\rm SI}_{\tilde{B}-N} \simeq 5 \times 10^{-45}{\rm cm}^2 \times \left[\left(\frac{C_{\tilde{B}\tilde{B}H_{\rm SM}}}{0.1}\right)\left(\frac{125 \text{ GeV}}{m_h}\right)^2 - \cot 2\beta \left(\frac{C_{\tilde{B}\tilde{B}H_{\rm NSM}}}{0.1}\right)\left(\frac{125 \text{ GeV}}{m_H}\right)^2\right]^2 .$$

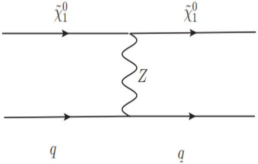
- Spin-dependent scattering:

$$\sigma^{\rm SD}_{\tilde{B}-N} \simeq C_N \times \left(\frac{C_{\tilde{B}\tilde{B}Z}}{0.1}\right)^2 .$$









Phenomenology-I: No secluded DM sector in MSSM

Cases where SI LZ constraints are stronger than LHC constraints.

m_A	Negative $m_{\tilde{\chi}_1^0}/\mu$	Positive $m_{\tilde{\chi}_1^0}/\mu$
1TeV	$tan\beta \lesssim 9$	$tan\beta \lesssim 25$
2TeV	$tan\beta \lesssim 10$	$tan\beta \lesssim 27$

Experimental Constraints:

- Higgs data fit: $m_h \simeq 125$ GeV;
- LHC search for extra Higgs bosons;
- LHC search for supersymmetry;
- DM direct detection experiment LZ.

Preferred mass spectra:

- **LHC search for SUSY**: $\mu \gtrsim 900$ GeV;
- **LZ SD constraints**: $\mu \gtrsim 700$ GeV for $\tan\beta > 4$;
- **LZ SI constraints**: Sometimes stronger than LHC constraints.

Phenomenology-I: No secluded DM sector in MSSM

Two Types of Fine-Tuning:

1. **Z boson mass**
   - Mass formula: $m_Z^2 = \frac{2(m_{H_d}^2+\Sigma_d)-2(m_{H_u}^2+\Sigma_u)\tan^2\beta}{\tan^2\beta-1} - 2\mu^2$;
   - 0.8% fine-tuning without considering $\tilde{g}$ effect from RGE!
2. **DM annihilation**
   - The $1/\mu$ suppression of $C_{\tilde{B}\tilde{B}Z}$ and $C_{\tilde{B}\tilde{B}h}$ couplings requires m_χ to be precisely around $m_Z/2$ or $m_h/2$ for correct Ωh^2;
   - New fine-tuning!

Conclusions:

1. The scenario suffers severe FT problem due to strong constraints;
2. LHC and LZ experiments are complementary in limiting SUSY parameter space;
3. **Latest results:** h **and** Z **resonant annihilation mechanisms have been excluded by LZ-2024 results alone;**
   $m_{\tilde{\chi}_1^0} \gtrsim 200$ GeV **from LHC search for compressed spectrum.**

Phenomenology-I: No secluded DM sector in MSSM

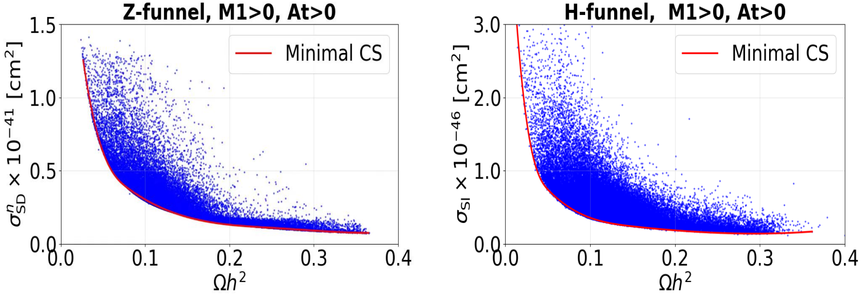


Figure: $\Omega h^2 - \sigma^{\rm SD}_{\tilde{\chi}^0_1 - N}$ **and** $\Omega h^2 - \sigma^{\rm SI}_{\tilde{\chi}^0_1 - N}$ **correlations for Z-funnel region (left panel) and h-funnel region (right panel). Radiative corrections to all involved quantities have been included.**

1. **Excessively large μ suppresses $C_{\tilde{B}\tilde{B}Z}$ and $C_{\tilde{B}\tilde{B}h}$, precluding the measured relic density even at resonance.**
2. **Conversely, the density requirement sets lower bounds on $\sigma^{\rm SD}_{\chi-N}$ and $\sigma^{\rm SI}_{\chi-N}$ at Z- and h-funnel regions, respectively.**

