Multi-reference in-medium similarity renormalization group method and its applications

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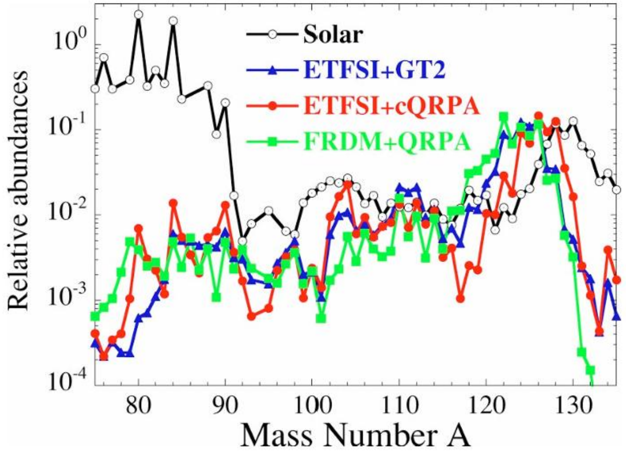
重庆 Jan. 11, 2026

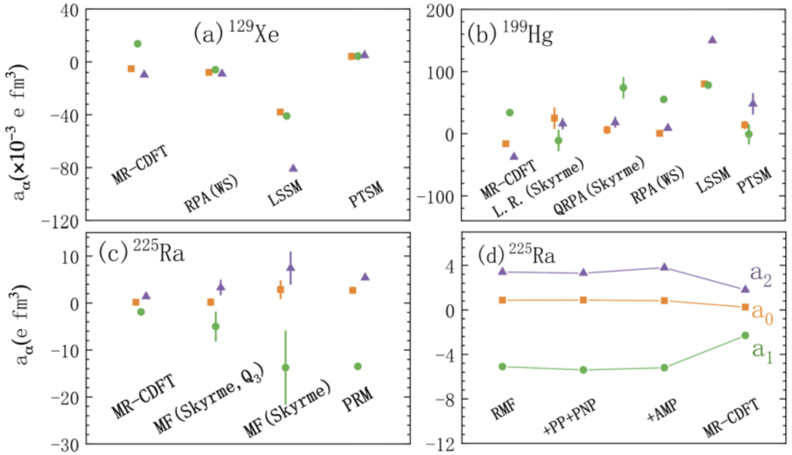
Theoretical description of NMEs: Uncertainties and Challenges

Impact of beta decay on abundances of r-process nuclides



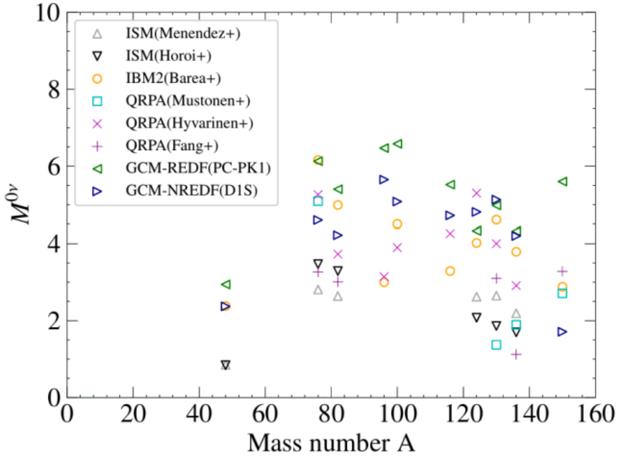
K. Langanke et al, Rev. Mod. Phys 75(2003)

Nuclear Schiff moments

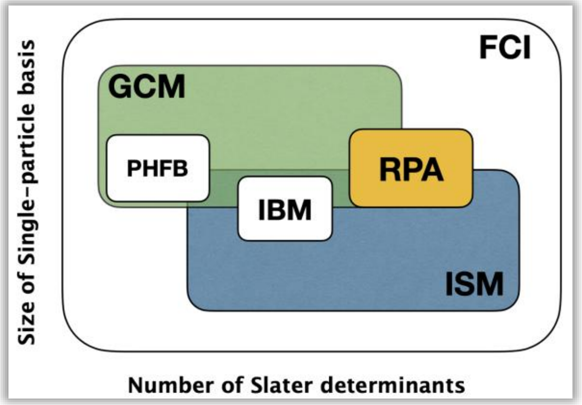


E. F. Zhou et al, arXiv2507.01369

Neutrinoless double-beta decay



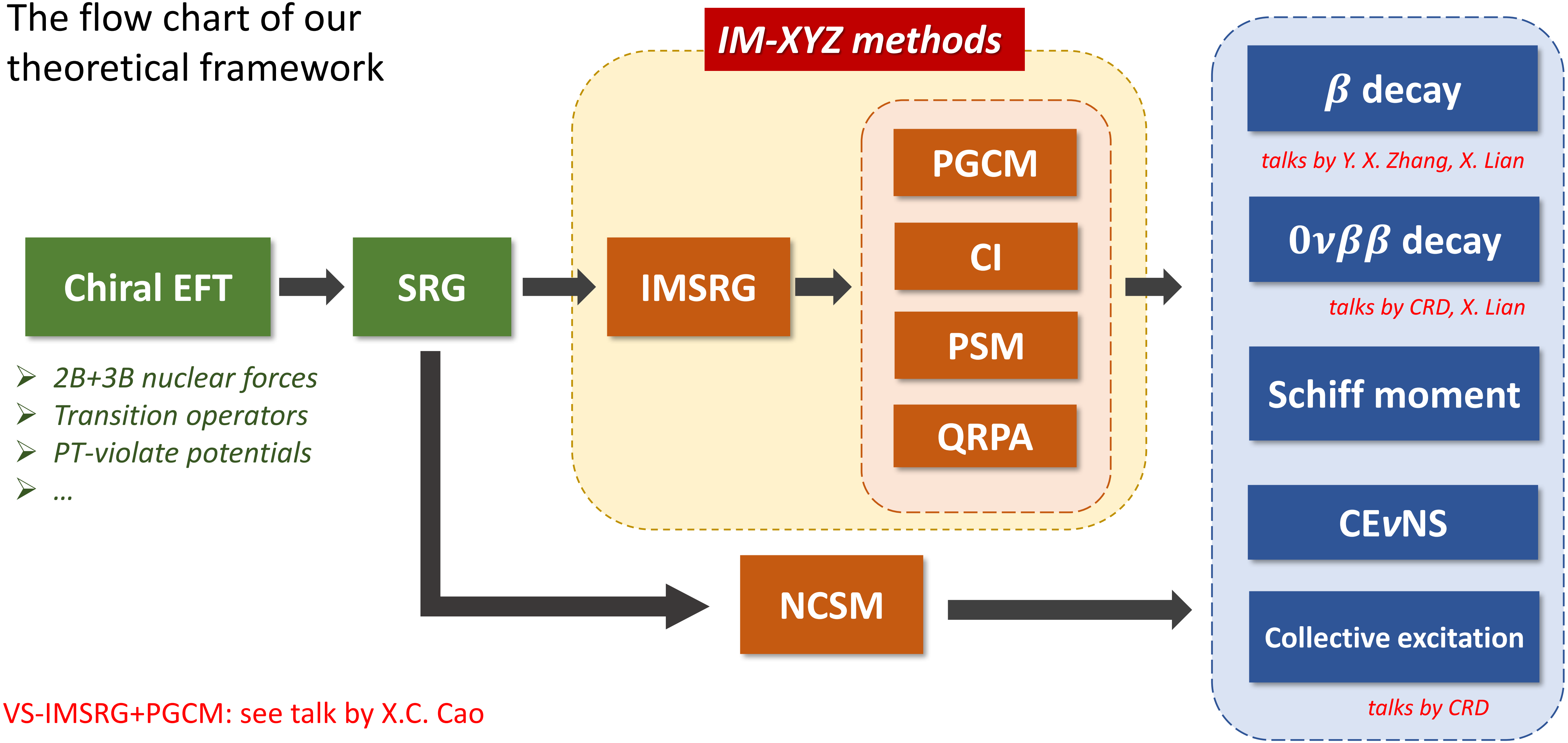
J. M. Yao, J. Meng, Y. F. Niu, P. Ring, PPNP 126, 103965 (2022)



