The primary HEFT as a precision benchmark for UV-HEFT matching

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Outline

- Introduction: the SM with scalar extensions, UV-HEFT matching
- A non-linear framework.
- Primary HEFT as a precision benchmark.
- Summary and Outlook.

The Standard Model Extended by Scalar Sectors

One singular Higgs doublet in SM,

Theo.

- Minimal but not explanatory
- Hierarchy problem

Pheo.

- Insufficient CP violation
- No strong first-order EW phase transition
- No dark matter candidate

An enlarged scalar sector can address these issues.

SM + singlet, 2HDM, SM + a real triplet, SM + a complex triplet, …

SUSY

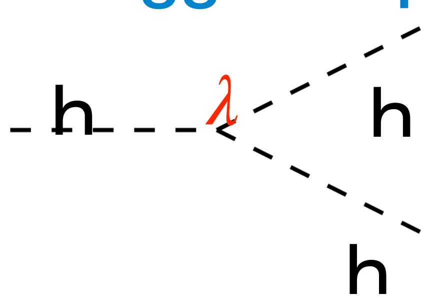
Type-II SEESAW Model

Theo.

- Additional scalar fields
- Non-trivial vacuum structure
- Multiple symmetry-breaking patterns
- Additional CP phases

Pheo.

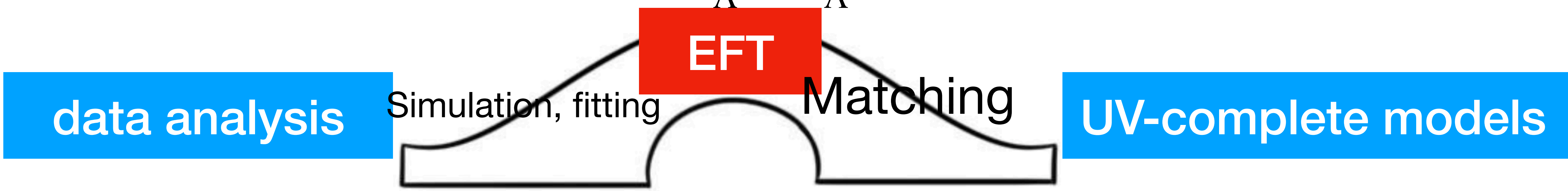
- Modified Higgs couplings
- Dark matter

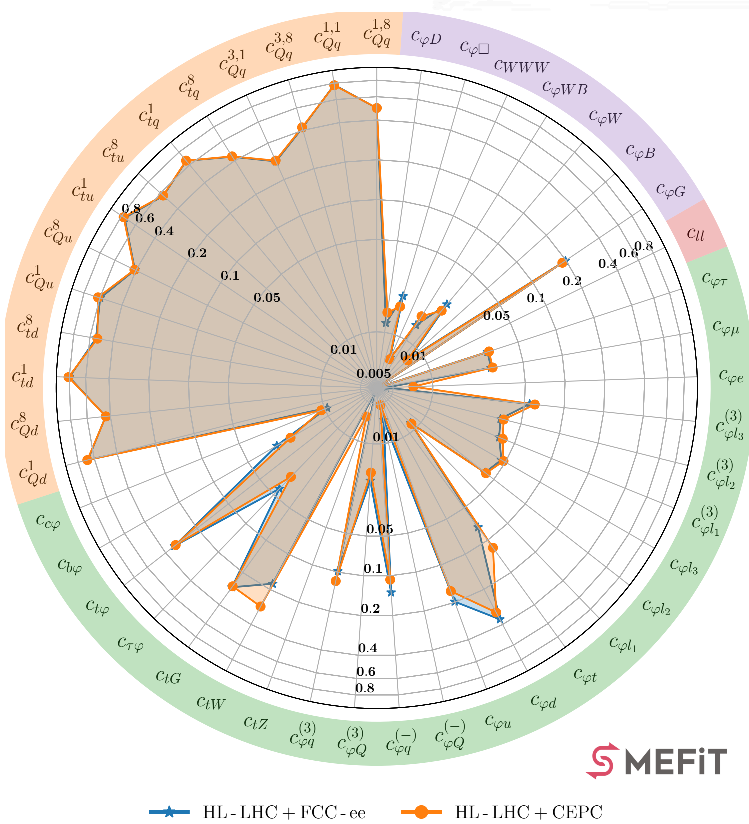


EFT (Effective Field Theory)

- UV theory is not known, use most general EFT operators to parameterize NP. (Bottom-up approach).
- UV theory is known, integrate out the heavy particles and match to EFT. (Top-down approach).

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda}\mathscr{O}_5 + \frac{1}{\Lambda^2}\mathscr{O}_6 + \cdots$$



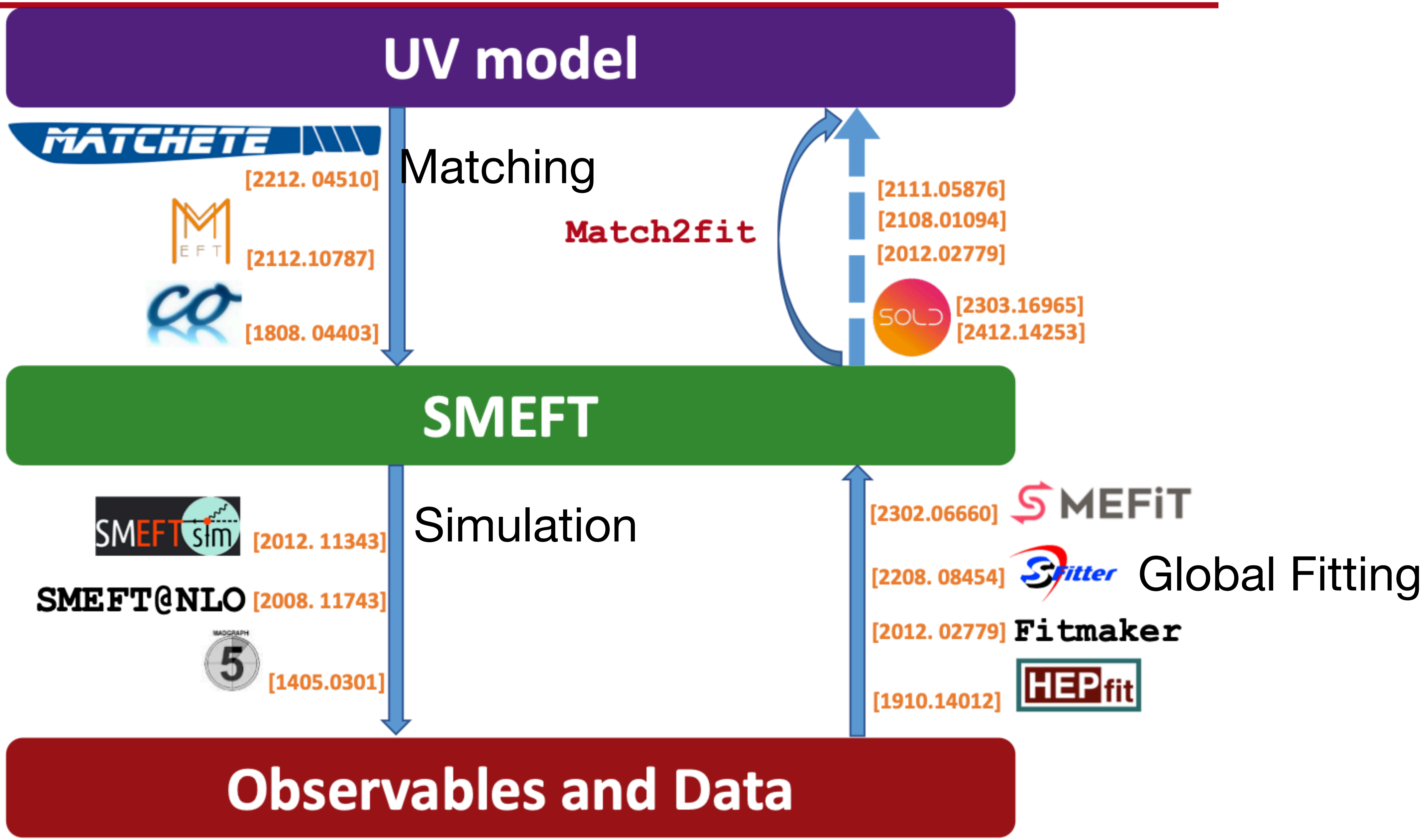


From Jaco ter Hoeve, ICHEP 2024

- Any EFT deviation from the SM must have a UV origin.
- UV–EFT matching provides the mapping between Wilson coefficients and UV parameters.

Automation of SMEFT

From Alejo N. Rossia's Silde, SMEFT-Tools,2025



SMEFT is not enough

- SMEFT, linear realization — Symmetric phase — Decoupling

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} G^+ \\ v + h + iG^0 \end{pmatrix}, \mathscr{L}_{\text{SMEFT}} \supset \frac{1}{2}D_\mu H^\dagger D^\mu H + \frac{m^2}{2}H^\dagger H - \lambda(H^\dagger H)^2 + \frac{C_H}{\Lambda^2}(H^\dagger H)^3 + \cdots$$

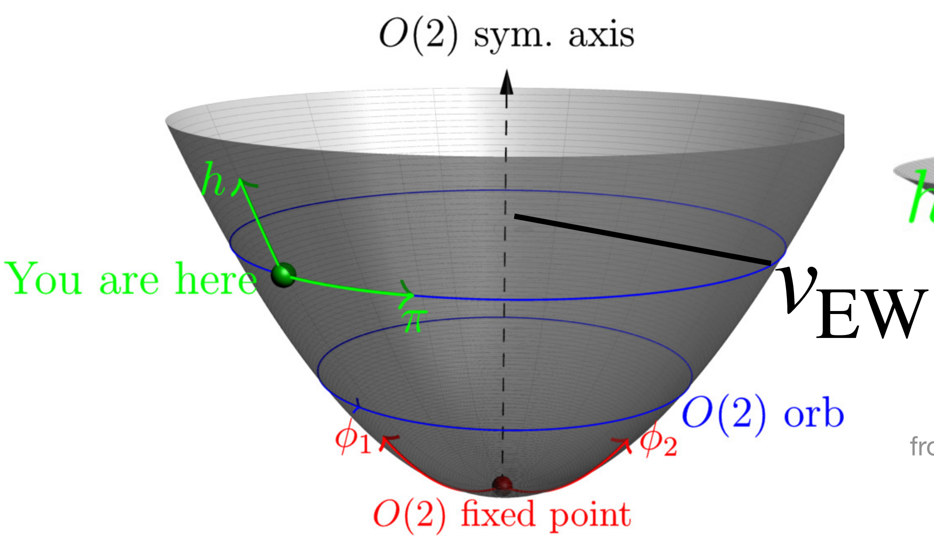
- HEFT, nonlinear realization — Broken phase — Non-decoupling

$$h, U \equiv \exp\left(\frac{i\pi_i\sigma_i}{v_{\text{EW}}}\right), \mathscr{L}_{\text{HEFT}}^{\text{LO}} \supset \frac{1}{2}D_\mu h D^\mu h - V(h) + \frac{v_{\text{EW}}^2}{4}F(h)\text{Tr}(D_\mu U^\dagger D^\mu U) + \cdots$$