Phenomenology-II: Secluded DM sector in GNMSSM

- **Chiral Superfields**

SF	Spin 0	Spin $\frac{1}{2}$	Generations	(U(1) ⊗ SU(2) ⊗ SU(3))
$\hat{q}$	$\tilde{q}$	q	3	$\left(\frac{1}{6},\mathbf{2},\mathbf{3}\right)$
$\hat{l}$	$\tilde{l}$	l	3	$\left(-\frac{1}{2},\mathbf{2},\mathbf{1}\right)$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$\left(-\frac{1}{2},\mathbf{2},\mathbf{1}\right)$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$\left(\frac{1}{2},\mathbf{2},\mathbf{1}\right)$
$\hat{d}$	$\tilde{d}_R^*$	d_R^*	3	$\left(\frac{1}{3},\mathbf{1},\overline{\mathbf{3}}\right)$
$\hat{u}$	$\tilde{u}_R^*$	u_R^*	3	$\left(-\frac{2}{3},\mathbf{1},\overline{\mathbf{3}}\right)$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$(1,\mathbf{1},\mathbf{1})$
$\hat{s}$	S	$\tilde{S}$	1	$(0,\mathbf{1},\mathbf{1})$

- **Superpotential — no ad hoc symmetry!**

$$W_{\rm GNMSSM} = W_{\rm Y} + \lambda \hat{S}\hat{H}_u\cdot\hat{H}_d + \frac{1}{3}\kappa\hat{S}^3 + \mu\hat{H}_u\cdot\hat{H}_d + \frac{1}{2}\mu'\hat{S}^2 + \xi\hat{S}$$

1. Solving domain wall and tadpole problems in Z_3-NMSSM.
2. Z_3-violating terms originate from unified theories with a Z_4^n or Z_8^n sym..
3. The $\xi\hat{S}$ term can be eliminated by field magentaefinitions.

GNMSSM: DM Mass and Couplings

Singlino-dominated DM

$\mu_{eff} \equiv \frac{\lambda}{\sqrt{2}} v_s$, $\mu_{tot} \equiv \mu + \mu_{eff}$.

- Neutralino mass matrix in basis $\left(\lambda_{\tilde{B}}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S}\right)$:

$$m_{\tilde{\chi}_i^0} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2} g_1 v_d & \frac{1}{2} g_1 v_u & 0 \\ 0 & M_2 & \frac{1}{2} g_2 v_d & -\frac{1}{2} g_2 v_u & 0 \\ -\frac{1}{2} g_1 v_d & \frac{1}{2} g_2 v_d & 0 & -\boldsymbol{\mu_{tot}} & -\frac{1}{\sqrt{2}} v_u \lambda \\ \frac{1}{2} g_1 v_u & -\frac{1}{2} g_2 v_u & -\boldsymbol{\mu_{tot}} & 0 & -\frac{1}{\sqrt{2}} v_d \lambda \\ 0 & 0 & -\frac{1}{\sqrt{2}} v_u \lambda & -\frac{1}{\sqrt{2}} v_d \lambda & \mathbf{m_N} \end{pmatrix}$$

Mass and couplings of the singlino-dominated DM are given by:

$$\mathbf{m_{\tilde{\chi}_1^0}} \quad \simeq \quad \mathbf{m_N} + \frac{\mathbf{1}}{\mathbf{2}} \frac{\boldsymbol{\lambda^2 v^2 (m_{\tilde{\chi}_1^0} - \mu_{tot} \sin 2\beta)}}{\boldsymbol{m^2_{\tilde{\chi}_1^0} - \mu^2_{tot}}} \simeq \mathbf{m_N}, \quad \mathbf{m_N} \equiv \boldsymbol{\sqrt{2} \kappa v_s + \mu'},$$

Theoretical features at the cost of model economy:

- **All particle masses are independent:** m_N, $\mu_{\rm tot}$, m_{h_s}, m_{A_s}, m_h, $\cdots$;
- **All couplings are independent:** λ, κ, g_1, g_2, g_s;
- **Singlet-related parameters are scarcely limited:** $m_{\tilde{\chi}_1^0}/m_N$, m_{h_s}, m_{A_s}, κ. **$\tilde{S}$-dominated DM: $|m_N| \lesssim \mu_{\rm tot}, |M_1|, |M_2|$;**

In contrast, Z_3-NMSSM: $2|\kappa| \lesssim \lambda$ to predict $\tilde{S}$-dominated DM.

Phenomenology-II: Secluded DM sector in GNMSSM

Scenario: GMSSM, $\tilde{\chi}_1^0 \lesssim 100$ GeV. $\tilde{\chi}_1^0$**: Bino- or Singlino-like**.

Key characteristics of DM physics:

- **For Bino-like DM**, they are same as those of the MSSM;
- **For Singlino-like DM**, $\tilde{S}$, h_s and A_s may form a **secluded** DM sector, specifically

  κ-controlled annihilations: $\tilde{S}\tilde{S} \to h_s A_s, h_s h_s, A_s A_s$;

  λ-controlled DM-nucleon scatterings: $\sigma^{\rm SI}_{\chi-N} \propto \lambda^2\kappa^2$, $\sigma^{\rm SD}_{\chi-N} \propto \lambda^4$.