Different models defined differently from the beginning

Modeling NMEs starting from operators from chiral EFT

The flow chart of our theoretical framework



VS-IMSRG+PGCM: see talk by X.C. Cao

Nuclear force

PGCM

Reference state

IMSRG

Operator

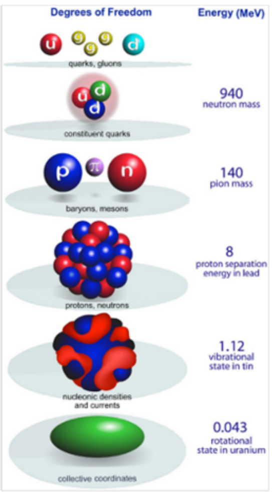
PGCM

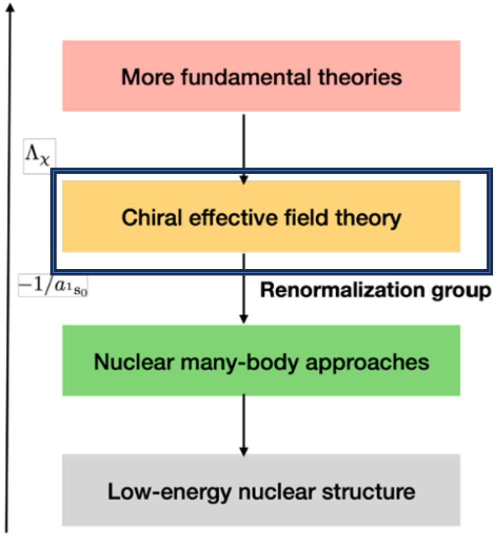
IMSRG

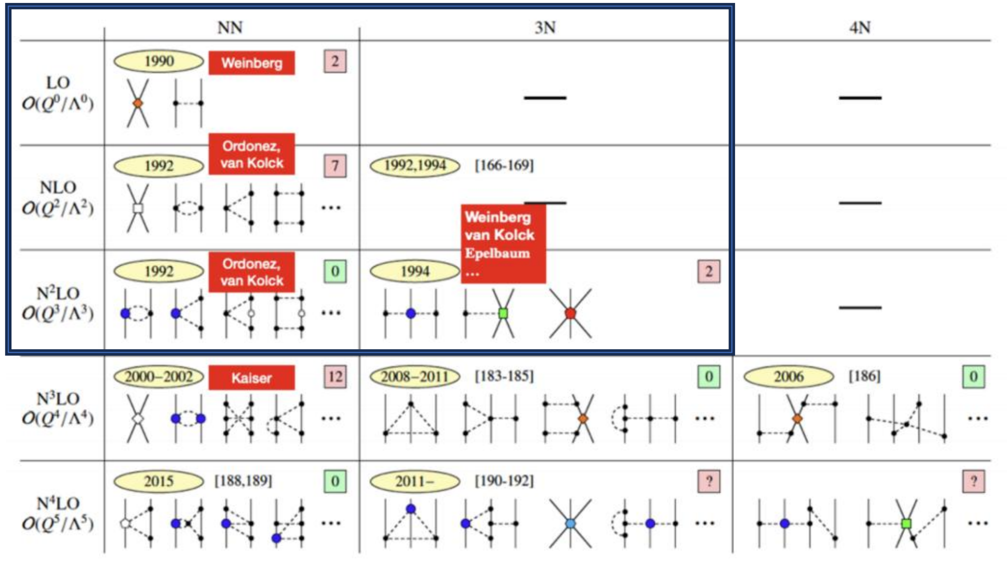
Wave function

Evolved operator

NME/ Observable







K. Hebeler, Phys. Rep. 890, 1 (2020)

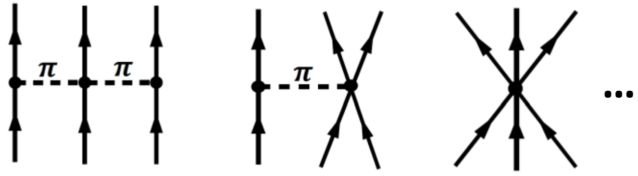
$$\hat{H}_0 = \left(1-\frac{1}{A}\right)T^{[1]} + \frac{1}{A}T^{[2]} + \boxed{\sum_{i<j} V_{ij}^{[2]} + \sum_{i<j<k} W_{ijk}^{[3]}}$$

$$V_{1\pi} + V_{2\pi} + \boldsymbol{V_{ct}} + \cdots$$

$$W_{2\pi} + W_{1\pi} + W_{ct} + \cdots$$

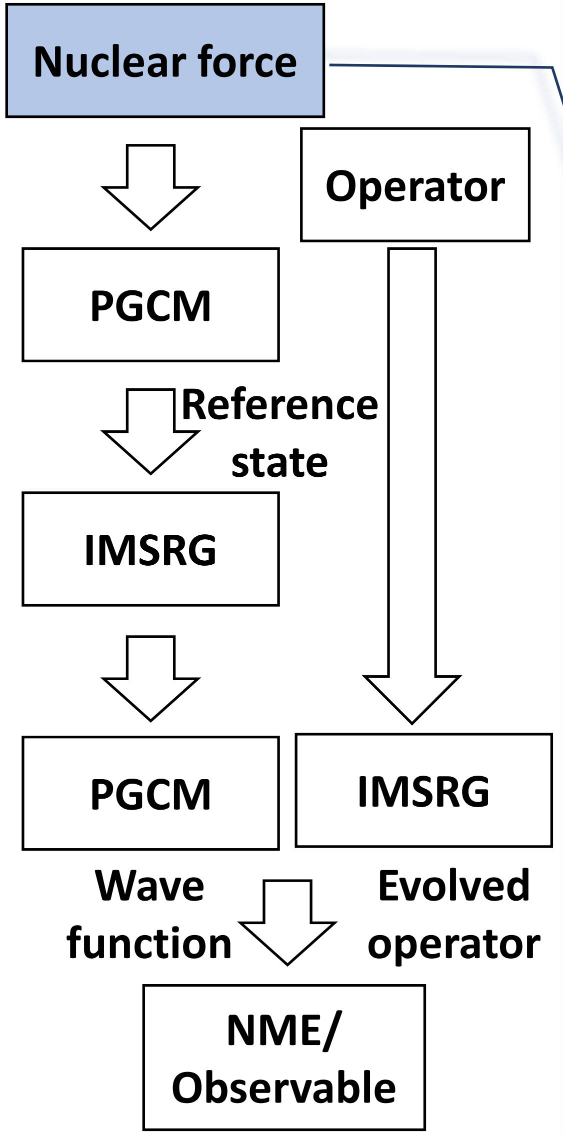
* Low energy constants $\tilde{C}_{1S0}, \tilde{C}_{3S1}, C_{1S0}, C_{3P0}, C_{1P1}, C_{3P1}, C_{3S1}, C_{3S1-3D1}, C_{3P2} \cdots$

Determined by fitting scattering phase shift.

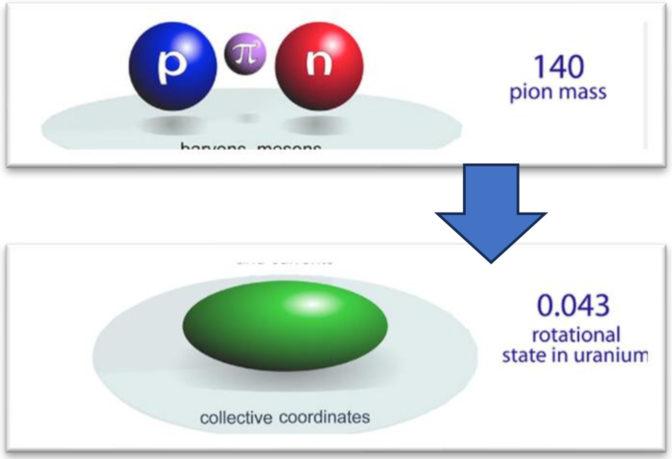


c_1, c_3, c_4 c_D c_E

Similarity Renormalization Group(SRG)



- Modeling nuclear structure and decays based on operators derived from chiral EFT across the energy scales from 10^2 MeV to 10^{-1} MeV

