Benchmarking the projected generator coordinate method for the single beta decay of odd-mass nuclei

Yanxin Zhang (张妍心)

School of Physics and Astronomy,
Sun Yat-sen University

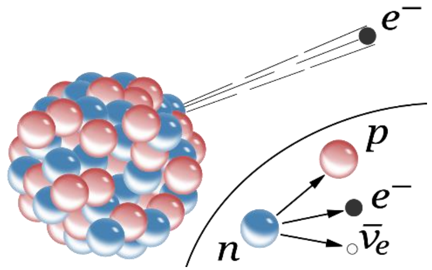
2026 年 1 月 10 日—1 月 12 日
中国·重庆

Outline

- Introduction
- Theoretical framework
- Result
- Summary

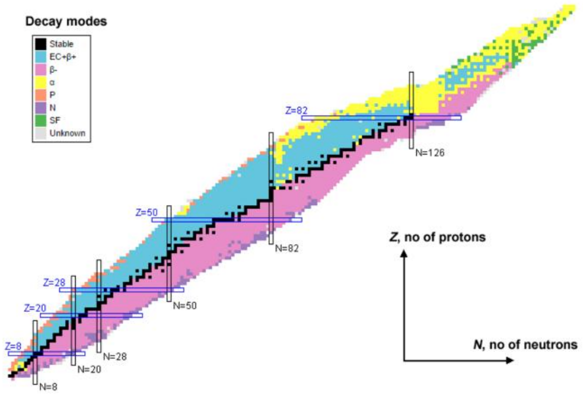
Introduction: Why β decay?

- Nuclear β decay (including beta+/- and EC) is one of the most important decay modes of atomic nuclei.
- Precise knowledge of nuclear β decay is essential for understanding the stability of atomic nuclei, the origin of elements heavier than iron, and the search for new physics beyond the Standard Model.

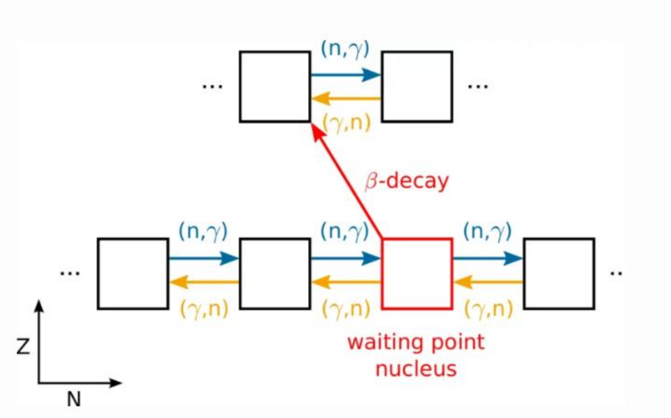


The CKM matrix: from flavor eigenstates to mass eigenstates :

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}.$$

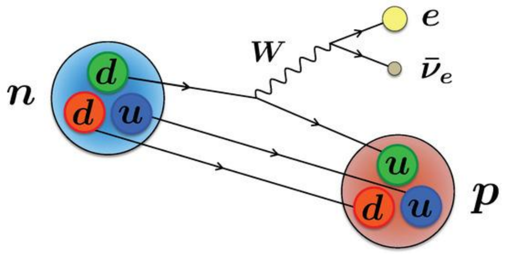


nuclear decay modes



rapid neutron capture process

Kratz et al., PPNP59,147(2007);
Cowan et al., Phys. Rep. 208 267(1991)

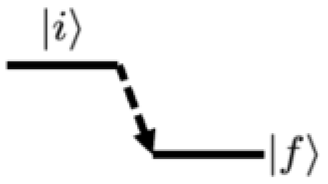


superallowed Fermi transition

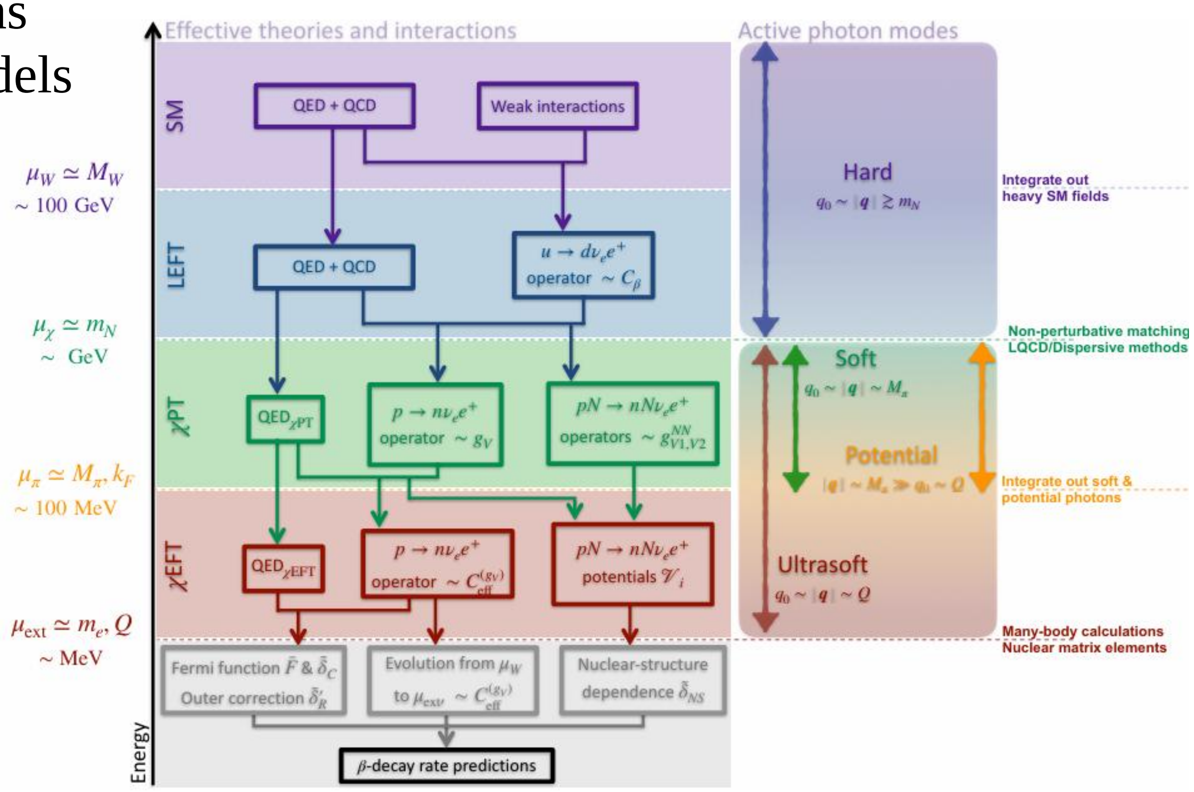
Severijns et al., RMP78, 991(2006)
J. C. Hardy et al., PRC91, 025501 (2015)