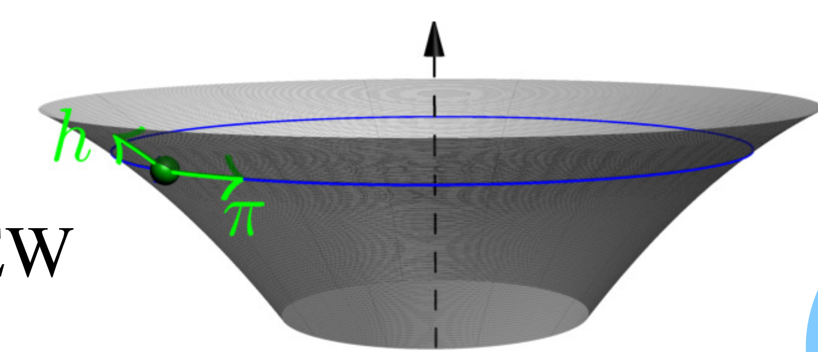
$$V(h) = \frac{1}{2}m_h^2 h^2[1 + (1 + \Delta\kappa_3)\frac{h}{v_{\text{EW}}} + \cdots], F(h) = 1 + 2(1 + \Delta a)\frac{h}{v_{\text{EW}}} + \cdots$$

[H. Sun, M.-L. Xiao, and J.-H. Yu, 2206.07722]

R. Alonso, E. Jenkins, A. Manohar [1511.00724,1605.03602]

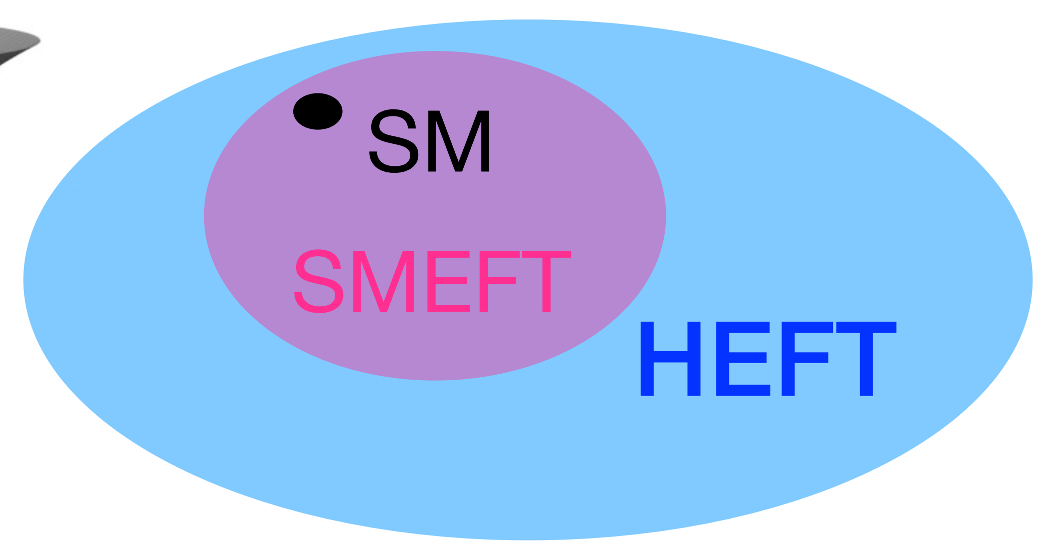


SMEFT/HEFT



from N. Craig, HEFT2021

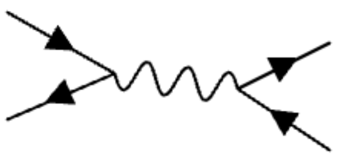
only HEFT



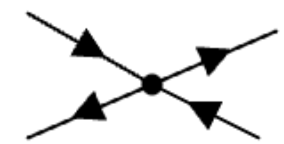
UV-EFT matching

$$\mathscr{L}_{\rm UV} = \mathscr{L}_{\rm SM} + \mathscr{L}_{\rm BSM}\left(\phi_{\rm SM}, \Phi_{\rm BSM}\right) \Longleftrightarrow \mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \sum_i C_i \mathscr{O}_i\left(\phi_{\rm SM}\right)$$

- Diagrammatic approach (amplitude)



$$\left\{\mathscr{A}_{\rm UV}\left(\lambda_{\rm UV}\right)\right\} = \left\{\mathscr{A}_{\rm SMEFT}\left(C_i\right)\right\}$$



- Need to know $\mathscr{O}_i$ in advance

- Functional method (action)

$$\Gamma_{\rm UV,1LPI}[\phi_{\rm SM}] = \Gamma_{\rm EFT,1PI}[\phi_{\rm SM}]$$

- Less intuitive
- Systematic operator generation
- Automation-friendly

Brian Henning, Xiaochuan Lu, and Hitoshi Murayama, 1412.1837

Aleksandra Drozd, John Ellis, Jérémi Quevillon and Tevong You, 1512.03003

Timothy Cohen,[1] Xiaochuan Lu,[1] and Zhengkang Zhang, 2012.07851

Complication of UV-HEFT matching

in functional method

- The non-linear structure

Higgs and Goldstones are separated, Goldstones are embedded in U matrix.

$$U \equiv \exp\left(\frac{i\pi_i\sigma_i}{v_{\rm EW}}\right)$$

- The broken phase

Many effective parameters after symmetry breaking, e.g. multiple scalar masses, mixing angles.
⇒ Multiple power-counting schemes.

Is HEFT unique? [S. Dawson et al, 2311.16897]

Non-linear Representation of the Singlet Model

The real singlet model

$$\mathcal{L}_{\rm Z2RSM} \supset (D^\mu H)^\dagger (D_\mu H) + \partial^\mu S\,\partial_\mu S - V(H,S),$$

$$V(H,S) = -\frac{\mu_1^2}{2} H^\dagger H - \frac{\mu_2^2}{2} S^2 + \frac{\lambda_1}{4}(H^\dagger H)^2 + \frac{\lambda_2}{4} S^4 + \frac{\lambda_3}{2} H^\dagger H\, S^2$$

$$H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v_{\rm EW} + h + iG^0) \end{pmatrix} \quad \longrightarrow \quad \boxed{\mathrm{H} = U\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_{\rm EW} + h^0 \end{pmatrix}, \quad U \equiv \exp\left(\frac{i\pi_i\sigma_i}{v_{\rm EW}}\right)}$$

- **Only suitable for the UV models with no extra VEVs,**

How to Get Non-linear Representation of 2HDM

2HDM

$$\mathcal{L}_{\rm 2HDM} \supset \left(D_\mu\Phi_1\right)^\dagger\left(D^\mu\Phi_1\right) + \left(D_\mu\Phi_2\right)^\dagger\left(D^\mu\Phi_2\right) - V_{\rm 2HDM},$$

$$V_{\rm 2HDM} = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - m_{12}^2\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) + \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$
$$+ \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + \left(\Phi_2^\dagger\Phi_1\right)^2\right],$$

$$v_{\rm EW}^2 = v_1^2 + v_2^2$$

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix},$$

Non-linear Representation of 2HDM

2HDM

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix},$$

U(2) symmetry among two doublets.

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix},$$

Higgs Basis:

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v_{\rm EW} + h_1^H + iG^0) \end{pmatrix}, H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2^H + iA) \end{pmatrix}$$

$$\mathscr{H}_1 = U \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{\rm EW} + \mathfrak{h}_1^H \end{pmatrix}, \qquad \mathscr{H}_2 = U \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\mathfrak{h}_2^H + iA) \end{pmatrix}$$

S. Dawson et al, 2305.07689

- No mass mixing between Goldstones and other scalars, because in the potential no U appears. E.g. $(\mathscr{H}_1^\dagger \mathscr{H}_2)$ term
- U could be considered as a rotation, which does not affect S-matrix.

How to Get Non-linear Representation of the Triplet Model

The Real Triplet Model

$$\mathcal{L} \supset (D_\mu H)^\dagger (D^\mu H) + \langle D_\mu \Sigma^\dagger D^\mu \Sigma \rangle - V_S - V_Q,$$

$$V_S = Y_1^2 H^\dagger H + Z_1 (H^\dagger H)^2 + Y_2^2 \langle \Sigma^\dagger \Sigma \rangle + Z_2 \langle \Sigma^\dagger \Sigma \rangle^2 + Z_3 H^\dagger H \langle \Sigma^\dagger \Sigma \rangle + 2 Y_3 H^\dagger \Sigma H,$$

$$V_Q = y_u \bar{Q}_L \tilde{H} u_R + y_d \bar{Q}_L H d_R + \text{h.c.},$$

$$\mathrm{H} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v_\mathrm{H} + h + iG^0) \end{pmatrix}, \Sigma = \frac{1}{2}\Sigma_i \sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -v_\Sigma - \Sigma^0 \end{pmatrix}, i = 1, 2, 3$$

$$v_\mathrm{EW}^2 = v_\mathrm{H}^2 + 4 v_\Sigma^2$$

An extra VEV, no U(2) symmetry rotates it away.