  Consequently, the light DM scenario easily aligns with experimental results.

Phenomenology-II: Secluded DM sector in GNMSSM

Other involved couplings: relying on $\mu_{\rm tot}$, λ, and κ.

$$C_{\widetilde{B}\widetilde{B}Re[S]} = iC_{\widetilde{B}\widetilde{B}Im[S]} = -\frac{\lambda g_1^2 \sin 2\beta}{2}\frac{v^2}{\mu_{\rm tot}^2},$$

$$C_{\widetilde{B}\widetilde{S}H_{\rm SM}} = -2\sqrt{2}\lambda g_1 \cos 2\beta \frac{v}{\mu_{\rm tot}}, \quad iC_{\widetilde{B}\widetilde{S}A_{\rm NSM}} = -\sqrt{2}\lambda g_1 \cos 2\beta \frac{\left(M_1 + M_{\widetilde{S}}\right) v}{\mu_{\rm tot}^2},$$

$$C_{\widetilde{B}\widetilde{S}H_{\rm NSM}} = \sqrt{2}\lambda g_1 \frac{v}{\mu_{\rm tot}}\left[2\sin 2\beta + \frac{\left(M_1 - M_{\widetilde{S}}\right)}{\mu_{\rm tot}}\right],$$

$$C_{\widetilde{B}\widetilde{S}Re[S]} = iC_{\widetilde{B}\widetilde{S}Im[S]} = \sqrt{2}\lambda^2 g_1 \cos 2\beta \frac{v^2}{\mu_{\rm tot}^2}, \quad C_{\widetilde{B}\widetilde{S}Z} = \frac{\sqrt{2}\lambda g_1^2 \sin 2\beta}{\sin\theta_W}\frac{v^2}{\mu_{\rm tot}^2},$$

$$C_{\widetilde{S}\widetilde{S}H_{\rm SM}} = -2\lambda^2 \sin 2\beta \frac{v}{\mu_{\rm tot}} + 2\lambda^2 \frac{m_{\tilde{S}} v}{\mu_{\rm tot}^2}, \quad C_{\widetilde{S}\widetilde{S}H_{\rm NSM}} = -2\lambda^2 \cos 2\beta \frac{v}{\mu_{\rm tot}},$$

$$C_{\widetilde{S}\widetilde{S}Re[S]} = iC_{\widetilde{S}\widetilde{S}Im[S]} = \lambda^3 \sin 2\beta \frac{v^2}{\mu_{\rm tot}^2} - 2\kappa\left(1 - \lambda^2, \frac{v^2}{\mu_{\rm tot}^2}\right),$$

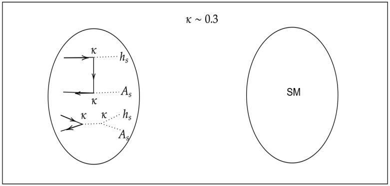
$$iC_{\widetilde{S}\widetilde{S}A_{\rm NSM}} = 2\lambda^2 \frac{v}{\mu_{tot}}\left[1 - \sin 2\beta \frac{m_{\tilde{S}}}{\mu_{\rm tot}}\right] \qquad C_{\widetilde{S}\widetilde{S}Z} = -\frac{\lambda^2 g_1 \cos 2\beta}{\sin\theta_W}\frac{v^2}{\mu_{\rm tot}^2}.$$

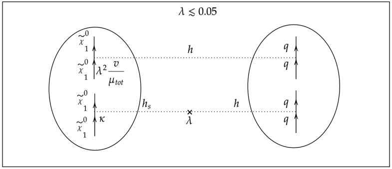
Formula for Direct Detections

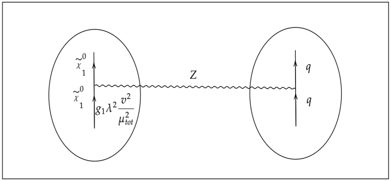
- Spin-independent scattering:

$$\sigma^{\rm SI}_{\tilde{\chi}-N} \simeq 5\times 10^{-45}{\rm cm}^2 \times \left[\left(\frac{125\ {\rm GeV}}{m_h}\right)^2\left(\frac{C_{\tilde{\chi}\tilde{\chi}h}}{0.1}\right) + \left(\frac{125\ {\rm GeV}}{m_H}\right)^2\left(\frac{C_{\tilde{\chi}\tilde{\chi}H}}{0.1}\right) + \left(\frac{125\ {\rm GeV}}{m_{h_s}}\right)^2\left(\frac{C_{\tilde{\chi}\tilde{\chi}h_s}}{0.1}\right)\right]^2.$$

- Spin-dependent scattering: $\sigma^{\rm SD}_{\tilde{\chi}-N} \simeq C_N \times \left(\frac{C_{\tilde{\chi}\tilde{\chi}Z}}{0.1}\right)^2.$