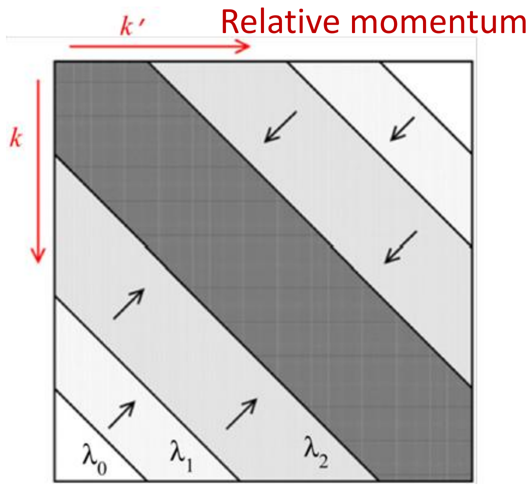


- **Similarity renormalization group (SRG)**

In the center-of-mass (CM) frame, apply unitary transformations U_s to decouple the coupling between high and low-momentum states

$$H_s = U_s H_0 U_s^\dagger = T_{rel} + V_s.$$

S. K. Bogner et al, PPNP 65(2010)94-147



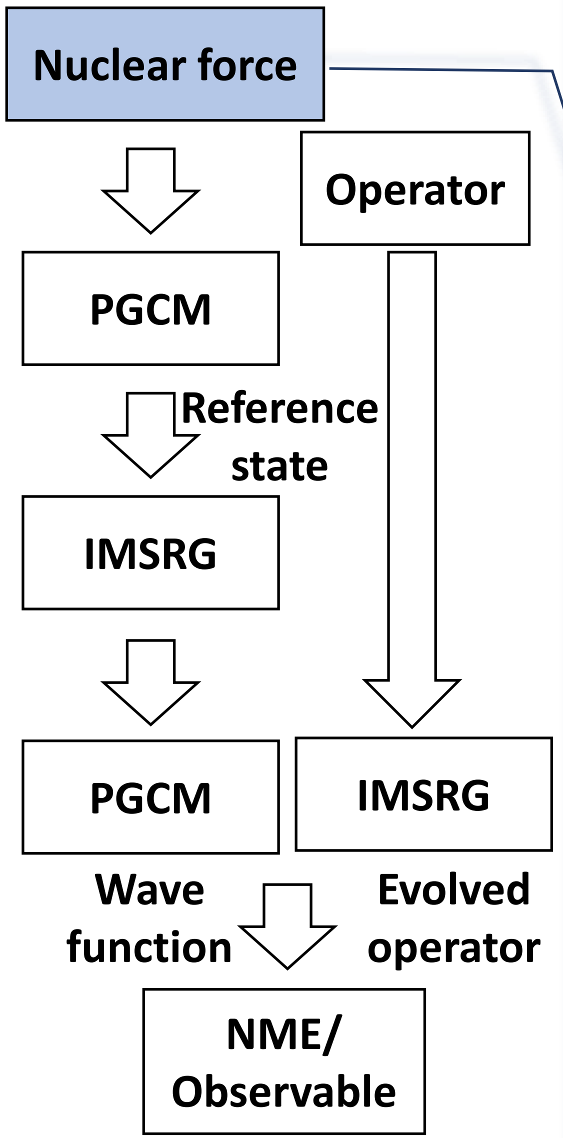
The flow equation $\dfrac{dH_s}{ds} = [\eta_s, H_s], \qquad \eta_s = [T_{rel}, H_s].$

Evolution of the potential

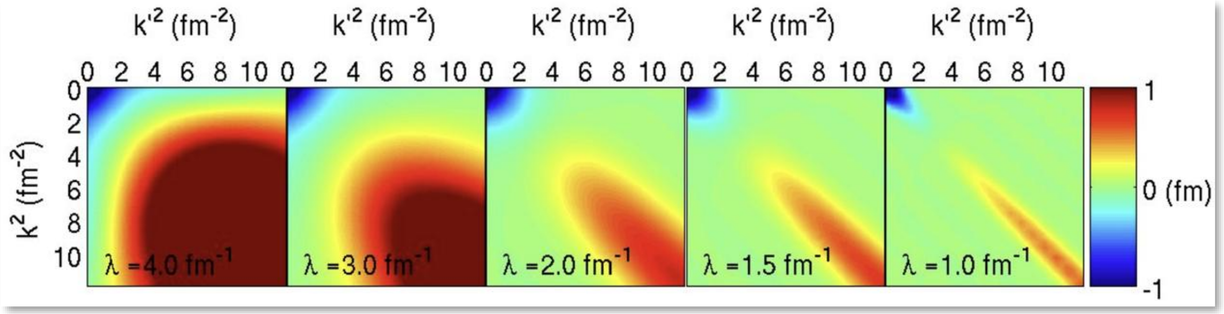
Flow parameter s is usually replaced with $\lambda = s^{-1/4}$ in units of fm^{-1}.

$$\frac{dV_s(k,k')}{ds} = -\left(k^2 - k'^2\right)V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2 dq\left(k^2 + k'^2 - 2q^2\right)V_s(k,q)V_s(q,k').$$

Similarity Renormalization Group(SRG)

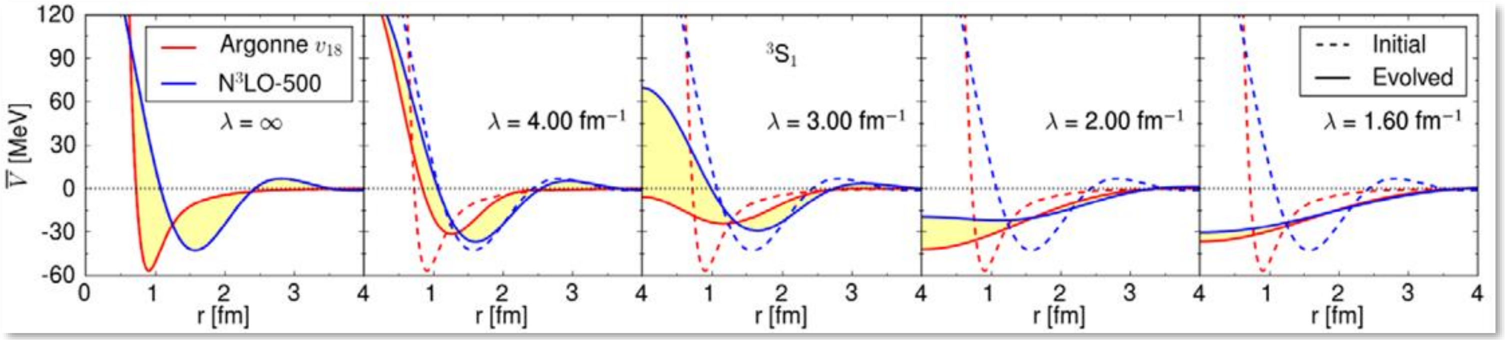


- N3LO(600MeV) momentum-space potential (1S0 channel)



S. K. Bogner et al, PPNP 65(2010)94-147

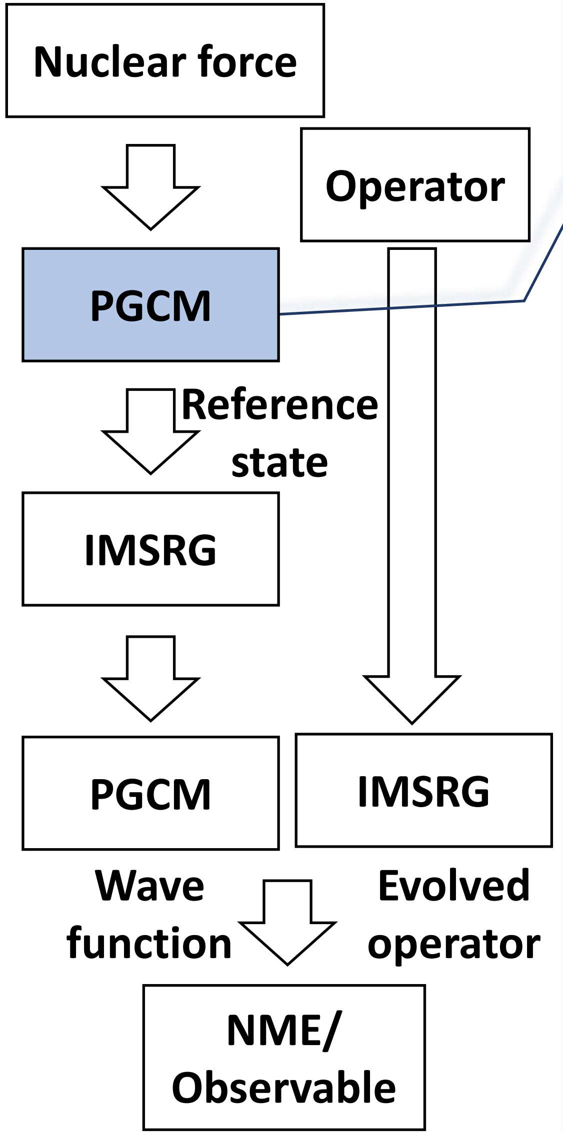
- AV18 and N3LO(500MeV) potential (3S1 channel)



K.A. Wendt et al, Phys.Rev.C 86(2012)014003

The hard core "disappears" in the softened interactions.