Non-linear Representation of the Triplet Model

The Real Triplet Higgs Model (RTHM)

$$\mathrm{H} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v_\mathrm{H} + h + iG^0) \end{pmatrix}, \Sigma = \frac{1}{2}\Sigma_i \sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -v_\Sigma - \Sigma^0 \end{pmatrix}, i = 1, 2, 3$$

$$v_\mathrm{EW}^2 = v_\mathrm{H}^2 + 4v_\Sigma^2$$

7 d.o.f

$$\mathrm{H} = U\frac{1}{\sqrt{2}}\begin{pmatrix} \chi^\pm \\ v_\mathrm{H} + h^0 + i\chi^0 \end{pmatrix}, \quad \Sigma = U\Phi U^\dagger, \ \Phi = \frac{1}{2}\phi_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \phi^0 & \sqrt{2}\phi^+ \\ \sqrt{2}\phi^- & -v_\Sigma - \phi^0 \end{pmatrix}$$

$$D_\mu H^\dagger D^\mu H \supset v_\mathrm{H}\epsilon_{3jk} D_\mu \chi_j D^\mu \pi_k / (2v_\mathrm{EW})$$

$$\langle D_\mu \Sigma^\dagger D^\mu \Sigma \rangle \supset -v_\Sigma \epsilon_{3jk} D_\mu \phi_j D^\mu \pi_k$$

$$\chi^\pm = 2\frac{v_\Sigma}{v_\mathrm{H}}\phi^\pm, \chi^0 = 0$$

1. **No mass mixing between Goldstones and other scalars.**
2. **Kinetic mixings are cancelled by special choice of χ states.**

U matrix is separated from heavy scalars.

Goldstone modes could also be excised in “Unitary basis”. (Cohen, Craig, Lu, and Sutherland 2021)
Similar ideas in the 2HDM .(P. Ciafaloni and D. Espriu,hep-ph/9612383; S. Dittmaier, et al, 2022)

General Scalar Extensions

$$\Phi_{ijklm\cdots} = U^i_{i_1} U^j_{j_1} U^k_{k_1} U^l_{l_1} U^k_{k_1} U^m_{m_1} \cdots \phi_{i_1 j_1 k_1 l_1 m_1 \cdots}$$

$\underbrace{1\cdots1}_{j-y+1}\underbrace{2\cdots2}_{j+y-1}$ for positive charge

$\underbrace{1\cdots1}_{j-y-1}\underbrace{2\cdots2}_{j+y+1}$ for negative charge

$\underbrace{1\cdots1}_{j-y}\underbrace{2\cdots2}_{j+y}$ Neutral

$$\begin{aligned}
&(D\Phi^{*i_1 i_2 i_3 i_4 i_5\cdots})(D\Phi_{i_1 i_2 i_3 i_4 i_5\cdots}) \\
=&(D\phi^{*i_1 i_2 i_3 i_4 i_5\cdots})(D\phi_{i_1 i_2 i_3 i_4 i_5\cdots}) + (DU^{*i_n}_{k_n} DU^{j_n}_{i_n})\phi^{*\cdots i_{n-1}k_n i_{n+1}\cdots}\phi_{\cdots i_{n-1}j_n i_{n+1}\cdots} \\
&+ (U^{*i_n}_{k_n} DU^{j_n}_{i_n} D\phi^{*\cdots i_{n-1}k_n i_{n+1}\cdots}\phi_{\cdots i_{n-1}j_n i_{n+1}\cdots} + DU^{*i_n}_{k_n} U^{j_n}_{i_n}\phi^{*\cdots i_{n-1}k_n i_{n+1}\cdots} D\phi_{\cdots i_{n-1}j_n i_{n+1}\cdots}) \\
&+ (U^{*i_m}_{k_m} DU^{j_m}_{i_m} DU^{*i_n}_{k_n} U^{j_n}_{i_n} + DU^{*i_m}_{k_m} U^{j_m}_{i_m} U^{*i_n}_{k_n} DU^{j_n}_{i_n}) \\
&\quad \phi^{*\cdots i_{m-1}k_m i_{m+1}\cdots i_{n-1}k_n i_{n+1}\cdots}\phi_{\cdots i_{m-1}j_m i_{m+1}\cdots i_{n-1}j_n i_{n+1}\cdots} \\
\supset& \frac{v_\phi}{\sqrt{2}}\left[(j+y)(U^\dagger DU)^2_2 + (j-y)(U^\dagger DU)^1_1\right] D\left(\phi^{0*} - \phi^0\right) \\
&+ v_\phi/\sqrt{2}(U^\dagger DU)^2_1\left[\sqrt{(j+y)(j-y+1)}D\phi^{+*} - \sqrt{(j-y)(j+y+1)}D\phi^-\right] \\
&\quad + v_\phi/\sqrt{2}(U^\dagger DU)^1_2\left[\sqrt{(j-y)(j+y+1)}D\phi^{-*} - \sqrt{(j+y)(j-y+1)}D\phi^+\right]
\end{aligned}$$

$$\chi^+ = \frac{v_\phi}{v_H}(\sqrt{(j-y)(j+y+1)}\phi^{-*} - \sqrt{(j+y)(j-y+1)}\phi^+) \quad \chi^0 = -\frac{2yv_\phi}{v_H}\eta^0$$

U (Goldstones) and heavy states are separate, matching become straight and simple, further programmable.

H. Song, XW, 2412.00355(*JHEP* 06 (2025) 021)

On Power Counting schemes in the Broken Phase

Is HEFT unique? [S. Dawson et al, 2311.16897]

Establishing the Primary HEFT as a Precision Benchmark for UV-HEFT Matching. [Zizhou Ge, Huayang Song, XW, 2602.14418]

Power counting in SMEFT

Take triplet model as an example

$$\mathscr{L}^{\Sigma} = \frac{1}{2}\vec{\Sigma}^{T}(-D_{\mu}D^{\mu} - Y_2^2 - Z_3\, \mathrm{H}^{\dagger}\mathrm{H})\vec{\Sigma} + Y_3\vec{\Sigma}\cdot \mathrm{H}^{\dagger}\vec{\sigma}\mathrm{H} - \frac{1}{4}Z_2(\vec{\Sigma}\cdot\vec{\Sigma})^2$$

EoM of Σ:

$$(-D_{\mu}D^{\mu} - Y_2^2 - Z_3\, \mathrm{H}^{\dagger}\mathrm{H})\vec{\Sigma}_c = -Y_3\mathrm{H}^{\dagger}\vec{\sigma}\mathrm{H} + Z_2(\vec{\Sigma}_c\cdot\vec{\Sigma}_c)\vec{\Sigma}_c$$

$$\vec{\Sigma}_c = -\frac{1}{-D_{\mu}D^{\mu} - Y_2^2 - Z_3\,\mathrm{H}^{+}\mathrm{H}}Y_3\mathrm{H}^{+}\vec{\sigma}\mathrm{H} + \frac{1}{-D_{\mu}D^{\mu} - Y_2^2 - Z_3\,\mathrm{H}^{+}\mathrm{H}}Z_2(\vec{\Sigma}_c\cdot\vec{\Sigma}_c)\vec{\Sigma}_c$$

Expansion with $1/Y_2^2$

Power Counting

$$\mathscr{L}_{\mathrm{SMEFT}} = \frac{1}{2Y_2^2}Y_3^2\mathrm{H}^{\dagger}\vec{\sigma}H\cdot\mathrm{H}^{\dagger}\vec{\sigma}\mathrm{H} + \frac{1}{2}(\mathrm{H}^{\dagger}\vec{\sigma}H)^{T}\frac{1}{Y_2^2}(-D_{\mu}D^{\mu} - Z_3H^{\dagger}H)\frac{1}{Y_2^2}\mathrm{H}^{\dagger}\vec{\sigma}H + \cdots$$

T. Corbett, A. Helset, A. Martin, M. Trott, [2102.02819]
J. Ellis, K. Mimasu, F. Zamperdri, [2304.06663]

Parameter Sets of RHTM

RHTM

$$\mathcal{L} \supset (D_\mu H)^\dagger (D^\mu H) + \langle D_\mu \Sigma^\dagger D^\mu \Sigma \rangle - V_S - V_Q,$$

$$V_S = Y_1^2 H^\dagger H + Z_1 (H^\dagger H)^2 + Y_2^2 \langle \Sigma^\dagger \Sigma \rangle + Z_2 \langle \Sigma^\dagger \Sigma \rangle^2 + Z_3 H^\dagger H \langle \Sigma^\dagger \Sigma \rangle + 2 Y_3 H^\dagger \Sigma H,$$

$$V_Q = y_u \bar{Q}_L \tilde{H} u_R + y_d \bar{Q}_L H d_R + \text{h.c.},$$

$$H = U \begin{pmatrix} \chi^+ \\ \frac{1}{\sqrt{2}}(v_H + h^0 + i\chi^0) \end{pmatrix}, \quad U \equiv \exp\left(\frac{i\pi_i \sigma_i}{v_{\rm EW}}\right) \qquad \chi^\pm = 2\frac{v_\Sigma}{v_H}\phi^\pm, \quad \chi^0 = 0,$$

$$\Sigma = U \Phi U^\dagger, \quad \Phi = \phi_i \sigma_i / 2 = \frac{1}{2}\begin{pmatrix} v_\Sigma + \phi^0 & \phi_1 - i\phi_2 \\ \phi_1 + i\phi_2 & -v_\Sigma - \phi^0 \end{pmatrix}$$

$$\{Z_1, Z_2, Z_3, Y_1, Y_2, Y_3\}$$

Minimum condition: $Y_1^2 = -Z_1 v_H^2 - \frac{Z_3 v_\Sigma^2}{2} + Y_3 v_\Sigma, \quad Y_2^2 = -Z_2 v_\Sigma^2 - \frac{Z_3 v_H^2}{2} + \frac{Y_3 v_H^2}{2 v_\Sigma}$ $\qquad \{Z_1, Z_2, Z_3, Y_3, v_H, v_\Sigma\}$

$$\mathcal{L}_{\rm mass} = \frac{1}{2}\begin{pmatrix} h^0 & \phi^0 \end{pmatrix} \begin{pmatrix} 2 Z_1 v_H^2 & v_H (Z_3 v_\Sigma - Y_3) \\ v_H (Z_3 v_\Sigma - Y_3) & 2 Z_2 v_\Sigma^2 + \frac{Y_3 v_H^2}{2 v_\Sigma} \end{pmatrix} \begin{pmatrix} h^0 \\ \phi^0 \end{pmatrix} + (v_H^2 + 4 v_\Sigma^2)^2 \frac{Y_3}{2 v_\Sigma v_H^2} \phi^+ \phi^-$$