Phenomenology-II: Secluded DM sector in GNMSSM

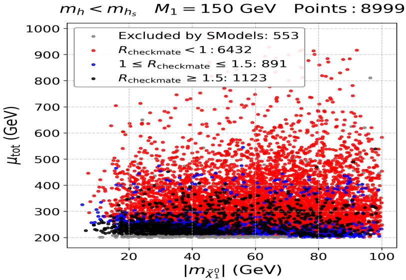


Figure: **Samples, surviving all experimental constraints except those from LHC, are projected onto $m_{\tilde{\chi}_1^0} - \mu_{\rm tot}$ plane.**

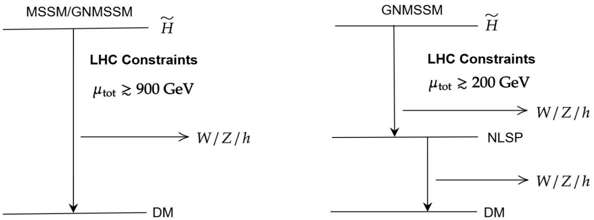


Figure: **Different cases of LHC constraints.**

Phenomenology-II: Secluded DM sector in GNMSSM

Conclusions:

1. Interactions of **weak magnitude** in the DM sector to predict measured relic density.
2. **Feeble connections** between SM and DM sectors to suppress the DM-nucleon scattering.
   Three portals: Higgs portal, Neutrino portal, and Gauge portal; **All can significantly suppress the scatterings**.
3. Constraints from LHC and LZ are **both significantly relaxed**, while they remain sensitive to complementary parameters.
   Example: Heavy sparticles prefer to decay first into intermediate non-LSP states,
   and **consequently**
   $\mu_{\rm tot}$ **as low as** 200 GeV **is experimentally allowed!**
4. Measured Z-boson mass and DM observables are **free from the FT problem**.
5. **Cost: Loss of economy in model construction;** μ **problem.**

Section V

B-L NMSSM

General Discussion

Requirements of natural SUSY:

- Naturally generate the μ **parameter**;
- Naturally explain the measurements in **DM physics**;
- Naturally accommodate the **LHC results** with EW physics;
- Generate **neutrino masses and mixings** dynamically;
- **Economy** in model-building.

Lessons from GNMSSM:

- The **minimal** framework to realize secluded DM sector;
- **Many theoretical advantages**, such as stable vacuum and significant alleviation of the LHC constraints;
- **However, DM-nucleon scatterings require** $\lambda \lesssim 0.05$**, implying** $\mu_{\rm eff} \ll \mu_{\rm tot}$**, where** $\mu_{\rm eff} \equiv \lambda/\sqrt{2} v_s$ **and** $\mu_{\rm tot} \equiv \mu + \mu_{\rm eff}$;
  Only a small portion of μ **parameter is dynamically generate**
- **More singlet fields are needed (preferred by both** μ **and DM problems), which may be motivated by symmetry!**

Advantages

B-L NMSSM: Personal opinion

Most promising theory among minimal frameworks!

Key Features of B-L NMSSM:

- **New $U(1)_{B-L}$ gauge symmetry ⇒ Origin of R-symmetry;**
- **New Higgs fields to break symmetry ⇒ Good motivation to introduce more SM singlet Higgs fields;**
- **Right-handed neutrino to cancel gauge anomalies ⇒ Neutrino masses and mixings;**
- **Lepton number violation ⇒ Matter-antimatter asymmetry;**
- **Abundant singlet Higgs fields and neutralinos ⇒ Rich Higgs, DM, and sparticle physics;**
- **Model characteristics:**
  Naturally generate μ parameter at TeV scale without domain-wall problem, naturally suppress DM-nucleon scattering, many applications at colliders and in cosmology (inflation, PT, and GW), $\cdots$.

Model Structure

- **Chiral Superfields**

SF	Spin 0	Spin $\frac{1}{2}$	Generations	$(\mathrm{U}(1)\otimes\mathrm{SU}(2)\otimes\mathrm{SU}(3)\otimes\mathrm{U}(1))$
$\hat{q}$	$\tilde{q}$	q	3	$\left(\frac{1}{6},\mathbf{2},\mathbf{3},\frac{1}{6}\right)$
$\hat{l}$	$\tilde{l}$	l	3	$\left(-\frac{1}{2},\mathbf{2},\mathbf{1},-\frac{1}{2}\right)$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$\left(-\frac{1}{2},\mathbf{2},\mathbf{1},0\right)$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$\left(\frac{1}{2},\mathbf{2},\mathbf{1},0\right)$
$\hat{d}$	$\tilde{d}_R^{0,*}$	d_R^*	3	$\left(\frac{1}{3},\mathbf{1},\overline{\mathbf{3}},-\frac{1}{6}\right)$
$\hat{u}$	$\tilde{u}_R^{0,*}$	u_R^*	3	$\left(-\frac{2}{3},\mathbf{1},\overline{\mathbf{3}},-\frac{1}{6}\right)$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$\left(1,\mathbf{1},\mathbf{1},\frac{1}{2}\right)$
$\hat{\nu}$	$\tilde{\nu}_R^*$	ν_R^*	**3**	$\left(0,\mathbf{1},\mathbf{1},\frac{1}{2}\right)$
$\hat{\eta}$	η	$\tilde{\eta}$	**1**	$(0,\mathbf{1},\mathbf{1},-1)$
$\hat{\bar{\eta}}$	$\bar{\eta}$	$\tilde{\bar{\eta}}$	**1**	$(0,\mathbf{1},\mathbf{1},1)$
$\hat{s}$	$\tilde{S}$	S	**1**	$(0,\mathbf{1},\mathbf{1},0)$