Introduction: Theoretical studies of nuclear weak processes

- Nuclear matrix elements are crucial for interpreting experimental signals.



- Towards an accurate calculation of NME
  - Transition operators: determined by weak interactions
  - Wave functions: obtained from nuclear structure models

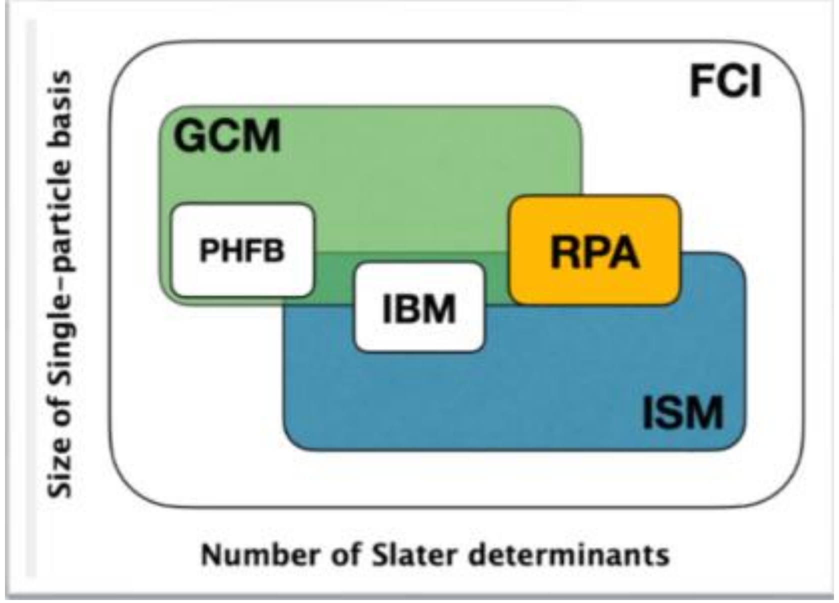


- Challenges in theoretical models:
  - Strong interaction: non-perturbative at **nuclear energy scale.**
  - Quantum many-body problem: **complex**

Introduction: Theoretical studies of nuclear weak processes

□Most widely used methods to compute beta decay properties:

- Interacting shell model (ISM)
- Proton-neutron (Q)RPA
- Projected shell model (PSM)
- Projected generator coordinate method (PGCM)

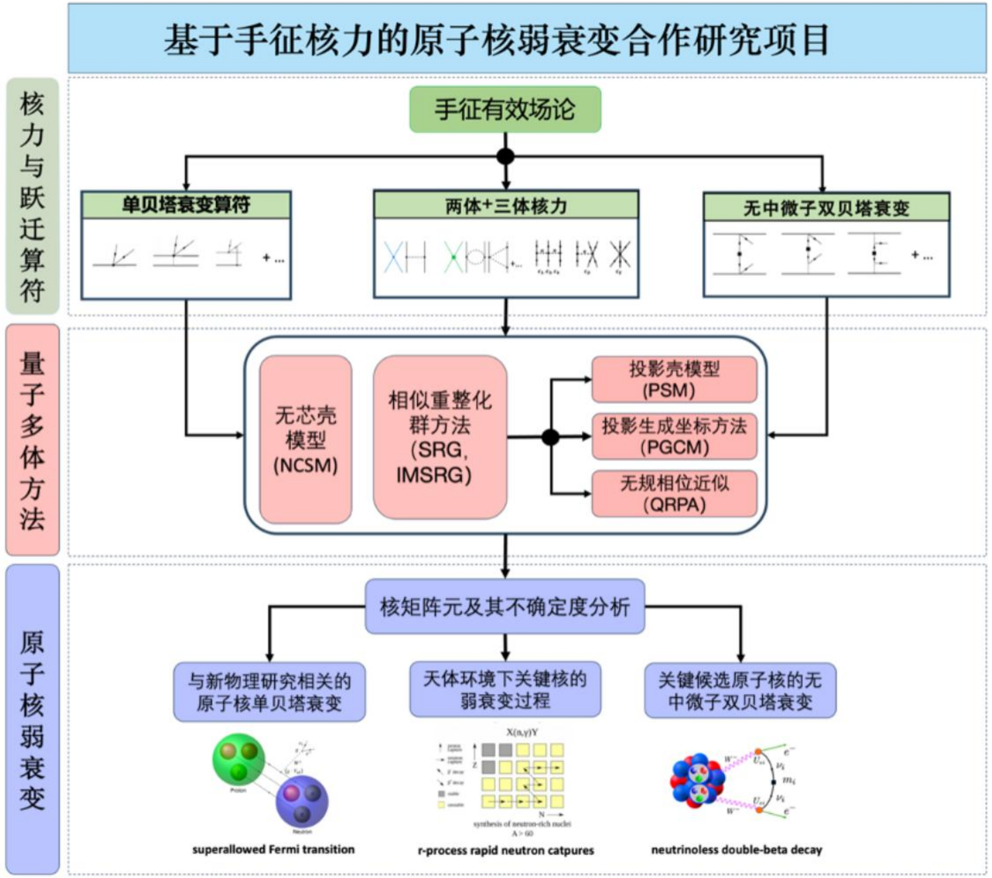


Its capability to microscopically describe collective behaviors and its adaptability to explore different degrees of freedom in nuclei.