Mass eigenstates:

$$\begin{pmatrix} h \\ K \end{pmatrix} = \begin{pmatrix} c_\gamma & -s_\gamma \\ s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} h^0 \\ \phi^0 \end{pmatrix}, \quad \tan(2\gamma) = \frac{v_H (Z_3 v_\Sigma - Y_3)}{Z_2 v_\Sigma^2 - Z_1 v_H^2 + \frac{Y_3 v_H^2}{4 v_\Sigma}}, \qquad s_\gamma \equiv \sin(\gamma)$$

$$m_{h,K}^2 = Z_1 v_H^2 + Z_2 v_\Sigma^2 + \frac{Y_3 v_H^2}{4 v_\Sigma} \mp \sqrt{\left(Z_1 v_H^2 - Z_2 v_\Sigma^2 - \frac{Y_3 v_H^2}{4 v_\Sigma}\right)^2 + v_H^2 (Z_3 v_\Sigma - Y_3)^2}.$$

$$m_{\phi^\pm}^2 = (v_H^2 + 4 v_\Sigma^2) \frac{Y_3}{2 v_\Sigma} = v_{\rm EW}^2 \frac{Y_3}{2 v_\Sigma}.$$

$$\{m_h, m_K, m_{\phi^\pm}, s_\gamma, v_H, v_\Sigma\}$$

Power Countings

$$\mathcal{P} = \{m^2_{\phi^\pm}, m^2_K, m_h, v_{\rm EW}, s_\gamma, \xi\},$$

$$\xi = v_\Sigma / v_H \qquad v_{\rm EW}^2 = v_H^2 + 4 v_\Sigma^2$$

$$\text{pHEFT:}\quad m^2_{\phi^\pm} \sim m^2_K \sim \mathcal{O}(t^{-1}), \quad m_h \sim v_{\rm EW} \sim s_\gamma \sim \xi \sim \mathcal{O}(t^0),$$

$$\text{dHEFT:}\quad m^2_{\phi^\pm} \sim m^2_K \sim \mathcal{O}(t^{-1}), \quad m_h \sim v_{\rm EW} \sim \mathcal{O}(t^0), \quad s_\gamma \sim \xi \sim \mathcal{O}(t^1).$$

$$\mathcal{P}_{Z_2} = \{m^2_{\phi^\pm}, m^2_K, m_h, v_{\rm EW}, Z_2, \xi\}$$

$$Z_2\text{-HEFT:}\quad m^2_{\phi^\pm} \sim m^2_K \sim \mathcal{O}(t^{-1}), \quad m_h \sim v_{\rm EW} \sim Z_2 \sim \xi \sim \mathcal{O}(t^0).$$

$$\mathcal{P}_\xi = \{Z_1, Z_2, Z_3, Y_3, v_H, \xi\}$$

$$\xi\text{-HEFT:}\quad Z_1 \sim Z_2 \sim Z_3 \sim Y_3 \sim v_H \sim \mathcal{O}(t^0), \quad \xi \sim \mathcal{O}(t^1).$$

$$\mathcal{P}_{Y_2} = \{Z_1, Z_2, Z_3, Y_1, Y_2, Y_3\}.$$

$$Y_2\text{-HEFT:}\quad Z_1 \sim Z_2 \sim Z_3 \sim Y_1 \sim Y_3 \sim \mathcal{O}(t^0), \quad Y_2^2 \sim \mathcal{O}(t^{-1}).$$

Power Countings

$$\mathcal{P} = \{m_{\phi^\pm}^2, m_K^2, m_h, v_{\rm EW}, s_\gamma, \xi\},$$

$\xi = v_\Sigma / v_H$ $\quad v_{\rm EW}^2 = v_H^2 + 4v_\Sigma^2$

Non-decoupling

pHEFT: $m_{\phi^\pm}^2 \sim m_K^2 \sim \mathcal{O}(t^{-1}),\quad m_h \sim v_{\rm EW} \sim s_\gamma \sim \xi \sim \mathcal{O}(t^0),$

dHEFT: $m_{\phi^\pm}^2 \sim m_K^2 \sim \mathcal{O}(t^{-1}),\quad m_h \sim v_{\rm EW} \sim \mathcal{O}(t^0),\quad s_\gamma \sim \xi \sim \mathcal{O}(t^1).$

Decoupling

$$\mathcal{P}_{Z_2} = \{m_{\phi^\pm}^2, m_K^2, m_h, v_{\rm EW}, Z_2, \xi\}$$

Z_2-HEFT: $m_{\phi^\pm}^2 \sim m_K^2 \sim \mathcal{O}(t^{-1}),\quad m_h \sim v_{\rm EW} \sim Z_2 \sim \xi \sim \mathcal{O}(t^0).$

$$\mathcal{P}_\xi = \{Z_1, Z_2, Z_3, Y_3, v_H, \xi\}$$

ξ-HEFT: $Z_1 \sim Z_2 \sim Z_3 \sim Y_3 \sim v_H \sim \mathcal{O}(t^0),\quad \xi \sim \mathcal{O}(t^1).$ **Decoupling**

H. Song, XW, 2503.00707(*JHEP* 06 (2025) 249)

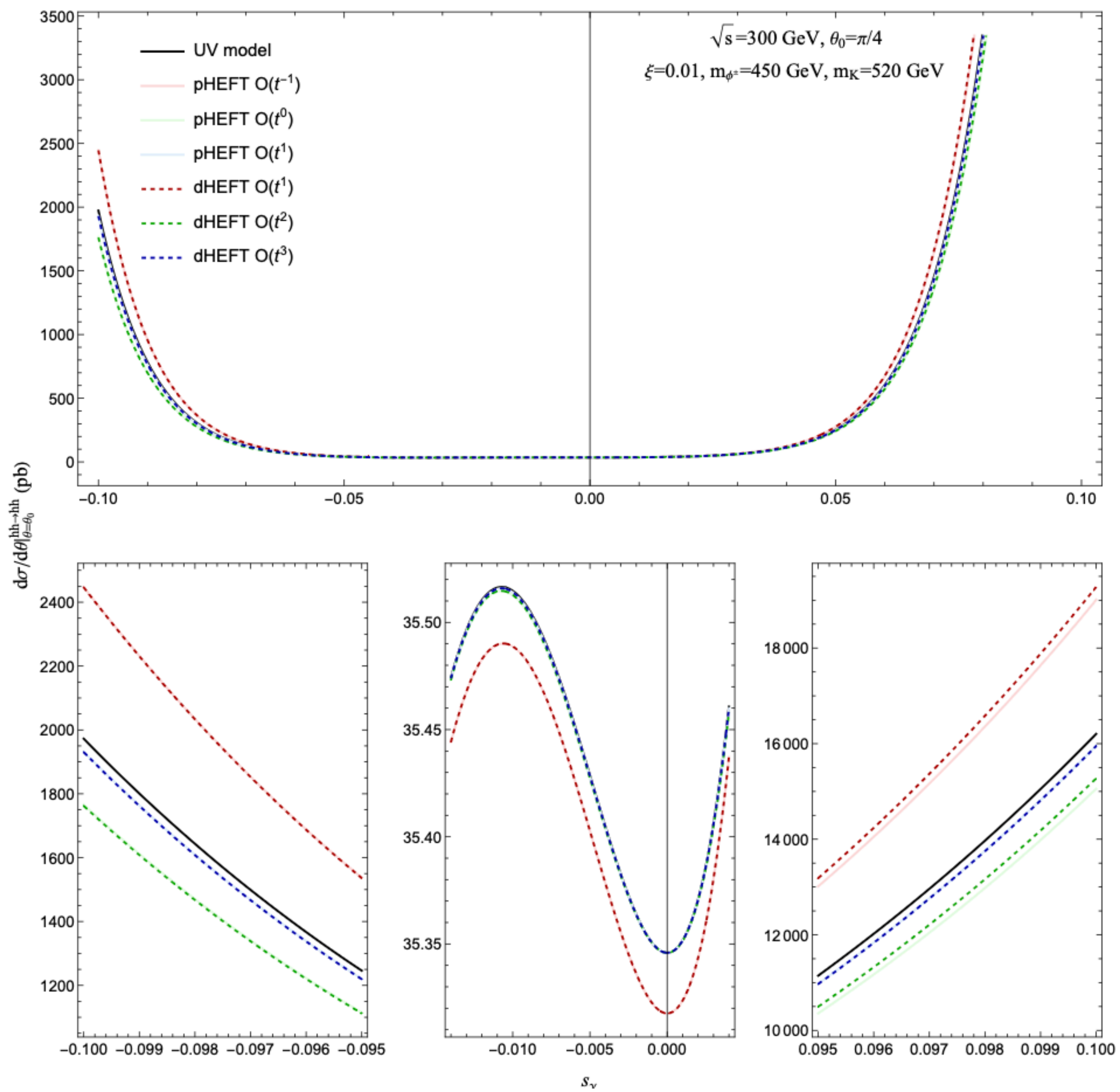
$$\mathcal{P}_{Y_2} = \{Z_1, Z_2, Z_3, Y_1, Y_2, Y_3\}.$$

Y_2-HEFT: $Z_1 \sim Z_2 \sim Z_3 \sim Y_1 \sim Y_3 \sim \mathcal{O}(t^0),\quad Y_2^2 \sim \mathcal{O}(t^{-1}).$ **SMEFT-like**