- **Superpotential**

$$W_{\mathrm{B-L}} = W_{\mathrm{Y}} + \lambda\hat{\mathbf{s}}\hat{\mathbf{H}}_{\mathbf{u}}\cdot\hat{\mathbf{H}}_{\mathbf{d}} + \frac{\mathbf{1}}{\mathbf{3}}\kappa\hat{\mathbf{s}}\hat{\mathbf{s}}^{\mathbf{2}} - \lambda_{\mathbf{2}}\hat{\mathbf{s}}\hat{\eta}\hat{\bar{\eta}} + \mathbf{Y}_{\mathbf{x}}\hat{\nu}\hat{\eta}\hat{\nu}$$

- **Crucial Observations**
  - **Z_3-invariant theory; Domain-wall problem like Z_3-NMSSM?**
  - **$\hat{\eta}$ and $\hat{\bar{\eta}}$ are symmetric for small Y_x, so $\tan\beta' \simeq 1$;**
  - **$\hat{s}$ transmits information of B-L and S sectors to EW sector!**

Key Features Inferred by Naturalness

1. **Three Scale Theory:** $v_{\rm B-L} \gg v_{\rm s} \gg v_{\rm EW}$.
   - $v_{\rm B-L} \gtrsim 15$ TeV: **LHC search for** Z'.
   - $v_{\rm s} \sim (1-10)$ TeV: **Suppress** $C_{\tilde{S}\tilde{S}h}$; **generate** μ **with small** λ.

     $C_{\tilde{S}\tilde{S}h} = \lambda \mathbf{v} \times \frac{\mathbf{1}}{\mu} \times \lambda = \lambda \times \frac{\mathbf{v}}{\mathbf{v}_{\rm s}}$ by mass insertion approximation.

     - ✓ λv: **Singlino-Higgsino Transition;**
     - ✓ $1/\mu$: **Higgsino Propagator;**
     - ✓ λ: $\tilde{H} - \tilde{S} - h$ **coupling.**
   - **Generate** μ**-parameter without domain-wall problem:** **B-L breaking generate** $\hat{s}$ **tadpole term!**
2. **Coupling Hierachy:**
   $\kappa \gg \lambda \gg \lambda_2$ **to stabilize** $v_{\rm s}$ **and** $v_{\rm EW}$ **by tadpole equations.**
   - ◇ $\kappa \sim \mathcal{O}(1)$: **Describe singlet self coupling;**
   - ◇ $\lambda \sim \mathcal{O}(0.1)$: **Transmit Singlet breaking to EW sector;**
   - ◇ $\lambda_2 \sim \mathcal{O}(0.01)$: **Transmit B-L breaking to both Singlet and EW sectors.**

Key Features Inferred by Naturalness

3. Singlino-Modulated and Sequestered DM:

- **Mass matrix for Neutralinos in the basis** $\left(\lambda_{\tilde{B}}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \lambda_{\tilde{B}'}, \tilde{\eta}, \tilde{\bar{\eta}}, \tilde{S}\right)$

$$\begin{pmatrix}
M_1 & 0 & -\frac{1}{2}g_1 v_d & \frac{1}{2}g_1 v_u & M_{BB'} & -g_{BY}v_\eta & g_{BY}v_{\bar{\eta}} & 0 \\
0 & M_2 & \frac{1}{2}g_2 v_d & -\frac{1}{2}g_2 v_u & 0 & 0 & 0 & 0 \\
-\frac{1}{2}g_1 v_d & \frac{1}{2}g_2 v_d & 0 & -\frac{1}{\sqrt{2}}\lambda v_s & -\frac{1}{2}g_{YB}v_d & 0 & 0 & -\frac{1}{\sqrt{2}}\lambda v_u \\
\frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & -\frac{1}{\sqrt{2}}\lambda v_s & 0 & \frac{1}{2}g_{YB}v_u & 0 & 0 & -\frac{1}{\sqrt{2}}\lambda v_d \\
M_{BB'} & 0 & -\frac{1}{2}g_{YB}v_d & \frac{1}{2}g_{YB}v_u & M_{BL} & -g_B v_\eta & g_B v_{\bar{\eta}} & 0 \\
-g_{BY}v_\eta & 0 & 0 & 0 & -g_B v_\eta & 0 & -\frac{1}{\sqrt{2}}\lambda_\eta v_s & -\frac{1}{\sqrt{2}}\lambda_\eta v_{\bar{\eta}} \\
g_{BY}v_{\bar{\eta}} & 0 & 0 & 0 & g_B v_{\bar{\eta}} & -\frac{1}{\sqrt{2}}\lambda_\eta v_s & 0 & -\frac{1}{\sqrt{2}}\lambda_\eta v_\eta \\
0 & 0 & -\frac{1}{\sqrt{2}}\lambda v_u & -\frac{1}{\sqrt{2}}\lambda v_d & 0 & -\frac{1}{\sqrt{2}}\lambda_\eta v_{\bar{\eta}} & -\frac{1}{\sqrt{2}}\lambda_\eta v_\eta & \sqrt{2}\kappa v_s
\end{pmatrix}$$