Projected Generator Coordinate Method



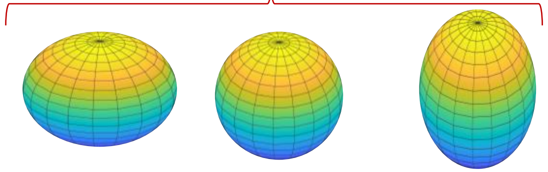
- **On HO basis, constrained Hamiltonian**

Ring and Schuck, *The nuclear many-body problem*, 1980

$$\langle\Phi|\hat{H}|\Phi\rangle = \left\langle\Phi\left|\hat{H}_0 - \sum_{\tau=n,p}\lambda_\tau \hat{N}\right|\Phi\right\rangle - \underbrace{\frac{1}{2}\lambda_Q\left(\langle\Phi|\hat{Q}_{20}|\Phi\rangle - q_{20}\right)^2}$$

Solve HFB equation and obtain HFB states with different deformations.

$|\Phi(\boldsymbol{q})\rangle =$

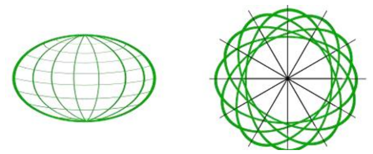


- **Wave function of a nuclear state**

$$|\Psi_\sigma^{JMNZ}\rangle = \sum_{\boldsymbol{q}} f_\sigma^J(\boldsymbol{q})\, |JMNZ,\boldsymbol{q}\rangle$$

Projection

$$|JMNZ,\boldsymbol{q}\rangle = \hat{P}^J_{M0}\,\hat{P}^N\,\hat{P}^Z|\Phi(\boldsymbol{q})\rangle$$



Solve Hill-Wheeler-Griffin (HWG) equation

$$\sum_{\boldsymbol{q}}\left[H^J_{00}(\boldsymbol{q},\boldsymbol{q}') - E^J_\sigma N^J_{00}(\boldsymbol{q},\boldsymbol{q}')\right] f^J_\sigma(\boldsymbol{q}') = 0$$

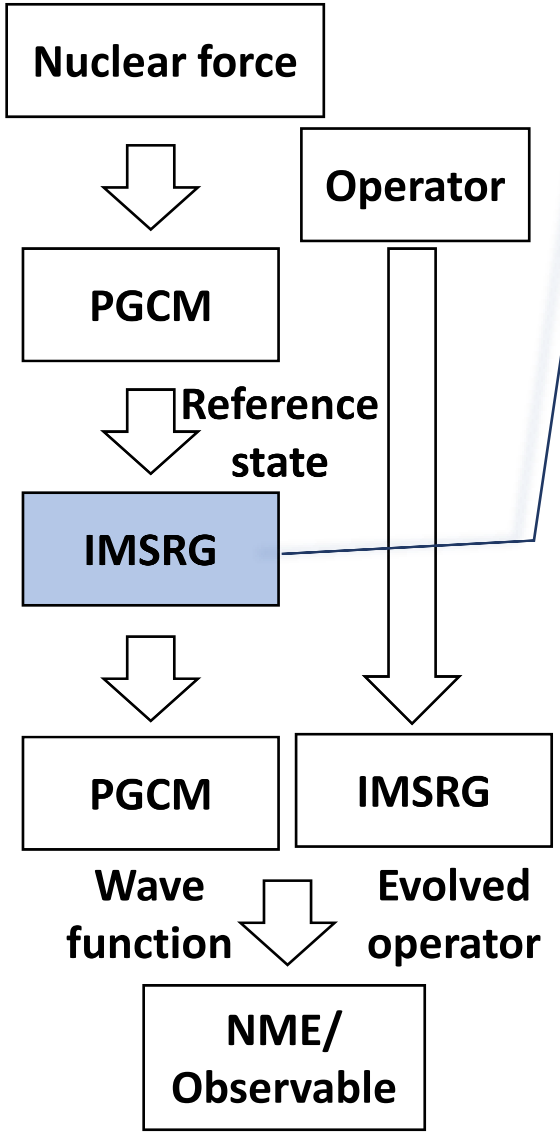
$$H^J_{00}(\boldsymbol{q},\boldsymbol{q}') = \langle\Phi(\boldsymbol{q})|\hat{H}\hat{P}^J_{00}\,\hat{P}^N\,\hat{P}^Z|\Phi(\boldsymbol{q}')\rangle$$

$$N^J_{00}(\boldsymbol{q},\boldsymbol{q}') = \langle\Phi(\boldsymbol{q})|\hat{P}^J_{00}\,\hat{P}^N\,\hat{P}^Z|\Phi(\boldsymbol{q}')\rangle$$

- **The 1B density** $\left[\rho_{ij}\right]^\lambda_\mu = \left\langle\Psi_\sigma^{J'M'NZ}\right|\left[c_i^\dagger c_j\right]^\lambda_\mu\left|\Psi_\sigma^{JMNZ}\right\rangle$

  The 2B density $\left[\rho_{ijkl}\right]^\lambda_\mu = \left\langle\Psi_\sigma^{J'M'NZ}\right|\left[c_i^\dagger c_j^\dagger c_l c_k\right]^\lambda_\mu\left|\Psi_\sigma^{JMNZ}\right\rangle$

In Medium SRG



- **Unitary transformation** $H_s = U_s H U_s^\dagger$

  Flow equation $\dfrac{dH_s}{ds} = [\eta_s, H_s]$

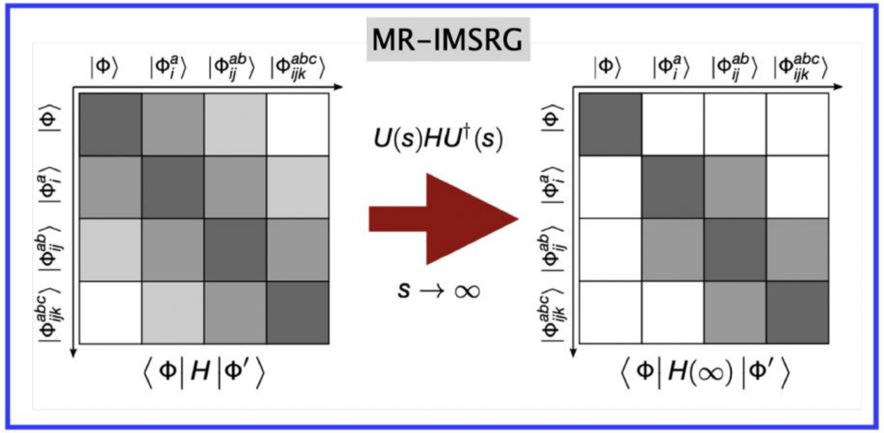
- **Generator η_s :** chosen either to decouple a given reference state from its excitations or to decouple the valence space from the excluded spaces.

If choosing a HF state as the reference state, one has

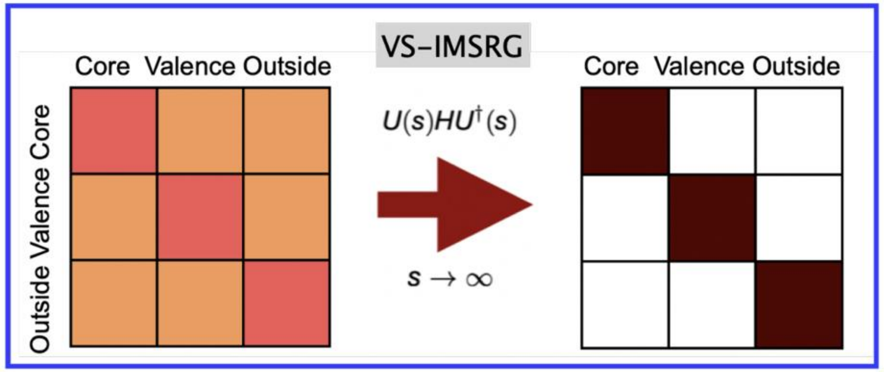
$$\hat{H}_0 \left|\Psi^{(0)}_{\text{Exact}}\right\rangle = E_0 \left|\Psi^{(0)}_{\text{Exact}}\right\rangle$$

$$\Downarrow$$

$$\hat{H}(\infty)\left|\Phi_{\text{HF}}\right\rangle = E_0 \left|\Phi_{\text{HF}}\right\rangle$$