pHEFT ⊃ dHEFT ⊃ ξ-HEFT

$$
\begin{aligned}
\mathcal{L}^{p}_{\rm HEFT}(t^{-1}) &= \frac{v_H^2\left[(4\xi^2+1)m_K^2(s_\gamma+\xi c_\gamma)^2-\xi^2 m_{\phi^\pm}^2\right]}{8(4\xi^2+1)} - \frac{h^3 m_{\phi^\pm}^2 s_\gamma^2(2\xi c_\gamma+s_\gamma)}{2\xi(4\xi^2+1)v_H} \\
&\quad - \frac{h^4}{8\xi^2(4\xi^2+1)^2 v_H^2 m_K^2}\Big\{m_{\phi^\pm}^2 s_\gamma^2\Big[(4\xi^2+1)m_K^2\Big(6(2\xi^2-1)c_\gamma^4+7c_\gamma^2+18\xi c_\gamma^3 s_\gamma-4\xi c_\gamma s_\gamma-1\Big) \\
&\qquad + m_{\phi^\pm}^2\Big(9(1-4\xi^2)c_\gamma^4+3(8\xi^2-3)c_\gamma^2-36\xi c_\gamma^3 s_\gamma+12\xi c_\gamma s_\gamma-4\xi^2\Big)\Big]\Big\} + \mathcal{O}(h^5),
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
\mathcal{L}^{p}_{\rm HEFT}(t^{0}) &= \frac{1}{2}\langle V_\mu\sigma_3\rangle\langle V^\mu\sigma_3\rangle\left\{\xi^2 v_H^2-2h\xi v_H s_\gamma+h^2 s_\gamma\left[s_\gamma^3-\xi c_\gamma^3+\frac{c_\gamma m_{\phi^\pm}^2(3c_\gamma s_\gamma+4\xi-6\xi s_\gamma^2)}{(4\xi^2+1)m_K^2}\right]+\mathcal{O}(h^3)\right\} \\
&\quad + \frac{1}{4}\langle V_\mu V^\mu\rangle\Big\{-(4\xi^2+1)v_H^2-2hv_H(c_\gamma-4\xi s_\gamma)+\frac{h^2}{\xi}\Big[c_\gamma s_\gamma(4\xi^2 c_\gamma^2+s_\gamma^2) \\
&\qquad + \xi(-5s_\gamma^4+2s_\gamma^2-1)+\frac{m_{\phi^\pm}^2 s_\gamma\big(3(8\xi^2-1)c_\gamma s_\gamma^2-16\xi^2 c_\gamma+2\xi(9s_\gamma^2-8)s_\gamma\big)}{(4\xi^2+1)m_K^2}\Big]+\mathcal{O}(h^3)\Big\} \\
&\quad + \frac{1}{2}D_\mu h D^\mu h+\frac{1}{8}m_h^2 v_H^2(c_\gamma-\xi s_\gamma)^2-\frac{1}{2}h^2 m_h^2+\frac{h^3 m_h^2(s_\gamma^3-\xi c_\gamma^3)}{2\xi v_H} \\
&\quad + \frac{h^4 m_h^2}{24\xi^2(4\xi^2+1)^2 v_H^2 m_K^4}\Big\{4m_{\phi^\pm}^4 s_\gamma^2(3c_\gamma s_\gamma+4\xi-6\xi s_\gamma^2)^2 \\
&\qquad + (4\xi^2+1)^2 m_K^4\Big[s_\gamma^2\Big(38\xi c_\gamma^3 s_\gamma+25\xi^2+19(\xi^2-1)s_\gamma^4+(16-41\xi^2)s_\gamma^2\Big)-3\xi^2\Big] \\
&\qquad - 20(4\xi^2+1)m_K^2 m_{\phi^\pm}^2 s_\gamma^2\Big[\xi c_\gamma s_\gamma(7-9s_\gamma^2)-c_\gamma^2\Big((6\xi^2-3)s_\gamma^2-4\xi^2\Big)\Big]\Big\}+\mathcal{O}(h^5) \\
&\quad + \bar{Q}_L U\begin{pmatrix} y_u & 0 \\ 0 & y_d\end{pmatrix}Q_R\times\frac{1}{\sqrt{2}}\Big\{-v_H-hc_\gamma \\
&\qquad + \frac{h^2 s_\gamma^2\left[(4\xi^2+1)c_\gamma m_K^2(\xi c_\gamma+s_\gamma)-m_{\phi^\pm}^2(3c_\gamma s_\gamma+4\xi-6\xi s_\gamma^2)\right]}{2\xi(4\xi^2+1)v_H m_K^2}+\mathcal{O}(h^3)\Big\}+\text{h.c.},
\end{aligned} \tag{23}
$$



Non-decoupling effects are all kept.

The pHEFT is most accurate.

pHEFT vs. Z_2-HEFT

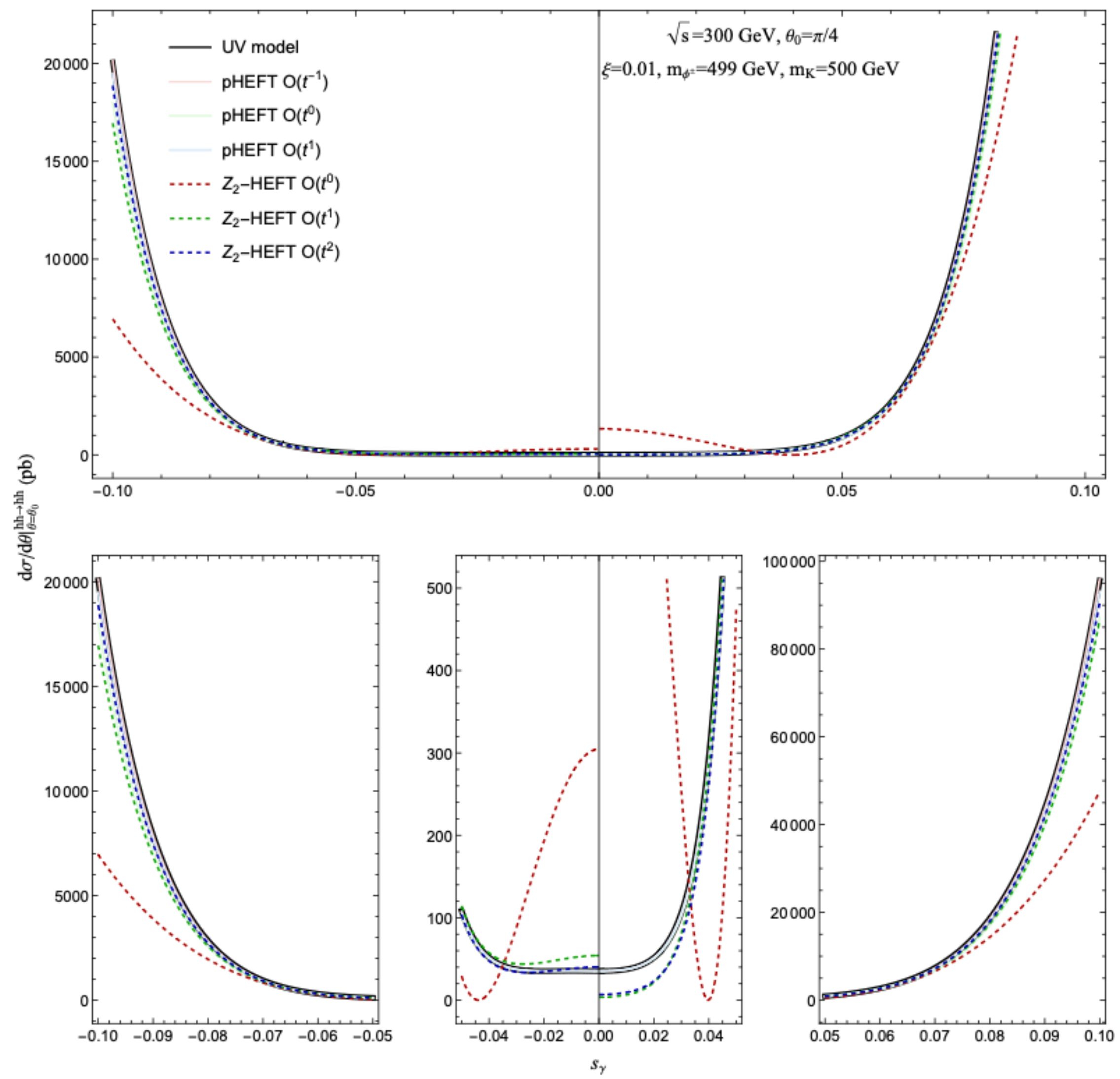
$$\mathcal{P} = \{m^2_{\phi^\pm}, m^2_K, m_h, v_{\rm EW}, \underline{s_\gamma}, \xi\},$$

$$\text{pHEFT:}\quad m^2_{\phi^\pm} \sim m^2_K \sim \mathcal{O}(t^{-1}), \quad m_h \sim v_{\rm EW} \sim s_\gamma \sim \xi \sim \mathcal{O}(t^0),$$

$$\mathcal{P}_{Z_2} = \{m^2_{\phi^\pm}, m^2_K, m_h, v_{\rm EW}, \underline{Z_2}, \xi\}$$

$$Z_2\text{-HEFT:}\quad m^2_{\phi^\pm} \sim m^2_K \sim \mathcal{O}(t^{-1}), \quad m_h \sim v_{\rm EW} \sim Z_2 \sim \xi \sim \mathcal{O}(t^0).$$

$$s_\gamma = \pm\sqrt{\frac{2\xi^2 Z_2 v^2_{\rm EW} - (4\xi^2+1)m^2_K + m^2_{\phi^\pm}}{(4\xi^2+1)(m^2_h - m^2_K)}}.$$



Expansion criteria:

$$m^2_h \ll m^2_K.$$

$$\left|2\xi^2 Z_2 v^2_{\rm EW}\right| \ll \left|\left(4\xi^2+1\right) m^2_K - m^2_{\phi^\pm}\right|$$

Why pHEFT keep maximal UV's information?

$$\mathcal{L} \supset (D_\mu H)^\dagger (D^\mu H) + \langle D_\mu \Sigma^\dagger D^\mu \Sigma \rangle - V_S - V_Q,$$

$$V_S = Y_1^2 H^\dagger H + Z_1 (H^\dagger H)^2 + Y_2^2 \langle \Sigma^\dagger \Sigma \rangle + Z_2 \langle \Sigma^\dagger \Sigma \rangle^2 + Z_3 H^\dagger H \langle \Sigma^\dagger \Sigma \rangle + 2 Y_3 H^\dagger \Sigma H,$$

$$V_Q = y_u \bar{Q}_L \tilde{H} u_R + y_d \bar{Q}_L H d_R + \text{h.c.},$$

$$\mathcal{L}_{\text{mass}} = \frac{1}{2} \begin{pmatrix} h^0 & \phi^0 \end{pmatrix} \begin{pmatrix} 2 Z_1 v_H^2 & v_H (Z_3 v_\Sigma - Y_3) \\ v_H (Z_3 v_\Sigma - Y_3) & 2 Z_2 v_\Sigma^2 + \frac{Y_3 v_H^2}{2 v_\Sigma} \end{pmatrix} \begin{pmatrix} h^0 \\ \phi^0 \end{pmatrix} + \left(v_H^2 + 4 v_\Sigma^2\right)^2 \frac{Y_3}{2 v_\Sigma v_H^2} \phi^+ \phi^-$$

Masses are eigenvalues.

$$Y_3 = \frac{2 v_\Sigma}{v_H^2 + 4 v_\Sigma^2} m_{\phi^\pm}^2$$

$$Z_1 = \frac{1}{2 v_H^2} \left(c_\gamma^2 m_h^2 + s_\gamma^2 m_K^2\right)$$

$$Z_2 = \frac{1}{2 v_\Sigma^2} \left(s_\gamma^2 m_h^2 + c_\gamma^2 m_K^2 - \frac{v_H^2}{v_H^2 + 4 v_\Sigma^2} m_{\phi^\pm}^2\right)$$

$$Z_3 = \frac{1}{v_H v_\Sigma} \left[s_\gamma c_\gamma \left(m_K^2 - m_h^2\right) + \frac{2 v_H v_\Sigma}{v_H^2 + 4 v_\Sigma^2} m_{\phi^\pm}^2 \right].$$

The linear relation between the UV Lagrangian's couplings and heavy masses, expansion of $1/M^2$ cause no extra truncations except for the propagator structure.