Introduction: PGCM for nuclear weak processes

- The PGCM, combined with ab initio in-medium SRG, has been applied to compute the nuclear matrix elements of neutrinoless double-beta decay.

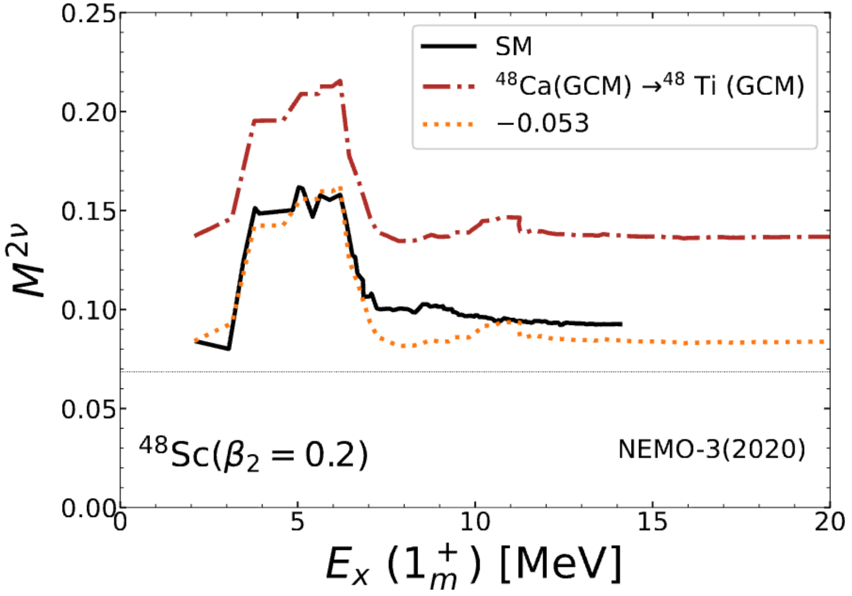
However, the extension of the PGCM for single-beta decay and two-neutrino double-beta decay is challenging, as many states of either odd-mass or odd-odd nuclei need to be determined precisely.

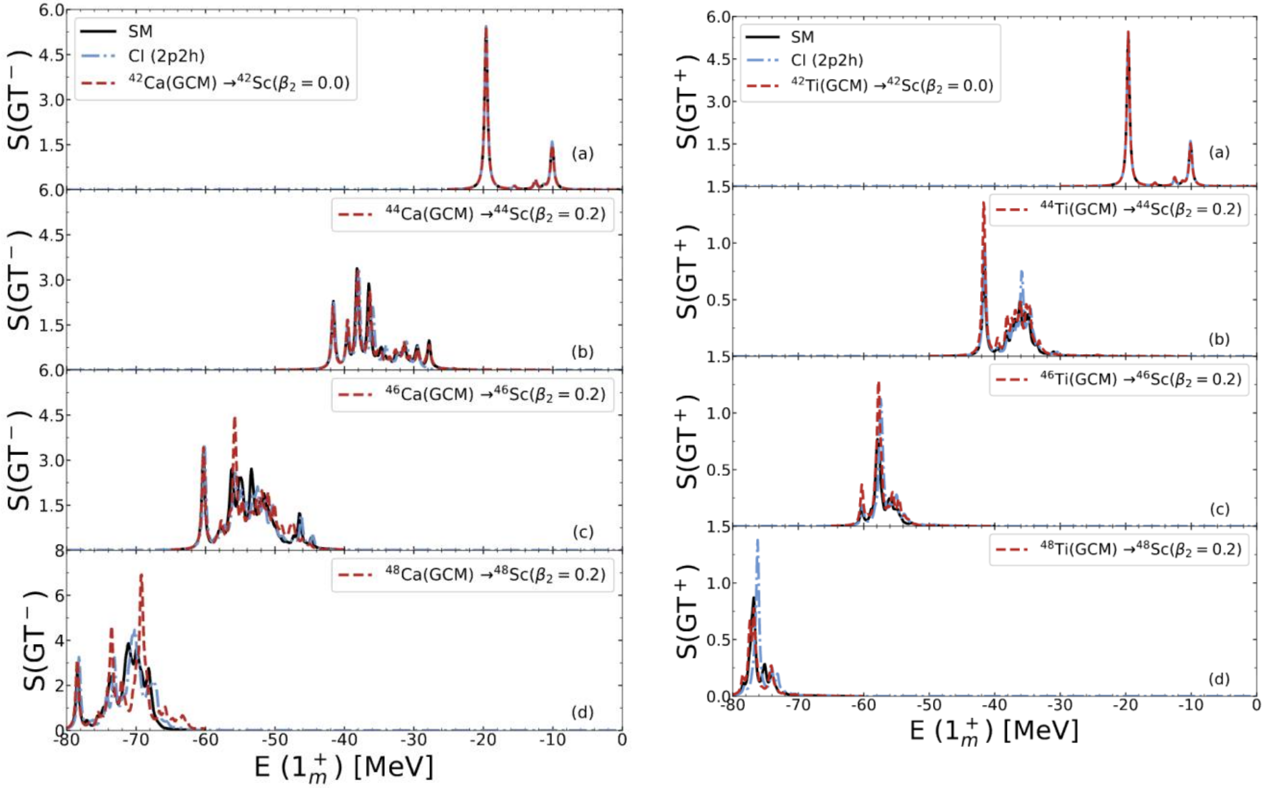


Vincenzo Cirigliano et al, Phys.Rev.C 110 (2024)

- Recently, the PGCM has been extended to describe Gamow-Teller (GT) transition strengths in even-even nuclei.