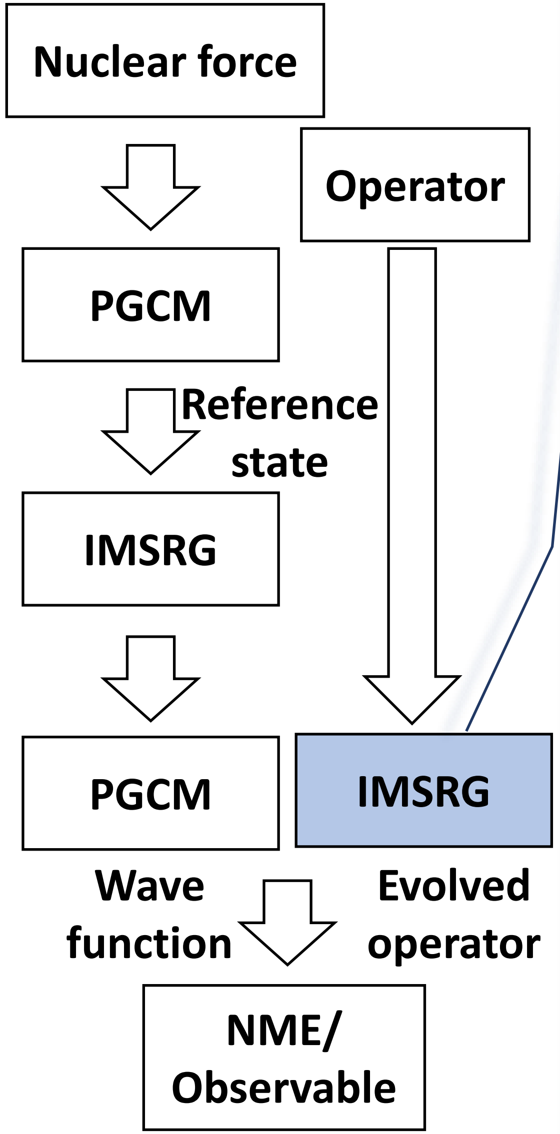
H. Hergert et al., Phys. Rep. 621, 165 (2016)



S. R. Stroberg et al., Annu. Rev. Nucl. Part. Sci. 69, 307 (2019)

About VS-IMSRG: also see the talk by X.C. Cao

Evolution of Operators



During IMSRG process

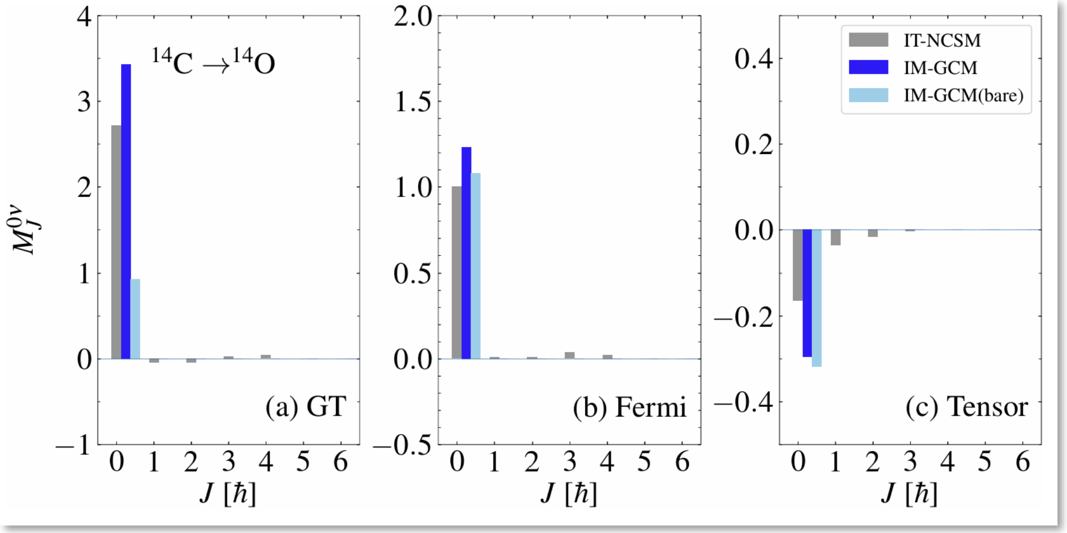
$$\langle \Phi_0 | \hat{H}_0 | \Phi_0 \rangle = \left\langle \Phi_s \middle| U_s \hat{H}_0 U_s^\dagger \middle| \Phi_s \right\rangle = \langle \Phi_s | \hat{H}_s | \Phi_s \rangle \Rightarrow |\Phi_s\rangle = U_s |\Phi_0\rangle$$

For operators

$$\langle \Phi_0 | \hat{O}_0 | \Phi_0 \rangle = \left\langle \Phi_s \middle| U_s \hat{O}_0 U_s^\dagger \middle| \Phi_s \right\rangle = \langle \Phi_s | \hat{O}_s | \Phi_s \rangle \Rightarrow \hat{O}_s = U_s \hat{O}_0 U_s^\dagger$$

In Magnus expansion

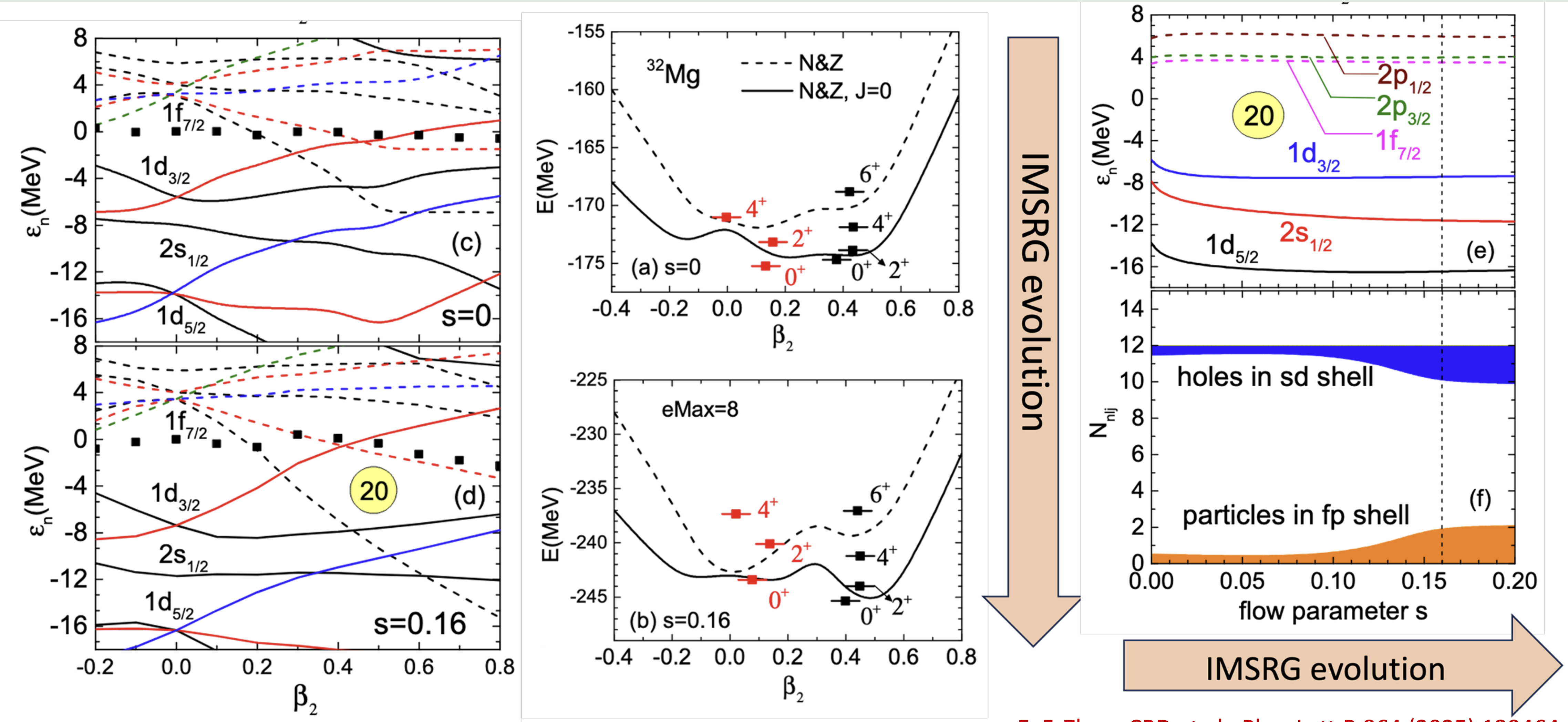
$$\boxed{e^{\Omega(s)} \hat{O} e^{-\Omega(s)} = \hat{O} + [\Omega(s), \hat{O}] + \frac{1}{2!}[\Omega(s), [\Omega(s), \hat{O}]] + \ldots}$$