2HDM

$$V^{\rm H}_{\rm 2HDM} = Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + \left(Y_3 H_1^\dagger H_2 + {\rm h.c.}\right) + \frac{Z_1}{2}\left(H_1^\dagger H_1\right)^2 + \frac{Z_2}{2}\left(H_2^\dagger H_2\right)^2 + Z_3\left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + Z_4\left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right) + \left\{\frac{Z_5}{2}\left(H_1^\dagger H_2\right)^2 + Z_6\left(H_1^\dagger H_1\right)\left(H_1^\dagger H_2\right) + Z_7\left(H_2^\dagger H_2\right)\left(H_1^\dagger H_2\right) + {\rm h.c.}\right\},$$

$$Y_1 = -\frac{Z_1}{2}v^2, \quad Y_3 = -\frac{Z_6}{2}v^2,$$

$$Z_1 = \frac{s^2_{\beta-\alpha} m_h^2 + c^2_{\beta-\alpha} m_H^2}{v^2},$$

$$Z_2 = \frac{1}{2v^2 t_\beta^3}\Big[c^2_{\beta-\alpha} t_\beta \left(3t_\beta^4 - 8t_\beta^2 + 3\right)\left(m_h^2 - m_H^2\right) + 2t_\beta\left(t_\beta^2 - 1\right)^2\left(m_H^2 - Y_2\right) - m_h^2\left(t_\beta^5 - 4t_\beta^3 + t_\beta\right) + s_{\beta-\alpha}c_{\beta-\alpha}\left(t_\beta^6 - 7t_\beta^4 + 7t_\beta^2 - 1\right)\left(m_h^2 - m_H^2\right)\Big],$$

$$Z_3 = \frac{2}{v^2}\left(m^2_{H^\pm} - Y_2\right),$$

$$Z_4 = \frac{c^2_{\beta-\alpha}\left(m_h^2 - m_H^2\right) + m_A^2 + m_H^2 - 2m^2_{H^\pm}}{v^2},$$

$$Z_5 = \frac{c^2_{\beta-\alpha}\left(m_h^2 - m_H^2\right) - m_A^2 + m_H^2}{v^2},$$

$$Z_6 = \frac{c_{\beta-\alpha}s_{\beta-\alpha}\left(m_h^2 - m_H^2\right)}{v^2},$$

$$Z_7 = \frac{1}{2v^2 t_\beta^2}\Big[-3c^2_{\beta-\alpha} t_\beta\left(t_\beta^2 - 1\right)\left(m_h^2 - m_H^2\right) + t_\beta\left(t_\beta^2 - 1\right)\left(m_h^2 - 2m_H^2 + 2Y_2\right) - s_{\beta-\alpha}c_{\beta-\alpha}\left(t_\beta^4 - 4t_\beta^2 + 1\right)\left(m_h^2 - m_H^2\right)\Big].$$

Linear relations also exist. Therefore a primary HEFT of 2HDM exists.

Y_2-HEFT vs. SMEFT

$$\mathcal{P} = \{m_{\phi^\pm}^2, m_K^2, m_h, v_{\rm EW}, s_\gamma, \xi\},$$
$$\text{pHEFT:} \quad m_{\phi^\pm}^2 \sim m_K^2 \sim \mathcal{O}(t^{-1}), \quad m_h \sim v_{\rm EW} \sim s_\gamma \sim \xi \sim \mathcal{O}(t^0),$$

$$\mathcal{P}_{Y_2} = \{Z_1, Z_2, Z_3, Y_1, Y_2, Y_3\}.$$
$$Y_2\text{-HEFT:} \quad Z_1 \sim Z_2 \sim Z_3 \sim Y_1 \sim Y_3 \sim \mathcal{O}(t^0), \quad Y_2^2 \sim \mathcal{O}(t^{-1}).$$

UV→pHEFT → Y_2-HEFT (mimic SMEFT's power counting)

UV→SMEFT

$$\mathscr{L}_{\rm SMEFT} \supset \mathscr{L}_{\rm SM} + \frac{C_H}{\Lambda^2}(H^\dagger H)^3 + \frac{C_{H\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H) + \frac{C_{HD}}{\Lambda^2}(D_\mu H^\dagger H)(H^\dagger D^\mu H) + \cdots$$

dim-4	
C_{H^4}	$-Z_1 + \frac{Y_3^2}{2Y_2^2} + \frac{2Y_1^2 Y_3^2}{Y_2^4} + \frac{6Y_1^4 Y_3^2}{Y_2^6}$
dim-6	
C_H	$\frac{Y_3^2}{2Y_2^4}\left[(8Z_1 - Z_3)\left(1 + \frac{4Y_1^2}{Y_2^2}\right) - \frac{4Y_3^2}{Y_2^2}\right]$
C_{HD}	$-\frac{2Y_3^2}{Y_2^4}\left(1 + \frac{4Y_1^2}{Y_2^2}\right)$
$C_{H\Box}$	$\frac{Y_3^2}{2Y_2^4}\left(1 + \frac{4Y_1^2}{Y_2^2}\right)$
dim-8	
C_{H^8}	$\frac{Y_3^2}{2Y_2^6}(4Z_1 - Z_3)^2$
$C_{H^6}^{(1)}$	0
$C_{H^6}^{(2)}$	$\frac{2Y_3^2}{Y_2^6}(-4Z_1 + Z_3)$
$C_{H^4}^{(1)}$	$\frac{4Y_3^2}{Y_2^6}$
$C_{H^4}^{(3)}$	$-\frac{2Y_3^2}{Y_2^6}$

After symmetry breaking of the SMEFT, the trilinear and quartic Higgs couplings are

$$\Delta\kappa_3^{\rm SMEFT} = \frac{Y_1^2 Y_3^2(2Z_1 - Z_3)}{2Z_1^2 Y_2^4} + \frac{Y_1^2 Y_3^4(Z_1 - Z_3) + Y_1^4 Y_3^2(9Z_1 Z_3 - 12Z_1^2 - 2Z_3^2)}{2Z_1^3 Y_2^6},$$
$$\Delta\kappa_4^{\rm SMEFT} = \frac{Y_1^2 Y_3^2(22Z_1 - 9Z_3)}{3Z_1^2 Y_2^4} + \frac{Y_1^2 Y_3^4(11Z_1 - 9Z_3) + Y_1^4 Y_3^2(123Z_1 Z_3 - 164Z_1^2 - 24Z_3^2)}{3Z_1^3 Y_2^6},$$

which is exactly same as in Y_2-HEFT.

→ **SMEFT is a limit of the pHEFT.**

Summary and outlook

- For the SM with scalar extensions, we construct a general non-linear representation to facilitate UV–HEFT matching.

- We establish the primary HEFT as a precision benchmark.
  - pHEFT keeps maximal UV's information.
  - Technically, pHEFT serves as a reference framework, enabling systematic and automatable mapping.

- Next: UV-HEFT matching for Type-II seesaw model (include a complex Higgs triplet), 1-loop matching.

Thanks for your attention!

Backups

Decoupling HEFT (dHEFT)

$$\mathcal{P} = \{m^2_{\phi\pm}, m^2_K, m_h, v_{\rm EW}, s_\gamma, \xi\},$$
$$\text{pHEFT:} \quad m^2_{\phi\pm} \sim m^2_K \sim \mathcal{O}(t^{-1}), \quad m_h \sim v_{\rm EW} \sim s_\gamma \sim \xi \sim \mathcal{O}(t^0),$$
$$\text{dHEFT:} \quad m^2_{\phi\pm} \sim m^2_K \sim \mathcal{O}(t^{-1}), \quad m_h \sim v_{\rm EW} \sim \mathcal{O}(t^0), \quad s_\gamma \sim \xi \sim \mathcal{O}(t^1).$$

$$\mathcal{L}^{\rm d}_{\rm HEFT}(t^0) = \frac{1}{2}D_\mu h D^\mu h - \frac{1}{4}(v_{\rm EW}+h)^2 \langle V_\mu V^\mu\rangle + \frac{m_h^2 (v_{\rm EW}+h)^2(v_{\rm EW}^2 - 2hv_{\rm EW} - h^2)}{8v_{\rm EW}^2} - \frac{v_{\rm EW}+h}{\sqrt{2}}\bar{Q}_L U \begin{pmatrix} y_u & 0 \\ 0 & y_d \end{pmatrix} Q_R + \text{h.c.}, \tag{25}$$

$$\mathcal{L}^{\rm d}_{\rm HEFT}(t^1) = \frac{1}{8}v_{\rm EW}^2\left[m_K^2(\xi+s_\gamma)^2 - \xi^2 m^2_{\phi\pm}\right] - \frac{h^3 m^2_{\phi\pm} s_\gamma^2(2\xi+s_\gamma)}{2\xi v_{\rm EW}} + \frac{h^4 m^2_{\phi\pm}s_\gamma^2\left[m^2_{\phi\pm}(4\xi+3s_\gamma)^2 - m_K^2(12\xi^2+14\xi s_\gamma+5s_\gamma^2)\right]}{8\xi^2 v_{\rm EW}^2 m_K^2} + \mathcal{O}(h^5), \tag{26}$$