Provides a reliable description of GT transitions for nuclei not far from closed shells.





R. N. Chen, X. Lian, J. M. Yao, C. L. Bai, arXiv:2601.05058 [nucl-th] (2026)

Introduction: PGCM for nuclear weak processes

- This work: Extension of the PGCM for the single-beta decay of odd-mass nuclei.

- We benchmark the PGCM calculation using a shell-model Hamiltonian for which exact solutions are available.

Outline

- Introduction
- Theoretical framework
- Result
- Summary

The PGCM framework

- The wave functions of low-lying states :

$$|\Psi_{\alpha}^{NZJM}\rangle = \sum_{K,q} f_{K,q}^{NZJM\alpha} |NZJM, q\rangle$$

linear combination coefficients of the linear combination

- Symmetry-projected quasiparticle vacuum:

$$|NZJM, q\rangle = \hat{P}_{MK}^{J} \hat{P}_{K}^{N} \hat{P}^{Z} |\Phi(q)\rangle$$

- The mean-field configuration for an odd-mass nucleus:

$$\left|\Phi_{k}^{(\mathrm{OA})}(\mathbf{q})\right\rangle = \alpha_{k}^{\dagger}|\Phi_{(k)}(\mathbf{q})\rangle$$

- Exchanging the k-column of the U and V matrices in the HFB wave function:

$$(U_{pk}, V_{pk}) \longleftrightarrow (V_{pk}^{*}, U_{pk}^{*}).$$

The PGCM framework

PNVAP+HFB：

$$\delta \frac{\left\langle \Phi_k^{(\mathrm{OA})}(\mathbf{q}) \middle| \hat{H}\hat{P}^N\hat{P}^Z \middle| \Phi_k^{(\mathrm{OA})}(\mathbf{q}) \right\rangle}{\left\langle \Phi_k^{(\mathrm{OA})}(\mathbf{q}) \middle| \hat{P}^N\hat{P}^Z \middle| \Phi_k^{(\mathrm{OA})}(\mathbf{q}) \right\rangle}$$

Constraints q :quadrupole deformations β; octupole deformations; pairing content …

The weight functions are determined by varying the ground-state energy for HWG equation：

$$\sum_{K',q'} [\hat{\mathcal{H}}^{NZJ}_{KK'}(q,q') - E_\alpha \hat{\mathcal{N}}^{NZJ}_{KK'}(q,q')] f^{NZJM\alpha}_{K',q'} = 0,$$

$$\begin{aligned}
\mathcal{O}^{NZJ}_{KK'}(q,q') &= \langle NZJK, q \,|\, \hat{O} \,|\, NZJK', q' \rangle \\
&= \frac{2J+1}{8\pi^2} \int d\Omega\, D^{J*}_{KK'}(\Omega) \int_0^{2\pi} d\varphi_n \frac{e^{-iN\varphi_n}}{2\pi} \int_0^{2\pi} d\varphi_p \frac{e^{-iZ\varphi_p}}{2\pi} \\
&\quad \times \left\langle \Phi_k^{(OA)}(\mathbf{q}) \middle| \hat{R}(\Omega)\, e^{iZ\varphi_p} e^{iN\varphi_n} \middle| \Phi_{k'}^{(OA)}(\mathbf{q}') \right\rangle.
\end{aligned}$$

Hamiltonian and norm kernels

Nuclear beta decay

- The half-life of nuclear β decay

$$T_{1/2} = \frac{\ln 2}{\sum_F \lambda_F}$$

$$\lambda_F = \frac{\ln 2}{\kappa} \sum_n f(\Omega_n) B_n^F$$

phase-space factor (electron)

nuclear transition strength

- The transition operators

Fermi

Gamow-Teller

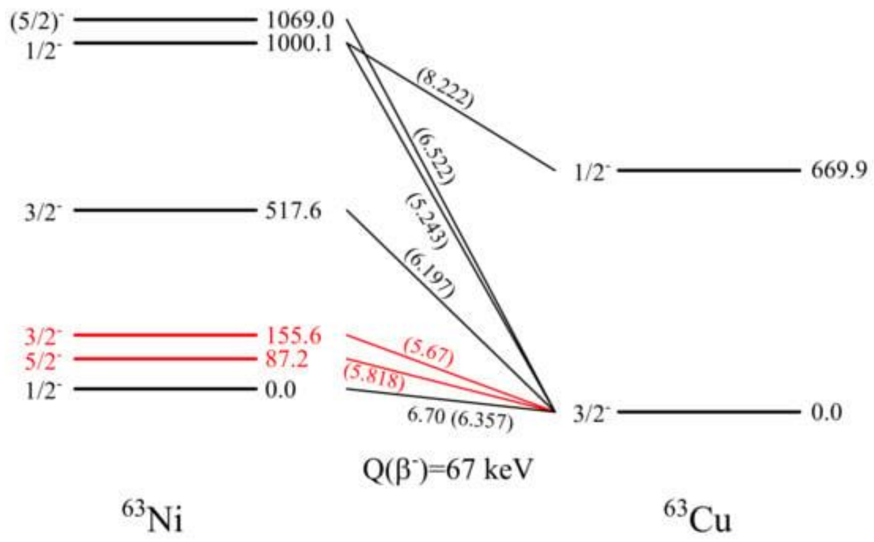
$$\hat{F}_{\text{Fermi}} = \sum_i g_V \tau_-(i), \quad \hat{F}_{\text{GT}} = \sum_i g_A \sigma(i) \tau_-(i),$$