$0\nu\beta\beta$ decay NME

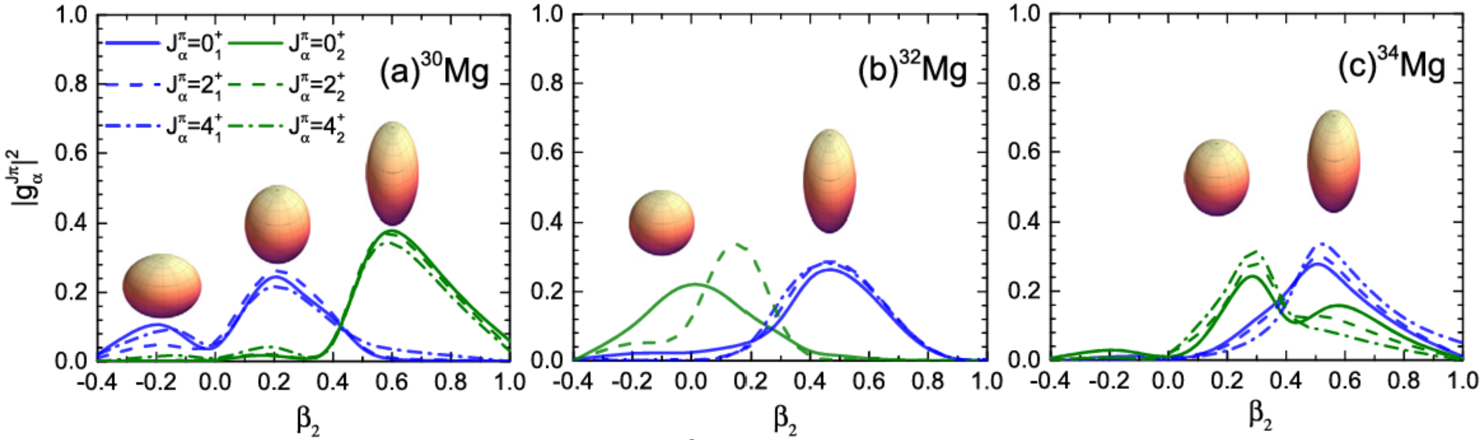
J. M. Yao et al., Phys.Rev.C 103 (2021) 1, 014315

Application: Low-lying States

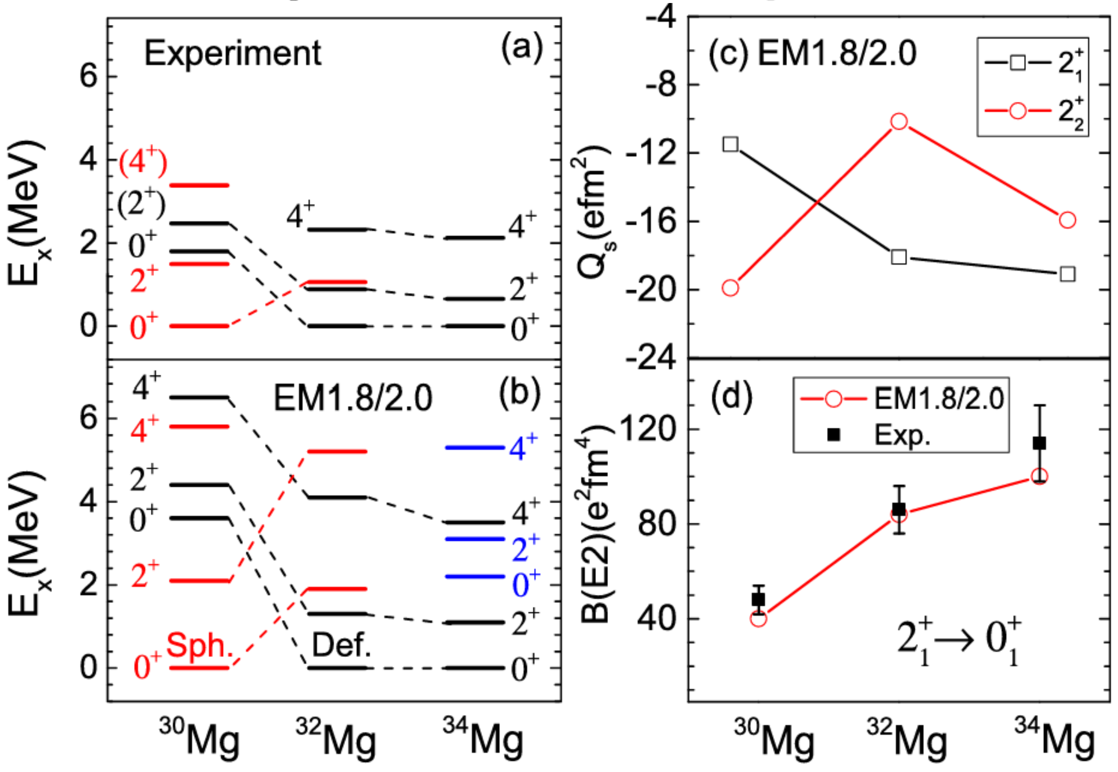


E. F. Zhou, CRD et al., Phys.Lett.B 864 (2025) 139464

- These pictures reveal how deformed ground states emerge naturally with the IMSRG evolution of the initial chiral two- plus three-nucleon interaction EM1.8/2.0.



E. F. Zhou, CRD et al.,
Phys.Lett.B 864 (2025) 139464



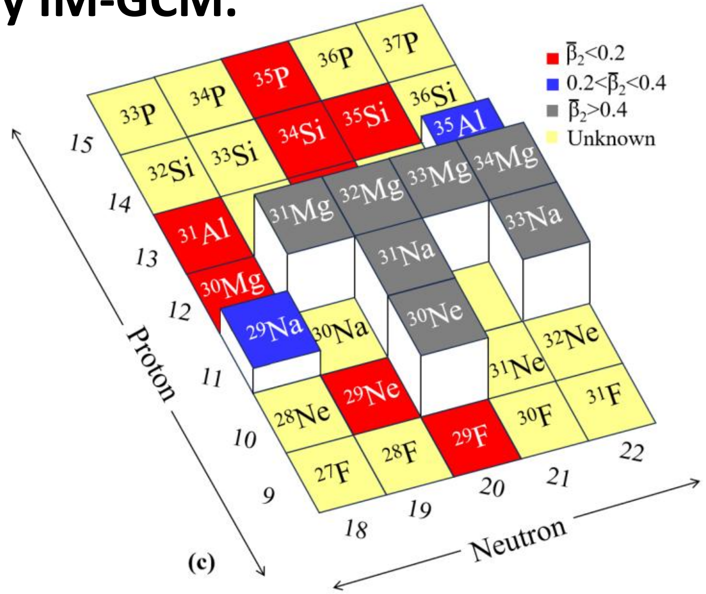
- ☐ **Onset of strong collectivity from 30Mg to 34Mg: increase in B(E2) and decrease in $\mathrm{E_x}(2_1^+)$.**
- ☐ **The weakly prolate and oblate deformed ground state of 30Mg, coexisting with strongly prolate deformed second 0^+state.**
- ☐ **The strongly prolate deformed ground states of 32Mg, 34Mg, coexisting with weakly deformed second 0^+ state.**