Preferred parameter regions:

- ◇ $g_{BY} \equiv 0$: Rotated away due to redefinition of gauge fields!
- ◇ $g_{YB} \ll g_2$: RGE generated (loop effects);
- ◇ $\lambda \sim 0.1$: DM direct detection experiment such as LZ;
- ◇ $M_{BB'}$: Safely neglected, as inferred by MIA, or equivalently integrating out massive fields.
  - ✓ $M_{BB'}$ is REG generated (loop effects);
  - ✓ $\lambda_{\tilde{B}'}$ is tremendously massive.

Key Features Inferred by Naturalness

3. Singlino-Modulated and Sequestered DM:

- **Reduced neutralino mass matrix in the bases** $\left(\lambda_{\tilde{B}'}, \tilde{\chi}_A = -\sin\beta' \tilde{\chi}_\eta + \cos\beta' \tilde{\chi}_{\bar{\eta}}, \tilde{\chi}_{\mathbf{B}} = \cos\beta' \tilde{\chi}_\eta + \sin\beta' \tilde{\chi}_{\bar{\eta}}, \tilde{S}\right)$**:**

$$\begin{pmatrix} M_{BL} & g_B v_{BL} & 0 & 0 \\ g_B v_{BL} & \frac{\lambda_2}{\sqrt{2}} v_s \sin 2\beta' & -\frac{\lambda_2}{\sqrt{2}} v_s \cos 2\beta' & 0 \\ 0 & -\frac{\lambda_2}{\sqrt{2}} v_s \cos 2\beta' & -\frac{\lambda_2}{\sqrt{2}} v_s \sin 2\beta' & -\frac{\lambda_2}{\sqrt{2}} v_{BL} \\ 0 & 0 & -\frac{\lambda_2}{\sqrt{2}} v_{BL} & \sqrt{2}\kappa v_s \end{pmatrix},$$

Terms in black are small since $\tan\beta' \simeq 1$**,** $\kappa \sim \mathcal{O}(1)$**, and** $\lambda_2 \sim 10^{-2}$**:**

$$\begin{pmatrix} M_{BL} & g_B v_{BL} & 0 & 0 \\ g_B v_{BL} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\lambda_2}{\sqrt{2}} v_{BL} \\ 0 & 0 & -\frac{\lambda_2}{\sqrt{2}} v_{BL} & \sqrt{2}\kappa v_s \end{pmatrix}.$$

Key Features Inferred by Naturalness

3. Singlino-Modulated and Sequestered DM:

- **Reduced neutralino mass matrix in the bases** $\left(\lambda_{\tilde{B}'}, \tilde{\chi}_A = -\sin\beta' \tilde{\chi}_\eta + \cos\beta' \tilde{\chi}_{\bar{\eta}}, \tilde{\chi}_{\mathbf{B}} = \cos\beta' \tilde{\chi}_\eta + \sin\beta' \tilde{\chi}_{\bar{\eta}}, \tilde{S}\right)$**:**

$$\begin{pmatrix} M_{BL} & g_B v_{BL} & 0 & 0 \\ g_B v_{BL} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\lambda_2}{\sqrt{2}} v_{BL} \\ 0 & 0 & -\frac{\lambda_2}{\sqrt{2}} v_{BL} & \sqrt{2}\kappa v_s \end{pmatrix}.$$

Important conclusions:

- ◇ $\lambda_{\tilde{B}'}$ **mixes with** $\tilde{\chi}_A$ **to form very massive mass eigenstates!**
- ◇ **DM is** $\tilde{\chi}_B$**-dominated, containing sizable** $\tilde{S}$ **component.**
  - ✓ **Mass:** $m_{\tilde{\chi}_1^0} \simeq -\lambda_2^2 v_{BL}^2/(2\sqrt{2}\kappa v_s)$ via seesaw mechanism;
  - ✓ **Mixing:** $\tilde{\chi}_1^0 \simeq \cos\theta \tilde{\chi}_B + \sin\theta \tilde{S}$ with $\sin\theta \simeq -\lambda_2 v_{BL}/(2\kappa v_s) \lesssim 0.3$;
  - ✓ **Interactions:** Sequestered since $Q_{BL} \simeq 0$ and $\tilde{S}$-modulated;
  - ✓ Mass and interactions are **independent**, similar to GNMSSM.