- The transition strength and matrix element

$$B_n^F = |\langle \Psi_n[{}^A(Z+1)] | \hat{F} | \Psi_0[{}^A Z] \rangle|^2$$

wave functions of the initial and final states

log *f t*



Outline

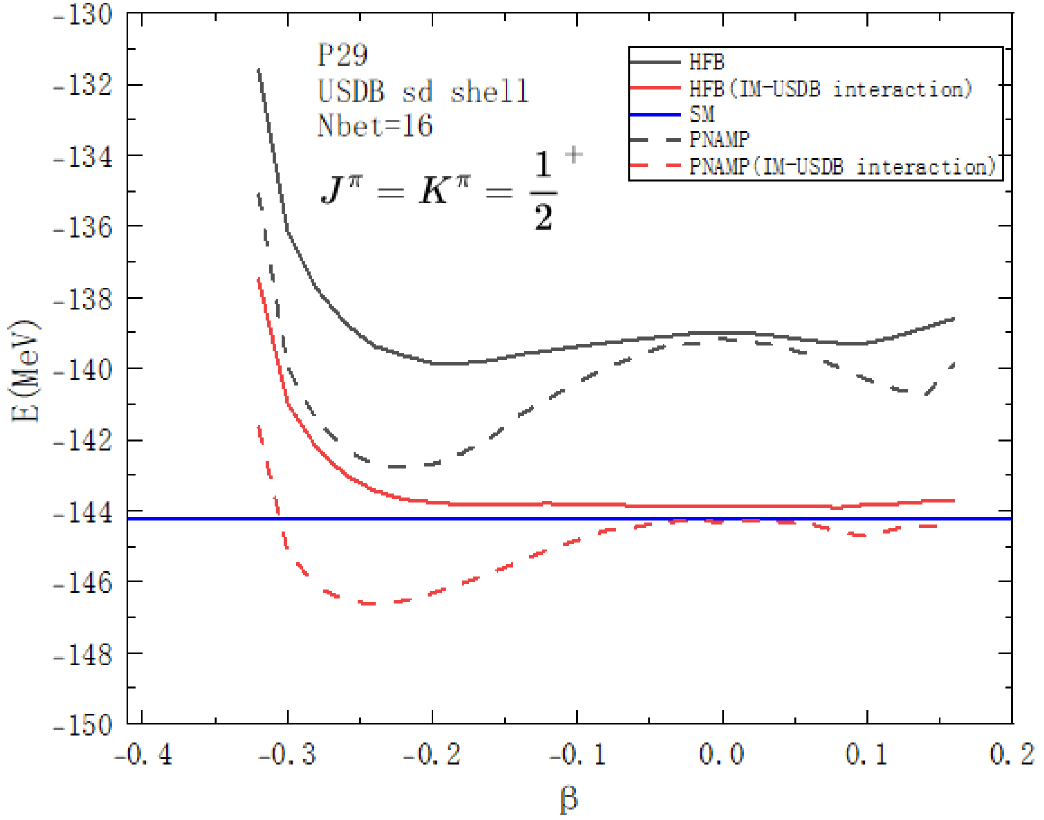
- Introduction
- Theoretical framework
- Results and discussion
- Summary

Numerical details

Comparison between Shell Model (SM) calculations and PGCM approach.

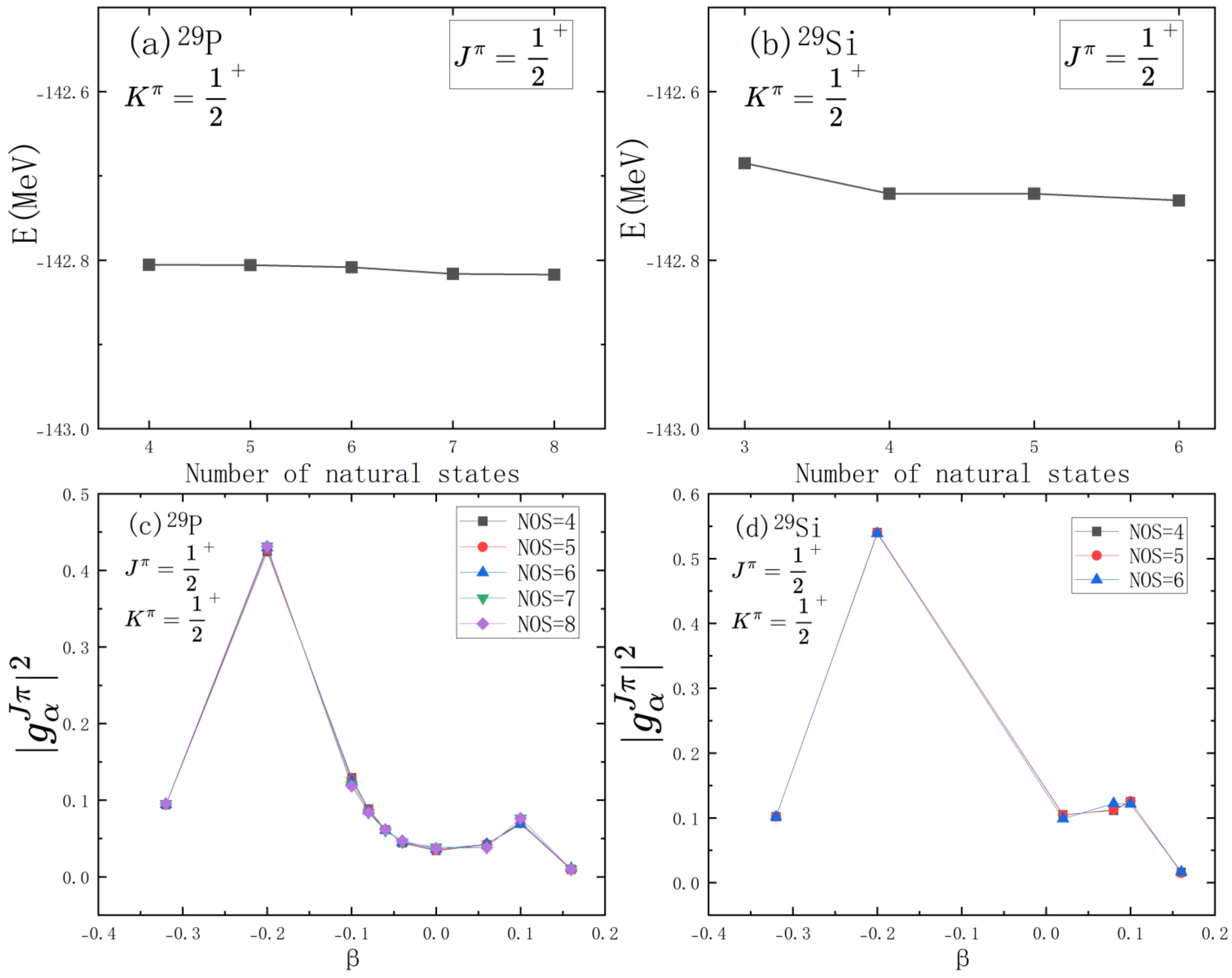
- Same effective Hamiltonian defined in the same valence space (USDB interaction in the sd-shell)
- Benchmark of the PGCM method against exact SM calculations results.
- Exact SM calculations performed with the *Bigstick* program.