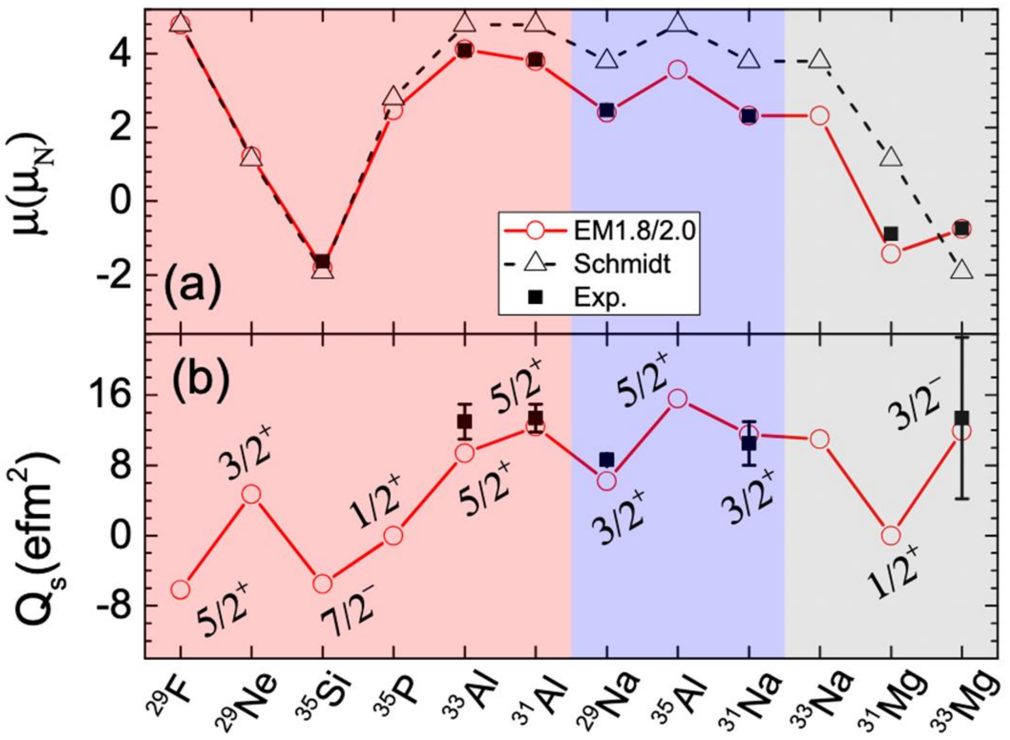
Boundary of Island of Inversion

* **Island of inversion** (IOI): a part of an “archipelago of islands” associated with **the breaking of magic numbers**

The intrusion of fp orbitals into the sd shell in deformed Nilsson levels

- **For nuclei around N=20, the boundary of the IOI is predicted by IM-GCM.**





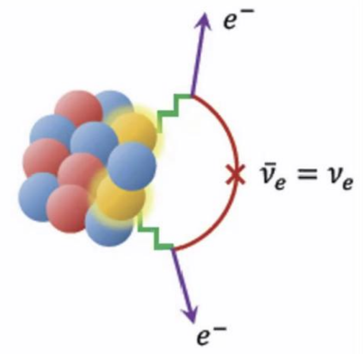
E. F. Zhou, CRD et al., in preparation

- **Nuclei inside the island of inversion: 30Ne, 29,31,33Na, 31,32,33,34Mg, and 35Al.**
- **Nuclei outside: 29F, 29Ne, 30Mg, 31,33Al, 34,35Si, and 35P.**

Application to $0\nu\beta\beta$ Decay

- **$0\nu\beta\beta$ decay:**

$$X_Z^A \rightarrow Y_{Z+2}^A + e^- + e^-$$



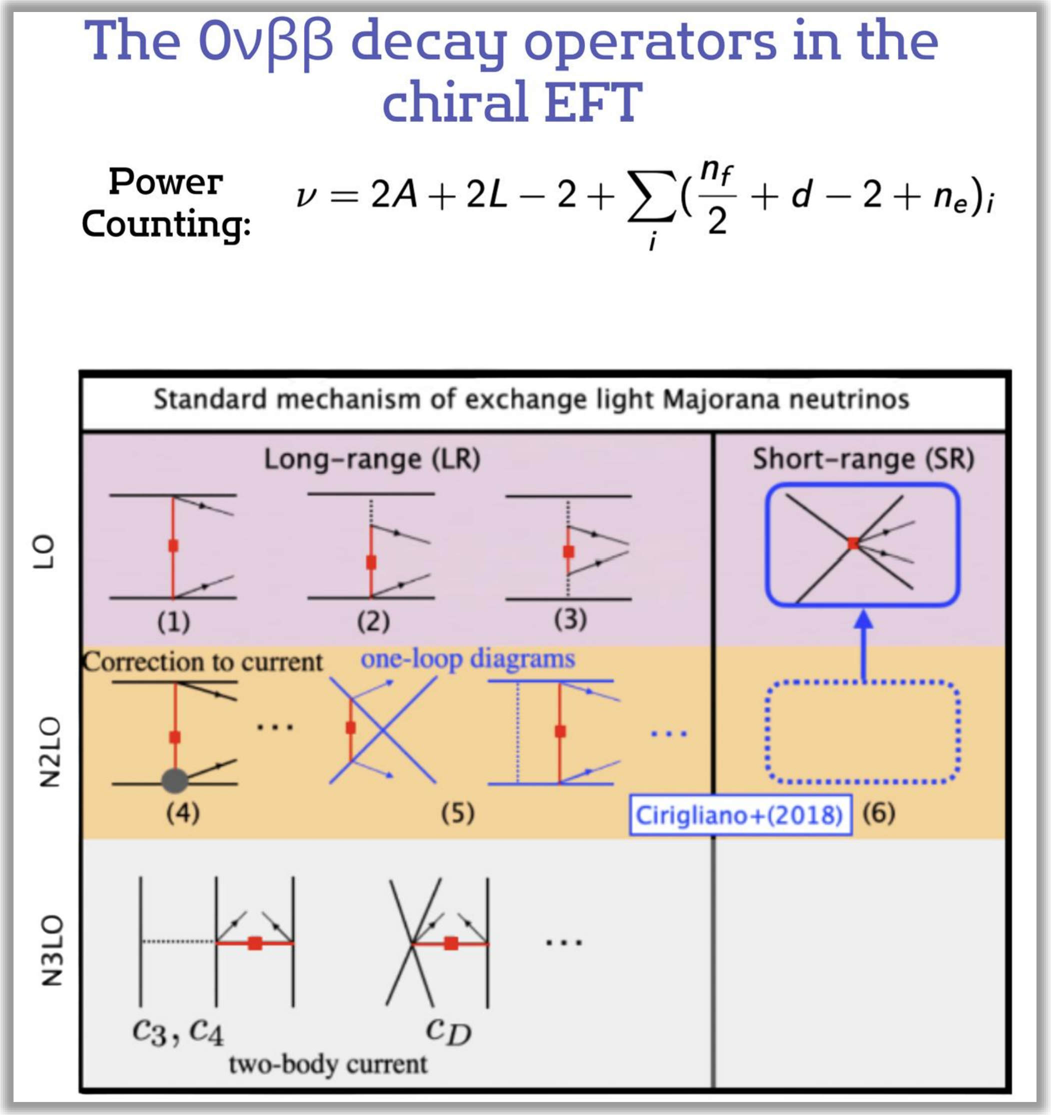
- **Effective neutrino mass:**

$$|\langle m_{\beta\beta}\rangle| = \left|\sum_{j=1}^{3} U_{ej}^2\, m_j\right| = \left[\frac{m_e^2}{g_A^4 G_{0\nu} T_{1/2}^{0\nu} |M^{0\nu}|^2}\right]^{1/2}$$