$$\begin{aligned}\mathcal{L}^{\rm d}_{\rm HEFT}(t^2) = {}& \frac{1}{2}\langle V_\mu \sigma_3\rangle\langle V^\mu\sigma_3\rangle\left\{\xi^2 v_{\rm EW}^2 - 2h\xi v_{\rm EW}s_\gamma + \frac{h^2 s_\gamma\left[m^2_{\phi\pm}(4\xi+3s_\gamma)-\xi m_K^2\right]}{m_K^2} + \mathcal{O}(h^3)\right\} \\ &+ \frac{1}{4}\langle V_\mu V^\mu\rangle\left\{h v_{\rm EW}(4\xi^2+8\xi s_\gamma+s_\gamma^2) + \frac{h^2 s_\gamma\left[m_K^2(4\xi^2+2\xi s_\gamma+s_\gamma^2) - m^2_{\phi\pm}(4\xi+s_\gamma)(4\xi+3s_\gamma)\right]}{\xi m_K^2} + \mathcal{O}(h^3)\right\} \\ &- \frac{1}{8}v_{\rm EW}^2 m_h^2(4\xi^2+2\xi s_\gamma+s_\gamma^2) + \frac{h^3 m_h^2(-4\xi^3+3\xi s_\gamma^2+2s_\gamma^3)}{4\xi v_{\rm EW}} \\ &+ \frac{h^4 m_h^2}{24\xi^2 v_{\rm EW}^2 m_K^4}\left[m_K^4(-12\xi^4+25\xi^2 s_\gamma^2+38\xi s_\gamma^3+16 s_\gamma^4) - 20 m_K^2 m^2_{\phi\pm}s_\gamma^2(\xi+s_\gamma)(4\xi+3s_\gamma) + 4m^4_{\phi\pm}s_\gamma^2(4\xi+3s_\gamma)^2\right] + \mathcal{O}(h^5) \\ &+ \bar{Q}_L U\begin{pmatrix} y_u & 0 \\ 0 & y_d\end{pmatrix} Q_R \times \sqrt{2}\left\{\xi^2 v_{\rm EW} + \frac{h s_\gamma^2}{4} + \frac{h^2 s_\gamma^2\left[m_K^2(\xi+s_\gamma) - m^2_{\phi\pm}(4\xi+3s_\gamma)\right]}{4\xi v_{\rm EW} m_K^2} + \mathcal{O}(h^3)\right\} + \text{h.c.},\end{aligned} \tag{27}$$

$$c_\gamma = 1 - \frac{s_\gamma^2}{2} - \frac{s_\gamma^4}{8} + \cdots,$$
$$\frac{1}{1+4\xi^2} = 1 - 4\xi^2 + 16\xi^4 + \cdots,$$

subset

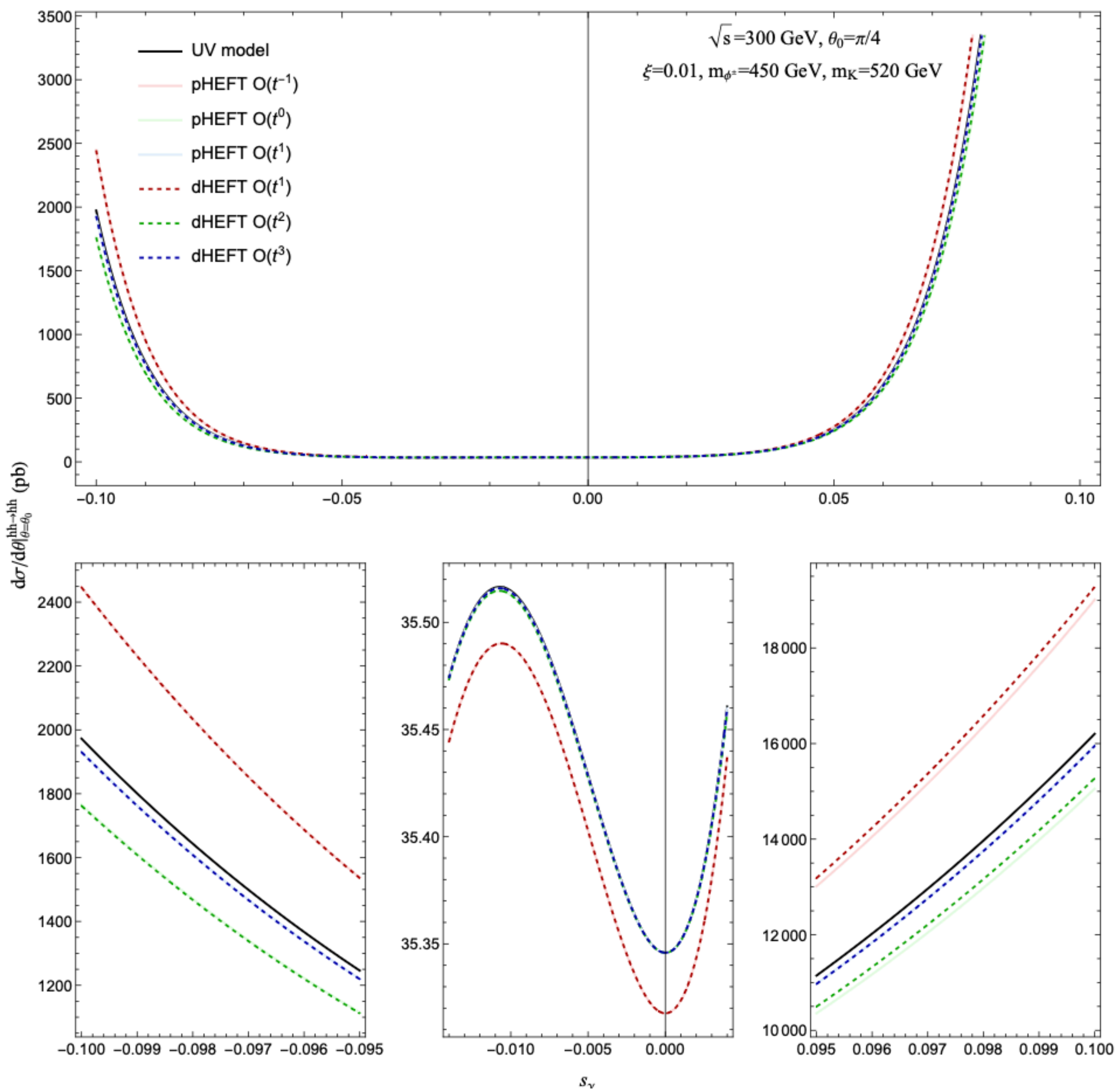
$$\mathcal{L}^{\rm p}_{\rm HEFT}(t^{-1}) \to \mathcal{L}^{\rm d}_{\rm HEFT}(t^1) + \mathcal{S}_1\mathcal{L}^{\rm d}_{\rm HEFT}(t^3) + \cdots,$$
$$\Delta\mathcal{L}^{\rm p}_{\rm HEFT}(t^0) \to \mathcal{L}^{\rm d}_{\rm HEFT}(t^2) + \mathcal{S}_1\mathcal{L}^{\rm d}_{\rm HEFT}(t^4) + \cdots,$$
$$\mathcal{L}^{\rm p}_{\rm HEFT}(t^1) \to \mathcal{S}_2\mathcal{L}^{\rm d}_{\rm HEFT}(t^3) + \mathcal{S}_2\mathcal{L}^{\rm d}_{\rm HEFT}(t^5) + \cdots,$$

pHEFT vs. dHEFT

$$\mathcal{L}^{\mathrm{p}}_{\mathrm{HEFT}}(t^{-1}) \supset \mathcal{L}^{\mathrm{d}}_{\mathrm{HEFT}}(t^{1}),$$

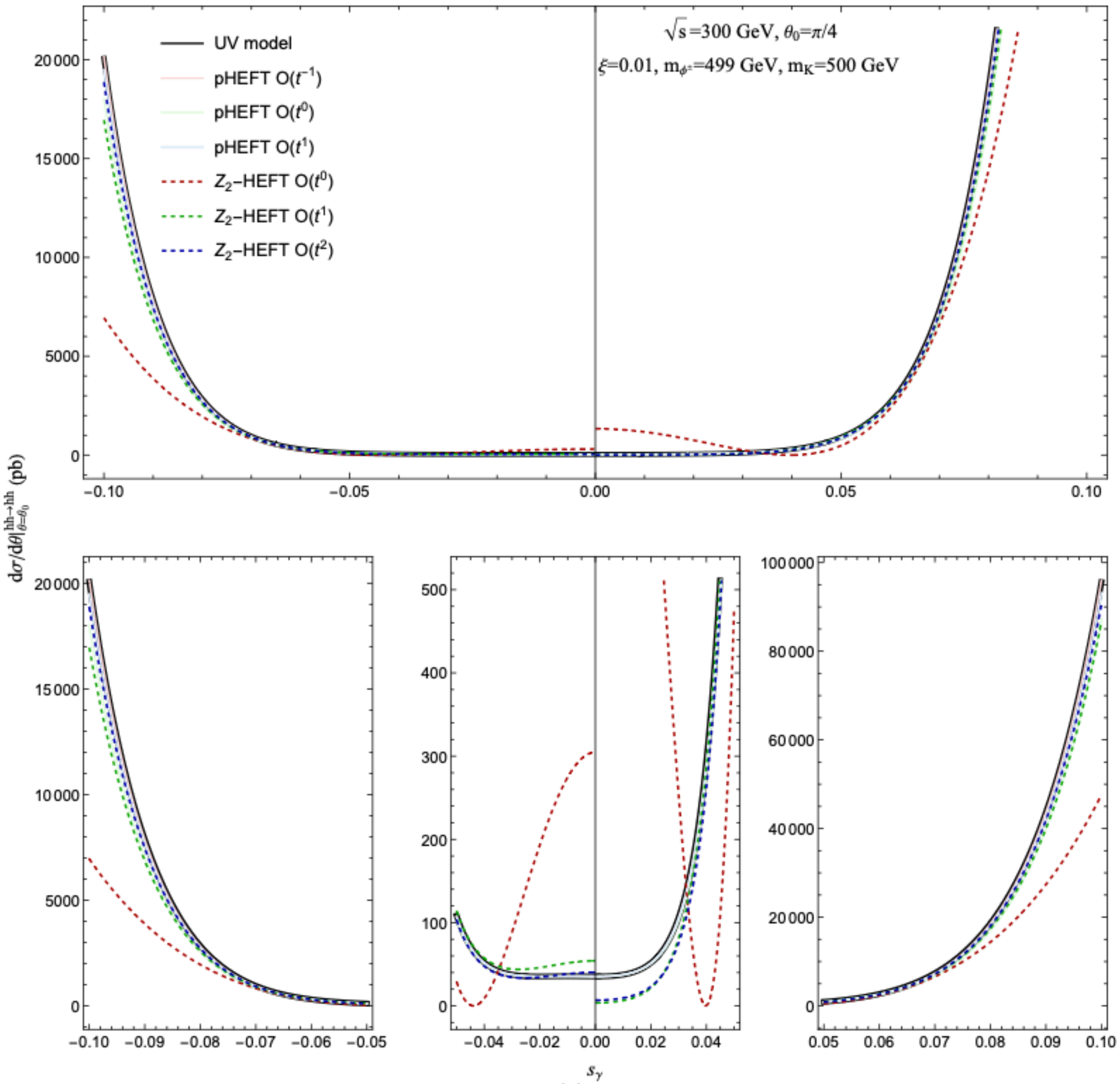
$$\mathcal{L}^{\mathrm{p}}_{\mathrm{HEFT}}(t^{-1}+t^{0}) \supset \mathcal{L}^{\mathrm{d}}_{\mathrm{HEFT}}(t^{1}+t^{2}),$$

$$\mathcal{L}^{\mathrm{p}}_{\mathrm{HEFT}}(t^{-1}+t^{0}+t^{1}) \supset \mathcal{L}^{\mathrm{d}}_{\mathrm{HEFT}}(t^{1}+t^{2}+t^{3}),\ldots$$



pHEFT is more accurate.