Application to ^{29}P and ^{29}Si



- Energies of one-quasiparticle states with $K^\pi = 1/2^+$ from the HFB calculation and the energies of low-lying state with $J^\pi = K^\pi = 1/2^+$ from the PNAMP calculation, with projection onto correct particle numbers and different angular momentum $J\pi$, as a function of the quadrupole deformation parameter β2.

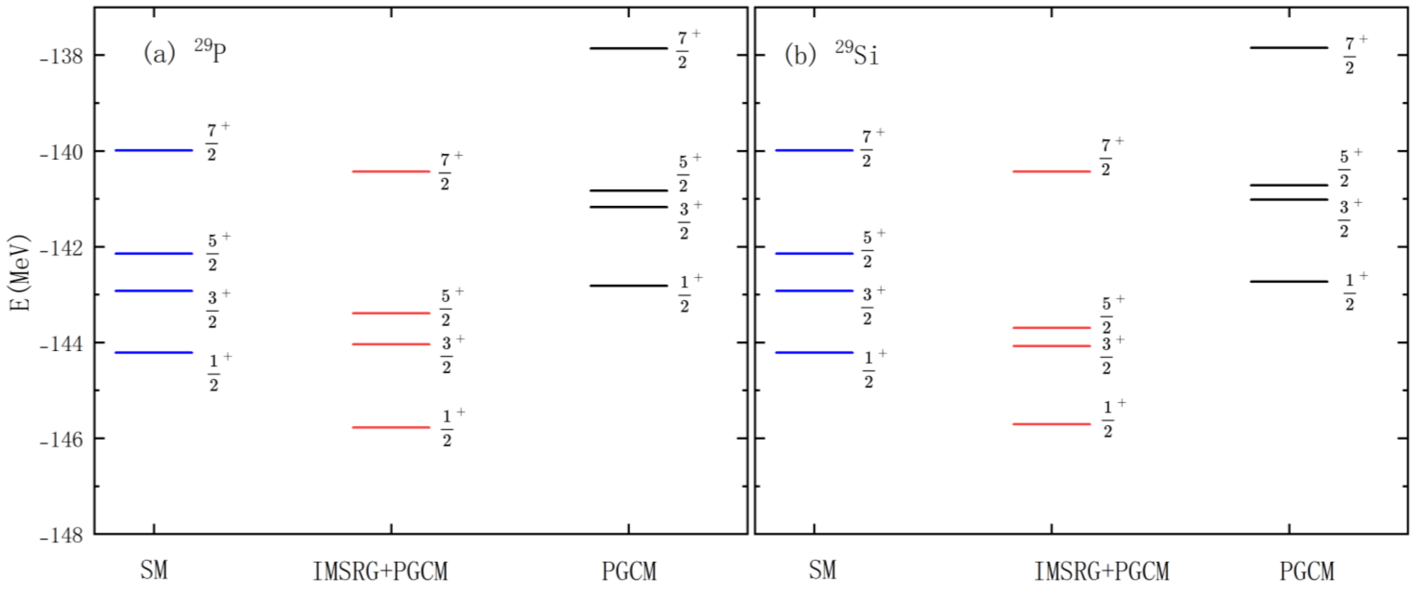
Numerical details



- Check the plateau condition for the ground-state (1/2+) energy in the GCM calculation.
- Different choices of the number of natural states in the GCM calculations have a negligible impact on the distribution of the collective wave functions.