- **Nuclear matrix element(NME):**

$$M^{0\nu} = \langle \Psi_F \boxed{|\hat{O}^{0\nu}|} \Psi_I \rangle$$

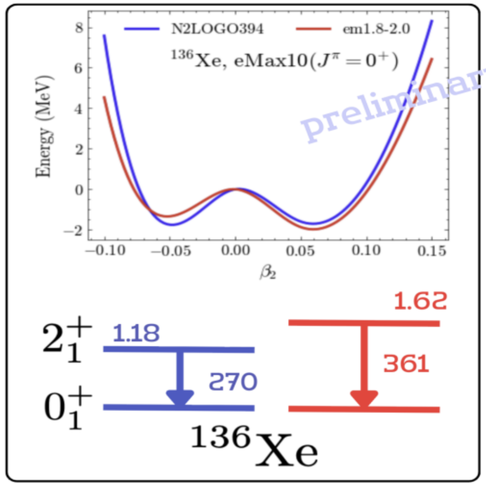
Decay operator

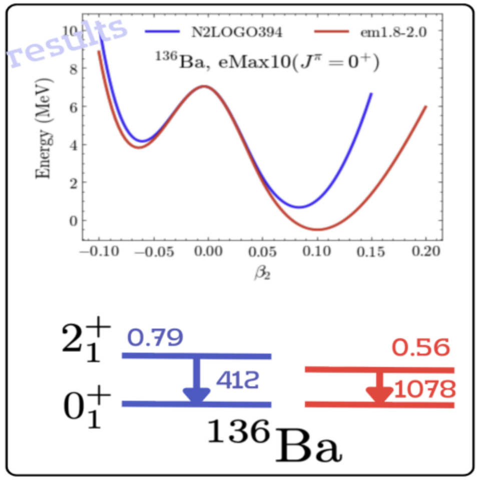


IM-GCM for the NME of 136Xe

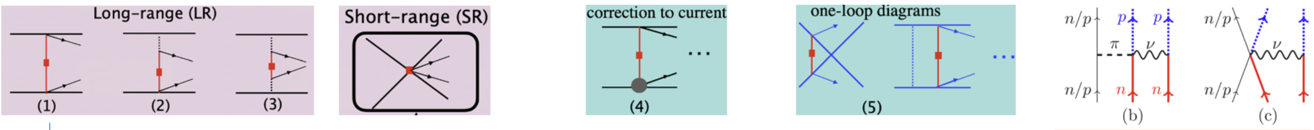
- The NME by the MR-IMSRG + PGCM

CRD et al., in progress





q^0/Λ^0 q^2/Λ^2 q^3/Λ^3

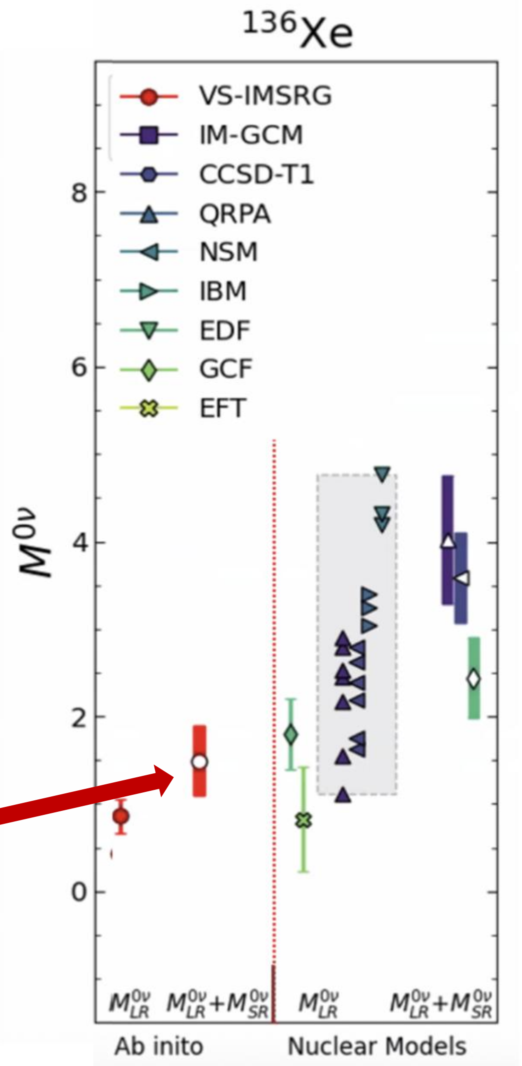


	LO long-range neutrino potential	LO contact term neutrino potential	Part of N²LO neutrino potential	Residual N²LO neutrino potential	NO2B N3LO neutrino potential	Total NME
EM1.8-2.0 eMax10 $q^{4/3}/\Lambda^{4/3}$	1.07	0.42(8)	-0.07	0.17	-0.32(3)	1.27(11)
N2LOGO394 eMax10 q^3/Λ^3	0.88	0.39 (7)	-0.07	0.17	-0.09(1)	1.28(8)

Hoferichter et al, PhysRevD.102.074018(2020)

$$g_A(q^2, 2b) = g_A(q^2) + \delta_a(q), \qquad g_P(q^2, 2b) = g_P(q^2) - \frac{2m_N}{q^2}\delta_a^P(q).$$

The NME by the VS-IMSRG: [1.08-1.90]
a factor of about two uncertainty



A. Belley et al., arXiv:2307.15156(2023)

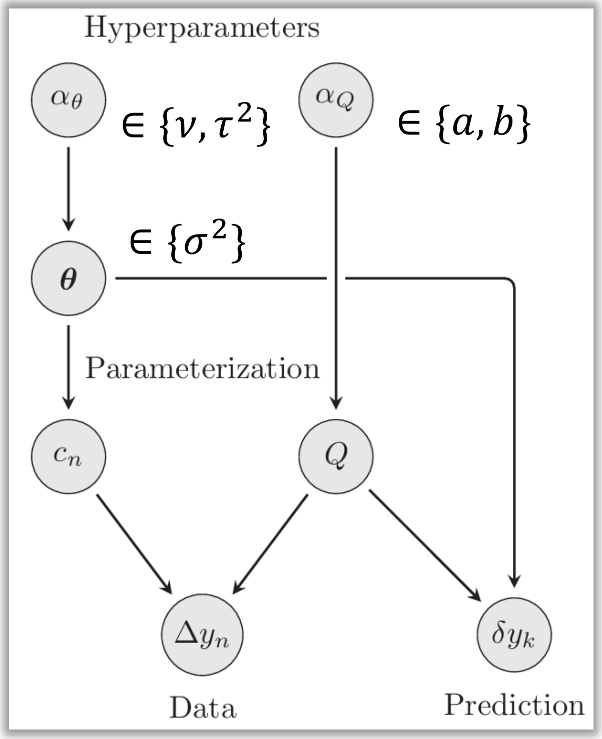
Uncertainty Quantification of EFT Truncation

Assume the NME could be described order by order by $y = y_{ref} \sum_{n=0}^{\infty} c_n Q^n$

- c_n **is the coefficient with natural size.**
- Q **is the expansion factor.**

If we can calculate NME up to k**-th order, how to predict the truncation error** $\Delta y_k = y_{ref} \sum_{n=k+1}^{\infty} c_n Q^n$

☐ Bayesian inference



$P(A|B) \propto P(B|A)P(A)$ **Prior**

Posterior **Likelihood**

$\theta|\alpha_\theta \sim \chi^{-2}(\nu_0, \tau_0^2)$

$c_\theta|\theta \sim N(0, \sigma^2)$

Conjugate Prior

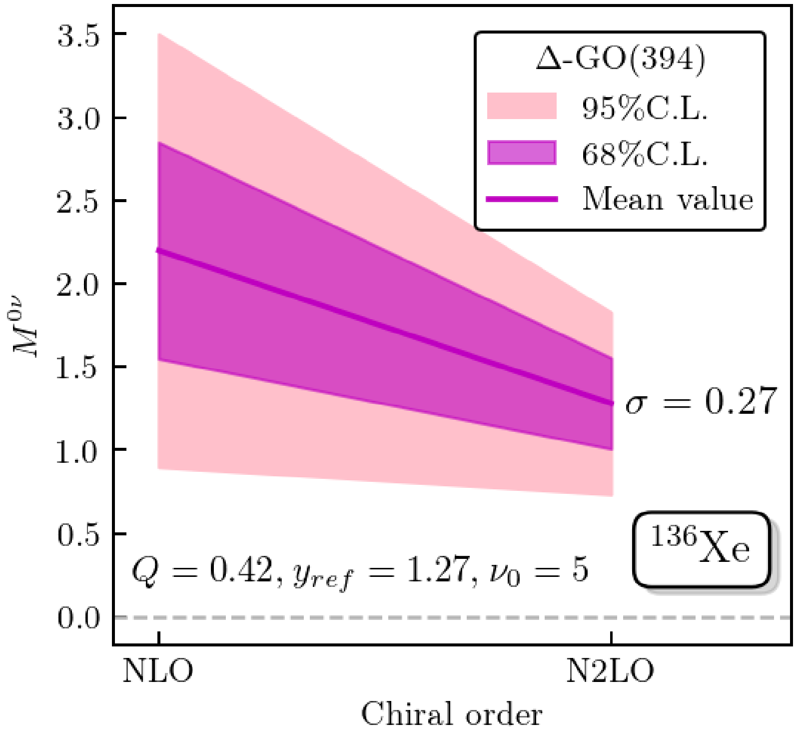
J. A. Melendez et al., PRC100(2019)044001

After the inference

$\theta|\vec{c}_k \sim \chi^{-2}(\nu, \tau^2)$,

$\nu = \nu_0 + n_k, \nu\tau^2 = \nu_0\tau_0^2 + \vec{c}_k^{\,2}$

CRD et al., in progress



*** Uncertainty quantification by the BUQEYE method.**

MR-IMSRG: Developments & Applications

Developments

MR-IMSRG(3)
Led by L. H. Chen

Quasi-particle excitations
Led by Y. Li

Emulator
Led by CRD

MR-IMSRG +GCM

Applications

Beta decay
Led by Y. X. Zhang

Schiff moment
Led by Q. Y. Luo

Neutrino scattering
Led by Q. L. Feng, B. L. Zhang

Collaborators: B. Bally, S. K. Bogner, H. Hergert, C. F. Jiao, H. Z. Liang, T. R. Rodríguez, C. C. Wang, J. M. Yao, E. F. Zhou.

Thank you for your attention!