pHEFT vs. Z_2-HEFT



pHEFT vs. ξ-HEFT

$$\mathcal{P}_\xi = \{Z_1, Z_2, Z_3, Y_3, v_H, \xi\}$$

$$\xi\text{-HEFT:} \quad Z_1 \sim Z_2 \sim Z_3 \sim Y_3 \sim v_H \sim \mathcal{O}(t^0), \quad \xi \sim \mathcal{O}(t^1).$$

ξ <0.015 by electroweak precision measurements

$$\mathcal{L}^\xi_{\rm HEFT}(t^0) = \frac{1}{2} D_\mu h D^\mu h - \frac{1}{4} Z_1 \left(-v_H^4 + 4h^2 v_H^2 + 4h^3 v_H + h^4\right) - \frac{1}{4}(v_H + h)^2 \langle V_\mu V^\mu \rangle$$

$$\mathcal{L}^\xi_{\rm HEFT}(t^1) = \frac{\xi Y_3}{4 v_H} \left(-v_H^4 + 4h^2 v_H^2 + 4h^3 v_H + h^4\right)$$

$$\begin{aligned}\mathcal{L}^\xi_{\rm HEFT}(t^2) = \frac{\xi^2}{4v_H^2} \Big\{ & v_H^6 Z_3 + 8h^2 v_H^4 (2Z_1 - Z_3) + 8h^3 v_H^3 (5Z_1 - 2Z_3) \\ & + \frac{14}{3} h^4 v_H^2 (8Z_1 - 3Z_3) - 4\left(v_H^4 + 3h v_H^3 + 4h^2 v_H^2\right) \langle V_\mu V^\mu \rangle \\ & + 2\left(v_H^4 + 4h v_H^3 + 6h^2 v_H^2\right) \langle V_\mu \sigma_3 \rangle \langle V^\mu \sigma_3 \rangle \Big\},\end{aligned}$$

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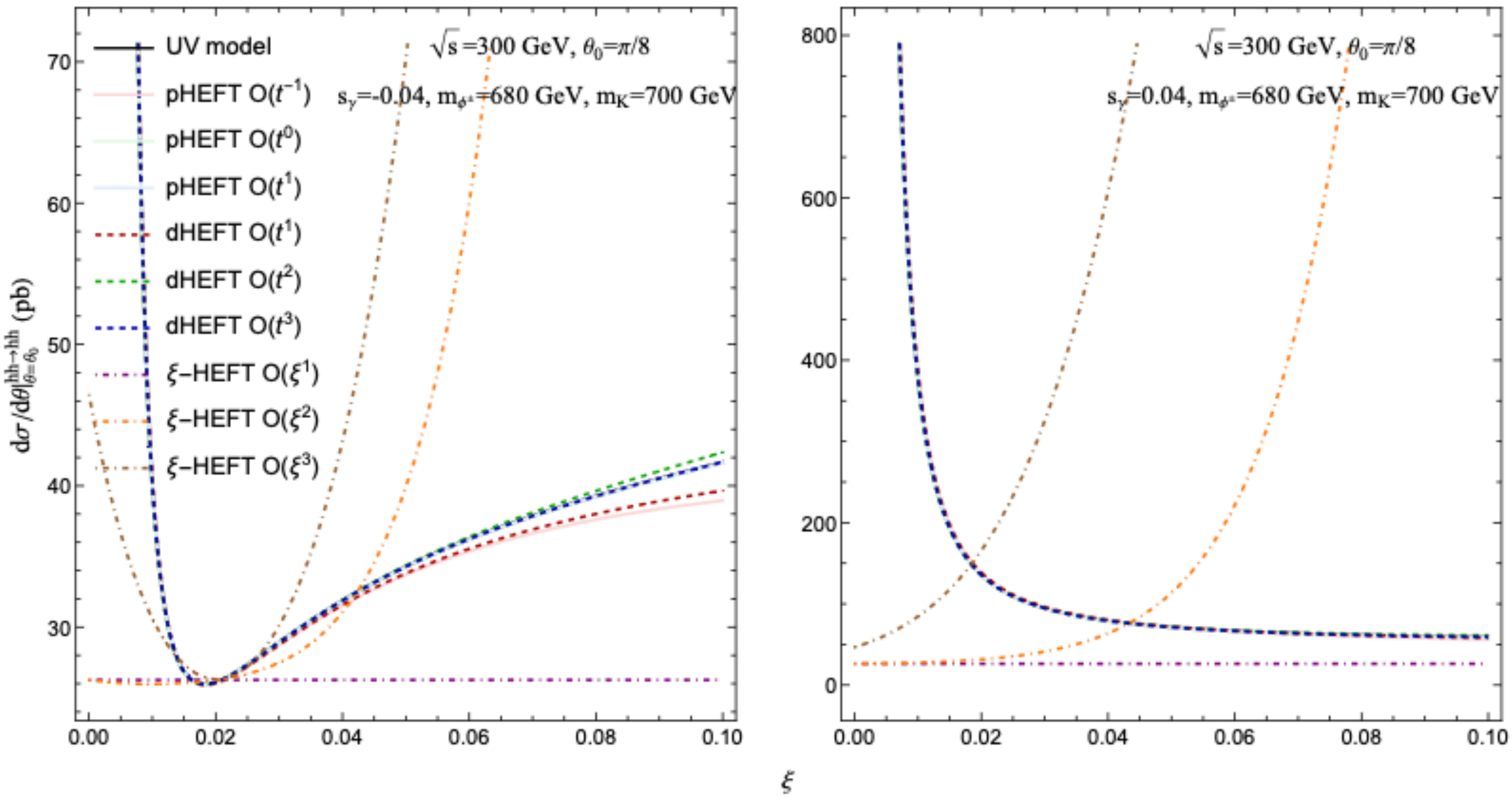
Rederive same results from pHEFT by

$$\begin{aligned} m^2_{\phi^\pm} &\to \frac{Y_3 v_H}{2\xi} + 2\xi Y_3 v_H, \\ m^2_K &\to \frac{Y_3 v_H}{2\xi} + 2\xi Y_3 v_H + 2\xi^2 v_H^2 (4Z_1 + Z_2 - 2Z_3) + O(\xi^3), \\ m_h^2 &\to 2Z_1 v_H^2 - 2\xi Y_3 v_H - 4\xi^2 v_H^2 (2Z_1 - Z_3) + O(\xi^3), \\ s_\gamma &\to -2\xi - \frac{2(4Z_1 - Z_3)\xi^2 v_H}{Y_3} + O(\xi^3), \\ v_{\rm EW} &\to v_H + 2\xi^2 v_H + O(\xi^3). \end{aligned}$$

a decoupling scheme.

pHEFT vs. ξ-HEFT



- ξ-HEFT is suitable in a limited parameter region.
- pHEFT ⊃ dHEFT ⊃ ξ-HEFT

Y_2-HEFT

$$\mathcal{P}_{Y_2} = \{Z_1, Z_2, Z_3, Y_1, Y_2, Y_3\}.$$

$$Y_2\text{-HEFT:} \quad Z_1 \sim Z_2 \sim Z_3 \sim Y_1 \sim Y_3 \sim \mathcal{O}(t^0), \quad Y_2^2 \sim \mathcal{O}(t^{-1}).$$

Mimic SMEFT's power counting

Y_2-HEFT is obtained from pHEFT by

$$m_{\phi^\pm}^2 \to Y_2^2 - \frac{Y_1^2 Z_3}{2Z_1} - \frac{Y_3^2 Y_1^2 \left(4Z_1 + Z_3\right)}{4Y_2^2 Z_1^2} + \mathcal{O}(Y_2^{-4}),$$

$$m_K^2 \to Y_2^2 - \frac{Y_1^2 Z_3}{2Z_1} - \frac{Y_3^2 Y_1^2 (4Z_1 + Z_3)}{4Y_2^2 Z_1^2} + \mathcal{O}(Y_2^{-4}),$$

$$m_h^2 \to -2Y_1^2 + \frac{Y_1^4 Y_3^2 \left(3Z_3 - 8Z_1\right)}{4Y_2^4 Z_1^2} + \mathcal{O}(Y_2^{-4}),$$

$$s_\gamma \to -\frac{\sqrt{-Y_1^2} Y_3}{Y_2^2 \sqrt{Z_1}} - \frac{\sqrt{-Y_1^2} Y_3 \left(4Y_1^2 \left(Z_3 - 2Z_1\right) + Y_3^2\right)}{4Y_2^4 Z_1^{3/2}} + \mathcal{O}(Y_2^{-6}),$$

$$\xi \to \frac{\sqrt{-Y_1^2}\, Y_3}{2\, Y_2^2 \sqrt{Z_1}} + \frac{\sqrt{-Y_1^2}\, Y_3 \left(2Y_1^2 Z_3 + Y_3^2\right)}{8\, Y_2^4\, Z_1^{3/2}} + \mathcal{O}(Y_2^{-6}),$$

$$v_{\mathrm{EW}} \to \frac{\sqrt{-Y_1^2}}{\sqrt{Z_1}} + \frac{\sqrt{-Y_1^2} Y_3^2}{4Y_2^2 Z_1^{3/2}} + \frac{\sqrt{-Y_1^2} \left(2Y_1^2 Y_3^2 \left(3Z_3 - 8Z_1\right) + 3Y_3^4\right)}{32 Y_2^4 Z_1^{5/2}} + \mathcal{O}(Y_2^{-6}),$$

Thanks for your attention!