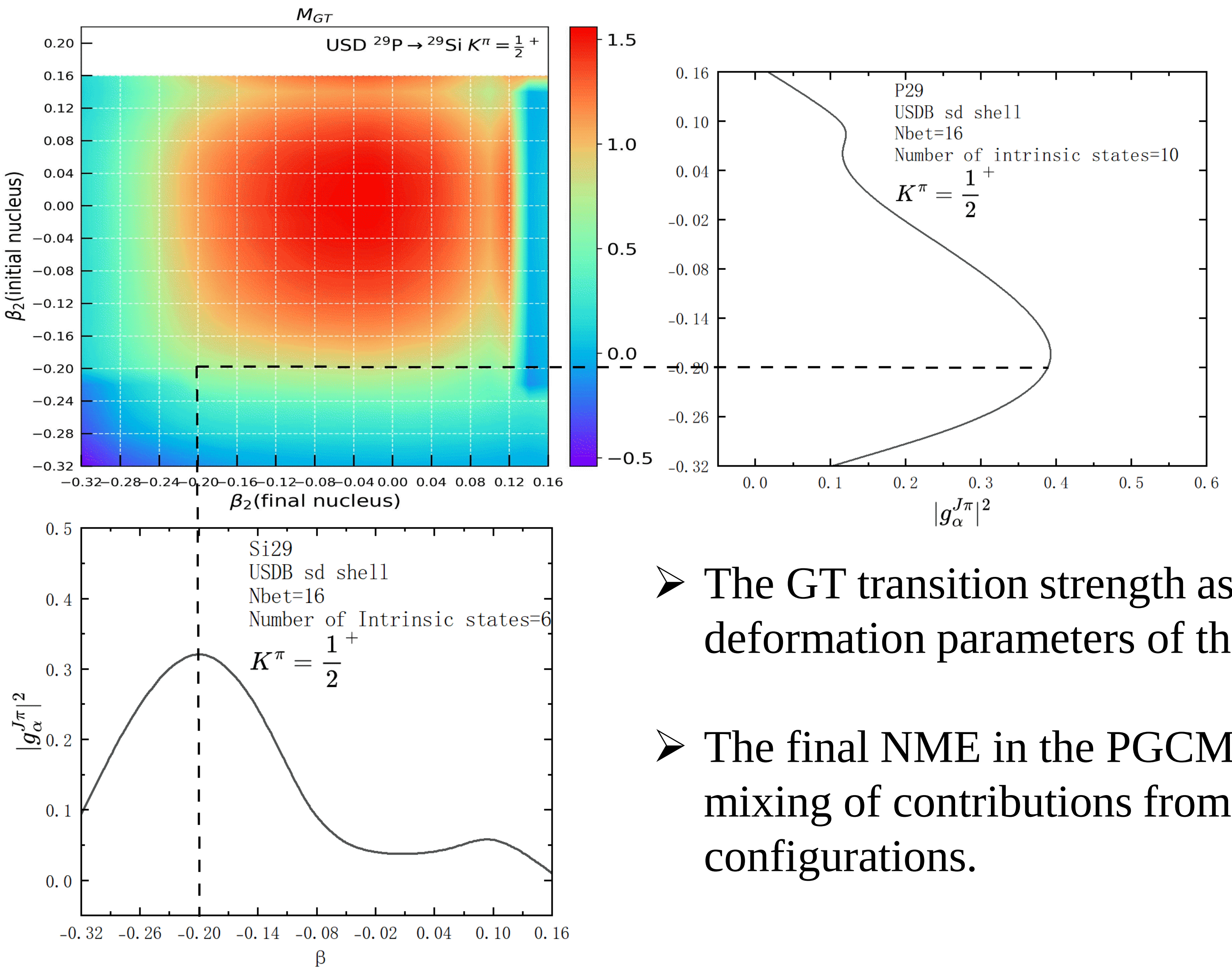
Application to the low-lying states of ^{29}P and ^{29}Si

	SM	IMSRG+GCM	GCM	HFB(Sph.)
^{29}P	−144.21	−145.71	−142.82	−139.00
^{29}Si	−144.21	−145.78	−142.73	−138.98



➢ The IMSRG evolution shifts down the entire low-lying states, even though too much, which is probably attributed to the NO2B approximation in the IMSRG.

Application to Gamow-Teller transition



➤ The GT transition strength as a function of the quadrupole deformation parameters of the initial and final nuclei.

➤ The final NME in the PGCM calculation is given by the mixing of contributions from different deformed configurations.