Key Features Inferred by Naturalness

3. Singlino-Modulated and Sequestered DM:

- **DM-nucleon scattering**

  Formula for scattering cross-sections:

$$\sigma^{\rm SD}_{\tilde\chi_1^0-N} \simeq C_N \times \left(\frac{C_{\tilde\chi_1^0\tilde\chi_1^0 Z}}{0.1}\right)^2 ,$$

$$\sigma^{\rm SI}_{\tilde\chi_1^0-N} \simeq 5\times 10^{-45}{\rm cm}^2 \times \left(\frac{125~{\rm GeV}}{m_h}\right)^4 \left(\frac{C_{\tilde\chi_1^0\tilde\chi_1^0 h}}{0.1}\right)^2 .$$

  Given $\tilde\chi_1^0 \simeq \cos\theta\tilde\chi_B + \sin\theta\tilde S$,

  - ◇ $C_{\tilde\chi_1^0\tilde\chi_1^0 Z} = \sin\theta \times C_{\tilde S\tilde S Z} \times \sin\theta \simeq g_1 \cos 2\beta \sin^{\mathbf{2}}\theta / \sin\theta_W \times v^2/v_s^2$;
  - ◇ $C_{\tilde\chi_1^0\tilde\chi_1^0 h} = \sin\theta \times C_{\tilde S\tilde S h} \times \sin\theta \simeq \lambda v \sin^{\mathbf{2}}\theta / v_s$;
  - ◇ **For** $\tilde\chi_1^0 - \tilde H$ **coannihilation,** $C_{\tilde\chi_1^0\tilde\chi_1^0 h} \simeq 2\sqrt{2} \times \kappa^{\mathbf{2}} \sin^{\mathbf{6}}\theta \times v/|m_{\tilde\chi_1^0}|$,

    $C_{\tilde\chi_1^0\tilde\chi_1^0 Z} \simeq 2g_1 \cos 2\beta / \sin\theta_W \times \kappa^{\mathbf{2}} \sin^{\mathbf{6}}\theta \times v^2/m^2_{\tilde\chi_1^0}$;
  - ◇ **DM-nucleon scattering: naturally suppressed for small λ and $\sin\theta$ and large v_s!**
  - ◇ $\mu \equiv \lambda v_s/\sqrt{2}$ **varies from** 100 GeV **to 1** TeV, **solving μ-problem!**

Key Features Inferred by Naturalness

3. Singlino-Modulated and Sequestered DM:

- **Relic density:**
  - ◇ $\tilde{\chi}_1^0 - \tilde{H}$ **co-annihilation**

    Condition: $|m_{\tilde{\chi}_1^0}| \simeq \mu$, **or equivalently** $v_s^2 \simeq |\lambda_2^2 v_{BL}^2/(2\sqrt{2}\lambda\kappa)|$.
    - ✓ **Not fine-tuned, as inferred from tadpole equations!**

      $$v_s^2 = \frac{\lambda_2 \kappa v_\eta v_{\bar{\eta}}(1+\alpha_2) - m_S^2}{\kappa^2(1+\alpha_\kappa)}, \quad \Delta_{FT}^{\rm BG} \sim \mathcal{O}(1).$$
    - ✓ **Bayesian evidence suppressed given** $0 < \delta < 0.1$ **in** $\lambda = 2\kappa(1+\delta)\sin^2\theta$.

      However, besides relic density, this suppressed parameter phase space is also preferred by EW FT, LZ experiment, and LHC search for SUSY, or equivalently, by likelihood function in global fitting of the theory!
  - ◇ A_s**-resonant annihilation:** $\Delta_{FT}^{\rm BG}$ **from 100 to 1000.**
    - ✓ Since $C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 A_s} \simeq \kappa \sin^2\theta$ is suppressed, m_{A_s} close to $2 m_{\tilde{\chi}_1^0}$;
    - ✓ Predicting a relatively light A_s by cancellation!

Key Features Inferred by Naturalness

4. Collider Signals:

Similar to those of MSSM with $\tilde{H}$-dominated DM for $\tilde{\chi}_1^0 - \tilde{H}$ co-annihilation.

- **Heavy sparticle will first decay to lighter ones other than DM due to gauge singlet nature of DM;**
- **Only Higgsinos as NLSP directly decay to DM:**
  (1) **Missing momentum;**
  (2) **Displaced vertex;**
  (3) **Missing track;**
  (4) **"Orphan" signal;**

  which depends on Higgsino mass spectrum and lifetime, and also on LHC search strategy.
- **Higgs bosons beyond those of MSSM are hard to detect due to their $SU(2)_L$ gauge singlet nature.**

Key Features Inferred by Naturalness

5. Other Aspects:

- **Secluded DM sector for righthanded sneutrino DM;**
  - Annihilation: $\tilde{\nu}\tilde{\nu} \to A_{BL} A_{BL}$ by contact interactions;
  - Measured density: $Y_x \sim 0.3$;
  - Predicting a Sub-TeV DM needs cancellations, e.g., $\Delta^{\rm BG}_{FT} \sim 100$ in acquiring $m_{\tilde{\nu}} = 500$ GeV.
- **Parameters exhibit hierarchy structure:**
  - $\kappa \sim \mathcal{O}(1)$, $\lambda \sim \sin^2\theta$, $\lambda_2 \sim \sin^3\theta$;
  - $v_s \sim v_{BL}\sin^2\theta$, $v_{\rm EW} \sim v_{BL}\sin^4\theta$, $|m_{\tilde{\chi}^0_1}| \sim v_{BL}\sin^4\theta$.