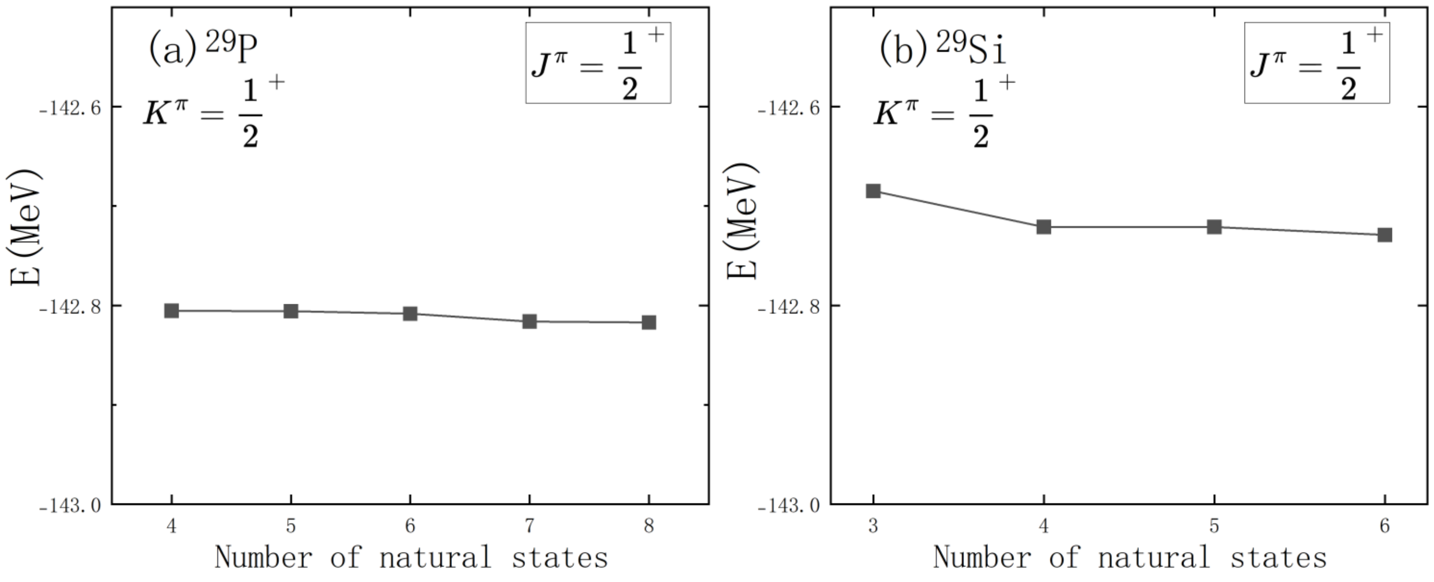
Application to Gamow-Teller transition

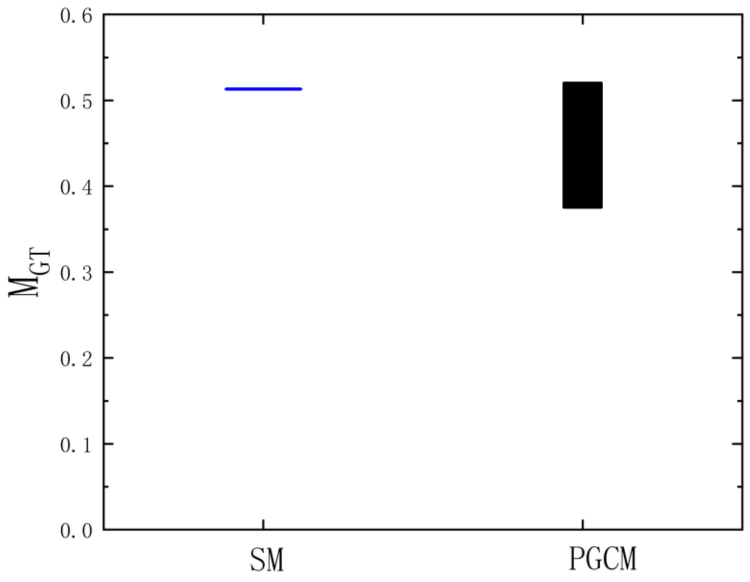
- The NMEs of Fermi and GT transitions by the GCM and IMSRG+GCM are as follows:

	Exp.	SM	IMSRG+GCM	GCM
M(F)	1	1	0.995	0.992
M(GT)	0.5380(21)	0.513	0.478	0.519

- The NMEs by GCM with and without the IMSRG are not much different from the results of SM, even though the IMSRG evolution worsens the agreement for the GT transition.

Uncertainty analysis





- The uncertainty in the NMEs from the GCM calculation is up to about 0.1, which is much larger than the effect of the IMSRG.

Outline

- Introduction
- Theoretical framework
- Result
- Summary and outlook

Summary and outlook

- We have extended the PGCM for the beta decay of odd-mass nuclei.

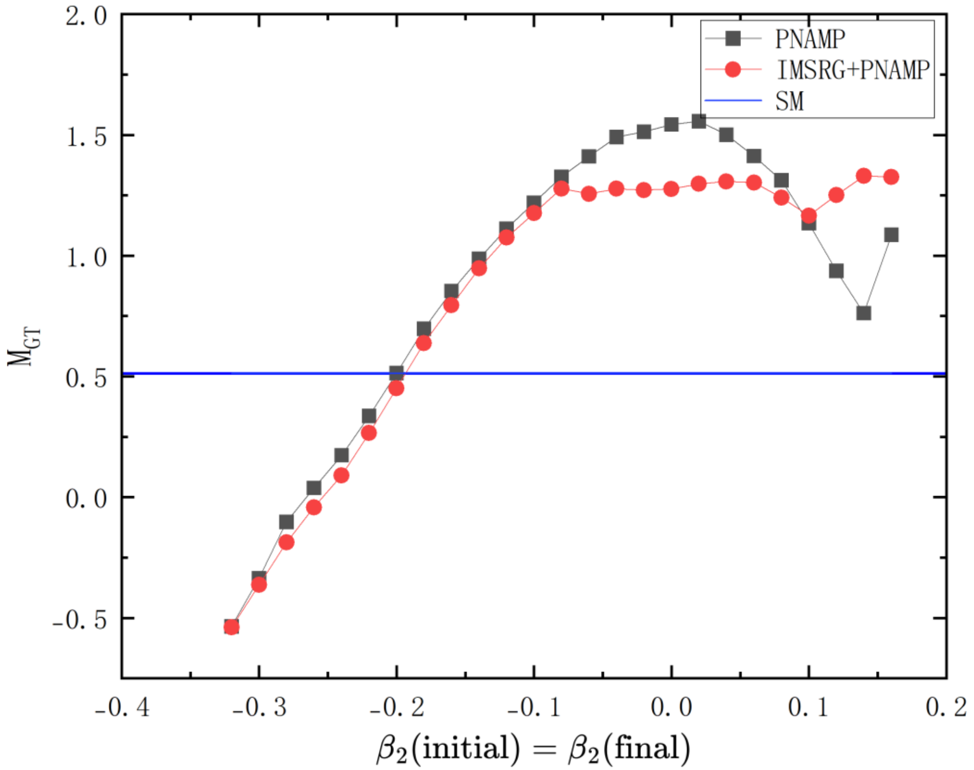
- We have benchmarked the GT transition of ^{29}P using the USDB shell Hamiltonian. We observed some discrepancies between the results of the PGCM and SM calculations. However, we also observed a large uncertainty in the NME originating from the PGCM calculation.

- Outlook:

➢ Reduce the uncertainty in the NME by the PGCM (Enlarge the model space).

➢ Implementation of operators from chiral effective field theory.

Thank you for your attention!


PNAMP
IMSRG+PNAMP
SM
M_GT
2.0
1.5
1.0
0.5
0.0
-0.5
-0.4
-0.3
-0.2
-0.1
0.0
0.1
0.2
β2(initial) = β2(final)