  Generated by Froggatt-Nielson Mechanism?
- **Economy but very Effective in model building!**
  No dimensional parameters in the superpotential due to ad hoc Z_3-symmetry, like Z_3-NMSSM;
  If abandoning the symmetry, no benefit to enrich phenomenology, but worsen the theory's other advantages!

Section VI

Conclusions

Conclusion: MSSM

MSSM is disfavored by naturalness argument!

- Owing to intrinsic structure in its theoretical framework, **the fine-tuning of MSSM** is at 0.01% level, i.e., $\Delta_{FT}^{\rm BG} \sim 10000$;
  - Barbieri-Giudice Measurement:

    $$\Delta_{FT}^{\rm BG} \equiv max\left(\frac{\partial \ln O}{\partial \ln p_i}\right)$$

    O: physics observable, such as m_Z, Ωh^2, etc.;
    p_i: ith input of the theory.
  - Expected $\Delta_{FT}^{\rm BG}$ during LEP era: $\Delta_{FT}^{\rm BG} \lesssim 10$;
  - Acceptable $\Delta_{FT}^{\rm BG}$ in post-LHC era: $\Delta_{FT}^{\rm BG} \lesssim 300$.
- Constraints from **only** Ωh^2 measurement and LZ experiment **have excluded** Z- and h-resonant annihilation mechanisms;
- **Furthermore**, LHC searches for compressed spectrum have set a lower bound on DM mass, $m_{\tilde{\chi}_1^0} \gtrsim 200$ GeV.

Conclusion: Beyond MSSM

Unexpected theoretical advantages in gauge extensions;
B-L extended NMSSM: the most economic extension to significantly alleviate FT problem of MSSM.

1. **Economic extension of MSSM** - No dimensional parameters in its superpotential;
2. **Elegant solution to μ-problem** - **Free from domain-wall problem with minimal framework**;
3. **Explaining the origin of R-symmetry;**
4. **Generate dynamically neutrino mass and mixing;**
5. **Singlino-modulated and sequestered DM** - **Naturally suppressed DM-nucleon scattering**;
6. **Well-motivated DM annihilation** - **BLino - Higgsino coannihilation**;
7. **Simple phenomenology** - **Similar to that of MSSM with Higgsino acting as DM candidate.** ⋯

Conclusion: WIMP

Recent advancements in WIMP DM theory:

- **Keep the idea of freeze-out and thermal relic;**
- **More complex structure to break crossing symmetry between DM annihilation and DM-nucleon scattering;**
- **Phenomenology significantly differs from traditional MSSM prediction due to gauge singlet nature of DM, which results in lengthened decay chain of sparticles.**
  - No hints of DM and sparticles in experiments are consistent!

Examples

1. **Secluded DM Sector**
   Singlino-dominated DM in GNMSSM:
   $$pp \to \tilde{\chi}_i^0 \tilde{\chi}_j^\pm \to \mathbf{4\{W/Z/h\}} E_T^{\rm Miss}$$
   τ-flavored righthanded sneutrino DM in seesaw extension:
   $$pp \to \tilde{\chi}_i^0 \tilde{\chi}_j^\pm \to \mathbf{2\tau 2\{W/Z/h\}} E_T^{\rm Miss}$$

Conclusion: WIMP

Example

2. **Co-annihilation**
   - ◇ **Most natural co-annihilation: Same gauge multiplet;**
     - ✓ Higgsino $SU(2)_L$ doublet, $m_{\tilde{\chi}_1^0} \simeq 1.1$ TeV for measured Ωh^2;
     - ✓ Wino $SU(2)_L$ triplet, $m_{\tilde{\chi}_1^0} \simeq 3.0$ TeV for measured Ωh^2, **excluded by DM indirect search experiments**.
   - ◇ **Another natural co-annihilation: Higgsino-partnered.** **BLino-Higgsino co-annihilation in B-L NMSSM, well-motivated, any mass value for measured Ωh^2.**

Phenomenology:

Similar to that of MSSM with **Higgsino** acting **as DM candidate** except for the subtlety about Higgsino signal:

(1) Missing momentum; **(2) Displaced vertex;**

(3) Missing track; **(4) "Orphan" signal,**

which depends on Higgsino mass spectrum and lifetime, and also on LHC search strategies.

thanks!